# Boston University
# Economics 702
# Fall 2008

## Introductory Study Problem #2:
## A simple model of labor supply

Suppose that an individual has a utility function of the form

$$u(c,l) = \log(c) - \frac{\chi}{1+\gamma}(1-l)^{1+\gamma}$$

where $c$ is consumption and $l$ is leisure. The utility function parameters $\chi$ and $\gamma$ are both positive. The household also faces the constraint that the amount of its market work

$$n = 1 - l$$

cannot exceed one (the endowment of time).

The individual's budget constraint is

$$c = wn + \pi$$

where $\pi$ is nonwage income.

(2-a) The Lagrangian suitable for an analysis of the optimal interior consumption ($c > 0$) and leisure ($0 < l < 1$) decisions is given by

$$L = u(c,l) + \lambda[w(1-l) + \pi - c]$$

Discuss how this is similar to the standard "two good" problem for a consumer with endowments. What is the numeraire? What are the endowments?

(2-b) Find the first order conditions to the problem in (2-a). Solve one for consumption given $\lambda$ and the other for work given $\lambda$ and $w$. Given these functions, how would $\lambda$ be determined? Would a higher value of $\lambda$ raise or lower consumption? work?

(2-c) Given $\lambda$, what is the elasticity of labor supply with respect to the wage rate?

(2-d) Suppose now that there is only labor income ($\pi = 0$). What is the elasticity of labor supply with respect to the wage rate if $\lambda$ is endogenously determined?

(2-e) If the individual in part (d) were given a small amount of nonwage income, would he work harder or less hard? Show your answer in a figure and using "comparative statics".