Asset Market Equilibrium under Rational Inattention*

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Abstract

We propose a noisy rational expectations equilibrium model of asset markets with rationally inattentive investors. We incorporate any finite number of assets with arbitrary correlation. We also do not restrict the signal form and show that investors optimally choose a single signal, which is a noisy linear combination of all risky assets. This generates comovement of asset prices and contagion of shocks, even when asset payoffs are negatively correlated. The model also provides testable predictions of the impact of risk aversion, aggregate risk, and information capacity on the security market line, the portfolio dispersion, and the abnormal return.

Keywords: Rational Inattention, Information Choice, Asset Pricing, Portfolio Choice

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1 Introduction

Since the seminal contributions by Grossman and Stiglitz (1980), there has been a growing literature in economics and finance that incorporates asymmetric information in the noisy rational expectations equilibrium framework.[1] This literature typically assumes that information structure is exogenously given. However, agents may endogenously process information given their limited information capacity. For example, investors in financial markets typically pay attention to some individual assets or some portfolios (linear combinations) of assets. They can acquire signals about individual assets or portfolios of assets.

In this paper we propose a multiple asset, noisy rational expectations equilibrium model with rationally inattentive investors. The model features any finite number of risky assets with arbitrarily correlated payoffs and a continuum of ex ante identical investors who face information-processing constraints as in Sims (2003, 2011). Investors observe asset prices and acquire private signals about the assets to reduce uncertainty about their portfolio. We do not restrict the signal form except that it is a noisy linear transformation of asset payoffs. Investors optimally choose both the linear transformation and the precision of the signal subject to an entropy-based information constraint. After allocating their attention, investors incorporate the information from their private signals and asset prices through Bayesian updating to form their posterior beliefs about the asset payoffs and then choose their optimal asset holdings.

We show that each investor will optimally choose a one-dimensional signal that is a noisy linear combination of all risky assets. In a symmetric linear equilibrium, all investors choose the same signal form. We find that an unconditional CAPM holds in our model, but is rejected by econometricians (Type I error). As argued by Andrei, Cujean, and Wilson (henceforth ACW) (2020), there is an information distance between econometricians and the average investor. Thus the security market line (SML) from the econometrician's point of view is different from the true SML in the model. We find that the econometrician's SML may not be linear and can be flatter or steeper than the true SML, unlike the result in ACW (2020). Moreover, an increase in the information capacity shifts down both the true SML and the econometrician's SML. The reason is that higher capacity allows investors to process more precise signals and hence reduce asset uncertainty more.

We also show that there is excess comovement in asset prices relative to asset fundamentals, as in Mondria (2010). We illustrate this result using a numerical example with three risky assets. One asset is independent of the other two and the other two are arbitrarily correlated. We find that the asset prices can be positively correlated even when asset payoffs are negatively correlated. The reason is that each investor receives a single signal, that is a noisy linear combination of all risky

[1]See Veldkamp (2011) and Angeletos and Lian (2016) for recent surveys of this literature.

assets with all positive coefficients. If there is good news about one asset, then investors observe a high realization of the private signal and they attribute part of the effect to one asset and the rest to the other two assets. This leads to an increase in the prices of all three assets and thus price comovement of these assets.

We also use the above numerical example to illustrate the contagion effect. We show that an increase in the variance of one independent asset payoff causes the prices of the other two assets to decline. This is true even when the payoffs of the other two assets are negatively correlated.

We finally study the implications for the portfolio dispersion. We find that the portfolio holdings dispersion in the case without uninformed investors declines in recessions when risk aversion (a proxy for the price of risk) or aggregate volatility is high. The portfolio return dispersion declines with risk aversion, but increases with the aggregate volatility. Intuitively, higher risk aversion leads to more conservative portfolio choices, and hence a smaller dispersion of portfolio holdings. On the other hand, higher risk aversion or higher aggregate risk leads to higher market risk premium. The portfolio return dispersion reflects the combined effects of portfolio holdings and risk premium. The impact of the portfolio return dispersion depends on which effect dominates.[2]

Our paper is closely related to the literature on asset pricing models with rational inattention (Peng 2005, Peng and Xiong 2006, van Nieuwerburgh and Veldkamp 2009, 2010, Mondria 2010, and Kacperczyk, van Nieuwerburgh, and Veldkamp (henceforth KVV) 2016).[3] Because of the difficulty in the case of multiple assets, this literature typically makes the signal independence assumption. That is, investors are assumed to process information about one asset (or one risk factor) at a time. The signal vector is equal to the unobservable asset payoff (or risk factor) vector plus a noise. An undesirable feature is that ex ante independent assets remain ex post independent, and hence such an assumption cannot explain asset comovement. KVV (2016) relax this assumption by imposing an invertibility assumption on the signal form. They derive some results different from ours as detailed in Section 4. Mondria (2010) does not restrict the signal form, but his approach only applies to the two-asset case with ex ante independent assets.

An important contribution of our paper is to analyze the general case with any finite number of correlated assets. We solve for both the linear transformation and the precision of the signal vector. The difficulty is that the information choice problem is nonconvex, unlike the multivariate linear-quadratic-Gaussian framework of Sims (2003), Miao (2019), and Miao, Wu, and Young (2021). We

[2]In our working paper version, we extend our model to incorporate a fraction of uninformed investors and study how informed investors can profit from their information advantage. We find that an informed investor earns an abnormal return. The abnormal return increases with the aggregate risk and the information capacity, and decreases with the fraction of informed investors. Moreover, it has a U-shaped relationship with risk aversion. We also find that the portfolio holdings dispersion rises in recessions when risk aversion or aggregate volatility is high, if the fraction of uninformed investors is large enough.

[3]See Matějka and McKay (2005), Luo (2008), Woodford (2009), Maćkowiak and Wiederholt (2009), Caplin, Dean, and Leahy (2019), Miao (2019), Miao, Wu, and Young (2021) for other models with rational inattention. See Sims (2011) and Maćkowiak, Matějka and Wiederholt (2021) for surveys and additional references cited therein.

are able to derive a closed-form solution and find that the optimal signal is one dimensional. In particular, the optimal signal is a noisy linear combination of risky assets, as in Mondria (2010) for the two asset-case. Van Nieuwerburgh and Veldkamp (2010) and KVV (2016) also find that the information choice problem is nonconvex, but their problem is different from ours and their results and solution methods are also different from ours. In particular, they find that investors specialize in learning about only one asset or only one risk factor, while we find that they acquire a single signal about a linear combination of assets. They find that portfolio holdings dispersion rises in recessions, while we show that it may decline in recessions.[4] We also study implications for SMLs absent from Mondria (2010) and KVV (2016). Relative to ACW (2020), our model delivers implications for attention allocation, comovements, and portfolio dispersions, absent from their paper.

Most studies in the literature on information acquisition assume that investors choose the signal precision only and do not consider the optimal signal form as a linear transformation of states. Unlike the rational inattention framework of Sims (2003, 2011), some researchers impose information cost other than the entropy-based cost. This literature is too large for us to cite it all. Recent contributions include Huang and Liu (2007), Vives (2010), KVV (2016), Andrei and Hasler (2019), and Vives and Yang (2019), among others.

2 Model

Consider a three-date economy populated by a continuum of ex ante identical investors of measure one, indexed by $i \in [0,1]$. Investors are endowed with initial wealth at date 1. They first choose their private signals given their limited capacity to process information at date 1. At date 2, they decide on the optimal portfolios given the observation of their private signals and the asset price. At the last date, investors consume the payoff of their portfolios.

There are n risky assets and one riskless asset. The riskless asset pays constant R_f units of the consumption good at date 3. The n risky assets pay F units of the consumption good at date 3, where F is a random column vector that is normally distributed with mean $\overline{F}$ and covariance matrix Σ_F (denoted by $F \sim N\left(\overline{F}, \Sigma_F\right)$). Assume that Σ_F is positive definite, denoted by $\Sigma_F \succ 0$.[5] Following KVV (2016), we may use the risk factor representation $F = \Gamma Y$, where $Y = [Y_1, ..., Y_N]'$ represents a column vector of N independent risk factors. If a risk factor Y_j appears in all asset payoffs, then it is called a common aggregate risk factor. If it appears only in asset j's payoff, then it is called an asset-specific (idiosyncratic) risk factor. The matrix Γ represents factor loadings. If $n = N$ and Γ is an identity matrix, then all risk factors are asset-specific. Let P denote the

[4]In an extended model of our working paper version, we show that this result also holds true if the fraction of uninformed investors is small enough.

[5]The notation $A \succeq B$ means that the matrix $A - B$ is positive semidefinite.

price vector of the n risky assets. To prevent the equilibrium price from being fully revealing, we introduce noisy asset supply. Suppose that the supply of the risky assets is given by a random vector $Z \sim N\left(\overline{Z}, \Sigma_Z\right)$, where $\Sigma_Z \succ 0$. Assume that Z and F are independent.

2.1 Information Cost

Investors want to acquire information about the risky assets to reduce the uncertainty of their portfolios. They have a limited capacity to process information about asset payoffs. They observe the asset prices and use asset prices and acquired signals to reduce payoff uncertainty. Assume that investor i can choose signals of the following form

$$S_i = C_i F + \epsilon_i, \tag{1}$$

where C_i is an n_s by n matrix, the noise $\epsilon_i \sim N(0, \Sigma_{\epsilon i})$ is independent of F and Z, and $\Sigma_{\epsilon i} \succ 0$. We call $(C_i, \Sigma_{\epsilon i})$ an information (signal) structure of investor i, which will be chosen endogenously. Notice that we do not impose any assumption on C_i. In particular, C_i may not be a square matrix in that $n_s \neq n$. Given F and ϵ_i are Gaussian, the signal vector S_i is also Gaussian. As is common in the literature, assume that investors do not process information about the asset supply.

Each investor i faces the following information-processing constraint

$$H(F) - H(F|S_i, P) \leq \kappa, \tag{2}$$

where $\kappa > 0$ is the parameter of the channel capacity, H denotes the Shannon entropy, and $H(\cdot|\cdot)$ denotes the conditional entropy. Intuitively, entropy $H(F)$ measures the amount of uncertainty about asset payoff F. After acquiring information (S_i, P), uncertainty $H(F|S_i, P)$ is reduced. Inequality (2) says that the reduction of uncertainty is limited by the capacity κ.

For any n-dimensional multivariate normal random variable $X \sim N\left(\overline{X}, \Sigma\right)$, its Shannon entropy is given by

$$H(X) = \frac{1}{2} \log\left((2\pi e)^n \det \Sigma\right),$$

where $\det(\Sigma)$ denotes the determinant of Σ. We will show later that the equilibrium price P is Gaussian and hence F is Gaussian conditional on S_i and P. Thus we can simplify the constraint (2) as

$$\log\det(\mathrm{Var}(F)) - \log\det(\mathrm{Var}(F|S_i, P)) \leq 2\kappa, \tag{3}$$

where Var denotes the variance-covariance operator.

2.2 Decision Problem

Following van Nieuwerburgh and Veldkamp (2009, 2010) and Mondria (2010), assume that each investor i has the following utility function:

$$U_i = \mathbb{E}\left\{-\log \mathbb{E}\left[\exp\left(-\frac{W_i}{\rho}\right)|S_i, P\right]\right\}, \tag{4}$$

where $\rho > 0$ is the risk tolerance parameter and W_i denotes the final wealth level at date 3. The parameter $1/\rho$ represents risk aversion, which also measures the price of risk as in KVV (2016).

Investor i faces the following budget constraint

$$W_i = R_f W_{i0} + X_i' R^e, \tag{5}$$

where $R^e \equiv F - PR_f$ denotes the excess (dollar) return, W_{i0} denotes investor i's initial wealth level, and X_i denotes the vector of his/her risky asset holdings.

Each investor i first chooses a signal structure $(C_i, \Sigma_{\epsilon i})$ at date 1 and then chooses a portfolio demand X_i given the information conveyed by the signal S_i and the price vector P at date 2 to maximize his/her utility in (4) subject to the budget constraint in (5). When solving this problem, investor i takes the price vector P and other investors' information structures as given.

2.3 Equilibrium

An asset market equilibrium under rational inattention consists of a price vector P, a signal structure $(C_i, \Sigma_{\epsilon i})$, and a portfolio demand X_i, for each investor $i \in [0,1]$, such that (i) each investor i solves his/her decision problem in the previous subsection taking the price P and other investors' signal structures as given, and (ii) the asset market clears in that

$$\int_0^1 X_i di = Z. \tag{6}$$

Investors make decisions in two separate stages (dates). In stage 1 (date 1), investors simultaneously acquire signals (information structures) subject to information processing constraints. This choice reflects rational inattention introduced by Sims (2003). In stage 2, investors choose portfolios taking private signals acquired in stage 1 and asset prices as given. Investors behave competitively (non-strategically) when choosing portfolios. In comparison, investors behave strategically when choosing private signals in that they consider others' choices when making their own choices. Given this structure of the model, a natural solution can be found through subgame perfection (backward induction): solve for market equilibrium in stage 2 taking private information acquisition decisions as given; determine optimal information acquisition decisions in stage 1 with investors' anticipations that decisions in stage 1 will be followed by market equilibrium in stage 2.

3 Model Solution

When the information structure is exogenously given, the model is essentially the same as that of Admati (1985). Thus we solve our model in two steps as discussed in Section 2.3. First, we derive equilibrium with a fixed information structure as in Admati (1985) and the associated utility level of each investor. Second, we solve for the optimal information structure for each investor to maximize his/her expected utility taken the other investors' information structures as given.

3.1 Equilibrium with Fixed Information Structure

Suppose that the information structure $(C_i, \Sigma_{\epsilon i})$ for each i is exogenously given. As in Admati (1985), we can show that the equilibrium asset price takes the following linear form

$$P = A_0 + A_1 F - A_2 Z, \tag{7}$$

where

$$A_0 = \frac{\rho}{R_f}\left(\rho\Sigma_F^{-1} + \rho\Pi\Sigma_Z^{-1}\Pi + \Pi\right)^{-1}\left(\Sigma_F^{-1}\overline{F} + \Pi\Sigma_Z^{-1}\overline{Z}\right), \tag{8}$$

$$A_1 = \frac{1}{R_f}\left(\rho\Sigma_F^{-1} + \rho\Pi\Sigma_Z^{-1}\Pi + \Pi\right)^{-1}\left(\Pi + \rho\Pi\Sigma_Z^{-1}\Pi\right), \tag{9}$$

$$A_2 = \frac{1}{R_f}\left(\rho\Sigma_F^{-1} + \rho\Pi\Sigma_Z^{-1}\Pi + \Pi\right)^{-1}\left(I + \rho\Pi\Sigma_Z^{-1}\right). \tag{10}$$

Notice that A_2 is invertible and satisfies $A_2^{-1}A_1 = \Pi$, where

$$\Pi = \rho\int_0^1 C_i'\Sigma_{\epsilon i}^{-1}C_i di, \tag{11}$$

and $C_i'\Sigma_{\epsilon j}^{-1}C_i$ is called the signal-to-noise ratio (SNR) for investor i in the engineering literature of information theory. The SNR is a positive semidefinite matrix and describes the information content of the signal. The equilibrium price is determined by the aggregate SNR, but not a particular C_i or $\Sigma_{\epsilon i}$.

Solving for investor i's optimal portfolio choice yields the familiar mean-variance rule:

$$X_i = \rho\left[\mathrm{Var}\left(R^e|S_i, P\right)\right]^{-1}\mathbb{E}\left[R^e|S_i, P\right]. \tag{12}$$

Imposing the market-clearing condition in (6), computing the optimal wealth level in (5), and substituting the resulting expression in (4), we obtain the following result as in Mondria (2010):

Proposition 1 *Investor i's utility for a fixed information structure $(C_i, \Sigma_{\epsilon i})$ for all $i \in [0,1]$ is given by*[6]

$$U_i = \frac{W_{i0}R_f}{\rho} - \frac{n}{2} + \frac{1}{2}\mathrm{Tr}\left(V_i^{-1}\left(V_e + \overline{R}^e\overline{R}^{e\prime}\right)\right), \tag{13}$$

where

$$\overline{R}^e \equiv \mathbb{E}\left[R^e\right] = \left(\rho\Sigma_F^{-1} + \rho\Pi\Sigma_Z^{-1}\Pi + \Pi\right)^{-1}\overline{Z}, \tag{14}$$

$$V_e \equiv \mathrm{Var}\left(R^e\right) = \Sigma_F + R_f^2 A_1\Sigma_F A_1' + R_f^2 A_2\Sigma_Z A_2' - R_f A_1\Sigma_F - R_f\Sigma_F A_1', \tag{15}$$

$$V_i \equiv \mathrm{Var}\left(R^e|S_i, P\right) = \left(\Sigma_F^{-1} + \Pi\Sigma_Z^{-1}\Pi + C_i'\Sigma_{\epsilon i}^{-1}C_i\right)^{-1}. \tag{16}$$

Here $\overline{R}^e$ denotes the vector of unconditional expected excess returns, V_e denotes the unconditional covariance matrix of excess returns., and V_i denotes the conditional covariance matrix of excess returns given investor i's information. Proposition 1 gives investor i's ex ante expected utility in equilibrium given a fixed information structure.

[6]Notice that $\mathrm{Tr}(\cdot)$ denotes the trace operator.

3.2 Optimal Information Structure

In this subsection we solve for the optimal information structure $(C_i, \Sigma_{\epsilon i})$ for each infinitesimal investor i. The decision problem is given by

$$\max_{C_i, \Sigma_{\epsilon i} \succ 0} U_i$$

subject to (3), where U_i is given in (13) and equations (14), (15), and (16) hold. When solving this problem, investor i takes the other investors' information structures as given. In particular, he takes Π, $\overline{R}^e$, and V_e as given.

Define

$$\Omega = V_e + \overline{R}^e \overline{R}^{e\prime}. \tag{17}$$

Since $V_e \succ 0$, we have $\Omega \succ 0$. Define investor i's precision matrix of the excess return as $K_i = V_i^{-1}$. Then we use Proposition 1 to transform his/her information choice problem above into the following problem

$$\max_{K_i} \operatorname{Tr}(K_i \Omega) \tag{18}$$

subject to

$$\log\det(\Sigma_F) + \log\det(K_i) \leq 2\kappa, \tag{19}$$

$$K_i \succeq G, \tag{20}$$

where we define

$$G \equiv \Sigma_F^{-1} + \Pi \Sigma_Z^{-1} \Pi \succ 0. \tag{21}$$

Inequality (19) follows from the information-processing constraint (3) as $V_i = \operatorname{Var}(R^e | S_i, P) = \operatorname{Var}(F | S_i, P)$ and inequality (20) is the no-forgetting constraint analogous to that in Sims (2003). The no-forgetting constraint follows from (16) and $C_i' \Sigma_{\epsilon i}^{-1} C_i \succeq 0$:

$$K_i = V_i^{-1} = \Sigma_F^{-1} + \Pi \Sigma_Z^{-1} \Pi + C_i' \Sigma_{\epsilon i}^{-1} C_i \succeq G. \tag{22}$$

It says that the excess return after acquiring private signal S_i is less uncertain than that without acquiring any signal. Intuitively, the matrix G represents the precision of the excess return when no private information $C_i' \Sigma_{\epsilon i}^{-1} C_i$ is acquired.

We have eliminated the choice of the matrices C_i and $\Sigma_{\epsilon i}$ in the above problem by using matrix inequality (20) to replace equality (16). After obtaining K_i or $V_i = K_i^{-1}$, we use (16) to recover the optimal information structure $(C_i, \Sigma_{\epsilon i})$:

$$C_i' \Sigma_{\epsilon i}^{-1} C_i = K_i - G. \tag{23}$$

Notice that the problem in (18) is not a concave optimization problem because the constraint set in (19) and (20) is not convex as noticed by van Nieuwerburgh and Veldkamp (2010). This is different from the linear-quadratic-Gaussian framework studied by Sims (2003), Miao (2019), and Miao, Wu, and Young (2021). In particular, the solution to the problem in (18) may be at the corner.

Now we solve the problem in (18). Consider the eigen-decomposition:

$$G^{\frac{1}{2}}\Omega G^{\frac{1}{2}} = U\Omega_d U', \tag{24}$$

where U is an orthogonal matrix and Ω_d is a diagonal matrix $\Omega_d = diag\,(d_j)_{j=1}^n$ with $d_1 > 0, ...,$ and $d_n > 0$ denoting eigenvalues of $G^{\frac{1}{2}}\Omega G^{\frac{1}{2}}$. Without loss of generality, let $d_1 = d_2 = ... = d_m$ be the identical largest elements of Ω_d.

Define the matrix

$$\widetilde{K}_i = U'G^{-\frac{1}{2}}K_i G^{-\frac{1}{2}}U.$$

Then $K_i = G^{\frac{1}{2}}U\widetilde{K}_i U'G^{\frac{1}{2}}$. Substituting this equation into (18), (19), and (20), we find that the problem in (18) becomes

$$\max_{\widetilde{K}_i} \text{Tr}\left(\widetilde{K}_i\Omega_d\right) \tag{25}$$

subject to $\widetilde{K}_i \succeq I$ and

$$\log\det\left(\Sigma_F\right) + \log\det\left(G\right) + \log\det\left(\widetilde{K}_i\right) \le 2\kappa. \tag{26}$$

Given the objective function in (25), only diagonal elements of $\widetilde{K}_i$ matters for the optimization. Thus we can focus only on the diagonal matrix for $\widetilde{K}_i$. We have the following result:

Proposition 2 *Suppose that $d_1, d_2, ..., d_m$ are the m identical largest eigenvalues of the matrix $G^{\frac{1}{2}}\Omega G^{\frac{1}{2}}$, where G is given in (21) and $1 \le m \le n$. Suppose that*

$$\lambda^* \equiv \frac{\exp\left(2\kappa\right)}{\det\left(I + \Sigma_F \Pi \Sigma_Z^{-1}\Pi\right)} > 1. \tag{27}$$

If $m = 1$, then the solution to (25) is unique and given by

$$\widetilde{K}_i = I + diag\left(\lambda^* - 1, 0, ..., 0\right).$$

If $m \ge 2$, then there are multiple solutions to (25) given by

$$\widetilde{K}_i = I + \left(\lambda^* - 1\right)v^* v^{*\prime},$$

where $v^* = [a_1^*, a_2^*, ..., a_n^*]'$ is a column vector satisfying $\sum_{j=1}^m \left(a_j^*\right)^2 = 1$ and $a_j^* = 0$ for $j > m$. The optimal precision matrix is given by

$$V_i^{-1} = K_i = G^{\frac{1}{2}}U\widetilde{K}_i U'G^{\frac{1}{2}},$$

The optimal information structure $(C_i, \Sigma_{\epsilon i})$ satisfies

$$C_i'\Sigma_{\epsilon i}^{-1}C_i = \left(\lambda^* - 1\right)G^{\frac{1}{2}}Uv^* v^{*\prime}U'G^{\frac{1}{2}}, \tag{28}$$

and the optimal signal is one-dimensional.

By equation (21), condition (27) is equivalent to the condition

$$\log\det(\Sigma_F) + \log\det(G) < 2\kappa, \tag{29}$$

which says that the information capacity parameter κ must be large enough such that there is a nontrivial feasible choice of $\widetilde{K}_i \succeq I$ for problem (25) subject to (26). If this condition is violated, then either there would be no feasible choice such that $\widetilde{K}_i \succeq I$ when $\lambda^* < 1$ or there would be a trivial solution such that $\widetilde{K}_i = I$ when $\lambda^* = 1$.

Now we can interpret Proposition 2 as follows. If the largest eigenvalue of $G^{\frac{1}{2}}\Omega G^{\frac{1}{2}}$ is unique (i.e., $m = 1$), then let d_1 be the unique largest eigenvalue without loss of generality. The matrix $\widetilde{K}_i$ is diagonal. Every investor i will allocate all attention to d_1 to reduce uncertainty associated with d_1 so that the first diagonal element of $\widetilde{K}_i$ is equal to λ^*. Moreover, every investor i will not attend to other eigenvalues by setting the kth diagonal element of $\widetilde{K}_i$ to 1 for any $k \neq 1$. Thus the diagonal matrix $\widetilde{K}_i$ is the same for every investor i and hence the optimal precision matrix of the excess return is also the same for every investor i. It follows from (28) that the SNR $C_i'\Sigma_{\epsilon i}^{-1}C_i$ is also the same for every investor i as $v^*v^{*\prime} = [1, 0, ..., 0]' [1, 0, ..., 0]$.

Notice that the optimal information structure $(C_i, \Sigma_{\epsilon i})$ is not unique. A particular solution is given by

$$\Sigma_{\epsilon i} = (\lambda^* - 1)^{-1} > 0,$$

and

$$C_i = [1, 0, ..., 0]_{1\times n} U' G^{\frac{1}{2}}.$$

Thus C_i is the first principal component of the matrix $G^{\frac{1}{2}}\Omega G^{\frac{1}{2}}$. For any solution, the optimal signal is one dimensional. Investor i learns a linear combination of all risky assets in that C_i is a one-dimensional vector. We can write investor i's one-dimensional signal as

$$S_i = \sum_{j=1}^{n} C_{ij} F_j + \epsilon_i,$$

where C_{ij} and F_j are the jth components of the vectors C_i and F, respectively.

Following Mondria (2010), we can normalize the signal weight on the first risky asset to one by setting $\overline{C}_i = C_i / C_{i1}$ if $C_{i1} \neq 0$. Then the new signal structure is $\left(\overline{C}_i, \overline{\Sigma}_{\epsilon i}\right)$, where the noise variance is given by $\overline{\Sigma}_{\epsilon i} = \Sigma_{\epsilon i} C_{i1}^2$. Alternatively, we can normalize $\Sigma_{\epsilon i}$ to 1 and let $C_{i1} > 0$.

If $G^{\frac{1}{2}}\Omega G^{\frac{1}{2}}$ has $m \geq 2$ identical largest eigenvalues, then the optimal information structure is not unique even after normalization. The optimal signal is still one dimensional because the dimension is determined by the rank of $v^*v^{*\prime}$ as shown in (28). Each investor acquires a signal that is a linear combination of the risky assets, but the normalized signal may not be identical for all investors. We will show next that this is the source of the existence of an asymmetric equilibrium.

3.3 Algorithm to Compute Equilibrium under Rational Inattention

The existence of an equilibrium under rational inattention depends on the existence of an optimal information structure $(C_i, \Sigma_{\epsilon i})$ for all i that satisfies equation (28). This is a fixed point problem. We are unable to prove the existence for the general case.[7] Here we provide an algorithm to solve for an equilibrium. Specifically, let $\Phi_i \equiv C_i' \Sigma_{\epsilon i}^{-1} C_i$ denote the SNR for all $i \in [0,1]$. The algorithm consists of the following steps:

Step 1. Given a guess (an $n \times n$ matrix) for Φ_i for each i, we can determine Π in equation (11).

Step 2. Solve for A_0, A_1, and A_2 in equations (8), (9), and (10).

Step 3. Solve for $\overline{R}^e$, V_e, Ω, and G using equations (14), (15), (17), and (21).

Step 4. Compute the eigen-decomposition (24) and derive U and Ω_d.

Step 5. Derive $\widetilde{K}_i$ and K_i, and use (28) to determine an update of Φ_i for all i, denoted by Φ_i^N.

Step 6. Iterate the above steps until convergence. Specifically, if the relative difference between Φ_i and Φ_i^N under the matrix Frobenius norm is less than 10^{-10}, then stop; otherwise replace Φ_i by $(1-a)\,\Phi_i + a\Phi_i^N$ for some $a \in (0,1]$ and go to step 1.

As discussed in the previous subsection, if $G^{\frac{1}{2}}\Omega G^{\frac{1}{2}}$ has a unique largest eigenvalue, then all investors choose the same information structure up to normalization. In this case, if an equilibrium exists, then it must be symmetric. If $G^{\frac{1}{2}}\Omega G^{\frac{1}{2}}$ has multiple largest eigenvalues, then different investors may choose different normalized information structures. Thus an asymmetric equilibrium may arise. We will focus on linear symmetric equilibrium in which $\Phi_i = \Phi$, $C_i = C$, and $\Sigma_{\epsilon i} = \Sigma_\epsilon$ for all $i \in [0,1]$. Once the SNR Φ is computed, we can use the singular value decomposition to determine C and Σ_ϵ. For all our numerical examples in the next section, we find that there exists a unique largest eigenvalue. As an accuracy check of our solution method, we find that our algorithm delivers almost the same numerical solutions as those in Mondria (2010), which provides a closed-form solution for the two-asset case. While it might be interesting to study asymmetric equilibria, solving for such equilibria analytically or numerically for the general multiple-asset case is technically challenging and is beyond the scope of this paper. We leave such an analysis for future research.

4 Properties of Equilibrium

In this section we analyze properties of the equilibrium under rational inattention if it exists. We will focus on the unique linear symmetric equilibrium only. Because a formal existence proof is unavailable due to the complexity of our model, we use the algorithm in Section 3.3 to find equilibria for all numerical examples in this section. Because all numerical solutions satisfy the convergence criterion of Step 6, we have verified the existence of equilibrium numerically.

[7]See Mondria (2010) for a proof for the case with two risky assets.

4.1 Asset Returns

Since the random supply vector Z and the market portfolio are not observable, the CAPM is difficult to test empirically. We thus derive an unconditional CAPM, similar to ACW (2020). As is well known, it is analytically more convenient to work with the dollar return instead of the rate of return in the CARA-normal framework because the rate of returns is not Gaussian. Recall that $R^e = F - R_f P$ is the vector of excess dollar returns on the n risky assets. Define $\overline{R}_m^e = \overline{Z}' R^e$ as the excess dollar return on the average market portfolio. We then obtain the following result.

Proposition 3 *In the linear symmetric equilibrium the excess (dollar) return and the average dollar market return satisfy the following unconditional CAPM:*

$$\mathbb{E}\left[R^e\right] = \beta_m \mathbb{E}\left[\bar{R}_m^e\right], \tag{30}$$

where

$$\beta_m = \frac{\mathrm{Cov}(R^e, \bar{R}_m^e | S_i, P)}{\mathrm{Var}(\bar{R}_m^e | S_i, P)} = \frac{\mathrm{Var}\left(R^e | S_i, P\right) \overline{Z}}{\overline{Z}' \mathrm{Var}\left(R^e | S_i, P\right) \overline{Z}}.$$

Notice that in the linear symmetric equilibrium the above expression for the vector β_m is independent of investor i. Computing β_m only requires to know the conditional variance of excess returns $\mathrm{Var}\left(R^e | S_i, P\right)$ for the average investor. If we plot $\mathbb{E}\left[R_j^e\right]$ against β_{mj} for different asset j, we obtain the security market line (SML). The slope of this line is the market risk premium $\mathbb{E}\left[\bar{R}_m^e\right]$. The market beta is equal to 1. From an econometrician's point of view, beta is computed as the linear regression coefficient of the realized excess return R^e on the average market return $\bar{R}_m^e$:

$$\widetilde{\beta}_m = \frac{\mathrm{Cov}\left(R^e, \bar{R}_m^e\right)}{\mathrm{Var}\left(\bar{R}_m^e\right)}.$$

This vector of betas is different from the true vector of betas β_m from the investors' point of view in the model. As ACW (2020) point out, there is an information distance between econometricians and investors because the unconditional covariance matrix of excess returns satisfies

$$\mathrm{Var}\left(R^e\right) = \mathrm{Var}\left[\mathbb{E}\left(R^e | S_i, P\right)\right] + \mathbb{E}\left[\mathrm{Var}\left(R^e | S_i, P\right)\right].$$

Investors' betas are computed based on the unexplained component $\mathbb{E}\left[\mathrm{Var}\left(R^e | S_i, P\right)\right] = \mathrm{Var}\left(R^e | S_i, P\right)$.

ACW (2020) show that their model delivers a linear relation between $\mathbb{E}\left[R_j^e\right]$ and $\widetilde{\beta}_{mj}$ but the slope is flatter than that of the true SML. The perceived SML rotates clockwise around the market portfolio, which flattens its slope and creates a positive intercept. We find that their result does not hold in our model. The main reason is that they assume that there is only one unobservable common risk factor in asset payoffs and investors receive signals about the common factor only. By contrast, we assume that investors receive signals about the unobservable asset payoffs which may contain several risk factors and asset specific idiosyncratic risks. Moreover, the information structure is endogenous in our model.

Now we use some numerical examples to study the impact of information choice on asset returns. Consider a factor specification of asset payoffs as in KVV (2016). There are five risky assets in the market, with asset payoffs given by $F = \Gamma Y$, where Y represents the risk factors and Γ represents the risk loadings. Let

$$\Gamma = \begin{bmatrix} 1 & 0 & 0 & 0 & 0.1 \\ 0 & 1 & 0 & 0 & 0.2 \\ 0 & 0 & 1 & 0 & 0.3 \\ 0 & 0 & 0 & 1 & 0.4 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

As in KVV (2016), we can interpret the last component Y_5 of Y as the aggregate risk factor because all of the risky assets are exposed to Y_5. The component Y_j represents the idiosyncratic risk for asset $j = 1, 2, 3, 4$. Since Γ is invertible, KVV (2016) construct from the original assets a set of synthetic assets whose payoffs are given by $Y = \Gamma^{-1} F$. The resulting payoff covariance matrix is diagonal. They also assume that the supply of each synthetic assets is independent and then work on the space of synthetic assets. Our solution method directly works with the original assets.

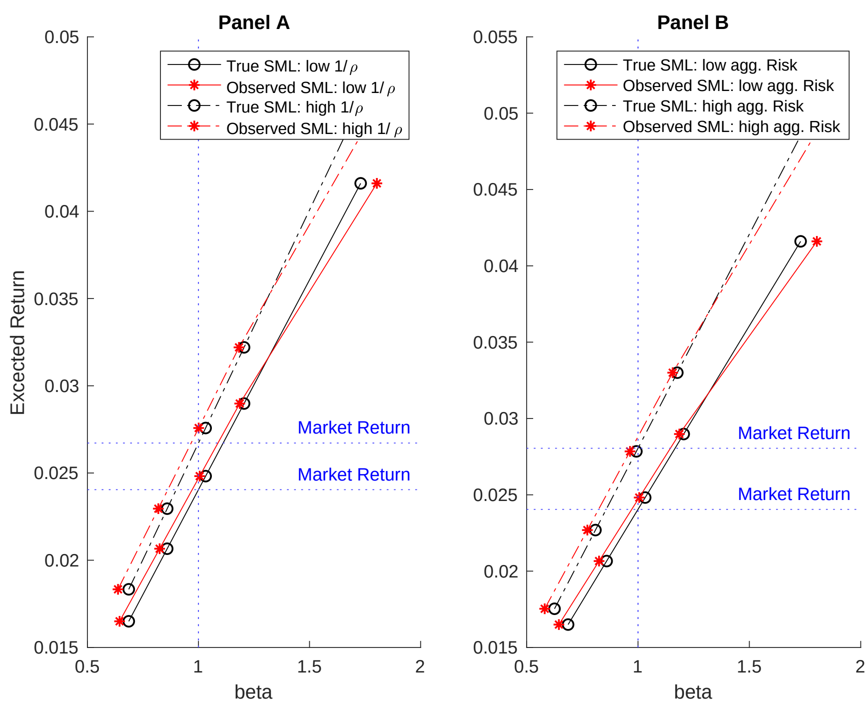

Figure 1: CAPM distortions

As baseline values, set the risk tolerance parameter as $\rho = 0.25$, and the information capacity parameter as $\kappa = 0.1$. The payoffs of synthetic assets are Gaussian with mean $[1, 1, 1, 1, 1]'$ and covariance matrix $diag(0.15^2, 0.15^2, 0.15^2, 0.15^2, 0.18^2)$. The supply of synthetic assets (denoted by Z_Y) is Gaussian with mean $[0.15, 0.15, 0.15, 0.15, 0.4]'$ and covariance matrix $diag(0.1^2, 0.1^2, 0.1^2, 0.1^2, 0.5^2)$. Following KVV (2016), we deduce that the supply of the original risky assets is given by $Z = \Gamma Z_Y$.

Panel A of Figure 1 shows that the observed SML is flatter than the true SML, a result that is discussed in ACW (2020). If we raise the degree of risk aversion from $1/\rho = 1/0.25$ to $1/\rho = 1/0.225$, both of the true SML and the observed SML shift up. This is because investors will demand higher excess returns if they are more risk averse. Panel B of Figure 1 shows the impact when the aggregate volatility is raised from 0.18 to 0.20. In this case, the market risk premium is higher and both the true SML and the observed SML shift up. This figure also shows that the observed SML may not be linear.

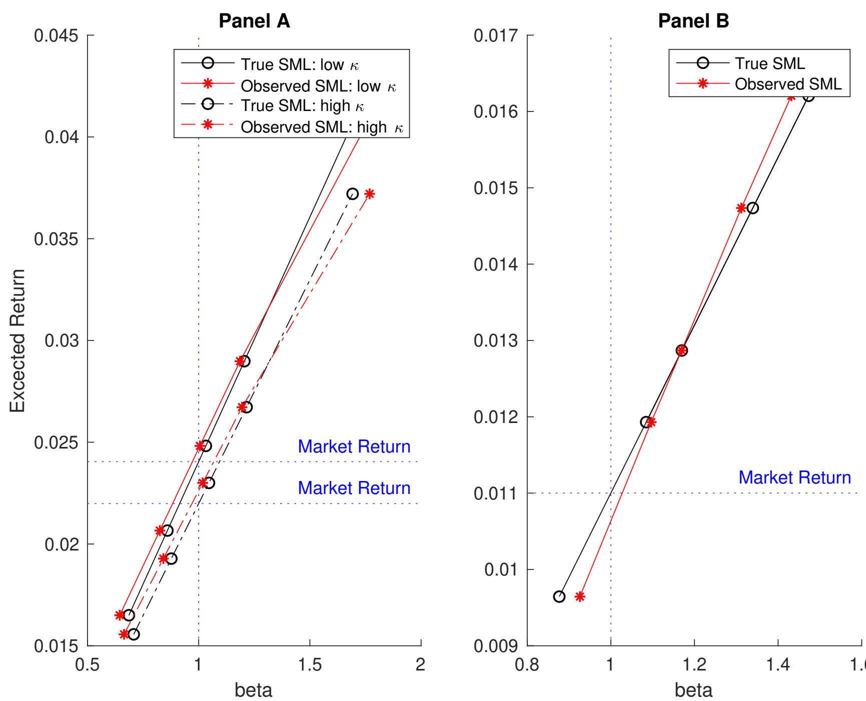

Figure 2: CAPM distortions

Figure 2 shows the impact of information capacity on asset returns. We find that an increase in information capacity κ causes both the true SML and the observed SML to shift down. Intuitively, a higher information capacity helps investors to reduce uncertainty through learning. Therefore, investors demand lower risk premium on risky assets, leading to lower SMLs. Panel B of Figure 2 shows that the observed SML can be steeper than the true SML, unlike the result in ACW(2020). The parameter values of this panel are the same as the baseline values except that we set the aggregate volatility as 0.10 and the covariance matrix of Z_Y as $diag(0.4^2, 0.3^2, 0.2^2, 0.1^2, 0.5^2)$.

Our model differs from ACW (2020) in three dimensions: (i) our model allows for multiple common risk factors while ACW only has one; (ii) agents in our model learn about asset payoffs F as in (1), instead of the common risk factor as in ACW (2020); and (iii) information structure is endogenously chosen in our model. We find that the second dimension is critical to generate the steeper observed SML in Panel B of Figure 2.[8] As shown in Section 3.1, the equilibrium price with fixed information structure given in (1) depends crucially on the SNR, which determines the information content of the signal. We can construct many numerical examples with particular SNRs to generate steeper SMLs.

For example, we suppose that all assets are ex ante identical in that $F = Y$, where each Y_j is an independently and identically distributed normal asset-specific risk factor across assets. The supply Z_j of asset j is also independently and identically distributed across assets. Assume that each investor i observes a one-dimensional signal $S_i = \sum_{j=1}^{n} C_j F_j + \epsilon_i$, where $0 < C_1 < C_2 < \cdots < C_n$ and ϵ_i is an independent noise with a normal distribution $N(0,1)$. Then we can construct numerical examples with steeper observed SMLs. This result is available upon request. Intuitively, asset n with the largest C_n has the lowest equilibrium excess return as investors can learn most information from this asset and the information distance between investors and econometricians for this asset is the largest. Asset n's beta estimated by econometricians is larger than the true beta. The opposite holds for asset 1 with the smallest C_1. Thus the observed SML is steeper than the true SML.

Notice that endogenous information structure is not critical for this result. If we allow investors to acquire information endogenously, then Proposition 2 shows that each investor will acquire a one-dimensional signal. Given the symmetric setup, we can verify numerically that the optimal signal takes the normalized form $S_i = \sum_{i=1}^{n} Y_i + \varepsilon_i$, where ε_i is normal with identical variance for each investor i. Then by symmetry all assets have the same beta and the same return so that the SML collapses to a single point.

4.2 Comovement and Contagion

Since investors optimally acquire a noisy linear combination of asset payoffs as their private signals, investors are unable to distinguish among various sources of asset payoff shocks when processing private signals. This generates potential asset price comovement and provides a new channel of volatility transmission (Mondria 2010). In this section we use some numerical examples to illustrate the role of endogenous information choice in generating asset price comovement and financial contagion.

By (7), we can derive the unconditional expectation of asset prices

$$\mathbb{E}[P] = A_0 + A_1 \overline{F} - A_2 \overline{Z}. \tag{31}$$

[8] ACW (2020) show that it is also possible to generate a steeper SML when there are many common risk factors and when all agents learn about these factors instead of asset payoffs.

and the unconditional covariance matrix of asset prices

$$\mathrm{Var}\,(P) = A_1 \Sigma_F A_1' + A_2 \Sigma_Z A_2'. \tag{32}$$

Following Mondria (2010), we use (32) to derive the unconditional correlation of asset prices to characterize asset price comovement.

Consider the following three-asset example with the covariance matrix of asset payoffs given by

$$\Sigma_F = \begin{bmatrix} 0.15^2 & 0 & 0 \\ 0 & 0.15^2 & 0.15^2\phi \\ 0 & 0.15^2\phi & 0.15^2 \end{bmatrix},$$

where ϕ denotes the payoff correlation between assets 2 and 3, and asset 1's payoff is independent of these two assets. Other parameter values are set as $\kappa = 0.1$, $\rho = 0.25$, $\overline{F} = [1, 1, 1]'$, $\overline{Z} = [1/3, 1/3, 1/3]'$, and $\Sigma_Z = diag(0.10^2, 0.10^2, 0.10^2)$. We consider two cases with $\phi = 0.75$ and $\phi = -0.75$.

As shown in Section 3.2, investors choose the same information structure in the unique linear symmetric equilibrium. The optimal signal structure is a noisy linear combination of the three assets with the coefficient vector denoted by $C = [C_1, C_2, C_3]$. We normalize the noise variance Σ_ϵ to 1 and let $C_1 > 0$. Since assets 2 and 3 are symmetric ex-ante, they have the same equilibrium properties. More specifically, investors will allocate the same attention between these two assets, i.e. $C_2 = C_3$. Furthermore, assets 2 and 3 have the same unconditional expected prices, $\mathbb{E}[P_2] = \mathbb{E}[P_3]$, and the same correlation with asset 1, $\mathrm{Corr}(P_1, P_2) = \mathrm{Corr}(P_1, P_3)$.

We study the impact of asset 1's payoff volatility σ_{F1} on the attention allocation C_1 and C_2, the prices of assets 2 and 3, and the correlations of asset prices. We also study the comparative statics with respect to ρ and κ.

Figure 3 shows that prices of assets 1 and 2 (or 3) are positively correlated when asset 1 is independent of assets 2 and 3. Mondria (2010) finds this result for the two-asset case. We also find that prices of assets 2 and 3 are positively correlated even when their payoffs are negatively correlated, provided that payoff uncertainty is not too high.[9] To see the intuition, we first consider the attention allocation described by the endogenous signal structure. We find that investors acquire a one-dimensional signal that is a noisy linear combination of the three assets with positive coefficients $C_1 > 0$, $C_2 > 0$, and $C_3 = C_2 > 0$. The top panel of Figure 3 presents C_1 and $C_2 = C_3$ for different values of σ_{F_1}. Positive coefficients imply that, conditional on the noisy asset supply shock Z, a high realization of the signal could be attributed to a high payoff realization for any of the three assets, leading to comovement of prices of these three assets. Then we can obtain the conditional comovement result as in Proposition 4 of Mondria (2010).

[9] If payoff uncertainty is too high, the private signal will be less reliable. Therefore, the learning-based contagion effect is weakened and dominated by the negative correlation of asset payoffs.

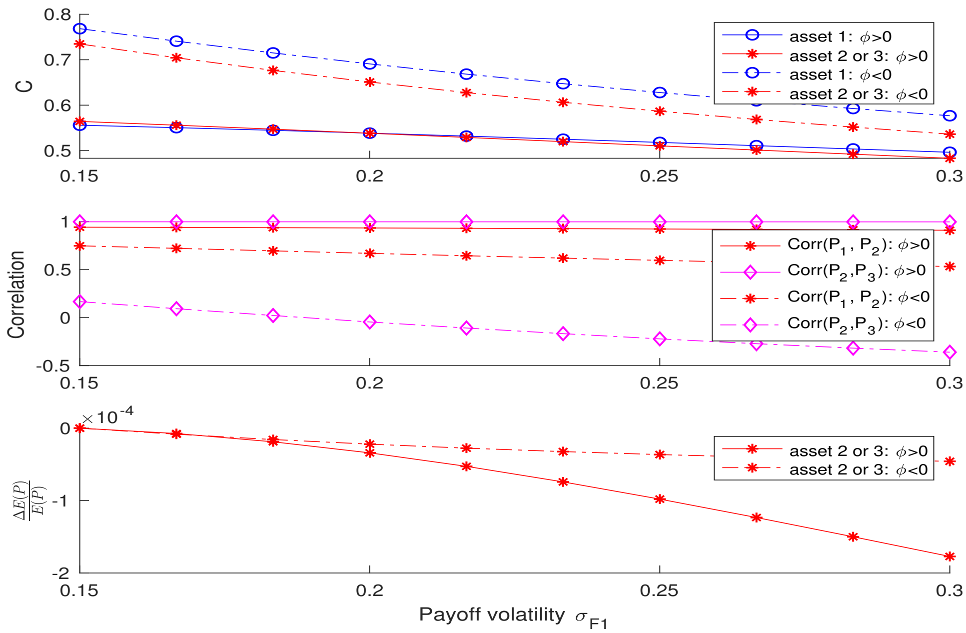

Figure 3: Correlation and contagion: transmission of payoff shocks. The top panel plots C_1 and $C_2 = C_3$ against σ_{F_1}. The middle plots the asset price correlation against σ_{F_1}. The bottom panel plots expected price changes against σ_{F_1}.

Since a positive supply shock Z decreases the asset price, it dampens the comovement effect unconditionally. Through extensive numerical examples available upon request, we find that if the asset supply effect is weak enough (i.e., asset supply variances are small enough) and the risk tolerance ρ is high enough, we will obtain the unconditional comovement result. Proposition 5 of Mondria (2010) gives an explicit condition for the two-asset case. Given the complexity of our multiple-asset model, we are unable to provide an analogue explicit condition. Notice that if all assets contain a common aggregate risk factor, then we can easily generate comovement and the comovement is stronger if aggregate risk is higher during recessions.

The comovement is illustrated in the middle panel of Figure 3, which also shows that as asset 1's payoff volatility increases, the correlations become smaller. This is essentially because higher payoff uncertainty tightens the information-processing constraint (2) or (19). Due to a smaller information capacity, private signals become less precise.

We also find that there exist transmissions of payoff risk. More specifically, when the payoff risk of asset 1 increases, not only the prices of asset 1 but also those of assets 2 and 3 decrease as illustrated in the bottom panel of Figure 3. This is due to the reallocation of investors' attention. Intuitively, investors pay more attention to asset 1 relative to the other two assets when asset 1's payoff volatility is relatively larger. As a result, the payoff risks of assets 2 and 3 are perceived to be higher. Higher posterior risks make assets 2 and 3 less desirable and reduce their prices. This result is analogous to Propositions 7 and 8 of Mondria (2010) for the two-asset case.

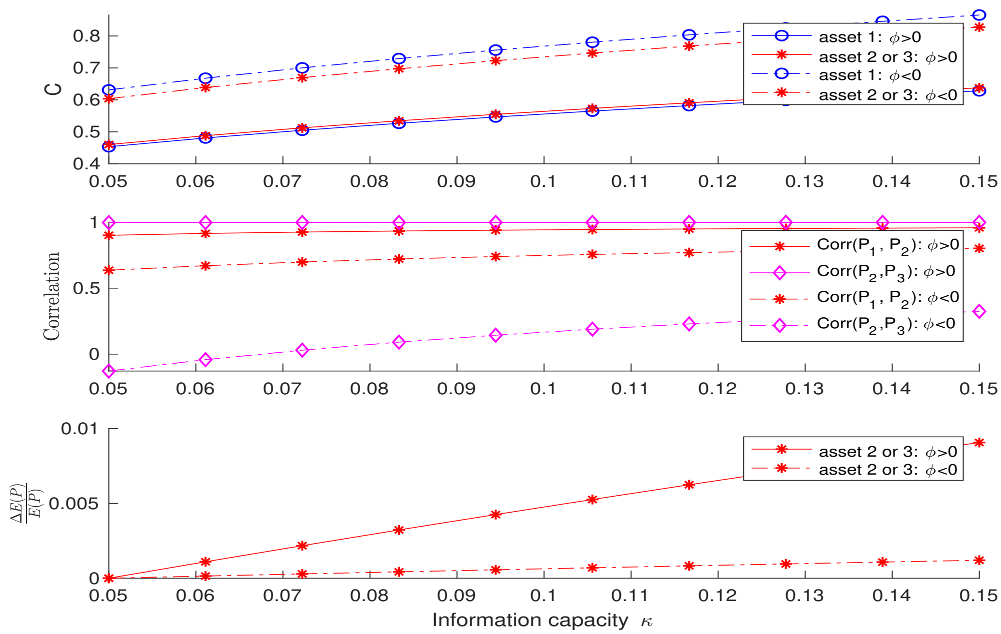

Figure 4: Correlation and contagion: information capacity κ. The top panel plots C_1 and $C_2 = C_3$ against κ. The middle plots the asset price correlation against κ. The bottom panel plots expected price changes against κ.

Figure 4 shows the impact of the information capacity κ. We find that asset price correlations increase with the information capacity κ. As κ increases, private signals become more precise and eventually become the dominating force, which leads to positive price correlations, even when the asset payoffs are negatively correlated. Moreover, as investors can process more information, the posterior variances of asset prices become smaller, leading to higher expected asset prices.

Figure 5 shows the impact of risk aversion. We find that as investors are more risk averse, asset price correlations are smaller. Intuitively, more risk averse investors will be less responsive to their private signals when making portfolio choice decisions. This makes asset prices less informative about future payoffs. Thus the price correlations become weaker. Moreover, since risk premium increases with risk aversion, expected asset prices decreases with risk aversion. We also find that asset price correlations are more sensitive to risk aversion if asset payoffs are negatively correlated. This is because when payoffs are ex-ante negatively correlated, the asset payoff correlations and the private signals work towards different directions. With higher risk aversion, private signals are less important and so asset prices decrease faster due to the negative payoff correlation.

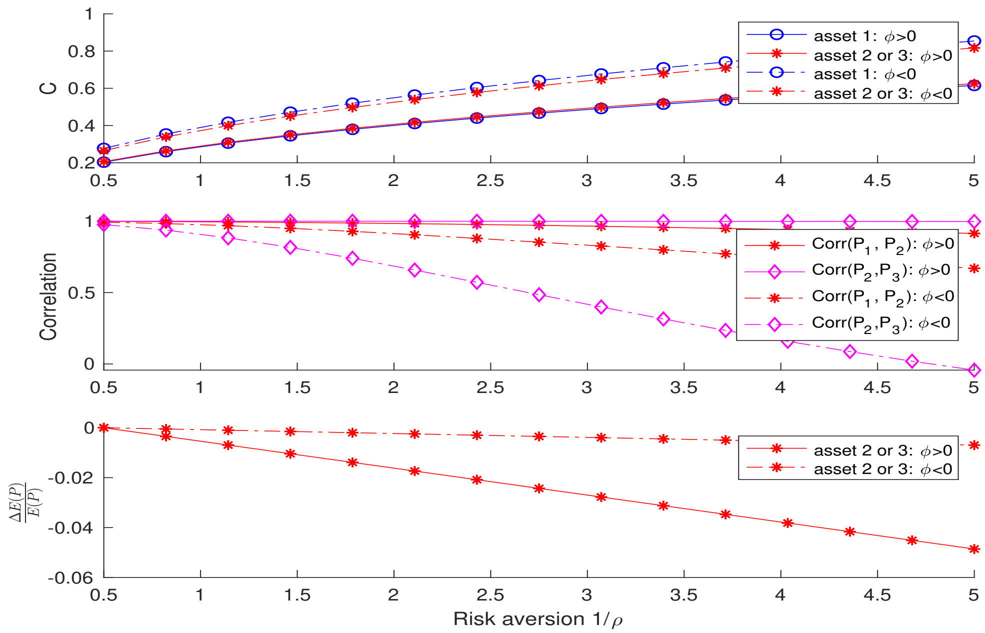

Figure 5: Correlation and contagion: risk aversion. The top panel plots C_1 and $C_2 = C_3$ against $1/\rho$. The middle plots the asset price correlation against $1/\rho$. The bottom panel plots expected price changes against $1/\rho$.

4.3 Portfolio Dispersion

In this section we study our model implications for the dispersion across investor portfolio holdings and portfolio excess returns. As KVV (2016) show, both portfolio dispersions would fall if investment strategies were passive during recessions. However, when investors endogenously process information and actively trade based on that information, the prediction may be different.

We first use equation (12) to derive the equilibrium portfolio strategies.

Proposition 4 *In the symmetric linear equilibrium investor i's holdings of risky assets are given by*

$$\begin{aligned} X_i &= \rho \left[\text{Var}\left(R^e|S_i, P\right)\right]^{-1} \left[\mathbb{E}\left(F|S_i, P\right) - \mathbb{E}^m(F)\right] + Z \\ &= \rho C' \Sigma_\epsilon^{-1} \epsilon_i + Z, \end{aligned} \tag{33}$$

where $\mathbb{E}^m(F) = \int_0^1 \mathbb{E}[F|S_i, P]di$ *denotes the market average expectation of the payoff vector.*

As in Biais, Bossaerts, and Spatt (2010), there is a winner's curse problem for the equilibrium portfolios: investor i invests more than the market portfolio Z in asset j when his/her expectation about the payoffs $\mathbb{E}\left(F|S_i, P\right)$ is greater than the average expectation $\mathbb{E}^m(F)$, while he invests less otherwise. Moreover, investor i's equilibrium portfolio responds to his/her idiosyncratic signal noise, conditional on the noisy supply Z.

Equation (33) holds for any fixed information structure (C, Σ_ϵ). Under rational inattention, information structure is endogenous in the sense that both C and Σ_ϵ are endogenously chosen. In particular, we have shown that the private signal is one dimensional so that ϵ_i is a scalar noise. In response to the signal noise, investor i will adjust holdings of all assets because he may believe the noise comes from the payoff shock to any asset in his/her portfolio. By contrast, if investors acquire a separate signal for each asset, i.e., C is an $n-$dimensional identity matrix, then they would adjust only one particular asset holdings in response to the signal noise on that asset.

To understand the aggregate implications, we follow KVV (2016) to define the dispersion of portfolio holdings as

$$\int_0^1 \mathbb{E}\left[(X_i - Z)'(X_i - Z)\right] di, \tag{34}$$

and define the dispersion of the portfolio excess return as

$$\int_0^1 \mathbb{E}\left[\left((X_i - Z)' R^e\right)^2\right] di. \tag{35}$$

Then we have the following result:

Proposition 5 *In the symmetric linear equilibrium, the dispersion of portfolio holdings is given by*

$$\int_0^1 \mathbb{E}\left[(X_i - Z)'(X_i - Z)\right] di = \rho^2 \mathrm{Tr}(C'\Sigma_\epsilon^{-1}C),$$

and the dispersion of portfolio returns is given by

$$\int_0^1 \mathbb{E}\left[\left((X_i - Z)' R^e\right)^2\right] di = \rho^2 \mathrm{Tr}\left(C'\Sigma_\epsilon^{-1}C\mathbb{E}[R^e R^{e\prime}]\right) = \rho \mathrm{Tr}\left(\Pi\Omega\right).$$

This proposition shows that the SNR $C'\Sigma_\epsilon^{-1}C$ shows up in both dispersion measures. The formulas above apply to any fixed information structure. When the information structure is chosen endogenously, changes in the SNR affect both dispersions. Since we are unable to derive analytical comparative statics results, we use numerical examples to illustrate the impact of information capacity κ, risk aversion $1/\rho$, and aggregate risk on portfolio dispersion. We still adopt the five-asset specification as in Section 4.1.

Panels A and B of Figure 6 show that a higher information capacity raises both the portfolio holdings dispersion and the portfolio return dispersion. Intuitively, a higher information capacity allows investors to process more precise private signals. Thus investors' portfolio holdings will be more responsive to private signals, leading to a rise in the dispersion of portfolio holdings, and hence in the dispersion of portfolio excess returns.

Panels C and D of Figure 6 show that both the portfolio holdings dispersion and the portfolio return dispersion decrease with risk aversion. Intuitively, if investors are more risk averse, then

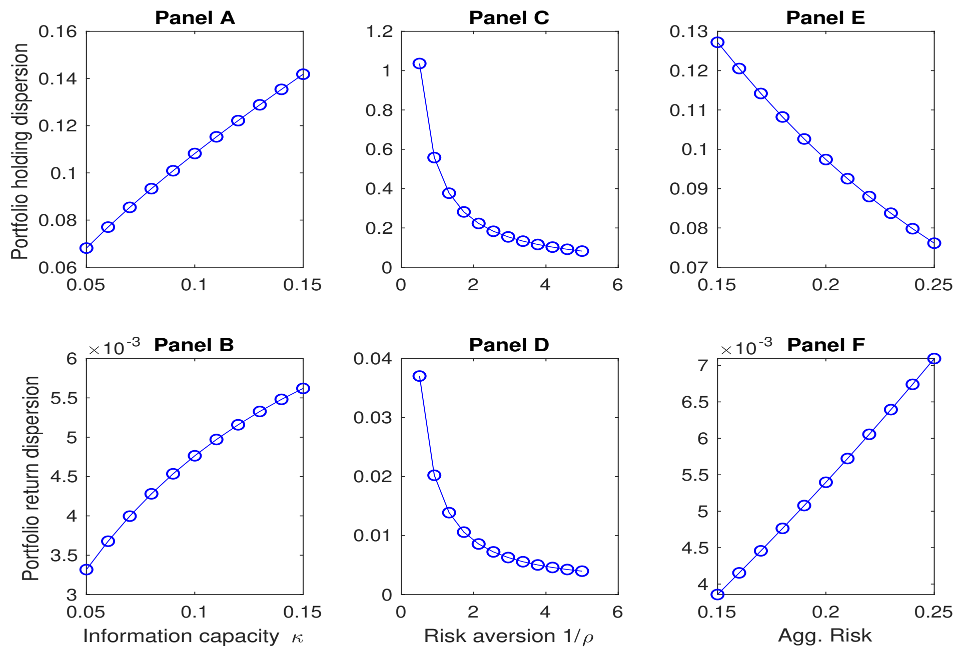

Figure 6: The impact of information capacity κ, risk aversion $1/\rho$, and aggregate risk on the portfolio dispersion.

their portfolio choices will be less responsive to their private signals. The common prior beliefs and the public price signal will drive investors to make similar portfolio choices. Hence there will be less heterogeneity in portfolio holdings. The portfolio return dispersion depends on both portfolio holdings and risk premium. Higher risk aversion raises risk premium. We find that the effect of a smaller portfolio holdings dispersion dominates the effect of higher risk premium, causing the dispersion of portfolio returns to decrease with risk aversion.

This result is qualitatively different from Proposition 4 in KVV (2016), which shows that under appropriate conditions, higher risk aversion leads to a higher dispersion of portfolio returns. In the KVV model, the impact of risk aversion on the portfolio return dispersion is mainly driven by the changes in risk premium. By contrast, our numerical examples show that this effect is dominated when there is no uninformed investor.

Finally we study the impact of aggregate risk on portfolio dispersions. All other things being equal, we increase the aggregate volatility from 0.15 to 0.25. Panels E and F of Figure 6 show that with higher aggregate risk, the dispersion of portfolio holdings is smaller but the dispersion of portfolio excess returns is larger.

To understand this result, we notice that with higher aggregate risk the information-processing constraint is effectively tighter. Investors have to reduce the precision of private signals to respect the information-processing constraint. Investors will then be less responsive to their private signals.

Therefore there will be smaller dispersion of portfolio holdings. Moreover, with higher aggregate risk, investors will demand higher risk premium. The impact of higher risk premium dominates the impact of lower portfolio holdings dispersion, causing the dispersion of portfolio returns to increase.

This result is in contrast to Proposition 3 in KVV (2016), which shows that under suitable conditions, an increase in the payoff uncertainty for any risk factors will weakly increase both the portfolio holdings dispersion and the portfolio return dispersion. Their model differs from ours in several ways, which are discussed further in our working paper version.

5 Conclusion

We have analyzed a noisy rational expectations equilibrium model with rationally inattentive investors. We have solved the difficult problem with any finite number of assets with arbitrary correlation by relaxing the signal independence assumption. Our solution approach is useful to analyze other finance models with multiple assets. We have also derived some testable predictions that are different from the existing literature. It would be interesting to test these predictions for future research.

A Appendix: Proofs

Proof of Proposition 1: The proof essentially follows from Mondria (2010). For completeness, we sketch the key steps. First, by the projection theorem, we can compute

$$V_i = \mathrm{Var}\left(R^e|S_i,P\right) = \mathrm{Var}\left(F|S_i,P\right) = \left(\Sigma_F^{-1} + \Pi\Sigma_Z^{-1}\Pi + C_i'\Sigma_{\epsilon i}^{-1}C_i\right)^{-1}.$$

Taking expectations on the two sides of the market-clearing condition yields

$$\overline{Z} = \mathbb{E}\left[\int X_i di\right] = \left(\rho\int V_i^{-1}di\right)\mathbb{E}\left\{\mathbb{E}\left[R^e|S_i,P\right]\right\} = \left(\rho\Sigma_F^{-1} + \rho\Pi\Sigma_Z^{-1}\Pi + \Pi\right)\overline{R}^e$$

where $\overline{R}^e \equiv \mathbb{E}\left[R^e\right]$. Thus we obtain

$$\overline{R}^e = \left(\rho\Sigma_F^{-1} + \rho\Pi\Sigma_Z^{-1}\Pi + \Pi\right)^{-1}\overline{Z}.$$

Using the budget constraint, we compute

$$\mathbb{E}\left[W_i|S_i,P\right] = W_{i0}R_f + X_i'\mathbb{E}\left[R^e|S_i,P\right] = W_{i0}R_f + \rho\mathbb{E}\left[R_e|S_i,P\right]'\left[\mathrm{Var}\left(R^e|S_i,P\right)\right]^{-1}\mathbb{E}\left[R^e|S_i,P\right],$$

where we have plugged in the optimal portfolio rule given in (12). Similarly, we compute

$$\mathrm{Var}\left[W_i|S_i,P\right] = X_i'\mathrm{Var}\left[R^e|S_i,P\right]X_i = \rho^2\mathbb{E}\left[R^e|S_i,P\right]'\left[\mathrm{Var}\left(R^e|S_i,P\right)\right]^{-1}\mathbb{E}\left[R^e|S_i,P\right].$$

Thus the initial utility at date zero is given by

$$\begin{aligned} U_i &= \frac{1}{\rho}\mathbb{E}\left(\mathbb{E}\left[W_i|S_i,P\right] - \frac{1}{2\rho}\mathrm{Var}\left[W_i|S_i,P\right]\right) \\ &= \frac{W_{i0}R_f}{\rho} + \frac{1}{2}\mathbb{E}\left\{\mathbb{E}\left[R^e|S_i,P\right]'\left[\mathrm{Var}\left(R^e|S_i,P\right)\right]^{-1}\mathbb{E}\left[R^e|S_i,P\right]\right\} \end{aligned}$$

Notice that $\mathbb{E}\left[R^e|S_i,P\right]$ is normal with mean $\overline{R}^e = \mathbb{E}\left[R^e\right]$ and variance $\mathrm{Var}\left(\mathbb{E}\left[R^e|S_i,P\right]\right)$. Notice that

$$\mathrm{Var}\left(\mathbb{E}\left[R^e|S_j,P\right]\right) = \mathrm{Var}\left(R^e\right) - \mathrm{Var}\left(R^e|S_i,P\right).$$

Moreover, if $x = (x_1,...,x_n)' \sim N(\mu,V)$ and $q = x'Ax$, then $\mathbb{E}[q] = \mathrm{Tr}(AV) + \mu'A\mu$. Using the preceding two formulas, we compute

$$\begin{aligned} & \mathbb{E}\left\{\mathbb{E}\left[R^e|S_i,P\right]'\left[\mathrm{Var}\left(R^e|S_i,P\right)\right]^{-1}\mathbb{E}\left[R^e|S_i,P\right]\right\} \\ =\ & \mathrm{Tr}\left(\left[\mathrm{Var}\left(R^e|S_i,P\right)\right]^{-1}\mathrm{Var}\left(\mathbb{E}\left[R^e|S_i,P\right]\right)\right) + \overline{R}^{e\prime}\left[\mathrm{Var}\left(R^e|S_i,P\right)\right]^{-1}\overline{R}^e \\ =\ & \mathrm{Tr}\left(\left[\mathrm{Var}\left(R^e|S_i,P\right)\right]^{-1}\mathrm{Var}\left(R^e\right) - I\right) + \overline{R}^{e\prime}\left[\mathrm{Var}\left(R^e|S_i,P\right)\right]^{-1}\overline{R}^e \\ =\ & \mathrm{Tr}\left(V_i^{-1}\left(V_e + \overline{R}^e\overline{R}^{e\prime}\right)\right) - n \end{aligned}$$

where we define $V_i \equiv \mathrm{Var}\left(R^e|S_i,P\right)$ and

$$V_e \equiv \mathrm{Var}\left(R^e\right) = \Sigma_F + R_f^2A_1\Sigma_FA_1' + R_f^2A_2\Sigma_ZA_2' - R_fA_1\Sigma_F - R_f\Sigma_FA_1'. \qquad \text{(A.1)}$$

We then obtain the utility value in the proposition. Q.E.D.

Proof of Proposition 2: Given the discussion in Section 3.2, we only need to study the following problem:

$$\max_{\widetilde{K}_i} \operatorname{Tr}\left(\widetilde{K}_i \Omega_d\right) \tag{A.2}$$

subject to $\widetilde{K}_i \succeq I$ and

$$\log\det\left(\Sigma_F\right) + \log\det\left(G\right) + \log\det\left(\widetilde{K}_i\right) \leq 2\kappa. \tag{A.3}$$

Since $\widetilde{K}_i = U'G^{-\frac{1}{2}}K_iG^{-\frac{1}{2}}U$, $\widetilde{K}_i$ is a real symmetric matrix. If $\widetilde{K}_i$ is not a diagonal matrix, we consider the eigen-decomposition $\widetilde{K}_i = Q'\Lambda Q$, where Q is an orthogonal matrix and Λ is a diagonal matrix whose diagonal elements are the eigenvalues of $\widetilde{K}_i$, denoted by $\lambda_1, \lambda_2, ...,$ and λ_n. Since

$$\operatorname{Tr}\left(\widetilde{K}_i \Omega_d\right) = \operatorname{Tr}\left(Q'\Lambda Q\Omega_d\right) = \operatorname{Tr}\left(\Lambda Q\Omega_d Q'\right),$$

$$\widetilde{K}_i \succeq I \Longleftrightarrow Q'\Lambda Q \succeq I \Longleftrightarrow \Lambda \succeq QQ' = I,$$

$$\log\det\left(\widetilde{K}_i\right) = \log\det\left(Q'\Lambda Q\right) = \log\det\left(\Lambda\right),$$

we can rewrite the problem above as

$$\max_{\Lambda, Q} \operatorname{Tr}\left(\Lambda Q\Omega_d Q'\right) \tag{A.4}$$

subject to $\Lambda \succeq I$ and

$$\log\det\left(\Sigma_F\right) + \log\det\left(G\right) + \log\det\left(\Lambda\right) \leq 2\kappa. \tag{A.5}$$

Let the diagonal elements of the matrix $Q\Omega_d Q'$ be $\omega_1, \omega_2, ...,$ and ω_n. Then problem (A.4) is equivalent to the following one:

$$\max_{\{\lambda_j\}_{j=1}^n, Q} \quad \sum_{i=1}^n \lambda_i \omega_i \tag{A.6}$$

subject to $\lambda_j \geq 1$, $j = 1, ..., n$,

$$\lambda_1 \lambda_2 \cdots \lambda_n \leq \frac{\exp\left(2\kappa\right)}{\det\left(\Sigma_F G\right)}.$$

Notice that the constraint set is not convex and is nonempty by condition (27). Thus the solution must be at the corner. Without loss of generality, suppose that $\omega_1, ..., \omega_\ell$ are the identical maximum among $\omega_1, \omega_2, ...,$ and ω_n. Then the solution to problem (A.6) for any given $\omega_1, \omega_2, ...,$ and ω_n is not unique and given by

$$\lambda_k^* = \lambda^* \equiv \frac{\exp\left(2\kappa\right)}{\det\left(\Sigma_F G\right)} > 1 \text{ for } 1 \leq k \leq \ell \text{ and } \lambda_j^* = 1 \text{ for } j \neq k. \tag{A.7}$$

If $\ell = 1$, the solution is unique with $\lambda_1^* = \lambda^*$ and $\lambda_j^* = 1$ for all $j > 1$.

Next we solve for Q. Let $\Lambda^* = diag\left(\lambda_j^*\right)_{j=1}^n$ and $Q' = \left(a_{sj}\right)_{n\times n}$. Since the jth diagonal element of the matrix $Q'\Lambda^* Q$ is $\sum_{s=1}^n \lambda_s^* a_{js}^2$, we can rewrite (A.6) or (A.4) as

$$\max_Q \operatorname{Tr}\left(Q'\Lambda^* Q\Omega_d\right) = \max_Q \; d_1 \sum_{s=1}^n \lambda_s^* a_{1s}^2 + d_2 \sum_{s=1}^n \lambda_s^* a_{2s}^2 + ... + d_n \sum_{s=1}^n \lambda_s^* a_{ns}^2, \tag{A.8}$$

where $d_1, d_2, ...,$ and d_n are the diagonal elements of Ω_d. Since Q is an orthogonal matrix, we have $\sum_{s=1}^{n} a_{js}^2 = 1$ for any $j = 1, 2..., n$. Thus it follows from (A.7) that

$$\sum_{s=1}^{n} \lambda_s^* a_{js}^2 = \lambda_k^* a_{jk}^2 + \left(1 - a_{jk}^2\right).$$

Then we can derive that

$$\begin{aligned} \max_Q \operatorname{Tr}\left(Q'\Lambda^* Q\Omega_d\right) &= \max_Q \; d_1 \; \left(\lambda_k^* a_{1k}^2 + 1 - a_{1k}^2\right) \\ &\quad + d_2 \left(\lambda_k^* a_{2k}^2 + 1 - a_{2k}^2\right) + ... + d_n \left(\lambda_k^* a_{nk}^2 + 1 - a_{nk}^2\right). \end{aligned} \tag{A.9}$$

Since Q is an orthogonal matrix, we have

$$\sum_{j=1}^{n} a_{jk}^2 = 1. \tag{A.10}$$

Without loss of generality, let $d_1, ..., d_m$ be the identical largest eigenvalues of Ω_d. Then problem (A.9) becomes

$$\begin{aligned} \max_Q \operatorname{Tr}\left(Q'\Lambda^* Q\Omega_d\right) &= \max_{a_{jk}} \; d_1 \left[\lambda_k^* \left(a_{1k}^2 + ... + a_{mk}^2\right) + m - \left(a_{1k}^2 + ... + a_{mk}^2\right)\right] \\ &\quad + d_{m+1} \left(\lambda_k^* a_{m+1,k}^2 + 1 - a_{m+1,k}^2\right) + ... + d_n \left(\lambda_k^* a_{nk}^2 + 1 - a_{nk}^2\right) \end{aligned} \tag{A.11}$$

subject to (A.10). Since $\lambda_k^* > 1$, the solution to the above problem is given by $a_{1k}^2 + ... + a_{mk}^2 = 1$ and thus $a_{m+1,k}^2 = ... = a_{nk}^2 = 0$. There is no restriction on the other elements of Q except that Q must be an orthogonal matrix. Thus

$$d_1 \lambda_k^* + (m-1)\, d_1 + d_{m+1} + ... + d_n = \max_Q \operatorname{Tr}\left(Q'\Lambda^* Q\Omega_d\right),$$

where Q is an orthogonal matrix.

Note that we can show that

$$\widetilde{K}_i = Q'\Lambda^* Q = Q'Q + Q'\left(\Lambda^* - I\right) Q = I + \left(\lambda_k^* - 1\right) v_k v_k',$$

where $Q = [v_1, ..., v_n]'$ with all $v_j = [a_{1j}, a_{2j}, ..., a_{nj}]'$ being column vectors. Let $v^* = v_k$. We then obtain the optimal signal structure stated in the proposition. Moreover, the dimension of the optimal signal is determined by the rank of $\widetilde{K}_i - I$ or $v^* v^{*\prime}$ by (23) or (28), which is equal to 1.

If $m = 1$, we have $a_{1k} = 1$ and $a_{jk} = 0$ for all $j \geq 1$. Then we have $\widetilde{K}_i = Q'\Lambda^* Q = diag\left(\lambda_k^*, 1, ..., 1\right)$, where λ_k^* is given by (A.7). The solution for $\widetilde{K}_i$ is unique.

If $m \geq 2$, the solution for $\widetilde{K}_i$ is not unique. For example, let $Q' = \left(a_{sj}\right)_{n\times n}$ be an elementary matrix where row 1 and row k are switched where $1 \leq k \leq \ell$. Then $Q\Omega_d Q'$ is the same as Ω_d except that the elements d_k and d_1 are switched. But the largest elements of $Q\Omega_d Q'$ and Ω_d are the same so that $\ell = m \geq 2$. We have $\widetilde{K}_i = Q'\Lambda^* Q$, which is the same as the diagonal matrix Λ^* except that the elements λ_k^* and λ_1^* are switched. Notice that a non-diagonal solution $\widetilde{K}_i = I + \left(\lambda_k^* - 1\right) v_k v_k'$ is also possible. Q.E.D.

Proof of Proposition 3: From the first-order condition for investor i's optimization problem, we have

$$\mathbb{E}\left[u'\left(W_i\right) R_j^e | S_i, P\right] = 0,$$

where

$$u\left(W\right) = -\exp\left(-\frac{W}{\rho}\right).$$

Using the covariance decomposition yields

$$\mathbb{E}\left[u'\left(W_i\right) | S_i, P\right] \mathbb{E}\left[R_j^e | S_i, P\right] = -\text{Cov}\left(u'\left(W_i\right), R_j^e | S_i, P\right).$$

By Stein's Lemma, we have

$$\mathbb{E}\left[u'\left(W_i\right) | S_i, P\right] \mathbb{E}\left[R_j^e | S_i, P\right] = -\mathbb{E}\left[u''\left(W_i\right) | S_i, P\right] \text{Cov}\left(W_i, R_j^e | S_i, P\right).$$

By the specification of the CARA utility, we have

$$\frac{-\mathbb{E}\left[u''\left(W_i\right) | S_i, P\right]}{\mathbb{E}\left[u'\left(W_i\right) | S_i, P\right]} = \frac{1}{\rho}.$$

Thus we obtain

$$\mathbb{E}\left[R_j^e | S_i, P\right] = \frac{1}{\rho} \text{Cov}\left(W_i, R_j^e | S_i, P\right).$$

By the budget constraint (5), we have

$$\mathbb{E}\left[R_j^e | S_i, P\right] = \frac{1}{\rho} X_i' \text{Cov}\left(F, R_j^e | S_i, P\right),$$

where we have taken X_i' out of the conditional covariance operator as X_i is measurable with respect the investor i's information set $\{S_i, P\}$. Integrating over i yields

$$\int \mathbb{E}\left[R_j^e | S_i, P\right] di = \frac{1}{\rho} \int X_i' \text{Cov}\left(F, R_j^e | S_i, P\right) di.$$

Notice that we have

$$\text{Var}(F | S_i, P) = (\Sigma_F^{-1} + \Pi \Sigma_Z^{-1} \Pi + C_i' \Sigma_{\epsilon i}^{-1} C_i)^{-1}.$$

In a symmetric equilibrium $C_i' \Sigma_{\epsilon i}^{-1} C_i$ is identical for all investors and thus $\text{Var}(F|S_i, P)$ and $\text{Cov}\left(F, R_j | S_i, P\right)$ are independent of investor i. It follows from the market-clearing condition that

$$\int \mathbb{E}\left[R_j^e | S_i, P\right] di = \frac{1}{\rho} \text{Cov}\left(F_j, F | S_i, P\right) Z. \tag{A.12}$$

Taking unconditional expectations on the two sides of equation (A.12) yields

$$\mathbb{E}\left[R_j^e\right] = \frac{1}{\rho} \text{Cov}\left(F_j, F | S_i, P\right) \overline{Z}. \tag{A.13}$$

Using the Gaussian property we rewrite the above equation in the vector form as

$$\mathbb{E}\left[R^e\right] = \frac{1}{\rho} \text{Cov}\left(F - R_f P, F - R_f P | S_i, P\right) \overline{Z} = \frac{1}{\rho} \text{Var}\left(R^e | S_i, P\right) \overline{Z}. \tag{A.14}$$

Pre-multiplying both sides of the equation above by $\overline{Z}'$ yields

$$\mathbb{E}\left(\overline{R}_m^e\right) = \overline{Z}' \mathbb{E}\left(R^e\right) = \frac{1}{\rho} \overline{Z}' \text{Var}(R^e | S_i, P) \overline{Z}.$$

Combining the above two equations yields (30). Q.E.D.

Proof of Proposition 4: From investor i's first-order condition, we have

$$\begin{aligned} X_i &= \rho \left[\operatorname{Var}\left(R^e|S_i, P\right)\right]^{-1} \mathbb{E}\left[R^e|S_i, P\right] \\ &= \rho \left[\operatorname{Var}\left(R^e|S_i, P\right)\right]^{-1} \left[\mathbb{E}\left(F|S_i, P\right) - PR_f\right]. \end{aligned} \tag{A.15}$$

From the market-clearing condition, we obtain

$$PR_f = \int \mathbb{E}(F|S_i, P)di - \frac{1}{\rho}\operatorname{Var}(F|S_i, P)Z. \tag{A.16}$$

Substituting this equation into (A.15) shows

$$\begin{aligned} X_i &= \rho \left[\operatorname{Var}\left(R^e|S_i, P\right)\right]^{-1} \left[\mathbb{E}\left(F|S_i, P\right) - \int \mathbb{E}(F|S_i, P)di + \frac{1}{\rho}\operatorname{Var}(F|S_i, P)Z\right] \\ &= \rho \left[\operatorname{Var}\left(R^e|S_i, P\right)\right]^{-1} \left[\mathbb{E}\left(F|S_i, P\right) - \int \mathbb{E}(F|S_i, P)di\right] + Z. \end{aligned} \tag{A.17}$$

Given that S_i, P and F are jointly normal, we know that $\mathbb{E}(F|S_i, P)$ has the following representation

$$\mathbb{E}(F|S_i, P) = B_0 + B_1 S_i + B_2 P, \tag{A.18}$$

where B_0, B_1 and B_2 are constant matrices. From Admati (1985), we have

$$B_1 = \operatorname{Var}(F|S_i, P)C'\Sigma_\epsilon^{-1}, \tag{A.19}$$

in the symmetric linear equilibrium.

Combining (A.18) and (A.19) yields

$$\mathbb{E}\left(F|S_i, P\right) - \int \mathbb{E}(F|S_i, P)di = B_1\left[\epsilon_i - \int \epsilon_i di\right] = \operatorname{Var}(F|S_i, P)C'\Sigma_\epsilon^{-1}\epsilon_i, \tag{A.20}$$

where we have use the fact that the integration of noises is zero.

Substituting the above equation into (A.17) leads to the desired result. Q.E.D.

Proof of Proposition 5: We focus on the symmetric linear equilibrium in which $C_i = C$ and $\Sigma_{\epsilon i} = \Sigma_\epsilon$ for all i. To get a more explicit expression of the portfolio holdings dispersion, substituting (33) into (34) yields

$$\int \mathbb{E}\left[\left(X_i - Z\right)'\left(X_i - Z\right)\right] di = \rho^2 \int \mathbb{E}\left[\epsilon_i'\Sigma_\epsilon^{-1}CC'\Sigma_\epsilon^{-1}\epsilon_i\right] di = \rho^2 \operatorname{Tr}(\Sigma_\epsilon^{-1}CC'),$$

where we notice that $\epsilon_i'\Sigma_\epsilon^{-1}CC'\Sigma_\epsilon^{-1}\epsilon_i$ follows a central χ^2 distribution. By the cyclic property of trace, the preceding equation can be rewritten as

$$\int \mathbb{E}\left[\left(X_i - Z\right)'\left(X_i - Z\right)\right] di = \rho^2 \operatorname{Tr}(C'\Sigma_\epsilon^{-1}C) = \rho \operatorname{Tr}(\Pi),$$

which says that the portfolio dispersion is simply the trace of signal-to-noise ratio adjusted by ρ^2.

For the dispersion of portfolio excess returns, we have

$$(X_i - Z)'R^e = \rho\epsilon_i'\Sigma_\epsilon^{-1}CR^e.$$

It follows that

$$\begin{aligned}\int \mathbb{E}\left[\left((X_i - Z)' R^e\right)^2\right] di &= \rho^2 \int \mathbb{E}\left[\epsilon_i'\Sigma_\epsilon^{-1}CR^eR^{e\prime}C'\Sigma_\epsilon^{-1}\epsilon_i\right] di \\ &= \rho^2 \int \mathbb{E}\left[\mathbb{E}(\epsilon_i'\Sigma_\epsilon^{-1}CR^eR^{e\prime}C'\Sigma_\epsilon^{-1}\epsilon_i|Z,F)\right] di. \end{aligned} \tag{A.21}$$

Conditional on Z and F, $\epsilon_i'\Sigma_\epsilon^{-1}CR^eR^{e\prime}C'\Sigma_\epsilon^{-1}\epsilon_i$ follows a central χ^2 distribution. This implies that

$$\mathbb{E}(\epsilon_i'\Sigma_\epsilon^{-1}CR^eR^{e\prime}C'\Sigma_\epsilon^{-1}\epsilon_i|Z,F) = \mathrm{Tr}\left(\Sigma_\epsilon^{-1}CR^eR^{e\prime}C'\right).$$

Substitution of the preceding equation into (A.21) shows

$$\begin{aligned}\int \mathbb{E}\left[\left((X_i - Z)' R^e\right)^2\right] di &= \rho^2 \int \mathbb{E}\left[\mathrm{Tr}\left(\Sigma_\epsilon^{-1}CR^eR^{e\prime}C'\right)\right] di \\ &= \rho^2 \int \mathbb{E}\left[\mathrm{Tr}\left(C'\Sigma_\epsilon^{-1}CR^eR^{e\prime}\right)\right] di.\end{aligned}$$

By the linearity of expectation and trace, we obtain

$$\begin{aligned}\int \mathbb{E}\left[\left((X_i - Z)' R^e\right)^2\right] di &= \rho^2 \int \mathrm{Tr}\left(C'\Sigma_\epsilon^{-1}C\mathbb{E}\left[R^eR^{e\prime}\right]\right) di \\ &= \rho\mathrm{Tr}\left(\int \rho C'\Sigma_\epsilon^{-1}Cdi\mathbb{E}\left[R^eR^{e\prime}\right]\right) \\ &= \rho\mathrm{Tr}\left(\Pi\mathbb{E}[R^eR^{e\prime}]\right). \end{aligned} \tag{A.22}$$

Furthermore, we notice that

$$V_e = \mathrm{Var}(R^e) = \mathbb{E}[R^eR^{e\prime}] - \mathbb{E}[R^e]\mathbb{E}[R^{e\prime}] = \mathbb{E}[R^eR^{e\prime}] - \overline{R}^e\overline{R}^{e\prime}.$$

Replacing $\mathbb{E}[R^eR^{e\prime}]$ in (A.22) yields

$$\int \mathbb{E}\left[\left((X_i - Z)' R^e\right)^2\right] di = \rho\mathrm{Tr}\left(\Pi(V_e + \overline{R}^e\overline{R}^{e\prime})\right) = \rho\mathrm{Tr}\left(\Pi\Omega\right).$$

The proof is completed. Q.E.D.

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