BU Macroeconomics, Lecture 3: Equilibrium over time

Corrections on pages highlighted in red

(equation changes mostly,

but these are not in color)

Outline

1. An environment
2. Consumption smoothing
3. Recursive market equilibrium, as envisioned by Irving Fisher: a one period loan/bond market each period
4. Initial date markets equilibrium, as developed by John Hicks: outcomes are sequences of prices and quantities
5. A more modern description of recursive equilibrium – outcomes are functions of a state vector -- but not a fully recursive one

1. Environment

- The economy lasts for an even number of periods (t=0,1,…T)
- There are many identical agents, each with the utility function

$$General: \quad U(\{c_t\})$$

$$Specific: \quad \sum_{t=0}^{T} \beta^t \frac{1}{1-\sigma} c_t^{1-\sigma} \qquad \sigma > 0$$

- Increasing in c, strictly concave, and differentiable as needed

Environment (cont'd)

Words

- Each individual owns a firm which produces a time sequence of output (a "tree")
- The time series of output is periodic, but has a convenient present value of "y" if the market discount factor is β , as at right
- There is no storage or capital accumulation

Equations

$$y_t = h > y \quad \text{if t is even}$$

$$y_t = l < y \quad \text{if t is odd}$$

$$\sum_{t=0}^{T} \beta^t y_t = \sum_{t=0}^{T} \beta^t y = \frac{1-\beta^{T+1}}{1-\beta} \text{y}$$

$$\text{Implication} \quad (h-y)+\beta(l-y)=0$$

$$\text{or} \quad h+\beta l=(1+\beta)y$$

2. Consumption smoothing

- While our individual faces periodic income, suppose that he has access to borrowing and lending at an interest rate $(1+r)=1/\beta$ in every period.
- Then, our individual with the specific utility function will choose full consumption smoothing with $c_t=y$ every period.
- The individual will save in even periods (when income is at $h>y$) and dissave in odd periods, when his income is low.
- Mathematically, $a_{t+1}=h-y$ in even periods and $a_{t+1}=0$ in odd periods .
- Cannot be an equilibrium because all individuals are the same (no interpersonal borrowing and lending) and there is no storage or capital accumulation

Optimal saving

3. Fisherian equilibrium

- Fisher reasoned that at each date, individuals would equate the intertemporal marginal rate of subsitution to the interest rate as a condition of private efficiency
- He also reasoned that with a given endowment path that the interest rate had to adjust to consumption, rather than the other way around (as in the individual decision problem from lecture 1)

Fisher's equilibrium in a graph

Fisher's equilibrium in an equation

General: $$\frac{\partial U(\{c_j\}_{j=0}^{T})/\partial c_{t+1}}{\partial U(\{c_j\}_{j=0}^{T})/\partial c_t} = \frac{1}{1+r_t}$$

Specific: $$\frac{\beta^{t+1} c_{t+1}^{-\sigma}}{\beta^{t} c_t^{-\sigma}} = \frac{1}{1+r_t}$$

Either: evaluated at $c_j = y_j$ all j=0,1,...T

Fisher's equilibrium (cont'd)

Specific: $\dfrac{\beta^{t+1} y_{t+1}^{-\sigma}}{\beta^{t} y_{t}^{-\sigma}} = \dfrac{1}{1+r_{t\ t}}$

Even periods: $\dfrac{1}{1+r_t} = \dfrac{\beta l^{-\sigma}}{h^{-\sigma}} = \beta(\dfrac{l}{h})^{-\sigma}$

Odd periods: $\dfrac{1}{1+r_t} = \dfrac{\beta h^{-\sigma}}{l^{-\sigma}} = \beta(\dfrac{h}{l})^{-\sigma}$

Approximate for both: $\mathrm{r}_t = \upsilon + \sigma[\log(y_{t+1}) - \log(y_t)]$

with $\beta(1+\upsilon) = 1$

Fisher's equilibrium

- In even periods, individuals have high income relative to average and they try to lend (save), pushing down the interest rate ($1+r<\beta$)
- In odd periods, individuals have low income relative to average and they try to borrow (dissave), pushing up the interest rate ($1+r>\beta$)
- All borrowing or lending attempts must be frustrated in equilibrium
- Strength of interest rate response depends on how much individuals are willing to substitute across time (high σ is low substitution: it leads to more "volatile" interest rate)

4. Hick’s construction

- Fisher worked to use standard consumer theory, appropriately structured, to consider equilibrium over time in a sequential setting
- Hicks produced a tighter link with standard static consumer and general equilibrium in his classic treatise, *Value and Capital*.
- He envisioned a set of date 0 markets, in which the good at t would be traded at a price P_t.
- Then, the intertemporal problem was just like general equilibrium in a static endowment economy with many goods

Household's problem

$$\max_{\{c_t\}_{t=0}^{T}} U(\{c_t\}_{t=0}^{T})$$

$$s.t. \qquad \sum_{t=0}^{T} P_t c_t \leq \sum_{t=0}^{T} P_t y_t$$

Household's Lagrangian

$$\min_{\Lambda} \max_{\{c_t\}_{t=0}^{T}} U(\{c_t\}_{t=0}^{T}) + \Lambda[\sum_{t=0}^{T} P_t y_t - \sum_{t=0}^{T} P_t c_t]$$

FOC for an interior optimum: $\dfrac{\partial U(\{c_j\}_{j=0}^{T})}{\partial c_t} - \Lambda P_t = 0$

also BC

Hicksian initial equilibrium prices

- Prices lead individuals to consume endowment at each date
- At such prices, the budget constraint is satisfied with equality
- Standard indeterminacy of the units of prices: only relative prices matter

$$P_t = \frac{1}{\Lambda} \frac{\partial U(\{c_j\}_{j=0}^T)}{\partial c_t} \Big|_{\{c_j\}_{j=0}^T = \{y_j\}_{j=0}^T} \qquad P_t = \frac{1}{\Lambda} y_t^{-\sigma}$$

5. Towards a recursive equilibrium

- Asset accumulation is naturally recursive.
- Endowments as a recursion: note that $y_t = y_{t-2}$

$$\varsigma_t = \begin{bmatrix} y_t \\ y_{t-1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ y_{t-2} \end{bmatrix} = M \varsigma_{t-1} \qquad \varsigma_0 = \begin{bmatrix} h \\ l \end{bmatrix}$$

$$y_t = \pi s_t = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} y_t \\ y_{t-1} \end{bmatrix}$$

- Questions to think about: Does this structure really produce the series described above? What are eigenvalues?

Recursive equilibrium components

- Wealth accumulation, given the state of the economy

$$a_{t+1} = (1 + r(\varsigma_t))[a_t + y(\varsigma_t) - c_t]$$

$$\varsigma_{t+1} = M * \varsigma_t$$

- We previously learned what Fisher thought r(ς) was given our y(ς) specification

$$1 + r(\varsigma_t) = \frac{1}{\beta}(\frac{y_{t+1}}{y_t})^{\sigma} = \frac{1}{\beta}(\frac{\pi\varsigma_{t+1}}{\pi\varsigma_t})^{\sigma} = \frac{1}{\beta}(\frac{\pi M \varsigma_t}{\pi\varsigma_t})^{\sigma}$$

Recursive equilibrium components

- Consumers choose a policy c(ς,a) to maximize their welfare
- Equilibrium r(s) requires that c(ς ,a)=y(ς) and a=0
- Next time: dynamic programming