Lecture 6: Dynamic models of consumption and investment

Modern perspectives

- Consumption is forward-looking because people prefer smooth consumption profiles and can manage their saving to this end.

- Investment is forward-looking because firm’s demand for new capital goods depends on a present-value of profits from such investments.

Two models

- The modern variant of the “life-cycle/ permanent-income” model of consumption as developed by Hall

- The modern variant of investment with “capital adjustment costs” as developed by Lucas, Prescott and Hayashi.

General empirical problem

- Theory says that behavior depends on present-discounted value (pdv)
- Evaluating the theory requires modeling this pdv, which can be complicated.
- Two modern approaches circumvent this pdv modeling problem by clever, but different eliminations of it.

Approaches

- Consumption (Hall): work off efficiency condition (“euler equation”) and use general property of expectation forecasting errors, which is that errors should be unrelated to available information
- Investment (LHP): work off efficiency condition and relate unobserved pdv to observable variable, the ratio of firm value to replacement cost of capital (Tobin’s Q)

Earlier approaches (as applied to consumption)

- "Permanent income" model relates consumption to "annuity value of future income"

$$c_t = k * y_{pt}$$

$$[\sum_{j=0}^{\infty} b^j] y_{pt} = [\sum_{j=0}^{\infty} b^j E_t y_{t+j}]$$

Variants in prior work

- Friedman: treat permanent income as average of past income (only rational in some settings)

$$y_{pt} = \theta y_{p,t-1} + (1-\theta) y_t = (1-\theta)\sum_{j=0}^{\infty} \theta^j y_{t-j}$$

- Sargent: treat permanent income as outcome of forecasting with specific model

$$y_t = \pi s_t \qquad s_t = M s_{t-1} + G e_t$$

$$y_t = \sum_{j=0}^{\infty} b^j E_t y_{t+j} = \pi \sum_{j=0}^{\infty} b^j E_t s_{t+j} = \pi [I - bM]^{-1} s_t$$

A. Hall's work on consumption

- Consider dynamic model of optimal consumption over time

$$\max\ E_t[\sum_{j=0}^{\infty}\beta^j u(c_{t+j})]$$

$$s.t.\quad p(\varsigma_t)a_{t+1} = a_t + y(\varsigma_t) - c_t$$

$$where\ \ p_t = 1/R_t$$

- Note that income and interest rate depend on a set of state variables, but this dependence is not made explicit.

Euler equation

- Utility consequences of a little more wealth and consumption tomorrow, at expense of consumption today, must be zero at an optimum

$$-u_c(c_t)\,p(\varsigma_t) + \beta E_t[u_c(c_{t+1})] = 0$$

$$or$$

$$-u_c(c_t) + \beta R(\varsigma_t) E_t[u_c(c_{t+1})] = 0$$

Hall's observation

- Realized marginal utility is expected marginal utility plus an error term
- Under RE, this error term should be uncorrelated with available information (prior work on "efficient markets" in finance had exploited this observation as well)
- With utility that was quadratic (or approximately so) then marginal utility would be linear (or approximately so)
- With interest rate just offestting time preference, current and expected future consumption should then be equal under RE PIH model

Steps

$$u_c(c_t) = \beta R(\varsigma_t)E_t[u_c(c_{t+1})]$$

$$R\beta = 1 \Rightarrow \quad u_c(c_t) = E_t[u_c(c_{t+1})]$$

$$\Rightarrow \quad u_c(c_{t+1}) = u_c(c_t) + \xi_{t+1}$$

$$u_c = \phi - \gamma c_t \Rightarrow c_{t+1} = c_t - \frac{1}{\gamma}\xi_{t+1}$$

Test

- Linear regression, testing various x's
  - Additional lags of consumption
  - Additional variables
    - Past income
    - Past wealth

$$c_t = c_{t-1} + \theta x_{t-1} + e_t$$

$$PIH \Rightarrow \theta = 0$$

Initial findings

- Changes in consumption were suprisingly, largely unpredictable: Hall found consumption was “random walk”
- Sometimes written in log form: growth rate of consumption unpredictable
- Tests on micro data by Hall and Mishkin
  – Surprisingly hard to predict individual consumption changes also

Extensions

- Time varying interest rate (under loglinearity)

$$\log(c_{t+1}) - \log(c_t) = \frac{1}{\sigma}[\log(R_t / \beta)] + e_{t+1}$$

  - Result is small intertemporal substitution elasticity 1/σ is small.
  - Estimated using an instrumental variables approach (more details in semester 2)

- Nonlinear estimation (Hansen-Singleton) applied to Euler equations for multiple assets

Challenges

- Campbell-Mankiw studied effect of a measure of expected income growth

$$\log(c_{t+1}) - \log(c_t) = \frac{1}{\sigma}[\log(R_t / \beta)] + \kappa[E_t \log(y_{t+1} / y_t)] + e_{t+1}$$

- Estimate “big” (κ about .4) and “significant” coefficient. Much dispute about interpretation, but rejection of basic model

B. Investment

- Consider a firm seeking to maximize its present value, subject to an accumulation equation that penalizes large movements in capital (h is positive, increasing, and strictly concave).
- Firm faces time-varying productivity of capital and time-varying investment good price (p). Constant discount factor for simplicity only

$$\max \sum_{j=0}^{\infty} b^j E_t(a_t k_t - p_t i_t)$$

$$s.t.$$

$$k_{t+1} - k_t = h(\frac{i_t}{k_t})k_t$$

Key features

- Homogeneity of firm's problem: a kind of dynamic constant-returns to scale.
  - Motivated by applied work showing growth rates of firms do not depend importantly on level.
- Implications for value of firm: $v_t = w_t k_t$ with w_t not depending on capital stock k_t.

Dynamic program

$$v(k_t, \varsigma_t) = \max_{k_{t+1}, i_t} [a(\varsigma_t)k_t - p(\varsigma_t)i_t]$$
$$+ bE_t v(k_{t+1}, \varsigma_{t+1})$$

$$s.t. \quad k_{t+1} - k_t = h(\frac{i_t}{k_t})k_t$$

Lagrangian

$$L = [a(\varsigma_t)k_t - p(\varsigma_t)i_t] + bE_t v(k_{t+1}, \varsigma_{t+1})$$

$$+ \lambda_t [h(\frac{i_t}{k_t})k_t + k_t - k_{t+1}]$$

FOCs and ET

$$i_t : -p(\varsigma_t) + \lambda_t h_z(z_t) = 0 \qquad with\ z_t = \frac{i_t}{k_t}$$

$$k_{t+1} : -\lambda_t \ + bE_t v_k(k_{t+1}, \varsigma_{t+1})$$

$$ET:\ v_k(k_t, \varsigma_t) = a(\varsigma_t) + \lambda_t[1 + h(z_t) - z_t h_z(z_t)]$$

So z=(i/k)

- Can be “explained” entirely by p/λ
  - Multiplier is “marginal value” of another unity of capital tomorrow, p is current cost of terms of investment
- Depends on form of h(z), particularly its extent of decreasing returns to z.
- Problem: multiplier is unobservable

Market value

- Suppose firm pays out all profits as dividends. Then, its “ex dividend” market value is

$$
\begin{aligned}
v_t - [a_t k_t - p_t i_t] &= bE_t v_{t+1} && \text{Ex dividend value} \\
&= bk_{t+1} E_t v_{k,t+1} && \text{Homogeneity} \\
&= k_{t+1} \lambda_t && \text{Efficient Investment}
\end{aligned}
$$

Why was it ok to assume that value was proportional to k?
If future value is proptional to k’
then current value will be proportional to k.

$$\frac{v(k_t,\varsigma_t)}{k_t} = \max_{\frac{k_{t+1}}{k_t},\frac{i_t}{k_t}} [a(\varsigma_t) - p(\varsigma_t)\frac{i_t}{k_t}]$$

$$+ bE_t \frac{v(k_{t+1},\varsigma_{t+1})}{k_{t+1}} \frac{k_{t+1}}{k_t}$$

$$s.t. \quad \frac{k_{t+1}}{k_t} = [h(\frac{i_t}{k_t}) + 1]$$

Tobin's Q

- James Tobin hypothesized that investment would be related to

$$Q = \frac{\text{market value of firm}}{\text{replacement cost of capital}}$$

- This model (and others like it) delivers Tobin's view since
  –

$$Q = \frac{v_t - \pi_t}{p_t k_{t+1}} = \frac{\lambda_t k_{t+1}}{p_t k_{t+1}} = \frac{\lambda_t}{p_t}$$

Theory and tests

- Theory predicts i/k depends on Q and only Q (perfect fit)

- Tests find some association between i/k and Q, but far from perfect fit
  - Mismeasurement of capital stock?
  - Firm value depends on other factors (eg patents)
  - Model wrong on other dimensions (homogeneity, instant adjustment of i, …)

“Blackboard” work

- Adding a stochastic discount factor
- Implications for DP
- Implications for investment rule
- Links to Lucas-Prescott