Student Name:
Student ID:

Economics 702
Macroeconomic Theory
Practice Examination #2
October 2008

Instructions: There are a total of 60 points on this examination and you will have 60 minutes to complete it. The first 10 minutes of the exam period will be allocated to students reading the exam for the purpose of asking questions about it, with no writing permitted, but students may use their study materials during that time. These 10 minutes do not count toward the total length of the exam.

Exams will be collected after exactly 60 minutes with no exceptions.

VERY IMPORTANT:

(1) Write your name on the **top of each page** of the exam before starting it: these exams will be copied and we do not want to lose any pages.

(2) Write only on the **front** of each page of the exam.

(3) Don't be afraid to write **short answers** to a question, describing the main idea of the answer. Then, if you have extra time of the end of the exam or want to perfect your answer, you can add further detail.

(4) Don't write everything that you know which is related to the topic of the answer: no credit will be given for correct material that is irrelevant to the question at hand and **some credit will be subtracted for incorrect material that is irrelevant**.

(5) If you need to add detail on any question beyond the allotted space, you may use the **one extra page** that appears at the end of this exam. Please be sure to indicate to which question or questions your additional work applies.

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1. Rational expectations models (15 points)

The following model describes how endogenous variables (k,i,p) respond to an exogenour variable x_t which varies stochastically.

$$\begin{aligned} k_{t+1} &= i_t + (1-\delta)k_t \\ k_{t+1} &= \beta E_t p_{t+1} - p_t \\ ak_t + x_t &= i_t - \gamma p_t \end{aligned}$$

In this model, all variables are interpreted as deviations from a stationary position.

(a) (5 points) Place this model in first-order form, $AEY_{t+1} = BY_t + Cx_t$.

(b) (5 points) Explain intuitively how many (finite) eigenvalues this model should have. Derive a polynomial that restricts these eigenvalues.

(c) (5 points) Assume that $0 < \beta < 1$ and that $a - \delta > 0$. Supposing that only k_t is predetermined, indicate the conditions for a unique, stable rational expectations solution to this model. Are there the required number of stable roots?

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2. Dynamic optimization (15 points)

Consider a family that maximizes a weighted sum of its member's utilities, attaches weight

$$\sum_{t=0}^{T} \beta^t \theta_{it} u(c_{it})$$

where θ_{it} is the weight attached to the momentary utility flow for its member i derives from his consumption c_{it}. In addition, assume that each household member has an momentary objective of the form

$$u(c) = \frac{1}{1-\sigma} c^{1-\sigma}$$

Suppose further that the household faces a budget constraint of the form

$$\sum_{t=0}^{T} \beta^t \sum_{i=1}^{I} c_{it} \leq \Omega$$

where Ω is the level of family wealth.

(a) (5 points) Find a general family efficiency condition for c_{it} that is directly obtained from a Lagrangian.

(b) (5 points) Explain intuitively why $\theta_{it} = \theta$ should lead to consumption that is constant across individuals and time. Show that the efficiency condition in (a) supports that finding.

(c) (5 points) Suppose that there are two individuals $i = 1, 2$ that have paths which satisfy

$$\theta_{1t} = \alpha\theta_{2t}$$

What can you say about the level of the consumption of individual 1 relative to that of individual 2 if $\alpha > 1$? What can you say about the relative growth rates of their consumptions?

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3. Dynamic programming (30 points).

Suppose that there is an individual who can hold either of two risky assets. The prices of these assets are governed by

$$p_j(\varsigma_t)$$

i.e., they depend on the state of the economic system ς_t. The dividend payouts on these securities d_{jt} and the individual's income also depend on this state.

The individual has an objective of the form

$$E_t\{\sum_{t=0}^{\infty}\beta^t u(c_t)\}$$

and a budget constraint of the form

$$\sum_{j=1}^{2}[p_j(\varsigma_t)+d_j(\varsigma_t)]z_{jt}+y(\varsigma_t)\geq c_t+\sum_{j=1}^{2}p_j(\varsigma_t)z_{j,t+1}$$

where z_{jt} is the amount of the asset j held from t-1 to t (and thus $z_{j,t+1}$ is the amount held from t to t+1).

(a) (8 points) Place this model in dynamic programming form by writing a Bellman equation for the individual's consumption, saving and portfolio choice problem. Please indicate the controlled and exogenous states of this problem.

(b) (8 points) Find the efficiency conditions for consumption (c_t) and portfolio holdings ($z_{j,t+1}$).

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(c) (8 points) What is the marginal value of having a little bit more of z_{1t} according to the envelope theorem? What is the underlying economic reasoning behind this result?

(d) (6 points) Suppose that the individual was constrained to hold positive quantities of both assets (no short-sales were allowed). How would that alter your answers to (a)?

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One extra page