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Informed Trading When Information Becomes Stale

DAN BERNHARDT and JIANJUN MIAO*

ABSTRACT

This paper characterizes informed trade when speculators can acquire distinct signals of varying quality about an asset's value at different dates. The most reasonable characterization of private information about stocks is that while information is long-lived, new information will arrive over time, information that may be acquired by others. Hence, while a speculator may know more than others at a moment, in the future, his information will become stale, but not valueless. In an environment that allows for arbitrary correlations among signals, we characterize equilibrium outcomes including trading, prices, and profits. We provide explicit numerical characterizations for different informational environments.

ONE OF THE MOST IMPORTANT OPEN QUESTIONS in theoretical market microstructure finance is: How do informed speculators trade in a dynamic environment when different speculators receive distinct signals at different dates?

The most interesting and reasonable characterization of private information about stocks is that, while information is long-lived, new information will arrive over time and this information may be acquired by other speculators. In practice, individuals only periodically engage in detailed research on a particular stock. At the moment a speculator does this research, he may know that he is better informed than everyone else with regard to the stock, but he also knows that in the future, his information will become dated, and others will acquire fresher information. Still, the speculator can continue to trade profitably on his information in the future; that is, even though his information will become stale, it will still have value.

An informed speculator must determine how intensively to trade on his information at each date. To do this he must use the information contained in both his signal and market prices to forecast (i) the information that each differentially informed agent has about the asset value, and (ii) the trades of other informed agents, because those trades influence prices. In such an environment, one wishes to determine how the strategic interplay among differentially

*Bernhardt is with the Department of Economics, University of Illinois at Urbana-Champaign. Miao is with the Department of Economics, Boston University. We thank Shmuel Baruch, Oleg Bondarenko, Larry Epstein, Terrence Henderschott, Eric Hughson, Leslie Marx, and a particularly conscientious referee for helpful comments. We are especially grateful to Motohiro Sato. Our analysis was tremendously facilitated by Motohiro Sato's extended solution to a homework problem in a market microstructure finance course at Queen's University in 1997.

informed agents affects trading behavior, market prices, volume and profits over time.

This problem has defied solution. Existing attempts at characterizing speculative informed trade have taken one of two basic approaches. The approach adopted by Kyle (1985), Back, Cao, and Willard (2000), Baruch (2002), Holden and Subrahmanyam (1992), and Foster and Viswanathan (1996) is to assume that (i) information arrives only at the beginning of the game; and (ii) the date 0 signals that the informed agents receive about the long lived value of the asset, although perhaps correlated, are *identically* distributed. These extreme modeling assumptions are made solely for reasons of tractability: "The assumption of symmetry, i.e., that all traders have the same initial variance of information, same cross covariance between signals and the same covariance between signals and true value is *critical* for the analysis" (Foster and Viswanathan (1996)). The alternative approach has been to assume that private information is short-lived, so that old information is immediately revealed to the market as soon as new private information becomes available (Admati and Pfleiderer (1988)).

What has made this problem so difficult is that each informed agent's trading problem is intrinsically different because the nature of each agent's information is different. One must simultaneously solve each informed agent's trading problem and solve for the consistent equilibrium pricing at each date. To do so, one must solve the forecasting and information extraction problems of these agents, and address how they trade over time.

This paper solves this problem. We consider an environment in which new, normally distributed innovations to the asset's value arrive in each period. We allow the correlation between the innovation in the asset's value in one period with the innovations in other periods to be arbitrary. In a base formulation, in each period t, a single new informed agent receives one of the following pieces of information: (i) the period t innovation to the asset's value; (ii) the period t asset value (i.e., the sum of all innovations up to period t); or (iii) the period t history of innovations (i.e., the vector containing each innovation up to period t's innovation). Going from the first formulation to the third, agents who acquire information later are increasingly better informed than earlier agents. Our analysis extends these base formulations to allow for arbitrary numbers of informed agents at each date, and to allow agents who become informed at the same date to see different innovations.

In sum, the informational environment we consider is very general. To determine how much an informed agent should trade in each period given his private information and past history of prices, the agent must first use this information to forecast both the information about the asset's value that other informed agents receive, and the trades by other informed agents, since those trades will affect prices. What complicates the informational extraction problem is a formidable "forecasting the forecasts of others" problem: Each agent's trades will depend on their forecasts of other agents' trades; since trades depend on forecasts, so too do prices, and hence agents must forecast the forecasts of others, and so on.

The key step in our analysis is to first conjecture and then verify that: (i) an equilibrium pricing rule is a linear function of net order flow; (ii) the optimal trading strategy of an informed agent is a linear function of each piece of his unrevealed private information, that is, the difference between each signal and the expectation of that signal given public information; and, (iii) an informed agent's expected future trading profits are a quadratic function of this private information.

Several driving forces underlie our findings. The first concerns how agents use the information contained in each signal that they receive to forecast the asset's value, the information of other agents, and other agents' trades. Given the linear pricing rule and the linear strategies of other informed agents, an agent's best estimates of both the difference between the asset's true value and the market's expectation of that value, and the private information of other agents, are linear functions of each piece of unrevealed private information that an agent has. That is, agents essentially run multivariate linear forecasting regressions. The conjectured linear strategies therefore render unnecessary the task of directly solving the forecasting-the-forecasts problem. Second, given the conjectured linear strategies of other agents, an agent's expected period profits are a quadratic function of each piece of his private information. Quadratic objectives yield linear first-order conditions, and thus an informed agent's optimal period trade is a linear function of his unrevealed private information. In turn, the linear trading strategies and linear pricing functions imply that period profits, and hence expected lifetime trading profits, are quadratic functions of the agent's unrevealed private information.

Importantly, this analysis also highlights that one cannot generally write an agent's optimal trade as a linear function of the difference between his forecast of the asset's value given both his private information and trade history, and the market's best forecast given the trade history. This is because the intensities with which agents compete on each piece of private information differ. For example, if agents face greater competition on "older" information that more agents have acquired, then each agent reduces the intensity with which he trades on that piece of information.

In Section I, we first provide explicit equilibrium characterizations for the problem in which informed agents observe period innovations. The model is very general since we assume that an arbitrary number of informed agents can acquire information at each date, these informed agents may see different innovations, and the correlations among innovations are arbitrary. We first detail the evolution of public information in the process of trading, and the difference between the private and public informational structures, and then provide a theorem characterizing the equilibrium. The linear equilibrium characterization is along the lines of Kyle (1985), Holden and Subrahmanyam (1992), or Foster and Viswanathan (1994, 1996): We provide a system of difference equations, the solution to which characterizes equilibrium informed trading and pricing.

Our results are foreshadowed by the analysis of Foster and Viswanathan (1994). Foster and Viswanathan consider an environment with two informed agents; one agent knows a date-1 innovation to an asset's value, while the other agent knows that innovation and an additional innovation. What our paper shows is that the nature of Foster and Viswanathan's solution in their one-period special case extends to very general finite-horizon contexts.

In Section II, we show how equilibrium outcomes are affected when there is a positive probability that an agent's private information may leak out at a future date and become public information. This might occur, for example, when a firm announces quarterly earnings.

Section II also provides characterizations of outcomes when agents observe the period asset value rather than the period innovation to the asset value. Finally, we show that our analytical approach extends even if an insider's private information is multidimensional, perhaps because his information subsumes that of agents who acquire information at earlier dates. These latter characterizations permit us to determine how equilibrium outcomes are affected by the quality of information: An agent who observes the period t innovation is less well informed than one who learns the period t value of the asset, who, in turn, is less well informed than someone who sees the entire history of asset innovations through date t.

In Section III, we augment our theoretical analysis with numerical characterizations for cases in which: (i) there is a single innovation to the asset's value, but informed agents observe the innovation at different dates; and, (ii) innovations are independently distributed and an informed agent observes either the date t innovation, the date t asset value, or the entire history of innovations.

Our numerical characterizations reveal several important findings:

- To generate the U-shaped intradaily pattern in volume found in the data, it is important to allow for both sequential information acquisition and heterogeneous (sufficiently uncorrelated) information. Even when innovations are independently distributed, for volume to exhibit a U-shaped pattern, agents must acquire sufficient new information over time.
- Sequential information acquisition and sufficiently heterogeneous (uncorrelated) information are also required to generate, simultaneously, the widening bid–ask spreads, increasing volume and increasing price volatility that are found empirically as the timing of earnings announcements draws near (see Brooks (1994) and Krinsky and Lee (1996)).
- The nature of private information has surprising effects on equilibrium outcomes. For example, when innovations are independently distributed, an agent's expected profits are lower when he knows each innovation that together comprise the asset's value than they are when he knows only the total asset value. The reason is that an agent who observes only the asset value cannot make the size of his trade a function of each individual innovation, and this causes him to trade more aggressively on innovations that are privately observed by others. In turn, the equilibrium "best response" of other agents to this more aggressive trading is to trade less aggressively.
- The division of information across agents matters in a dynamic context, even when innovations are independently distributed. In a static

environment, when signals are independently distributed, pricing is unaffected by the division of information. However, in a dynamic context, agents learn from the price and their own private information about the signals of other agents, even if innovations are independently distributed. Recognizing that their signals are conditionally negatively correlated, agents compete less aggressively, reducing information release (Back et al. (2000) uncover a related result).

- Using a Kyle-style decomposition of time for the case in which there is a single innovation observed at different times, we show that the timing at which the second agent observes the second innovation endogenously determines the trading horizon of the first-informed agent in a Kyle environment: The first agent to acquire information will trade to exhaust all private information at the moment that the second agent learns the news. More generally, as one increases the number of trading periods in which the first-informed agent is the sole observer of his private information, equilibrium dynamics move smoothly from duopoly to monopoly dynamics.

These numerical findings highlight the importance of a theoretical analysis that allows both for heterogeneity in agents' signal qualities and intertemporal information arrival.

Section I outlines the base model, and characterizes the equilibrium. Section II extends the analysis to other informational environments. Section III provides the numerical characterization. A conclusion follows. Most proofs are placed in the Appendix.

I. A Base Model

A single risky asset is traded over a finite number of periods, N. We assume that the asset's final liquidation value is equal to $v = \bar{v} + \sum_{t=1}^{N} \delta_t$, where $\bar{v}$ is the (unconditional) mean of v and the innovations $(\delta_1, \ldots, \delta_N)$ are drawn from a multivariate normal distribution with mean zero and variance–covariance matrix Ψ. In the information structures that we consider, agents who are privately informed in a particular period observe in that period either a vector containing some subset of the δ's or a partial of sum the δ's, for example, $\sum_{j=1}^{L} \delta_j$, where $L \leq N$. At the end of period N, the asset's liquidation value, v, becomes public information. Risk-neutral informed traders and uninformed liquidity traders trade in a market made by risk-neutral, competitive, uninformed market makers who only observe the net order flow.

No structure is imposed on the covariance matrix between innovations, Ψ. Special cases of this informational environment include a single informational innovation as in Kyle (1985) and Holden and Subrahmanyam (1992), and symmetrically distributed innovations as in Foster and Viswanathan (1996), or Back et al. (2000). In addition to allowing for a far more general informational structure on innovations, our set-up allows agents to receive signals at different dates.

In the base formulation, an informed agent observes a single innovation, δ_i, $i \in \{1, 2, \ldots, N\}$, to the asset's value. However, we impose no restrictions on the number of agents who learn δ_i, nor on the dates at which agents acquire this information. Let $M_k^i \geq 0$ be the number of traders who learn δ_i at date k. Thus, $M \equiv \sum_{i=1}^N \sum_{k=1}^N M_k^i$ is the total number of informed agents. A timing interpretation for the arrival of innovations is preserved if $M_k^i = 0$, for $i > k$, so that agents do not observe innovation δ_i until at least date i. However, if $M_k^i \neq 0$, $i > k$, the formulation admits the interpretation that δ_i and δ_k are distinct innovations to the asset value that are realized at date 0, and are observed by different agents. In particular, with this interpretation, our formulation incorporates Kyle (1985), Holden and Subrahmanyam (1992), and Foster and Viswanathan (1996) as special cases. Kyle's (1985) model is obtained when the sole informed trader acquires information at date 1: $M_1^1 = 1, M_k^j = 0$ for $k, j \neq 1$, and $\delta_j = 0$ for $j > 1$. Holden and Subrahmanyam's (1992) model is obtained when more than one agent observes δ_1, so that $M_1^1 > 1, M_k^j = 0$ for $k, j \neq 1$, and $\delta_j = 0$ for $j > 1$. Foster and Viswanathan's (1996) numerical model is obtained when $M_1^i = 1, M_k^i = 0$ for $i = 1, 2, \ldots, M \leq N$, and $k > 1$, where innovations $(\delta_1, \ldots, \delta_M)$ are symmetrically distributed, and $\delta_j = 0$ for $j > M$.

In each period, t, in addition to trade from informed agents, there is also exogenous liquidity trade of u_t. We assume that net liquidity trade is independently and identically normally distributed each period according to $N(0, \sigma_u^2)$. Let x_{kt}^i be the date t trade of an informed agent who observes δ_i at date k. Then

$$X_t = \sum_{k=1}^N \sum_{i=1}^N M_k^i x_{kt}^i \tag{1}$$

is the total informed trade at date t. Competitive market making implies that the price of the asset in period t, p_t, equals the expectation of v conditional on the current information available to the market maker:

$$p_t = E[v \mid X_t + u_t, \Omega_{t-1}], \tag{2}$$

where Ω_{t-1} is the public information revealed prior to trading at period t, and $X_t + u_t$ is the total net order flow at t that the market maker observes. Thus, public information evolves according to $\Omega_t = \{\Omega_{t-1}, X_t + u_t\}$, where $\Omega_0 = \bar{v}$. The market efficiency condition (2) implies that p_t follows a martingale.

Consider an informed agent who learns δ_i at date k. The contribution of period t trading to the agent's total realized profits is $\pi_{kt}^i \equiv (v - p_t)x_{kt}^i$, and his cumulative trading profit from period t to N is $\Pi_{kt}^i \equiv \sum_{j=t}^N \pi_{kj}^i$. For $t < k$, this agent is still uninformed; he trades to maximize expected profits given the public information history of prices, maximizing $E[\Pi_{kt}^i \mid \Omega_{t-1}]$, and his optimal trading strategy x_{kt}^i is a measurable function of Ω_{t-1}. For $t \geq k$, the agent who learns δ_i at date k is informed at date t, and hence trades to maximize expected profits given both the public information history of prices and his private information, maximizing $E[\Pi_{kt}^i \mid \delta_i, \Omega_{t-1}]$. Furthermore, his optimal trading strategy is a measurable function of (δ_i, Ω_{t-1}).

An equilibrium can be defined by (i) the market efficiency condition, equation (2), and (ii) profit maximization by each informed agent at each date given the optimal strategies of other agents and the equilibrium pricing function.

We restrict attention to linear equilibria in which both the pricing function and informed agents' strategies are linear with respect to the information structure. In particular, we conjecture that the trading strategy of an informed trader who learns δ_i at date k takes the form:

$$\begin{aligned} x^i_{kt} = \beta^i_t(\delta_i - E[\delta_i \mid \Omega_{t-1}]) \qquad &\text{for} \quad t \geq k \text{ and } M^i_k > 0, \\ x^i_{kt} = 0 \qquad &\text{for} \quad t < k \text{ or } M^i_k = 0, \end{aligned} \tag{3}$$

where β^i_t is a constant, and that the equilibrium price functions take the form:

$$p_t = p_{t-1} + \lambda_t \cdot (X_t + u_t), \tag{4}$$

where λ_t is a constant and $p_0 = \bar{v}$. If prices evolve according to equation (4), and the strategies of informed agents take the form of equation (3), then p_t is a linear function of the net orders, so that the streams of net order flows and prices are informationally equivalent. Hence, without loss of generality we can let $\Omega_{t-1} \equiv \{\bar{v}, p_1, \ldots, p_{t-1}\}$. Further, given the linear strategies, past prices $(p_1, \ldots, p_t)$ are normally distributed with some strategy-dependent conditional mean and conditional variance–covariance matrix.

The standard approach (Kyle (1985), Holden and Subrahmanyam (1992), Back et al. (2000), Foster and Viswanathan (1996), etc.) is to conjecture that the trading strategies are linear functions of the difference between an agent's signal and the date $t-1$ price, the latter of which represents the market's expectation of the asset's value given the public information history. One might wonder why equilibrium strategies are not linear functions of the difference between an agent's expectation of the asset value and the market's date t assessment of this value, p_{t-1}. In fact, with symmetrically distributed signals Back et al. (2000) show that these strategy formulations are equivalent.

The "standard" approach works when agents have symmetrically distributed signals, and hence adopt identical strategies. As a result, p_{t-1} corresponds to the market's forecast of each agent's signal. Our conjectured form of trading strategies generalizes this approach in the natural way to environments in which informed agents are heterogeneous ex ante, both with regard to the timing of when they receive information, and to the distribution of their signals.

Our equilibrium concept mirrors that of Kyle (1985), Back et al. (2000), Holden and Subrahmanyam (1992) in that we characterize behavior only along the equilibrium path, in which in each past period, each agent followed his equilibrium linear trading rules. This ensures that the evolution of past prices is normally distributed.

A. Information

We now examine how private and public information evolves over time, and how the expectations of informed agents are formed. Since agents follow linear strategies, at period t, the past stream of prices $(p_1, \ldots, p_{t-1})$ is normally distributed along the equilibrium path. This allows us to exploit the projection theorem (see Grossman and Stiglitz (1980), or Greene (1993)).

The private information δ_i acquired by an agent may be at least partially revealed through trading at t through the price p_t. If δ_i is (unconditionally) correlated with other signals, say $\delta_j (j < i)$, then the market may provide information about δ_i prior to period i:

LEMMA 1: *The market's date t forecast of δ_i can be decomposed into the sum of the expectation at date $t-1$ plus a component reflecting information revealed by the date t price:*

$$E[\delta_i \mid \Omega_t] = E[\delta_i \mid \Omega_{t-1}] + \phi_t^i \{p_t - E[p_t \mid \Omega_{t-1}]\}, \tag{5}$$

$$\textit{where} \quad \phi_t^i = \frac{\operatorname{cov}(\delta_i, p_t \mid \Omega_{t-1})}{\operatorname{Var}(p_t \mid \Omega_{t-1})}. \tag{6}$$

The second term in equation (5) represents the adjustment of the expectation reflecting the difference between the realized value of p_t and the prior expectation $E[p_t \mid \Omega_{t-1}]$. Also, note that the conditional variance and covariance are not functions of Ω_{t-1} and neither is ϕ_t^i, so that this term can be treated as a parameter at any date prior to $t-1$. Because informed trading strategies are linear, net order flow is normally distributed. Consequently, the market's forecast of δ_i is obtained from a regression of the unobserved δ_i on the equilibrium price change (equivalently, net order flow) using population moments.

Analogously, an informed agent must forecast the information of other agents using both his private signal, δ_i, and the information contained in prices. In general, pricing will reflect an infinite hierarchy of forecasts by each agent: Each agent, when deciding how much to trade, forecasts the information of other agents, their forecasts of the information of other agents, their forecasts of other agents' forecasts, and so on. However, the conjectured linear form of optimal trading strategies implies that the relationship between $E[\delta_j \mid \Omega_{t-1}]$ and $E[\delta_j \mid \delta_i, \Omega_{t-1}]$ takes a simple form, as described by Lemma 2.

LEMMA 2: *The relationship between $E[\delta_j \mid \Omega_{t-1}]$ and $E[\delta_j \mid \delta_i, \Omega_{t-1}]$ is given by*

$$E[\delta_j \mid \delta_i, \Omega_{t-1}] = E[\delta_j \mid \Omega_{t-1}] + \theta_t^{ij} \{\delta_i - E[\delta_i \mid \Omega_{t-1}]\}, \tag{7}$$

$$\textit{where} \quad \theta_t^{ij} = \frac{\operatorname{cov}(\delta_i, \delta_j \mid \Omega_{t-1})}{\operatorname{Var}(\delta_i \mid \Omega_{t-1})}. \tag{8}$$

Here, $\delta_i - E[\delta_i \mid \Omega_{t-1}]$ represents the agent's informational advantage over the market. Using equation (7), which characterizes the agent's forecasts of those innovations that he does not observe, we derive his forecast of the asset value v conditional on δ_i and Ω_{t-1} :

$$E[v \mid \delta_i, \Omega_{t-1}] = \bar{v} + \sum_{j=1}^{N} E[\delta_j \mid \delta_i, \Omega_{t-1}] = p_{t-1} + \sum_{j=1}^{N} \theta_t^{ij} \{\delta_i - E[\delta_i \mid \Omega_{t-1}]\}, \quad (9)$$

where the final equality uses $p_{t-1} = \bar{v} + E[\sum_{j=1}^{N} \delta_j \mid \Omega_{t-1}]$.

This result implies that an equivalence result similar to that derived by Back et al. (2000) obtains in our environment if agents only observe one signal. That is, we could have formulated strategies as linear functions of the difference between an agent's forecast of the asset value given both his information and the history of prices, and the market's forecast of the asset value given the history of prices. This is because the difference between the agent's forecast and the market's forecast of the asset value turns out to be $\sum_{j=1}^{N} \theta_t^{ij}$ times the difference between an agent's signal and the market's forecast of the signal. If an agent sees more than one signal, then this equivalence result breaks down: An agent's trading strategy is not generally a linear function of the difference between his forecast of the asset value and the market's forecast.

Finally, we must characterize how the agent's information about other innovations evolves over time. To do this we must determine how covariances between innovations (conditioned both on public and on private information) are updated each period.

LEMMA 3: *Conditional on public information, the covariance between δ_j and δ_k evolves according to*

$$\operatorname{Cov}(\delta_j, \delta_k \mid \Omega_t) = \operatorname{Cov}(\delta_j, \delta_k \mid \Omega_{t-1}) - \frac{\operatorname{Cov}(\delta_j, p_t \mid \Omega_{t-1}) \operatorname{Cov}(\delta_k, p_t \mid \Omega_{t-1})}{\operatorname{Var}(p_t \mid \Omega_{t-1})}. \quad (10)$$

Conditional on private and public information, the covariance between δ_k and δ_j evolves according to

$$\operatorname{Cov}(\delta_j, \delta_k \mid \delta_i, \Omega_{t-1}) = \operatorname{Cov}(\delta_j, \delta_k \mid \Omega_{t-1}) - \frac{\operatorname{Cov}(\delta_j, \delta_i \mid \Omega_{t-1}) \operatorname{Cov}(\delta_k, \delta_i \mid \Omega_{t-1})}{\operatorname{Var}(\delta_i \mid \Omega_{t-1})}. \quad (11)$$

B. Informed Optimization and Equilibrium

We now turn to the optimization problem of an informed agent. For an informed agent who learns δ_i at date k, let $V_{kt}^i(\delta_i, \Omega_{t-1})$ be his date t future expected trading profits given δ_i and the history of past prices. This value function is the sum of the maximized expected trading profits from trading at $j \geq t$, evaluated at the beginning of period t. We conjecture that the informed agent's value function is a quadratic function of the difference between his private information, δ_i, and the expectation of that value given the history of prices, $E[\delta_i \mid \Omega_{t-1}]$:

$$\begin{aligned} V^i_{kt}(\delta_i, \Omega_{t-1}) = A^i_{t-1} + B^i_{t-1}(\delta_i - E[\delta_i \mid \Omega_{t-1}])^2 \quad & \text{for} \quad t \geq k \text{ and } M^i_k > 0, \\ V^i_{kt}(\cdot) = 0 \quad & \text{for} \quad M^i_k = 0, \end{aligned} \tag{12}$$

where $A^i_N = B^i_N = 0$. Kyle (1985), Holden and Subrahmanyam (1992), and others. conjecture that future expected profits are a quadratic function of $\delta_i - p_{t-1}$. Our conjectured form of $V^i_{kt}(\delta_i, \Omega_t)$ is the natural generalization to environments in which the market's assessment of agent i's private information, $E[\delta_i \mid \Omega_{t-1}]$, does not *necessarily* correspond to the past price, p_{t-1}.

We must also determine the expected future profits of agents who have yet to acquire private information. Let $V^i_{kt}(\Omega_{t-1})$ represent the maximized expected profits at date $t < k$ of an agent who has yet to acquire private information δ_i at date k, but who does see the history of prices. Specifically, $V^i_{kt}(\Omega_{t-1})$ corresponds to the expected value of his information once he obtains it: For $t < k$,

$$V^i_{kt}(\Omega_{t-1}) = V^i_{k-1k}(\Omega_{k-2}) \equiv E\left[V^i_{kk}(\delta_i, \Omega_{k-1}) \mid \Omega_{k-1}\right], \quad M^i_k > 0. \tag{13}$$

Proposition 1 characterizes decision making by informed agents and associated equilibrium pricing.

Proposition 1: *Suppose there are N periods and M^i_k informed traders observe δ_i at date k. Then the necessary and sufficient conditions for a linear equilibrium are as follows:*

(*a*) *The trading strategies of an informed agent who observes δ_i at date k take their conjectured form of equation* (3)*. The constants β^i_t, for $i = 1, \ldots, N$, $M^i_k > 0$, and $k \leq t$, satisfy:*

$$\mathbf{\Lambda_N} \begin{bmatrix} \beta^1_t \\ \vdots \\ \beta^N_t \end{bmatrix} = \frac{1}{\lambda_t} \begin{bmatrix} \sum_{j=1}^N \theta^{1j}_t \\ \vdots \\ \sum_{j=1}^N \theta^{Nj}_t \end{bmatrix} - 2 \begin{bmatrix} B^1_t \phi^1_t \\ \vdots \\ B^N_t \phi^N_t \end{bmatrix},$$

where $\Lambda_{\boldsymbol{N}}$ is an $N \times N$ matrix with elements:

$$\begin{aligned} a_{ii} &= 1 + M^i_k\left(1 - 2\lambda_t B^i_t \left(\phi^i_t\right)^2\right) \qquad (k \leq t) \quad \text{and} \\ a_{ij} &= \left(1 - 2\lambda_t B^i_t \left(\phi^i_t\right)^2\right) \sum_{l=1}^{t} M^j_l \theta^{ij}_t \quad (i \neq j \leq t). \end{aligned} \tag{14}$$

For $M^i_k = 0$ or $k > t$, $\beta^i_t = 0$. The second-order condition for optimization is given by

$$\lambda_t\left(1 - \lambda_t B^i_t \left(\phi^i_t\right)^2\right) > 0. \tag{15}$$

(*b*) *The value functions of the informed agent who observes δ_i at date k take the conjectured form of equations* (12) *and* (13), *where A^i_{t-1} evolves according to*

$$A^i_{t-1} = A^i_t + B^i_t\left(\phi^i_t\lambda_t\right)^2\left[\mathrm{Var}(X_t \mid \delta_i, \Omega_{t-1}) + \sigma_u^2\right], \tag{16}$$

and B^i_{t-1} evolves according to

$$B^i_{t-1} = \beta^i_t\left[\sum_{j=1}^{N}\theta^{ij}_t - \lambda_t\sum_{l=1}^{t}\sum_{j=1}^{N}M^j_l\theta^{ij}_t\beta^j_t\right] + B^i_t\left[1 - \phi^i_t\lambda_t\sum_{l=1}^{t}\sum_{j=1}^{N}M^j_l\theta^{ij}_t\beta^j_t\right]^2, \tag{17}$$

with $A^i_N = B^i_N = 0$.

(*c*) *The price function takes the form of equation* (4), *where*

$$\lambda_t = \frac{\mathrm{Cov}(v, X_t \mid \Omega_{t-1})}{\mathrm{Var}(X_t \mid \Omega_{t-1}) + \sigma_u^2}. \tag{18}$$

(*d*) *The conditional covariances between δ_i and δ_j based on public information and on private and public information evolve according to equations* (10) *and* (11), *respectively, and the conditional variance of the asset value Σ_t evolves according to:*

$$\mathrm{Var}(v \mid \Omega_t) \equiv \Sigma_t = \Sigma_{t-1} - \lambda_t^2\left[\mathrm{Var}(X_t \mid \Omega_{t-1}) + \sigma_u^2\right]. \tag{19}$$

The key to the proof is showing that the date t equilibrium trading strategy of any informed agent who has learned δ_i is a linear function of $\delta_i - E[\delta_i \mid \Omega_{t-1}]$, that is, the difference between the agent's private information and the market's expectation of his information. To do this, we first solve an informed agent's decision problem at each date recursively. We suppose that expected future trading profits take their conjectured quadratic form, and that the pricing function and trading strategies of other agents are linear functions. We then show that when agents substitute their best linear forecast (a constant times $\delta_i - E[\delta_i \mid \Omega_{t-1}]$) for the information of other agents, the resulting first-order conditions are linear functions of $\delta_i - E[\delta_i \mid \Omega_{t-1}]$. But, linear first-order conditions imply linear trading rules. Linear trading rules, in turn, imply linear forecasting by the market maker, and hence linear pricing. Finally, the resulting linear trading rules together with linear pricing generate quadratic expected period profits (linear order times expected linear price), and thus the conjectured quadratic form of expected future trading profits, verifying the consistent conjectures.

II. Other Informational Environments

Throughout this section, for expositional purposes, we will attach a temporal interpretation to δ_t. Specifically, we will refer to the t^{th} element in the vector $(\delta_1, \ldots, \delta_N)$ as the period t innovation. We will also assume that there are N informed agents and that informed agent t acquires private information at date t. The reader should note, however, that δ_t does not necessarily equal the period t change in the expectation of v for any particular informed agent. Since in our general formulation the δ's may be correlated, it is possible that part or all of δ_t is forecastable based on information received before period t. We will consider and contrast equilibrium outcomes when informed agent t's private information is only the period innovation δ_t versus when the agent sees the period t asset value, $v_t = \bar{v} + \sum_{j=1}^{t} \delta_j$, versus when he sees the period t history of innovations, $(\delta_1, \ldots, \delta_t)$. We will also consider how the possibility that private information is prematurely made public affects outcomes.

We let x_t^i and V_t^i denote agent i's date t trading strategy and value function, respectively.

A. Positive Probability of Information Revelation

Our analysis has thus far presumed that informed agents can keep their private information to themselves until the end of the trading horizon. However, our qualitative findings extend directly if private information up to period t; that is, $(\delta_1, \ldots, \delta_t)$ may be revealed publicly *after* period t trading and before date $t+1$ trading. Public information then evolves over time according to either $\Omega_t = (\Omega_{t-1}, p_t)$ or $\Omega_t = (\Omega_{t-1}, p_t, \delta_1, \ldots, \delta_t)$. Let μ_{t+1} be the probability that $(\delta_1, \ldots, \delta_t)$ is revealed before date $t+1$ trading occurs. For example, informed agents may have private information about a firm's quarterly earnings, and μ_{t+1} may correspond to the probability that the firm releases quarterly earnings at date $t+1$. This formulation admits the possibility that information release becomes more likely over time. As $\mu_{t+1} \to 1$, an insider's private information is almost certainly short-lived, so that equilibrium outcomes approach those in Admati and Pfleiderer (1988).

It is straightforward to show that x_t^i and V_t^i take the following forms:

$$x_t^i = \beta_t^i(\delta_i - E[\delta_i \mid \Omega_{t-1}]) \quad \text{if} \quad i \le t; \quad x_t^i = 0 \quad \text{otherwise, and} \tag{20}$$

$$V_t^i(\delta_i, \Omega_{t-1}) = A_{t-1}^i + B_{t-1}^i(\delta_i - E[\delta_i \mid \Omega_{t-1}])^2 \quad \text{if} \quad i \le t \quad \text{and} \quad \Omega_t = (\Omega_{t-1}, p_t); \tag{21}$$

$$V_t^i(\delta_i, \Omega_{t-1}) = 0 \quad \text{otherwise.} \tag{22}$$

That is, informed traders $(i \le t)$ participate in the t^{th} period trading and expected future trading profits are positive, if and only if their private information has not been revealed. Agents whose private information has been revealed do not trade. Consequently, for an agent with private information, $(1 - \mu_{t+1})$ essentially acts as a discount factor in his optimization problem. Because the agent may lose future profitable trading opportunities, he trades more aggressively. The optimization problem of an informed agent who has observed δ_i becomes:

$$\begin{aligned}&\max_{x_t^i} E\left[(v - p_t)x_t^i \,\middle|\, \delta_i, \Omega_{t-1}\right]\\&\quad - (1-\mu_{t+1})\cdot 2B_t^i\phi_t^i(\delta_i - E[\delta_i \mid \Omega_{t-1}])E[p_t - p_{t-1} \mid \delta_i, \Omega_{t-1}]\\&\quad + (1-\mu_{t+1})\cdot B_t^i\left(\phi_t^i\right)^2 E\left[(p_t - p_{t-1})^2 \,\middle|\, \delta_i, \Omega_{t-1}\right].\end{aligned} \tag{23}$$

That is, B_t^i is now replaced by $(1-\mu_{t+1})\cdot B_t^i$ in the insider's optimization. It follows that equilibrium outcomes described in Proposition 1 are qualitatively unchanged, except that B_t^i and A_t^i are replaced with $(1-\mu_{t+1})B_t^i$ and $(1-\mu_{t+1})A_t^i$, respectively.

B. Informed Agents Observe Asset Value

We next show how trading strategies are affected when the informed agent i observes the date i asset value, $v_i \equiv \bar{v} + \sum_{j=1}^{i}\delta_j$, $(v_N = v)$, rather than the date i asset innovation. Now, when informed agent i acquires his information, he becomes unambiguously better informed about the asset's value than agents who learn the asset value at earlier dates, $j < i$. This formulation therefore captures the feature that the information of agents who acquire information at earlier dates becomes stale as time passes *due to the later acquisition of private information by other agents*.

The qualitative properties of the equilibrium trading strategies extend in the natural way. Redefine equations (6) and (7) by replacing agent i's previous signal, δ_i, with v_i:

$$\phi_t^i = \frac{\mathrm{Cov}(v_i, p_t \mid \Omega_{t-1})}{\mathrm{Var}(p_t \mid \Omega_{t-1})}, \quad \theta_t^{ij} = \frac{\mathrm{Cov}(v_i, v_j \mid \Omega_{t-1})}{\mathrm{Var}(v_i \mid \Omega_{t-1})} = \frac{\sum_{l=1}^{i}\sum_{k=1}^{j}\mathrm{Cov}(\delta_l, \delta_k \mid \Omega_{t-1})}{\sum_{l=1}^{i}\sum_{k=1}^{i}\mathrm{Cov}(\delta_l, \delta_k \mid \Omega_{t-1})}.$$

Lemmas 1–3 are unchanged, save that δ_i, δ_j and δ_k are replaced by v_i, v_j and v_k, respectively.

Again, uninformed agents $i > t$ do not trade in period t and the equilibrium trading rule of informed agent $i \le t$ is a linear function of his private information, that is, the difference between his signal, v_i, and the market's forecast of his signal, $E[v_i \mid \Omega_{t-1}]$:

$$x_t^i = \beta_t^i(v_i - E[v_i \mid \Omega_{t-1}]), \qquad t \ge i; \quad x_t^i = 0, t < i. \tag{3$'$}$$

Analogously, the value function of an informed agent $i \le t$ takes the form

$$V_t^i(v_i, \Omega_{t-1}) = A_{t-1}^i + B_{t-1}^i(v_i - E[v_i \mid \Omega_{t-1}])^2, \tag{12$'$}$$

and the value function of an uninformed agent $i > t$ takes the form:

$$V_t^i(\Omega_{t-1}) = V_{i-1}^i(\Omega_{i-2}) = A_{i-1}^i + B_{i-1}^i \mathrm{Var}[v_i \mid \Omega_{i-1}], t < i. \tag{13$'$}$$

Finally, pricing functions take the form of (4), where total informed trade is

$$X_t = \sum_{i=1}^{t} \beta_t^i (v_i - E[v_i \mid \Omega_{t-1}]). \tag{24}$$

A formal statement and proof of this proposition is in a working paper draft. Both the statement and proof follow that of Proposition 1 directly, except that δ_i is replaced by v_i.

C. Later Information Subsumes Earlier Information

We now show how our analytical approach extends if informed agent *i*'s information subsumes that of all agents who acquired information at earlier dates. That is, we now suppose that informed agent i sees the entire *history* of past innovations, $(\delta_1, \ldots, \delta_i)$, at time i rather than just the period i innovation, δ_i, or the period i asset value, $v_i = \bar{v} + \sum_{j=1}^{i} \delta_j$. Note that if agent i sees the history of past asset values, $(\bar{v}, v_1, \ldots, v_i)$, then he can infer the innovation history.

In this informational environment, the quality of information that later agents acquire is unambiguously better than if they only observe the asset values, or a single-period innovation. Now the history of past innovations reveals not only the period i asset value v_i, but also, in equilibrium, the future trades of those agents who acquired information at earlier dates. Informed agents are now asymmetrically situated with respect to each other: Agents who acquire information at earlier dates must forecast the information of agents who acquire information at later dates, but the converse is not true.

We first detail how forecasts, given public and private information, are updated. Each agent forecasts the asset's value and the trades of other agents by running a multivariate regression on *each* piece of his private information:

LEMMA 4:

(*a*) *The market's expectation of the period i innovation is*

$$E[\delta_i \mid \Omega_t] = E[\delta_i \mid \Omega_{t-1}] + \phi_t^i (p_t - E[p_t \mid \Omega_{t-1}]),$$
$$\textit{where} \quad \phi_t^i = \frac{\mathrm{Cov}(\delta_i, p_t \mid \Omega_{t-1})}{\mathrm{Var}(p_t \mid \Omega_{t-1})}.$$

(*b*) *For* $j > i$, *the relationship between* $E[\delta_j \mid \Omega_{t-1}]$ *and* $E[\delta_j \mid \delta_1, \ldots, \delta_i, \Omega_{t-1}]$ *is given by*

$$E[\delta_j \mid \delta_1, \ldots, \delta_i, \Omega_{t-1}] = E[\delta_j \mid \Omega_{t-1}] + \sum_{k=1}^{i} \theta_t^{kj} (\delta_k - E[\delta_k \mid \Omega_{t-1}]),$$
$$\textit{where} \quad \theta_t^{kj} = \frac{\mathrm{Cov}(\delta_k, \delta_j \mid \Omega_{t-1})}{\mathrm{Var}(\delta_k \mid \Omega_{t-1})}.$$

(*c*) *The covariance between* δ_j *and* δ_k *conditional on public information evolves according to*

$$\mathrm{Cov}(\delta_j, \delta_k \mid \Omega_t) = \mathrm{Cov}(\delta_j, \delta_k \mid \Omega_{t-1}) - \frac{\mathrm{Cov}(\delta_j, p_t \mid \Omega_{t-1})\,\mathrm{Cov}(\delta_k, p_t \mid \Omega_{t-1})}{\mathrm{Var}(p_t \mid \Omega_{t-1})}.$$

At date $t \geq i$, *insider* i *observes* $\delta_1, \ldots, \delta_i$ *so that conditional on private and public information, the covariance between* δ_j *and* δ_k *evolves according to*

$$\begin{aligned}&\mathrm{Cov}(\delta_j, \delta_k \mid \delta_1, \ldots, \delta_i, \Omega_{t-1})\\ &\quad = \mathrm{Cov}(\delta_j, \delta_k \mid \Omega_{t-1}) - \sum_{\ell=1}^{i} \frac{\mathrm{Cov}(\delta_j, \delta_\ell \mid \Omega_{t-1})\,\mathrm{Cov}(\delta_k, \delta_\ell \mid \Omega_{t-1})}{\mathrm{Var}(\delta_\ell \mid \Omega_{t-1})}.\end{aligned}$$

The next proposition, which is proved in the Appendix, shows that our analysis extends in a straightforward way: At each date t, informed agent $i \leq t$'s optimal trading strategy is a linear function of *each* piece of his unrevealed private information, $(\delta_1 - E[\delta_1 \mid \Omega_{t-1}], \ldots, (\delta_i - E[\delta_i \mid \Omega_{t-1}])$, and his value function is a quadratic function of unrevealed private information.

PROPOSITION 2: *Suppose informed trader i observes asset innovations at earlier dates, observing* $\{\delta_1, \delta_2, \ldots, \delta_i\}, i = 1, \ldots, N$, *so that his information subsumes that of agents who acquired information at earlier dates. Then on an equilibrium path:*

(*a*) *The period t pricing function takes the form:*

$$p_t = p_{t-1} + \lambda_t(X_t + u_t), \quad \text{where } \lambda_t = \frac{\mathrm{Cov}(v, X_t \mid \Omega_{t-1})}{\mathrm{Var}(X_t \mid \Omega_{t-1}) + \sigma_u^2}. \tag{25}$$

(*b*) *Trading strategies of each agent i take the form:*

$$\begin{aligned} x_t^i(\delta_1, \ldots, \delta_i, \Omega_{t-1}) &= \sum_{j=1}^{i} \beta_{jt}^i(\delta_j - E[\delta_j \mid \Omega_{t-1}]) \\ &\qquad \text{for } i \leq t, \quad \text{where } \beta_{jt}^i \text{ is a constant} \\ &= 0 \qquad \text{for } i > t. \end{aligned} \tag{26}$$

(*c*) *The value functions take the form:*

$$\begin{aligned} V_t^i(\delta_1, \ldots, \delta_i, \Omega_{t-1}) &= A_{t-1}^i + \sum_{k=1}^{i}\sum_{j=1}^{i} B_{jt-1}^i C_{kt-1}^i(\delta_j - E[\delta_j \mid \Omega_{t-1}]) \\ &\qquad \times (\delta_k - E[\delta_k \mid \Omega_{t-1}]) \quad \text{for } i \leq t, \end{aligned} \tag{27}$$

where B_{jt-1}^i *and* C_{kt-1}^i *are constants,* $B_{jN}^i = C_{kN}^i = 0$, *and* $V_t^i(\Omega_{t-1}) = V_{i-1}^i$ *for* $i > t$.

The proof strategy is again a straightforward induction argument. We suppose that expected future trading profits take their conjectured quadratic form, and that the pricing function and trading strategies of other agents are linear functions. We then show that the linear structure of the first-order condition from optimization is preserved—agents either substitute the known value for an innovation, or their estimate of the innovation given their multivariate forecasting regression described in Lemma 4(b). Finally, we show that this linear structure induces the conjectured quadratic form of the value function. Thus, in equilibrium, trading strategies are linear and the value functions are quadratic.

Note that Proposition 2 provides an incomplete characterization of outcomes: Tractability, in general, precludes deriving, the difference equations governing the equilibrium evolution of the constants $\beta^i_{jt}, \lambda_t, A^i_{t-1}, B^i_{jt-1}$, and C^i_{kt-1}, which characterize the evolution of trading behavior, payoffs, and pricing.

It is important to note that our analytical approach is easily extended to situations where agents see a set of different (normally distributed) asset-value innovations, which do not necessarily correspond to the history of past innovations. That is, equilibrium strategic trading and pricing when agents have many sources of information at their disposal (perhaps acquiring information at different dates) is qualitatively unaffected by the number of pieces of information to which they have access. Again, agents' trading strategies are linear functions of each piece of information available to them; agents use each signal to forecast both the asset's value, and the trades of other agents, and pricing is a linear function of order flow. As such, this represents an important extension of the literature, which, to date, assumes that each agent has access to only one signal. We now consider a special case to provide sharper characterizations.

PROPOSITION 3: *Suppose there are two periods, two informed traders, innovations are independently distributed, informed agent* 1 *observes* δ_1 *in period* 1, *and informed agent* 2 *observes both* δ_1 *and* δ_2 *in period* 2. *Then:*

(*a*) *Equilibrium trading strategies are*

$$x^1_2 = \frac{1}{3\lambda_2}\left(\delta_1 - E[\delta_1 \mid \Omega_1]\right), \tag{28}$$

$$x^2_2 = \frac{1}{3\lambda_2}(\delta_1 - E[\delta_1 \mid \Omega_1]) + \frac{1}{2\lambda_2}\delta_2,$$

$$x^2_1 = 0, \quad \text{and} \quad x^1_1 = \frac{1 - 2\lambda_1/(9\lambda_2)}{2\lambda_1[1 - \lambda_1/(9\lambda_2)]}(\delta_1 - E[\delta_1 \mid \Omega_0]). \tag{29}$$

The second-order conditions are $\lambda_2 > 0$, and $0 < \lambda_1 < 9\lambda_2/2$.

(*b*) *The constants in the price function* (4) *solve*

$$\lambda_2 = \frac{1}{\sigma_u}\sqrt{\frac{2}{9}\text{Var}(\delta_1 \mid \Omega_1) + \frac{1}{4}\text{Var}(\delta_2)},$$

$$\begin{aligned}&\left(1 - 2\lambda_1/(9\lambda_2)\right)^2 \text{Var}(\delta_1) + 4(\lambda_1)^2(1 - \lambda_1/(9\lambda_2)^2\sigma_u^2 \\ &\quad = 2(1 - \lambda_1/(9\lambda_2))\left(1 - 2\lambda_1/(9\lambda_2\right)\text{Var}(\delta_1).\end{aligned} \tag{30}$$

Proof: See the Appendix for the proof and a more complete characterization. Q.E.D.

Proposition 3 implies that an agent's optimal trading strategy *cannot* be written as a linear function of the difference between his forecast of the asset value given his private information and trade history, and the market's forecast given the trade history. Essentially, the reason is that the difference between each signal that an agent sees and the market's forecast is a distinct strategic component. The intensity with which an agent trades on each signal varies with the competition on that information. Here, agent 2 faces competition from agent 1 on the older, date 1 information innovation, δ_1, but he does not face competition over his information about δ_2. The increased competition over δ_1 causes agent 2 to reduce the intensity with which he trades. He trades as a "monopolist" over δ_2, but as a "duopolist" over δ_1.

III. Numerical Results

To shed more light on how the informational environment affects outcomes, we now consider three different informational environments.

First, to illustrate how the timing of information acquisition affects outcomes, we consider two agents who acquire the same information but at different points in time. We show that as the number of trading periods in which the first informed agent has private information to himself rises, equilibrium dynamics move smoothly from duopoly to monopoly dynamics. Indeed, as we divide the trading horizon up more finely, à la Kyle, as the number of trading periods grows large, we endogenize the trading horizon of a Kyle monopolist.

Next, we consider an environment in which innovations are independently distributed, and each agent sees one innovation. We then consider how the division of information between the agents affects outcomes. Three key observations emerge. First, less information is revealed in dynamic settings when information is more evenly divided, because this leads private information to be more negatively conditionally correlated. Second, the price impact of order flow falls with time if agents acquire information simultaneously, but the price impact rises with time if agents do not acquire information at the same time and the agent acquiring information at a later date acquires "enough" of it. Third, to generate the U-shaped intradaily pattern in volume found in the data, agents must again acquire their information at different dates, and the agent acquiring information at a later date, must acquire an "intermediate" amount of information.

Finally, we consider how the nature of information that agents acquire affects outcomes. Specifically, in a two-period environment with independently distributed innovations we consider how outcomes are affected when agent 2 observes (a) δ_2, (b), $v = \delta_1 + \delta_2$, and (c) $\{\delta_1, \delta_2\}$. We contrast outcomes when agent 2 acquires his information at date 1 with those when he acquires his information at date 2. Several observations emerge. Again, sequential information acquisition can generate simultaneously both the increased bid–ask spreads and increased trading volumes, and hence price volatilities, while simultaneous information acquisition cannot. Second, improving agent 2's information need not increase even his own profits; his profits are higher when he sees $v = \delta_1 + \delta_2$ than when he sees $\{\delta_1, \delta_2\}$. Third, improving agent 2's information always leads to more information being revealed through price. Finally, the nature and timing of information acquisition interact in subtle ways on the trading intensities of the two informed agents.

A. *Timing of Information*

To determine the impact of acquiring information at different dates, we first generalize Holden and Subrahmanyam's (1992) environment. Suppose innovations are perfectly correlated, $v = \bar{v} + n\delta$, and $L_j = \sum_{i=1}^{n} M_j^i$ traders observe δ at date j. Since signals are perfectly correlated, each informed agent can use his signal to infer the signal that other informed agents have received or will receive. Proposition 4 in the Appendix characterizes the unique equilibrium. This environment is general enough to include several scenarios of interest. First, if $n = 1$ and $L_1 > 0$, then Proposition 4 reduces to Proposition 2 in Holden and Subrahmanyam (1992). Second, if $n > 1$ and $L_j = L > 0, \forall j$, then Proposition 4 captures the case where L newly informed traders learn δ each period. Finally, we can capture different degrees of delayed information acquisition. For example, $M = 2, L_1 = 1, L_t = 1$, and $L_j = 0$ for $j \notin \{1, t\}$, captures an environment in which one informed agent acquires information with a considerable lag.

Figure 1 contrasts equilibrium outcomes for different specifications of information acquisition in a six-period model: (a) a single agent who observes δ in period 1, as in Kyle (1985); (b) two informed agents who observe δ in period 1, as in Holden and Subrahmanyam (1992); (c) two informed agents, one of whom observes δ in period 1, the other observes δ one period later; and, (d) two informed agents, one of whom observes δ in period 1, the other does not observe δ until period 5. In all of our numerical examples we set the variances of both the asset value and liquidity trade to equal 1.

This figure suggests that differences in the timing of information acquisition become important if there is a substantial delay in information acquisition. If there is only a one-period difference, outcomes are similar to those where agents acquire information simultaneously. In contrast, if the second informed agent does not acquire information until date 5, outcomes approximate Kyle's monopoly outcome. Relative to a monopolist informed agent, the first agent to acquire information trades more aggressively on his information at earlier dates, because he anticipates greater future competition. This effect is especially significant if the first informed agent anticipates competition shortly after information acquisition. As a result, the price impact of order flow (λ_t) is greater both prior to the entry of the second informed agent, and immediately thereafter as the agents compete away their informational rents. Eventually, the greater information decay due to the more aggressive trading reduces the price impact below the level that obtained when there is a single informed agent. Figure 1 also reveals that the four-period delay in information

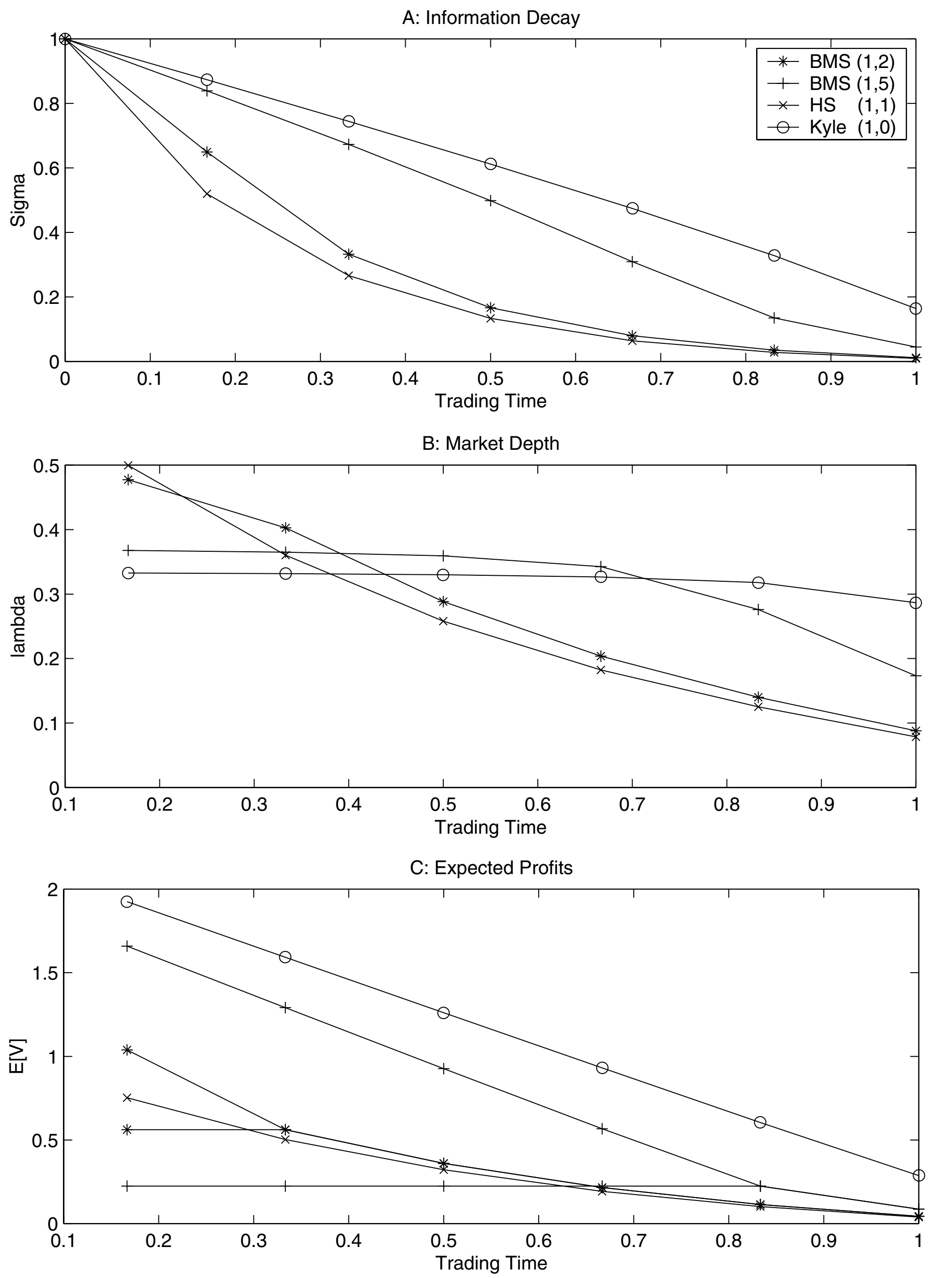

Figure 1. Perfectly correlated innovations and six trading periods. The figure contrasts equilibrium outcomes for different specifications of information acquisition. BMS (1,2): agent 1 (2) acquires information at date 1 (2). BMS (1,5): agent 1 (2) acquires information at date 1 (5). HS (1,1): both agents acquire information at date 1. Kyle (1,0): Kyle's model. Σ: conditional variance of the asset value given public information. λ: the price impact of order flow. E[V]: the expected lifetime profits.

acquisition means that it takes much longer for information to be revealed to the market through prices.

Figure 2 illustrates how outcomes are affected by sequential information acquisition if we take limits à la Kyle and approximate the continuous trading environment by dividing the trading interval [0, 1] into 2^J equal subintervals, so that liquidity trade in each period is $\sigma_u^2/2^J$ for $J = 4, 8, 16$. This figure contrasts the Kyle monopoly outcome with that where the second informed agent acquires information in period $2^{J-1} + 1$, after half of the trading opportunities have passed.

Figure 2 reveals that if the first informed agent has enough periods to trade before the second agent acquires information, then he essentially behaves as a Kyle monopolist facing a trading horizon that corresponds to the period before the second agent acquires information (here $t = 0.5$). In particular, as $J \to \infty$, the first informed agent trades so that all of his private information is just revealed at the moment that the second agent acquires information. In this way, anticipation of the competition induced by information acquisition by the second agent effectively endogenizes the terminal condition for the Kyle monopolist. Because the first informed agent trades more aggressively prior to $t = 0.5$, the limit value of the pricing parameter before date $t = 0.5$ exceeds that for a single Kyle monopolist.

Figure 3 shows further how the possibility that agents' private information may be publicly revealed at an early date (recall that μ is the probability each period that the information is revealed publicly) causes informed agents to trade more aggressively, which leads to more rapid information release through trade. Trading intensities and hence pricing (conditional on information *not* being revealed) are relatively similar when (i) agents acquire information at nearly the same time and there is no possibility that their information becomes public and when (ii) there is a long delay in the information acquisition by a second informed agent and a 0.5 probability each period that private information is made public. The strategic presence of a second informed agent leads to slightly more information release through trading than does "competition from God" in the form of a second nonstrategic "agent" who reveals private information publicly with probability 0.5.

B. Division and Timing of Information

Now consider the polar case where asset innovations are independently distributed, and let $v = \bar{v} + \delta_1 + \delta_2$. Since innovations are independently distributed, the current price contains no information about future asset innovations. Proposition 5 in the Appendix details equilibrium outcomes in a three-period environment with independently distributed innovations and two informed agents, both when agent i observes δ_i at date 1, and when agent i observes δ_i at date i. The first setting is a generalization of Foster and Viswanathan (1996) in that the innovations can have arbitrary variances.

Although innovations are independently distributed, the strategic trading behavior of the informed agents leads the covariance between δ_1 and δ_2 conditional

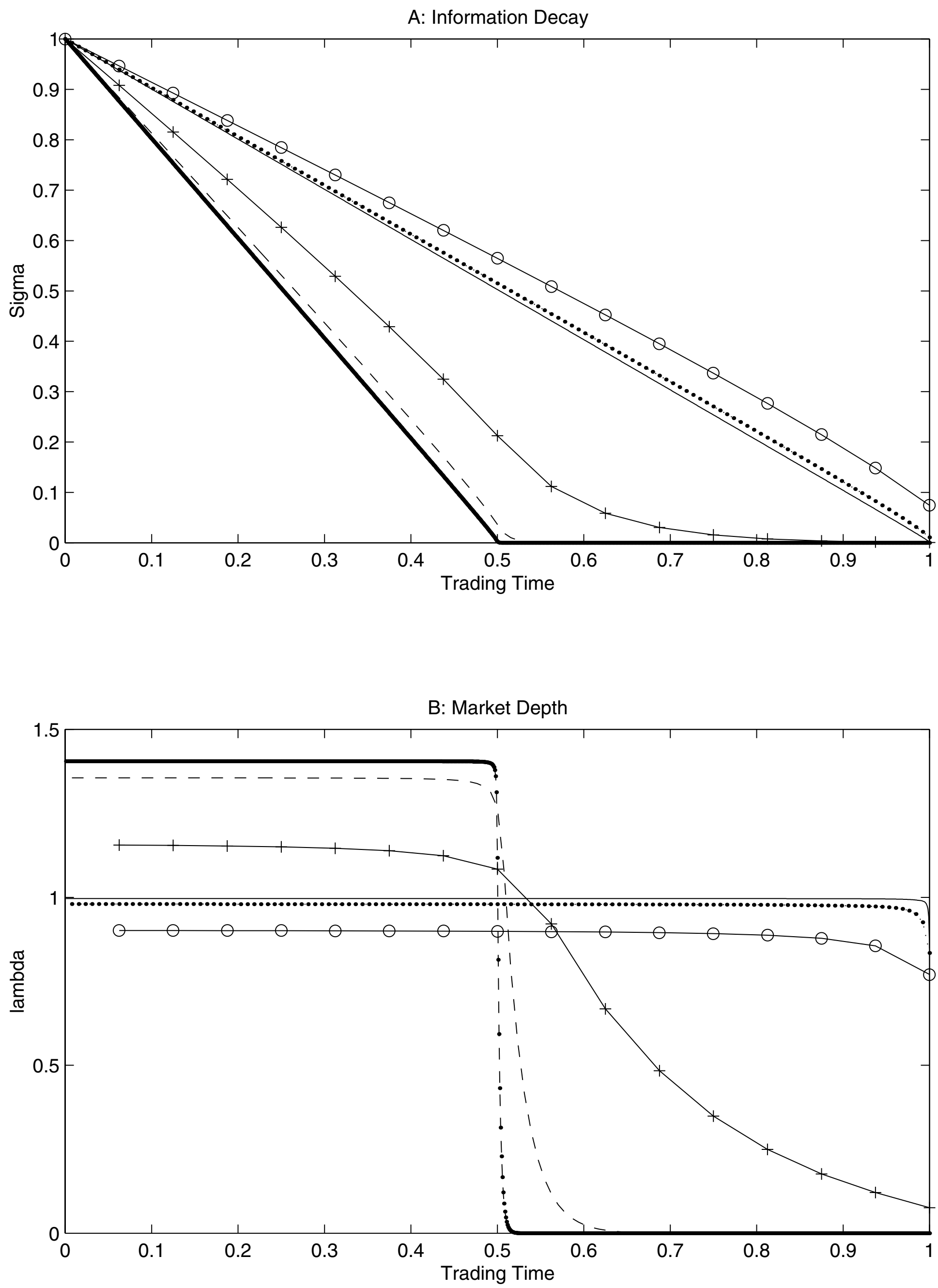

Figure 2. Perfectly correlated innovations and convergence. The figure illustrates limiting results by dividing the trading interval [0,1] into 2^J equal subintervals so that liquidity trade in each period is $\sigma_u^2/2^J$ for $J = 4, 8, 16$. Σ: conditional variance of the asset value given public information. λ: the price impact of order flow.

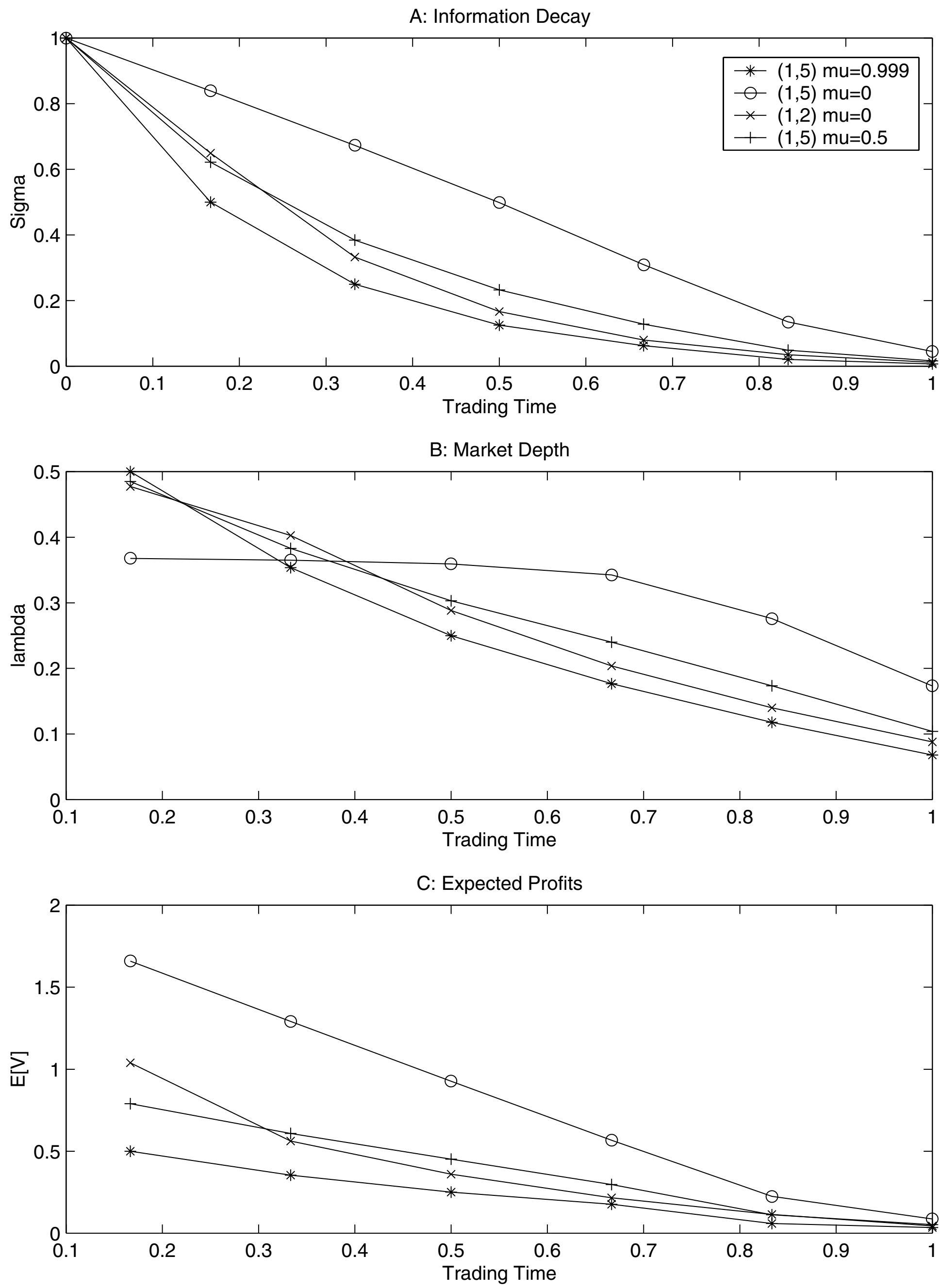

Figure 3. Positive probability of information revelation. The figure illustrates equilibrium outcomes when innovations are perfectly correlated and private information is publicly revealed with probability μ. Σ: conditional variance of the asset value given public information. λ: the price impact of order flow. E[V]: the expected lifetime profits.

on date 1 public information to be *negative*. Intuitively, this is because the market maker is forecasting both δ_1 and δ_2 using total net order flow: If he attributes too much of the order flow to δ_1, then he is attributing too little to δ_2. This negative covariance is easiest to see when information is acquired sequentially. Proposition 5(c) in the Appendix yields

$$\mathrm{Cov}(\delta_1, \delta_2 \mid \Omega_2) = -\frac{\beta_2^1 \beta_2^2 \,\mathrm{Var}(\delta_1 \mid \Omega_1)\,\mathrm{Var}(\delta_2 \mid \Omega_1)}{\left(\beta_2^1\right)^2 \mathrm{Var}(\delta_1 \mid \Omega_1) + \left(\beta_2^2\right)^2 \mathrm{Var}(\delta_2 \mid \Omega_1) + \sigma_u^2} < 0.$$

Further, the difference in informed trading intensities at date 3, $\beta_3^1 - \beta_3^2$, is proportional to

$$\theta_3^{12} - \theta_3^{21} = \mathrm{Cov}(\delta_1, \delta_2 \mid \Omega_2)\left[\frac{1}{\mathrm{Var}(\delta_1 \mid \Omega_2)} - \frac{1}{\mathrm{Var}(\delta_2 \mid \Omega_2)}\right].$$

Since δ_1 and δ_2 are conditionally negatively correlated, it follows that $\beta_3^1 < \beta_3^2$ if and only if $\mathrm{Var}(\delta_1 \mid \Omega_2) < \mathrm{Var}(\delta_2 \mid \Omega_2)$. That is, as private information is revealed through trading prior to the final period (and thus as the conditional variance decreases), insider i's trading strategy becomes less elastic with respect to his unrevealed private information, $\delta_i - E[\delta_i \mid \Omega_2]$.

Figures 4 and 5 illustrate outcomes when both agents acquire information at date 1, while Figures 6 and 7 illustrate outcomes where agent j observes δ_j at date j, $j = 1, 2$. We contrast this case where innovations are identically distributed, $\mathrm{Var}(\delta_1) = \mathrm{Var}(\delta_2) = 0.5$, with the case where informed agent 2 has better information than agent 1, $\mathrm{Var}(\delta_1) = 0.1$ and $\mathrm{Var}(\delta_2) = 0.9$, and Kyle's framework where $\mathrm{Var}(\delta_1) = 1.0$ and $\mathrm{Var}(\delta_2) = 0.0$. Importantly, our solution technique does not require symmetrically distributed signals.

Note that even though innovations are independently distributed, agents trade less aggressively when information is more evenly divided among agents. In a one-period trading environment, trading intensities and pricing would not depend on how information is divided among informed agents. However, in a dynamic context, each informed agent uses the price and his own private signal to update his forecast of the other agent's signal. Signals are more conditionally negatively correlated in periods 2 and 3 when information is more evenly divided (the date 2 and date 3 conditional covariances are -0.0862 and -0.1454 when $\mathrm{Var}(\delta_1) = \mathrm{Var}(\delta_2) = 0.5$ versus -0.0380 and -0.0563 when $\mathrm{Var}(\delta_1) = 0.9, \mathrm{Var}(\delta_2) = 0.1$). As a result, when signals are more symmetrically distributed, agents trade less intensively on their information at dates 2 and 3 (see Figure 5). In turn, less information is revealed through trade (see Figure 4, Panel A). This augments a finding by Back et al. (2000), who find that less information is revealed when innovations are independently and symmetrically distributed between two informed agents than when a single trader sees the sum of the innovations. More equal signal divisions also reduce total informed profits, but the impact is not spread uniformly across periods. Total second-period profits are greatest with more equal distributions

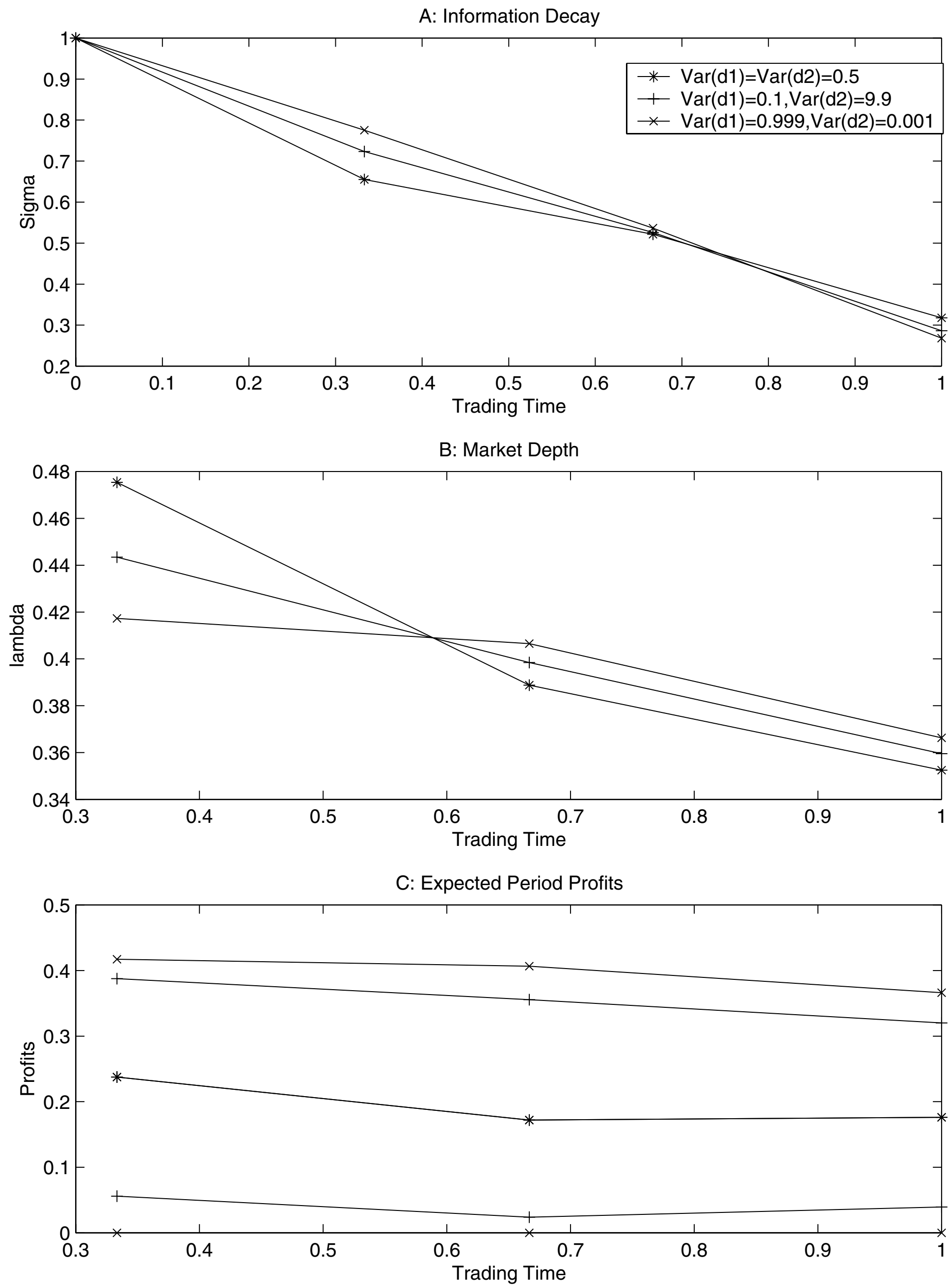

Figure 4. Simultaneous information acquisition. The figure illustrates equilibrium outcomes when innovations are independently distributed and agent i acquires information δ_i at date 1 for $i = 1, 2$. Σ: conditional variance of the asset value given public information. λ: the price impact of order flow. Var(d1) (Var(d2)): Var(δ_1)(Var(δ_2)).

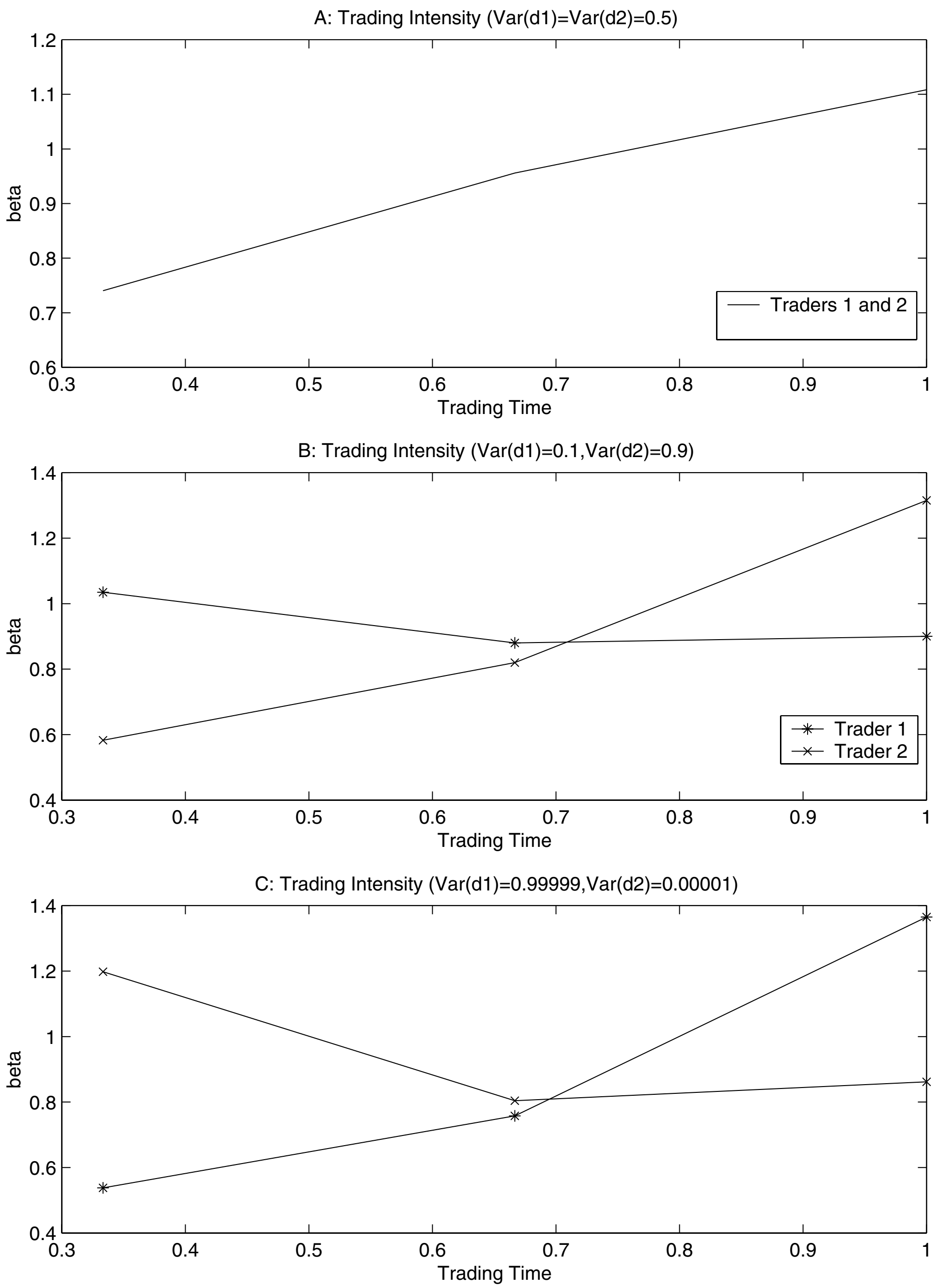

Figure 5. Simultaneous information and trading intensities. The figure illustrates trading intensities for the two agents when innovations are independently distributed and agent i acquires information δ_i at date 1 for $i = 1, 2$. Var(d1) (Var(d2)): Var(δ_1)(Var(δ_2)).

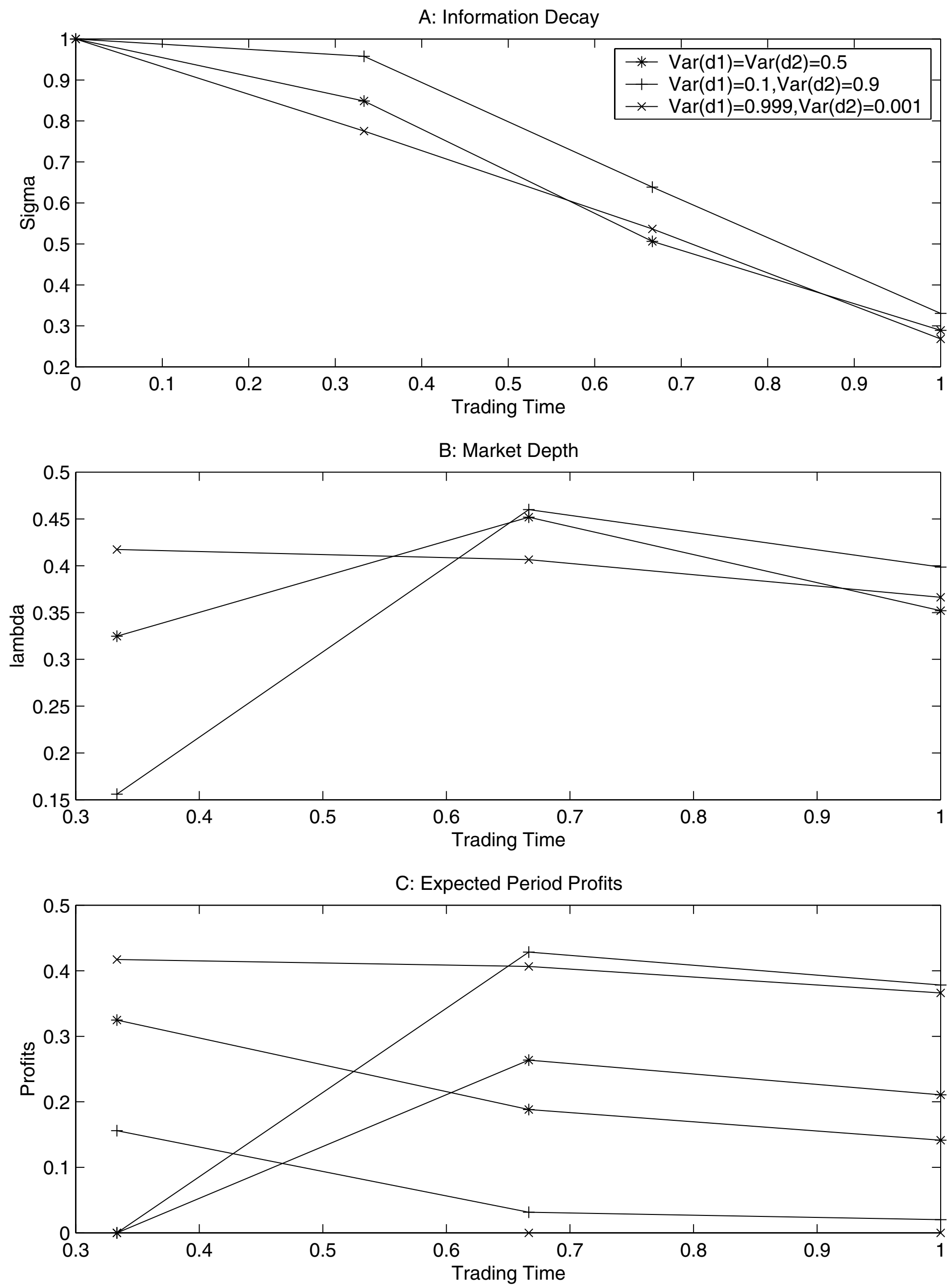

Figure 6. Sequential information acquisition. The figure illustrates equilibrium outcomes when innovations are independently distributed and agent i acquires information δ_i at date i for $i = 1, 2$. Σ: conditional variance of the asset value given public information. λ: the price impact of order flow.

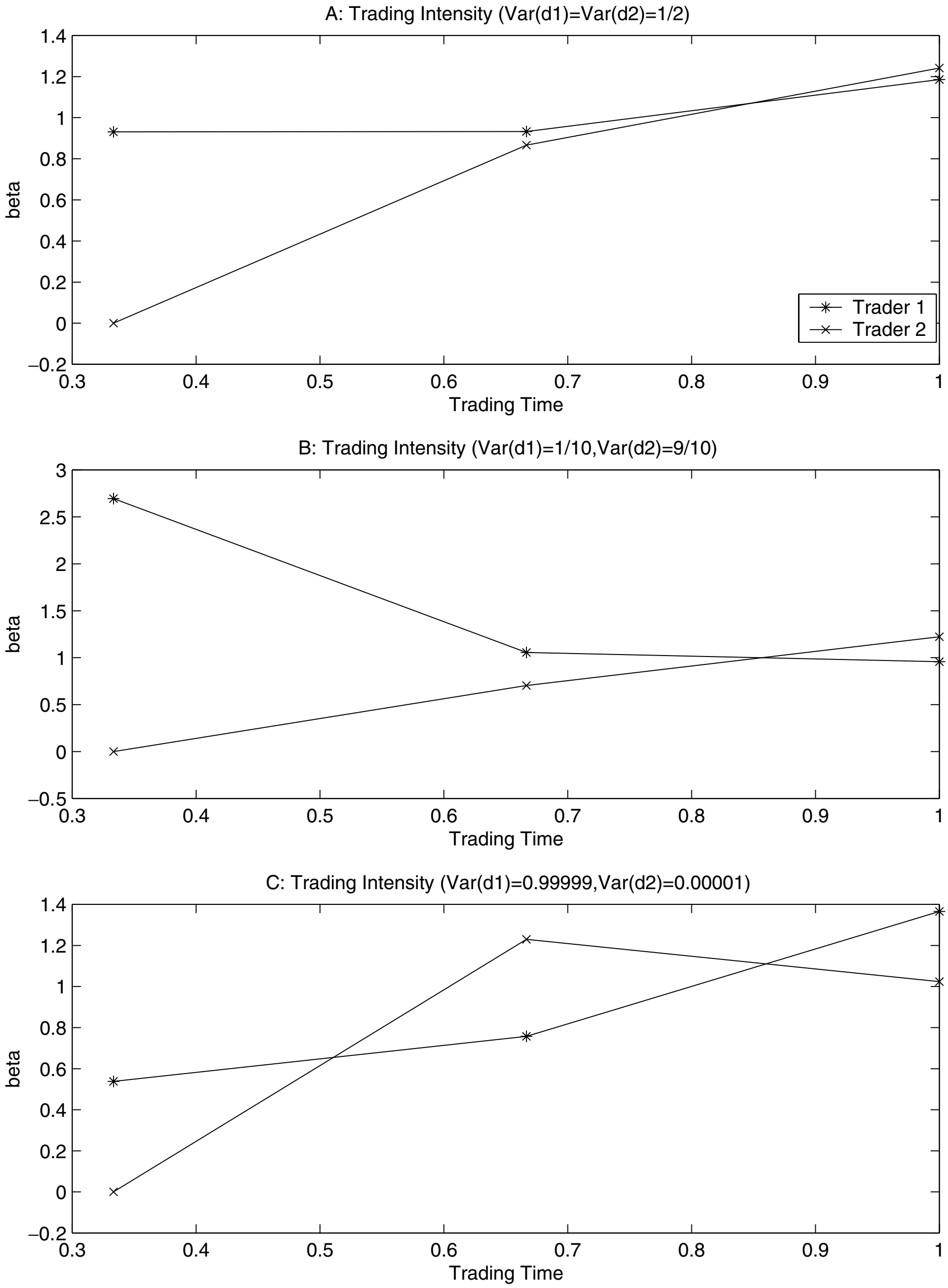

Figure 7. Sequential information and trading intensities. The figure illustrates trading intensities for the two agents when innovations are independently distributed and agent i acquires information δ_i at date i for $i = 1, 2$. Var(d1) (Var(d2)): Var(δ_1) (Var(δ_2)).

of information, while first- and third-period profits are smallest (see Figure 4, Panel C).

Figure 4, Panel B reveals that the price impact of order flow (λ_t) falls with time if both agents acquire information at date 1. In sharp contrast, Figure 6 reveals that if information is acquired sequentially, then as long as informed agent 2's signal contains enough information, the price impact of trade rises in period 2 and then falls. This reflects that while some of agent 1's information has been revealed at date 1, (i) the private information is further increased at date 2 because of agent 2's newly acquired information, and (ii) (see Figure 7) both agents trade more aggressively at date 2 because the trading horizon is shorter. At date 3, the price impact falls because period 2 trading reveals enough private information to offset the greater informed trading intensities. Thus, sequential information arrival helps generate the widening bid-ask spreads and price volatility observed in the data as the moment at which information becomes public approaches.

Finally, Figure 8 illustrates the evolution of trading volume, which we measure as the square root of the expected net order flow,

$$\sqrt{\mathrm{Var}(X_t + u_t)} = \sqrt{\mathrm{Var}\left[\sum_{i=1}^{I} \beta_t^i(\delta_i - E[\delta_i \mid \Omega_{t-1}]) + u_t\right]}$$
$$= \left[\sum_{j=1}^{I}\sum_{i=1}^{I} \beta_t^i \beta_t^j \,\mathrm{Cov}(\delta_i, \delta_j \mid \Omega_{t-1}) + \sigma_u^2\right]^{1/2}. \tag{31}$$

Importantly, if agents acquire information sequentially and agent 2 has better information ($\mathrm{Var}(\delta_1) = 0.1$, $\mathrm{Var}(\delta_2) = 0.9$), then the model can generate the U-shaped pattern of intraday trading volume found in the data (see, e.g., Bernhardt and Hughson (2002)). The behavior of trading intensities (Figures 5 and 7) underlies this result. To generate the U-shaped pattern, there must be both sufficient new information arrival at the opening and in the middle of the day. Then, the initial volume is relatively high, driven by new information arrival; volume at period 2 falls as the new information arrival causes the agent who obtained information at the beginning of the day to reduce sharply his trading intensity; and, volume rises again at period 3 because the trading horizon ends, so there ceases to be a reason to restrict trade to support future profits. In sharp contrast, if both agents acquire information at date 1, or if innovations are sufficiently correlated, then volume rises over time as informed trading intensities rise due to the reduced trading horizon. A nice feature of our model is that it can generate the U-shaped intradaily volume even when information arrival is high in the middle of the day, which is presumably when analysts are busiest gathering information. This contrasts with Bernhardt and Hughson (2002), who provide structural estimates of intradaily information arrival. Specifically, in Bernhardt and Hughson (2002) there is a one-to-one link between information arrival and volume, so that they necessarily require information production to be lowest in the middle of the day.

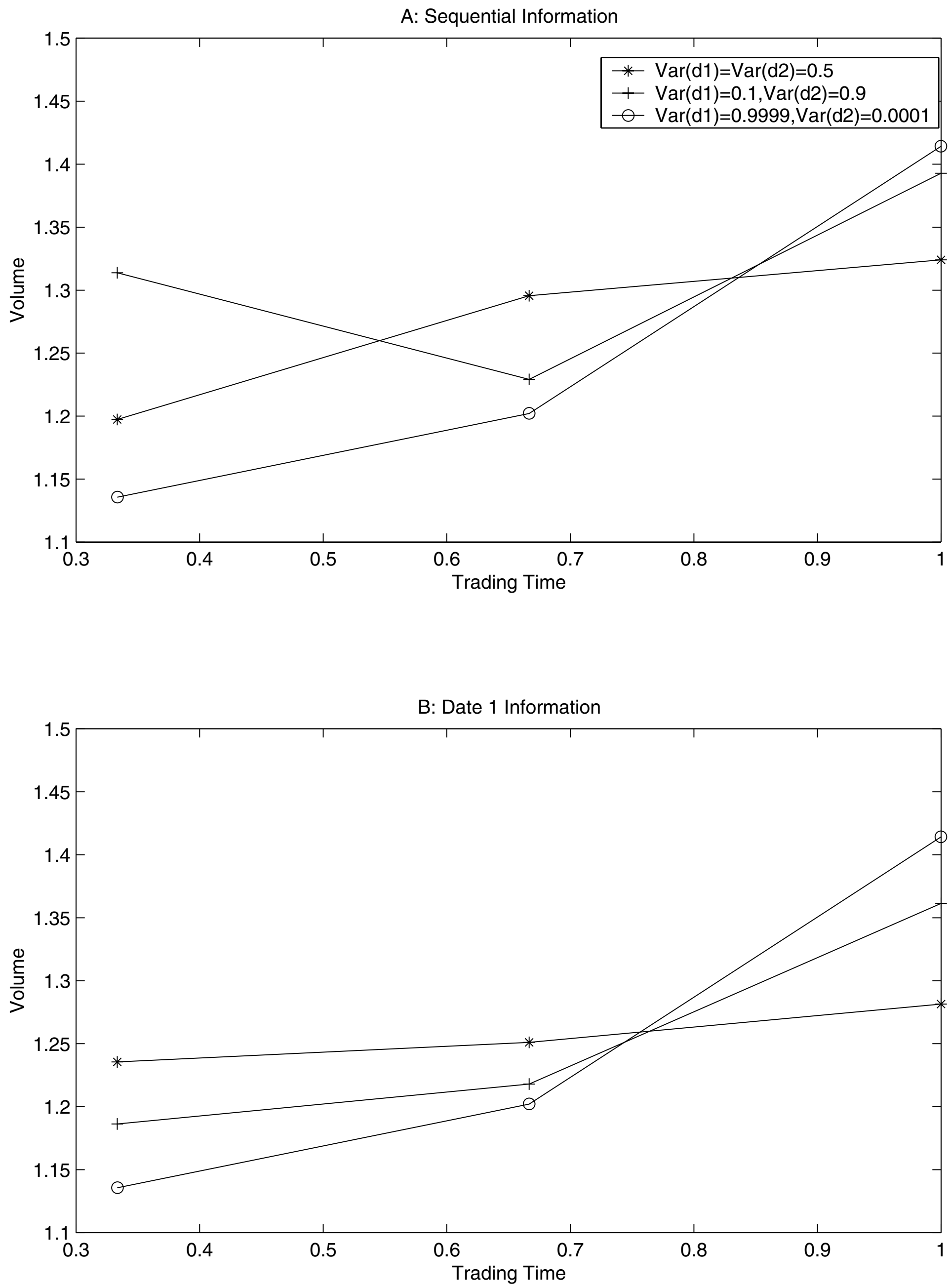

Figure 8. Trading volume. The figure illustrates trading volume when innovations are independently distributed and two agents acquire information simultaneously at date 1 or sequentially. Var(d1) (Var(d2)): Var(δ_1) (Var(δ_2)).

C. Nature of Information

Finally, we characterize how the nature of information that agents acquire affects equilibrium outcomes. Specifically, let the asset value be $v = \delta_1 + \delta_2$, where δ_1 and δ_2 are independently distributed. Within a two-period trading environment with two informed agents, we contrast outcomes when agent 1 sees δ_1 and agent 2 observes

- Only the date 2 asset innovation, δ_2.
- The asset value $v = \delta_1 + \delta_2$.
- Both asset innovations, $\{\delta_1, \delta_2\}$, so that his information subsumes agent 1's.

For each informational environment, we also determine how outcomes are affected by the timing of agent 2's information acquisition. The first set of columns of Table I details equilibrium outcomes when agent 2 does not acquire information until period 2, while the last two columns present outcomes when the agents simultaneously acquire information in the first period, as in Foster and Viswanathan (1994).

This table shows that the timing of information acquisition does not affect the ordering of the impact of the quality of agent 2's information on key variables: total information revealed through trade, individual and total informed profit, and the price impact of information. Of course, the timing of information acquisition has distinct impacts within periods. The most important impact

Table I

Impact of Information Quality and Timing of Information Acquisition on Equilibrium Outcomes

The table illustrates equilibrium outcomes when agent 1 observes δ_1 and agent 2 observes δ_2, $v = \delta_1 + \delta_2$, or $\{\delta_1, \delta_2\}$. Σ : conditional variance of the asset value given public information. λ : price impact of order flow. V_t^j : trader j's profit at date t. β : trading intensity.

	Agent 2's Info.	Sequential Information		Simultaneous Information	
		Period 1	Period 2	Period 1	Period 2
	δ_2	0.81	0.41	0.59	0.34
Σ	v	0.77	0.35	0.56	0.26
	$\{\delta_1, \delta_2\}$	0.78	0.34	0.55	0.25
	δ_2	0.34	0.45	0.49	0.38
λ	v	0.353	0.42	0.465	0.36
	$\{\delta_1, \delta_2\}$	0.352	0.43	0.473	0.37
		(V_1^1, V_1^2)	(V_2^1, V_2^2)	(V_1^1, V_1^2)	(V_2^1, V_2^2)
	δ_2	(0.52, 0.28)	(0.17, 0.28)	(0.44, 0.44)	(0.19, 0.19)
V_t^j	v	(0.40, 0.37)	(0.05, 0.37)	(0.17, 0.66)	(0.03, 0.34)
	$\{\delta_1, \delta_2\}$	(0.42, 0.36)	(0.07, 0.36)	(0.19, 0.65)	(0.04, 0.33)
		(β^1, β^2)	(β^1, β^2)	(β^1, β^2)	(β^1, β^2)
	δ_2	(1.11, 0)	(1.11, 1.11)	(0.83, 0.83)	(1.12, 1.12)
β	v	(1.32, 0)	(0.65, 1.08)	(0.76, 0.57)	(0.49, 1.29)
	$\{\delta_1, \delta_2\}$	(1.28, {0,0})	(0.77, {0.77,1.42})	(0.74, {0.54,0.62})	(0.61, {1.06,1.36})

of the timing of information acquisition is on the price impact of order flow (λ_t). Again, we see that sequential information acquisition generates a rising price impact over time, while simultaneous information acquisition leads to a declining price impact.

Table I also reveals that, quite surprisingly, anticipated monotonic relationships do not obtain. For example, as agent 2 gets access to increasingly better information, his expected profits do not rise monotonically. Specifically, his expected profits are lower when he observes $\{\delta_1, \delta_2\}$ than they are when he just observes the asset value, $v = \delta_1 + \delta_2$.

What causes agent 2's profit to fall as his information improves is that seeing only the asset value rather than its constituent components confers a strategic advantage on him. When agent 2 sees each component, he trades more aggressively on δ_2 than δ_1 because he has a monopoly on δ_2 information, whereas he is a duopolist on δ_1 information. However, when agent 2 only sees the asset value v, he cannot disentangle the contribution of δ_2, on which he would like to trade more aggressively, from that of δ_1. As a result, agent 2 trades more aggressively on the δ_1 component of asset value than he would if he saw each component separately. In turn, agent 1's "best response" to this more aggressive trading is to reduce his trading intensity, thereby raising agent 2's expected profits. Indeed, this strategic effect is magnified when both agents acquire information at date 1, because the net private information of the two agents given date 1 public information becomes negatively correlated. This shifts back agent 1's date 2 trading intensity, but does not shift agent 2's, as he knows v.

Of course, when agent 2 sees the asset value, his information is worse than when he sees each component of the asset value distinctly. So the question becomes: Which dominates, in terms of impact on expected profits, information release, and price impact—the strategic advantage conferred by seeing v, or the reduced information quality? It turns out that the answer depends on the variable in question.

C.1. Informed Lifetime Profits

As was just noted, the third panel of Table I reveals that for agent 2's expected lifetime profits, the "strategic advantage" of seeing $v = \delta_1 + \delta_2$ rather than $\{\delta_1, \delta_2\}$ dominates: Agent 2's expected lifetime profits are *higher* when he only sees $v = \delta_1 + \delta_2$.

Agent 2's higher profit due to seeing $v = \delta_1 + \delta_2$ rather than $\{\delta_1, \delta_2\}$ comes at the expense of both agent 1 and total informed profit. Total informed profit is reduced by about 1.5 percent when agent 2 sees $v = \delta_1 + \delta_2$ rather than $\{\delta_1, \delta_2\}$, independent of the timing of information acquisition. Thus, the reduction in total informed profit is on the same order of magnitude as agent 2's profit increase from seeing v rather than $\{\delta_1, \delta_2\}$.

Obviously, agent 1's lifetime profit is highest when he has a monopoly over δ_1 information (23 percent higher for sequential information, 232 percent for simultaneous information). However, total informed profit is but 3–4 percent higher in this case; most of agent 1's increased profit is simply a redistribution from agent 2.

C.2. Trading Intensities

In the last panel of Table I, three features of informed trading intensities stand out:

1. Agent 2's second-period trading intensity on δ_1 is far higher when he sees $v = \delta_1 + \delta_2$ than when he sees $\{\delta_1, \delta_2\}$ and hence can distinguish the contribution of each innovation, and identify agent 1's information.
2. The flip side is that agent 1's second-period trading intensity is far lower when agent 2 sees $v = \delta_1 + \delta_2$ than when agent 2 sees $\{\delta_1, \delta_2\}$.
3. Agent 1's trading intensity rises over time if agent 2 only sees δ_2, but drops significantly if agent 2 sees δ_1 and , δ_2 separately, and drops tremendously if agent 2 sees $v = \delta_1 + \delta_2$.

It is the first two trading intensity relationships that underlie the strategic benefit accruing to agent 2 from seeing the asset value $v = \delta_1 + \delta_2$, rather than each of its constituent components. To glean intuition, contrast the second-period first-order conditions when agents acquire information sequentially. When agent 2 sees $v = \delta_1 + \delta_2$, the first-order conditions are

$$\text{Agent 1:} \quad 2\beta_2^1 = \frac{1}{\lambda_2} - \beta_2^2, \tag{32}$$

$$\text{Agent 2:} \quad 2\beta_2^2 = \frac{1}{\lambda_2} - \beta_2^1 \frac{\text{Var}(\delta_1|\Omega_1)}{\text{Var}(\delta_1|\Omega_1) + \text{Var}(\delta_2|\Omega_1)}, \tag{33}$$

where $\frac{\text{Var}(\delta_1\,|\,\Omega_1)}{\text{Var}(\delta_1|\Omega_1) + \text{Var}(\delta_2|\Omega_1)} = 0.348$, whereas when agent 2 sees $\{\delta_1, \delta_2\}$, the first-order conditions for trade on $\delta_1 - p_1$ are

$$\text{Agent 1:} \quad 2\beta_2^1 = \frac{1}{\lambda_2} - \beta_{12}^2,$$

$$\text{Agent 2:} \quad 2\beta_{12}^2 = \frac{1}{\lambda_2} - \beta_2^1.$$

Thus, agent 1's first-order conditions are the same for both informational environments (treating λ_2 and agent 2's trade as parameters). However, when agent 2 only sees v, he places a reduced weight on β_2^1 compared to when he sees δ_1 distinctly because δ_1 is only a fraction of the information on which he is trading. Therefore, agent 2's trading intensity on δ_1 information is shifted out sharply. As a result, agent 1's equilibrium best response is to cut back on his trading intensity.

This result also helps explain why, in the sequential information acquisition environment, agent 1's trading intensity falls over time when agent 2 acquires any information about δ_1: At date 1, agent 1 anticipates future competition on his information, and this leads him to ratchet up his date 1 trading intensity in order to profit when he has an informational monopoly.

Obviously, when information is acquired simultaneously, a different rationale must underlie the intertemporal decline in agent 1's trading intensity, as competition is immediate. Again, the second-period first-order conditions provide the explanation. For example, when agent 2 sees $v = \delta_1 + \delta_2$, the first-order conditions become

$$\text{Agent 1:} \quad 2\beta_2^1 = (1 + \theta_2^{12})\left(\frac{1}{\lambda_2} - \beta_2^2\right), \tag{34}$$

$$\text{Agent 2:} \quad 2\beta_2^2 = \frac{1}{\lambda_2} - \beta_2^1 \frac{\text{Var}(\delta_1 \mid \Omega_1) + \text{Cov}(\delta_1, \delta_2 \mid \Omega_1)}{\text{Var}(\delta_1 \mid \Omega_1) + 2\text{Cov}(\delta_1, \delta_2 \mid \Omega_1) + \text{Var}(\delta_2 \mid \Omega_1)}, \tag{35}$$

where $\frac{\text{Var}(\delta_1|\Omega_1)+\text{Cov}(\delta_1,\delta_2|\Omega_1)}{\text{Var}(\delta_1|\Omega_1)+2\text{Cov}(\delta_1,\delta_2|\Omega_1)+\text{Var}(\delta_2|\Omega_1)} = 0.343$, and $\theta_2^{12} = \frac{\text{Cov}(\delta_1,\delta_2|\Omega_1)}{\text{Var}(\delta_1|\Omega_1)} = -0.326$ in agent 1's forecast of v reflects the negative correlation in the agents' net private information conditional on period-1 information. Comparing the first-order conditions when information is acquired sequentially with those when information is acquired simultaneously reveals that agent 2's first-order conditions are virtually the same (equations (33) and (35)), but the right-hand side of agent 1's first-order condition is shifted down by about θ_2^{12} (approximately 33 percent) when information is simultaneously acquired (equations (32) and (34)), implying a greatly reduced equilibrium trading intensity for agent 1. Thus, it is this negative conditional correlation in information that underlies the intertemporal decline in agent 1's trading intensity when agent 2 has information about δ_1. Finally, there is no such decline when agent 2 only sees δ_2, because when information is simultaneously acquired, agent 2 has an offsetting reduction in second-period trading intensity, which reflects his forecasting of δ_1.

C.3. Pricing and Information Revelation

The second panel of Table I reveals that the price impact of information falls over time if the agents both acquire information at date 1, reflecting the release of information over time. If, instead, agent 2 does not acquire his information until date 2, then this new information leads to a greater date 2 price impact. Thus, sequential information acquisition can generate simultaneously the increased bid–ask spreads, increased trading intensities, and increased price volatilities found in the data as the timing of earnings announcements is approached, while simultaneous information acquisition does not. Other models that generate widening bid–ask spreads (see, e.g., Back et al. (2000), and Foster and Viswanathan (1996)) also have the (counterfactual) feature that informed order flow vanishes as the announcement date draws near.

The first panel of Table I reveals that the pattern of information revelation through trade does not match the pattern of informed profit. Less information is revealed through price if agent 2 only observes δ_2 than if he has some information about δ_1. This reflects the facts that both agents have less information, and each agent has a monopoly over his signal, which means that total trading intensities and hence information release are more sharply restricted.

More interestingly, by the end of the trading horizon, slightly more information is released through trade when agent 2's information is perfect than when he only observes the asset value, $v = \delta_1 + \delta_2$. In particular, for simultaneous information acquisition, roughly 3 percent more information is revealed through trade when agent 2 sees $\{\delta_1, \delta_2\}$, and for sequential information acquisition about 2 percent more information is revealed. That is, the direct effect of the higher quality of 2's information dominates the indirect effect on the reduced trading intensities for the total amount of information revealed through trade.

C.4. Homogeneous Good Oligopoly

To provide added intuition for our findings, we compare these results with those that obtain in a homogeneous good oligopoly under different competitive environments. One can draw parallels between: (a) the environment in which informed agent 1 has a monopoly over δ_1 information and an output monopolist in the "good 1" market; (b) the environment in which agent 2 also sees δ_1 (and δ_2) distinctly and an oligopoly featuring Cournot competition in the "good 1" market; and, (c) the environment in which agent 2 sees $\delta_1 + \delta_2$ and there is a Stackelberg oligopoly in which agent 2 is the leader and agent 1 is the follower, reflecting the strategic effect of agent 2 averaging his trading intensity between δ_1 and δ_2, which shifts back agent 1's best response.

In the output market, total firm profit is greatest when firm 1 has a monopoly and output (total trading intensity) is at its least. Total firm profit is next highest and output (total trading intensity) increases when the firms engage in Cournot competition, as they compete away some of the monopoly rents. Firm 1 reduces its output (trading intensity) relative to a monopoly, but does not reduce it by enough to offset 2's output. Finally, total firm profit is least and output (total trading intensity) is highest when firm 2 is a Stackelberg leader. Relative to the Cournot setting, firm 2 increases its output (trading intensity); firm 1's best response is to reduce its output (trading intensity), but again, not by enough to offset 2's increase in output. By increasing its trading intensity, firm 2 raises its profits above those in the Cournot setting, but it inefficiently extracts its greater profit from the economy, not only at the expense of firm 1's profit, but at the expense of total firm profit.

As Table I reveals, the strategic considerations highlighted in the oligopoly environment dominate the profit and trading intensity outcomes in our economy. Comparisons of information release, however, reveal that more than these strategic considerations matter; when agent 2's information is perfect, slightly more information about the asset's value is revealed through price than when agent 2 only observes the total asset value. That is, the difference in information qualities more than offsets the strategic impact of trading intensities on the total amount of information revealed through price.

IV. Conclusion

This paper characterizes informed trading and pricing when agents can acquire distinct signals of varying quality about an asset's value at different dates. Incorporating these features into a model is of both fundamental theoretical and empirical importance; it is important to allow for the possibility that agents who acquire information at later dates have access to better information and hence that these agents are better informed.

We first characterize the form of equilibrium trading strategies, the market maker's pricing function and trader's profits. We then determine the quantitative impacts of different informational environments on the time paths of (i) information release, (ii) pricing, (iii) trading intensities, and (iv) informed profits. Our main findings are:

- The price impact of order flow crucially depends on both how information is divided among informed agents and the timing of information acquisition. Specifically, for the price impact of order flow to rise with time, the sequential arrival of sufficiently heterogeneous information is necessary. More generally, such information arrival can generate the U-shaped intradaily volume found in the data, and the increasing spreads, volume, and price volatility that arise as an event window approaches.
- Because of the strategic ramifications of the nature of private information on the trading behavior of other agents, an informed agent's trading profits need not rise with the quality of his information. For example, knowing only the asset value, and not the components that make up the asset value, can raise profits: An agent who observes only the asset value cannot decompose his trade on each innovation, thus causing him to trade more aggressively on commonly observed innovations. In turn, the equilibrium "best response" by other agents is to trade less aggressively.

Appendix

Proof of Proposition 1: (a) *Step 1: Informed agent at date t who observes δ_i at date $k \leq t$.* Given the conjectured value function (12) at date $t+1$, the forecast of the continuation profits V^i_{kt+1} conditional on (δ_i, Ω_{t-1}) at date t is given by

$$\begin{aligned} V^i_{kt+1}(\delta_i, \Omega_{t-1}) &\equiv E\left[V^i_{kt+1}(\delta_i, \Omega_t) \,\middle|\, \delta_i, \Omega_{t-1}\right] \\ &= A^i_t + B^i_t E\left[(\delta_i - E[\delta_i \mid \Omega_t])^2 \,\middle|\, \delta_i, \Omega_{t-1}\right] \\ &= A^i_t + B^i_t E\left[(\delta_i - E[\delta_i \mid \Omega_{t-1}] - \phi^i_t(p_t - p_{t-1}))^2 \,\middle|\, \delta_i, \Omega_{t-1}\right] \\ &= A^i_t + B^i_t \big\{(\delta_i - E[\delta_i \mid \Omega_{t-1}])^2 - 2\phi^i_t(\delta_i - E[\delta_i \mid \Omega_{t-1}]) \\ &\quad \times E[p_t - p_{t-1} \mid \delta_i, \Omega_{t-1}] + \left(\phi^i_t\right)^2 \cdot E[(p_t - p_{t-1})^2 \mid \delta_i, \Omega_{t-1}]\big\}, \end{aligned} \tag{A1}$$

where we use Lemma 1 in the third equality.

Hence, at date t, the agent seeks to maximize:

$$\begin{aligned} V^i_{kt}(\delta_i, \Omega_{t-1}) &= \max_{x^i_{kt}} E\left[(v - p_t)x^i_{kt} + V^i_{kt+1}(\delta_i, \Omega_t) \,\middle|\, \delta_i, \Omega_{t-1}\right] \\ &= \max_{x^i_{kt}} E\left[(v - p_t)x^i_{kt} \,\middle|\, \delta_i, \Omega_{t-1}\right] - 2B^i_t\phi^i_t(\delta_i - E[\delta_i \mid \Omega_{t-1}]) \\ &\quad \cdot E[p_t - p_{t-1} \mid \delta_i, \Omega_{t-1}] + B^i_t\left(\phi^i_t\right)^2 E\left[(p_t - p_{t-1})^2 \,\middle|\, \delta_i, \Omega_{t-1}\right], \end{aligned} \tag{A2}$$

where, without loss of generality, the first two terms of (A1) are suppressed since they are constant at the beginning of date t.

Each insider takes into account how his order affects price through the conjectured price function, equation (4). Each informed agent also believes that the trading strategies of other agents, $x_{lt}^{j}(j \neq i,\, l = 1, \ldots, N)$, take the form of equation (3); these beliefs are rational in equilibrium. Using the conjectured pricing function (4), the first-order condition becomes:

$$0 = E\left[v - p_{t-1} - \lambda_t \sum_{j=1}^{N} \sum_{l=1}^{N} \left(M_l^j x_{lt}^j - x_{kt}^i \right) \,\middle|\, \delta_i, \Omega_{t-1} \right] - 2\lambda_t x_{kt}^i$$
$$- 2\lambda_t B_t^i \phi_t^i (\delta_i - E[\delta_i \mid \Omega_{t-1}]) + 2\lambda_t B_t^i \cdot \left(\phi_t^i\right)^2 \cdot E[(p_t - p_{t-1}) \mid \delta_i, \Omega_{t-1}]. \tag{A3}$$

The second-order condition is:

$$-2\lambda_t + 2\lambda_t^2 B_t^i \left(\phi_t^i\right)^2 < 0, \quad \text{or,} \quad \lambda_t \left(1 - \lambda_t B_t^i \cdot \left(\phi_t^i\right)^2\right) > 0, \tag{A4}$$

which gives (15). Substituting for $E[(p_t - p_{t-1}) \mid \delta_i, \Omega_{t-1}]$ using the conjectured linear forms for informed trading and pricing, equations (3) and (4), we rewrite the first-order condition as:

$$\begin{aligned} &\left[1 + M_k^i \left(1 - 2\lambda_t B_t^i \left(\phi_t^i\right)^2\right)\right] x_{kt}^i \\ &\quad = E[v - p_{t-1} \mid \delta_i, \Omega_{t-1}] - 2\lambda_t B_t^i \phi_t^i (\delta_i - E[\delta_i \mid \Omega_{t-1}]) \\ &\qquad + \lambda_t \left(2\lambda_t B_t^i \left(\phi_t^i\right)^2 - 1\right) \sum_{l=1}^{t} \sum_{j \neq i} M_l^j E\left[x_{lt}^j \,\middle|\, \delta_i, \Omega_{t-1}\right]. \end{aligned} \tag{A5}$$

We next exploit the market efficiency condition and Lemma 2 to rewrite $E[v - p_{t-1} \mid \delta_i, \Omega_{t-1}]$ as:

$$\begin{aligned} E\left[v - p_{t-1} \,\middle|\, \delta_i, \Omega_{t-1}\right] &= E[v - E[v \mid \Omega_{t-1}] \mid \delta_i, \Omega_{t-1}] \\ &= \sum_{j=1}^{N} E[\delta_j - E[\delta_j \mid \Omega_{t-1}] \mid \delta_i, \Omega_{t-1}] \\ &= \sum_{j=1}^{N} \theta_t^{ij} \cdot \{\delta_i - E[\delta_i \mid \Omega_{t-1}]\}. \end{aligned} \tag{A6}$$

Using Lemma 2 and the conjectured linear form of informed trade, equation (3), the last term on the right-hand side of (A5) can be written as:

$$\begin{aligned} \sum_{l=1}^{t} \sum_{j \neq i} M_l^j E\left[x_{lt}^j \mid \delta_i, \Omega_{t-1}\right] &= \sum_{l=1}^{t} \sum_{j \neq i} M_l^j \beta_{lt}^j E[(\delta_j - E[\delta_j \mid \Omega_{t-1}]) \mid \delta_i, \Omega_{t-1}] \\ &= \sum_{l=1}^{t} \sum_{j \neq i} M_l^j \beta_t^j \theta_t^{ij} \cdot \{\delta_i - E[\delta_i \mid \Omega_{t-1}]\}. \end{aligned} \tag{A7}$$

Substituting for the first and third terms of (A5), the first-order condition becomes

$$\Big[1 + M_k^i\Big(1 - 2\lambda_t B_t^i (\phi_t^i)^2\Big)\Big] x_{kt}^i$$
$$= \left\{ \sum_{j=1}^{N} \theta_t^{ij} - 2\lambda_t B_t^i \phi_t^i + \lambda_t \Big(2\lambda_t B_t^i (\phi_t^i)^2 - 1\Big) \sum_{l=1}^{t} \sum_{j \neq i} M_l^j \beta_t^j \theta_t^{ij} \right\} (\delta_i - E[\delta_i \mid \Omega_{t-1}]). \tag{A8}$$

Thus, given the conjectured form of continuation profits, we see that x_{kt}^i is proportional to unrevealed private information, $\delta_i - E[\delta_i \mid \Omega_{t-1}]$, verifying the conjectured form of the linear trading strategies in equation (3). We thus have the following system of t equations:

$$\Big[1 + M_k^i\Big(1 - 2\lambda_t B_t^i (\phi_t^i)^2\Big)\Big]\beta_t^i + \lambda_t \Big(1 - 2\lambda_t B_t^i (\phi_t^i)^2\Big) \sum_{l=1}^{t} \sum_{j \neq i} M_l^j \theta_t^{ij} \beta_t^j$$
$$= \sum_{j=1}^{N} \theta_t^{ij} - 2\lambda_t B_t^i \phi_t^i. \tag{A9}$$

The equilibrium values of β_t^i are given by the solution to these simultaneous equations. Letting $\Lambda_{\boldsymbol{N}}$ be the $N \times N$ matrix such that

$$a_{ii} = 1 + M_k^i \Big(1 - 2\lambda_t B_t^i (\phi_t^i)^2\Big) \quad \text{and}$$
$$a_{ij} = \Big(1 - 2\lambda_t B_t^i (\phi_t^i)^2\Big) \sum_{l=1}^{N} \sum_{j \neq i} M_l^j \theta_t^{ij} \quad (i \neq j) \tag{A10}$$

yields equation (14) in the proposition. In turn, these linear trading strategies imply that expected period t profits are a quadratic function of unrevealed private information, so the conjectured form of continuation profits is correct.

Step 2: Uninformed agent at date $t < k$ who learns δ_i at date k. At date $k-1$, this agent has yet to receive private information, so he uses public information Ω_{k-2} to infer his future expected trading profits:

$$\begin{aligned}
V_{kk}^i(\Omega_{k-2}) &\equiv E\big[V_{kk}^i(\delta_k, \Omega_{k-1}) \,\big|\, \Omega_{k-2}\big] \\
&= E\big[A_{k-1}^i + B_{k-1}^i (\delta_i - E[\delta_i | \Omega_{k-1}])^2 \,\big|\, \Omega_{k-2}\big] \\
&= A_{k-1}^i + B_{k-1}^i E\big[(\delta_i - E\big[\delta_i \mid \Omega_{k-1}])^2 \,\big|\, \Omega_{k-2}\big] \\
&= A_{k-1}^i + B_{k-1}^i E\big[E\big[(\delta_i - E[\delta_i \mid \Omega_{k-1}])^2 \,\big|\, \Omega_{k-1}\big] \,\big|\, \Omega_{k-2}\big] \\
&= A_{k-1}^i + B_{k-1}^i \mathrm{Var}(\delta_i \mid \Omega_{k-1}) \\
&= A_{k-1}^i + B_{k-1}^i \left\{ \mathrm{Var}(\delta_i \mid \Omega_{k-2}) - \frac{\mathrm{Cov}^2(\delta_i, p_{k-1} \mid \Omega_{k-2})}{\mathrm{Var}(p_{k-1} \mid \Omega_{k-2})} \right\} \\
&= A_{k-1}^i + B_{k-1}^i \left\{ \mathrm{Var}(\delta_i \mid \Omega_{k-2}) - \frac{\mathrm{Cov}^2(\delta_i, X_{k-1} \mid \Omega_{k-2})}{\mathrm{Var}(X_{k-1} \mid \Omega_{k-2}) + \sigma_u^2} \right\},
\end{aligned} \tag{A11}$$

where the third equality follows from the law of iterated expectations, the fourth equality from the fact that $\mathrm{Var}(\delta_i \mid \Omega_{i-1})$ is a deterministic number, the fifth from Lemma 3, and the last equality from the price function (4). Thus, the agent's optimization problem at date $k-1$ is

$$\begin{aligned}
V^i_{kk-1}(\Omega_{k-2}) &= \max_{x^i_{kk-1}} E\left[(v-p_{k-1})x^i_{kk-1} + V^i_{kk-1}(\delta_i,\Omega_{k-1}) \,\middle|\, \Omega_{k-2}\right] \\
&= \max_{x^i_{kk-1}} E\left[(v-p_{k-2}-\lambda_{k-1}(X_{k-1}+u_{k-1}))x^i_{kk-1} + V^i_{kk}(\delta_i,\Omega_{k-1}) \,\middle|\, \Omega_{k-2}\right] \\
&= \max_{x^i_{kk-1}} E[(v-p_{k-2}) \mid \Omega_{k-2}]x^i_{kk-1} - \lambda_{k-1}\left(x^i_{kk-1}\right)^2 \\
&\quad - \lambda_{k-1}\sum_{j=1}^{k-1} E\left[x^j_{kk-1} \,\middle|\, \Omega_{k-2}\right] + E\left[V^i_{kk}(\delta_i,\Omega_{k-1}) \,\middle|\, \Omega_{k-2}\right] \\
&= \max_{x^i_{kk-1}} -\lambda_{k-1}\left(x^i_{kk-1}\right)^2 + V^i_{kk}(\Omega_{k-2}) \\
&= \max_{x^i_{kk-1}} -\lambda_{k-1}\left(x^i_{kk-1}\right)^2 + A^i_{k-1} \\
&\quad + B^i_{k-1}\left\{\mathrm{Var}(\delta_i \mid \Omega_{k-2}) - \frac{\mathrm{Cov}^2(\delta_i, X_{k-1} \mid \Omega_{k-2})}{\mathrm{Var}(X_{k-1} \mid \Omega_{k-2}) + \sigma_u^2}\right\},
\end{aligned} \tag{A12}$$

where the second equality follows from the substitution of the conjectured form of price function (4), the third equality from the definition of total informed trade X_{i-1}, the fourth equality from the market efficiency condition and the conjectured form of trading strategies of other traders (3), and the last equality from the preceding equation.

Since x^i_{kk-1} is Ω_{k-2}-measurable, the last expression in the preceding equation is not a function of x^i_{kk-1}. It follows that the optimal trade by an agent who has yet to acquire information is $x^i_{kk-1} = 0$, and thus the expected value of his future trading opportunities is $V^i_{kk-1}(\Omega_{k-2})$.

Step 3: Uninformed agent at dates $t < k-1$ who learns δ_i at date k. At time $t = k-2$,

$$E\left[V^i_{kk-1}(\Omega_{k-2}) \,\middle|\, \Omega_{k-3}\right] = V^i_{kk-1}(\Omega_{k-2}), \tag{A13}$$

since $V^i_{kk-1}(\Omega_{k-2})$ is a deterministic number from Step 2. Thus,

$$\begin{aligned}
V^i_{kk-2}(\Omega_{k-3}) &= \max_{x^i_{kk-2}} E\left[(v-p_{k-2})x^i_{kk-2} + V^i_{kk-1}(\Omega_{k-2}) \,\middle|\, \Omega_{k-3}\right] \\
&= \max_{x^i_{kk-2}} -\lambda_{k-2}\left(x^i_{kk-2}\right)^2 + V^i_{kk-1}(\Omega_{k-2}).
\end{aligned} \tag{A14}$$

Hence, $x^i_{kk-2} = 0$, and $V^i_{kk-2}(\Omega_{k-3}) = V^i_{kk-1}(\Omega_{k-2})$. Continuing this argument inductively, it follows that at any time $t < k$, the optimal uninformed trade is

$x^i_{kt} = 0$ and the expected future value of his information does not change, that is, $V^i_{kt}(\Omega_{t-1}) = V^i_{kk-1}(\Omega_{k-2})$.

(b) For $t < k$, $V^i_{kt}(\Omega_{t-1})$ has already been derived. We now verify that the agent's value function at date $t \geq k$ takes the conjectured form of (12). Using the expected continuation profits (A1), in equilibrium the value function at date $t \geq k$ must satisfy

$$\begin{aligned} V^i_{kt}(\delta_i, \Omega_{t-1}) &= E\left[(v - p_t)x^i_{kt} \,\middle|\, \delta_i, \Omega_{t-1}\right] + V^i_{kt+1}(\delta_i, \Omega_{t-1}) \\ &= E[v - p_t \mid \delta_i, \Omega_{t-1}]x^i_{kt} + A^i_t + B^i_t(\delta_i - E[\delta_i \mid \Omega_{t-1}])^2 \\ &\quad - 2B^i_t\phi^i_t E[p_t - p_{t-1} \mid \delta_i, \Omega_{t-1}](\delta_i - E[\delta_i \mid \Omega_{t-1}]) \\ &\quad + B^i_t\left(\phi^i_t\right)^2 E\left[(p_t - p_{t-1})^2 \,\middle|\, \delta_i, \Omega_{t-1}\right]. \end{aligned} \tag{A15}$$

Using the conjectured form of trading strategies, equation (3), we can derive

$$\begin{aligned} E[X_t \mid \delta_i, \Omega_{t-1}] &= \sum_{l=1}^{t}\sum_{j=1}^{N} M^j_k E\left[\beta^j_t(\delta_j - E[\delta_j \mid \Omega_{t-1}]) \,\middle|\, \delta_i, \Omega_{t-1}\right] \\ &= \sum_{j=1}^{N}\sum_{k=1}^{N} M^j_k \beta^j_t \theta^{ij}_t \cdot (\delta_i - E[\delta_i \mid \Omega_{t-1}]). \end{aligned} \tag{A16}$$

Thus, using the conjectured form of the price function we derive:

$$\begin{aligned} E[v - p_t \mid \delta_i, \Omega_{t-1}] &= E[v - p_{t-1} - \lambda_t(X_t + u_t) \mid \delta_i, \Omega_{t-1}] \\ &= \sum_{j=1}^{N} E[\delta_j - E[\delta_j \mid \Omega_{t-1}] \mid \delta_i, \Omega_{t-1}] \\ &\quad - \lambda_t \sum_{l=1}^{t}\sum_{j=1}^{N} M^j_l \beta^j_t E[\delta_j - E[\delta_j \mid \Omega_{t-1}] \mid \delta_i, \Omega_{t-1}] \\ &= \sum_{j=1}^{N} \theta^{ij}_t \cdot (\delta_i - E[\delta_i \mid \Omega_{t-1}]) \\ &\quad - \lambda_t \sum_{l=1}^{t}\sum_{j=1}^{N} M^j_l \beta^j_t \theta^{ij}_t \cdot (\delta_i - E[\delta_i \mid \Omega_{t-1}]), \end{aligned} \tag{A17}$$

$$\begin{aligned} E[p_t - p_{t-1} \mid \delta_i, \Omega_{t-1}] &= \lambda_t E[X_t + u_t \mid \delta_i, \Omega_{t-1}] = \lambda_t E[X_t \mid \delta_i, \Omega_{t-1}] \\ &= \lambda_t \sum_{j=1}^{N}\sum_{k=1}^{N} M^j_k \beta^j_t \theta^{ij}_t (\delta_i - E[\delta_i \mid \Omega_{t-1}]), \end{aligned} \tag{A18}$$

$$\begin{aligned} E\left[(p_t - p_{t-1})^2 \,\middle|\, \delta_i, \Omega_{t-1}\right] &= \lambda^2_t E\left[(X_t + u_t)^2 \,\middle|\, \delta_i, \Omega_{t-1}\right] \\ &= \lambda^2_t E\left[(X_t)^2 \,\middle|\, \delta_i, \Omega_{t-1}\right] + \lambda^2_t \sigma^2_u \\ &= \lambda^2_t\left(\mathrm{Var}(X_t \mid \delta_i, \Omega_{t-1}) + (E[X_t \mid \delta_i, \Omega_{t-1}])^2\right) + \lambda^2_t \sigma^2_u \end{aligned}$$

$$= \lambda_t^2\big(\mathrm{Var}(X_t \mid \delta_i, \Omega_{t-1}) + \sigma_u^2\big) + \lambda_t^2\left(\sum_{l=1}^{t}\sum_{j=1}^{N} M_l^j\beta_t^j\theta_t^{ij}\right)^2(\delta_i - E[\delta_i \mid \Omega_{t-1}])^2. \quad \text{(A19)}$$

Substituting these expressions along with (3) and (12) into (A1) and (A15) yields

$$\begin{aligned} V_{kt}^i(\delta_i, \Omega_{t-1}) = {} & A_t^i + B_t^i\big(\phi_t^i\lambda_t\big)^2\big(\mathrm{Var}(X_t \mid \delta_i, \Omega_{t-1}) + \sigma_u^2\big) \\ & + \beta_t^i\left(\sum_{j=1}^{N}\theta_t^{ij} - \sum_{l=1}^{t}\sum_{j=1}^{N} M_l^j\lambda_t\beta_t^j\theta_t^{ij}\right)(\delta_i - E[\delta_i \mid \Omega_{t-1}])^2 \\ & + \left[B_t^i - 2B_t^i\lambda_t\phi_t^i\sum_{l=1}^{t}\sum_{j=1}^{N} M_l^j\beta_t^j\theta_t^{ij} + B_t^i\left(\lambda_t\phi_t^i\sum_{l=1}^{t}\sum_{j=1}^{N} M_l^j\beta_t^j\theta_t^{ij}\right)^2\right] \\ & \times(\delta_i - E[\delta_i \mid \Omega_{t-1}])^2. \end{aligned} \quad \text{(A20)}$$

Thus, the value function at date $t \geq i$ takes the conjectured form of (12). Substituting this form (12) of $V_t^i(\delta_i, \Omega_{t-1})$, we can derive the difference equations, (16) and (17), governing A_t^i and B_t^i.

(c) By the projection theorem and the market efficiency condition,

$$\begin{aligned} p_t &= E[v \mid X_t + u_t, \Omega_{t-1}] \\ &= E[v \mid \Omega_{t-1}] + \frac{\mathrm{Cov}(v, X_t + u_t \mid \Omega_{t-1})}{\mathrm{Var}(X_t + u_t \mid \Omega_{t-1})}\big(X_t + u_t - E[X_t + u_t \mid \Omega_{t-1}]\big) \\ &= p_{t-1} + \frac{\mathrm{Cov}(v, X_t \mid \Omega_{t-1})}{\mathrm{Var}(X_t \mid \Omega_{t-1}) + \sigma_u^2}(X_t + u_t). \end{aligned} \quad \text{(A21)}$$

The final equality uses the fact that $E[X_t \mid \Omega_{t-1}] = 0$ under (1) and (3).

(d) Equation (19) follows from equation (18) and the projection theorem:

$$\Sigma_t = \mathrm{Var}(v \mid \Omega_{t-1}) - \frac{\mathrm{Cov}^2(v, X_t \mid \Omega_{t-1})}{\mathrm{Var}(X_t + u_t \mid \Omega_{t-1})}. \quad \text{(A22)}$$

Q.E.D.

Proof of Proposition 2: Using the proof method of Proposition 1, one can show that for $t < i, x_t^i = 0$ and expected future trading profits do not vary with information revealed in trade.

For $t \geq i$, proceed as before. Given the conjectured value function (27) at date $t+1$, forecasted continuation profits are

$$\begin{aligned} & V_{t+1}^i(\delta_1, \ldots, \delta_i, \Omega_{t-1}) \\ & \quad = E\big[V_{t+1}^i(\delta_1, \ldots, \delta_i, \Omega_t) \mid \delta_1, \ldots, \delta_i, \Omega_{t-1}\big] \end{aligned}$$

$$= A_t^i + \sum_{k=1}^{i}\sum_{j=1}^{i} B_{jt}^i C_{kt}^i E[(\delta_j - E[\delta_j \mid \Omega_t])(\delta_k - E[\delta_k \mid \Omega_t]) \mid \delta_1, \ldots, \delta_i, \Omega_{t-1}]$$

$$= A_t^i + \sum_{k=1}^{i}\sum_{j=1}^{i} B_{jt}^i C_{kt}^i E\big[\big(\delta_j - E[\delta_j \mid \Omega_{t-1}] - \phi_t^j (p_t - p_{t-1})\big)$$
$$\cdot \big(\delta_k - E[\delta_k \mid \Omega_{t-1}] - \phi_t^k (p_t - p_{t-1})\big) \,\big|\, \delta_1, \ldots, \delta_i, \Omega_{t-1}\big], \tag{A23}$$

where the last equality uses Lemma 4(a). Thus, the decision problem of insider $i \le t$ is

$$V_t^i(\delta_1, \ldots, \delta_i, \Omega_{t-1})$$
$$= \max_{x_t^i} E\left[(v - p_t)x_t^i + V_{t+1}^i(\delta_1, \ldots, \delta_i, \Omega_t) \mid \delta_1, \ldots, \delta_i, \Omega_{t-1}\right]$$
$$= \max_{x_t^i} E\left[\left(v - p_{t-1} - \lambda_t \sum_{j \neq i} x_t^j\right) x_t^i \,\middle|\, \delta_1, \ldots, \delta_i, \Omega_{t-1}\right] - \lambda_t \big(x_t^i\big)^2$$
$$+ \sum_{k=1}^{i}\sum_{j=1}^{i} B_{jt}^i C_{kt}^i E\big[\big(\delta_j - E[\delta_j \mid \Omega_{t-1}] - \phi_t^j (p_t - p_{t-1})\big)$$
$$\times \big(\delta_k - E[\delta_k \mid \Omega_{t-1}] - \phi_t^k (p_t - p_{t-1})\big) \,\big|\, \delta_1, \ldots, \delta_i, \Omega_{t-1}\big]. \tag{A24}$$

Given the conjectured form of price function (4) and the conjectured form of trading strategies of other traders $x_t^j, j \neq i$, (26), the first-order condition is

$$0 = E[v - p_{t-1} \mid \delta_1, \ldots, \delta_i, \Omega_{t-1}] - \lambda_t E\left[\sum_{j \neq i, j \le t} x_t^j \,\middle|\, \delta_1, \ldots, \delta_i, \Omega_{t-1}\right]$$
$$- 2\lambda_t x_t^i - 2\lambda_t \sum_{k=1}^{i}\sum_{j=1}^{i} B_{jt}^i C_{kt}^i \phi_t^k$$
$$\times E\big[\delta_j - E[\delta_j \mid \Omega_{t-1}] - \phi_t^j (p_t - p_{t-1}) \,\big|\, \delta_1, \ldots, \delta_i, \Omega_{t-1}\big]. \tag{A25}$$

Rearranging yields

$$2\lambda_t \left(1 - \lambda_t \sum_{k=1}^{i}\sum_{j=1}^{i} B_{jt}^i C_{kt}^i \phi_t^j \phi_t^k\right) x_t^i$$
$$= E[v - p_{t-1} \mid \delta_1, \ldots, \delta_i, \Omega_{t-1}] - 2\lambda_t \sum_{j=1}^{i}\sum_{k=1}^{i} B_{jt}^i C_{kt}^i \phi_t^k (\delta_j - E[\delta_j \mid \Omega_{t-1}])$$
$$+ \lambda_t \left[2\lambda_t \sum_{j=1}^{i}\sum_{k=1}^{i} B_{jt}^i C_{kt}^i \phi_t^j \phi_t^k - 1\right] E\left[\sum_{j \neq i, j \le t} x_t^j \,\middle|\, \delta_1, \ldots, \delta_i, \Omega_{t-1}\right]. \tag{A26}$$

Using market efficiency and Lemma 4(b), rewrite the first term on the right-hand side as

$$\begin{aligned}E[v - p_{t-1} \mid \delta_1, \ldots, \delta_i, \Omega_{t-1}] &= E[v - E[v \mid \Omega_{t-1}] \mid \delta_1, \ldots, \delta_i, \Omega_{t-1}] \\ &= \sum_{j=1}^{N} E[\delta_j - E[\delta_j \mid \Omega_{t-1}] \mid \delta_1, \ldots, \delta_i, \Omega_{t-1}] \\ &= \sum_{k=1}^{i}\sum_{j=1}^{N} \theta_t^{kj}(\delta_k - E[\delta_k \mid \Omega_{t-1}]), \qquad \text{(A27)}\end{aligned}$$

and the last term on the right-hand side can be written as:

$$\begin{aligned}&E\left[\sum_{k\neq i, k\leq t} x_t^k \mid \delta_1, \ldots, \delta_i, \Omega_{t-1}\right] \\ &\quad = E\left[\sum_{k\neq i}\sum_{m=1}^{k} \beta_{mt}^k(\delta_m - E[\delta_m \mid \Omega_{t-1}]) \,\middle|\, \delta_1, \ldots, \delta_i, \Omega_{t-1}\right] \\ &\quad = \sum_{j=1}^{i}\left(\sum_{m=1}^{k}\sum_{k\neq i, k\leq t} \beta_{mt}^k \theta_t^{jm}\right)(\delta_j - E[\delta_j \mid \Omega_{t-1}]), \qquad \text{(A28)}\end{aligned}$$

where we have substituted the conjectured form of trading strategies of other traders, (26). Thus, given the conjectured form of the value function, equation (27), the price function, equation (4), and the trading strategies of other agents, equation (26), trader i's optimal trading strategy x_t^i takes the conjectured form of equation (26).

Lastly, we show the value function takes the conjectured form of equation (27). Substituting the optimal trading strategies into the continuation value function, equation (A24), yields

$$\begin{aligned}&V_t^i(\delta_1, \ldots, \delta_i, \Omega_{t-1}) \\ &\quad = E\left[(v - p_t)x_t^i \,\middle|\, \delta_1, \ldots, \delta_i, \Omega_{t-1}\right] + V_{t+1}^i(\delta_1, \ldots, \delta_i, \Omega_{t-1}) \\ &\quad = E\left[(v - p_t)x_t^i \,\middle|\, \delta_1, \ldots, \delta_i, \Omega_{t-1}\right] + A_t^i \\ &\qquad + \sum_{k=1}^{i}\sum_{j=1}^{i} B_{jt}^i C_{kt}^i E\big[\big(\delta_j - E[\delta_j \mid \Omega_{t-1}] - \phi_t^j(p_t - p_{t-1})\big) \\ &\qquad \times\big(\delta_k - E[\delta_k \mid \Omega_{t-1}] - \phi_t^k(p_t - p_{t-1})\big) \,\big|\, \delta_1, \ldots, \delta_i, \Omega_{t-1}\big]. \qquad \text{(A29)}\end{aligned}$$

Thus, we only need to show that the first term and the last term on the right-hand side are quadratic functions of unrevealed private information, $(\delta_1 - E[\delta_1 \mid \Omega_{t-1}], \ldots, \delta_i - E[\delta_i \mid \Omega_{t-1}])$.

First, using the market efficiency condition, the conjectured price function (4), and the conjectured form of trading strategies (26), we derive

$$
\begin{aligned}
&E[v - p_t \mid \delta_1, \ldots, \delta_i, \Omega_{t-1}] \\
&\quad = E[v - p_{t-1} - \lambda_t(X_t + u_t) \mid \delta_1, \ldots, \delta_i, \Omega_{t-1}] \\
&\quad = \sum_{j=1}^{N} E[\delta_j - E[\delta_j \mid \Omega_{t-1}] \mid \delta_1, \ldots, \delta_i, \Omega_{t-1}] - \lambda_t E\left[\sum_{k=1}^{N} x_t^k \,\middle|\, \delta_1, \ldots, \delta_i, \Omega_{t-1}\right] \\
&\quad = \sum_{j=1}^{N}\sum_{k=1}^{i} \theta_t^{kj}(\delta_k - E[\delta_k \mid \Omega_{t-1}]) \\
&\qquad - \lambda_t E\left[\sum_{k=1}^{t}\sum_{m=1}^{k} \beta_{mt}^k(\delta_m - E[\delta_m \mid \Omega_{t-1}]) \,\middle|\, \delta_1, \ldots, \delta_i, \Omega_{t-1}\right] \\
&\quad = \sum_{j=1}^{i}\sum_{k=1}^{N} \theta_{t-1}^{jk}(\delta_j - E[\delta_j \mid \Omega_{t-1}]) - \lambda_t \sum_{j=1}^{i}\sum_{k=1}^{t}\sum_{m=1}^{k} \beta_{mt}^k \theta_t^{jm}(\delta_j - E[\delta_j \mid \Omega_{t-1}]) \\
&\quad = \sum_{j=1}^{i}\left[\sum_{k=1}^{N} \theta_t^{jk} - \lambda_t \sum_{k=1}^{t}\sum_{m=1}^{k} \beta_{mt}^k \theta_t^{jm}\right](\delta_j - E[\delta_j \mid \Omega_{t-1}]). \qquad \text{(A30)}
\end{aligned}
$$

Hence,

$$
\begin{aligned}
&E\left[(v - p_t)x_t^i \,\middle|\, \delta_1, \ldots, \delta_i, \Omega_{t-1}\right] \\
&\quad = \left[\sum_{\ell=1}^{i} \beta_{\ell t}^i(\delta_\ell - E[\delta_\ell \mid \Omega_{t-1}])\right] \\
&\qquad \times \left[\sum_{j=1}^{i}\left[\sum_{k=1}^{N} \theta_t^{jk} - \lambda_t \sum_{k=1}^{t}\sum_{m=1}^{k} \beta_{mt}^k \theta_t^{jm}\right]\big(\delta_j - E[\delta_j \mid \Omega_{t-1}]\big)\right], \qquad \text{(A31)}
\end{aligned}
$$

is a quadratic function of unrevealed private information $(\delta_1 - E[\delta_1 \mid \Omega_{t-1}], \ldots, \delta_i - E[\delta_i \mid \Omega_{t-1}])$.

Finally, we compute the last term on the right-hand side of (A29). We can similarly derive that for $j, k \leq i$,

$$
\begin{aligned}
&E\big[\big(\delta_j - E[\delta_j \mid \Omega_{t-1}] - \phi_t^j(p_t - p_{t-1})\big)\big(\delta_k - E[\delta_k \mid \Omega_{t-1}] \\
&\qquad - \phi_t^k(p_t - p_{t-1})\big) \,\big|\, \delta_1, \ldots, \delta_i, \Omega_{t-1}\big] \\
&\quad = E\big[\big(\delta_j - E[\delta_j \mid \Omega_{t-1}] - \phi_t^j \lambda_t(X_t + u_t)\big)\big(\delta_k - E[\delta_k \,|\, \Omega_{t-1}] \\
&\qquad - \phi_t^k \lambda_t(X_t + u_t)\big) \mid \delta_1, \ldots, \delta_i, \Omega_{t-1}\big]
\end{aligned}
$$

$$
\begin{aligned}
&= E[(\delta_j - E[\delta_j \mid \Omega_{t-1}])(\delta_k - E[\delta_k \mid \Omega_{t-1}]) \mid \delta_1, \ldots, \delta_i, \Omega_{t-1}] \\
&\quad - \phi_t^j \lambda_t E[(X_t + u_t)(\delta_k - E[\delta_k \mid \Omega_{t-1}]) \mid \delta_1, \ldots, \delta_i, \Omega_{t-1}] \\
&\quad - \phi_t^k \lambda_t E[(X_t + u_t)(\delta_j - E[\delta_j \mid \Omega_{t-1}]) \mid \delta_1, \ldots, \delta_i, \Omega_{t-1}] \\
&\quad + \phi_t^k \phi_t^j \lambda_t^2 E\left[(X_t + u_t)^2 \,\middle|\, \delta_1, \ldots, \delta_i, \Omega_{t-1}\right] \\
&= (\delta_j - E[\delta_j \mid \Omega_{t-1}])(\delta_k - E[\delta_k \mid \Omega_{t-1}]) \\
&\quad - \phi_t^j \lambda_t E\left[\sum_{k=1}^{t} x_t^i (\delta_k - E[\delta_k \mid \Omega_{t-1}]) \,\middle|\, \delta_1, \ldots, \delta_i, \Omega_{t-1}\right] \\
&\quad - \phi_t^k \lambda_t E\left[\sum_{k=1}^{t} x_t^i (\delta_j - E[\delta_j \mid \Omega_{t-1}]) \,\middle|\, \delta_1, \ldots, \delta_i, \Omega_{t-1}\right] \\
&\quad + \phi_t^k \phi_t^j \lambda_t^2 \left(E\left[X_t^2 \,\middle|\, \delta_1, \ldots, \delta_i, \Omega_{t-1}\right] + \sigma_u^2\right) \\
&= (\delta_j - E[\delta_j \mid \Omega_{t-1}])(\delta_k - E[\delta_k \mid \Omega_{t-1}]) \\
&\quad - \phi_t^j \lambda_t (\delta_k - E[\delta_k \mid \Omega_{t-1}]) \sum_{j=1}^{i} \sum_{\ell=1}^{t} \sum_{m=1}^{\ell} \beta_{mt}^{\ell} \theta_t^{jm} (\delta_j - E[\delta_j \mid \Omega_{t-1}]) \\
&\quad - \phi_t^k \lambda_t (\delta_j - E[\delta_j \mid \Omega_{t-1}]) \sum_{j=1}^{i} \sum_{\ell=1}^{t} \sum_{m=1}^{\ell} \beta_{mt}^{\ell} \theta_t^{jm} (\delta_j - E[\delta_j \mid \Omega_{t-1}]) \\
&\quad + \phi_t^k \phi_t^j \lambda_t^2 \left(E\left[X_t^2 \,\middle|\, \delta_1, \ldots, \delta_i, \Omega_{t-1}\right] + \sigma_u^2\right). \qquad \text{(A32)}
\end{aligned}
$$

It remains to be shown that $E[X_t^2 \mid \delta_1, \ldots, \delta_i, \Omega_{t-1}]$ is a quadratic function of unrevealed private information $(\delta_1 - E[\delta_1 \mid \Omega_{t-1}], \ldots, \delta_i - E[\delta_i \mid \Omega_{t-1}])$. Using the forecasting rules described in Lemma 4 and the conjectured form of trading strategies (26), we derive

$$
\begin{aligned}
&E\left[X_t^2 \,\middle|\, \delta_1, \ldots, \delta_i, \Omega_{t-1}\right] \\
&\quad = E\left[\left(\sum_{k \le t} \sum_{j=1}^{k} \beta_{jt}^k (\delta_j - E[\delta_j \mid \Omega_{t-1}])\right)^2 \,\middle|\, \delta_1, \ldots, \delta_i, \Omega_{t-1}\right] \\
&\quad = E\Bigg[\Bigg(\sum_{k \le t} \sum_{j=1}^{k} \beta_{jt}^k \Bigg(\delta_j - E[\delta_j \mid \delta_1, \ldots, \delta_i, \Omega_{t-1}] \\
&\qquad + \sum_{\ell=1}^{i} \theta_t^{\ell j} (\delta_\ell - E[\delta_\ell \mid \Omega_{t-1}])\Bigg)\Bigg)^2 \,\Bigg|\, \delta_1, \ldots, \delta_i, \Omega_{t-1}\Bigg] \\
&\quad = \sum_{m=1}^{t} \sum_{k=1}^{t} \sum_{j=1}^{k} \sum_{\ell=1}^{m} \beta_{jt}^k \beta_{\ell t}^m \operatorname{Cov}(\delta_j, \delta_\ell \mid \delta_1, \ldots, \delta_i, \Omega_{t-1}) \\
&\qquad + \left[\sum_{k \le t} \sum_{j=1}^{k} \sum_{\ell=1}^{i} \beta_{jt}^k \theta_t^{\ell j} (\delta_\ell - E[\delta_\ell \mid \Omega_{t-1}])\right]^2 \qquad \text{(A33)}
\end{aligned}
$$

Since $\mathrm{Cov}(\delta_j, \delta_\ell \mid \delta_1, \ldots, \delta_i, \Omega_{t-1})$ is a deterministic constant, the last expression on the right-hand side is indeed a quadratic function of unrevealed private information. Q.E.D.

Proof of Proposition 3:

Period 2. Trader 2's objective is

$$\begin{aligned} V_2^2(\delta_1, \delta_2, \Omega_1) &= \max_{x_2^2} E\big[(v - p_2)x_2^2 \,\big|\, \delta_1, \delta_2, \Omega_1\big] \\ &= \max_{x_2^2} E\big[\big(v - p_1 - \lambda_2\big(x_2^1 + x_2^2 + u_3\big)\big)x_2^2 \,\big|\, \delta_1, \delta_2, \Omega_1\big] \\ &= \max_{x_2^2} \big[v - p_1 - \lambda_2\big(x_2^1 + x_2^2\big)\big]x_2^2. \end{aligned} \tag{A34}$$

Solving the first-order condition yields

$$x_2^2 = \frac{v - p_1}{2\lambda_2} - \frac{1}{2}x_2^1. \tag{A35}$$

The second-order condition is $\lambda_2 > 0$. Similarly, trader 1's decision problem is

$$\begin{aligned} V_2^1(\delta_1, \Omega_1) &= \max_{x_2^1} E\big[\big(v - p_2\big)x_2^1 \,\big|\, \delta_1, \Omega_1\big] \\ &= \max_{x_2^1} E\big[\big(v - p_1 - \lambda_2\big(x_2^1 + x_2^2\big)\big)x_2^1 \,\big|\, \delta_1, \Omega_1\big]. \end{aligned} \tag{A36}$$

The first-order condition is

$$E[v - p_1 \mid \delta_1, \Omega_1] - 2\lambda_2 x_2^1 - \lambda_2 E\big[x_2^2 \,\big|\, \delta_1, \Omega_1\big] = 0. \tag{A37}$$

Substituting for x_2^2 using (A35) yields

$$2\lambda_2 x_2^1 = E[v - p_1 \mid \delta_1, \Omega_1] - \lambda_2 E\left[\frac{v - p_1 - \lambda_2 x_2^1}{2\lambda_2} \,\middle|\, \delta_1, \Omega_1\right]. \tag{A38}$$

Using the market efficiency condition and independence, we have

$$E[v - p_1 \mid \delta_1, \Omega_1] = E[v - E[v \mid \Omega_1] \mid \delta_1, \Omega_1] = \delta_1 - E[\delta_1 \mid \Omega_1]. \tag{A39}$$

Thus, we can solve for the expression for x_2^1, (28), from the first-order condition. The expression for x_2^2 follows from (28) and (A35). The second-order condition is $\lambda_2 > 0$.

To solve for the value functions, substitute for x_2^1 and x_2^2 into trader 2's objective to obtain

$$\begin{aligned} V_2^2(\delta_1, \delta_2, \Omega_1) &= \left[v - p_1 - \lambda_2\left(x_2^1 + x_2^2\right)\right]x_2^2 \\ &= \frac{1}{\lambda_2}\left[\frac{1}{3}(\delta_1 - E[\delta_1 \mid \Omega_1]) + \frac{1}{2}(\delta_2 - E[\delta_2 \mid \Omega_1])\right]^2. \end{aligned} \tag{A40}$$

Similarly, trader 1's value function is

$$\begin{aligned} V_2^1(\delta_1, \Omega_1) &= \max_{x_2^1} E\left[\left(v - p_1 - \lambda_2\left(x_2^1 + x_2^2\right)\right)x_2^1 \,\middle|\, \delta_1, \Omega_1\right] \\ &= (\delta_1 - E[\delta_1 \mid \Omega_1])\, x_2^1 - \lambda_2 x_2^1 E\left[\left(x_2^1 + x_2^2\right) \,\middle|\, \delta_1, \Omega_1\right] \\ &= \frac{1}{9\lambda_2}(\delta_1 - E[\delta_1 \mid \Omega_1])^2. \end{aligned} \tag{A41}$$

The expression for λ_2 follows from (18).

Period 1. In period 1, trader 1's decision problem is

$$\begin{aligned} V_1^1(\delta_1, \Omega_0) &= \max_{x_1^1} E\left[(v - p_1)x_1^1 + V_2^1(\delta_1, \Omega_1) \,\middle|\, \delta_1, \Omega_0\right] \\ &= \max_{x_1^1} E\left[\left(v - p_0 - \lambda_1\left(x_1^1 + u_1\right)\right)x_1^1 \,\middle|\, \delta_1, \Omega_0\right] + E\left[V_2^1(\delta_1, \Omega_1) \,\middle|\, \delta_1, \Omega_0\right] \\ &= \max_{x_1^1} E\left[(v - p_0)x_1^1 - \lambda_1\left(x_1^1\right)^2 \,\middle|\, \delta_1, \Omega_0\right] \\ &\quad + \frac{1}{9\lambda_2}E\left[(\delta_1 - E[\delta_1 \mid \Omega_1])^2 \,\middle|\, \delta_1, \Omega_0\right]. \end{aligned} \tag{A42}$$

Using Lemma 1, the last expectation can be written as

$$\begin{aligned} E\left[(\delta_1 - E[\delta_1 \,|\, \Omega_1])^2 \,\middle|\, \delta_1, \Omega_0\right] = E\Big[\Big(\delta_1 - E[\delta_1 \mid \Omega_0] \\ - \phi_1^1(p_1 - p_0)\Big)^2 \,\Big|\, \delta_1, \Omega_0\Big]. \end{aligned} \tag{A43}$$

Noting that $p_1 - p_0 = \lambda_1(x_1^1 + u_1)$, we can obtain the first-order condition

$$\begin{aligned} 0 &= E[v - p_0 \mid \delta_1, \Omega_0] - 2\lambda_1 x_1^1 \\ &\quad - \frac{2}{9\lambda_2}\phi_1^1\lambda_1 E\left[\delta_1 - E[\delta_1 \mid \Omega_1] - \phi_1^1(p_1 - p_0) \,\middle|\, \delta_1, \Omega_0\right]. \end{aligned} \tag{A44}$$

Substituting the price function (4) and using

$$E[v - p_0 \mid \delta_1, \Omega_0] = E[v - E[v \mid \Omega_0] \mid \delta_1, \Omega_0] = \delta_1 - E[\delta_1 \mid \Omega_0], \tag{A45}$$

we obtain

$$2\lambda_1\left[1-\frac{\lambda_1}{9\lambda_2}(\phi_1^1)^2\right]x_1^1=\left[1-\frac{2\lambda_1}{9\lambda_2}\phi_1^1\right](\delta_1-E[\delta_1\mid\Omega_0]). \tag{A46}$$

The expression for x_1^1 then follows from

$$\phi_1^1=\frac{\mathrm{Cov}(\delta_1,p_1)}{\mathrm{Var}(p_1)}=\frac{\mathrm{Cov}(v,X_1\mid\Omega_0)}{\lambda_1\left[\mathrm{Var}(X_1\mid\Omega_0)+\sigma_u^2\right]}=1. \tag{A47}$$

Clearly, the second-order condition is $\lambda_1(1-\lambda_1/(9\lambda_2))>0$.

The equation for λ_1, (30), follows from (16). Inspecting equation (30) and the above second-order condition, we must have $0<\lambda_1<9/2\lambda_2$. Moreover, equation (30) implies a unique solution for λ_1 in $(0,9/2\lambda_2)$.

We now solve for the value function. Using Lemma 1, the pricing function (4), and the expression for x_1^1, we have

$$\begin{aligned}E\left[(\delta_1-E[\delta_1\mid\Omega_1])^2\,\middle|\,\delta_1,\Omega_0\right]&=E\left[(\delta_1-E[\delta_1\mid\Omega_0]-\phi_1^1(p_1-p_0))^2\,\middle|\,\delta_1,\Omega_0\right]\\&=\frac{1}{4(1-\lambda_1/(9\lambda_2))^2}(\delta_1-E[\delta_1\mid\Omega_0])^2+\lambda_1^2\sigma_u^2.\end{aligned} \tag{A48}$$

The value function then follows from substituting the above expression, the expression for x_1^1, and the pricing function (4) into the maximized objective function:

$$\begin{aligned}V_1^1(\delta_1,\Omega_0)&=E\left[(v-p_1)x_1^1+V_2^1(\delta_1,\Omega_1)\,\middle|\,\delta_1,\Omega_0\right]\\&=E\left[(v-p_0-\lambda_1x_1^1\,\middle|\,\delta_1,\Omega_0\right]x_1^1+\frac{1}{9\lambda_2}E\left[(\delta_1-E[\delta_1\mid\Omega_1])^2\,\middle|\,\delta_1,\Omega_0\right].\end{aligned} \tag{A49}$$

Finally, the expressions for $\mathrm{Var}(\delta_1\mid\Omega_1)$ follow from (a) Lemma 3, (b) the expression for x_1^1, and (c) the pricing function (4)

$$\mathrm{Var}(\delta_1\mid\Omega_1)=\mathrm{Var}(\delta_1)-\frac{\mathrm{Cov}^2(\delta_1,p_1\mid\Omega_0)}{\mathrm{Var}(p_1\mid\Omega_0)}=\mathrm{Var}(\delta_1)-\frac{\mathrm{Cov}^2\left(\delta_1,x_1^1\mid\Omega_0\right)}{\mathrm{Var}\left(x_1^1\mid\Omega_0\right)+\sigma_u^2}. \tag{A50}$$

Q.E.D.

PROPOSITION 4: *Suppose innovations are perfectly correlated and L_j traders observe δ at date j. Let $q_t=\lambda_tB_t$. Then:*

(*a*) *The unique equilibrium is obtained by recursively solving for the unique root in* $(0, n^2/2)$ *of the cubic equation*

$$2\sum_{m=1}^{n\wedge t} L_m \sum_{m=1}^{n\wedge(t-1)} L_m q_{t-1}^3 - \left[\sum_{m=1}^{n\wedge(t-1)} L_m + 1\right]\sum_{m=1}^{n\wedge t} L_m n^2 q_{t-1}^2 - 2\sum_{m=1}^{n\wedge(t-1)} L_m k_t q_{t-1} + \sum_{m=1}^{n\wedge(t-1)} L_m n^2 k_t = 0, \quad \textit{for } t \geq 2, \tag{A51}$$

$$\textit{where} \quad q_N = 0,\ k_t = \frac{(n^2 - q_t)^2}{(1 - 2q_t/n^2)\left[\sum_{m=1}^{n\wedge t} L_m(1 - 2q_t/n^2) + 1\right]^2},$$

and $t \wedge n \equiv \min(t, n)$.

(*b*) *The trading strategy of any trader with information at date j takes the form*

$$x_{jt}^i = \beta_t(\delta - E[\delta \mid \Omega_{t-1}]), \quad j \leq n \wedge t, x_t^i \equiv x_{tj}^i = 0, t < j \leq n, \tag{A52}$$

$$\textit{where} \qquad \beta_t = \frac{n - 2q_t/n}{\lambda_t\left[\sum_{m=1}^{n\wedge t} L_m(1 - 2q_t/n^2) + 1\right]}.$$

(*c*) *For* $t \geq 1$, *the constants* Σ_t *and* λ_t *are given by*

$$\Sigma_t = \frac{1}{\sum_{m=1}^{n\wedge t} L_m(1 - 2q_t/n^2) + 1}\Sigma_{t-1},$$

$$\lambda_t = \left[\frac{\sum_{m=1}^{n\wedge t} L_m(1 - 2q_t/n^2)\Sigma_t}{\sigma_u^2\left(\sum_{m=1}^{n\wedge t} L_m(1 - 2q_t/n^2) + 1\right)}\right]^{1/2}, \tag{A53}$$

with boundary condition $\Sigma_0 = n^2\mathrm{Var}(\delta)$.

(*d*) *The parameters in the value functions are given by*

$$B_{t-1}^i = B_{t-1} = \frac{n^2 - q_t}{\lambda_t\left[\sum_{m=1}^{n\wedge t} L_m(1 - 2q_t/n^2) + 1\right]^2},$$

$$A_{t-1}^i = A_{t-1} = A_t + B_t(\lambda_t/n)^2\sigma_u^2, \tag{A54}$$

with boundary conditions $B_N = 0$ *and* $A_N = 0$.

Proof: Since $\delta_i = \delta$, $\Sigma_t = \mathrm{Var}(v \mid \Omega_t) = n^2\mathrm{Var}(\delta \mid \Omega_t)$. Using (25), we derive

$$\lambda_t = \frac{n\sum_{j=1}^{n\wedge t} L_j\beta_t \mathrm{Var}(\delta \mid \Omega_{t-1})}{\left(\sum_{j=1}^{n\wedge t} L_j\right)^2(\beta_t)^2\mathrm{Var}(\delta \mid \Omega_{t-1}) + \sigma_u^2} = \frac{\sum_{j=1}^{n\wedge t} L_j\beta_t\Sigma_{t-1}/n}{\left(\sum_{j=1}^{n\wedge t} L_j\right)^2(\beta_t)^2\Sigma_{t-1}/n^2 + \sigma_u^2}, \tag{A55}$$

$$\operatorname{Var}(X_t \mid \Omega_{t-1}) = \left(\sum_{j=1}^{n\wedge t} L_j\right)^2 (\beta_t)^2 \operatorname{Var}(\delta \mid \Omega_{t-1})$$
$$= \left(\sum_{j=1}^{n\wedge t} L_j\right)^2 (\beta_t)^2 \Sigma_{t-1}/n^2. \tag{A56}$$

Thus, from (19) we can derive

$$\Sigma_t = \Sigma_{t-1} - \lambda_t^2 \left[\operatorname{Var}(X_t \mid \Omega_{t-1}) + \sigma_u^2\right]$$
$$= \Sigma_{t-1} - \lambda_t^2 \left[\left(\sum_{j=1}^{n\wedge t} L_j\right)^2 (\beta_t)^2 \Sigma_{t-1}/n^2 + \sigma_u^2\right]$$
$$= \frac{\Sigma_{t-1}\sigma_u^2}{\left(\sum_{j=1}^{n\wedge t} L_j\right)^2 (\beta_t)^2 \Sigma_{t-1}/n^2 + \sigma_u^2}. \tag{A57}$$

Using the above expressions, rewrite λ_t and Σ_t as

$$\lambda_t = \frac{\sum_{j=1}^{n\wedge t} L_j \beta_t \Sigma_{t-1}/n}{\left(\sum_{j=1}^{n\wedge t} L_j\right)^2 (\beta_t)^2 \Sigma_{t-1}/n^2 + \sigma_u^2} = \frac{\sum_{j=1}^{n\wedge t} L_j \beta_t}{n\sigma_u^2}\Sigma_t \quad \text{and}$$
$$\Sigma_t = \left(1 - \sum_{j=1}^{n\wedge t} L_j \beta_t \lambda_t/n\right)\Sigma_{t-1}. \tag{A58}$$

Then $q_t = \lambda_t B_t$ and Proposition 1 together imply that

$$\beta_t = \frac{n(1 - 2q_t/n^2)}{\lambda_t\left[\sum_{k=1}^{n\wedge t} L_k(1 - 2q_t/n^2) + 1\right]}. \tag{A59}$$

Substituting for β_t into the value function yields

$$B_{t-1} = \beta_t\left[n - \sum_{k=1}^{n\wedge t} L_k \lambda_t \beta_t\right] + B_t\left[1 - \sum_{k=1}^{n\wedge t} L_k \beta_t \lambda_t/n\right]^2$$
$$= \frac{n(1 - 2q_t/n^2)}{\lambda_t\left[\sum_{k=1}^{n\wedge t} L_k(1 - 2q_t/n^2) + 1\right]}\left[n - \frac{\sum_{k=1}^{n\wedge t} L_k n(1 - 2q_t/n^2)}{\sum_{k=1}^{n\wedge t} L_k(1 - 2q_t/n^2) + 1}\right]$$
$$+ B_t\left[1 - \frac{\sum_{k=1}^{n\wedge t} L_k(1 - 2q_t/n^2)}{\sum_{k=1}^{n\wedge t} L_k(1 - 2q_t/n^2) + 1}\right]^2$$
$$= \frac{n^2 - q_t}{\lambda_t\left[\sum_{k=1}^{n\wedge t} L_k(1 - 2q_t/n^2) + 1\right]^2}. \tag{A60}$$

Multiplying both sides by λ_{t-1} and using $q_{t-1} = \lambda_{t-1} B_{t-1}$ yields

$$\frac{\lambda_t}{\lambda_{t-1}} = \frac{n^2 - q_t}{q_{t-1}\left[\sum_{k=1}^{n\wedge t} L_k\left(1 - 2q_t/n^2\right) + 1\right]^2}. \tag{A61}$$

But, equation (A58) also implies that

$$\frac{\lambda_t}{\lambda_{t-1}} = \frac{\sum_{k=1}^{n\wedge t} L_k \beta_t \Sigma_t}{\sum_{k=1}^{n\wedge(t-1)} L_k \beta_{t-1} \Sigma_{t-1}} = \frac{\sum_{k=1}^{n\wedge t} L_k \beta_t}{\sum_{k=1}^{n\wedge(t-1)} L_k \beta_{t-1}} \left[1 - \sum_{k=1}^{n\wedge t} L_k \beta_t \lambda_t / n\right]. \tag{A62}$$

Thus, using the above expression for β_t we can derive

$$\left(\frac{\lambda_t}{\lambda_{t-1}}\right)^2 = \frac{\sum_{k=1}^{n\wedge t} L_k}{\sum_{k=1}^{n\wedge(t-1)} L_k} \frac{1 - 2q_t/n^2}{\left[\sum_{k=1}^{n\wedge t} L_k\left(1 - 2q_t/n^2\right) + 1\right]^2} \times \frac{\sum_{k=1}^{n\wedge(t-1)} L_k(1 - 2q_{t-1}/n^2) + 1}{1 - 2q_{t-1}/n^2}. \tag{A63}$$

Using the above two expressions for λ_t/λ_{t-1} we can derive the cubic equation (A51). Then it is easy to derive the expressions for Σ_t, λ_t, A_t, and B_t in the proposition.

It remains to show that the equilibrium is unique. Looking at the expression for λ_t and the second-order conditions, we see that they are satisfied only if $0 < q_t < n^2/2$. Hence, it suffices to show that there is a unique root to the cubic equation (A51) in the interval $(0, n^2/2)$. The left-hand side of equation (A51) is negative for q_{t-1} sufficiently small, and positive at $q_{t-1} = 0$, so there is one negative root. Further, it is negative at $q_{t-1} = n^2/2$, so there is a root in the interval $q_{t-1} \in (0, n^2/2)$. Finally, since the left-hand side is positive for q_{t-1} sufficiently large, it follows that the third root is for $q_{t-1} > n^2/2$. Q.E.D.

PROPOSITION 5: *Suppose there are three periods and two informed traders who observe δ_i at date 1, $i = 1, 2$, respectively, and assume further that the innovations are distributed independently. Then in equilibrium:*

(*a*) *The constants in the trading strategies* (3) *are given by*

$$\begin{bmatrix} \beta_3^1 \\ \beta_3^2 \end{bmatrix} = \frac{1}{\lambda_3} \begin{bmatrix} 2 & \theta_3^{12} \\ \theta_3^{21} & 2 \end{bmatrix}^{-1} \begin{bmatrix} 1 + \theta_3^{12} \\ 1 + \theta_3^{21} \end{bmatrix}, \tag{A64}$$

$$\begin{bmatrix} \beta_2^1 \\ \beta_2^2 \end{bmatrix} = \begin{bmatrix} 2 - 2\lambda_2 B_2^1 (\phi_2^1)^2 & (1 - 2\lambda_2 B_2^1 (\phi_2^1)^2)\theta_2^{12} \\ (1 - 2\lambda_2 B_2^2 (\phi_2^2)^2)\theta_2^{21} & 2 - 2\lambda_2 B_2^2 (\phi_2^2)^2 \end{bmatrix}^{-1} \times \left(\frac{1}{\lambda_2} \begin{bmatrix} 1 + \theta_2^{12} \\ 1 + \theta_2^{21} \end{bmatrix} - 2 \begin{bmatrix} B_2^1 \phi_2^1 \\ B_2^2 \phi_2^2 \end{bmatrix}\right),$$

$$\begin{bmatrix} \beta_1^1 \\ \beta_1^2 \end{bmatrix} = \begin{bmatrix} 2 - 2\lambda_1 B_1^1 (\phi_1^1)^2 & 0 \\ 0 & 2 - 2\lambda_1 B_1^2 (\phi_1^2)^2 \end{bmatrix}^{-1} \left(\frac{1}{\lambda_1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} - 2 \begin{bmatrix} B_1^1 \phi_1^1 \\ B_1^2 \phi_1^2 \end{bmatrix} \right),$$

where

$$\theta_3^{12} = \frac{\mathrm{Cov}(\delta_1, \delta_2 \mid \Omega_2)}{\mathrm{Var}(\delta_1 \mid \Omega_2)}, \quad \theta_3^{21} = \frac{\mathrm{Cov}(\delta_1, \delta_2 \mid \Omega_2)}{\mathrm{Var}(\delta_2 \mid \Omega_2)},$$

$$\theta_2^{12} = \frac{\mathrm{Cov}(\delta_1, \delta_2 \mid \Omega_1)}{\mathrm{Var}(\delta_1 \mid \Omega_1)}, \quad \theta_2^{21} = \frac{\mathrm{Cov}(\delta_1, \delta_2 \mid \Omega_1)}{\mathrm{Var}(\delta_2 \mid \Omega_1)},$$

$$\phi_2^i = \frac{\beta_2^i \mathrm{Var}(\delta_i \mid \Omega_1) + \beta_2^j \mathrm{Cov}(\delta_i, \delta_j \mid \Omega_1)}{\beta_2^1 \mathrm{Var}(\delta_1 \mid \Omega_1) + (\beta_2^1 + \beta_2^2) \mathrm{Cov}(\delta_1, \delta_2 \mid \Omega_1) + \beta_2^2 \mathrm{Var}(\delta_2 \mid \Omega_1)}, \quad \textit{and}$$

$$\phi_1^i = \frac{\beta_1^i \mathrm{Var}(\delta_i)}{\beta_1^1 \mathrm{Var}(\delta_1) + \beta_1^2 \mathrm{Var}(\delta_2)}.$$

(*b*) *The period pricing constants*, λ_t, *are given by*

$$\lambda_3 = \frac{\beta_3^1 \mathrm{Var}(\delta_1 \mid \Omega_2) + (\beta_3^2 + \beta_3^1) \mathrm{Cov}(\delta_1, \delta_2 \mid \Omega_2) + \beta_3^2 \mathrm{Var}(\delta_2 \mid \Omega_2)}{(\beta_3^1)^2 \mathrm{Var}(\delta_1 \mid \Omega_2) + 2\beta_3^1 \beta_3^2 \mathrm{Cov}(\delta_1, \delta_2 \mid \Omega_2) + (\beta_3^2)^2 \mathrm{Var}(\delta_2 \mid \Omega_2) + \sigma_u^2},$$

$$\lambda_2 = \frac{\beta_2^1 \mathrm{Var}(\delta_1 \mid \Omega_1) + (\beta_2^1 + \beta_2^2) \mathrm{Cov}(\delta_1, \delta_2 \mid \Omega_1) + \beta_2^2 \mathrm{Var}(\delta_2 \mid \Omega_1)}{(\beta_2^1)^2 \mathrm{Var}(\delta_1 \mid \Omega_1) + 2\beta_2^1 \beta_2^2 \mathrm{Cov}(\delta_1, \delta_2 \mid \Omega_1) + (\beta_2^2)^2 \mathrm{Var}(\delta_2 \mid \Omega_1) + \sigma_u^2},$$

and

$$\lambda_1 = \frac{\beta_1^1 \mathrm{Var}(\delta_1) + \beta_1^2 \mathrm{Var}(\delta_2)}{(\beta_1^1)^2 \mathrm{Var}(\delta_1) + (\beta_1^2)^2 \mathrm{Var}(\delta_2) + \sigma_u^2}. \tag{A65}$$

(*c*) *The conditional covariances between innovations are:*

$$\mathrm{Cov}(\delta_1, \delta_2 \mid \Omega_2) = \frac{[\beta_2^1 \beta_2^2 \mathrm{Cov}(\delta_1, \delta_2 \mid \Omega_1) + \sigma_u^2] \mathrm{Cov}(\delta_1, \delta_2 \mid \Omega_1) - \beta_2^1 \beta_2^2 \mathrm{Var}(\delta_1 \mid \Omega_1) \mathrm{Var}(\delta_2 \mid \Omega_1)}{(\beta_2^1)^2 \mathrm{Var}(\delta_1 \mid \Omega_1) + 2\beta_2^1 \beta_2^2 \mathrm{Cov}(\delta_1, \delta_2 \mid \Omega_1) + (\beta_2^2)^2 \mathrm{Var}(\delta_2 \mid \Omega_1) + \sigma_u^2},$$

and

$$\mathrm{Cov}(\delta_1, \delta_2 \mid \Omega_1) = \frac{-\beta_1^1 \beta_1^2 \mathrm{Var}(\delta_1) \mathrm{Var}(\delta_2)}{(\beta_1^1)^2 \mathrm{Var}(\delta_1) + (\beta_1^2)^2 \mathrm{Var}(\delta_2) + \sigma_u^2}. \tag{A66}$$

Proof: Follows as a special case of Proposition 1. To use Proposition 5 to characterize outcomes where informed agent 1 learns δ_1 in the first period and informed trader 2 learns δ_2 in the second period, set $\beta_1^2 = 0$, so informed trader 2 does not trade at date 1. Q.E.D.

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