

Name:

Economics 702
Sample Exam #1
October 2003

There are 75 points on this examination, which is one point per minute. Please budget your time carefully and write your answers legibly in the space provided.

Part A. Short answer questions

These questions are each worth 15 points (if there are subcomponents within a question then these are weighted equally). They can be answered with a few sentences and an occasional formula. Do not write outside the space provided.

1. *Forecasting.* Suppose that the short-term interest rate, R_t , is governed by

$$\begin{aligned} R_t &= \pi s_t \\ s_t &= M s_{t-1} + G e_t \end{aligned}$$

where e_t is serially uncorrelated.

- (a) Show how to forecast R_{t+k} given this model.

- (b) Suppose also that the yield on an n period bond is given by $R_{nt} = \frac{1}{n}[R_t + E_t R_{t+1} + \dots E_t R_{t+n-1}]$. Work out the rational expectations solution that links R_{nt} to s_t .

2. *Money, Interest and the Price Level.* A basic model of price level determination is comprised of the following pair of equations, interpretable as a monetary equilibrium condition – with exogenous money M_t – and as a Fisher equation linking the nominal interest rate R_t to the expected inflation rate.

$$\begin{aligned} M_t - P_t &= -\alpha R_t \\ R_t &= E_t P_{t+1} - P_t \end{aligned}$$

- (a) Combine the equations to yield an expectational difference equation in the price level. Assuming that the price level P_t is not predetermined, show that there is a unique nonexplosive solution so

long as $\alpha > 0$.

(b) Write the model as a linear difference equation system of the form $AE_t Y_{t+1} = BY_t + CX_t$.

(c) What are the roots of $|Az-B|$? What are the roots of $|Bz-A|$? What is the significance of the values of these roots?

(d) If $\alpha > 0$ is this model uniquely solvable?

3. *Business Cycles and Statistical Models.*

Suppose that output takes the form

$$y_t = \rho y_{t-1} + e_{yt}$$

and that employment takes the form

$$n_t = \lambda y_{t-1} + e_{nt}$$

with $0 < \rho < 1$ and $\lambda > 0$. Suppose further that the e_t are serially and mutually uncorrelated

(a) In what sense can the time series on output look like a business cycle, to a Burns-Mitchell analyst?

(b) What does this model predict about the relative variances of output and employment?

(c) What does this model predict about the correlation of output and employment?

B. Longer Question

4. *Choice over time* (30 points, 6 points per part): Consider a household that is maximizing its lifetime utility,

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, l_t^*, l_t)$$

where c_t is consumption, l_t is leisure and

$$l_{t+1}^* = \rho l_t^* + \theta l_t$$

depends on current and past leisure ($l_t = 1 - n_t$, where n_t is market work). Suppose further that the household is optimizing within a regime of sequential markets, with its wealth evolving according to

$$a_{t+1} = (1 + r_{t,t+1})[a_t + w_t n_t - c_t + \pi_t]$$

- (a) What economic considerations might lead l_t^* to enter in the utility function?
- (b) Which variables are controlled state variables from the point of household? What choices affect the motion of these controlled states?
- (c) Supposing that $1 + r_{t,t+1}$, w_t , and π_t are all functions of exogenous state variables δ_t , write a Bellman equation suitable for this problem. Append each of the dynamic equations to form a Lagrangian, with a different multiplier attached to each state evolution equation.

(d) Find the first-order conditions that describe the efficient choices.

(e) Use the envelope theorem to determine how the first derivatives of the value function depend on economic variables, including the multipliers from part (c).