Student Name:
Student ID:

Economics 702
Macroeconomic Theory
Practice Examination #1
October 2008

Instructions: There are a total of 60 points on this examination and you will have 60 minutes to complete it. The first 10 minutes of the exam period will be allocated to students reading the exam for the purpose of asking questions about it, with no writing permitted, but students may use their study materials during that time. These 10 minutes do not count toward the total length of the exam.

Exams will be collected after exactly 60 minutes with no exceptions.

VERY IMPORTANT:

(1) Write your name on the **top of each page** of the exam before starting it: these exams will be copied and we do not want to lose any pages.

(2) Write only on the **front** of each page of the exam.

(3) Don't be afraid to write **short answers** to a question, describing the main idea of the answer. Then, if you have extra time of the end of the exam or want to perfect your answer, you can add further detail.

(4) Don't write everything that you know which is related to the topic of the answer: no credit will be given for correct material that is irrelevant to the question at hand and **some credit will be subtracted for incorrect material that is irrelevant**.

(5) If you need to add detail on any question beyond the allotted space, you may use the **one extra page** that appears at the end of this exam. Please be sure to indicate to which question or questions your additional work applies.

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Part A: Shorter Questions

Each question below is worth 15 points. If there are components to a question, then these are weighted equally within each question..

1. *Stock prices, dividends and forecasting.* Suppose that a stock's price is related to its dividends by the present value formula

$$p_t = \frac{1}{R}[E_t p_{t+1} + E_t d_{t+1}] = \sum_{j=1}^{\infty} (\frac{1}{R})^j E_t d_{t+j}$$

with $R > 1$. Suppose further that dividends are generated by the stochastic difference equation

$$(d_t - d) = \rho(d_{t-1} - d) + e_t$$

(a) Use some method (undetermined coefficients, evaluation of discounted sums) to find a solution of the form

$$p_t = a + b(d_t - d)$$

(b) Consider the b and d coefficients in this solution. Show that $b < a$ if $0 < \rho < 1$. Discuss why $b = a$ if $\rho = 1$ relying on a graph of the impulse response of d_{t+k} to e_t

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2. *Dynamic optimization.* Suppose that there is a decisionmaker that has a dynamic decision problem of the form,

$$\max_{\{c_t\}} \sum_{t=0}^{T} \beta^t \pi(k_t, i_t)$$

where $\pi(k_t) = bk_t - i_t$. Suppose further that there is a state variable governed by

$$k_{t+1} = (\frac{i_t}{k_t} - \theta)^\nu k_t$$

with given initial condition k_0 and terminal condition k_{T+1}.

(a) Manipulate the state transition equation so that you can substitute it into the objective and derive the Euler equation. .

(b) Form a Lagrangian for this problem, attaching a multiplier $\beta^t \lambda_t$ to the date t state equation. Derive the first order conditions for this optimization problem.

(c) What is the form of the Bellman equation for this dynamic optimization problem?.(You may choose either to consider a finite horizon [__] or infinite horizon [__] problem, but you must indicate which one by placing an "x" inside the brackets above.).

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Part B: Longer Question

3. *A basic model of consumption growth.*

   This question is worth 30 points in total: there are five subcomponents that are weighted equally.

   Consider a one person economy which has a production function which takes the form

   $$y_t = ak_t$$

   where k_t is a stock of capital and y_t is the output produced by that stock of capital. The parameter a is positive. Capital accumulates according to

   $$k_{t+1} - k_t = i_t$$

   i.e., the change in the capital stock is equal to investment. Finally, consumption plus investment must be less than or equal to output,

   $$c_t + i_t \leq y_t$$

   (a) Combine the equations above to obtain a restriction that eliminates investment and takes the form

   $$g(k_t, c_t) - k_{t+1} \geq 0$$

   for each date $t = 0, 1, ...$ Determine the form of this equation.

   (b) Suppose that individuals value a marginal unit of consumption positively, so that the above equation holds with equality. Solve the resulting difference equation for k_{t+j} as a function of k_t and other variables.

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(c) Suppose that individuals have the utility function

$$\sum_{t=0}^{\infty} \beta^t u(c_t)$$

with $u(c) = \frac{1}{1-\sigma} c^{1-\sigma}$ with $0 < \sigma < 1$. Set up a dynamic optimization problem of the "Lagrangian" form.

(d) Find the first-order necessary conditions and determine the growth rate of consumption as a function of the model's parameters.

(e) This model of economic growth in an economy is identical to another model which we have studied. What is this other model? Contrast the state evolution equation in the two models. How are they similar? How are they different?

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One extra page