PEER-TO-PEER MARKETS WITH BILATERAL RATINGS*

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ABSTRACT

Peer-to-peer (P2P) markets have become a critical aspect of the modern economy. We consider a P2P market where a time-sensitive service is provided through a platform that matches providers of varying qualities to customers of varying costs. The P2P platform features bilateral ratings, which distinguishes itself from a traditional market: ratings of a provider reveal his service quality and ratings of a customer reveal her service cost. The existence of a cost measure in the P2P market leads to novel pricing considerations: a provider can attract low-cost customers by charging a low price, leading to an "endogenous composition effect". As a result, equilibrium prices may *decrease* as customers become more costly to serve or as the platform's commission rate gets higher. Under certain conditions, high-quality providers may even charge a *lower* equilibrium price than low-quality providers in order to cherrypick low-cost customers. Exploratory analysis reveals that compared with unilateral ratings, bilateral ratings may soften provider competition and raise equilibrium prices as the providers target customers in different cost segments.

Keywords: *user-generated content, customer reviews, decentralized matching, peer-to-peer market, platform design*

1 Introduction

The past two decades have witnessed an explosion of information technology that has given birth to larger, faster, and more geographically diverse marketplaces. Peer-to-peer (P2P) platforms, in particular, can provide consumers with access to either material goods such as vehicles (e.g., Toru, RVShare), or services such as accommodation (e.g., Airbnb), care-giving (e.g., Rover, Care.com), home improvement (e.g., Thumbtack, TaskRabbit), freelance work (e.g., Upwork, Fiverr) and personal loans (e.g., Prosper, LendingClub) . Due to the broad adoption of P2P platforms, they have become an important part of a modern consumer's life and a critical aspect of the sharing economy.

P2P markets have two distinctive features when compared to traditional markets. First, the service provided in a P2P market is often time-sensitive, which creates a capacity constraint for the service and may lead to significant coordination frictions: some customers may fail to match with any provider and some providers may end up with no customer to serve. Recent empirical studies (e.g., Horton 2014; Fradkin 2015) have, for example, systematically documented that popular providers on oDesk and Airbnb get contacted by multiple buyers and often reject certain guests due to capacity constraints.[1] When a failure of match occurs, the customer would often have to seek service from other channels (e.g., family and friends, traditional businesses), do it themselves (e.g., home repair, data entry), or simply abort the need. Acknowledging such coordination frictions enables us to understand why customers and providers in P2P markets have a strong incentive to maintain a good reputation so that they are more likely to be matched.

A bilateral reputation system is the second distinctive feature of P2P markets. In such a system, both customers and providers are rated by the other side of the market, whereas in a typical traditional market only the providers are rated. The service orientation of P2P markets and the capacity constraint on such time-sensitivity service make it important for providers to learn the cost to serve a particular customer. When aggregated, ratings can significantly resolve uncertainties around the quality of a provider's service and the cost to serve a customer.[2] While many potential factors may hinder truthful reporting, the flourishing of peer-to-peer markets is in itself evidence that bilateral reputation systems work well to screen out dysfunctional actors and deter fraudulent behavior (Resnick et al. 2002; Dellarocas 2003; Cabral and Hortacsu 2010; Ke and Zhu 2021).

We build a decentralized matching model to explore the effects of these two distinctive features of P2P markets. Our one-shot model captures the time-sensitivity of P2P services and features large numbers of customers and providers. We find two types of equilibria. In the first type, low-cost customers apply only to high-quality providers while high-cost customers apply to all providers. Intuitively, low-cost customers are more attractive than high-cost cus-

[1] Similarly, Liu et al. (2020) empirically document that providers tend to refuse to serve customers with a weak reputation. Brown (2023) finds that discrimination against black taxi riders results in far higher rates of trip cancellation and longer wait times compared with white riders. By contrast, ride-hailing dramatically reduces differences between riders. Scott et al. (2016) find that in the context of service refusals in supply chains, spot prices for rejected truckload services average about 62% higher than contract rates.

[2] In addition to organic reviews on peer-to-peer platforms, certain service providers can also use third-party review sites such as ratemycustomers.com, badbuyerlist.org, and customers2avoid.com to avoid costly customers.

tomers and more likely to be accepted by high-quality providers, who have more attractive applicants, longer queues, and lower acceptance rates than low-quality providers. A high-cost customer may, interestingly, get *over-penalized* for her reputation with such a low acceptance rate that at the application stage, she would find it worthwhile to reimburse the provider of her entire service cost if she could then be treated as a low-cost customer. In the second type of equilibria, all customers apply to all types of providers but low-cost customers still have higher acceptance rates than high-cost ones.

We highlight in our analysis a unique *endogenous composition effect*: the expected service cost faced by a provider is not an exogenous market feature but rather depends on the composition of customers that apply to him, which in turn depends on the price set by the provider. More specifically, as a provider lowers his price, he attracts a pool of applicants with a lower average service cost. In a traditional market, a lower price has two effects, a decrease in the profit margin accompanied by an increase the demand, whereas in our model, it continues to have these two effects but also has a third strategic effect of lowering the cost of service which can help reduce or even reverse the loss in profit margin.

These novel pricing considerations lead to interesting comparative statics. To begin, equilibrium prices may decrease with customers' service costs and the platform's commission rate. These two patterns are counter-intuitive: one would expect that higher costs and fees would drive up prices in order for the providers to cover their expenses. In the presence of the endogenous composition effect, however, providers may lower prices instead if the strategic benefit of attracting low-cost customers outweighs the immediate loss in their profit margin.

Allowing for incomplete market coverage, we also find that high-quality providers may charge a price that is *lower* than that of low-quality providers. Given the prevalence of high-cost customers, the high-quality providers in this case prioritize the cultivation of a low-cost applicant pool through charging a low price, while low-quality providers anticipate only applications from more costly customers and are forced to charge a high price in order to make up for the high service costs of his applicants.

Finally, we characterize the market equilibrium under unilateral ratings of providers, and find through numerical comparisons that bilateral ratings tend to raise market prices as a result of cost-based customer segmentation and the subsequent reduction in provider competition. The platform may hence benefit from bilateral ratings as it takes a fraction of the increased provider revenue, but the implications of bilateral ratings on consumer welfare remain unclear: matches indeed become more informed but prices are increased as well.

1.1 Literature Review

Our research contributes to the growing literature of pricing, matching and P2P markets. In the literature of behavior-based price discrimination (BBP), firms typically discriminate based on customers' willingness-to-pay that they learn from the customers' past purchase behavior. For example, a firm may discriminate between its own and the competitors' customers (e.g., Fudenberg and Tirole 2000; Pazgal and Soberman 2008; Shin and Sudhir 2010; Zhang 2011), and a firm can charge returning and new customers different prices (e.g., Hart and Tirole 1988; Schmidt 1993; Villas-Boas 2004). In our model, providers cannot charge different prices based on customers' willingness to pay or service cost, but they can decide whether to serve a specific customer based on her service cost. In this regard, our study relates to Shin et al. (2012) who show that a monopolist may find it optimal to fire some high-cost customers even if they are profitable. While our results share the general intuition from the BBP literature that sellers' enhanced ability to track customer behavior may intensify price competition, our study points out that bilateral ratings may also relax competition through cost-based segmentation.

Another related branch of the pricing literature focuses on cross-brand retail pass-through: retailers may change prices of multiple products in response to a change in the wholesale cost of a single product (e.g., Nakamura 2008), and there can be positive (negative) cross-brand pass-through where the retail price of an item would decrease (increase) as the wholesale price of another item decreases (e.g., Besanko et al. 2005). While our study also features competition between providers who sell through the same platform (e.g., Hu et al. 2022), our main focus is on how providers can influence the composition of their own customer base. This departs from the pass-through literature which highlights retailers' profit maximization as the driving force behind co-movement of prices (e.g., Moorthy 2005).

Within the context of decentralized matching, our model builds on the competitive search literature in labor economics pioneered by Peters (1991) and Montgomery (1991). Early contributions to this literature (e.g., Burdett et al. 2001) focus on homogeneous agents on both sides of the market while more recent papers have started to incorporate heterogeneous agents (e.g., Eeckhout and Kircher 2010; Chade et al. 2017). Compared with other search and matching models, the labor-economics framework explicitly models the matching process (see Rogerson et al. (2005) for a survey). Shi (2002), in particular, considers a directed search model with coordination frictions in which workers with heterogeneous skills apply to firms with heterogeneous technologies. Shi assumes that high-skill workers generate higher output when they are matched with high-technology firms, which leads to positive assortment. The absence of an equivalent assumption in our model enables us to shift the focus from positive assortment to the endogenous composition effect. To this regard, our model also makes methodological contribution to the competitive search literature by incorporating a continuous distribution of customer types, which has two advantages: first, it allows us to establish an elegant relationship between an individual provider's profit and his acceptance rates for all types of customers, which facilitates tractable analysis; second, it enables a straightforward and visual characterization of market segmentation based on customer costs.

Finally, our paper makes an important contribution to the emerging literature on P2P platforms, collaborative consumption and sharing economy (e.g., Einav et al. 2016; Veiga and Weyl 2017). Existing research in this literature has generally focused on how peer-to-peer markets differ from traditional markets, discussing the impact of peer-to-peer markets on traditional markets (e.g., Zervas et al. 2016; Jiang and Tian 2018; Tian and Jiang 2018; Gong et al. 2017), the value of flexible work (e.g., Chen et al. 2019), and search frictions (e.g., Horton 2014; Fradkin 2015; Arnosti et al. 2021).[3] By viewing bilateral ratings in our model as a form of information

[3]Other studies have examined buyers' incentives to leave feedback (e.g., Bolton et al. 2013; Nosko and Tadelis 2015; Horton and Golden 2015), the impact of choice sets (e.g., Halaburda et al. 2018), potential incentive misalignment between customers and the platform (e.g., Armstrong and Zhou 2011; Eliaz and Spiegler 2011; Hagiu and

disclosure, we recognize Romanyuk and Smolin (2019) as the closest to our paper in this literature, who show that full information disclosure on a platform with nonstrategic buyers may lead to a market failure due to excessive rejections by the sellers. To our best knowledge, the current paper is the first theoretical attempt to understand how a bilateral reputation mechanism affects strategic behavior of agents on both sides of a P2P platform.

The remainder of the paper proceeds as follows. Section 2 introduces the main model and Section 3 characterizes the equilibrium outcomes and comparative statics. Section 4 explores providers' pricing strategies when market coverage is incomplete. Section 5 studies a market with unilateral ratings and compares the equilibrium outcomes with those from the main model. Section 6 tests the robustness of our findings under alternative assumptions. Section 7 concludes and points out directions for future work.

2 Main Model

We consider a P2P market where M providers and N customers gather on a platform to trade with each other for a time-sensitive service. Following the literature on matching markets and consistent with our observation of P2P markets, we assume that the market is large, i.e., $M, N \to \infty$, and neither side of the market is infinitely larger than the other side, i.e., $0 < n \equiv N/M < \infty$.[4]

To explore the effects of a bilateral reputation system, we incorporate quality differentiation for providers and cost differentiation for customers into our model.[5] On the provider side, we assume that a fraction γ of the providers are of high (H) quality q_H and the remaining $1-\gamma$ fraction are of low (L) quality q_L, with $\gamma \in (0,1)$ and $q_H \geq q_L > 0$.[6] On the customer side, a customer's utility from receiving service of quality q at price p is $u(p,q) = q - p$.[7] To serve a customer of cost type θ, a provider of quality q incurs a variable cost of $\theta g(q)$, where $g(\cdot) > 0$, $g'(\cdot) \geq 0$, and θ follows a distribution function $F(\cdot)$ in $[0, \bar{\theta}]$ with PDF $f(\theta) \equiv F'(\theta)$ being

Jullien 2011; De Cornière and Taylor 2014), and comparison of different pricing formats such as auctions, posted prices and surge pricing (e.g., Einav et al. 2018; Gomez Lemmen Meyer 2015; Cullen and Farronato 2021; Guda and Subramanian 2019; Castillo et al. 2017).

[4]This assumption facilitates the tractability of the model by essentially approximating binomial distribution of the number of applications received by a provider by exponential distribution (Butters 1977). An equivalent representation of the model is to have a set of providers of measure 1 and a set of customers of measure n. Each provider or customer can then be interpreted as an infinitesimally small subset. To facilitate analysis, we take the limit of $M, N \to \infty$ when analyzing the equilibrium instead of first solving the equilibrium for finite N and M and then taking the limit. In Section 6.1, we analyze an alternative market setup with $M = N = 2$ and find that a pure-strategy price equilibrium does not exist.

[5]We focus on vertical differentiation for two reasons. First, the most salient features of customer reviews in the real world are often the average rating and the number of ratings, both of which mainly reflect vertical quality of the provider's service. Second, we want to explore the possibility of a higher-quality provider charging a lower price in an environment with endogenous composition effect. If providers are horizontally rather than vertically differentiated, our intuition is that the endogenous composition effect would still persist: providers may respond to higher commission rates and average service cost by lowering their prices.

[6]As the driving force of our results is coordination frictions that are rooted in time-sensitivity of the service provided, we do not expect the main results to change for alternative model setups with, for example, a continuum of quality types. The discrete setup of provider qualities is in line with classic vertical differentiation models (e.g., Moorthy 1988), which makes our result directly comparable with the classic results in the literature.

[7]If customers have different sensitivities towards quality, the high-quality providers may have an incentive to charge a higher price to cream skim the high-sensitivity customers who have higher willingness to pay for quality. This new incentive could balance against these providers' incentive to cherry-pick the low-cost customers through lower prices, leading to a weaker endogenous composition effect.

positive and finite for $\forall\theta \in [0, \bar{\theta}]$. As one can see, customers with a higher θ are more costly to serve. Note that the cost to serve a particular customer may be the same if $g(q) = 1$, or different across providers if for example, $g(q) = q$, under which, service cost is higher to a high-quality provider than to a low-quality provider. This happens when high-quality providers value their time more than the low-quality ones and hence regard service as more costly.[8] The fixed cost of service is normalized to zero.

To keep the analysis tractable, we assume complete market coverage in the main model: service costs are low enough in comparison to its value so that even the most costly customer would be served in equilibrium. We will analyze the case of incomplete market coverage in Section 4. We also assume that the platform implements a *bilateral* rating system that fully reveals providers' service quality and customers' service cost.

We consider a matching game in three stages. First, all providers post prices simultaneously, which are observed by all agents in the marketplace. Each provider posts a single price and there is no price discrimination based on service costs. Second, all customers simultaneously submit their applications to providers. Each customer can only submit at most one application but is allowed to use a mixed strategy.[9] Lastly, a provider decides which application to accept if he receives one or more applications. An important assumption we make here is that each provider can serve at most one customer. The capacity constraint enables us to capture the scarce, time-sensitive nature of services in many P2P markets and is also recognized as an important feature of matching models in the literature (e.g., Burdett et al. 2001). Given the capacity constraint, upon receiving multiple applications, a provider will accept the customer with the lowest θ, because the price has been set and customers are homogeneous along all other dimensions. Once acceptance occurs, service is provided and payment is made to the provider, who then pays the platform a fraction $\delta \in [0, 1)$ of the transaction price as a commission. If a provider receives no applications from customers or turns down all the applications he receives, he receives zero payoff. Similarly, if a customer does not submit an application or has her application rejected, she gets zero payoff.

3 Equilibrium Analysis

We solve the game with backward induction and focus on symmetric equilibria in which all providers of type $j \in \{H, L\}$ post the same price p_j, and all customers of type $\theta \in [0, \bar{\theta}]$ use the same application strategy $a_j(\theta)$, $j \in \{H, L\}$, which denotes the probability of a type θ customer submitting an application to a provider of type j. Following the literature, we define the *queue length* of a provider of type j from customer type θ as $x_j(\theta) \equiv Nf(\theta)a_j(\theta)$. Subsequently, $x_j(\theta)d\theta$ is the number of applications that provider j receives from customers in $[\theta, \theta+d\theta]$. As M and N go to infinity, $a_j(\theta)$ goes to zero while $x_j(\theta)$ converges to a finite positive number. Therefore, it is easier to work with $x_j(\theta)$ than $a_j(\theta)$.[10] As there is no submission

[8] We consider the case of $g'(\cdot) < 0$ in Section 6.4.

[9] We allow a customer to make multiple applications in Section 6.2.

[10] In a large market, if a subset of customers with zero measure change their application strategies, the equilibrium remains unchanged. It is hence reasonable and technically convenient to stipulate that $x_H(\theta)$ and $x_L(\theta)$ are piece-wise continuous with a finite number of discontinuities.

cost, each customer always submits one application and we have the normalization condition $\gamma M a_H(\theta) + (1-\gamma) M a_L(\theta) = 1$, or equivalently,

$$\gamma x_H(\theta) + (1-\gamma) x_L(\theta) = n f(\theta). \tag{1}$$

3.1 Customer's Problem

A customer maximizes her expected utility by deciding which provider to apply to. Let $U(\theta)$ denote the maximum expected utility of customers of type θ. A customer of this type submits an application to a provider of type j with positive probability iff $b_j(\theta)(q_j - p_j) \geq U(\theta)$, where the left-hand side of the inequality is her expected utility from that provider, p_j denotes the posted price from providers of type j and $b_j(\theta)$ denotes the probability that an application from a customer of type θ gets accepted by a provider of type j.

To calculate $b_j(\theta)$, notice that the provider will accept the customer if and only if he receives no application from customers with a lower cost:[11]

$$b_j(\theta) = \lim_{N\to\infty} \prod_{t=0}^{\theta} (1 - a_j(t))^{Nf(t)\mathrm{d}t} = \lim_{N\to\infty} \prod_{t=0}^{\theta} \left(1 - \frac{x_j(t)}{Nf(t)}\right)^{Nf(t)\mathrm{d}t} = \prod_{t=0}^{\theta} e^{-x_j(t)\mathrm{d}t} = e^{-\int_0^{\theta} x_j(t)\mathrm{d}t}. \tag{2}$$

Obviously, $b_j(\theta)$ decreases with θ, which implies that a customer of higher cost expects a lower acceptance rate. Intuitively, low-cost customers are less likely to be rejected, because they get rejected only when there exists at least one other customer with even lower cost who applies to the same provider.

In equilibrium, it cannot be the case that $b_j(\theta)(q_j - p_j) > U(\theta)$; otherwise, customers of type θ would apply to that provider with probability one, which would then drive down the acceptance probability $b_j(\theta)$ until $b_j(\theta)(q_j - p_j) = U(\theta)$. This equation illustrates a customer's tradeoff: on one hand, she prefers providers that generate a higher *potential payoff* upon acceptance, $q_j - p_j$; on the other hand, such high-payoff providers may receive many applications and hence have a lower acceptance rate $b_j(\theta)$. As low-cost customers are less likely to be rejected, they would be more confident to apply to providers with higher potential payoffs, which keeps driving down the acceptance rate of those providers until the high-cost customers become indifferent between the two types of providers. We formalize the intuition in the following proposition.

Proposition 1. *When one type of providers generate a significantly greater potential payoff than the other type, all customers apply only to the high-payoff providers. When one type of providers generate a slightly greater potential payoff than the other type, customers of cost types below a certain threshold apply only to high-payoff providers, and the other customers are indifferent between the two types of providers. When all providers provide the same potential payoff, all customers are indifferent between the two types of providers.*

See all proofs in the Appendix, which characterizes customers' application strategies by calculating $x_L(\theta)$, $x_H(\theta)$ and $U(\theta)$ explicitly. Figure 1 illustrates Proposition 1, with the closed-

[11]Notice that the product is taken over the series of dt increments in equation (2); the geometric integral has finite bounds so it can be exchanged with the limit of $N \to \infty$.

form expressions for the thresholds of θ_L and θ_H provided in Appendix. One can see an interesting market segmentation arising in this figure: desirable providers may serve customers in the entire cost spectrum, while the other providers serve only the undesirable, high-cost customers.

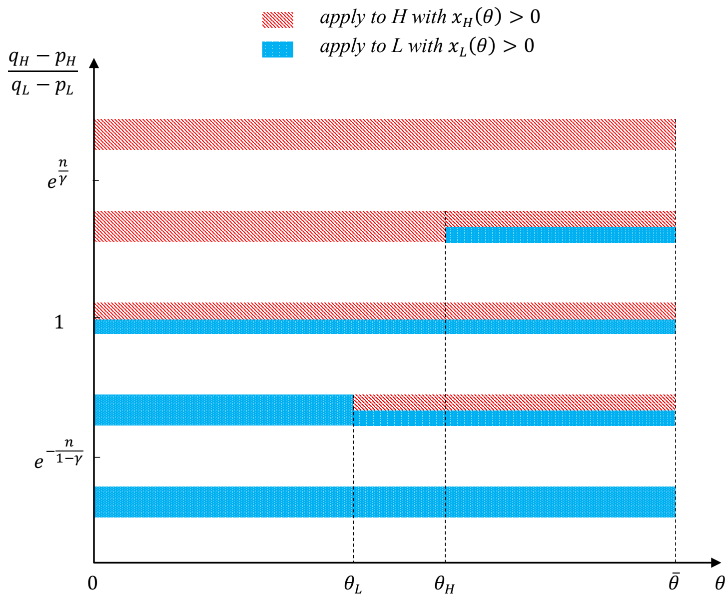

Figure 1: Possible Market Segmentations.

It is worth noting that we have only considered customers' application strategies when all providers of the same type post the same price, i.e., symmetric pricing. When we consider an individual provider's pricing decision as a next step, we have to consider his deviation of price with respect to other providers, which leads to analysis of asymmetric prices. How can we utilize customers' application strategies derived only under symmetric pricing in Proposition 1 to inform an individual provider's pricing decision? The key lies in the large market assumption that $M, N \to \infty$. Under this assumption, an individual provider's deviation in price would not change individual customers' maximum expected utility $U(\theta)$, because this provider can serve at most one customer: even if he offers the best deal in the market, a customer rationally expects that there is little chance that she would be the lucky one to get the deal. We are now ready to analyze providers' pricing problem.

3.2 Providers' Problem

Let us consider an individual provider of type j, who posts price p_j^0 for $j \in \{H, L\}$. Given all other providers' prices of p_H and p_L, $U(\theta)$ is determined by Proposition 1 as can be seen in the proof of the proposition. $U(\theta)$ is taken as given by the provider when choosing his price p_j^0. Now consider customers' application strategy for this provider. If $U(\theta) \leq q_j - p_j^0$, customers of type θ apply to him until $b_j^0(\theta)(q_j - p_j^0) = U(\theta)$, where $b_j^0(\theta)$ is his acceptance rate; on the other hand, if $U(\theta) > q_j - p_j^0$, customers of type θ apply not to him but to other providers with a higher expected utility. It is easy to show that $U(\theta)$ is a strictly decreasing function, and therefore we have,

$$b_j^0(\theta) = \frac{U(\theta)}{q_j - p_j^0} \text{ for } \theta \in [\theta^0, \bar{\theta}], \text{ where } \theta^0 = \begin{cases} 0, & q_j - p_j^0 > U(0), \\ U^{-1}(q_j - p_j^0), & \text{otherwise.} \end{cases} \tag{3}$$

Notice that by definition, $b_j^0(\theta)$ is the probability that the provider has not received an application from customers of types lower than θ. Correspondingly, $1 - b_j^0(\theta)$ is the probability that the provider has received at least one application from these customers. Therefore, $\mathrm{d}\left(1 - b_j^0(\theta)\right) = -\frac{\mathrm{d}b_j^0(\theta)}{\mathrm{d}\theta}\mathrm{d}\theta$ is the probability that the provider has received at least one application from customers of types in $[\theta, \theta + \mathrm{d}\theta]$ and has not received any application from customers with type lower than θ.

When a provider receives no application, he ends up with no match and thus zero profit. When a provider of type j receives and accepts an application from a customer of type θ, he earns revenue p_j^0, pays the commission fee of δp_j^0 to the platform, and incurs serving cost of $\theta g(q_j)$. Therefore, we can write down the provider's expected profit $\pi_j^0(p_j^0; p_H, p_L)$ as follows, given his price p_j^0 as well as other providers' prices:

$$\pi_j^0(p_j^0; p_H, p_L) = \int_{\theta^0}^{\bar{\theta}} \left[(1-\delta)p_j^0 \; - \theta g(q_j)\right] \cdot \left(-\frac{\mathrm{d}b_j^0(\theta)}{\mathrm{d}\theta}\right) \mathrm{d}\theta. \tag{4}$$

This equation establishes an explicit relationship between a provider's profit and his acceptance rate for each type of customers, and it showcases the analytical tractability of our model that comes from incorporating a continuous distribution of service costs.[12] The provider's objective is to maximize his expected profit $\pi_j^0(p_j^0; p_H, p_L)$ by choosing the posted price p_j^0. In equilibrium, we must have $p_j^0 = p_j$. A pure-strategy Nash equilibrium (p_H^*, p_L^*) is determined by

$$p_j^* = \arg\max_{p_j^0} \pi_j^0(p_j^0; p_H^*, p_L^*), \text{ for } j \in \{H, L\}. \tag{5}$$

As common for competitive search models with heterogeneous agents, there is no closed-form expression for the equilibrium prices (p_H^*, p_L^*). To facilitate equilibrium analysis, we restrict our attention to the uniform distribution of cost types for the following analysis, where $F(\theta) = \theta/\bar{\theta}$.[13] The following proposition characterizes the equilibrium.

Proposition 2. *Assume uniform distribution of cost type $F(\cdot)$ and that the ratio of customers to providers n is sufficiently large.*

- *There always exist an infinite number of solutions to the problem in equation (5), which satisfy $q_H - p_H^* = q_L - p_L^* = \varepsilon$ for $\forall \varepsilon \in (0, \bar{\varepsilon}]$.*
- *When $g(q) = 1$ or $g(q) = q$, high quality providers generate higher potential payoff in equilibrium: $q_H - p_H^* \geq q_L - p_L^*$.*

The proof of the proposition and the closed-form expression of $\bar{\varepsilon} > 0$ are relegated to the

[12] Note that the profit function above reflects our assumption that the customer market is fully covered, $p_j^0 \geq \bar{\theta} g(q_j)/(1-\delta)$. We verify this assumption in equilibrium below.

[13] Section 6.3 explores alternative distributions of F.

Appendix.[14] Proposition 2 reveals that there exist multiple equilibria. In fact, our model setup belongs to a general class of games with strategic complementarities and multiple equilibria are a well-known common potential outcome for this type of games (e.g. Vives 2005). Following the tradition of the literature, we will work with multiple equilibria directly instead of imposing specific rules of equilibrium selection.

The first result in the proposition points out that $(p_H^*, p_L^*) = (q_H - \varepsilon, q_L - \varepsilon)$ is always an equilibrium for ε positive but sufficiently small.[15] This result implies that coordination frictions can give rise to monopolistic power for individual providers in a decentralized P2P market. Since the customer is left with almost no surplus, the two kinds of providers are equally attractive. The second result suggests that as long as service cost weakly increases with service quality, a customer would expect a weakly higher potential payoff from providers of higher quality. That is, high-quality providers are more attractive in this case than low-quality ones.

Following the analysis, there are two possible market segmentations in equilibrium, as shown by Figure 2: either all customers apply to both types of providers, or low-cost customers apply solely to high-quality providers and high-cost customers apply to both types of providers.

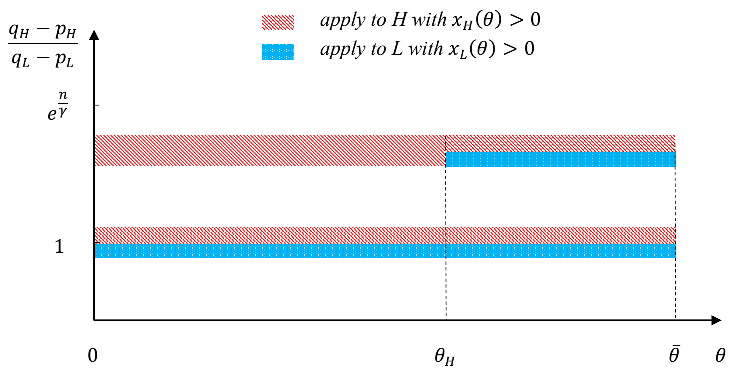

Figure 2: Equilibrium Market Segmentations.

Figure 3 illustrates all price equilibria under one parameter setting with $n = 1$, $\gamma = 0.5$, $q_H = 2q_L$, $\delta = 0.1$, $\bar{\theta} = 0.2$, and $g(q) = 1$, which will be used for all following illustrations and referred to as "the parameter setting". We can see that under this parameter setting, both types of price equilibria are possible: there are infinite equilibria with both types of providers generating the same potential payoff, as marked by the solid line above the kink, and there are also infinite equilibria with higher-quality providers generating a higher potential payoff, as marked by the solid line below the kink. Moreover, equilibrium prices are lower in the second type of equilibria.

By combining Propositions 1 and 2, we characterize customers' equilibrium application strategies, acceptance rates, and expected utilities in the following proposition, where we unify

[14] The lower bound on n comes from the second-order optimality condition in optimization problem (5). That is, we require the profit function $\pi_j^0(p_j^0; p_H, p_L)$ in equation (4) to be concave in $p_j^0 \in [q_j - U(0), q_j]$.

[15] We do not consider the case with $\varepsilon = 0$, because in this case, customers expect zero utility from all providers. Consequently, each customer is indifferent between the two types of providers, and there is arbitrariness in customers' application strategy.

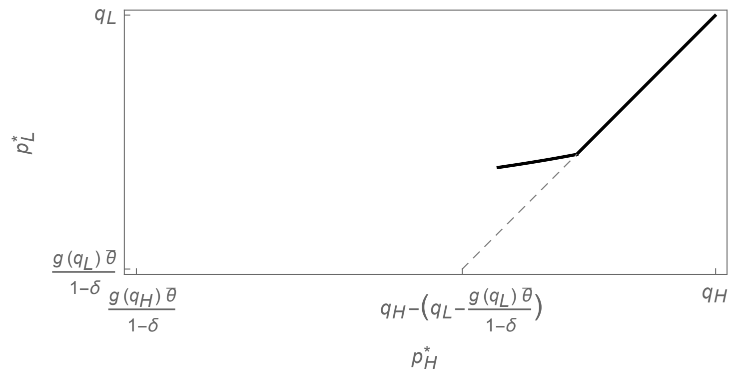

Figure 3: Equilibrium Prices under the Parameter Setting with $n = 1$, $\gamma = 0.5$, $q_H = 2q_L$, $\delta = 0.1$, $\overline{\theta} = 0.2$ and $g(q) = 1$. The dashed line represents $q_H - p_H^* = q_L - p_L^*$.

the two cases in Figure 2 by noting that when $q_H - p_H^* = q_L - p_L^*$, $\theta_H = 0$ and $[0, \theta_H) = \emptyset$.

Proposition 3. *Assume uniform distribution of $F(\cdot)$, n is sufficiently large, and $g(q) = 1$ or $g(q) = q$. In equilibrium, customers with $\theta \in [0, \theta_H)$ apply only to high-quality providers, and customers with $\theta \in [\theta_H, \bar{\theta}]$ apply to all providers. Prices satisfy $q_H - p_H^* \geq q_L - p_L^*$. Queue lengths, acceptance rates, and customers' expected utilities are as follows,*

$$x_H(\theta) = \begin{cases} \dfrac{n}{\gamma\bar{\theta}}, & 0 \leq \theta \leq \theta_H \\ \dfrac{n}{\bar{\theta}}, & \theta_H < \theta \leq \bar{\theta} \end{cases}, \quad x_L(\theta) = \begin{cases} 0, & 0 \leq \theta \leq \theta_H \\ \dfrac{n}{\bar{\theta}}, & \theta_H < \theta \leq \bar{\theta} \end{cases};$$

$$b_H(\theta) = \begin{cases} e^{-\frac{n\theta}{\gamma\bar{\theta}}}, & 0 \leq \theta < \theta_H \\ \left(\dfrac{q_H - p_H^*}{q_L - p_L^*}\right)^{-(1-\gamma)} e^{-\frac{n\theta}{\bar{\theta}}}, & \theta_H \leq \theta \leq \bar{\theta} \end{cases}, \quad b_L(\theta) = \begin{cases} 1, & 0 \leq \theta < \theta_H \\ \left(\dfrac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma} e^{-\frac{n\theta}{\bar{\theta}}}, & \theta_H \leq \theta \leq \bar{\theta} \end{cases};$$

$$U(\theta) = \begin{cases} (q_H - p_H^*)e^{-\frac{n\theta}{\gamma\bar{\theta}}}, & 0 \leq \theta \leq \theta_H \\ (q_H - p_H^*)\left(\dfrac{q_H - p_H^*}{q_L - p_L^*}\right)^{-(1-\gamma)} e^{-\frac{n\theta}{\bar{\theta}}}, & \theta_H < \theta \leq \bar{\theta} \end{cases}.$$

Figure 4 plots queue lengths, acceptance rates, and expected utility in the aforementioned two types of equilibria. When all providers generate the same potential payoff (panel on the left), queue lengths and acceptance rates are the same for the two types of providers. When high-quality providers generate a higher potential payoff than low-quality providers (panel on the right), only low-cost customers enter the queue for high-quality providers, who have a lower acceptance rate than low-quality providers for all types of customers. As one can see from the figure, when high-quality providers provide a higher, rather than the same, potential payoff than low-quality providers, the difference in the two types of providers' queue length, acceptance rates and expected utility generated for the customers widens. This is intuitive as the high-quality providers essentially become more attractive to the customers than low-quality providers. Notice that in all equilibria, customers with higher costs expect lower utility. In fact, a customer would rather reimburse her provider of her service cost if she could then be treated as a customer with zero cost. We formalize this result below.

Corollary 1. *Under the same assumptions in Proposition 3 with n sufficiently large, $U(\theta) < U(0) - \theta$.*

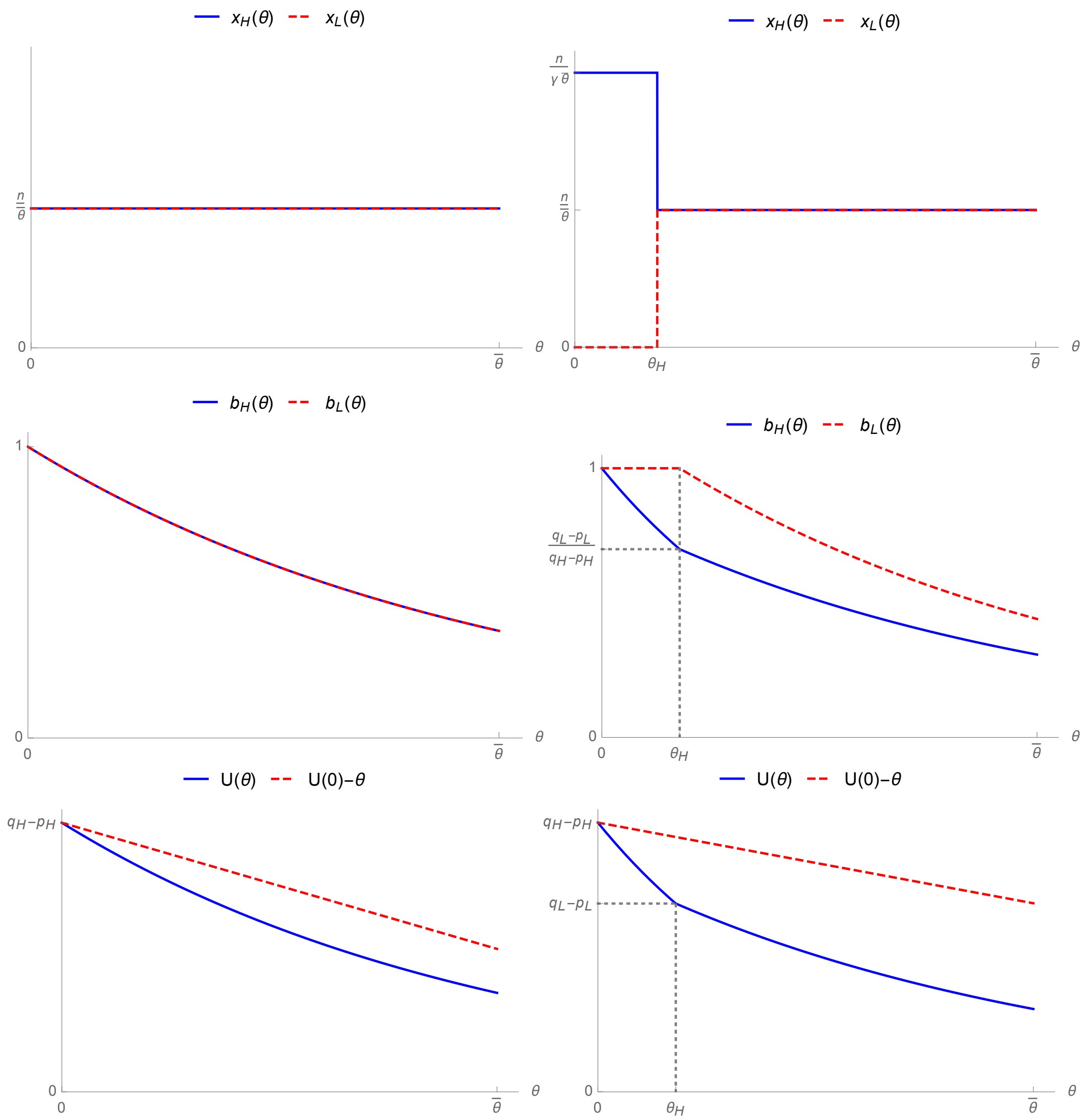

Figure 4: Equilibrium Queue Length $x(\theta)$, Acceptance Rate $b(\theta)$, and Customers' Expected Utility $U(\theta)$ under the Parameter Setting. In the left panel, $(p_H^*, p_L^*) = (0.787q_H, 0.573q_L)$ with $q_H - p_H^* = q_L - p_L^*$ so that all providers generate the same potential payoff; in the right panel, $(p_H^*, p_L^*) = (0.666q_H, 0.533q_L)$ with $q_H - p_H^* > q_L - p_L^*$ so that high-quality providers generate a higher potential payoff than low-quality providers.

The result highlights that a high-cost customer is subject to disproportionately low acceptance rate in equilibrium and customers would in general find it rather beneficial to maintain a good reputation in a P2P market.

3.3 Comparative Statics

Given the existence of multiple equilibria, we leverage the general monotone comparative statics results for supermodular games to prove the following results.[16]

Proposition 4. *Under uniform distribution of $F(\cdot)$, the game described by equation (5) is supermodular. Equilibrium prices p_H^* and p_L^* increase with n, decrease with γ, and for n sufficiently high, decrease with $\bar{\theta}$ and δ. Given multiple equilibria of (p_H^*, p_L^*), an increasing (decreasing) relationship with a parameter is defined as:*

1. *the largest and smallest equilibrium points increase (decrease);*
2. *starting from any equilibrium, the best-response dynamics lead to a weakly larger (smaller) equilibrium point following the parameter change.*

We plot the comparative statics numerically in Figure 5. Similarly, we see two types of equilibria under all parameter settings, where the solid line above the kink represents the equilibrium with equal potential payoff for both types of providers, and the solid line below the kink represents the equilibrium with high-type providers generating a higher potential payoff.

Several patterns in Proposition 4 and Figure 5 are noteworthy. First, as the ratio of customers to providers n gets larger, providers' market becomes less competitive and the equilibrium prices increase. Similarly, as the fraction of high-quality providers γ gets larger, high-quality providers' market becomes more competitive, and their price p_H^* decreases, which has a spillover effect that decreases the low-quality providers' price. Note that prices decrease despite the fact that low-quality providers' market becomes less competitive: in essence, the increase in competition among high-quality providers dominates the decrease in competition among low-quality providers.

Second, when service cost $\bar{\theta}$ increases, equilibrium prices decrease as long as n is sufficiently large, which stands in stark contract to how production costs typically pass through positively to prices. The intuition goes back to the endogenous composition effect: as an individual provider raises his price, he loses applications from customers with low costs and gains applications from those with high costs.[17] On the other hand, by setting a lower price, a provider can get more applications and cherry-pick the lowest-cost customer in an enlarged pool. The incentive to lower his price becomes stronger, when there are enough customers in the market to cherry-pick from (n is large) and when service costs become higher are more spreaded out so that it is important to distinguish the low-cost customers from the high-cost ones. Similarly, when the platform's commission rate (δ) increases, equilibrium prices decrease

[16] The technique is developed by Topkis (1978) and Topkis (1979), and applied to economics by Vives (1990) and Milgrom and Roberts (1990). Our proof in Appendix follows the user manual outlined by Vives (2005).

[17] Particularly, from equation (3), we find that the demand from customer θ, $-\frac{\mathrm{d}b_j^0(\theta)}{\mathrm{d}\theta}\mathrm{d}\theta$ decreases (increases) with the type-j provider's price p_j^0 for relatively small (large) θ.

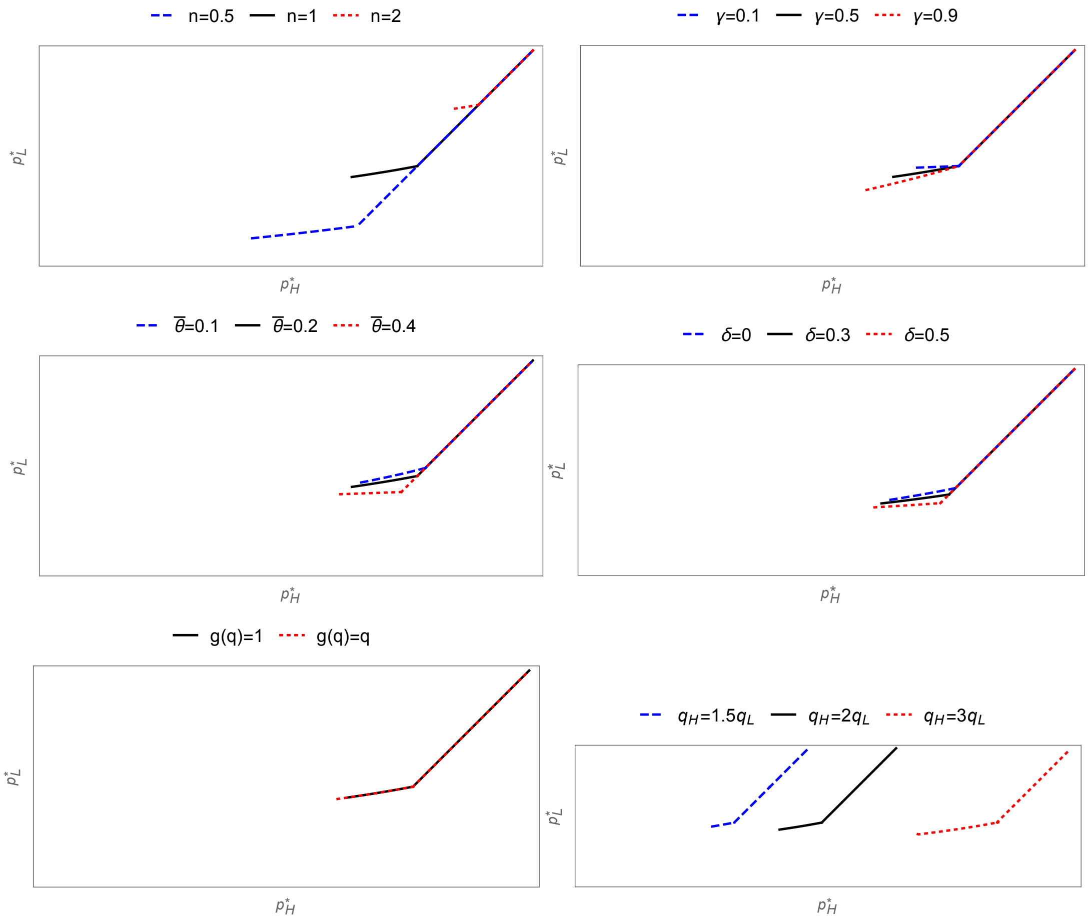

Figure 5: Comparative Statics of Equilibrium Prices under the Parameter Setting unless Otherwise Noted.

as long as n is sufficiently large, since providers now have a stronger incentive to attract the low-cost customers.

In the last two plots in Figure 5, we illustrate how equilibrium prices may depend on $g(\cdot)$ and q_H/q_L. Compared with the case $g(q) = 1$, we find that prices with $g(q) = q$ are lower. The intuition is as follows. In the latter case, the difference in the service costs between a low-cost and a high-cost customer for a high-quality provider is higher than that for a low-quality provider , which motivates high-quality providers to compete more aggressively for low-cost customers, pushing low-quality providers to also lower their prices. Finally, as service quality q_H becomes higher, equilibrium price p_H^* also increases, which is intuitive.

3.4 Platform Strategy

When the platform raises its commission rate δ, it has two effects. The direct effect is that it would get a larger share of the total profit, and the indirect effect, as discussed above, comes from the providers' reducing prices in order to attract lower-cost customers. Formally, the platform's profit under the equilibrium is given by,

$$PP \equiv \delta M \left[(1-\gamma) p_L^* \left(1 - e^{-n} \left(\frac{q_H - p_H^*}{q_L - p_L^*} \right)^{\gamma} \right) + \gamma p_H^* \left(1 - e^{-n} \left(\frac{q_H - p_H^*}{q_L - p_L^*} \right)^{-(1-\gamma)} \right) \right]. \quad (6)$$

While an analytical characterization of the total effect is not tractable, Figure 6 illustrates the equilibrium prices from the lowest-price equilibrium. We see that prices indeed decrease with the commission rate but since the decrease is relatively mild, the direct effect overall dominates the indirect effect, and the platform's profit increases with its commission rate. To arrive at an optimal commission rate, however, one would need to incorporate the entry of customers and providers into the analysis, which is beyond the scope of the current paper.

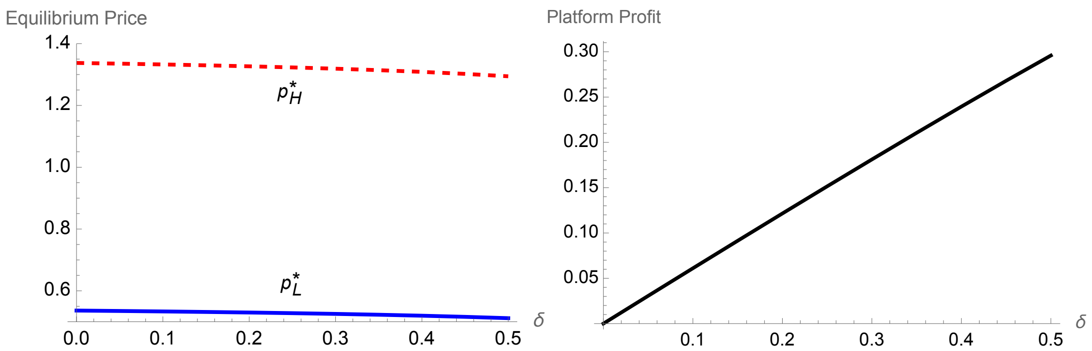

Figure 6: Equilibrium Prices and Platform Profit on Commission Rate under The Parameter Setting.

4 Incomplete Market Coverage

In this section, we consider the possibility that there may exist some customers who are so costly that providers would rather not serve them.[18] In particular, we consider the more interesting equilibria in which low-quality providers are willing to serve all customers and high-quality providers are willing to serve only customers of low costs, i.e.,

$$p_H < \bar{\theta} g(q_H)/(1-\delta) \text{ and } p_L \geq \bar{\theta} g(q_L)/(1-\delta). \quad (7)$$

We consider both cases $g(q) = 1$ and $g(q) = q$, under which it is not hard to show that the incomplete market coverage condition in equation (7) implies that $q_H - p_H > q_L - p_L$: high-quality providers would generate a higher potential payoff than low-quality providers. Similar to Proposition 1, we are able to completely characterize customers' application strategies given providers' posted prices p_H and p_L (details provided in the Online Appendix). A key difference here is that customers of type $\theta \in \left((1-\delta)p_H/g(q_H), \bar{\theta}\right]$ would be rejected by high-quality providers and hence have no other choice but to apply to low-quality providers.

[18]The market coverage condition refers to whether the costliest customer will be served by a provider if he has no other applicant. It is not directly linked to comparison between the number of providers, M and the number of customers, N. In fact, due to coordination friction, there will almost surely exist some customers unserved regardless of the comparison between M and N.

Figure 7 characterizes the resulting market segmentation: low-cost customers apply to high-quality providers only, medium-cost customers apply to both types of providers, and high-cost customers apply to low-quality providers only.

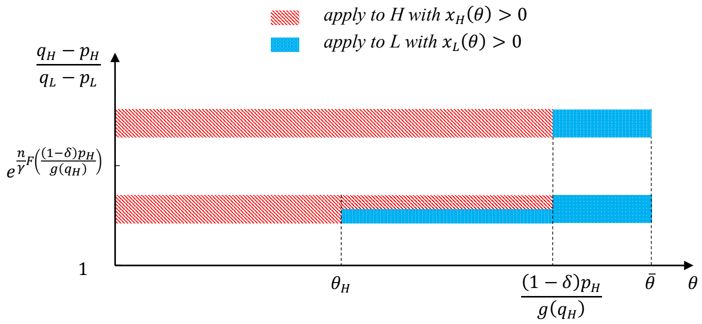

Figure 7: Possible Market Segmentations under Incomplete Market Coverage.

Given customers' application strategies, we explore providers' pricing decisions in the Online Appendix. While a full characterization of the equilibrium is not tractable, we are able to highlight one interesting finding numerically, as demonstrated in Figure 8. Unlike before, in this figure, there is only one type of equilibrium, in which high-quality providers generate a higher potential payoff than low-quality providers. Meanwhile, as one can see from the figure, for the part of the solid line to the left of the dashed line, $p_L > p_H$ while for the part to the right, $p_L < p_H$. In other words, it is possible that high-payoff, high-quality providers may charge a *lower* price than low-quality providers in equilibrium. This is a rather surprising finding, as quality is observed to be positively correlated with price in most markets. The intuition of our result is as follows: since high-cost customers are unprofitable for high-quality providers, low-quality providers face less competition in serving these customers. As a result, they charge a high price to cover the high service cost of their clientele while high-quality providers charge a low price to compete for the lowest-cost customers. Note that the parameter setting in this figure indeed features high service costs and a high fraction of high-quality providers, which is consistent with our intuition that intense competition and spreaded-out service costs drive high-quality providers' cherry-picking of the lowest-cost customers through low prices.

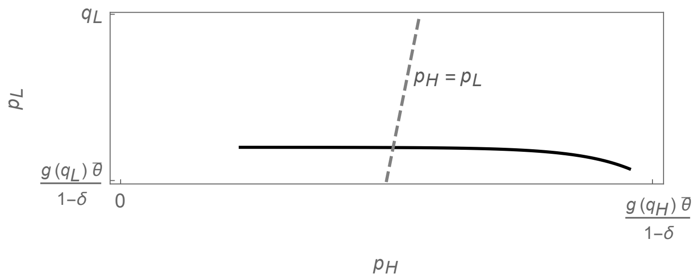

Figure 8: Equilibrium Prices under Incomplete Market Coverage under $n = 1$, $\gamma = 0.9$, $q_H = 2q_L$, $\delta = 0.1$, $\overline{\theta} = 0.8$ and $g(q) = q$.

5 Unilateral Ratings

To understand the role of a bilateral rating system, we consider in this section P2P markets with a unilateral rating system, in which only providers' quality is publicly observable and customers' cost is not. With unilateral ratings, providers cannot discern low-cost customers from high-cost ones, so they will randomly select a customer when receiving multiple applications.

Denote the probability that a customer submits an application to a provider of type j by A_j, for $j \in \{H, L\}$ and the queue length at a provider of type j by $X_j = NA_j$. We now have the normalization condition:

$$\gamma X_H + (1-\gamma)X_L = n. \tag{8}$$

Given the providers' prices p_H and p_L, a customer's acceptance rate by a provider of type j conditioning on that the customer has submitted her application to the provider is

$$B_j = \lim_{N,M\to\infty} \sum_{i=0}^{N-1} \binom{N-1}{i} A_j^i (1-A_j)^{N-1-i} \frac{1}{i+1} = \lim_{N_j,M\to\infty} \frac{1-(1-A_j)^N}{NA_j} = \frac{1-e^{-X_j}}{X_j}. \tag{9}$$

With unilateral ratings, low-cost customers lose preferential treatment from providers, and all customers have the same acceptance rate. Similar as before, a customer's expected utility from the two types of providers must be the same in equilibrium: $U = B_H(q_H - p_H) = B_L(q_L - p_L)$, or equivalently,

$$U = (q_H - p_H)\frac{1-e^{-X_H}}{X_H} = (q_L - p_L)\frac{1-e^{-X_L}}{X_L}. \tag{10}$$

By combining equations (8) and (10), we can solve for X_H, X_L and determine U.

Consider now an individual provider j's profit maximization problem. Given his posted price p_j^0, his acceptance rate is

$$B_j^0 = \min\left\{\frac{U}{q_j - p_j^0}, 1\right\}. \tag{11}$$

Define function $\phi(x) \equiv [1-e^{-x}]/x$ for $x \in (0,\infty)$ and $\phi(0) \equiv \lim_{x\to 0}\phi(x) = 1$. $\phi(x)$ is then a continuous and strictly decreasing function on $[0,\infty)$. Given the provider's acceptance rate B_j^0, his queue length can be written as $X_j^0 = \phi^{-1}(B_j^0)$. The provider's expected profit, given his posted price p_j^0 and the market prices p_H and p_L, is

$$\Pi_j^0(p_j^0; p_H, p_L) = \left[(1-\delta)p_j^0 - \frac{\bar{\theta}}{2}g(q_j)\right] X_j^0 B_j^0. \tag{12}$$

Note that H and L providers now have the same customer composition and there is no cost-based segmentation of customers with unilateral ratings. In the appendix, we show that $\Pi_j^0(p_j^0; p_H, p_L)$ is concave in p_j^0 and there exists a unique solution to the first-order optimality condition, $\partial\Pi_j^0(p_j^0; p_H, p_L)/\partial p_j^0|_{p_j^0=p_j} = 0$, which can be written as

$$\frac{q_j - p_j}{U} - \frac{(1-\delta)(q_j-p_j)}{(1-\delta)q_j - \frac{\bar{\theta}}{2}g(q_j)} = \ln\left(\frac{q_j-p_j}{U}\right) - \ln\left[\frac{(1-\delta)(q_j-p_j)}{(1-\delta)q_j - \frac{\bar{\theta}}{2}g(q_j)}\right] \geq 0,\ j \in \{H, L\}. \tag{13}$$

The equilibrium is the set of (p_H, p_L, X_H, X_L, U) that satisfies equations (8), (10) and (13). There are five equations in total to determine five unknown variables. The following proposition characterizes the existence and some properties of the equilibrium.

Proposition 5. *With unilateral ratings, when equations (8), (10) and (13) are not degenerate, there exists a unique pure-strategy Nash equilibrium (p_H^{**}, p_L^{**}). If $q_H > q_L$ and $g(q) = 1$ or q, we have $q_H - p_H^{**} > q_L - p_L^{**}$ in equilibrium.*

5.1 Comparison of The Two Rating Systems

In the absence of closed-form expressions of equilibrium prices with either bilateral or unilateral ratings, we use Figure 9 to plot the prices numerically. As suggested by the proposition above, there is a unique equilibrium with unilateral ratings. What is more interesting is perhaps that compared with unilateral ratings, bilateral ratings may lead to *higher* equilibrium prices. At the first sight, this may contradict our prior intuition that information on service costs may facilitate cherry-picking through low prices. The dominant effect here is, however, that bilateral ratings can facilitate market segmentation and soften the competition between the two types of providers. Specifically, as high-quality providers focus on competing for low-cost customers, the competition between high- and low-quality providers is softened. Consistent with the intuition, we solve the equilibrium for all parameter settings used in Figure 5 and find that equilibrium prices with bilateral ratings are always higher than those with unilateral ratings.

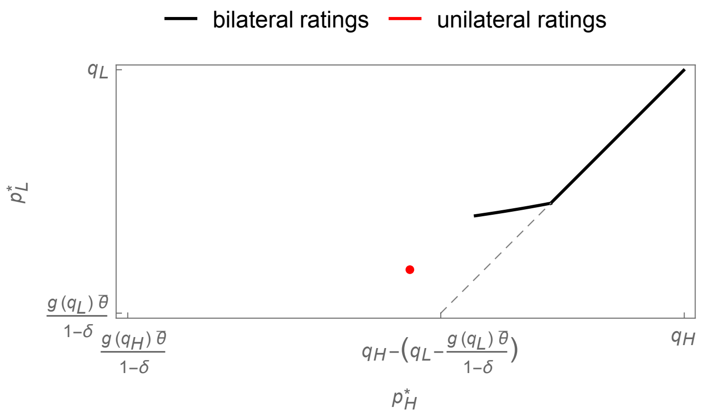

Figure 9: Equilibrium Prices with Bilateral and Unilateral Ratings

Parameters: $n = 1$; $\gamma = 0.5$; $q_H = 2q_L$; $\delta = 0.1$; $\bar\theta = 0.2$; $g(q) = 1$

One may also wonder about the impact of different rating systems on customer surplus. In Figure 10, we show that bilateral ratings may lower total customer surplus. In particular, for low-cost customers, bilateral ratings increase their surplus due to higher acceptance rates; for high-cost customers, however, bilateral ratings decrease their surplus due to higher prices. Overall, in this particular example, bilateral ratings decreases total customer surplus. In other parameter settings, however, we find that it is possible for customer surplus to be higher under bilateral ratings.

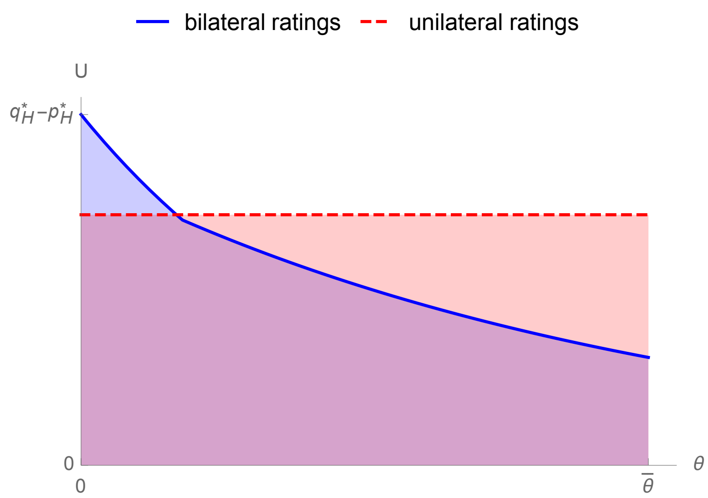

Figure 10: Comparison of Customer Surplus (Shaded Areas) under the Parameter Setting.

In sum, a numerically robust finding is that bilateral ratings may lead to higher prices than unilateral ratings. Furthermore, one may think that customers may have higher reputation concerns with bilateral ratings and hence take precautions to reduce their service costs. This would further increase the equilibrium prices with bilateral ratings, as suggested by Proposition 4, making our prediction stronger. The welfare comparison, on the other hand, remains unclear: bilateral ratings could either increase or decrease customer surplus as well as total surplus.

6 Alternative Model Assumptions

In this section, we explore several alternative model assumptions and show that the endogenous composition effect persists and plays similar roles as in our main model.

6.1 A Small Marketplace

In this section, we analyze a small P2P market with $M = N = 2$: there is one high-quality provider, one low-quality provider, one high-cost customer with service cost $c > 0$, and one low-cost customer with service cost normalized to zero. We assume $q_L > c$ so that there is positive social surplus for the match between a low-quality provider and a high-cost customer. If $q_L \leq c$, we have the uninteresting equilibrium, where the high-quality provider prices at $p_H^* = q_H - q_L$ and serves the low-cost customer, and the low-quality provider is out of the market. As a tie-breaking rule consistent with the main model, we assume that when a customer is indifferent between the two providers, she chooses one randomly.

We can solve the game with backward induction. First, notice that when a provider receives applications from two customers, he accepts the one with a lower service cost. This implies that the low-cost customer applies to the provider with a higher potential payoff, $q_i - p_i$. The high-cost customer hence applies to the other provider that is not chosen by the low-cost customer. Given the customers' application strategies, we can write down provider i' profit function as,

$$\pi_i(p_i) = \begin{cases} p_i, & \text{if } q_i - p_i > q_j - p_j; \\ \frac{1}{2}p_i + \frac{1}{4}\max\{p_i - c, 0\}, & \text{if } q_i - p_i = q_j - p_j; \\ \max\{p_i - c, 0\}, & \text{otherwise.} \end{cases}$$

Bertrand competition for the low-cost customer would keep driving the two providers' prices down to $p_H = q_H - c$ and $p_L = q_L - c$, at which point each provider would have an incentive to raise his price and serve the high-cost customer instead. As a result, there is no pure-strategy pricing equilibrium and the following result characterizes the mixed-strategy equilibrium (see proof in the Online Appendix).

Proposition 6. *There is a unique equilibrium in which both providers use a mixed pricing strategy. The equilibrium price distribution for provider $i \in \{H, L\}$ is*

$$F_i(p_i) = \frac{p_i + c - q_i}{p_i - q_i + q_j - \max\{p_i - q_i + q_j - c, 0\}}, \text{ for } p_i \in [q_i - c, q_i] \text{ and } i \neq j \in \{H, L\}. \quad (14)$$

Correspondingly, provider i's expected equilibrium profit is $q_i - c$.

When $q_L \geq 2c$, we have complete market coverage as the high-cost customer is always served. In fact, we can obtain $F_i(p_i) = 1 - (q_i - p_i)/c$ in this case, which means that both providers' prices follow uniform distributions. Since $1 - F_H(p) = (q_H - p)/c \geq (q_L - p)/c = 1 - F_L(p)$, the high-quality provider charges a higher price than the low-quality provider in the sense of first-order stochastic dominance.

On the other hand, when $q_L < 2c$, the high-cost customer may get rejected by the low-quality provider at certain price levels, similar to the incomplete market coverage case in Section 4. Through equation (14), we can show that we no longer have the first-order stochastic dominance and similar to what we find in Section 4, the high-quality provider's price may be lower than that of the low-quality provider.

6.2 Multiple Applications

In this section, we consider the possibility that a customer can submit multiple applications. This introduces new complexities when a provider decides which customer to make an offer to upon receiving multiple applications, because he expects that a low-cost customer may turn his offer down if the customer has received multiple offers. In the extreme case where a customer can freely submit an application to every provider, the model in essence would turn into one of *competitive auctions*. We do not expect all of our findings to extend to this setting. As each customer applies to every provider, there is no incentive for high-quality providers to set a low price to attract applications from low-cost customers; instead, these providers can make an offer to low-cost customers directly. In some freelance P2P marketplaces that are more aligned with the competitive auction model, casual observations suggest that high-quality providers ("experts") tend to charge a higher price than the low-quality ones ("beginners") in general (Ke and Zhu 2021).

What would happen in less extreme cases? We examine here a case in which a customer can submit two applications (see analysis in the Online Appendix) and find that a pure-strategy equilibrium does not exist. While mixed-strategy equilibrium outcomes make comparison to the main model difficult, our analysis suggests that in the revised game, high-quality providers still have incentives to lower their prices to attract low-cost customers (in particular, those with $\theta = 0$) to apply. In equilibrium, the lowest-cost customers would always submit one of their two applications to the provider with the highest potential payoff, which in turn gives providers incentives to compete for such customers. The intuition is hence broadly consistent with the endogenous composition effect in our main model.

6.3 Alternative Distribution of Customer Types

To demonstrate that our results do not depend on the customer cost type being uniformly distributed, we numerically solve for the equilibrium under a generalized distribution function $F(\theta) = (1-\alpha)\theta/\bar{\theta}+\alpha(\theta/\bar{\theta})^2$, $|\alpha| < 1$. When $\alpha = 0$, $F(\cdot)$ reverts back to uniform distribution. The probability density function, $F'(\theta) = (1-\alpha)/\bar{\theta} + 2\alpha\theta/\bar{\theta}^2$ increases with θ for $\alpha > 0$ and decreases with θ for $\alpha < 0$, allowing for the possibility of either more high-cost or more low-cost customers in the market.

We are able to numerically verify that in equilibrium, $q_H - p_H^* \geq q_L - p_L^*$ for all $|\alpha| < 1$. Figure 11 plots the comparative statics of equilibrium prices with respect to $\bar{\theta}$ under $\alpha = \pm 0.5$. As before, equilibrium prices decrease when service cost increases , which points again to the importance of the endogenous composition effect.

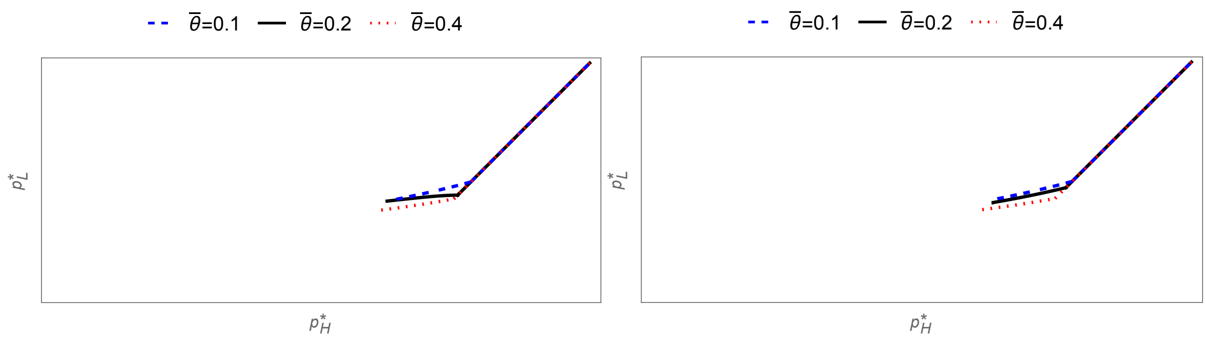

Figure 11: Equilibrium Prices with Non-Uniformly Distributed Customers under the Parameter Setting with $\alpha = 0.5$ (left) and $\alpha = -0.5$ (right).

6.4 Alternative Service Cost Structure

We now explore the possibility that the cost to serve the same customer is lower for a high-quality provider: $g(q) = 1/q$. This could be due to the provider being more experienced and hence more efficient at providing the service. Under complete market coverage, Figure 12 presents equilibrium prices and comparative statics with respect to the highest service cost $\bar{\theta}$. Similar as in the main model, a higher $\bar{\theta}$ lowers equilibrium prices due to the endogenous composition effect.

Under incomplete market coverage, if an equilibrium exists where the market coverage is incomplete for high-quality providers and complete for low-quality providers, we can show

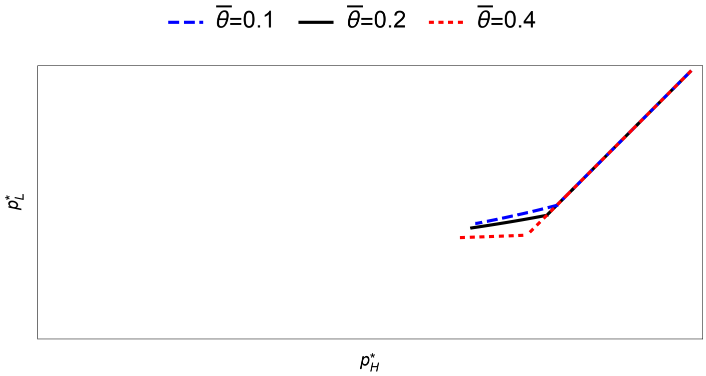

Figure 12: Equilibrium Prices under the Parameter Setting Except for $g(q) = 1/q$.

that the high-quality providers would charge a lower equilibrium price than the low-quality ones.[19] However, we are unable to identify a numerical example, which suggests that such equilibria may not exist. Intuitively, as high-quality providers already incur lower service cost for the same customer than low-quality providers, their incentive to compete for low-cost customers weakens.

7 Conclusion

This paper considers a peer-to-peer market that matches capacity-constrained providers with customers. We explore how bilateral ratings, which reveal providers' service quality and customers' service costs, influence price competition and market segmentation. We identify a unique endogenous composition effect: a provider can attract certain customers to apply to him by varying his price. This effect leads to important and novel patterns in our analysis. First, a higher average service cost or a higher platform commission fee may lead to lower, instead of, higher equilibrium prices as providers compete more aggressively for low-cost customers. A platform hence needs to consider how the lower prices may affect its profit when raising the commission rate. Second, a high-quality provider may charge a lower, instead of higher, equilibrium price than a low-quality provider. Finally, compared with unilateral ratings, bilateral ratings may soften provider competition and lead to higher equilibrium prices.

As a final remark, it is worth noting that in our model, market frictions come not from agents' search costs but rather from coordination frictions—more than one customer may apply to the same provider, and some providers may receive no application. We believe that such coordination frictions are significant in P2P markets in the real world: providers often end up with no customers at certain time points and customers also exit the market if they find it too hard to find a matching provider.

If we allow unmatched agents to play the same matching game again, some of them would get matched. In theory, we can extend the current one-shot game into multiple rounds, and that would reduce coordination frictions. While a formal analysis is beyond the scope of the current paper, we offer some intuition here for a two-round game. The possibility for

[19] Equation (7) implies that $p_H q_H < (1-\delta)\bar{\theta} < p_L q_L$, which further implies that $p_H < p_L$ given $q_H > q_L$.

an unmatched provider to get matched in the second period would serve as an outside option for the provider, which in the first period reduces his incentive to price low to attract customers. This effect is weaker for high-quality providers than for low-quality ones, as the former are more likely to get matched in the first round and do not value the outside option as much. Therefore, the price increase for high-quality providers would be smaller than that for low-quality ones. As a result, if high-quality providers are already charging a lower price than low-quality providers in a one-shot game, the price gap may widen in a two-round game. There is, however, an opposite force at work too. Customers are now less worried about being rejected in the first period because they have a second chance. They are more likely to apply to high-quality providers in the first period, which reduces price competition among these providers, leading to higher prices. This effect would narrow the aforementioned price gap, making the total effect ambiguous. It may be interesting for future studies to investigate the dynamic matching process and characterize the trade-off between these two opposing effects.

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Appendix

Proof of Proposition 1:

Proof. In mathematical terms, we write the proposition as follows. Suppose $q_j - p_j > 0$ for $j \in \{H, L\}$. Define

$$\theta_H \equiv F^{-1}\left[\frac{\gamma}{n}\ln\left(\frac{q_H - p_H}{q_L - p_L}\right)\right] \text{ and } \theta_L \equiv F^{-1}\left[\frac{1-\gamma}{n}\ln\left(\frac{q_L - p_L}{q_H - p_H}\right)\right].$$

1. If $\frac{q_H - p_H}{q_L - p_L} \leq e^{-\frac{n}{1-\gamma}}$, all customers apply to providers of type L with $x_H(\theta) = 0$ and $x_L(\theta) = \frac{n}{1-\gamma} f(\theta)$ for $\theta \in \left[0, \bar{\theta}\right]$.
2. If $e^{-\frac{n}{1-\gamma}} < \frac{q_H - p_H}{q_L - p_L} < 1$, customers with type $\theta \in [0, \theta_L]$ apply to providers of type L with $x_H(\theta) = 0$ and $x_L(\theta) = \frac{n}{1-\gamma} f(\theta)$, and customers with $\theta \in \left(\theta_L, \bar{\theta}\right]$ apply to both types of providers with $x_H(\theta) = x_L(\theta) = nf(\theta)$.
3. If $\frac{q_H - p_H}{q_L - p_L} = 1$, all customers apply to both types of providers with $x_H(\theta) = x_L(\theta) = nf(\theta)$ for $\theta \in \left[0, \bar{\theta}\right]$.
4. If $1 < \frac{q_H - p_H}{q_L - p_L} < e^{\frac{n}{\gamma}}$, customers with type $\theta \in [0, \theta_H]$ apply to providers of type H only with $x_H(\theta) = \frac{n}{\gamma} f(\theta)$ and $x_L(\theta) = 0$, and customers with $\theta \in \left(\theta_H, \bar{\theta}\right]$ apply to both types of providers with $x_H(\theta) = x_L(\theta) = nf(\theta)$.
5. If $\frac{q_H - p_H}{q_L - p_L} \geq e^{\frac{n}{\gamma}}$, all customers apply to providers of type H with $x_H(\theta) = \frac{n}{\gamma} f(\theta)$ and $x_L(\theta) = 0$ for $\theta \in \left[0, \bar{\theta}\right]$.

Correspondingly,

$$U(\theta) = \begin{cases} (q_L - p_L)e^{-\frac{n}{1-\gamma}F(\theta)}, & \text{if } \dfrac{q_H - p_H}{q_L - p_L} < 1 \text{ and } 0 \leq \theta \leq \min\left\{\theta_L, \bar{\theta}\right\}, \\ (q_L - p_L)\left(\dfrac{q_H - p_H}{q_L - p_L}\right)^{\gamma} e^{-nF(\theta)}, & \text{if } e^{-\frac{n}{1-\gamma}} < \dfrac{q_H - p_H}{q_L - p_L} < 1 \text{ and } \theta_L < \theta \leq \bar{\theta}, \\ (q_H - p_H)e^{-nF(\theta)}, & \text{if } \dfrac{q_H - p_H}{q_L - p_L} = 1, \\ (q_H - p_H)\left(\dfrac{q_H - p_H}{q_L - p_L}\right)^{-(1-\gamma)} e^{-nF(\theta)}, & \text{if } 1 < \dfrac{q_H - p_H}{q_L - p_L} < e^{\frac{n}{\gamma}} \text{ and } \theta_H < \theta \leq \bar{\theta}, \\ (q_H - p_H)e^{-\frac{n}{\gamma}F(\theta)}, & \text{if } \dfrac{q_H - p_H}{q_L - p_L} > 1 \text{ and } 0 \leq \theta \leq \min\left\{\theta_H, \bar{\theta}\right\}. \end{cases}$$

We start the proof by first proving two lemmas.

Lemma 1. *Suppose $q_L - p_L > 0$. If there exists $\theta' \in \left[0, \bar{\theta}\right]$ such that $x_H(\theta') > 0$ and $x_L(\theta') = 0$, then we have that $x_H(\theta) > 0$ and $x_L(\theta) = 0$ for $\forall \theta \in [0, \theta']$.*

Proof. The lemma is obviously true for $\theta' = 0$. For $\theta' > 0$, we prove the lemma by contradiction. Suppose there exists $\theta' \in \left(0, \bar{\theta}\right]$ such that $x_H(\theta') > 0$ and $x_L(\theta') = 0$, and there exists $\theta'' \in [0, \theta')$ such that $x_H(\theta'') = 0$ or $x_L(\theta'') > 0$. By equation (1) and $f(\cdot) > 0$, the condition $x_H(\theta'') = 0$ or $x_L(\theta'') > 0$ is equivalent to $x_L(\theta'') > 0$.

Based on the main text, it is easy to see that customers' application strategy must satisfy that

$$x_j(\theta) = \begin{cases} \in (0, \infty), & \text{if } b_j(\theta)(q_j - p_j) = U(\theta); \\ 0, & \text{if } b_j(\theta)(q_j - p_j) < U(\theta). \end{cases} \tag{15}$$

According to equation (15), the conditions $x_H(\theta') > 0$ and $x_L(\theta') = 0$ imply that

$$\begin{aligned} U(\theta') = e^{-\int_0^{\theta'} x_H(t)\mathrm{d}t}(q_H - p_H) > e^{-\int_0^{\theta'} x_L(t)\mathrm{d}t}(q_L - p_L), \\ \text{i.e., } e^{\int_0^{\theta'}(x_H(t) - x_L(t))\mathrm{d}t} < \frac{q_H - p_H}{q_L - p_L}. \end{aligned} \tag{16}$$

Similarly, by applying equation (15) to the condition $x_L(\theta'') > 0$, we have that

$$\begin{aligned} U(\theta'') = e^{-\int_0^{\theta''} x_L(t)\mathrm{d}t}(q_L - p_L) \geq e^{-\int_0^{\theta''} x_H(t)\mathrm{d}t}(q_H - p_H), \\ \text{i.e., } e^{\int_0^{\theta''}(x_H(t) - x_L(t))\mathrm{d}t} \geq \frac{q_H - p_H}{q_L - p_L}. \end{aligned} \tag{17}$$

Combining equations (16) and (17), we have that

$$\begin{aligned} e^{\int_0^{\theta''}(x_H(t) - x_L(t))\mathrm{d}t} \geq \frac{q_H - p_H}{q_L - p_L} > e^{\int_0^{\theta'}(x_H(t) - x_L(t))\mathrm{d}t}, \\ \text{i.e., } \int_{\theta''}^{\theta'} (x_H(t) - x_L(t))\,\mathrm{d}t < 0. \end{aligned} \tag{18}$$

Meanwhile, by the normalization condition in equation (1), we know that $x_H(\theta') - x_L(\theta') = \frac{n}{\gamma} f(\theta') > 0$. Since $x_H(\theta)$ and $x_L(\theta)$ are both piecewise continuous, $x_H(\theta) - x_L(\theta)$ is also piecewise continuous. Thus, there exists a neighborhood around θ' such that for all θ within the neighborhood, $x_H(\theta) - x_L(\theta) > \frac{n}{2\gamma} f(\theta') > 0$. Without loss of generality, assume that $x_H(\theta) - x_L(\theta)$ is left-continuous, so the neighborhood takes the form of $[\theta' - \varepsilon, \theta']$, where $\varepsilon > 0$. Similarly, if $x_H(\theta) - x_L(\theta)$ is right-continuous instead, the neighborhood takes the form of $[\theta', \theta' + \varepsilon]$, under which case we can redefine θ' as $\theta' + \varepsilon$ for the following discussion.

To summarize, we have shown that $x_H(\theta) - x_L(\theta) > \frac{n}{2\gamma} f(\theta') > 0$ for $\theta \in [\theta' - \varepsilon, \theta']$. Now we can rewrite the inequality (18) as follows,

$$\begin{aligned} 0 > \int_{\theta''}^{\theta'} (x_H(t) - x_L(t))\,\mathrm{d}t &= \int_{\theta''}^{\theta' - \varepsilon} (x_H(t) - x_L(t))\,\mathrm{d}t + \int_{\theta' - \varepsilon}^{\theta'} (x_H(t) - x_L(t))\,\mathrm{d}t \\ &> \int_{\theta''}^{\theta' - \varepsilon} (x_H(t) - x_L(t))\,\mathrm{d}t + \frac{n}{2\gamma} f(\theta')\varepsilon > 0, \end{aligned}$$

which is a contradiction. The last inequality above is due to the fact that inequality (18) is valid for any θ' and θ'' that satisfy their definitions, so we can let θ'' and $\theta' - \varepsilon$ be infinitely close to each other, and $\int_{\theta''}^{\theta' - \varepsilon} (x_H(t) - x_L(t))\,\mathrm{d}t$ infinitely close to zero. Therefore, we have proved the original statement in Lemma 1. □

With the same logic, we can show the following lemma, whose proof is omitted.

Lemma 2. *Suppose $q_H - p_H > 0$. If there exists $\theta' \in [0, \bar{\theta}]$ such that $x_L(\theta') > 0$ and $x_H(\theta') = 0$,* *then we have $x_L(\theta) > 0$ and $x_H(\theta) = 0$ for $\forall\theta \in [0, \theta']$.*

We next prove Proposition 1. Consider first the case $q_H - p_H \geq q_L - p_L > 0$. By the normalization condition in equation (1), we have $0 \leq x_H(\theta) \leq \frac{n}{\gamma} f(\theta)$ and $0 \leq x_L(\theta) \leq \frac{n}{1-\gamma} f(\theta)$. This implies that

$$\begin{aligned}
b_H(\theta)(q_H - p_H) &= e^{-\int_0^\theta x_H(t)\mathrm{d}t}(q_H - p_H) \\
&\geq e^{-\int_0^\theta \frac{n}{\gamma} f(t)\mathrm{d}t}(q_H - p_H) \\
&= e^{-\frac{n}{\gamma}F(\theta)}(q_H - p_H) \\
&\geq e^{-\frac{n}{\gamma}F(\theta)} \frac{q_H - p_H}{q_L - p_L}(q_L - p_L) e^{-\int_0^\theta x_L(t)\mathrm{d}t} \\
&= e^{-\frac{n}{\gamma}F(\theta)} \frac{q_H - p_H}{q_L - p_L} \times b_L(\theta)(q_L - p_L) \\
&> b_L(\theta)(q_L - p_L), \text{ when } \theta < \theta_H.
\end{aligned}$$

By equation (15), the above inequality implies that $x_H(\theta) = \frac{n}{\gamma} f(\theta)$ and $x_L(\theta) = 0$ for $\theta \in [0, \theta_H)$. If $\theta_H \geq \bar{\theta}$, we have effectively determined $x_j(\theta)$ ($j \in \{H, L\}$) for all $\theta \in \left[0, \bar{\theta}\right]$. If $\theta_H < \bar{\theta}$, we still need to determine $x_j(\theta)$ ($j \in \{H, L\}$) for $\theta \in \left[\theta_H, \bar{\theta}\right]$. For $\theta_H < \bar{\theta}$, we pin down $x_j(\theta)$ ($j \in \{H, L\}$) for $\theta \in \left(\theta_H, \bar{\theta}\right]$ first and then determine $x_j(\theta_H)$ ($j \in \{H, L\}$).

First, we know that it is impossible that $x_L(\theta) > 0$ and $x_H(\theta) = 0$ for $\theta \in \left(\theta_H, \bar{\theta}\right]$, because by Lemma 2 this would imply $x_L(\theta) > 0$ and $x_H(\theta) = 0$ for $\theta \in [0, \theta_H)$, which is a contradiction. Second, we also know that it is impossible that $x_H(\theta) > 0$ and $x_L(\theta) = 0$ for $\theta \in \left(\theta_H, \bar{\theta}\right]$, because by Lemma 1 this would imply that for $\theta \in \left(\theta_H, \bar{\theta}\right]$,

$$b_H(\theta)(q_H - p_H) = e^{-\int_0^\theta \frac{n}{\gamma} f(t)\mathrm{d}t}(q_H - p_H) = e^{-\frac{n}{\gamma}F(\theta)} \frac{q_H - p_H}{q_L - p_L}(q_L - p_L) < (q_L - p_L) = b_L(\theta)(q_L - p_L),$$

which is a contradiction. Therefore, we must have $x_H(\theta) > 0$ and $x_L(\theta) > 0$ for $\theta \in \left(\theta_H, \bar{\theta}\right]$. By equation (15), this implies that

$$\begin{aligned}
e^{-\int_0^\theta x_L(t)\mathrm{d}t}(q_L - p_L) &= e^{-\int_0^\theta x_H(t)\mathrm{d}t}(q_H - p_H), \\
\text{i.e., } e^{-\int_{\theta_H}^\theta x_L(t)\mathrm{d}t}(q_L - p_L) &= e^{-\int_0^{\theta_H} x_H(t)\mathrm{d}t} e^{-\int_{\theta_H}^\theta x_H(t)\mathrm{d}t}(q_H - p_H), \\
\text{i.e., } e^{-\int_{\theta_H}^\theta x_L(t)\mathrm{d}t} &= e^{-\int_{\theta_H}^\theta x_H(t)\mathrm{d}t}, \\
\text{i.e., } \int_{\theta_H}^\theta \left(x_H(t) - x_L(t)\right)\mathrm{d}t &= 0.
\end{aligned}$$

Notice that for the equality above to be valid for $\forall\theta \in \left(\theta_H, \bar{\theta}\right]$, we must have $x_H(\theta) = x_L(\theta)$ for $\forall\theta \in \left(\theta_H, \bar{\theta}\right]$. By the piece-wise continuity of $x_H(\theta)$ and $x_L(\theta)$, we must also have that $x_H(\theta_H) = x_L(\theta_H)$. By the normalization condition in equation (1), we then have $x_H(\theta) = x_L(\theta) = nf(\theta)$ for $\forall\theta \in \left[\theta_H, \bar{\theta}\right]$.

We have completely proved the proposition for the case $q_H - p_H \geq q_L - p_L > 0$ above. The proof for the other case with $q_L - p_L \geq q_H - p_H > 0$ follows the same logic and is thus omitted. Finally, $U(\theta)$ can be calculated by equation (15) given $x_H(\theta)$ and $x_L(\theta)$. □

Proof of Proposition 2:

Proof. The expression of $\bar{\varepsilon}$ is

$$\bar{\varepsilon} = \min_{j \in \{H,L\}} e^{-n} q_j + \frac{1-(n+1)e^{-n}}{(1-\delta)n} \bar{\theta} g(q_j) \geq \frac{\bar{\theta} q(g_L)}{2(1-\delta)n} > 0.$$

To start the proof, we calculate $\pi_j^0(p_j^0; p_H, p_L)$ based on equation (4) as follows,

$$\begin{aligned}
\pi_j^0(p_j^0; p_H, p_L) &= \left[(1-\delta)p_j^0 - g(q_j)\theta^0\right] b_j^0(\theta^0) - \left[(1-\delta)p_j^0 - g(q_j)\bar{\theta}\right] b_j^0(\bar{\theta}) - g(q_j) \int_{\theta^0}^{\bar{\theta}} b_j^0(\theta) \mathrm{d}\theta \\
&= \begin{cases}
-(1-\delta)\left[U(0) - U(\bar{\theta})\right] - \dfrac{\int_0^{\bar{\theta}} \left[(1-\delta)q_j - g(q_j)\theta\right] U'(\theta) d\theta}{q_j - p_j^0}, & \text{if } p_j^0 < q_j - U(0); \\
(1-\delta)U(\bar{\theta}) + (1-\delta)p_j^0 - g(q_j)U^{-1}(q_j - p_j^0) & \\
\quad - \dfrac{(1-\delta)U(\bar{\theta})q_j + g(q_j)\left[\int_{U^{-1}(q_j - p_j^0)}^{\bar{\theta}} U(\theta)\mathrm{d}\theta - \bar{\theta}U(\bar{\theta})\right]}{q_j - p_j^0}, & \text{otherwise.}
\end{cases}
\end{aligned} \tag{19}$$

The first equality above is due to integration by part and rearrangement of terms; to get the second equality, we use $b_j^0(\theta)$ in equation (3). Notice from equation (19) that $\pi_j^0(p_j^0; p_H, p_L)$ increases with p_j^0 when $p_j^0 < q_j - U(0)$. This is because $U'(\theta) < 0$ and $(1-\delta)q_j - g(q_j)\theta > (1-\delta)p_j^0 - g(q_j)\bar{\theta} \geq 0$. Therefore, we only need to consider the case $p_j^0 \geq q_j - U(0)$. Correspondingly, $\pi_j^0(p_j^0; p_H, p_L)$ is given by the second case in equation (19).

The solution to the optimization problem in equation (5) must be either a corner solution with $p_j^0 = q_j - U(0)$ or an interior solution with $p_j^0 > q_j - U(0)$. Notice that the corner solution of $p_j^0 = q_j - U(0)$ is possible, because $\pi_j^0(p_j^0; p_H, p_L)$ is continuous at $p_j^0 = q_j - U(0)$ and $\partial_{p_j^0} \pi_j^0(p_j^0; p_H, p_L)$ jumps by $-(1-\delta)U(0)p_j^0/(q_j - p_j^0)^2 < 0$ when p_j^0 increases from $(q_j - U(0))^-$ to $(q_j - U(0))^+$. Meanwhile, from Proposition 1, we know that $U(0) = \max\{q_H - p_H, q_L - p_L\}$.

There are three cases to consider. First, suppose in equilibrium, $q_H - p_H^* > q_L - p_L^*$, which immediately implies that $U(0) = q_H - p_H^*$, or equivalently, $p_H^* = q_H - U(0)$. This implies that the maximizing point of $\pi_H^0(p_H^0; p_H, p_L)$ must be the corner solution of $p_H^0 = p_H^* = q_H - U(0)$. Correspondingly, the first-order optimality condition for the corner solution is $\partial_{p_H^0} \pi_H^0(p_H^0; p_H, p_L)|_{p_H^0 = p_H^*} \leq 0$. Moreover, we have $p_L^* > q_L - (q_H - p_H^*) = q_L - U(0)$, which implies that the maximizing point of $\pi_L^0(p_L^0; p_H, p_L)$ must be the interior solution of $p_L^0 = p_L^* > q_L - U(0)$. Correspondingly, the first-order condition for the interior solution is $\partial_{p_L^0} \pi_L^0(p_L^0; p_H, p_L)|_{p_L^0 = p_L^*} = 0$. Similarly, we can analyze the other two cases with $q_H - p_H^* < q_L - p_L^*$ and $q_H - p_H^* = q_L - p_L^*$. The first-order condition for the optimization problem in equation (5) is

$$\begin{cases}
q_H - p_H^* < q_L - p_L^* \\
(1-\delta)(q_H - p_H^*)^2 - \left[(1-\delta)q_H - g(q_H)\bar{\theta}\right] U(\bar{\theta}) - g(q_H) \displaystyle\int_{\theta_L}^{\bar{\theta}} U(\theta)\mathrm{d}\theta = 0 \\
(1-\delta)(q_L - p_L^*)^2 - \left[(1-\delta)q_L - g(q_L)\bar{\theta}\right] U(\bar{\theta}) - g(q_L) \displaystyle\int_0^{\bar{\theta}} U(\theta)\mathrm{d}\theta \leq 0
\end{cases}, \text{ or} \tag{20}$$

$$\begin{cases} q_H - p_H^* = q_L - p_L^* \\ (1-\delta)(q_H - p_H^*)^2 - \left[(1-\delta)q_H - g(q_H)\bar{\theta}\right] U(\bar{\theta}) - g(q_H) \int_0^{\bar{\theta}} U(\theta)\mathrm{d}\theta \le 0 \\ (1-\delta)(q_L - p_L^*)^2 - \left[(1-\delta)q_L - g(q_L)\bar{\theta}\right] U(\bar{\theta}) - g(q_L) \int_0^{\bar{\theta}} U(\theta)\mathrm{d}\theta \le 0 \end{cases}, \text{ or} \tag{21}$$

$$\begin{cases} q_H - p_H^* > q_L - p_L^* \\ (1-\delta)(q_H - p_H^*)^2 - \left[(1-\delta)q_H - g(q_H)\bar{\theta}\right] U(\bar{\theta}) - g(q_H) \int_0^{\bar{\theta}} U(\theta)\mathrm{d}\theta \le 0 \\ (1-\delta)(q_L - p_L^*)^2 - \left[(1-\delta)q_L - g(q_L)\bar{\theta}\right] U(\bar{\theta}) - g(q_L) \int_{\theta_H}^{\bar{\theta}} U(\theta)\mathrm{d}\theta = 0 \end{cases}. \tag{22}$$

$U(\theta)$ in equations (20), (21) and (22) is given in cases 2, 3, and 4 respectively at the beginning of the proof of Proposition 1. Cases 1 and 5 never occur in equilibrium, because all customers apply only to one type of providers in these cases. Consequently, the other type of providers get zero profit. It is of each of these individual providers' interest to decrease his price until either case 2 or 4 is holding. In writing down equations (20)-(22), we utilize the expressions of $U(\theta)$ in the proof of Proposition 1. Particularly, in equation (20), $U^{-1}(q_L - p_L^*) = \theta_H$ according to case 2 in the proof, and in equation (22), $U^{-1}(q_H - p_H^*) = \theta_L$ according to case 4.

Next, we consider the second-order optimality condition. We require $\pi_j^0(p_j^0; p_H, p_L)$ in equation (19) to be concave in $p_j^0 \in [q_j - U(0), q_j]$, which holds if and only if for $j \in \{H, L\}$,

$$\begin{aligned} nf(\theta) &\ge \left\{ \begin{array}{ll} \dfrac{(1-\gamma)U(\theta)}{2\left[\left(\frac{(1-\delta)q_j}{g(q_j)} - \bar{\theta}\right) U(\bar{\theta}) + \int_\theta^{\bar{\theta}} U(t)\mathrm{d}t\right]}, & \text{for } \forall\theta \in [0, \theta_L] \\ \dfrac{U(\theta)}{2\left[\left(\frac{(1-\delta)q_j}{g(q_j)} - \bar{\theta}\right) U(\bar{\theta}) + \int_\theta^{\bar{\theta}} U(t)\mathrm{d}t\right]}, & \text{for } \forall\theta \in [\theta_L, \bar{\theta}] \end{array} \right\} \text{if } q_H - p_H \le q_L - p_L; \\ nf(\theta) &\ge \left\{ \begin{array}{ll} \dfrac{\gamma U(\theta)}{2\left[\left(\frac{(1-\delta)q_j}{g(q_j)} - \bar{\theta}\right) U(\bar{\theta}) + \int_\theta^{\bar{\theta}} U(t)\mathrm{d}t\right]}, & \text{for } \forall\theta \in [0, \theta_H] \\ \dfrac{U(\theta)}{2\left[\left(\frac{(1-\delta)q_j}{g(q_j)} - \bar{\theta}\right) U(\bar{\theta}) + \int_\theta^{\bar{\theta}} U(t)\mathrm{d}t\right]}, & \text{for } \forall\theta \in [\theta_H, \bar{\theta}] \end{array} \right\} \text{otherwise.} \end{aligned} \tag{23}$$

To see where equation (23) comes from, notice that for $p_j^0 \in [q_j - U(0), q_j]$, $\pi_j^0(p_j^0; p_H, p_L)$ is given by the second case in equation (19). $\pi_j^0(p_j^0; p_H, p_L)$ is concave if and only if

$$\begin{aligned} \frac{\partial \pi_j^0(p_j^0; p_H, p_L)}{\partial (p_j^0)^2} = -\frac{1}{(q_j - p_j^0)^3} \Bigg\{ & 2\left[(1-\delta)q_j - \bar{\theta} g(g_j)\right] U(\bar{\theta}) \\ & + g(q_j) \left[2\int_{U^{-1}(q_j - p_j^0)}^{\bar{\theta}} U(\theta)\mathrm{d}\theta + \frac{(q_j - p_j^0)^2}{U'\left(U^{-1}(q_j - p_j^0)\right)} \right] \Bigg\} \le 0. \end{aligned} \tag{24}$$

From equation (15), we know that $U(\theta) = (q_{j^*} - p_{j^*})e^{-\int_0^\theta x_{j^*}(t)\mathrm{d}t}$, where $j^* \in \{H, L\}$ is defined by $x_{j^*} > 0$. Therefore, $U'(\theta) = -U(\theta)x_{j^*}(\theta)$. By substituting this equality back to inequality

(24), we can rearrange and rewrite (24) as

$$x_{j^*}\left(U^{-1}(q_j-p_j^0)\right) \geq \frac{(q_j-p_j^0)}{2\left[\left(\frac{(1-\delta)q_j}{g(q_j)}-\bar{\theta}\right)U(\bar{\theta})+\int_{U^{-1}(q_j-p_j^0)}^{\bar{\theta}}U(t)\mathrm{d}t\right]}.$$

Denote $\theta = U^{-1}(q_j - p_j^0)$, which ranges in $[0,\bar{\theta}]$. Notice that x_{j^*} is given in the proof of Proposition 1 and we can rewrite the inequality above as inequality (23).

With the first- and second-order conditions above, we are ready to prove the proposition. To show the existence of Nash equilibria with $q_H - p_H^* = q_L - p_L^* = \varepsilon \in (0, \bar{\varepsilon}]$, we need to show that condition (21) is satisfied for $\varepsilon \in (0,\bar{\varepsilon}]$. In fact, condition (21) can be simplified as

$$\begin{cases} q_H - p_H^* = q_L - p_L^* = \varepsilon, \\ (1-\delta)\varepsilon - \dfrac{1}{n}\bar{\theta}g(q_j) - e^{-n}\left[(1-\delta)q_j - \dfrac{n+1}{n}\bar{\theta}g(q_j)\right] \leq 0,\ j \in \{H, L\}, \end{cases}$$

which implies that $\varepsilon \leq \bar{\varepsilon}$. To show that $\bar{\varepsilon} \geq \frac{\bar{\theta}g(g_L)}{2(1-\delta)n} > 0$, we can set $\theta = 0$ and $q_H - p_H^* = q_L - p_L^*$ in concavity condition (23).

To show that in equilibrium, $q_H - p_H^* \geq q_L - p_L^*$, we need to show that equation (20) never holds in equilibrium given condition (23).

By substituting the expression of $U(\theta)$ from the second case in the proof of Proposition 1, the above statement can be equivalently written as:

$$\text{If}\begin{cases} q_H - p_H^* < q_L - p_L^* \\ (1-\delta)(q_H - p_H^*) - \dfrac{1}{n}g(q_H)\bar{\theta} - \left[(1-\delta)q_H - \dfrac{n+1}{n}g(q_H)\bar{\theta}\right]e^{-n}\left(\dfrac{q_H-p_H^*}{q_L-p_L^*}\right)^{-(1-\gamma)} = 0 \end{cases},$$

$$\text{then } (1-\delta)(q_L-p_L^*) - \frac{1}{n}g(q_L)\bar{\theta}\left[(1-\gamma)+\gamma\left(\frac{q_H-p_H^*}{q_L-p_L^*}\right)\right]$$
$$- \left[(1-\delta)q_L - \left(1+\frac{1}{n}\right)g(q_L)\bar{\theta}\right]e^{-n}\left(\frac{q_H-p_H^*}{q_L-p_L^*}\right)^{\gamma} > 0.$$

In fact, we have that

$$\begin{aligned}
&(1-\delta)(q_L-p_L^*) - \frac{1}{n}g(q_L)\bar{\theta}\left[(1-\gamma)+\gamma\left(\frac{q_H-p_H^*}{q_L-p_L^*}\right)\right] \\
&- \left[(1-\delta)q_L - \left(1+\frac{1}{n}\right)g(q_L)\bar{\theta}\right]e^{-n}\left(\frac{q_H-p_H^*}{q_L-p_L^*}\right)^{\gamma} \\
=&\left(\frac{q_L-p_L^*}{q_H-p_H^*}\right)\left\{(1-\delta)(q_H-p_H^*) - \frac{1}{n}g(q_L)\bar{\theta}\left[(1-\gamma)+\gamma\left(\frac{q_H-p_H^*}{q_L-p_L^*}\right)\right]\left(\frac{q_H-p_H^*}{q_L-p_L^*}\right)\right. \\
&\left.+\frac{1}{2n}g(q_L)\bar{\theta}e^{-n}\left(\frac{q_H-p_H^*}{q_L-p_L^*}\right)^{\gamma+1} - \left[(1-\delta)q_L - \left(1+\frac{1}{2n}\right)g(q_L)\bar{\theta}\right]e^{-n}\left(\frac{q_H-p_H^*}{q_L-p_L^*}\right)^{\gamma+1}\right\} \\
\geq&\left(\frac{q_L-p_L^*}{q_H-p_H^*}\right)\left\{(1-\delta)(q_H-p_H^*) - \frac{1}{n}g(q_L)\bar{\theta}\left[(1-\gamma)+\gamma\left(\frac{q_H-p_H^*}{q_L-p_L^*}\right)\right]\left(\frac{q_H-p_H^*}{q_L-p_L^*}\right)\right. \\
&\left.+\frac{1}{2n}g(q_L)\bar{\theta}e^{-n}\left(\frac{q_H-p_H^*}{q_L-p_L^*}\right)^{\gamma+1} - \left[(1-\delta)q_H - \left(1+\frac{1}{2n}\right)g(q_H)\bar{\theta}\right]e^{-n}\left(\frac{q_H-p_H^*}{q_L-p_L^*}\right)^{\gamma+1}\right\}
\end{aligned} \tag{25}$$

$$
\begin{aligned}
&= \left(\frac{q_L - p_L^*}{q_H - p_H^*}\right) \left\{ (1-\delta)(q_H - p_H^*) - \frac{1}{n} g(q_L)\bar{\theta} \left[(1-\gamma) + \gamma \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right) \right] \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right) \right. \\
&+ \frac{1}{2n} g(q_L)\bar{\theta} e^{-n} \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma+1} - \left[(1-\delta) q_H - \left(1 + \frac{1}{2n}\right) g(q_H)\bar{\theta} \right] e^{-n} \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma+1} \\
&- \left. \left[(1-\delta)(q_H - p_H^*) - \frac{1}{n} g(q_H)\bar{\theta} - \left[(1-\delta) q_H - \left(1 + \frac{1}{n}\right) g(q_H)\bar{\theta} \right] e^{-n} \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{-(1-\gamma)} \right] \right\} \\
&= \left(\frac{q_L - p_L^*}{q_H - p_H^*}\right) \left\{ \frac{1}{n} g(q_H)\bar{\theta} \left[1 - \frac{1}{2} e^{-n} \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma+1} \right] \right. \\
&- \frac{1}{n} g(q_L)\bar{\theta} \left[(1-\gamma) + \gamma \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right) - \frac{1}{2} e^{-n} \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma} \right] \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right) \\
&+ \left. \left[(1-\delta) q_H - \left(1 + \frac{1}{n}\right) g(q_H)\bar{\theta} \right] e^{-n} \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{-(1-\gamma)} \left[1 - \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{2} \right] \right\} \\
&\geq \left(\frac{q_L - p_L^*}{q_H - p_H^*}\right) \left\{ \frac{1}{n} g(q_H)\bar{\theta} \left[1 - \frac{1}{2} e^{-n} \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma+1} \right] \right. \\
&- \frac{1}{n} g(q_L)\bar{\theta} \left[(1-\gamma) + \gamma \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right) - \frac{1}{2} e^{-n} \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma} \right] \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right) \\
&- \left. \frac{1}{2n} g(q_H)\bar{\theta} \left[1 - \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{2} \right] \right\}. \qquad (26)
\end{aligned}
$$

Inequality (25) is due to that

$$
(1-\delta) q_L - \left(1 + \frac{1}{2n}\right) g(q_L)\bar{\theta} \leq (1-\delta) q_H - \left(1 + \frac{1}{2n}\right) g(q_H)\bar{\theta}.
$$

This inequality is obviously true if $g(q) = 1$. To show that it is true if $g(q) = q$, we need to show that $(1-\delta) - \left(1 + \frac{1}{2n}\right)\bar{\theta} \geq 0$. Letting $\theta = \bar{\theta}$ in equation (23), we have that

$$
\frac{n}{\bar{\theta}} \geq \frac{1}{2\left(\frac{(1-\delta)q_j}{g(q_j)} - \bar{\theta}\right)},
$$

which implies that $(1-\delta) - \left(1 + \frac{1}{2n}\right)\bar{\theta} \geq 0$ when $g(q) = q$.

Inequality (26) is due to that

$$
\left[(1-\delta) q_H - \left(1 + \frac{1}{n}\right) g(q_H)\bar{\theta} \right] e^{-n} \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{-(1-\gamma)} \geq -\frac{1}{2n} g(q_H)\bar{\theta}.
$$

This inequality can be obtained by setting $\theta = \theta_L$ and $j = H$ in inequality (23).

To continue the derivation following inequality (26), we have

$$
\begin{aligned}
&\left(\frac{q_L - p_L^*}{q_H - p_H^*}\right) \left\{ \frac{1}{n} g(q_H)\bar{\theta} \left[1 - \frac{1}{2} e^{-n} \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma+1} \right] \right. \\
&- \frac{1}{n} g(q_L)\bar{\theta} \left[(1-\gamma) + \gamma \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right) - \frac{1}{2} e^{-n} \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma} \right] \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right) \\
&- \left. \frac{1}{2n} g(q_H)\bar{\theta} \left[1 - \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{2} \right] \right\}
\end{aligned}
$$

$$
\begin{aligned}
&= \left(\frac{q_L - p_L^*}{q_H - p_H^*}\right) \left\{ \frac{1}{n} g(q_H)\bar{\theta} \left[\frac{1}{2} + \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^2 - \frac{1}{2} e^{-n} \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma+1} \right] \right. \\
&\quad \left. - \frac{1}{n} g(q_L)\bar{\theta} \left[(1-\gamma) + \gamma \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right) - \frac{1}{2} e^{-n} \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma} \right] \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right) \right. \\
&\geq \left(\frac{q_L - p_L^*}{q_H - p_H^*}\right) \left\{ \frac{1}{n} g(q_L)\bar{\theta} \left[\frac{1}{2} + \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^2 - \frac{1}{2} e^{-n} \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma+1} \right] \right. \\
&\quad \left. - \frac{1}{n} g(q_L)\bar{\theta} \left[(1-\gamma) + \gamma \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right) - \frac{1}{2} e^{-n} \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma} \right] \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right) \right. \qquad (27) \\
&= \left(\frac{q_L - p_L^*}{q_H - p_H^*}\right) \frac{1}{2n} g(q_L)\bar{\theta} \left[1 - \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right) \right] \left[1 - \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right) + 2\gamma \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right) \right] > 0. \qquad (28)
\end{aligned}
$$

Inequality (27) is due to $g(q_H) \geq g(q_L)$, and inequality (28) is due to $0 < \frac{q_H - p_H^*}{q_L - p_L^*} < 1$. □

Proof of Proposition 3:

Proof. The proof is straightforward by combining Propositions 1 and 2. We only need to notice that we can unify the two types of price equilibria by observing that when $q_H - p_H = q_L - p_L$, we have $\theta_H = 0$ and $[0, \theta_H) = \emptyset$. □

Proof of Corollary 1:

Proof. Let's consider the case with $0 \leq \theta \leq \theta_H$ first. We have

$$
U(0) - U(\theta) = (q_H - p_H^*) \left(1 - e^{-\frac{n\theta}{\gamma\bar{\theta}}}\right) \geq (q_H - p_H^*) \frac{n\theta}{\gamma\bar{\theta}} > g(q_j)\theta, \text{ if } n > \frac{\gamma g(q_j)\bar{\theta}}{q_H - p_H^*}.
$$

That is, for n sufficiently high, we have $U(0) - U(\theta) > g(q_j)\theta$. The other case with $\theta_H < \theta \leq \bar{\theta}$ can be proved following the same logic. □

Proof of Proposition 4:

Proof. As pointed out in the proof of Proposition 2, we only need to consider the case $p_j^0 \geq q_j - U(0)$, and correspondingly, by equation (19), we have

$$
\begin{aligned}
\pi_j^0(p_j^0; p_H^*, p_L^*) &= (1-\delta)U(\bar{\theta}) + (1-\delta)p_j^0 - g(q_j)U^{-1}(q_j - p_j^0) \\
&\quad - \frac{(1-\delta)U(\bar{\theta})q_j + g(q_j)\left[\int_{U^{-1}(q_j - p_j^0)}^{\bar{\theta}} U(\theta)\mathrm{d}\theta - \bar{\theta}U(\bar{\theta})\right]}{q_j - p_j^0}.
\end{aligned}
$$

Hence,

$$
\frac{\partial \pi_j^0(p_j^0; p_H^*, p_L^*)}{\partial p_j^0} = (1-\delta) - \frac{g(q_j)\int_{U^{-1}(q_j - p_j^0)}^{\bar{\theta}} U(\theta)\mathrm{d}\theta + \left[(1-\delta)q_j - \bar{\theta}g(q_j)\right]U(\bar{\theta})}{\left(q_j - p_j^0\right)^2}.
$$

Notice that $\partial_{p_j^0}\pi_j^0(p_j^0; p_H^*, p_L^*)$ depends on p_H^* and p_L^* only via $U(\cdot)$. Under the market coverage condition, we have $(1-\delta)q_j - \bar{\theta}g(q_j) \geq 0$. It is then obvious that $\partial_{p_j^0}\pi_j^0(p_j^0; p_H^*, p_L^*)$ decreases with

$U(\cdot)$. Meanwhile, Cases 3 and 4 in the proof of Proposition 1 are the only possible equilibria, under which we have:

1. If $\frac{q_H - p_H^*}{q_L - p_L^*} = 1$,
$$U(\theta) = (q_H - p_H^*)e^{-\frac{n\theta}{\bar{\theta}}}.$$

2. If $1 < \frac{q_H - p_H^*}{q_L - p_L^*} < e^{\frac{n}{\gamma}}$,
$$U(\theta) = \begin{cases} (q_H - p_H^*)e^{-\frac{n\theta}{\gamma\bar{\theta}}}, & 0 \le \theta \le \frac{\gamma}{n}\ln\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)\bar{\theta} \\ (q_H - p_H^*)\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{-(1-\gamma)} e^{-\frac{n\theta}{\bar{\theta}}}, & \frac{\gamma}{n}\ln\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)\bar{\theta} < \theta \le \bar{\theta} \end{cases}.$$

It is straightforward to show that $U(\cdot)$ decreases with p_H^* and p_L^*. Therefore, $\partial_{p_j^0}\pi_j^0(p_j^0; p_H^*, p_L^*)$ increases with p_H^* and p_L^*. This proves that the game is supermodular by the definition on page 446 in Vives (2005).

Next, we leverage Result 5 on page 450 in Vives (2005) to prove the comparative statics. We only need to show that given fixed p_j^0, p_H^* and p_L^*, $\partial_{p_j^0}\pi_j^0(p_j^0; p_H^*, p_L^*)|_{p_j^0 = p_j^*}$ (1) increases with n and decreases with γ, (2) decreases with δ for n sufficiently high, and (3) decreases with $\bar{\theta}$ for n sufficiently high.

For (1), notice that $\partial_{p_j^0}\pi_j^0(p_j^0; p_H^*, p_L^*)$ depends on n and γ only via $U(\cdot)$, and thus we only need to show that given fixed p_H^* and p_L^*, $U(\cdot)$ decreases with n and increases with γ. Given the expressions of $U(\cdot)$, it is easy to verify that this is true.

For (2), notice that $U(\cdot)$ does not depend on δ. We have
$$\frac{\partial^2\pi_j^0(p_j^0; p_H^*, p_L^*)}{\partial p_j^0 \partial\delta}\Big|_{p_j^0 = p_j^*} = -1 + \frac{q_j U(\bar{\theta})}{\left(q_j - p_j^*\right)^2}.$$

As $n \to \infty$, $U(\bar{\theta}) \to 0$, so we have for sufficiently high n, $\frac{\partial^2\pi_j^0(p_j^0; p_H^*, p_L^*)}{\partial p_j^0 \partial\delta}|_{p_j^0 = p_j^*} \le 0$.

For (3), notice that $U(\bar{\theta})$ does not depend on $\bar{\theta}$. There are two cases to consider depending on the expression of $U(\cdot)$. In the first case with $\frac{q_H - p_H^*}{q_L - p_L^*} = 1$, we have
$$\frac{\partial^2\pi_j^0(p_j^0; p_H^*, p_L^*)}{\partial p_j^0 \partial\bar{\theta}}\Big|_{p_j^0 = p_j^*} = -\frac{g(q_j)\int_0^{\bar{\theta}} \partial_{\bar{\theta}} U(\theta)\mathrm{d}\theta}{\left(q_j - p_j^*\right)^2} \le 0,$$
where the inequality is due to $\partial_{\bar{\theta}} U(\theta) \ge 0$.

In the second case with $1 < \frac{q_H - p_H^*}{q_L - p_L^*} < e^{\frac{n}{\gamma}}$, similar to the first case, we have
$$\frac{\partial^2\pi_H^0(p_H^0; p_H^*, p_L^*)}{\partial p_H^0 \partial\bar{\theta}}\Big|_{p_H^0 = p_H^*} = -\frac{g(q_j)\int_0^{\bar{\theta}} \partial_{\bar{\theta}} U(\theta)\mathrm{d}\theta}{\left(q_H - p_H^*\right)^2} \le 0;$$
moreover,
$$\frac{\partial^2\pi_L^0(p_L^0; p_H^*, p_L^*)}{\partial p_L^0 \partial\bar{\theta}}\Big|_{p_L^0 = p_L^*} = -\frac{g(q_L)\left[1 - (n+1)e^{-n}\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma}\right]}{n\left(q_L - p_L^*\right)},$$

which is negative when n is sufficiently high. $\square$

Proof of Proposition 5:

Proof. Based on equation (12), to ensure that the provider has a nonnegative profit margin, we must have $p_j^0 \geq \bar{\theta} g(q_j)/[2(1-\delta)]$; to ensure that the provider has positive demand, we must have $p_j^0 \leq q_j - U$. Therefore, we must have $\bar{\theta} g(q_j)/[2(1-\delta)] \leq q_j - U$ to ensure that the provider is willing to participate in the market. We verify this condition in equilibrium, which is similar to the full market coverage condition in the case of bilateral ratings. Given the participation constraint, the provider only considers $p_j^0 \in \left[\bar{\theta} g(q_j)/[2(1-\delta)], q_j - U\right]$, because $\Pi_j^0\left(\bar{\theta} g(q_j)/[2(1-\delta)]; p_H, p_L\right) = \Pi_j^0(q_j - U; p_H, p_L) = 0$.

Next, we show that $\Pi_j^0(p_j^0; p_H, p_L)$ is concave in p_j^0:

$$
\begin{aligned}
\frac{\partial^2 \Pi_j^0}{\partial (p_j^0)^2} = & -\frac{W\left(-\frac{q_j - p_j^0}{U} e^{-\frac{q_j - p_j^0}{U}}\right)\left[\frac{q_j - p_j^0}{U} + W\left(-\frac{q_j - p_j^0}{U} e^{-\frac{q_j - p_j^0}{U}}\right)\right]}{2(q_j - p_j^0)^3\left[1 + W\left(-\frac{q_j - p_j^0}{U} e^{-\frac{q_j - p_j^0}{U}}\right)\right]^3} \\
& \times \Bigg\{ -4(1-\delta)U(q_j - p_j^0)\left[1 + W\left(-\frac{q_j - p_j^0}{U} e^{-\frac{q_j - p_j^0}{U}}\right)\right]^2 \\
& - 2U\left[(1-\delta)p_j^0 - \frac{\bar{\theta}}{2} g(q_j)\right]\left[\frac{q_j - p_j^0}{U} + 3W\left(-\frac{q_j - p_j^0}{U} e^{-\frac{q_j - p_j^0}{U}}\right) + 2W\left(-\frac{q_j - p_j^0}{U} e^{-\frac{q_j - p_j^0}{U}}\right)^2\right] \Bigg\},
\end{aligned}
$$

where $W(z)$ is the product logarithm function, which is defined as the upper branch of the inverse function of $z = We^W$. We note that $\frac{q_j - p_j^0}{U} \geq 1$, so $0 > W\left(-\frac{q_j - p_j^0}{U} e^{-\frac{q_j - p_j^0}{U}}\right) \geq -1 \geq -\frac{q_j - p_j^0}{U}$. We can further show that $\frac{q_j - p_j^0}{U} + 3W\left(-\frac{q_j - p_j^0}{U} e^{-\frac{q_j - p_j^0}{U}}\right) + 2W\left(-\frac{q_j - p_j^0}{U} e^{-\frac{q_j - p_j^0}{U}}\right)^2 \geq 0$. We also notice that $(1-\delta)p_j^0 - \frac{\bar{\theta}}{2} g(q_j) \geq 0$. According to all these inequalities, it is straightforward to verify that $\frac{\partial^2 \Pi_j^0}{\partial (p_j^0)^2} \leq 0$ and thus $\Pi_j^0(p_j^0; p_H, p_L)$ is a concave function in p_j^0.

Because $\Pi_j^0(p_j^0; p_H, p_L)$ is continuous in p_j^0, p_H and p_L, and concave in p_j^0, the existence of a pure-strategy Nash equilibrium is guaranteed by classic results, such as Proposition 8.D.3 on page 260 of Mas-Collell et al. (1995), except that now we have symmetric infinite games. Cheng et al. (2004) extend the classic results to consider symmetric infinite games and show that a symmetric pure-strategy Nash equilibrium exists with compact, convex strategy spaces and continuous, quasiconcave utility functions.

We prove $q_H - p_H^{**} > q_L - p_L^{**}$ by contradiction. Suppose $q_H - p_H^{**} \leq q_L - p_L^{**}$. We know that $(1-\delta)q_H - \frac{\bar{\theta}}{2} g(q_H) > (1-\delta)q_L - \frac{\bar{\theta}}{2} g(q_L)$ when $q_H > q_L$, and $g(q) = 1$ or $g(q) = q$. Therefore, we have

$$
\frac{(1-\delta)(q_H - p_H)}{(1-\delta)q_H - \frac{\bar{\theta}}{2} g(q_H)} < \frac{(1-\delta)(q_L - p_L)}{(1-\delta)q_L - \frac{\bar{\theta}}{2} g(q_L)} < 1,
$$

which implies that

$$\frac{(1-\delta)(q_H-p_H)}{(1-\delta)q_H-\frac{\bar{\theta}}{2}g(q_H)}-\ln\left[\frac{(1-\delta)(q_H-p_H)}{(1-\delta)q_H-\frac{\bar{\theta}}{2}g(q_H)}\right]>\frac{(1-\delta)(q_L-p_L)}{(1-\delta)q_L-\frac{\bar{\theta}}{2}g(q_L)}-\ln\left[\frac{(1-\delta)(q_L-p_L)}{(1-\delta)q_L-\frac{\bar{\theta}}{2}g(q_L)}\right].$$

Meanwhile, we know that

$$1\leq\frac{q_H-p_H}{U}\leq\frac{q_L-p_L}{U},$$

which implies that,

$$\frac{q_H-p_H}{U}-\ln\left(\frac{q_H-p_H}{U}\right)\leq\frac{q_L-p_L}{U}-\ln\left(\frac{q_L-p_L}{U}\right).$$

By equation (13), we know that

$$\frac{(1-\delta)(q_H-p_H)}{(1-\delta)q_H-\frac{\bar{\theta}}{2}g(q_H)}-\ln\left[\frac{(1-\delta)(q_H-p_H)}{(1-\delta)q_H-\frac{\bar{\theta}}{2}g(q_H)}\right]=\frac{q_H-p_H}{U}-\ln\left(\frac{q_H-p_H}{U}\right),$$

which implies that

$$\frac{(1-\delta)(q_L-p_L)}{(1-\delta)q_L-\frac{\bar{\theta}}{2}g(q_L)}-\ln\left[\frac{(1-\delta)(q_L-p_L)}{(1-\delta)q_L-\frac{\bar{\theta}}{2}g(q_L)}\right]<\frac{q_L-p_L}{U}-\ln\left(\frac{q_L-p_L}{U}\right).$$

This is a contradiction to equation (13). Therefore, we have that $q_H-p_H^{**}>q_L-p_L^{**}$. □

Online Appendix of *Peer-to-Peer Markets with Bilateral Ratings*

Analysis of Incomplete Market Coverage:

The following proposition characterizes customers' application strategy with a proof similar to that of Proposition 1 and thus omitted.

Proposition OA1. *Under the incomplete market condition (7) and with $\bar{\theta}+\delta \leq 1$, we have $q_H - p_H > q_L - p_L$.*

1. *If $\frac{q_H-p_H}{q_L-p_L} < e^{\frac{n}{\gamma}F\left(\frac{(1-\delta)p_H}{g(q_H)}\right)}$, customers with type $\theta \in [0, \theta_H]$ apply to providers of type H, customers with $\theta \in (\theta_H, (1-\delta)p_H/g(q_H)]$ apply to both types of providers, and customers with $\theta \in \left(1-\delta)p_H/g(q_H), \bar{\theta}\right]$ apply to providers of type L. For $\theta \in \left[0, \bar{\theta}\right]$,*

$$x_H(\theta) = \begin{cases} \dfrac{n}{\gamma}f(\theta), & 0 \leq \theta \leq \theta_H \\ nf(\theta), & \theta_H < \theta \leq \dfrac{(1-\delta)p_H}{g(q_H)} \\ 0, & \dfrac{(1-\delta)p_H}{g(q_H)} < \theta \leq \bar{\theta} \end{cases}; \quad x_L(\theta) = \begin{cases} 0, & 0 \leq \theta \leq \theta_H \\ nf(\theta), & \theta_H < \theta \leq \dfrac{(1-\delta)p_H}{g(q_H)} \\ \dfrac{n}{1-\gamma}f(\theta), & \dfrac{(1-\delta)p_H}{g(q_H)} < \theta \leq \bar{\theta} \end{cases};$$

$$U(\theta) = \begin{cases} (q_H - p_H)e^{-\frac{n}{\gamma}F(\theta)}, & 0 \leq \theta \leq \theta_H \\ (q_H - p_H)\left(\dfrac{q_H - p_H}{q_L - p_L}\right)^{-(1-\gamma)} e^{-nF(\theta)}, & \theta_H < \theta \leq \dfrac{(1-\delta)p_H}{g(q_H)} \\ (q_H - p_H)\left(\dfrac{q_H - p_H}{q_L - p_L}\right)^{-(1-\gamma)} e^{\frac{\gamma}{1-\gamma}nF\left(\frac{(1-\delta)p_H}{g(q_H)}\right)} e^{-\frac{1}{1-\gamma}nF(\theta)}, & \dfrac{(1-\delta)p_H}{g(q_H)} < \theta \leq \bar{\theta} \end{cases}.$$

2. *Otherwise, if $\frac{q_H-p_H}{q_L-p_L} \geq e^{\frac{n}{\gamma}F\left(\frac{(1-\delta)p_H}{g(q_H)}\right)}$, customers with type $\theta \in [0, (1-\delta)p_H/g(q_H)]$ apply to providers of type H, and customers with $\theta \in \left(1-\delta)p_H/g(q_H), \bar{\theta}\right]$ apply to providers of type L. For $\theta \in \left[0, \bar{\theta}\right]$,*

$$x_H(\theta) = \begin{cases} \dfrac{n}{\gamma}f(\theta), & 0 \leq \theta \leq \dfrac{(1-\delta)p_H}{g(q_H)} \\ 0, & \dfrac{(1-\delta)p_H}{g(q_H)} < \theta \leq \bar{\theta} \end{cases}; \quad x_L(\theta) = \begin{cases} 0, & 0 \leq \theta \leq \dfrac{(1-\delta)p_H}{g(q_H)} \\ \dfrac{n}{1-\gamma}f(\theta), & \dfrac{(1-\delta)p_H}{g(q_H)} < \theta \leq \bar{\theta} \end{cases};$$

$$U(\theta) = \begin{cases} (q_H - p_H)e^{-\frac{n}{\gamma}F(\theta)}, & 0 \leq \theta \leq \dfrac{(1-\delta)p_H}{g(q_H)} \\ (q_L - p_L)e^{\frac{1}{1-\gamma}nF\left(\frac{(1-\delta)p_H}{g(q_H)}\right)} e^{-\frac{1}{1-\gamma}nF(\theta)}, & \dfrac{(1-\delta)p_H}{g(q_H)} < \theta \leq \bar{\theta} \end{cases}.$$

Next, we study the providers' problem. Consider a provider of type j posting price p_j^0. Similar to equation (19) in the case of complete market coverage, a low-type provider's profit function can be written as

$$
\pi_L^0(p_L^0; p_H, p_L) = \begin{cases} -(1-\delta)\left[U(0) - U(\bar{\theta})\right] \\ \quad - \dfrac{\int_0^{\bar{\theta}} \left[(1-\delta)q_L - g(q_L)\theta\right] U'(\theta) d\theta}{q_L - p_L^0}, & \text{if } p_L^0 \leq q_L - U(0); \\ (1-\delta)U(\bar{\theta}) + (1-\delta)p_L^0 - g(q_L)U^{-1}(q_L - p_L^0) \\ \quad - \dfrac{(1-\delta)U(\bar{\theta})q_L + g(q_L)\left[\int_{U^{-1}(q_L - p_L^0)}^{\bar{\theta}} U(\theta)\mathrm{d}\theta - \bar{\theta}U(\bar{\theta})\right]}{q_L - p_L^0} & \text{otherwise.} \end{cases} \tag{1}
$$

Different from the case of complete market coverage, $U(\cdot)$ is no longer a continuous function when $\frac{q_H - p_H}{q_L - p_L} \geq e^{\frac{n}{\gamma}F\left(\frac{(1-\delta)p_H}{g(q_H)}\right)}$. Therefore, $U^{-1}(\cdot)$ is not well defined. By redefining $U^{-1}(u) = \inf\left\{x \in [0, \bar{\theta}] \middle| U(x) \leq u\right\}$, one can show that equation (1) still holds.

Consider now a high-type provider, whose profit function is different from equation (19) in that the provider would not serve any customer with cost type $\theta \in ((1-\delta)p_H/g(q_H), \bar{\theta}]$. Therefore, the upper limit of the integral in equation (19) is replaced by $(1-\delta)p_H/g(q_H)$ as we write down the provider's profit function:

$$
\pi_H^0(p_H^0; p_H, p_L) = \begin{cases} -(1-\delta)\left[U(0) - U\left(\dfrac{(1-\delta)p_H^0}{q_H}\right)\right] \\ \quad - \dfrac{\int_0^{\frac{(1-\delta)p_H^0}{q_H}} \left[(1-\delta)q_H - g(q_H)\theta\right] U'(\theta) d\theta}{q_H - p_H^0}, & \text{if } p_H^0 \leq q_H - U(0); \\ (1-\delta)U\left(\dfrac{(1-\delta)p_H^0}{q_H}\right) + (1-\delta)p_H^0 \\ \quad - g(q_H)U^{-1}(q_H - p_H^0) - \dfrac{(1-\delta)U\left(\frac{(1-\delta)p_H^0}{q_H}\right)q_H}{q_H - p_H^0} \\ \quad - \dfrac{g(q_H)\left[\int_{U^{-1}(q_H - p_H^0)}^{\frac{(1-\delta)p_H^0}{q_H}} U(\theta)\mathrm{d}\theta - \frac{(1-\delta)p_H^0}{q_H}U\left(\frac{(1-\delta)p_H^0}{q_H}\right)\right]}{q_H - p_H^0}, & \text{otherwise.} \end{cases} \tag{2}
$$

Following similar analysis in Section 3.2, we can write down the first-order conditions to the profit maximization problems whose objectives are given in equations (1) and (2), along with the incomplete market coverage condition, as follows,

$$
\begin{cases} p_H^* < \bar{\theta}g(q_H)/(1-\delta), \\ p_L^* \geq \bar{\theta}g(q_L)/(1-\delta), \\ (1-\delta)(q_H - p_H^*)^2 - (1-\delta)(q_H - p_H^*)U\left(\dfrac{(1-\delta)p_H^*}{q_H}\right) - g(q_H)\displaystyle\int_0^{\frac{(1-\delta)p_H^*}{q_H}} U(\theta)\mathrm{d}\theta \leq 0, \\ (1-\delta)(q_L - p_L^*)^2 - \left[(1-\delta)q_L - g(q_L)\bar{\theta}\right] U(\bar{\theta}) - g(q_L)\displaystyle\int_{\min\left\{\theta_H, \frac{(1-\delta)p_H^*}{q_H}\right\}}^{\bar{\theta}} U(\theta)\mathrm{d}\theta = 0. \end{cases} \tag{3}
$$

Proof of Proposition 6: Analysis for a Small Marketplace

Proof. We first prove there is no pure-strategy equilibrium. Suppose p_H^* and p_L^* are an equilibrium pair of prices. There are two cases to consider. First, $q_H - p_H^* = q_L - p_L^*$, in which case the high-quality provider can increase his profit by lowering his price by a small amount to get the low-cost customer for sure. Therefore, p_H^* is not in equilibrium. Second, $q_i - p_i^* > q_j - p_j^*$, in which case provider i gets the low-cost customer and provider j gets the high-cost customer. We must have $p_j^* = q_j$ given that the high-cost customer has no other choice except for the outside option. Given $p_j^* = q_j$, we must have $p_i^* = q_i - \varepsilon$ for $\varepsilon > 0$ and $\varepsilon \to 0$. Given $p_i^* = q_i - \varepsilon$, provider j has an incentive to choose price $p_j^* = q_j - \varepsilon - \eta$ for $\eta > 0$ and $\eta \to 0$, so as to get the low-cost customer. Therefore, p_j^* is not in equilibrium.

Denote S_i as the support for provider i's mixed pricing strategy, for $i \in \{H, L\}$. Further denote $U_i \equiv \{q_i - p_i | p_i \in S_i\}$. Following the proof for Proposition 2 in Narasimhan (1988), we can show that both U_i and $U_i - U_i \cap U_j$ are convex, and thus U_i is convex. Therefore, S_i is convex, and we can denote $S_i = [\underline{p}_i, \overline{p}_i]$.

We now prove that $q_H - \underline{p}_H = q_L - \underline{p}_L > 0$. In fact, suppose $q_i - \underline{p}_i > q_j - \underline{p}_j$. Then, provider i can increase his profit by raising $\underline{p}_i$ to $\underline{p}_i' = \underline{p}_i + [(q_i - \underline{p}_i) - (q_j - \underline{p}_j)]/2$. Therefore $q_H - \underline{p}_H = q_L - \underline{p}_L$, which is greater than zero because both providers' profits are positive. Furthermore, there is no mass point for provider i's price distribution at $\underline{p}_i$. Suppose provider i has a mass point at $\underline{p}_i$. Provider j can then increase his profit by shifting some probability from above $\underline{p}_j$ to below $\underline{p}_j$. Following a similar argument, one can prove that $\overline{p}_i = q_i$ and it is not an equilibrium for both providers to have a mass point at $\overline{p}_i$ (see more detailed argument in the proof of Proposition 3 in Narasimhan 1988).

We are now ready to calculate the providers' price distributions. Given that there is at least one provider who has no mass point at $\overline{p}_i$, without loss of generality, we assume that it is the low-quality provider who has no mass point at $\overline{p}_L = q_L$. This implies that by charging $\overline{p}_H = q_H$, the high-quality provider would serve the high-cost customer almost surely. The provider is indifferent towards charging any price $p \in [\underline{p}_H, q_H]$. This implies that provider H's profit is

$$\pi_H = q_H - c = [1 - F_L(p - (q_H - q_L))]\, p + F_L(p - (q_H - q_L)) \max\{p - c, 0\}.$$

By solving $F_L(\cdot)$ from the equation above, we have

$$F_L(p) = \frac{p + c - q_L}{p + q_H - q_L - \max\{p + q_H - q_L - c, 0\}}.$$

Notice that there is no mass point at $\underline{p}_L$, so we have $F_L(\underline{p}_L) = 0$, which enables us to solve $\underline{p}_L = q_L - c$. Provider L's profit is hence

$$\pi_L = \underline{p}_L = q_L - c = [1 - F_H(p + (q_H - q_L)))\, p + F_H(p + (q_H - q_L)] \max\{p - c, 0\}.$$

By solving $F_H(\cdot)$ from the equation above, we have

$$F_H(p) = \frac{p + c - q_H}{p - q_H + q_L - \max\{p - q_H + q_L - c, 0\}}.$$

By $q_H - \underline{p}_H = q_L - \underline{p}_L$, we have $\underline{p}_H = q_H - c$. Moreover, one can see that $F_H(\overline{p}_H) = 1$, so there is no mass point at $\overline{p}_H = q_H$ either. $\square$

Analysis for Two Applications

Proof. Consistent with the main model, we assume that the number of customers per provider, n is sufficiently high so that when a provider receives multiple applications, he chooses the one with the lowest θ because he is not concerned about the possibility that the customer gets better offers from other providers and rejects his offer. If it happens that the customer indeed rejects his offer, it is assumed that the provider has no option to make another offer and would stay unmatched.

Customers' Problem

Consider first the customers' application strategy. A customer has three possible application strategies: (1) submitting both applications to H providers, (2) submitting both applications to L providers, (3) submitting one application to an H provider and one to an L provider. Assume that among providers of the same type $j \in \{H, L\}$, a customer of type θ still uses a symmetric mixed strategy represented by $a_j(\theta)$. A customer is allowed to further mix among the three types of application strategies above.

Similar to the main model, we define $x_j(\theta) = Nf(\theta)a_j(\theta)$ and obtain the following normalization condition:

$$\gamma x_H(\theta) + (1-\gamma)x_L(\theta) = 2nf(\theta).$$

It is straightforward to see that the acceptance rate is still given by $b_j(\theta)$ in equation (2). When a customer of type θ uses application strategies (1), (2) and (3), her expected utilities are, respectively,

$$\begin{aligned}
U_{HH}(\theta) &= \left(1 - (1 - b_H(\theta))^2\right)(q_H - p_H);\\
U_{LL}(\theta) &= \left(1 - (1 - b_L(\theta))^2\right)(q_L - p_L);\\
U_{HL}(\theta) &= \begin{cases} b_H(\theta)(q_H - p_H) + (1 - b_H(\theta))b_L(\theta)(q_L - p_L), & \text{if } q_H - p_H \geq q_L - p_L;\\ b_L(\theta)(q_L - p_L) + (1 - b_L(\theta))b_H(\theta)(q_H - p_H), & \text{otherwise.} \end{cases}
\end{aligned}$$

Two patterns are noteworthy. First, the acceptances of a customer's two applications are not independent events with a finite number of providers, but become asymptotically independent as the number of providers goes to infinity (Albrecht et al. 2004). Second, the calculation of customer's expected utilities above illustrates the ex-post competition among providers after they make offers, because if a customer receives two offers, she would pick the better one, a new feature to this setting. The customer chooses among the three application strategies to maximize her expected utility,

$$U(\theta) = \max\left\{U_{HH}(\theta), U_{LL}(\theta), U_{HL}(\theta)\right\}.$$

With a similar proof, we can show that except for the expression of $U(\theta)$, everything else in Proposition 1 still applies here as long as we replace n by $2n$ and the notations of θ_H and θ_L by θ_{HH} and θ_{LL}. That is,

$$\theta_{HH} \equiv F^{-1}\left[\frac{\gamma}{2n}\ln\left(\frac{q_H - p_H}{q_L - p_L}\right)\right] \text{ and } \theta_{LL} \equiv F^{-1}\left[\frac{1-\gamma}{2n}\ln\left(\frac{q_L - p_L}{q_H - p_H}\right)\right].$$

Consider the five cases in the proof of Proposition 1, customers in Cases 1 and 5, those with $\theta \leq \theta_{LL}$ in Case 2, and those with $\theta \leq \theta_{HH}$ in Case 4 would submit both applications to the same type of providers; for the remaining customers (those in Case 3, those with $\theta > \theta_{LL}$ in Case 2, and those with $\theta > \theta_{HH}$ in Case 4), they would submit one application to providers of type j that offer a higher potential payoff $q_j - p_j$ and are indifferent between the two types of providers for their second application.

To understand the intuition behind this result, consider the case of $q_H - p_H > q_L - p_L$ as an example. Given $q_H - p_H > q_L - p_L$, customers with low θ submit both their applications to H providers, because $U_{HH}(\theta)$ dominates $U_{LL}(\theta)$ and $U_{HL}(\theta)$ for $b_H(\theta)$ and $b_L(\theta)$ close to one. Low-θ customers' application strategy drives down $b_H(\theta)$, the acceptance rate of H providers, while $b_L(\theta)$ remains at one, till $\theta = \theta_{HH}$, at which point $U_{HH}(\theta_{HH}) = \left(1-(1-b_H(\theta_{HH}))^2\right)(q_H - p_H) = U_{HL}(\theta_{HH}) = b_H(\theta_{HH})(q_H - p_H) + (1 - b_H(\theta_{HH}))(q_L - p_L)$. Customers with $\theta > \theta_{HH}$ are indifferent between application strategies (1) and (3) so that $U_{HH}(\theta) = U_{HL}(\theta)$, which implies that $x_H(\theta) = x_L(\theta)$.

Providers' Problem

To simplify exposition, we continue to focus on the case with $q_H - p_H > q_L - p_L$. We show next that a pure-strategy price equilibrium does not exist. To this end, consider a focal individual provider of type H, who posts price $p_H^0 < p_H$ and is deviating from equilibrium. Given $q_H - p_H > q_L - p_L$, we have $U(\theta) = U_{HH}(\theta) = \left(1-(1-b_H(\theta))^2\right)(q_H - p_H)$ according to the analysis above. Consider a customer of type θ who submits one application to the focal provider and the other application to an H provider. Given the focal provider's acceptance probability $b_H^0(\theta)$, the customers' expected utility is $U^0(\theta) = b_H^0(\theta)(q_H - p_H^0) + (1 - b_H^0(\theta))b_H(\theta)(q_H - p_H)$. Notice that for any $b_H^0(0) > 0$, we have $U^0(0) > U(0)$. This implies that all customers of type $\theta = 0$ would submit one application to the focal provider, which makes his acceptance probability $b_H^0(0) = 0$. Moreover, $b_H^0(\theta) \geq 0$ and is weakly increasing in θ by definition, so we have $b_H^0(\theta) = 0$ for all $\theta \in [0, \overline{\theta}]$. In other words, by deviating to a price with $p_H^0 < p_H$, the provider profits from a discrete upward jump in the number of applications. Furthermore, $p_H = 0$ cannot constitute an equilibrium, because the provider would raise the price to profit from a positive acceptance probability. Therefore, a pure-strategy price equilibrium does not exist.

$\square$