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Competitive Mobile Geo Targeting

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Abstract. We investigate in a competitive setting the consequences of mobile geo targeting, the practice of firms targeting consumers based on their real-time locations. A distinct market feature of mobile geo targeting is that a consumer could travel across different locations for an offer that maximizes his total utility. This mobile-deal seeking opportunity motivates firms to carefully balance prices across locations to avoid intrafirm cannibalization, which in turn mitigates interfirm price competition and prevents firms from going into a prisoner's dilemma. As a result, a firm's profit can be higher under mobile geo targeting than under uniform or traditional targeted pricing. We extend our model in three different directions: (a) a fraction of consumers are not aware of mobile offers outside of their permanent locations, (b) mobile offers can be collected when consumers travel for other reasons, and (c) firms use both permanent and real-time locations when setting prices. Our findings have important managerial implications for marketers who are interested in optimizing their mobile geo-targeting strategies.

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Keywords: targeted pricing • mobile targeting • geo targeting • analytical models

1. Introduction

People are spending more time with their mobile devices: U.S. adults, for example, are estimated to spend an average of 2 hours and 51 minutes per day on mobile devices in 2014 (*eMarketer* 2014a). According to Ninth Decimal, 55% of consumers have purchased a retail product as a result of seeing a mobile ad (*NinthDecimal* 2014). The same study finds that when mobile users are asked what information they are most likely to respond to in a retail-related mobile ad, the highest ranked answer is discounts/sales, topping other answers such as product reviews or giveaways. The 2015 Global Shopper Study finds that 37% of the surveyed consumers use mobile coupons sent as text/email messages and when asked the question of "How likely would you be to use the following in-store services offered on your smartphones?" 51% of the consumers said yes to "Location-based coupons" (*Zebra Technologies* 2015). Marketers are quick to follow the eyeballs: global mobile ad spending more than doubled from 2013 to 2014, projected to reach $94 billion in 2018 (*eMarketer* 2014b). In particular, 170 brands in the United States, including Adidas, Pinkberry, Walmart, and Outback Steakhouse, are known to be using location-based mobile targeting technologies in their marketing campaigns.[1]

The fast growth of mobile ad spending has triggered an increasing body of empirical research on the topic, especially on location-based mobile targeting, i.e., mobile geo targeting. Ghose et al. (2013) are among the first to show that search costs may be higher on mobile phones because of the small screen size, and stores located in close proximity to a user's home are much more likely to be clicked on. Luo et al. (2014) investigate the location and timing of mobile offers on movie tickets and find that it can be profitable for firms to allow more time when targeting nonproximal consumers. Danaher et al. (2015) consider the redemption of mobile coupons distributed within a shopping mall and find that redemption is more likely if the offer has a higher face value and is received at a location that is closer to the store. Fong et al. (2015) examine the effectiveness of geo-conquesting promotions with mobile offers that target consumers located near a competitor's store. They find that firms may benefit from such promotions and the optimal discount depth varies with the distance from a firm. Besides location and timing, other factors that have been shown to have an impact on the effectiveness of mobile marketing include the product category (Bart et al. 2014) and contextual factors such as crowdedness (Andrews et al. 2016) and shoppers' in-store paths (Hui et al. 2013).

While empirical research quickly accumulates, little research is done in the theoretical domain on mobile geo targeting. We aim to fill the gap in this paper by providing insights on how firms can optimize mobile geo targeting in a *competitive* environment. As existing research often utilizes field experiments to gauge causal effects of mobile geo targeting, it can be hard to get two competing firms to participate in the same study. Our theory can therefore help marketers understand the incentives for competing firms to adopt the mobile geo-targeting technology and conditions under which the technology enhances their profitability.

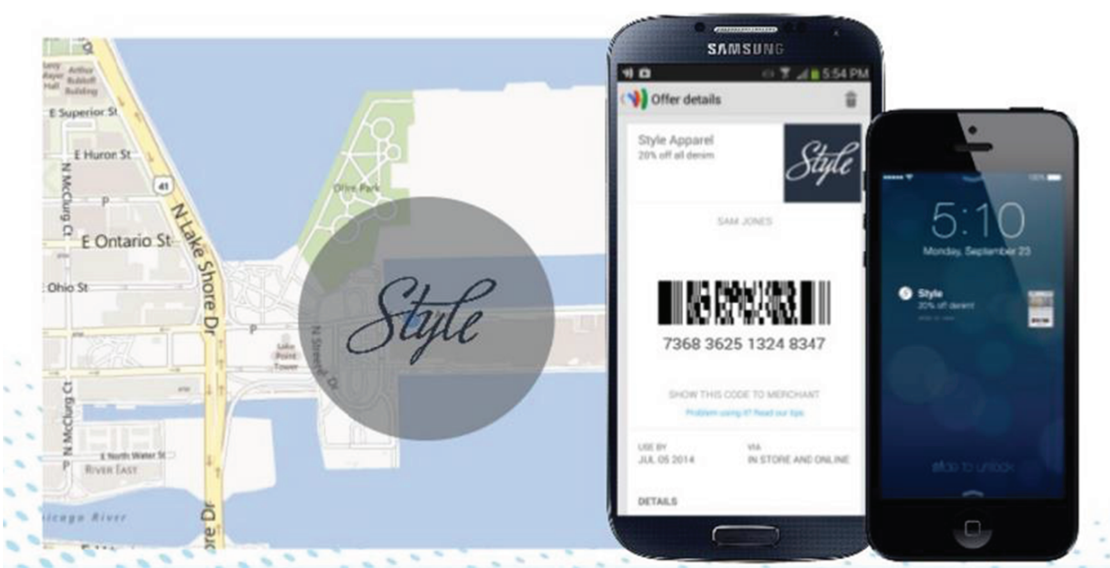
We focus on an important feature of mobile geo targeting that distinguishes it from traditional targeting (e.g., mailed coupons): mobile geo targeting is often based on a consumer's real-time location rather than his permanent/home location. Our survey of 158 Amazon Mechanical Turk subjects shows that 54% of consumers have used location-based mobile coupons. Among the coupon users, 68% are aware of at least one of the coupons before getting them and 60% have travelled to a particular location to obtain such a coupon. Among the nonusers, around half (49%) are aware of the existence of such coupons. Across all of the consumers we have surveyed, when asked "Would you be willing to travel to a particular location to obtain such a coupon?" the vast majority selected either "Yes" (28%) or "It depends on the value of the coupon and the distance I have to travel" (62%), and only 10% selected "No."

In practice, mobile offers are delivered through both "push" and "pull" technologies, although the boundary between pull and push is getting increasingly blurred. Once a consumer opts into a couponing/payment application, say Google Wallet or Apple Passbook, the app then pulls coupons from participating vendors based on the consumer's real-time location. In the example[2] shown in Figure 1, as soon as a consumer enters the shaded region on the map that is predetermined by the store, he receives a push notification from Google Wallet on the phone's home screen, which links to the redeemable coupon with a barcode.

Another way of implementing mobile geo targeting is to send location-based coupons through short message service (SMS) (Fong et al. 2015, Danaher et al. 2015). Users typically opt in to receive such messages beforehand so that their privacy is protected and messages are pushed to them once they enter the predetermined region of target. Mobile geo targeting can also be implemented through dynamic banner ads that link to location-based coupons. As a critical new feature of mobile geo targeting, the final price is determined by the consumer's real-time location and he could travel across different regions to obtain the best overall offer. The mobile-deal seeking (MDS) behavior, as demonstrated in our model, turns out to have profound implications for competing firms' pricing strategies and the consequent market outcomes.

Specifically, we consider a duopoly model in which each firm has consumers residing at its home base and there are also some consumers located in the middle of the two firms. Besides their permanent locations, consumers are also differentiated in their relative preferences for each of the two firms' product or service as taste heterogeneity has been shown to play an important role in the literature of competitive targeting (e.g., Fudenberg and Tirole 2000, Besanko et al. 2003, Shin and Sudhir 2010). We assume that the unit transportation cost on the location dimension is higher than the unit mismatch cost on the product preference dimension, so that the physical location is the primary source of differentiation between the two firms and mobile location-based targeting has a significant impact on how firms compete with each other.

Figure 1. (Color Online) Mobile Coupon Notification from Google Wallet



Under mobile geo targeting, a consumer could receive different price offers as his real-time location changes, and the final offer location can differ from both his permanent location and the actual purchase location. A consumer carefully evaluates his total utility of buying with each available offer based on his permanent location, the price at the offer location, and his product preference. Once he identifies the best offer that maximizes the total utility of buying, he makes a final decision on whether to buy a product, and if so, from which firm and with which offer.

Three main forces drive the equilibrium outcomes in our model. First, firms' ability to price discriminate helps them expand demand without having to charge lower prices to the local consumers. As we allow category demand to increase with targeted pricing, our model fits best with nonnecessity product categories with a reasonably high elasticity of demand, such as movies (Luo et al. 2014) and café snack foods (Danaher et al. 2015), for which a consumer may not make a purchase unless he receives a discount. It is important to note that while coupons of significant monetary value may provide stronger motivations for consumers to engage in mobile-deal seeking, coupons on small-value products such as snacks can also be attractive when the distance the consumer has to travel is small.[3]

Second, as documented in the literature, price discrimination could intensify interfirm price competition in each segment of the market and as a result, traditional targeted pricing is often found to lead to a prisoner's dilemma in which every firm is worse off (Thisse and Vives 1988, Shaffer and Zhang 1995, Corts 1998). The same force applies in our model as well.

Third, as a unique feature of our model, consumer mobile-deal seeking motivates each firm to balance its prices across different consumer segments (i.e., locations) to avoid intrafirm cannibalization. As a result, interfirm competition is mitigated at each location and mobile geo targeting could outperform uniform pricing in firm profitability, whereas traditional targeting typically underperforms uniform pricing.

We also examine three extensions of the main model. First, we investigate what happens when only a fraction of consumers are aware of all mobile offers and actively seek the best offer, while the other consumers are "naïve" and only know about the offers at their permanent locations. When the fraction of mobile-deal seekers is small, consumers travel for better mobile deals in equilibrium as firms find it more important to compete aggressively for the large amount of naïve consumers at the middle location with low prices than to prevent mobile-deal seeking. Interestingly, the equilibrium profit in this case decreases with the fraction of mobile-deal seekers, as the dominant effect of mobile-deal seeking in this case is intrafirm cannibalization. When the fraction of informed residents exceeds a certain threshold, potential cannibalization of high-margin sales from the local consumers becomes so significant that both firms raise their prices at the middle to prevent deal seeking from occurring in equilibrium, just like in our main model. Our results hence suggest that increasing the fraction of informed residents, through means such as direct advertising or supporting social media sites that promote information sharing among customers, can potentially improve firms' profits.

Second, we allow some residents to travel for reasons that are external to shopping and stumble on the best mobile offer without incurring additional travel costs. In this case, firms need to equalize their prices even more across locations and interfirm price competition is further mitigated. Interestingly, while uniform pricing arises when mobile-deal seeking is costless and traditional targeting arises when deal seeking is prohibitively costly, both strategies can be outperformed by mobile geo targeting in terms of profitability, a case in which deal seeking is costless for some consumers but prohibitively costly for others.

Finally, we explicitly compare the firm's equilibrium profit under traditional and mobile geo targeting. In addition, we look into the possibility of firms setting their prices based on both the permanent and real-time locations of a consumer. In equilibrium, given the same permanent (real-time) location of a consumer, the further away a consumer's real-time (permanent) location is from the firm, the lower the equilibrium price. Although each firm now optimizes a complicated price schedule based on many possible combinations of permanent and real-time locations, the equilibrium outcomes degenerate to those under traditional targeting: consumers use offers at their permanent locations and the equilibrium price each consumer pays is the same as that under traditional targeting. Intuitively, each firm has a strong incentive to use information on permanent location to directly prevent mobile-deal seeking and intrafirm cannibalization, but doing so turns out to hurt both firms because of intensified price competition.

Taken together, our analysis shows that mobile geo targeting, as a unique pricing mechanism that is based on consumers' real-time location, can benefit firms in a competitive setting. While a consumer's ability to obtain and use an offer outside his home location seems to have the obvious consequence of cannibalizing high-margin sales, this possibility may turn out to benefit both firms through equalizing prices across locations in each firm and limit the price competition between different firms.

Our paper contributes to the literature of competitive targeting, behavior-based price discrimination, and mobile marketing. The first literature tends to focus on the interaction of competition and price discrimination. Studies in this literature that explore third-degree price discrimination along a horizontal dimension such as location or taste often find that price discrimination increases interfirm competition and leads to a prisoner's dilemma in which all firms obtain lower profits, unless these firms are differentiated in other dimensions (e.g., Thisse and Vives 1988, Shaffer and Zhang 1995, Corts 1998). Lal and Rao (1997), for example, consider supermarket competition and show that a retailer may use price, service, and communications together as a positioning tool. As a result, retailers with different pricing strategies such as every-day-low-price and hi-lo promotions could use multidimensional targeting strategies that appeal to all consumer segments. Shaffer and Zhang (2002) show that one-to-one promotions have the potential to either increase or decrease a firm's equilibrium profit, depending on, for example, whether the firm has a larger market share than its competitor. On the other hand, Desai and Purohit (2004) and Desai et al. (2016) identify interesting scenarios, such as consumer haggling and firms offering exchange promotions, in which uniform pricing can also be the outcome of a prisoner's dilemma in a competitive setting.

By allowing consumers to self-select the best deal across different locations, we are essentially modeling a particular type of second-degree price discrimination in a horizontally differentiated market.[4] To our knowledge, there is very limited work in this direction, as most studies of second-degree price discrimination focus on the optimal levels of vertical attributes such as quality and quantity, and the corresponding price schedule (e.g., Spulber 1989). In an interesting paper by Desai (2001), the vertical competition between firms is modeled in a Hotelling framework (1929): both H-type and L-type consumers are allowed to have different preferences toward the two competing firms. Similar to other research on second-degree price discrimination, Desai (2001) focuses on firms' selection of the optimal quality-price bundles, whereas we focus on the optimal location-price bundles and consumers' self-selection on the horizontal, rather than vertical, dimension.

On a broader level, by focusing on horizontal locations as opposed to vertical quality, our model features a strong form of "best response asymmetry" in that firms are asymmetric in their rankings of strong and weak markets: one firm's strong market is the other firm's weak market (Stole 2007). While it is generally acknowledged in studies of third-degree price discrimination that such asymmetry can significantly change the equilibrium outcomes in a competitive setting, to our knowledge this is the first paper to incorporate this asymmetry in the context of second-degree price discrimination. Our results highlight that mobile-deal seeking has the potential to limit interfirm price competition to such a degree that mobile geo targeting, as a particular form of second-degree price discrimination, can outperform both uniform pricing and third-degree price discrimination in a competitive environment.

Behavior-based price discrimination refers to the practice of firms pricing consumers differently based on their behavior, which can serve as a signal of their underlying preferences. Early works in this literature argue that price discrimination based on past purchase behavior can lead to a prisoner's dilemma that ultimately lowers profits for competing firms (Fudenberg and Tirole 2000, Villas-Boas 1999). Zhang (2011) further argues that as forward-looking firms try to attenuate this intensified competition by altering product design in early periods, products turn out to be less differentiated, causing even stronger competition and lower profits for firms. Recent studies explore different situations in which behavior-based price discrimination might benefit firms, for example, when one firm is significantly more advanced in its capability to add benefits to previous customers (Pazgal and Soberman 2008), when customers differ in purchase quantity and their preferences change over time (Shin and Sudhir 2010), and when past purchase is positively correlated with the likelihood that a consumer has a high willingness to pay in a related product category (Shen and Villas-Boas 2017). Besides past purchases, studies have also investigated the consequences of pricing on other variables such as information related to customer cost to the firm (Shin et al 2012, Subramanian et al. 2014).

Our paper contributes to the literature of behavior-based pricing by adding another dimension of consumer behavior that firms can price on: a consumer's *real-time* location. As mobile-deal seeking helps limit interfirm price competition and makes it more likely for firms to benefit from location-based price discrimination, our paper is different from classic studies in the behavior-based pricing literature that rely on firm asymmetry, either in capability or in information about customer type, to soften competition. The stochastic consumer preference assumed in Shin and Sudhir (2010) is close to our setting in that consumer types can change over time. A key difference, however, is that consumer types change exogenously in their model, whereas mobile-deal seeking is endogenous in our model.

The literature of mobile geo targeting has also been growing quickly in the past few years. It provides empirical evidence that is consistent with our model. Luo et al. (2014), for example, find that it can be profitable for firms to allow more time when targeting nonproximal consumers, which is consistent with our assumption that consumers need to incur substantial travel cost to visit the store when they receive a mobile offer from far away. Consistent with our result that firms would offer a lower price to consumers who are located further away, Fong et al. (2015) find that the optimal discount is deeper at locations near the competitor's store than at locations near one's own store. Dubé et al. (2017) use a field experiment to estimate consumer demand and the best response functions of two competing firms engaging in price discrimination via mobile devices. They find that mobile geo targeting decreases firm profit while behavioral targeting based on the recency of last purchase increases profit. Different from our study, they do not consider the possibility of consumers actively seeking out the best mobile offer across locations, or firms targeting consumers located in the middle of two firms' home bases. As a result, mobile targeting is conceptually similar to traditional targeting in their study. As we make the first attempt to model competitive mobile geo targeting in a game-theoretical framework, our results yield interesting predictions for future empirical work in this domain.

The remainder of the paper proceeds as follows. In the next section, we introduce the main model of competitive mobile geo targeting. We then present the three extensions of the model, and conclude with a discussion of managerial implications and directions for future research.

2. A Model of Competitive Mobile Geo Targeting

We consider a spatial model of competitive mobile geo targeting. Two firms are located at the endpoints of the unit interval, Firm A at 0 and Firm B at 1. The production cost is normalized to zero without loss of generality. There are three groups of consumers, each with a mass of one. The first group resides at 0, the second group resides at 1, and a third group resides at ½. That is, each firm has a unit mass of home-base consumers, while another unit mass of consumers reside at the middle of the market an equal distance away from both firms.[5] We refer to a consumer's base location as his permanent or home location.

Residents at each location have heterogeneous preferences toward the two firms. At each location, they are distributed uniformly on a unit line in the preference dimension with utility $V - tx_1 - sy$ for Firm A and utility $V - tx_2 - s(1-y)$ for Firm B, where V is the consumers' reservation price for the product category, t is the unit transportation cost, s is the unit mismatch cost on the preference dimension, y is uniformly distributed in $[0,1]$ and captures the consumer's preference mismatch with Firm A relative to Firm B, and finally x_i, $i \in \{A, B\}$, is the *total* distance the consumer has to travel to buy from firm i, including the distance he travels to obtain the mobile offer, to visit the store to make the actual purchase, and to come back home once the purchase is made. A consumer purchases at most one unit of the product and gets zero utility without a purchase.

In this section, we consider the situation in which information on all of the offers (both their existence and the associated prices) can spread across the consumers through word of mouth.[6] Such information can disseminate through two channels. First, consumers would spontaneously spread word of mouth on these offers. Second, mobile apps such as FourSquare, Yowza[7] and Google's Field Trip[8] track location-based coupons and present them on a map on the user's phone for easy perusing. As consumers become more familiar with geo-targeted offers, we also expect them to become more aware of such offers.

If consumers are only aware of their home-location offers, the model becomes one of traditional targeting. If consumers know about the existence of the other offers but do not know the associated prices, the market is then subject to the classic hold-up problem discussed in the consumer search literature (e.g., Anderson and Renault 2006): firms will have an incentive to raise the price once consumers have incurred the travel cost to arrive at a particular location. Anticipating this, consumers would refrain from traveling for better offers to begin with, making offers outside of their home locations irrelevant.

To ensure that firms' physical locations constitute the main source of differentiation, we assume that $t > s$. This is consistent with the idea that location-based targeting is naturally more relevant for product categories in which location is actually important. To fix ideas, we focus on a particular range of V, s, and t to illustrate the key trade-offs in the main model. At the end of this section, we show that our results are robust in other parameter ranges. The parameter range for the main model is defined by $2s < t < 4s$ and $2t < V < 2t + s$. The first set of inequalities ensures that a pure-strategy equilibrium exists under mobile geo targeting and the equilibrium prices differ under traditional and mobile geo targeting. The second set of inequalities ensures that the category willingness to pay is high enough for a firm to target consumers located near its competitor and low enough so that there is still room for demand expansion.

Consider the benchmark scenario in which the technology of mobile geo targeting is not available and each firm charges a uniform price to all consumers. Suppose that firms simultaneously choose their prices before consumers decide whether to buy one unit of the product, and if so, from which firm. We characterize the equilibrium of this game below.

Proposition 1. *Under uniform pricing, each firm remains a local monopoly and sells to all of its home-base consumers. Residents at ½ do not purchase from either firm. A firm's optimal price and equilibrium profit are both $V - s$.*

Proofs of all propositions, lemmas, and corollaries are in the appendix. Proposition 1 suggests that a firm would set its price such that all local residents (i.e., residents at the firm's own location) would buy its product, while other consumers do not buy its product.

Suppose now the technology of mobile geo targeting becomes available to both firms, enabling them to set prices based on the real-time location of a consumer. Consider now the new game in which the firms simultaneously adopt mobile geo targeting with a separate price charged at each location (if the prices charged by a firm at all locations happen to be the same, then its mobile geo targeting degenerates to uniform pricing). As consumers obtain offers on their mobile devices, they can travel across different locations to maximize the total utility of buying. We assume that consumers have rational expectations on firms' location-based prices. As a tie-breaking rule, we also assume that consumers do not travel or choose to travel less when they are indifferent.

To keep the analysis tractable, we focus on deriving the symmetric equilibrium in which both firms adopt the same pricing strategy.[9] As a first step, we show that mobile geo targeting would disrupt the uniform pricing equilibrium.

Lemma 1. *When mobile geo targeting is available, there does not exist a symmetric equilibrium with uniform pricing.*

Intuitively, the ability to charge different prices at different locations gives a firm increased flexibility to compete with the other firm. For example, it could potentially enable a firm to increase demand at distance ½ without decreasing its home-base profit. As a result, a firm always finds it attractive to adopt the mobile geo-targeting technology once it becomes available.

Since uniform pricing is no longer part of the equilibrium, we investigate next whether an equilibrium in which both firms adopt mobile geo targeting can be sustained. To characterize the firms' pricing strategies under mobile geo targeting, consider consumers' total cost of buying that equals the price he pays plus the travel cost he has to incur. Table 1 summarizes this total cost for all consumers, with $\{p_0, p_{½}, p_1\}$ denoting a firm's prices for consumers located at *distances* $\{0, ½, 1\}$ from the firm in real time.

Based on the consumers' total cost of buying, we make the following observations.

Lemma 2. *A symmetric equilibrium with mobile geo targeting must satisfy the following properties*: (a) *demand at distance* 0 *is positive for each firm,* (b) *a resident at distance* 0 *from a firm has* (*weakly*) *lower total cost of buying when using the offer at his home location, i.e.,* $p_0 \leq \min\{p_{½} + t, p_1 + 2t\}$, (c) *a resident at distance ½ from a firm has* (*weakly*) *lower total cost to buy with his home-location offer than to buy with the firm's offer at distance* 1, *i.e.,* $p_{½} \leq p_1 + t$, *and* (d) *demand at distance* 1 *is* 0 *for each firm, i.e.,* $p_1 + 2t - p_0 \geq s$.

The intuition of Lemma 2 is as follows. First, demand from local residents has to be positive. Since travel cost is significant in our model, at least the perfectly matched local resident would find it optimal to buy from the firm at his home base. In addition, by preventing consumers from traveling for better offers, firms could profit from the consumers' saved travel costs. For example, a local consumer would not find it optimal to obtain the offer at distance ½ or 1. This is because if he does, then the firm would find it more profitable to lower the price at its home base to induce the consumer to buy at his home location. By doing so, the consumer can save on his travel cost and pay the firm a higher price at his home location. Similarly, if a resident at distance ½ travels to distance 1 to obtain the poaching offer there, the firm located at 0 could once again lower the price at distance ½ to induce the consumer to buy from it and profit from his saved travel cost. Finally, given the relative importance of the travel cost and taste mismatch, a firm finds it optimal to fight the competitor out of its home base as local residents yield the highest profit margin.

To capture how firms balance prices across locations to prevent mobile-deal seeking, we formalize their optimal pricing strategy under mobile geo targeting.

Proposition 2. *Under mobile geo targeting, each firm charges prices* $\{2t - s, t - s, 0\}$ *to consumers located at distances* $\{0, ½, 1\}$. *The profit for each firm is* $5t/2 - 3s/2$ *and all consumers are served in equilibrium.*

The firms' equilibrium prices above are driven by their incentive to use mobile geo targeting to expand

Table 1. Consumers' Total Cost of Buying under Mobile Geo Targeting

	Residents at 0	Residents at ½	Residents at 1
Firm A's price	p_0	$p_{½}$	p_1
Cost of buying from Firm A	$p_0, p_{½} + t, p_1 + 2t$	$p_0 + t, p_{½} + t, p_1 + 2t$	$p_0 + 2t, p_{½} + 2t, p_1 + 2t$
Firm B's price	p_1	$p_{½}$	p_0
Cost of buying from Firm B	$p_0 + 2t, p_{½} + 2t, p_1 + 2t$	$p_0 + t, p_{½} + t, p_1 + 2t$	$p_0, p_{½} + t, p_1 + 2t$

Notes. We list the total cost of buying with all three available offers, e.g., for top cell #1, the total cost is p_0 if the consumer simply buys from Firm A with its offer at location 0, $p_{½} + t$ if he travels to location ½ to get $p_{½}$ and then comes back to buy from Firm A, and $p_1 + 2t$ if he travels to location 1 to get p_1 and then comes back to buy from Firm A.

demand with lower targeted prices, and their incentive to prevent deal seeking from high-margin local residents. Intuitively, a positive price at the competitor's home base cannot be sustained as the two firms would fight a price war until the poaching firm is driven out of the focal firm's home base at price zero. As a result, a resident at 0 or 1 can always travel to the opposite end of the line to buy the product with a total cost of $2t$. To retain all of the local consumers, a firm has to charge a price that is not higher than $2t - s$. When $2t - s$ is charged at the home base, the price at the middle location needs to be at least $t - s$ to prevent local residents from traveling to the middle location for a better deal. These optimal prices, when put together, yield the following comparative statics on firm profit.

Corollary 1. *A firm's equilibrium price and profit under mobile geo targeting increase with t and decrease with s.*

Essentially, when t increases, it is harder for consumers to obtain mobile offers outside of their home locations, and firms can hence increase equilibrium prices. When s increases, on the other hand, firms have to lower their home-base prices to keep all local residents, obtaining a lower profit in equilibrium.

Comparing mobile geo targeting to uniform pricing, one can see that the technology lowers market price at all locations and increases overall market coverage. As with traditional forms of targeting, the ability for firms to price discriminate against different consumers allows them to expand their consumer base, although it comes at the cost of intensified price competition in each submarket. Different from traditional forms of targeting, however, the firms' incentive to balance prices across locations to prevent mobile-deal seeking helps limit interfirm price competition and enhance firm profitability.

Proposition 3. *Mobile geo targeting increases firms' equilibrium profit from uniform pricing iff $V < (5t - s)/2$.*

Proposition 3 suggests that mobile geo targeting enhances firm profitability when the consumers' willingness to pay for the product category is low, their travel cost is high, and their preference regarding different firms is weak. The intuition of this result lies in how firms trade off demand expansion, interfirm price competition, and their incentives to mitigate intrafirm competition across different locations as firms move from uniform pricing to mobile geo targeting. While each firm's demand in our model always increases from 1 to 1.5, the decrease in prices is less significant when the condition in Proposition 3 is satisfied. When the travel cost t is higher, consumers have a decreased ability to travel and firms can charge higher prices under mobile geo targeting. When consumer preference becomes weaker (i.e., s is lower), price increases for more consumers under mobile geo targeting than under uniform pricing, as demand is higher in the first case. Finally, when consumers have a lower willingness to pay, V, for the product category, their profit under mobile geo targeting remains unchanged as the equilibrium prices are determined by intrafirm price competition across locations. On the other hand, they make less profit under uniform pricing as price needs to be lowered to retain all local residents. Overall, demand increases and price decreases as firms adopt mobile geo targeting and consumers are better off at all locations. Given Proposition 3, that is to say, there exist conditions under which both firms and consumers are strictly better off under mobile geo targeting than under uniform pricing.

When only a fraction of consumers are mobile accessible to the firms, firms would treat the two types of consumers (nonaccessible and accessible) as two different markets in which they practice uniform pricing and mobile geo targeting, respectively. As a result, firms' prices depend not only on the consumer's location but also on his accessibility to mobile geo targeting: even consumers with zero distance to a firm may receive a discount from the firm once they become accessible to mobile geo targeting, which prevents them from seeking even bigger discounts at other locations. This insight is formalized below and provides a good explanation to why retailers such as Starbucks, Toys "R" Us, Talbots, Peets Coffee, and Kohl's, offer mobile-based discounts to consumers who have already arrived at their stores.[10]

Corollary 2. *If only a fraction of the consumers are accessible to mobile geo targeting, in every location, consumers who are accessible to mobile geo targeting pay a lower price than those who do not.*

Another factor that could affect the equilibrium prices under mobile geo targeting is the distribution of consumers across different locations. Suppose the mass of residents at location ½ is k ($k > 0$) and k is small enough so that the firms remain local monopolies in the uniform pricing equilibrium.[11] We find that when there are more consumers at the middle (i.e., k increases), firms' profit under mobile geo targeting would increase because the category demand expands more. If we fix the total market size, however, profit decreases with the proportion of residents at the middle because local residents yield a higher margin.

Last, we also explore what happens once we step out of the assumed parameter region, with detailed derivation of the equilibrium outcomes in the online appendix. In general, when s/t decreases, firms find it optimal to decrease equilibrium prices to $\{t + 3s, 3s, 0\}$ at distances $\{0, ½, 1\}$: a lower s/t means that the competition at any given location becomes more fierce as consumers care less about the difference between the two firms' offerings. When the category willingness to pay decreases, on the other hand, the firms lower equilibrium prices to $\{V-s, V-t-s, 0\}$ at distances $\{0, ½, 1\}$ to make sure that all consumers would make a purchase.

In summary, our exploration of the expanded parameter region suggests that as long as there exists a pure strategy equilibrium under both mobile geo targeting and uniform pricing, mobile geo targeting (M) outperforms uniform pricing (U) when the category reservation price is low, i.e., $V < \min\{t+11s/2,\ 5t/2-s/2\}$, and the general intuition that firms' incentives to avoid intrafirm cannibalization mitigate interfirm price competition remains to hold. As in our main model, in order for mobile geo targeting to outperform uniform pricing, we need the category willingness to pay to be reasonably low so that demand expansion can dominate the reduction in price. Again, movies and café snack foods could be good examples of such nonnecessity product categories.

3. Extensions

In this section, we develop three extensions of the main model. To keep the analysis tractable and make the results more comparable, we use the same parameter range as in the main model for all of the extensions. In addition, each extension extends the model in a different direction, so that new features introduced in one extension, such as naïve consumers and consumers who stumble on mobile offers, do not carry over to the other extensions unless otherwise mentioned.

3.1. Coexistence of Informed and Naïve Consumers

In our main model, all consumers are strategic and seek the best overall offer. The model fits a scenario in which information on all mobile offers is readily available to consumers. Currently, the technology may still be in its early stage and some consumers are familiarizing themselves with location-based offers. In this extension, we investigate a situation in which a fraction h $(0 \le h \le 1)$ of residents at each location are informed of all available mobile offers, while others are targeted by offers at their permanent locations only and are "naïve" and remain unaware of offers outside of their permanent locations. That is, a naïve resident at 0 is only aware of p_0 from Firm A and p_1 from Firm B. Similarly, a naïve resident at ½ is only aware of $p_{½}$ from both firms. By definition, the case of $h=0$ corresponds to traditional targeting, and that of $h=1$ corresponds to mobile geo targeting as in our main model.

Proposition 4. *When the fraction of informed residents is small* $(h < (3t-2s-2\sqrt{2}\sqrt{t(t-s)})/(2s))$, *equilibrium prices are* $\{2t-s, (1+2h)s, 0\}$ *at distances* $\{0, ½, 1\}$. *An informed (naïve) resident at* 0 *or* 1 *buys from his local firm with its mobile offer at* ½ *(his home location), and a resident at* ½ *buys from his preferred firm with its offer at his home location. Equilibrium profit is* $2(1-h)t+[h(3+2h)-1/2]s$ *and decreases with h. When the fraction of informed residents is large* $(h \ge (3t-2s-2\sqrt{2}\sqrt{t(t-s)})/(2s))$, *the equilibrium outcomes are the same as those in the main model. In both cases, the equilibrium profit under mobile geo targeting is greater than that under uniform pricing if the category willingness to pay is low.*

When the fraction of informed residents is small, firms' incentive to prevent deal seeking is weak as the loss from intrafirm cannibalization is limited and they compete aggressively for the large amount of naïve consumers. In equilibrium, informed residents at 0 and 1 travel to the middle location for the significantly better offer before making a purchase. Equilibrium profit in this case decreases with the fraction of informed residents as the dominant effect of deal seeking is to cannibalize high-margin sales from local informed residents.

Once the fraction of informed residents reaches a certain threshold, both firms find it optimal to raise prices substantially at the middle location, from $(1+2h)s$ to $t-s$, to prevent deal seeking from occurring in equilibrium. Both firms experience a significant increase in their profit at this threshold, due to the discontinuous jump in price at the middle and the complete prevention of equilibrium deal seeking.

Overall, while mobile-deal seeking could indeed occur in the early stages of mobile geo targeting when the fraction of informed residents is low, our general intuition that mobile geo targeting could outperform uniform pricing for low levels of category willingness to pay continues to hold. In particular, our results suggest that increasing the fraction of informed residents, through means such as direct advertising or supporting social media sites that promote information sharing among customers, can potentially benefit firms.

3.2. Consumer Travel for External Reasons

A consumer could stumble on mobile coupons when traveling for external reasons. For example, he may receive a mobile coupon for a movie on a business trip to a nearby office building. In this case, the cost of mobile-deal seeking becomes external to the purchase decision itself. To investigate the consequences of this possibility, we allow in this extension a fraction of the residents at each location to travel across all of the locations for reasons that are external to making a purchase from one of the two competing firms. They can collect the best mobile offer effortlessly when traveling for other purposes, and make a separate trip for the actual purchase later on.[12] In the example above, the consumer collects the best offer for the movie on his business trip, and makes a separate trip later on to watch the movie at the designated theatre. Formally, at each location, a fraction r $(0 < r \le 1)$ of the residents are mobile-deal collectors as described above, while the remaining residents remain unaware of mobile offers outside of their home locations. The endogenous travel cost for all residents, no matter which offers they collect or use, is the cost of visiting the firm of choice and coming back home afterward.

A critical difference between this extension and the previous one is that the cost of mobile-deal seeking now becomes negligible. In the following proposition, we discuss the impact of this change on the equilibrium outcomes.

Proposition 5. *When a fraction of the residents at each location can collect the best mobile offer when traveling for external reasons,* (a) *equilibrium prices at* 0 *and* 1 *are higher than when these residents have to incur a travel cost to obtain the offer,* (b) *the equilibrium price at distance* 1 *becomes* (*weakly*) *higher than the price at distance* ½, *and* (c) *the equilibrium profit under mobile geo targeting is* (*weakly*) *higher than that under uniform pricing.*

As deal collection becomes free, intrafirm price competition is intensified and firms equalize prices even further across locations. Because deal collectors at distances 0 and ½ no longer need to incur the travel cost to obtain p_1, lower levels of the poaching price p_1 would lead to more significant cannibalization of profits at these two locations. As a result, firms now offer their lowest prices at the middle location, ½, rather than at distance 1. The increase in the poaching price also makes the firms less defensive at their home bases. In equilibrium, prices at both firms' home locations, 0 and 1, are higher than when deal seeking is costly.

As in the previous extension, when the fraction of mobile-deal collectors is small, those collectors residing at 0 and 1 use the better middle-location offers as firms find it critical to compete for the large amount of naïve consumers at the middle. When the fraction of deal collectors exceeds a certain threshold, firms find it too costly to allow deal collections at the middle and charge the same price at all locations. In the extreme case of $r = 1$, all consumers can collect offers effortlessly and face the same effective price, and we are back to the uniform pricing benchmark.

It is noteworthy that mobile geo targeting in this extension is always weakly more profitable than uniform pricing. The intuition is that given its competitors' mobile offers in equilibrium, a firm can always deviate to the defensive strategy of charging the same price, $V - s$, across all locations. Since the competitor's poaching price is high, all local residents would purchase from the deviating firm, leading to a deviation profit of $V - s$. For the firm not to deviate, it must be that it can earn more than this level of profit under mobile geo targeting.

Putting this extension together with the previous extension, one can see that the general intuition that the ability of consumers to obtain offers outside of their home locations can mitigate interfirm competition at each location and potentially increase firm profit is robust to our assumptions on consumers' cost of obtaining the offers. Regardless of whether consumers internalize their travel costs to obtain the best offer or not, the mobile-deal seeking opportunity always tends to incentivize a firm to balance its prices across locations and hence weaken interfirm price competition.

3.3. Tracing Down Consumers' Base Locations

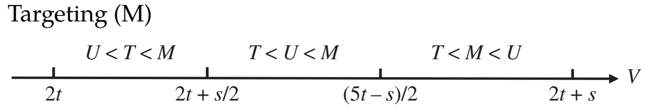
In this final extension, we consider the possibility for firms to trace down the consumers' home locations. Tracing can be implemented with either new technologies such as Placed and JiWire, which can be used to identify where consumers spend the bulk of their time and create audience profiles based on their location histories,[13] or traditional technologies such as obtaining a mailing list.

Suppose the firms can trace consumers and restrict them to obtain only offers at their home locations. If firms are already in the mobile geo-targeting equilibrium, they would have an incentive to adopt tracing if the fixed cost of the tracing technology is lower than $(t - 2s)^2/(8s)$.[14] Intuitively, tracing could immediately prevent deal seeking across locations and is hence attractive to the firms, although in a competitive setting it erodes equilibrium profit by intensifying price competition at each location. Conceptually, mobile targeting at consumers' home locations only, through tracing, is equivalent to traditional targeting, and firms in this case would charge prices $2t - s, s, 0$ to consumers at distances $0, ½, 1$.[15] The equilibrium profit is $2t - s/2$, which is lower than that under mobile geo targeting.

Integrating these results with Propositions 1 and 2, we present in Figure 2 the ranking of firm profit across the scenarios of uniform pricing (U), mobile geo targeting (M) as in the main model, and traditional targeting (T), which is equivalent to mobile geo targeting with tracing.

As can be seen from the figure, there exists a parameter range in which traditional targeting leads to lower profits than uniform pricing while mobile geo targeting leads to higher profits. Again, this is because while mobile geo targeting has a similar benefit to demand expansion as traditional targeting, it alleviates the negative effect of intensified price competition. As

Figure 2. (Color Online) Profit Ranking of Uniform Pricing (U), Traditional Targeting (T), and Mobile Geo Targeting (M)



Note. The third region above ($T < M < U$) appears only if $t < 3s$.

a result, although mobile and traditional targeting both reduce deadweight loss and expand market demand, the former can be more profitable in a competitive environment.

Finally, consider the possibility for firms to set their prices based on both the home and real-time locations of a consumer. Each firm in this case needs to determine nine prices based on the different combinations of three permanent locations and three real-time locations. A consumer can pull mobile offers at most one time. As in our main model, consumers are aware of the prices at different locations, based on which they decide whether to make a purchase and if so, from which firm and at which location to pull the offer.

Proposition 6. *Given any permanent (real-time) location, the equilibrium price from a firm decreases with the distance between the firm and the consumer's real-time (permanent) location. All consumers make a purchase with a home-location offer, and the price they pay and the firm from which they buy are all the same as under traditional targeting.*

Since location is the main source of firm differentiation in our model, prices decrease as consumers move further away from the firm. As mentioned before, pricing on the permanent location can directly prevent intrafirm mobile-deal seeking. Tempted by this immediate benefit, firms focus on the permanent locations in their pricing strategy and end up competing fiercely at each location. As a result, the equilibrium profit falls back to the level under traditional targeting.

4. Conclusion

In this paper, we show in a duopoly setting that targeting consumers based on their real-time locations on a mobile platform can increase firm profit from traditional targeting and uniform pricing. In essence, the ability for consumers to seek the best overall mobile deal incentivizes firms to balance prices across different locations and hence curtails interfirm price competition. We discuss conditions under which consumers travel for better deals in equilibrium, how firms' profit under mobile geo targeting varies with the fraction of informed consumers and their cost of seeking out the best deal, and the possibility for firms to price simultaneously on consumers' home and real-time locations.

Our results have important managerial implications for marketers who aim to optimize their mobile geo-targeting strategies. For example, managers should carefully trade off the benefit (i.e., demand expansion) and the cost (i.e., increased price competition) of mobile geo targeting when compared to uniform pricing. They should encourage more consumers to become mobile-deal accessible only if mobile geo targeting is beneficial, which tends to occur when the category willingness to pay is low, the transportation cost is high, and consumers' taste preference is weak. Mobile-deal seeking is more likely to benefit firms when the fraction of informed consumers is substantial. In addition, surprise and effortless collections of mobile deals tend to further equalize prices across locations and limit interfirm price competition. If the technology of tracing consumers' home locations is readily available at a low cost, firms may end up suffering from this technology because of the intensified price competition that it would bring about.

There are many interesting directions for future research. Some companies, for example, have been investing in connecting multiple devices of a consumer and building an integrated profile based on his purchase history, location history, demographics, and browsing habits. It would be useful to understand how these elements would interact with each other in shaping a user's purchase intent and correspondingly, the optimal way to target different users. Also, we have focused on a case in which consumers need to incur physical travel costs to visit the store to purchase the product, and as mobile payment matures, one can imagine situations in which consumers make purchases directly on their mobile device. In that case, the relevant cost might be the shipping cost, which could be partially absorbed by the firms. Finally, it may be worthwhile to investigate how asymmetry between firms may affect the effectiveness of competitive mobile geo targeting.

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Appendix

Proof of Proposition 1. If firms only serve the local residents, $\pi = p(V-p)/s$, where $V-s<p<V$. The profit maximizing price is $V/2$. Because $V/2<V-s$, the optimal price is $V-s$, and the corresponding profit is $V-s$. Given $s<t/2$, the middle location is not served because the utility of buying from either firm is $V-t-(V-s)<0$. To make sure that neither firm has an incentive to deviate to lowering its price to serve residents at the middle, i.e., $V-s>\max_{0\le\lambda\le1}(V-t-\lambda s)(1+\lambda)$, we need $V<s+2t$. This condition also ensures that neither firm has an incentive to serve the competitor's local residents, i.e., $V-s>\max_{0\le\lambda\le1}(V-2t-2\lambda s)(2+\lambda)$.

If firms serve residents at distances 0 and ½, but not residents at distance 1. There are two possible symmetric equilibra based on if residents at ½ are all served. As we show below, however, neither equilibrium exists.

First, if some residents at ½ do not buy from either firm, then we must have $V-t-p-s/2<0$, and so $p>V-t-s/2$.

In this case, firms' equilibrium profit is $p + p(V - t - p)/s$. The profit maximizing price is $(V - t - s)/2$, because $p > V - t - s/2 > (V - t - s)/2$, the optimal price is $V - t - s/2$. In this case, however, a firm has an incentive to deviate to charging $V - s$ because the deviation profit $V - s$ is higher than the equilibrium profit $3(V - t - s/2)/2$.

Second, if all residents at ½ are served, then $V - t - p - s/2 \geq 0$, and so $p \leq V - t - s/2$. Firms' profits are $\pi_A = p_A + p_A(p_B - p_A + s)/(2s)$ and $\pi_B = p_B + p_B(p_A - p_B + s)/(2s)$. Under the first-order condition, $p_A = (p_B + 3s)/2$, $p_B = (p_A + 3s)/2$, and the profit maximizing price is $3s$. If $3s \leq V - t - s/2$, i.e., $V \geq t + 7s/2$, then the equilibrium price is $3s$. In this case, however, a firm has an incentive to deviate to charging $V - s$ because the deviation profit $V - s$ is higher than the equilibrium profit $9s/2$ given $V \geq t + 7s/2$. If $3s > V - t - s/2$, then the equilibrium price is $V - t - s/2$ and from before, we know the equilibrium does not exist.

If firms serve all residents, then $p < V - 2t$. In this case, a firm has an incentive to deviate to charging $p + 2t - s$ because the lowest deviation profit $p + 2t - s$ is greater than the highest equilibrium profit $3p$, given $p < V - 2t$. Q.E.D.

Proof of Lemma 1. Consider the different ranges of prices that firms can charge in a symmetric uniform pricing equilibrium. First, suppose $p > V - s$. In this case we know from Proposition 1 that a firm can increase its profit by lowering its price to $p = V - s$. Second, suppose $V - t \leq p \leq V - s$ so that firms remain local monopolies. In this case a firm can profitably deviate to charging a different price at location ½, $p_{½} = V - t - s$, which would enable the firm to generate a positive profit at this location without decreasing its profit from local residents. Third, suppose $s < p < V - t$. In this case a firm can profit from lowering $p_{½}$ by a small number $\varepsilon > 0$ as the unconstrained optimal price at location ½ is s. Finally, suppose that $p \leq s$, a firm then can profitably deviate to charging $t + s$ at its own location. Q.E.D.

Proof of Lemma 2. To show Lemma 2(a), suppose in equilibrium the demand at distance 0 is zero for both firms. There are two possibilities. First, local residents do not buy from either firm. Then we must have $p_0 \geq V$, $p_{½} \geq V - t$, and $p_1 \geq V - 2t$. Then no one at any location buys from either firm. A firm can thus increase profit by setting p_0 lower than V. Second, at least some local residents buy from the competing firm (and the rest of the residents at distance 0, if there are any left, do not buy from either firm). These residents cannot buy from the competing firm at p_0 or $p_{½}$ because their total cost of shopping would be higher than if they buy from the local firm at p_0 or $p_{½}$. So these residents must buy from the competing firm at p_1. Then at least the local residents who have a perfect match with the local firm on the preference dimension should buy from the local firm at p_1. Therefore, neither possibility holds, so firms' demand at distance 0 must be positive.

To show Lemma 2(b), realize that if $p_0 > \min\{p_{½} + t, p_1 + 2t\}$, no one buys at p_0. Then a firm can increase profit by lowering p_0 to $\min\{p_{½} + t, p_1 + 2t\}$. When the firm does this, local residents switch from buying at $p_{½}$ or p_1 to buying at p_0, and the firm can gain an additional profit of t or $2t$ from these residents without affecting residents at other locations.

To show Lemma 2(c), realize that if $p_{½} > p_1 + t$, then no one buys at $p_{½}$. In this case, there are three possibilities. First, if $p_0 + 2t < p_1 + 2t$, then p_0 gives the lowest total cost of buying at all locations. This is equivalent to uniform pricing equilibrium, which, according to Lemma 1, does not exist under mobile geo targeting. Second, if $p_0 + t \leq p_1 + 2t \leq p_0 + 2t$, then p_0 gives the lowest total cost of buying at distance 0 and ½ and p_1 gives the lowest total cost of buying at distance 1. Because $p_0 + t \leq p_0 + 2t$, the demand at distance 1 must be zero because the difference in total cost of buying is greater than s. This is again equivalent to uniform pricing equilibrium, which, according to Lemma 1, does not exist under mobile geo targeting. Third, if $p_1 + 2t < p_0 + t$, then p_0 gives the lowest total cost of buying at distance 0 and p_1 gives the lowest total cost of buying at distance ½ and 1. A firm can deviate profitably by lowering $p_{½}$ to $p_1 + t$. In this deviation, consumers at distance 0 are not affected because of Lemma 2(b) and residents at distance 1 are not affected because $p_{½} + 2t$ would not be the lowest price, while at least some residents at location ½ would switch from buying at p_1 to buying at $p_{½}$, and the firm would hence make an additional profit of t on these consumers. Note that here, firms must have a positive demand at location ½ in equilibrium, as otherwise a firm has an incentive to lower $p_{½}$ to $p_0 - t (> p_1 \geq 0)$ to increase the firm's profit at ½, without affecting the profit at other locations.

To show Lemma 2(d), suppose without loss of generality that Firm B's demand at location 0 is not zero. We show below that it is profitable for Firm A in this case to lower p_0. Given Lemma 2(b) and Lemma 2(c), we have $p_0 \leq p_{½} + t \leq p_1 + 2t$. Therefore, a local resident would either buy from Firm A at p_0, or buy from Firm B. The only possible scenario in which demand for Firm B is positive is when the consumer buys from Firm B at price p_1 and $p_1 + 2t - s < p_0$, as the consumers' total cost of buying from Firm B at the other two prices are at least higher than p_0 by t $(t > s)$. Firm A's demand at location 0 is thus determined by $V - p_0 - sy \geq V - (p_1 + 2t) - s(1 - y)$, i.e., $y \leq (p_1 - p_0 + s + 2t)/(2s)$. Firm A's profit at location 0 is hence $p_0(p_1 - p_0 + s + 2t)/(2s)$ and it is optimal for the firm to decrease its price as long as $p_0 > (p_1 + s + 2t)/2$. Given $p_1 + 2t - s > (p_1 + s + 2t)/2$ whenever $p_1 > 0$, and that a decrease in p_0 for Firm A would not affect its profit at other locations as long as $p_0 > p_1 + t$ (implied by $p_0 > p_1 + 2t - s > p_1 + t$), Firm A finds it profitable to decrease its price. Q.E.D.

Proof of Proposition 2. From Lemma 2, we know that residents at location 0 and 1 buy at p_0 (although at this point we cannot exclude the possibility that some of them may not buy from either firm). In addition, we have $p_0 \leq p_{½} + t$ and $p_{½} \leq p_1 + t$, with at most one condition binding given $p_0 \leq 2t - s + p_1$.

(A) If $p_{½} \leq p_1 + t$ is binding, i.e., $p_{½} = p_1 + t$, we have $p_0 < p_{½} + t$ and $p_0 \leq 2t - s + p_1$. Since $p_1 \geq 0$, $p_{½} \geq t$. Since $t > s$ (s is the unconstrained optimal price at ½ for two competing firms), both firms have incentives to lower prices to get a higher profit from location ½; this can be done without affecting residents at distance 0 or 1 (because $p_0 < p_{½} + t$ is not binding). So this cannot be an equilibrium.

(B) If $p_0 \leq p_{½} + t$ is binding, i.e., $p_0 = p_{½} + t$, we have $p_{½} < p_1 + t$ and $p_0 \leq 2t - s + p_1$. In this case, firms' demand at distance 0 must be 1, i.e., $p_0 \leq V - s$. Suppose this is not true, i.e., if $V - s < p_0 < V$, then $V - t - s < p_{½} < V - t$ and $p_1 > V - 2t$. In this case, a firm can deviate and get a higher profit at distance 0 by setting $p_0 = V - s$, without affecting its profit at other locations.

Given $p_0 \le V - s$, we can further show that $p_0 \le 2t - s$, and then $p_{½} \le t - s$ and $p_{½} \le t - s + p_1$. So in equilibrium, firms' demand is 1 at distance 0 and ½ at distance ½, and firms' profit is $p_{½} + t + p_{½}/2 = t + 3p_{½}/2$. Suppose this is not true, i.e., if $2t - s < p_0 \le V - s$, then $t - s < p_{½} \le V - t - s$. Then a firm can deviate profitably by setting p_1 to be just below $p_0 - (2t - s)$ because by doing so, the firm can have a higher profit at distances 1 and ½ without affecting its profit at distance 0.

Given $p_0 \le 2t - s$ and $p_0 = p_{½} + t$, p_1 must be 0. Suppose this is not true, i.e., if $p_1 > 0$, then a firm can deviate profitably by increasing both p_0 and $p_{½}$: if the firm increases $p_{½}$ to p_E and increases p_0 to $p_E + t$, its deviation profit is $p_E(p_{½} - p_E + s)/(2s) + (p_E + t)$, which is higher than the equilibrium profit when $p_E > p_{½}$ as long as $p_{½} < 3s$, which is true since $p_{½} \le t - s < 3s$. Similarly we can show that given $p_1 = 0$, p_0 cannot be lower than $2t - s$, otherwise a firm can deviate profitably by raising p_0 and $p_{½}$. Therefore, the equilibrium prices are $\{2t - s, t - s, 0\}$ at distance $\{0, ½, 1\}$, and the equilibrium profit is $\pi = 5t/2 - 3s/2$.

Below we check a firm's incentives to deviate from these prices.

First, the firm has no incentive to lower p_0 because it would lead to a lower profit from distance 0 and at the same time, without affecting the profit from distance ½ or 1 (if p_0 is lowered to a price at least equal to $p_{½}$) or even decreasing the profit from some residents at distance ½ by t (if p_0 is lowered to a price less than $p_{½}$).

Second, the firm also has no incentive to raise p_0 because it would lead to a profit loss of t from distance 0 without getting additional profit from distance ½ or 1, unless it raises $p_{½}$ by the same amount and also changes p_1 if needed. In the latter case, there are two possibilities. (1) If after the raise, $p_0 = p_{½} + t \le p_1 + 2t$, then this deviation is not profitable at distance 0 (based on the proof for Lemma 2(d)) or distance ½ (because $p_{½}$ is further away from the optimal price s when considering the competition at this location alone). (2) If after the raise, $p_1 + 2t < p_0 = p_{½} + t$, then basically one firm deviates to uniform pricing, which is also unprofitable as we show at the end of this proof.

Third, the firm has no incentive to raise $p_{½}$ because it would deviate further from the optimal price s when considering the competition at this location alone, and the firm would therefore lose profit from distance ½ without getting additional profit from 0 or 1.

Fourth, if the firm lowers $p_{½}$, we derive the deviation profit for the firm as follows. If the firm deviates and lowers $p_{½}$ to p_D, it would get more consumers at ½, but lose profit t from consumers at distance 0 unless it lowers p_0 by the same amount; in the latter case, it would lose profit $p_{½} - p_D$ from consumers at distance 0. Because $p_{½} - p_D < t$, the firm would find it profitable to lower p_0 as well if it deviates. So the deviation profit is $p_D(p_{½} - p_D + s)/(2s) + (p_D + t)$, and this is less than the equilibrium profit when $p_D < p_{½}$ as long as $p_{½} < 3s$, which is satisfied since $3s > t - s$.

Last, the firm has no incentive to increase p_1 because it would not affect profits at any location.

(C) If neither condition is binding, i.e., $p_0 < p_{½} + t$ and $p_{½} < p_1 + t$, then we can show that it cannot be an equilibrium. If consumers cannot travel to get a lower price (i.e., there are constraints to keep the consumers from traveling), the optimal price at distance 0 is $2t - s$, and the optimal price at distance ½ is s. Since $2t - s > s + t$, if $p_0 \le p_{½} + t$ is not binding, it must be that at most one price is at its optimal price. Then the firm would have an incentive to move the other price closer to the optimal price. For example, if $p_{½} \le s$, then $p_0 < s + t < 2t - s$ and the firm would have an incentive to raise p_0 to get more profit from distance 0 without affecting its demand at distance ½. If $p_{½} > s$, then the firm would have an incentive to lower $p_{½}$ to get more profit from distance ½ without affecting its demand at distance 0 or 1. So this cannot be an equilibrium.

As a final step of the proof, we show that the equilibrium above would not break down because of firms deviating to the uniform pricing strategy. Without loss of generality, check Firm B's incentive to deviate. Given Firm A's equilibrium prices, the most profitable deviation in uniform pricing for Firm B is to charge $2t - s$ and sell to every consumer at distance 0. The deviation profit is $2t - s$, which is always lower than Firm B's equilibrium profit given $s < t$. Q.E.D.

Proof of Corollary 1. The corollary follows directly from Proposition 2.

Proof of Proposition 3. The firms' profit under uniform pricing is $V - s$, and their profit under mobile geo targeting is $5t/2 - 3s/2$. Therefore, profit under mobile geo targeting is higher if $5t/2 - 3s/2 > V - s$, which can be reduced to $V < 5t/2 - s/2$. Q.E.D.

Proof of Corollary 2. The corollary follows directly from Propositions 1 and 2.

Proof of Proposition 4. We show that the equilibrium outcomes are as follows.

(1) If $0 \le h < (3t - 2s - 2\sqrt{2}\sqrt{t(t-s)})/(2s)$, prices are $\{2t - s, (1 + 2h)s, 0\}$, and deal seekers at 0 choose the mobile offer at ½. Profit is $2(1 - h)t + (h(3 + 2h) - 1/2)s$, which is greater than $V - s$ if $V < (s + 6hs + 4h^2s + 4t - 4ht)/2$.

1. If $1 \ge h \ge (3t - 4s - \sqrt{t(5t - 8s)})/(4s)$, prices are $\{2t - s, t - s, 0\}$, and everyone uses the offer at his home location. Profit is $2t - s + (t - s)/2$, which is greater than $V - s$ if $V < (5t - s)/2$.

(2) If $(2s)^{-1}(3t - 2s - 2\sqrt{2}\sqrt{t(t-s)}) \le h < (4s)^{-1}(3t - 4s - \sqrt{t(5t - 8s)})$, prices are $\{2t - s, (1 + 2h)s, 0\}$ or $\{2t - s, t - s, 0\}$; the latter has a higher profit, so the best equilibrium is $\{2t - s, t - s, 0\}$, profit is $2t - s + (t - s)/2$, which is greater than $V - s$ if $V < (5t - s)/2$..

Below we explain how the equilibrium outcomes are derived. There are two possible symmetric equilibria:

(1) Some or all deal seekers travel for better deals in equilibrium.

(2) No one travels for better deals in equilibrium.

Case 1. *Some or all deal seekers travel for better deals in equilibrium.*

There are four possibilities:

(a) $p_0 > p_{½} + t$ and $p_{½} \le p_1 + t$.

In this case, the effective prices for the naïve residents are $\{p_0, p_{½}, p_1\}$, and the effective prices for the informed residents are $\{p_{½}, p_{½}, p_1\}$. That is, the naïve residents at 0 (1) can either buy from Firm A at p_0 (p_1) or buy from Firm B at p_1(p_0), and the naïve residents at ½ buy from either firm at $p_{½}$. The informed residents at 0 (1) can either buy from Firm A at $p_{½}$

(p_1) or buy from Firm B at $p_1(p_{1/2})$, and the informed residents at ½ can buy from either firm at $p_{1/2}$.

Because $p_0 > p_{1/2}+t$, more naïve residents at 0 (1) buy from Firm A (Firm B) than informed residents at 0 (1). The optimal p_0 and p_1 thus can be determined by solving the competition at location 0 (or location 1) for naïve residents, where we have $V - sy - p_0 = V - s(1-y) - 2t - p_1$, which leads to $y = (p_1 - p_0 + s + 2t)/(2s)$. Therefore, $p_0 = (p_1 + s + 2t)/2$, $p_1 = (p_0 + s - 2t)/2$, and $p_0^* = s + 2t/3$, $p_1^* = s - 2t/3$. Given $s < t/2$, $p_1^* < 0$. So we have to set $p_1^* = 0$. If $p_1^* = 0$, it can be derived that Firm A finds it optimal to sell to all local residents. So p_0 is derived by $V - s - p_0 = V - 2t - 0$, which leads to $p_0^* = 2t - s$. Because $p_0 > p_{1/2} + t$, we know that all informed residents at location 0 (location 1) will buy from Firm A (Firm B) as well.

From $p_0 > p_{1/2} + t$ and $p_{1/2} \le p_1 + t$, we know that $p_{1/2} < t - s$ and so the market at ½ is completely covered. To find the optimal price $p_{1/2}$, each firm considers both the competition at location ½ and its profit from local informed residents. Firm A maximizes $p_A(p_B - p_A + s)/(2s) + p_A h$ and Firm B maximizes $p_B(p_A - p_B + s)/(2s) + p_B h$. Therefore, $p_A = (p_B + s + 2hs)/2$, $p_B = (p_A + s + 2hs)/2$, and $p_A^* = p_B^* = (1+2h)s$, i.e., $p_{1/2}^* = (1+2h)s$.

Therefore, the equilibrium prices are $\{2t - s, (1+2h)s, 0\}$ and the equilibrium profit is $(2t-s)(1-h) + (1+2h)s/2 + h(1+2h)s = 2(1-h)t + (h(3+2h) - 1/2)s$. Because $p_{1/2} < t - s$, this equilibrium holds only if $h < t/(2s) - 1$. Below we check a firm's incentives to deviate from these prices and show that this equilibrium holds only if $h < (3t - 4s - \sqrt{t(5t-8s)})/(4s)$.

First, the firm has no incentive to increase p_1 because it would not change its profit.

Second, the firm has no incentive to increase p_0 because it would lead to a lower profit from naïve local residents without affecting the profit at other places or from informed residents.

Third, the firm has no incentive to decrease p_0 if $h < (t - \sqrt{4s^2 - 2st + t^2})/(2s)$. This can be shown as follows. If the firm lowers p_0, the most profitable deviation is to charge t + (1+2h)s so that the profit from the informed residents at distance 0 would increase by t. In this case, the deviation profit is $t + (1+2h)s + (1+2h)s/2$, which is smaller than the equilibrium profit only if $h < (t - \sqrt{4s^2 - 2st + t^2})/(2s)$.

Fourth, the firm has no incentive to increase $p_{1/2}$ if $h < (3t - 4s - \sqrt{t(5t-8s)})/(4s)$. This can be shown as follows. If the firm increases $p_{1/2}$, the most profitable deviation is to charge $t - s$ so that the profit from local informed residents would increase by t. In this case, the deviation profit is $(2t - s) + (t-s)((1+2h)s - (t-s) + s)/(2s)$, which is smaller than the equilibrium profit only if $h < (3t - 4s - \sqrt{t(5t-8s)})/(4s)$.

Last, the firm has no incentive to decrease $p_{1/2}$ because it would simply deviate from its optimal price at ½ without improving its profit at location 0 or 1.

Because

$$\frac{3t - 4s - \sqrt{t(5t-8s)}}{4s} < \frac{t - \sqrt{4s^2 - 2st + t^2}}{2s} \quad \text{and}$$
$$\frac{3t - 4s - \sqrt{t(5t-8s)}}{4s} < \frac{t}{2s} - 1,$$

we conclude that the equilibrium holds if $h < (3t - 4s - \sqrt{t(5t-8s)})/(4s)$.

(b) $p_0 > p_{1/2} + t$ and $p_{1/2} > p_1 + t$.

In this case, the effective prices for the naïve residents are $\{p_0, p_{1/2}, p_1\}$, and the effective prices for all informed residents are p_1. That is, the naïve residents at 0 (1) can either buy from Firm A at p_0 (p_1) or buy from Firm B at p_1 (p_0), and the naïve residents at ½ buy from either firm at $p_{1/2}$. The informed residents can buy from either firm at p_1. This equilibrium, however, cannot hold. This can be shown as follows. From $p_{1/2} > p_1 + t$, we know $p_{1/2} > t > s$, where s is the unconstrained optimal price at ½. So each firm has an incentive to decrease $p_{1/2}$ to get more naïve residents at ½; by doing so, the firm gets a higher profit from the naïve residents without affecting its profit from the informed residents, as long as the deviation price is higher than $p_1 + t$.

(c) $p_0 \le p_{1/2} + t$ and $p_{1/2} > p_1 + t$.

Depending on if p_0 is greater than $p_1 + 2t$, there are two possible cases. First, if $p_0 > p_1 + 2t$, we have $p_{1/2} + t \ge p_0 > p_1 + 2t$. In this case, the effective prices for the naïve residents are $\{p_0, p_{1/2}, p_1\}$, the effective prices for the informed residents are $\{p_1, p_1, p_1\}$ and $p_{1/2} > t > s$. Second, if $p_0 < p_1 + 2t$, we have $p_{1/2} + t > p_1 + 2t > p_0$. In this case, the effective prices for the naïve residents are $\{p_0, p_{1/2}, p_1\}$, the effective prices for the informed residents are $\{p_0, p_1, p_1\}$ and $p_{1/2} > t > s$. In both cases, the equilibrium cannot hold because each firm has an incentive to decrease $p_{1/2}$ to get more naïve residents at ½; by doing so, the firm gets a higher profit from the naïve residents without affecting its profit from the informed residents, as long as the deviation price is higher than $p_1 + t$.

(d) $p_0 \le p_{1/2} + t$ and $p_{1/2} \le p_1 + t$.

In this case, the effective prices for the naïve residents are $\{p_0, p_{1/2}, p_1\}$, and the effective prices for the informed residents are $\{p_0, p_0, p_0\}$. In this case, we must have $p_{1/2} = s$ and $p_1 = 0$, and therefore $p_0 \le s + t$. Thus, the equilibrium profit is $p_0(1 + h/2) + (1-h)s/2$ if $p_0 \le V - t - s/2$, or $p_0(1 + (V - t - p_0)/s) + (1-h)s/2$ if $V - t - s/2 < p_0 < V - t$, or $p_0 + (1-h)s/2$ if $V - t \le p_0 \le t + s$. So the highest possible equilibrium profit is either $(V - t - s/2)(1 + h/2) + (1-h)s/2$ or $t + s + (1-h)s/2$. Then each firm has an incentive to deviate to $p_0 = 2t - s$ because the deviation profit $2t - s + (1-h)s/2$ is greater than the highest possible equilibrium profit given $2s < t < 4s$ and $2t < V < 2t + s$.

Case 2. *No one travels for better deals in equilibrium*.

In this case, we must have $p_0 \le p_{1/2} + t$ and $p_{1/2} \le p_1 + t$. Then following the same proof for Lemma 2(d), we know that demand at distance 1 is 0 for both firms, i.e., $p_1 + 2t - p_0 \ge s$. We can then follow the same proof for Proposition 2 to show that the equilibrium is $\{2t - s, t - s, 0\}$ and no deviation from equilibrium requires $h \ge (3t - 2s - 2\sqrt{2}\sqrt{t(t-s)})/(2s)$. The proof is similar to the proof for Proposition 2 with the additional proof of no deviation, for which the detail is given below.

The no-deviation condition is derived by checking the firm's incentive to lower price $p_{1/2}$. If the firm deviates and lowers $p_{1/2}$ from $t - s$ to p_D without changing p_0, it would get more residents at ½, but lose profit from local informed residents. In this case, the deviation profit is $p_D(p_{1/2} - p_D + s)/(2s) + hp_D + (1-h)p_0$, which is less than the equilibrium profit for all possible values of $p_D < p_{1/2}$ if $h \ge (3t - 2s - 2\sqrt{2}\sqrt{t(t-s)})/(2s)$. If the firm deviates and lowers $p_{1/2}$ from $t - s$ to p_D and at the same time, lowers p_0 by the same amount, it would lose profit $p_{1/2} - p_D$ from local residents. In this case, the deviation profit is $p_D(p_{½} - p_D + s)/(2s) + (p_D + t)$, and this is less than the equilibrium profit when $p_D < p_{½}$ as long as $p_{½} < 3s$, which is satisfied since $3s > t - s$. Q.E.D.

Proof of Proposition 5. In equilibrium, the effective prices for the naïve residents are $\{p_0, p_{½}, p_1\}$, and the effective prices for the deal collectors are $\min\{p_0, p_{½}, p_1\}$ at all locations. That is, the naïve residents at 0 (1) can either buy from Firm A at p_0 (p_1) or buy from Firm B at $p_1(p_0)$, and the naïve residents at ½ buy from either firm at $p_{½}$. The deal collectors can buy from either firm at $\min\{p_0, p_{½}, p_1\}$.

We first note that in equilibrium, p_0 cannot be lower than $\min\{p_{½}, p_1\}$, otherwise either firm can get a higher profit by simply setting $p_1 = p_0$ and increasing p_0; by doing so, the firm's profit from their local naïve residents increases and its profit from other residents is unchanged.

Since $p_0 \geq \min\{p_{½}, p_1\}$, the optimal p_0 can be determined by solving the competition at location 0 (or location 1) for the naïve residents. From $V - sy - p_0 = V - s(1-y) - 2t - p_1$, we get $y = (p_1 - p_0 + s + 2t)/(2s)$. So the optimal $p_0 = (p_1 + s + 2t)/2$, which suggests that $y = (p_1 + s + 2t)/(4s) > 1$ given $p_1 > 0$ and $t > 2s$. Therefore, a firm always has an incentive to get all naïve local residents, so in equilibrium we must have $p_0 = \min\{p_1 + 2t - s, V - s\}$. This also implies that neither firm gets a positive demand from the naïve residents at distance 1.

Next, we show that in equilibrium we must have $p_1 \geq p_{½}$. Suppose in equilibrium, $p_1 < p_{½}$, then the effective prices for the informed residents are $\{p_1, p_1, p_1\}$. The optimal $p_{½}$ thus should equal the unconstrained optimal price at location ½ for the naïve residents, which is s. To solve for the optimal p_1, note there are two possible cases based on if location ½ is fully covered for the deal collectors who reside there. If it is fully covered, each firm considers both the competition at location ½ and its profit at distance 0 from the deal collectors. Firm A maximizes $rp_A(1 + (p_B - p_A + s)/(2s))$ and Firm B maximizes $rp_B(1 + (p_A - p_B + s)/(2s))$. Therefore, $p_A = (p_B + 3s)/2$, $p_B = (p_A + 3s)/2$, and $p_A^* = p_B^* = 3s$. This contradicts the presumption that $p_1 < p_{½}$. If location ½ is not fully covered, $p_1 > V - t - s/2 > s$ since $V > 2t > t + 3s/2$. This contradicts the presumption that $p_1 < p_{½}$.

Thus, we conclude that in equilibrium, the effective prices for the naïve residents are $\{p_0, p_{½}, p_1\}$, and the effective prices for deal collectors are $\{p_{½}, p_{½}, p_{½}\}$ and $p_1 \geq p_{½}$. We now solve the optimal prices based on whether location ½ is fully covered.

(1) Location ½ is fully covered.
In this case, Firm A maximizes

$$rp_A\left(1 + \frac{p_B - p_A + s}{2s}\right) + (1-r)p_A\frac{p_B - p_A + s}{2s},$$

and Firm B maximizes

$$rp_B\left(1 + \frac{p_A - p_B + s}{2s}\right) + (1-r)p_B\frac{p_A - p_B + s}{2s}.$$

Therefore,

$$p_A = (p_B + s + 2rs)/2, \quad p_B = (p_A + s + 2rs)/2, \quad \text{and}$$
$$p_A^* = p_B^* = (1 + 2r)s,$$

i.e., $p_{½} = (1 + 2r)s$. Because $p_1 \geq p_{½}$, and $p_0 = \min\{p_1 + 2t - s, V - s\}$, we have $p_0 = V - s$ given $V < 2t + s$. Therefore, the equilibrium prices are $\{V - s, (1 + 2r)s, \text{any price} \geq (1 + 2r)s\}$ and the equilibrium profit is $(1 - r)(V - s) + (1 + 2r)^2 s/2$.

For location ½ to be fully covered we need $t > 3s/2 + 2rs + t$. We then further check a firm's incentive to deviate from these prices under this condition. First, the firm has no incentive to increase p_0 because it reduces profit from the naïve local residents without affecting the profit from other residents. Second, the firm has no incentive to decrease p_0 because it reduces the profit from the naïve local residents and either does not affect the profit from other residents, or even decreases the profit from other residents if the lowered p_0 is smaller than $(1 + 2r)s$. Third, the firm has no incentive to decrease or increase $p_{½}$ as long as $p_1 \geq p_{½}$ because $(1 + 2r)$ is the optimal price for $p_{½}$ based on previous derivations. Fourth, if the firm changes both p_1 and $p_{½}$ so that $p_1 < p_{½}$, there are two possibilities. First, $p_1 \geq V - 2t$. In this case, the deviating firm still gets no consumers from distance 1, charges $p_{½}$ to naïve residents at location ½, and charges p_1 to deal collectors who reside at distance 0 and ½. We can show that the optimal deviation price p_1 is in fact larger than the optimal deviation price $p_{½}$, violating the assumption that $p_1 < p_{½}$. Second, $p_1 < V - 2t$. In this case, the deviating firm gets some naïve residents at distance 1 (but no informed residents there), charges $p_{½}$ to naïve residents at location ½, and charges p_1 to deal collectors who reside at distance 0 and ½. The highest deviation profit is when the deviation prices are $p_{½} = (1 + r)s$ and $p_1 = (2rs(2 + r) + (1 - r)(V - 2t))/2 < V - 2t$. This deviation profit is less than the equilibrium profit only if $r > 0.483$ or if $r < 0.483$ and $V < 2((r(2 + r)s)/(1 + r) + t)$, where $r = 0.483$ is the approximate solution to $1 + 1/(1 + r) = \sqrt{1 - (1 - r)r}/(1 - r)$. Last, the firm may deviate to uniform pricing $V - s$. The deviation profit $V - s$ is less than the equilibrium profit only if $V < ((1 + 6r + 4r^2)/(2r))s$.

In sum, the equilibrium holds if the following three conditions hold: (1) $V > 3s/2 + 2rs + t$, (2) $r > 0.483$ or both $r < 0.483$ and $V < 2((r(2 + r)s)/(1 + r) + t)$, and (3) $V < ((1 + 6r + 4r^2)/(2r))s$. Because $2((r(2 + r)s)/(1 + r) + t) < ((1 + 6r + 4r^2)/(2r))s$ if $r < (7s - 4t + \sqrt{41s^2 - 40st + 16t^2})/(8t - 4s) < 0.483$, $3s/2 + 2rs + t < ((1 + 6r + 4r^2)/(2r))s$ if $r < s/(-3s + 2t)$, and $3s/2 + 2rs + t < 2((r(2 + r)s)/(1 + r) + t)$ given $t/4 < s < t/2$, the three conditions can be combined into ($r < (7s - 4t + \sqrt{41s^2 - 40st + 16t^2})/(8t - 4s)$ and $3s/2 + 2rs + t < V < 2((r(2 + r)s)/(1 + r) + t)$) or ($(7s - 4t + \sqrt{41s^2 - 40st + 16t^2})/(8t - 4s) \leq r < s/(-3s + 2t)$ and $3s/2 + 2rs + t < V < ((1 + 6r + 4r^2)/(2r))s$). We then further verify if these conditions hold under our main model assumptions ($2t < V < 2t + s$). Because $3s/2 + 2rs + t < 2t + s$ if $r < t/(2s) - \frac{1}{4}$, $((1 + 6r + 4r^2)/(2r))s > 2t$ if $r < (2t - 3s - \sqrt{(2t - s)(2t - 5s)})/(4s)$, and $2((r(2 + r)s)/(1 + r) + t) > 2t$, the final conditions for the equilibrium to hold are ($r < (7s - 4t + \sqrt{41s^2 - 40st + 16t^2})/(8t - 4s)$ and $\max\{2t, 3s/2 + 2rs + t\} < V < \min\{2t + s, 2((r(2 + r)s)/(1 + r) + t)\}$), or ($(7s - 4t + \sqrt{41s^2 - 40st + 16t^2})/(8t - 4s) \leq r < \min\{s/(-3s + 2t), t/(2s) - \frac{1}{4}, (2t - 3s - \sqrt{(2t - s)(2t - 5s)})/(4s)\}$ and $\max\{2t, 3s/2 + 2rs + t\} < V < \min\{2t + s, ((1 + 6r + 4r^2)/(2r))s\}$.

(2) Location ½ is partially covered.
In this case, each firm maximizes

$$rp_{½}\left(1 + \frac{V - t - p_{½}}{s}\right) + (1 - r)p_A\frac{V - t - p_{½}}{s}.$$

Therefore, $p_{½} = (V - t + rs)/2$. Because residents at ½ get a positive utility at this price, violating the assumption that location is not fully covered, this cannot be an equilibrium. However, if the optimal price $(1+2r)s$ from Case 1 yields a nonpositive utility for the indifferent resident with $y = ½$ at location ½ (i.e., $V \le 3s/2 + 2rs + t$), a possible equilibrium exists here in which this resident receives exactly a utility of zero, i.e., $p_{½} = V - t - s/2$. Because $p_1 \ge p_{½}$, and $p_0 = \min\{p_1 + 2t - s, V - s\}$, we have $p_0 = V - s$ given $t > 2s$. Therefore, the equilibrium prices are $\{V - s, V - t - s/2$, any price $\ge V - t - s/2\}$ and the equilibrium profit is $(1-r)(V-s) + (V - t - s/2)(r + 1/2)$.

We then further check a firm's incentive to deviate from these prices. Similar to Case 1, we find that the firm has no incentive to decrease or increase p_0 and the firm also has no incentive to decrease or increase $p_{½}$ as long as $p_1 \ge p_{½}$. If the firm changes both p_1 and $p_{½}$ so that $p_1 < p_{½}$, there are two possibilities. First, $p_1 \ge V - 2t$. In this case, the deviating firm still gets no consumers from distance 1, charges $p_{½}$ to naïve residents at location ½, and charges p_1 to deal collectors who reside at distance 0 and ½. We can show that the optimal deviation price p_1 is in fact larger than the optimal deviation price $p_{½}$, violating the assumption that $p_1 < p_{½}$. Second, $p_1 < V - 2t$. In this case, the deviating firm gets some naïve residents from distance 1 (but no informed residents there), charges $p_{½}$ to naïve residents at location ½, and charges p_1 to deal collectors who reside at distance 0 and ½. We can show that the optimal deviation price p_1 is larger than $V - 2t$, violating the assumption that $p_1 < V - 2t$. Last, the firm may deviate to uniform pricing $V - s$. The deviation profit $V - s$ is less than the equilibrium profit only if $V > (s - 2rs + 2t + 4rt)/2$.

In sum the equilibrium holds if $\frac{1}{2}(s - 2rs + 2t + 4rt) < V \le 3s/2 + 2rs + t$. We then further verify if these conditions hold under our main model assumptions ($2t < V < 2t + s$). Because $(s - 2rs + 2t + 4rt)/2 < 3s/2 + 2rs + t$ if $r < s/(-3s + 2t)$, $(s - 2rs + 2t + 4rt)/2 < 2t + s$ if $r < \frac{1}{2} - s/(s - 2t)$, and $3s/2 + 2rs + t \ge 2t$ if $r \ge t/(2s) - \frac{3}{4}$, the final conditions for the equilibrium to hold are

$$\frac{t}{2s} - \frac{3}{4} \le r < \min\left[\frac{s}{-3s+2t}, \frac{1}{2} - \frac{s}{s-2t}\right] \quad \text{and}$$
$$\max\left[\frac{s - 2rs + 2t + 4rt}{2}, 2t\right] < V \le \min\left[2t + s, \frac{3s}{2} + 2rs + t\right].$$

(3) Location ½ is not covered.

In this case, each firm only sells to its own local residents. The equilibrium prices are $\{V - s, V - s, \ge V - s\}$, and the equilibrium profit is $V - s$. We then check a firm's incentive to deviate from these prices. Obviously, the firm has no incentive to increase or decrease its p_0. If the firm changes p_1 and $p_{½}$, there are two possibilities. First, in deviation, $V - 2t < p_{D_{½}} < V - t$ and $V - 2t < p_{D_1} < V - t$. In this case, the deviating firm sells to no residents at distance 1 but may sell to some residents at location ½. The deviation profit is the highest when the firm gets naïve residents from location ½ at $p_{D_{½}}$ and gets deal collectors who reside at distances 0 and ½ at p_{D_1}. To make sure that this deviation profit is lower than the equilibrium profit, i.e., $V - s > (1-r)(V-s) + \max_{0\le\lambda\le1}(V - t - \lambda s)r(1+\lambda) + \max_{0\le\lambda\le1}(V - t - \lambda s)(1-r)\lambda$, we need $t + 3s \le V < s + (1+r)t$ or $2t < V < \min\{t + 3s, ((-2+3r)s + rt + 2\sqrt{s(s - 2rs + r^2 t)})/r\}$. Second, in deviation, $V - 2t < p_{D_{½}} < V - t$ and $p_{D_1} \le V - 2t$. In this case, the deviating firm also sells to some residents at distance 1, and the deviation profit is the highest when the firm sells to naïve residents at location ½ at $p_{D_{½}}$, to naïve residents at distance 1 at p_{D_1}, and to deal collectors at p_{D_1}. To make sure that this deviation profit is lower than the equilibrium profit, i.e., $V - s > (1-r) \cdot (V-s) + \max_{0\le\lambda\le1}(V - 2t - 2\lambda s)(r + r + \lambda) + \max_{0\le\lambda\le1}(V - t - \lambda s)(1-r)\lambda$, we need $2t < V < 2t + 4rs$.

Because $2t + 4rs > s + (1+r)t$, $2t + 4rs > ((-2+3r)s + rt + 2\sqrt{s(s - 2rs + r^2 t)})/r$, $t + 3s \le ((-2+3r)s + rt + 2\sqrt{s(s - 2rs + r^2 t)})/r$ if $r \ge 2s/t$, $t + 3s \le s + (1+r)t$ if $r \ge 2s/t$, $s + (1+r)t \ge 2t$ if $r \ge (t-s)/t$ and $((-2+3r)s + rt + 2\sqrt{s(s - 2rs + r^2 t)})/r \ge 2t$ if $r \ge 4s/(9s - t)$, the region where the equilibrium holds is

(1) $r \ge \max\{(t-s)/t, 2s/t\}$ and $2t < V < s + (1+r)t$, or

(2) $4s/(9s - t) \le r < 2s/t$ and $2t < V < ((-2+3r)s + rt + 2\sqrt{s(s - 2rs + r^2 t)})/r$.

Combining cases (1)–(3), we have all of the equilibrium outcomes.

Proof of Proposition 6. Let $p^f_{i,j}$ represent Firm f's price for a consumer whose home location is at location i and who decides to pull the offer at location j. Below we derive the equilibrium price for each combination of i and j.

(1) Home location $i = 0$ and current location $j = 0$.

For these consumers, if they pull the prices, their utility of buying from Firm A is $V - p^A_{0,0} - s(1-y)$ and their utility of buying from Firm B is $V - 2t - p^B_{0,0} - s(1-y)$. To find the optimal prices, we have $V - p^A_{0,0} - sy = V - 2t - p^B_{0,0} - s(1-y)$, which leads to $y = (p^B_{0,0} - p^A_{0,0} + s + 2t)/(2s)$. Therefore, $p^A_{0,0} = (p^B_{0,0} + s + 2t)/2$, $p^B_{0,0} = (p^A_{0,0} + s - 2t)/2$, and $p^{A*}_{0,0} = s + 2t/3$, $p^{B*}_{0,0} = s - 2t/3$. Given $s < t/2$, $p^{B*}_{0,0} < 0$. So we have to set $p^{B*}_{0,0} = 0$. Now if $p^{B*}_{0,0} = 0$, Firm A would find it optimal to sell to all of these consumers and so $p^A_{0,0}$ is derived by $V - s - p^A_{0,0} = V - 2t - 0$, which leads to $p^{A*}_{0,0} = 2t - s$.

(2) Home location $i = ½$ and current location $j = 0$.

For these consumers, if they pull the prices, their utility of buying from Firm A will be $V - p^A_{½,0} - s(1-y)$ because they are at location 0 at the moment and their utility of buying from Firm B is $V - t - p^B_{½,0} - s(1-y)$ because they would go back home anyway and their travel cost to Firm B is thus only t. To find the optimal prices, we have $V - p^A_{½,0} - sy = V - t - p^B_{½,0} - s(1-y)$, which leads to $y = (p^B_{½,0} - p^A_{½,0} + s + t)/(2s)$. Therefore, $p^A_{½,0} = (p^B_{½,0} + s + t)/2$, $p^B_{½,0} = (p^A_{½,0} + s - t)/2$, and $p^{A*}_{½,0} = s + t/3$, $p^{B*}_{½,0} = s - t/3$. Thus, if $t \le 3s$, the equilibrium prices are $p^{A*}_{½,0} = s + t/3$, $p^{B*}_{½,0} = s - t/3$. If $t > 3s$, $p^{B*}_{½,0} < 0$. So we have to set $p^{B*}_{½,0} = 0$. Now if $p^{B*}_{½,0} = 0$, Firm A would find it optimal to sell to all of these consumers and so $p^A_{½,0}$ is derived by $V - s - p^A_{½,0} = V - t - 0$, which leads to $p^{A*}_{½,0} = t - s$.

(3) Home location $i = 1$ and current location $j = 0$.

For these consumers, if they pull the prices, their utility of buying from Firm A is $V - p^A_{1,0} - s(1-y)$ because they are at location 0 at the moment and their utility of buying from Firm B is $V - p^B_{1,0} - s(1-y)$ because they would go back home anyway and their travel cost to Firm B is thus also 0. To find the optimal prices, we have $V - p^A_{1,0} - sy = V - p^B_{1,0} - s(1-y)$, which leads to $y = (p^B_{1,0} - p^A_{1,0} + s)/(2s)$. Therefore, $p^A_{1,0} = (p^B_{1,0} + s)/2$, $p^B_{1,0} = (p^A_{1,0} + s)/2$, and $p^{A*}_{1,0} = p^{B*}_{1,0} = s$.

(4) Home location $i = 0$ and current location $j = ½$.

For these consumers, if they pull the prices, their utility of buying from Firm A is $V - p^A_{0,½} - s(1-y)$ because they would go back home anyway and their travel cost to Firm A is thus 0, and their utility of buying from Firm B will be $V - t - p^B_{0,½} - s(1-y)$ because they are at ½ and thus only need to occur an additional travel cost of t to go to Firm B. To find the optimal prices, we have $V - p^A_{0,½} - sy = V - t - p^B_{0,½} - s(1-y)$, which leads to $y = (p^B_{0,½} - p^A_{0,½} + s + t)/(2s)$. Therefore, $p^A_{0,½} = (p^B_{0,½} + s + t)/2$, $p^B_{0,½} = (p^A_{0,½} + s - t)/2$, and $p^{A*}_{0,½} = s + t/3$, $p^{B*}_{0,½} = s - t/3$. Thus, if $t \le 3s$, the equilibrium prices are $p^{A*}_{0,½} = s + t/3$, $p^{B*}_{0,½} = s - t/3$. If $t > 3s$, $p^{B*}_{0,½} < 0$. So we have to set $p^{B*}_{0,½} = 0$. Now if $p^{B*}_{0,½} = 0$, Firm A would find it optimal to sell to all of these consumers and so $p^A_{0,½}$ is derived by $V - s - p^A_{0,½} = V - t - 0$, which leads to $p^{B*}_{0,½} = t - s$.

(5) Home location $i = ½$ and current location $j = ½$.

For these consumers, if they pull the prices, their utility of buying from Firm A is $V - t - p^A_{½,½} - s(1-y)$ and that of buying from Firm B is $V - t - p^B_{½,½} - s(1-y)$. To find the optimal prices, we have $V - t - p^A_{½,½} - sy = V - t - p^B_{½,½} - s(1-y)$, which leads to $y = (p^B_{½,½} - p^A_{½,½} + s)/(2s)$. Therefore, $p^A_{½,½} = (p^B_{½,½} + s)/2$, $p^B_{½,½} = (p^A_{½,½} + s)/2$, and $p^{A*}_{½,½} = p^{B*}_{½,½} = s$.

(6) Home location $i = 1$ and current location $j = ½$.

For these consumers, if they pull the prices, their utility of buying from Firm A will be $V - t - p^A_{1,½} - s(1-y)$ because they are at ½ and thus only need to occur an additional travel cost of t to go to Firm A's location, and their utility of buying from Firm B is $V - p^B_{1,½} - s(1-y)$ because they would go back home anyway and their travel cost to Firm B is thus 0. To find the optimal prices, we have $V - t - p^A_{1,½} - sy = V - p^B_{1,½} - s(1-y)$, which leads to $y = (p^B_{1,½} - p^A_{1,½} + s - t)/(2s)$. Therefore, $p^A_{1,½} = (p^B_{1,½} + s - t)/2$, $p^B_{1,½} = (p^A_{1,½} + s + t)/2$, and $p^{A*}_{1,½} = s - t/3$, $p^{B*}_{1,½} = s + t/3$. Thus, if $t \le 3s$, the equilibrium prices are $p^{A*}_{1,½} = s - t/3$, $p^{B*}_{1,½} = s + t/3$. If $t > 3s$, $p^{A*}_{1,½} < 0$. So we have to set $p^{A*}_{1,½} = 0$. Now if $p^{A*}_{1,½} = 0$, Firm B would find it optimal to sell to all of these consumers and so $p^B_{1,½}$ is derived by $V - t - 0 = V - s - p^B_{1,½}$, which leads to $p^{B*}_{1,½} = t - s$.

(7) Home location $i = 0$ and current location $j = 1$.

For these consumers, if they pull the prices, their utility of buying from Firm A is $V - p^A_{0,1} - s(1-y)$ because they would go back home anyway and their travel cost to Firm A is thus 0, and their utility of buying from Firm B is $V - p^B_{0,1} - s(1-y)$ because they are at Firm B's location at the moment. To find the optimal prices, we have $V - p^A_{0,1} - sy = V - p^B_{0,1} - s(1-y)$, which leads to $y = (p^B_{0,1} - p^A_{0,1} + s)/(2s)$. Therefore, $p^A_{0,1} = (p^B_{0,1} + s)/2$, $p^B_{0,1} = (p^A_{0,1} + s)/2$, and $p^{A*}_{0,1} = p^{B*}_{0,1} = s$.

(8) Home location $i = ½$ and current location $j = 1$.

For these consumers, if they pull the prices, their utility of buying from Firm A is $V - t - p^A_{½,1} - s(1-y)$ because they would go back home anyway and thus only need to incur an additional travel cost of t to visit Firm A, and their utility of buying from Firm B is $V - p^B_{½,1} - s(1-y)$ because they are at Firm B's location at the moment. To find the optimal prices, we have $V - t - p^A_{½,1} - sy = V - p^B_{½,1} - s(1-y)$, which leads to $y = (p^B_{½,1} - p^A_{½,1} + s - t)/(2s)$. Therefore, $p^A_{½,1} = (p^B_{½,1} + s - t)/2$, $p^{f2}_{1,½} = (p^A_{½,1} + s + t)/2$, and $p^{A*}_{½,1} = s - t/3$, $p^{B*}_{½,1} = s + t/3$. Thus, if $t \le 3s$, the equilibrium prices are $p^{A*}_{½,1} = s - t/3$, $p^{B*}_{½,1} = s + t/3$. If $t > 3s$, $p^{A*}_{½,1} < 0$. So we have to set $p^{A*}_{½,1} = 0$. Now if $p^{A*}_{½,1} = 0$, Firm B would find it optimal to sell to all of these consumers and so $p^B_{½,1}$ is derived by $V - t - 0 = V - s - p^B_{½,1}$, which leads to $p^{B*}_{½,1} = t - s$.

(9) Home location $i = 1$ and current location $j = 1$.

For these consumers, if they pull the prices, their utility of buying from Firm A is $V - 2t - p^A_{1,1} - s(1-y)$ and their utility of buying from Firm B is $V - p^B_{1,1} - s(1-y)$. To find the optimal prices, we have $V - 2t - p^A_{1,1} - sy = V - p^B_{1,1} - s(1-y)$, which leads to $y = (p^B_{1,1} - p^A_{1,1} + s - 2t)/(2s)$. Therefore, $p^A_{1,1} = (p^B_{1,1} + s - 2t)/2$, $p^B_{1,1} = (p^A_{1,1} + s + 2t)/2$, and $p^{A*}_{1,1} = s - 2t/3$, $p^{B*}_{1,1} = s + 2t/3$. Given $s < t/2$, $p^{A*}_{1,1} < 0$. So we have to set $p^{A*}_{1,1} = 0$. Now if $p^{A*}_{1,1} = 0$, Firm A would find it optimal to sell to all of these consumers and so $p^B_{1,1}$ is derived by $V - 2t - 0 = V - s - p^B_{1,1}$, which leads to $p^{B*}_{1,1} = 2t - s$.

Now consider the decision of a resident at 0 who has rational expectations on the prices being offered at each location. Given the equilibrium prices that are set based on both home location and current location, the consumer has no incentive to travel to other locations for a different mobile offer because the total cost of buying will not be reduced: if the consumer uses the mobile offer at ½, the total cost of buying is $s + 4t/s$ if $t \le 3s$ and is $2t - s$ if $t > 3s$, both of which is no lower than $2t - s$; if the consumer uses the offer at 1, the total cost of buying is $s + 2tT$, higher than $2t - s$. The same conclusion can be drawn for residents at ½ or 1.

Endnotes

[1] http://www.adweek.com/news/technology/170-us-brands-are-already-using-ad-tech-can-target-people-specific-building-163272 (accessed October 2016).

[2] http://www.slideshare.net/Vibes_Thought_Leadership/vibes-abcs-of-google-wallet-and-passbook-30915033 (accessed December 2015).

[3] For example, one SMS text message in Danaher et al. (2015, p. 715) says, "Ever had a shake with Turkish Delight or Orange Tic Tacs? Milkshake Store has reinvented milkshakes. Visit Level 1 today for $1 off any icy-cold shake!" and a shopper in the mall can easily retrieve such a coupon from a pod near the mall entrance.

[4] We thank the associate editor for bringing this comparison with the second-degree price discrimination to our attention.

[5] The choice of the middle location is intuitive and also consistent with recent empirical studies of mobile targeting (e.g., Fong et al. 2015).

[6] In Extensions 1 and 2, we allow a fraction of consumers to remain "naïve" and unaware of offers outside of their home locations.

[7] https://play.google.com/store/apps/details?id=com.yowza&hl=en (accessed December 2015).

[8] https://play.google.com/store/apps/details?id=com.nianticproject.scout&hl=en and http://thenextweb.com/apps/2013/01/10/googles-location-aware-field-trip-tour-guide-app-serves-up-mobile-deals-courtesy-of-scoutmob/ (accessed December 2015).

[9] The analysis of a general equilibrium in which firms do not have to adopt the same pricing strategy is intractable.

[10] See more examples at http://www.thecouponsapp.com/ (accessed February 2015).

[11] Detailed derivation of conditions and results are available in the online appendix.

[12] If mobile-deal collectors do not incur any travel cost at all, then the location loses its significance and we are back to Bertrand-type competition for these type of consumers.

[13] See, for example, http://marketingland.com/how-location-evolved-into-audiences-for-mobile-ad-argeting-59126 (accessed February 2015).

[14] In the most profitable deviation, the deviating firm charges $2t - s$ to residents at 1, $t/2$ at location ½ and at location 0. Its deviation profit equals $t^2/(8s) + 2t - s$ minus the cost of tracing, and the deviation profit is higher than the mobile geo-targeting profit under this condition.

[15] This is equivalent to Extension 1 where $h = 0$, so the derivations can be found in the proof of Proposition 4.

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