BU Macro 2008: Lecture 1

Introduction to dynamic macroeconomics: Analysis of saving and investment

Background

- Modern macroeconomics is concerned with understanding the factors that influence development ("economic growth") and fluctuations ("business cycles")
- Since each of these topics involves <u>time</u> in an essential way, we use economic theories and empirical strategies that highlight this dimension.

Background (cont'd)

- Macroeconomic modeling sometimes abstracts from heterogeneity across households, firms, countries – but incorporating such heterogeneity in a tractable manner is the subject of much work on the the frontiers of macroeconomics
- The most basic dynamic question that has concerned (macro) economists is the evolution of saving and investment, so that we start with this topic. Elements of it will be present in many of the next few lectures and throughout EC702-4

Outline of lecture

- A. Basic facts about consumption and investment
- B. Core concepts in intertemporal models and some basic approaches to about intertemporal consumption choice

A. Basic Facts

- Consumption, investment and output all trend up together (log scale in figure below is normalized so all have same mean, emphasizing trend and cycle variation)
- Consumption is about 2/3 of output and investment is about 1/6 of output
- Investment is proportionately more volatile than output, which is in turn more volatile than consumption

Trends and cycles



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Ratios



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Implications for modeling

- Evidence of common trends (rough stability of ratios) has motivated development of models with such a property (steady state growth, balanced growth)
- First modeling attacks on capturing these saving and investment outcomes also employs idea that proportionate fluctuations are not related to level of development (this requires restrictions on preferences and possibilities)

B. Core ideas

- Economists have long been interested in dynamic economic problems
- Analysis of consumption and investment is hard because:
 - It involves the study of choices over time and under uncertainty
 - It involves general equilibrium (at a point in time and over time)
- Start by abstracting from GE and look just at an individual's consumption choice, given income and interest rates

B1. Irving Fisher on consumption and interest

- Interested in real and nominal interactions
 - The "Fisher equation" linking nominal rate (R), real rate (r) and expected inflation $E\pi$: R=r+E π
 - Monetary effects on economic activity: "I discovered the Phillips curve"
 - Invented "distributed lags" for this purpose
 - Index number theorist
 - Investment: practice as well as theory
- However, our focus is on his real theory, as described in *Theory of Interest*.

Fisher's Theory of Interest

- Consumption over time: used utility analysis (then applied to static demand), building in
 - Preference for current over future consumption
 - Desire for consumption smoothing
 - Willingness to substitute across time
- Equilibrium (which we will discuss later): theory of interest rate determination in sequential markets
 - First approximation: endowment economy (much progress)
 - Second approximation: production economy (little progress)

Two period model

- Preferences over time
 - specialize to $U(c_0,c_1)=u(c_0)+\beta u(c_1)$ with β reflecting preference for earlier consumption
 - [slope of indifference curve at $c_0=c_1$) is $1/\beta$]
 - "momentary" utility u is increasing and strictly concave so as to produce desire to smooth consumption and willingness to substitute across time
- Saving (z) at interest rate "r"
 - links current and future consumption: $c_0+z_0=y_0$ and $c_1=(1+r)z_0+y_1$ combine to yield $c_0+(1+r)c_1=y_1+(1+r)y_0$
 - The relative price of future consumption in terms of present consumption is (1+r)

Preference specification: time preference and slope of indifference curve

Theory of consumption over time:

(1) consumption level adjusts to lifetime resources;

(2) consumption differs across time based on interest rate

(3) saving is residual (z=y-c)

The present value budget constraint in a sequence of one-period bond markets

$$a_{t+1} = (1+r_t)(a_t + z_t) \Leftrightarrow \frac{1}{(1+r_t)}a_{t+1} = (a_t + z_t)$$

$$a_{T+1} \ge 0 \Leftrightarrow a_0 + \sum_{t=0}^T P_t z_t \ge 0$$

with $P_t = [(1 + r_0)(1 + r_1)...(1 + r_{t-1})]^{-1}$ an implicit present value price

Note:
$$\frac{1}{(1+r_t)}$$
 is a one period discount bond price

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Multiperiod Fisherian preferences

$$\sum_{t=0}^{T} \beta^{t} u(c_{t}) \qquad \beta = \frac{1}{1+\nu} < 1; \quad \nu > 0 \text{ is rate of pure time preference}$$

$$u(c) = \begin{cases} \frac{1}{1-\sigma} (c^{1-\sigma} - 1) \text{ with } \sigma > 0 \text{ and } \sigma \neq 1 \\ \log(c) \end{cases}$$

 σ controls elasticity of intertemporal substitution

Modern version of Fisher's dynamic optimization (via a Lagrangian, with integrated budget constraint)

$$L = \sum_{t=0}^{T} \beta^{t} u(c_{t}) + \Lambda[a_{0} + \sum_{t=0}^{T} P_{t}(y_{t} - c_{t})]$$

FOC:
$$\beta^t u_c(c_t) - \Lambda P_t = 0$$

Ratio (mrs):
$$\frac{\beta^{t+1}u_c(c_{t+1})}{\beta^t u_c(c_t)} = \frac{P_{t+1}}{P_t}$$
$$\Rightarrow \beta [\frac{c_{t+1}}{c_t}]^{-\sigma} = \frac{1}{1+r_t}$$

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Fisher's rule

 Can be used for theory of consumption growth and for theory of interest: just switch the dependent variable.

$$\log(c_{t+1}/c_t) = \frac{1}{\sigma}\log(\frac{1+r_t}{1+\upsilon}) \cong \frac{1}{\sigma}[r_t-\upsilon]$$

$$[r_t - \upsilon] \cong \log(\frac{1 + r_t}{1 + \upsilon}) = \sigma \log(y_{t+1} / y_t)$$

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B.2 Fisher's Heirs: Friedman and Modigliani

- Consumption smoothing implications for individuals and aggregates
 - Permanent income theory: differential response to sustained and transitory income shocks
 - Life cycle model: allocation of consumption over time given life cycle changes

Friedman's permanent income model

• Typically focused on special case in which interest rate equal time preference

$$c_t = r[a_t + \sum_{t=0}^{\infty} (\frac{1}{1+r})^j y_{t+j}] = y_t^p$$

modern description:

consumption is the annuity value of wealth (the constant level with same present value)

Making the permanent income model operational

• Friedman assumed that permanent income could be proxied by

$$y_t^p - y_{t-1}^p = \theta[y_t - y_{t-1}^p] \text{ with } 0 \le \theta \le 1$$

$$y_{t}^{p} = \theta y_{t} + (1 - \theta) y_{t-1}^{p}$$

= $\theta \sum_{j=0}^{J-1} (1 - \theta)^{j} y_{t-j} + (1 - \theta)^{j} y_{t-j}^{p}$

• That is: permanent income relates to actual income via a linear difference equation

Friedman's Theory of the consumption function

- Worked out against the backdrop of Keynesian c = a + b y and its relatives, with "b" being marginal propensity to consume.
- MPC smaller out of transitory income than out of permanent income
 - Occupations (farmers, dentists)
 - Trend versus cycle in macroeconomic data (note in figures above that c/y roughly invariant to trend, c deviations from trend less volatile than y deviations)

Modigliani's life cycle model

- Aimed at explaining micro data (crosssection, panel) of consumption and then aggregating up
- Fact 1: life cycle variations in income
- Fact 2: life cycle variations in household demographics and activities

Life cycle income profile

Modigliani's Lifecycle model of consumption

Note: sequence of constraints

max $U = \sum_{t=0}^{T} \beta^{t} u(c_{t}, x_{t})]$ with $u(c_{t}, x_{t}) = [x_{t}^{\sigma} \frac{1}{1 - \sigma} (c_{t}^{1 - \sigma} - 1)]$ subject to $a_{t} + y_{t} - c_{t} = \frac{1}{1 + r_{t}} a_{t+1} \text{ for } t = 0, 1, ...T$ and $a_{T+1} \ge 0 \text{ (enforced by credit market)}$

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Lagrangian for problem with sequence of constraints (multipliers scaled by β^t for convenient FOCs)

$$\begin{split} L &= \sum_{t=0}^{T} \beta^{t} u(c_{t}, x_{t})] \\ &+ \sum_{t=0}^{T} (\beta^{t} \lambda_{t}) [a_{t} + y_{t} - c_{t} - \frac{1}{1 + r_{t}} a_{t+1}] \\ &+ \Theta_{T+1} a_{T+1} \end{split}$$



Terminal wealth: the transversality condition

$$\frac{\partial L}{\partial \Theta_{T+1}} = a_{T+1} \ge 0$$
$$\Theta_{T+1} \frac{\partial L}{\partial \Theta_{T+1}} = \Theta_{T+1} a_{T+1} = 0$$

Formally: Kuhn-Tucker condition Economics: valued wealth should not be left at the end Either: $\Theta_{T+1} = 0$ or $a_{T+1} = 0$ or both Equivalently: $\beta^T \lambda_T a_{T+1} = 0$

Solving for optimal outcomes: Step 1 is conditionally optimal consumption

$$c_t : 0 = \beta^t [u_c(c_t, x_t) - \lambda_t] \implies c_t = c(\lambda_t, x_t)$$
$$c_t : 0 = \beta^t [x_t^{\sigma} c_t^{-\sigma} - \lambda_t] \implies c_t = x_t (\lambda_t)^{(-1/\sigma)}$$

Solving for optimal outcomes: Step 2 is optimal shadow price dynamics

$$a_{t+1}: 0 = \beta^t \left[-\lambda_t \frac{1}{1+r_t} + \beta \lambda_{t+1}\right] \implies \lambda_{t+1} = \left(\frac{1}{\beta(1+r_t)}\right) \lambda_t = \gamma_{\lambda t} \lambda_t$$

Words: shadow price falls at rate depending on gap between interest rate and time preference rate $[\log(\lambda_{t+1}/\lambda_t) \simeq -(r_t - \upsilon)]$

 $\lambda_t = [\gamma_{\lambda,t-1}\gamma_{\lambda,t-2}...\gamma_{\lambda,0}]\lambda_0$

Words: shadow price is fully determined by λ_0

Solving for optimal outcomes: Step 3 is optimal asset stock dynamics

#2: Once we know λ_0 , we know $\{\lambda_t\}$ path

#1: Once we know $\{\lambda_t\}$ and $\{x_t\}$ path, we know $\{c_t\}$ path

#3: Once we know { c_t } path, we determine asset path via $a_{t+1} = (1 + r_t)[a_t + y_t - c_t]$

from given a_0 for all dates.

However, an arbitrary λ_0 does not make $a_{T+1}=0$: only one does

Example

- Suppose interest rate (r) is constant over time and suppose β(1+r)=1: then λ is constant over time as well.
- Suppose x is constant over time.
- Higher λ implies lower consumption.
- There is just one level of c (of λ) such that assets are zero at T+1.

Example (cont'd in figure)

Example (cont'd in figure)

Summary of key points

- 1. General facts about consumption and investment
- 2. Consumption smoothing
- 3. Saving as a residual: a basic theory of investment
- 4. PDV budget constraint in sequential markets
- 5. Intertemporal substitution and the interest rate
- 6. The LC-PIH model of consumption
- 7. The transversality condition for a household

What's next?

- The analysis so far suggest why consumption might be smoother than income (output)
- But it determines either saving (investment) or the interest rate (with saving zero) but <u>not</u> both at the same time
- Our next step is to analyze investment in more detail
- Then we will turn to the general equilibrium determination of consumption, investment, output and the interest rate.