Available online at www.sciencedirect.com
SciVerse ScienceDirect
Journal of Economic Theory 149 (2014) 236–275
JOURNAL OF Economic Theory
www.elsevier.com/locate/jet

Advance information and asset prices ☆

Rui Albuquerque [a,b,c,d,*], Jianjun Miao [e,f,g]

[a] Boston University School of Management, United States
[b] Católica-Lisbon School of Business and Economics, Portugal
[c] CEPR, United Kingdom
[d] ECGI, Belgium
[e] Department of Economics, Boston University, 270 Bay State Road, Boston, MA 02215, United States
[f] CEMA, Central University of Finance and Economics, China
[g] AFR, Zhejiang University, China

Received 15 October 2010; final version received 25 September 2012; accepted 1 May 2013

Available online 5 June 2013

Abstract

This paper provides a dynamic rational expectations equilibrium model in which investors have heterogeneous information and investment opportunities. Informed investors privately receive advance information about future earnings that is unrelated to current earnings. In response to good advance information, stock prices increase and informed investors act as trend chasers, increasing their investment in stocks. Informed investors also buy other investment opportunities that are positively correlated with stocks, bearing more aggregate risk. The expected risk premium increases generating short-run momentum. Uninformed investors sell stocks, acting as contrarians. When the advance information materializes in the future, excess returns fall, generating long-run reversals.

JEL classification: G11; G12; G14

Keywords: Advance information; Rational expectations equilibrium; Momentum and reversal effects

☆ We would like to thank Christian Hellwig (editor), two referees and an associate editor to JET, Snehal Banerjee, Peter DeMarzo, Bernard Dumas, Darrell Duffie, Ilan Kremer, Simon Gervais, Denis Gromb, Erwan Morellec, Jiang Wang, and seminar participants at the Boston University Economics Department macro lunch, the Boston University School of Management, Copenhagen Business School, George Mason University, HEC Lausanne, Helsinki School of Economics, INSEAD, Yale University, the 2009 AFA meetings in San Francisco, the 2008 SED meetings in Boston, the 2008 NBER Asset Pricing program meeting in Chicago, Norwegian School of Management BI, and the 2008 Econometric Society meetings in New Orleans for comments. The usual disclaimer applies.
* Corresponding author at: Boston University, 595 Commonwealth Avenue, Boston, MA 02215, United States.
E-mail addresses: ralbuque@bu.edu (R. Albuquerque), miaoj@bu.edu (J. Miao).

0022-0531/$ – see front matter
http://dx.doi.org/10.1016/j.jet.2013.06.001

1. Introduction

Many empirical studies have documented evidence of momentum and reversal effects in aggregate and cross-sectional stock returns. The momentum effect refers to the phenomenon of excess stock returns tending to exhibit unconditional positive serial correlation in the short to medium run. The reversal effect refers to the phenomenon that excess stock returns are negatively correlated in the long run.[1] The momentum and reversal effects provide a serious challenge to the efficient markets hypothesis and to standard risk-based models.

In this paper, we propose a theory to explain the momentum and reversal effects, based on the hypothesis that some investors possess advance information—news about firms' future cash flows—in a rational expectations framework. Our model builds on the seminal study of Wang [47]. In the model, two types of investors, informed and uninformed, trade in the financial market. Uninformed investors can invest in public assets only; a risky stock and a risk-free bond. Their information consists of past earnings and stock price realizations. Based on this public information, they try to infer informed investors' private information. Informed investors can invest in both publicly traded assets and in a private investment opportunity whose returns are assumed to be positively correlated with earnings.[2] Informed investors have private information about earnings as well as about the return on the private investment.

We extend Wang's [47] model by assuming that informed investors also possess private advance information about a firm's future performance, such as future shocks to earnings, that are unrelated to current performance. In this case, informed investors also must solve a forecasting problem because their advance information contains noisy signals about the future. Uninformed investors solve their forecasting problems trying to learn informed investors' forecasts. The presence of advance information complicates our analysis and requires us to use a solution technique different from that in Wang [47]. The key to our method is to define a suitable state vector. The elements of this vector are persistent components of earnings and returns of private investment opportunities as well as a finite sequence of future innovations to earnings. We then conjecture that the equilibrium price function is a linear function of the informed and uninformed investors' forecasts of this state vector. This approach allows us to derive a linear rational expectations equilibrium in quasi-closed form up to a nonlinear system of algebraic equations. In this equilibrium, the price function contains not only current fundamental values, but also advance information signals that help forecast future earnings innovations.

We decompose the equilibrium stock price into three components: a "fundamental" component, an "information wedge" component, and a "risk premium" component. The fundamental component is equal to the expected present value of future dividends discounted by the riskfree rate. The information wedge is equal to the forecasting errors made by uninformed investors relative to informed investors. The risk premium component reflects the compensation to risk averse investors for bearing risks from trading stocks and nontraded assets. We show that the first two

1 See Cutler et al. [16], Jegadeesh and Titman [29], Chan et al. [14] and Rouwenhorst [42] for time-series and cross-sectional evidence on short term momentum, and Bernard and Thomas [9] for evidence on price continuation after public news events. For evidence on long term return reversals see De Bondt and Thaler [18], Fama and French [21], and Poterba and Summers [41], among others. For evidence on the negative association between stock returns and past price-scaled variables see De Bondt and Thaler [19], Fama and French [22] and La Porta et al. [33].

2 That informed investors have a larger investment opportunity set is in the spirit of Merton's [38] investor recognition hypothesis that some investors are better at identifying investment opportunities than others. This assumption is in line with having private equity, real estate, foreign exchange, and derivatives markets accessible to only a subset of investors.

components do not contribute to the serial correlation of excess stock returns. Only the risk premium component drives the serial correlation. We show that this component is determined by the persistent component of the return on the private investment opportunity and the advance information signal. They constitute two state variables that drive the movements in the risk premium, which generate momentum and reversals.

To see the intuition, consider the impact of an increase in the persistent component of the private investment return. Informed investors sell stocks to invest in the private investment opportunity. The current stock price falls to induce uninformed investors to buy, who in turn expect high stock returns in the next period. This trading pattern decreases the current stock return and raises next period's stock return if the private investment returns are independent over time. Thus, stock returns generated by rebalancing trades tend to reverse themselves. If the private investment returns are persistent, future private investment returns will change when the current private investment return changes; therefore, future stock prices will also change due to future rebalancing trades. This effect may dominate making stock returns positively serially correlated when the private investment return is sufficiently persistent. Thus, in the absence of advance information, the serial correlation of excess stock returns is determined solely by the persistence parameter of the private investment returns. This case cannot generate momentum in the short run and subsequent reversals in the long run as in the data.

In the presence of advance information, the advance information signal provides another state variable that drives the movements of excess returns. In this case, rebalancing trades induced by advance information help generate return continuation, even though the persistence of private investment returns is low. We explain this intuition by considering the effects of a good piece of one-period-ahead advance information. In response to this information shock, the current stock price and excess stock return rise, partially incorporating the good information. Informed investors buy the stock for speculative reasons. Uninformed investors do not observe the information shock and may associate the rise in the stock price to the fall of the private investment return. Consequently, they perceive the informed investors' speculative buy as rebalancing trades and hence they sell stocks at a high price to accommodate the informed investors' buy positions.

Importantly, a good piece of advance information about future stock earnings also signals high expected returns on the private investment opportunity because stock earnings and returns on the private investment opportunity are assumed to be positively correlated. As a result, informed investors raise their investments in the nontraded asset in response to a good advance information signal. Because informed investors hold both more stocks and more private investments, they bear more aggregate risk, leading to higher expected future excess stock returns and short-run momentum.

In short, informed investors are trend-chasers and uninformed investors are contrarians in response to good advance information. This trading behavior generates short-run momentum and is consistent with evidence on time series momentum and investor behavior presented in Moskowitz et al. [39]. Unlike a representative agent model, our heterogeneous-investor model shows that momentum occurs because informed trend-chasers are able to hide their information under the guise of rebalancing trades. We also discuss the relationship between volume and momentum and show that it is consistent with the evidence in Connolly and Stivers [15].

In addition to generating short-run momentum in excess stock returns, private advance information may also generate long-run excess stock return reversals. This is because the impact of advance information on stock prices dies out quickly once the advance information materializes, and subsequently excess returns revert themselves when the return on private investment opportunities is not persistent.

We show that with a single piece of advance information, the short-run momentum and long-run reversal effects can occur when this information is about next period's earnings innovations. An undesirable prediction of this case is that momentum lasts only for several periods, which seems inconsistent with empirical evidence. We thus extend this model by assuming that informed investors receive increasingly precise signals about earnings innovations as these get closer to materialize. In this case, stale information is useful for forecasting, and informed investors trade on this information.[3] As a result, the effects of advance information can last for a long period, causing long-lived momentum. Moreover, after a sustained streak of good news, the advance information materializes, future excess returns fall and the stock displays long-run reversals.

Rational models are generally unable to explain momentum and reversal effects in a unified way. First, some rational models have the mechanisms to explain momentum but not reversals. Berk, Green and Naik [8] show that a rich variety of return patterns, including momentum effects, result from the variation of risk exposures over the life-cycle of a firm's endogenously chosen projects (see also Gomes et al. [26]). Johnson [30] provides a standard model of firm cash flows discounted by an ordinary pricing kernel that are able to deliver the momentum effect. His key idea is that expected dividend growth rates vary over time and growth rate risk varies with the growth rates. Neither model, however, delivers the long-horizon reversal effect (see also Holden and Subrahmanyam [27], Biais et al. [11] and Makarov and Rytchkov [37]).

Second, some models have the mechanisms to explain reversals but not momentum. Lewellen and Shanken [34] predict return reversals due to correction of past forecast errors, but do not predict momentum. Fama and French [23,24] show that many of the long-horizon results—such as return reversals, the book-to-market effect, and the earnings-to-price-ratio effect—can be largely subsumed within their three-factor model. However, Fama and French [24] point out that the momentum result of Jegadeesh and Titman [29] constitutes the "main embarrassment" for their three-factor model (see also Fama and French [25]). Finally, some models can predict either momentum or reversals but not both (see Wang [46], Cespa and Vives [13] and our benchmark model discussed above). One exception is Vayanos and Woolley [45] who combines the assumptions of asymmetric information under delegated portfolio management with gradual portfolio adjustment. In Vayanos and Woolley, momentum and reversals are tied to cash flow shocks to assets held by an active fund and the trades triggered by these shocks which lead investors to update their beliefs about the managers' ability and migrate to an index fund.

Recently, Daniel and Titman [17] dispute both behavioral and risk-based interpretations that the reversal and book-to-market effects are a result of high expected returns on stocks of distressed firms with poor past performance. In their empirical study, they decompose individual firm returns into two components. One, which they call tangible information, is associated with past performance as measured by accounting performance variables. The other, which they call intangible information, is the component of returns that is orthogonal to the tangible information. They show that future returns are unrelated to tangible information, but are strongly negatively related to intangible information. We may interpret the advance information in our model as intangible information. This information is unrelated to past performance, but impacts prices. Using a rational-expectations model with asymmetric information, we show that the presence of intangible information is important for generating the momentum and reversal effects.

[3] See Bernhardt and Miao [10] for a strategic model of stock prices with stale information. See Tetlock [44] for empirical evidence of the importance of stale information.

Our modeling of advance information is similar to that in the macroeconomics literature on news shocks, e.g., Beaudry and Portier [6,7], Jaimovich and Rebelo [28] and Lorenzoni [36], based on the ideas starting from Pigou [40] and Keynes [32]. This literature typically assumes that the news shock is one period ahead. There is also an empirical literature suggesting that stock price movements reflect the market's expectations about future cash flows and growth opportunities in the economy (e.g., Fama [20] and Schwert [43]). However, there is no theoretical study in the literature that tries to explore the implications of investors' trading behavior when some of them possess news about future cash flows, referred to as advance information in this paper. Our paper fills this gap.

The market microstructure literature, e.g., Banerjee, Kaniel and Kremer [5], also models advance information. In this literature, there is typically a finite horizon T at which time the asset has a liquidation value of v_T. At time 0, investors receive a signal $S_0 = v_T + \varepsilon_0^S$. Because the horizon T is finite and v_T is exogenous, the implication that S_0 is useless after period T cannot be studied. Instead, we solve for a stationary equilibrium in an infinite horizon model and are able to analyze what happens after period T. Bacchetta and van Wincoop [3,4] construct models of investor heterogeneity with advance information to study the role of higher order expectations. However, they do not address the questions we study here.

We organize the rest of the paper as follows. Section 2 presents a simple model with a representative agent to illustrate the role of advance information, teasing out the effect of rebalancing trades. Section 3 presents the model with two types of investors. Section 4 studies the equilibrium properties of the model with and without advance information. Section 5 analyzes the predictions of the model for stock returns and trading behavior. Section 6 extends the model along two dimensions: allowing for intertemporal consumption and incorporating multiple pieces of advance information. Section 7 concludes the paper. Proofs are relegated to Appendices B–C.

2. A representative agent model

We first study a representative agent model to develop our intuition and key insights transparently. Consider an economy with a representative investor that derives utility over next period's wealth, W_{t+1}. Preferences are represented by[4]

$$E_t\left\{-e^{-\gamma W_{t+1}}\right\}, \tag{1}$$

where γ is the coefficient of constant absolute risk aversion and E_t is the expectation operator conditional on all available information at time t. The investor can invest in a riskless bond with gross return $R > 1$ that is in infinite elastic supply and in a stock that generates earnings D_t at time t and has fixed supply normalized to unity. The earnings process is described by

$$D_t = F_t + \varepsilon_t^D, \tag{2}$$

where F_t follows an AR(1) process,

$$F_t = a_F F_{t-1} + \varepsilon_t^F, \quad 0 < a_F < 1. \tag{3}$$

[4] The assumption of myopic preferences rules out dynamic hedging demands and simplifies our analysis significantly. Introducing a dynamic hedging demand, however, does not change our key insights as shown in Section 6.2. Other papers that also adopt myopic preferences for tractability include Bacchetta and van Wincoop [3,4], Campbell et al. [12] and LIorente et al. [35].

Earnings have both a persistent component F_t, with persistence given by a_F, and a temporary component ε_t^D. We assume that shocks to both components, ε_t^D and ε_t^F, are normal random variables with means of zero and variances σ_D^2 and σ_F^2, respectively. The firm is assumed to distribute 100 percent of its earnings as dividends. We therefore use the terms earnings and dividends interchangeably.

In addition to the publicly traded assets, a nontraded asset or a private investment technology is available to the investor. This technology has constant returns to scale and its return between period t and $t+1$ is $R + q_{t+1}$, where the excess return q_{t+1} satisfies,

$$q_{t+1} = Z_t + \varepsilon_{t+1}^q. \tag{4}$$

Here, ε_{t+1}^q is the transitory component and Z_t is the persistent component satisfying:

$$Z_t = a_Z Z_{t-1} + \varepsilon_t^Z, \quad 0 < a_Z < 1. \tag{5}$$

We assume that shocks to both components, ε_t^q and ε_t^Z are normal random variables with means of zero and variances σ_q^2 and σ_Z^2, respectively.

The investor observes all variable realizations up to time t and also receives news about future earnings announcements that we label as advance information. In this model, advance information is modeled as a noisy signal about next period's earnings innovations:

$$S_t = \varepsilon_{t+1}^D + \varepsilon_t^S, \tag{6}$$

where ε_t^S is a normal variable with mean zero and variance σ_S^2. All shocks are independently and identically distributed (i.i.d.) and uncorrelated with each other except for $E[\varepsilon_t^D \varepsilon_t^q] = \sigma_{Dq} > 0$. The last assumption follows from Wang [47], which implies that the nontraded asset and the stock are substitutes.[5]

Arguably, advance information is likely to be also about the persistent component of earnings, ε_t^F. For example, advance information may be about the outcome of a procurement contract, a future M&A event, or the outcome of a law suit, all of which could affect earnings over many periods. Since the critical channel for our results is through rebalancing trades in the nontraded asset in response to advance information signals, our results still hold in this case provided that $E[\varepsilon_t^F \varepsilon_t^q] > 0$.[6]

We focus on the rational expectations equilibrium in which the stock price is a stationary function of state variables. Let P_t denote the time t stock price and $Q_t = P_t + D_t - RP_{t-1}$ the stock's excess return at time t. The investor takes prices as given and maximizes (1) by choosing stock holdings, θ_t, and nontraded asset holdings, α_t, subject to the budget constraint:

$$W_{t+1} = \theta_t Q_{t+1} + \alpha_t q_{t+1} + W_t R. \tag{7}$$

Market clearing requires $\theta_t = 1$.

In Appendix A, we show that the equilibrium stock price can be written as

$$P_t = f_t + \pi_t,$$

where $f_t = E_t[\sum_{s=1}^{\infty} R^{-s} D_{t+s}]$ is a fundamental component equal to the expected present value of dividends discounted at the riskless rate and π_t denotes the "risk premium" component of the

[5] If $\sigma_{Dq} < 0$, then the nontrade asset and the stock are complements since $V_{Qq} < 0$ by Eq. (10). As can be seen from the analysis to follow, our key results carry over to this case with small changes in proofs.

[6] This analysis is contained in an online appendix.

price that is attributed to risk from dividends, nontraded asset returns and advance information signals. These two components are given by

$$f_t = R^{-1}\left(\frac{a_F}{1-R^{-1}a_F}F_t + \frac{\sigma_D^2}{\sigma_S^2+\sigma_D^2}S_t\right), \tag{8}$$

$$\pi_t = -\frac{\gamma(V_Q V_q - V_{Qq}^2)}{V_q}\frac{1}{R-1} - \frac{R^{-1}V_{Qq}}{V_q}\left(\frac{Z_t}{1-R^{-1}a_Z} + \bar{\sigma} S_t\right), \tag{9}$$

where $V_Q \equiv Var_t(Q_{t+1})$, $V_q \equiv Var_t(q_{t+1})$, and

$$\bar{\sigma} \equiv \frac{\sigma_{Dq}}{\sigma_S^2+\sigma_D^2}, \qquad V_{Qq} \equiv Cov_t(Q_{t+1}, q_{t+1}) = \frac{\sigma_{Dq}\sigma_S^2}{\sigma_S^2+\sigma_D^2}. \tag{10}$$

Since $\sigma_{Dq} > 0$, it follows that $V_{Qq} > 0$, indicating that the stock and the nontraded asset are substitutes. In this case, the investor has a hedging incentive to rebalance his portfolio. In Appendix A, we derive explicit expressions for V_Q and V_q and show that they are constant. The first term in Eq. (9) is a negative constant and reflects the price discount to compensate the risk-averse investor for bearing dividend risk. The other terms in Eq. (9) reveal that two sources of uncertainty affect π_t: expected return on the nontraded asset, Z_t, and the advance information signal, S_t. We will show below that changes in Z_t or S_t induce rebalance trades and hence affect stock prices as well.

The solution for the excess stock return can then be expressed as

$$Q_{t+1} = D_{t+1} + f_{t+1} - E_t(D_{t+1} + f_{t+1}) + \pi_{t+1} - R\pi_t, \tag{11}$$

from which we get the conditional expected excess return or risk premium:

$$\mu_{Qt} = E_t[Q_{t+1}] = E_t[\pi_{t+1} - R\pi_t] = \frac{\gamma(V_Q V_q - V_{Qq}^2)}{V_q} + \frac{V_{Qq}}{V_q}(Z_t + \bar{\sigma} S_t). \tag{12}$$

This equation reveals that the expected excess return is time varying and its movement is driven by both the expected return on the nontraded asset, Z_t, and the advance information signal, S_t.

We argue that variation in the expected excess return, μ_{Qt}, is determinant for momentum and reversal effects in stock returns and that its driving force is the advance information signal S_t.[7] To see this, we use the law of iterated expectations to derive $E[Q_{t+n+1}|Q_t] = E[\mu_{Qt+n}|Q_t]$, for any $n \geqslant 0$. We can then use (12) to derive that

$$E[Q_{t+1}|Q_t] = \frac{\gamma(V_Q V_q - V_{Qq}^2)}{V_q} + \frac{V_{Qq}}{V_q}\frac{Cov(Z_t + \bar{\sigma} S_t, Q_t)}{Var(Q_t)}Q_t, \tag{13}$$

$$E[Q_{t+n}|Q_t] = \frac{\gamma(V_Q V_q - V_{Qq}^2)}{V_q} + a_Z^{n-1}\frac{V_{Qq}}{V_q}\frac{Cov(Z_t, Q_t)}{Var(Q_t)}Q_t, \tag{14}$$

for $n \geqslant 2$. If $Cov(Q_{t+n}, Q_t) > (<)0$, we say that one-period excess returns exhibit momentum (reversals) at horizon n. The above two equations reveal that the momentum and reversal effects are determined by the signs of $Cov(Z_t, Q_t)$ and $Cov(S_t, Q_t)$. Using (11), we can compute

[7] In an online appendix, we show that removing advance information but introducing other correlation structure, e.g., $E[\varepsilon_t^F \varepsilon_t^Z] > 0$ cannot generate short-run momentum followed by long-run reversals. The reason is that there is only one shock Z_t that moves μ_{Qt}.

$$Cov(Z_t, Q_t) = \frac{V_{Qq}}{V_q} \frac{Ra_Z - 1}{R - a_Z} \frac{\sigma_Z^2}{1 - a_Z^2}, \tag{15}$$

$$Cov(S_t, Q_t) = R^{-1} \frac{\sigma_D^2 (1 - \rho_{Dq}^2)}{\sigma_S^2 + \sigma_D^2 (1 - \rho_{Dq}^2)} \left(\sigma_S^2 + \sigma_D^2\right) > 0, \tag{16}$$

where $\rho_{Dq} \in (0, 1)$ is the conditional correlation coefficient between ε_t^D and ε_t^q.

If there is no advance information, then Eq. (14) holds for all $n \geqslant 1$. Since $Cov(Z_t, Q_t) < 0$ if and only if $Ra_Z < 1$, Eq. (14) implies that $Cov(Q_{t+n}, Q_t) < 0$ if and only if $Ra_Z < 1$, for all $n \geqslant 1$. This means that one can get either momentum at all horizons or reversals at all horizons in the absence of advance information, contradicting the empirical evidence that momentum appears in the short run and reversals appear in the long run.

Turn to the case with advance information. Assuming $Ra_Z < 1$, we obtain $Cov(Q_{t+n}, Q_t) < 0$ for all $n \geqslant 2$ by (14) and (15). Since $Cov(S_t, Q_t) > 0$ by (16), it follows from (13) that advance information helps generate positive correlation between Q_t and Q_{t+1}. When a_Z is sufficiently close to $1/R$ from below, $Cov(Z_t + \bar{\sigma} S_t, Q_t)$ is sufficiently close to $Cov(\bar{\sigma} S_t, Q_t) > 0$. In this case, we can make $Cov(Z_t + \bar{\sigma} S_t, Q_t) > 0$ and hence $Cov(Q_{t+1}, Q_t) > 0$. Thus, the presence of advance information can generate one-period momentum and subsequent reversals.

The representative agent model with one-period-ahead advance information is helpful for understanding the economic mechanism behind our theory of momentum and reversals. Assume $Ra_Z < 1$. Consider an increase in the stock price in period t, which raises Q_t. This price increase could be due to a low expected nontraded asset return, Z_t. If this were true, less would be invested in the nontraded asset, so less aggregate risk would be borne in equilibrium and consequently the conditional expected excess return (or risk premium) would be low. Thus, high current excess returns would be associated with low expected future excess returns, generating reversals. In the presence of advance information, a good signal about future dividend innovations also leads to a contemporaneous increase in the stock price. In addition, good advance information signals that the nontraded asset return is also high in the future because dividend innovations are positively correlated with innovations in nontraded asset returns. This induces an increased investment in the nontraded asset and causes the investor to bear more aggregate risk, which causes the expected future excess return to rise. Thus advance information helps generate one-period momentum.

The special feature of the one-period-ahead advance information is that it materializes in the next period becoming useless. The future stock price then falls, causing the n-period excess returns ($n \geqslant 2$) to be negatively serially correlated with the current excess return, i.e., $Cov(Q_{t+n}, Q_t) < 0$, for any $n \geqslant 2$ if and only if $Ra_Z < 1$. Thus, our model with advance information can generate short-run momentum and long-run reversals simultaneously.

It is important that advance information induces rebalancing trades for the above mechanism to work. To see this formally, we show in Appendix A that

$$E[Q_{t+1} | Q_t] = E[\mu_{Qt} | Q_t] = \gamma\left(V_Q + V_{Qq} E[\alpha_t | Q_t]\right). \tag{17}$$

Momentum is explained by how investment in the nontraded asset, α_t, correlates with the current excess stock return, Q_t. Provided $V_{Qq} > 0$, momentum occurs if the investor puts more money in the nontraded asset when the current stock return is high. This is likely to occur in the presence of advance information because it drives both the future return on nontraded asset, q_{t+1}, and the current stock return, Q_t in the same direction.

Interestingly, the precision of advance information is important for the momentum effect. When $\sigma_S \to \infty$, advance information is too noisy to be useful. The model then reduces to the one without advance information. When $\sigma_S \to 0$, the investor knows ε_{t+1}^D precisely at date t. Thus, he has no hedging incentive as $V_{Qq} \to 0$ by (10). His investment in the nontraded asset does not respond to the movement in the stock return. As a result, the risk premium is a constant and there is no serial correlation of excess returns. Momentum occurs only for intermediate precision of advance information.

Consider now the intuition behind the condition $Ra_Z < 1$. When expected returns in the nontraded asset, Z_t, increase, the current stock price P_t and excess stock returns Q_t fall, *ceteris paribus*. On the other hand, a high Z_t tends to follow a high Z_{t-1}, because this process is persistent. A high Z_{t-1} causes period $t-1$ stock price P_{t-1} and excess stock returns Q_t to rise. The first effect dominates when the persistence of Z_t is low enough in that $Ra_Z < 1$, causing negative correlation between Z_t and Q_t, and hence reversals in stock returns. Otherwise, excess stock returns tend to be positively serially correlated.

While the representative agent model presented in this section illustrates the role of advance information, there are two limitations. First, the representative agent setup cannot address the trading behavior of different types of investors in the market. Second, the one-period-ahead advance information cannot generate a multi-period momentum effect. In the following analysis, we shall overcome these limitations.

3. A heterogeneous agent model

We shall study a full model that builds on the previous one by incorporating investor heterogeneity and information asymmetry and a more general process for advance information. These features are important for generating additional predictions regarding investor trading activity in the presence of momentum and reversals in stock returns and help understand who gains and who loses from trading on momentum and reversals, an important question that cannot be answered in representative agent models.

There are two types of infinitely-lived investors in the economy, informed and uninformed. Informed and uninformed investors differ in their information structure and investment opportunities. The fraction of informed investors is $\lambda \in (0, 1)$ and the fraction of uninformed investors is $1 - \lambda$. Both investor types share the same utility function (1). We use the superscript i to index a variable associated with an informed investor and the superscript u to index a variable associated with an uninformed investor.

Any investor can trade a riskless bond and a risky stock whose payoffs have been described in the preceding section. In addition, informed investors may also invest in the nontraded asset, which we call interchangeably the private investment opportunity, whose returns are described in Eqs. (4) and (5).[8]

3.1. Information structure

All investors observe the past and current realizations of earnings and of stock prices. As in Wang [47], informed investors have private information about the persistent component F_t of the stock and the expected return Z_t on the private investment, while uninformed investors do not.

8 This nontraded asset prevents the existence of a fully revealing equilibrium. An alternative approach in the noisy rational expectations equilibrium literature is to introduce exogenous noise trading or liquidity trading. As Wang [47] points out, since the noninformational component is exogenous, this approach is not preferable in studying trading behavior. In addition, this approach becomes problematic in welfare analysis.

In addition to these pieces of private information, informed investors receive advance information about future earnings announcements. The advance information is modeled as a noisy signal about future earnings innovations:

$$S_t = \varepsilon_{t+k}^D + \varepsilon_t^S, \tag{18}$$

where $k > 0$ and ε_t^S is an i.i.d. normal variable with mean zero and variance σ_S^2. The shock ε_t^S is assumed independent of all other shocks. Informed investors' information set is

$$\mathcal{F}_t^i = \{D_s, F_s, P_s, Z_s, S_s\colon\ s \leqslant t\}. \tag{19}$$

The information set of uninformed investors is given by

$$\mathcal{F}_t^u = \{D_s, P_s\colon\ s \leqslant t\}. \tag{20}$$

3.2. Equilibrium

A rational expectations equilibrium is defined in the usual way. Again, we focus on a stationary equilibrium for the stock price. The key step in defining an equilibrium is formulating the investors' portfolio choice problems. An informed investor maximizes (1), where the expectation is taken with respect to the information set $\mathcal{F}_t^i$, taking prices as given and subject to the budget constraint

$$W_{t+1}^i = \theta_t^i Q_{t+1} + \alpha_t^i q_{t+1} + W_t^i R.$$

Similarly, an uninformed investor maximizes (1), where the expectation is taken with respect to the information set $\mathcal{F}_t^u$, taking prices as given and subject to

$$W_{t+1}^u = \theta_t^u Q_{t+1} + R W_t^u. \tag{21}$$

Note that an uninformed investor cannot invest in the nontraded asset.

The market clearing condition is given by

$$\lambda\theta_t^i + (1-\lambda)\theta_t^u = 1. \tag{22}$$

4. Equilibrium properties

The key step to solve for the equilibrium is to construct suitable state variables that informed and uninformed investors may use in making their conditional forecasts of future returns. We define the state vector as:

$$\mathbf{x}_t = \left(F_t, Z_t, \varepsilon_{t+k}^D, \ldots, \varepsilon_t^D\right)^\intercal, \tag{23}$$

and the unforecastable (based on period $t-1$ information) shock vector as $\boldsymbol{\varepsilon}_t = (\varepsilon_{t+k}^D, \varepsilon_t^F, \varepsilon_t^Z, \varepsilon_{t+k}^q, \varepsilon_t^S)^\intercal$. Note that $\boldsymbol{\varepsilon}_t \sim N(0, \Sigma)$, where $\Sigma = E[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t^\intercal]$ is the covariance matrix, with the only nonzero covariance being $\sigma_{Dq} = E[\varepsilon_{t+k}^D \varepsilon_{t+k}^q] > 0$. The state vector includes all future realizations of the transitory shocks to earnings up to $t+k$. This is because informed investors can use their private information to forecast these values. For technical reasons, we also include ε_t^D as part of the state vector. We show below that ε_t^D is not priced in equilibrium and does not appear in the asset demand functions because ε_t^D has already been paid out in the form of earnings at time t and has no value for forecasting future earnings.

Throughout the paper, we benchmark our model against a model without advance information, which is a simplified version of Wang [47]. In the absence of advance information, i.e., $\sigma_S^2 = \infty$, we write the state vector as $\mathbf{x}_t = (F_t, Z_t)^\intercal$, and the unforecastable shock vector as $\boldsymbol{\varepsilon}_t = (\varepsilon_t^D, \varepsilon_t^F, \varepsilon_t^Z, \varepsilon_t^q)^\intercal$.

4.1. Stock price

We conjecture that the equilibrium stock price function takes the following form:

$$P_t = -p_0 + \mathbf{p}_i \hat{\mathbf{x}}_t^i + \mathbf{p}_u \hat{\mathbf{x}}_t^u, \tag{24}$$

where $\hat{\mathbf{x}}_t^i = E_t^i[\mathbf{x}_t]$, $\hat{\mathbf{x}}_t^u = E_t^u[\mathbf{x}_t]$, p_0 is a constant, and $\mathbf{p}_i = (p_{i1}, \ldots, p_{i,k+3})$ and $\mathbf{p}_u = (p_{u1}, \ldots, p_{u,k+3})$ are row vectors of constants to be determined in equilibrium. We set $p_{u2} = 0$. In general, one may include $\hat{Z}_t^u$ in the price function in that $p_{u2} \neq 0$. However, from the current price, P_t, the uninformed investors can infer the sum

$$P_t + p_0 - \mathbf{p}_u \hat{\mathbf{x}}_t^u = \mathbf{p}_i \hat{\mathbf{x}}_t^i \equiv \Pi_t, \tag{25}$$

since $\hat{\mathbf{x}}_t^u$ is observable by the uninformed investors. Thus, Π_t represents the information content of the equilibrium price. This implies that $\mathbf{p}_i \hat{\mathbf{x}}_t^i = \mathbf{E}_t^u[\mathbf{p}_i \hat{\mathbf{x}}_t^i] = \mathbf{p}_i \hat{\mathbf{x}}_t^u$. Therefore,

$$\hat{Z}_t^u = \frac{1}{p_{i2}} \mathbf{p}_i \hat{\mathbf{x}}_t^i - \frac{\mathbf{p}_i \mathbf{I}_{-2}}{p_{i2}} \hat{\mathbf{x}}_t^u, \tag{26}$$

where $\mathbf{I}_{-2}$ conforms with the state vector and denotes the matrix that is the same as the identity matrix, except that the $(2, 2)$ element equals zero. Thus, we can eliminate $\hat{Z}_t^u$ in the price function (24), and set $p_{u2} = 0$. Eq. (26) further indicates that uninformed investors' forecast errors are perfectly linearly correlated.

In Appendix B, we prove the following:

Proposition 1. *If there is a solution to the system of equations given in Appendix B, then the economy has a stationary rational expectations equilibrium in which the equilibrium stock price is given by*

$$P_t = -p_0 + \frac{a_F}{R - a_F} F_t - \frac{e_{i2}}{R - a_Z} Z_t - p_{u1}\left(F_t - \hat{F}_t^u\right) + \sum_{j=1}^{k} \left\{ \frac{1 - e_{i2} \frac{\sigma_{Dq}}{\sigma_D^2}}{R^j} E_t^i\left[\varepsilon_{t+j}^D\right] - p_{u,k+3-j}\left(E_t^i\left[\varepsilon_{t+j}^D\right] - E_t^u\left[\varepsilon_{t+j}^D\right]\right) \right\}, \tag{27}$$

where $p_0 > 0$. In addition, $e_{i2} > 0$ iff $Cov_t^i(Q_{t+1}, q_{t+1}) > 0$.

Using (27), we may decompose the stock price into three components,[9]

$$P_t = f_t + \pi_t + w_t.$$

The first is the expected present value of future dividends conditional on $\mathcal{F}_t^i$ discounted at the risk-free rate,

[9] See Albagli et al. [1] for a similar decomposition. We would like to thank the editor for pointing out this decomposition which helps us develop the intuition more transparently.

$$f_t = R^{-1}\left(\frac{a_F}{1-R^{-1}a_F}F_t + \sum_{j=1}^{k}\frac{1}{R^{j-1}}E_t^i\left[\varepsilon_{t+j}^D\right]\right).$$

The second denotes the risk premium component of price that is attributed to risk from dividends, private investment returns and advance information signals,

$$\pi_t = -p_0 - R^{-1}e_{i2}\left(\frac{1}{1-R^{-1}a_Z}Z_t + \sum_{j=1}^{k}\frac{1}{R^{j-1}}\frac{\sigma_{Dq}}{\sigma_D^2}E_t^i\left[\varepsilon_{t+j}^D\right]\right). \tag{28}$$

When $e_{i2} > 0$, the stock and the nontraded asset are substitutes.[10] When the expected return on the nontraded asset is high (i.e., Z_t is high or $E_t^i[\varepsilon_{t+j}^D]$ is high), informed investors increase their holdings on that asset and rebalance their portfolios by selling the stock to reduce their overall exposure to the same source of risk. This causes the stock price to drop because uninformed investors accept to bear more risk through holding more of the stock only if they buy at a lower price.

The third component, w_t, is an "information wedge" that gives rise to speculative trading by informed investors. When uninformed investors underestimate the persistence component of dividends and $F_t - \hat{F}_t^u > 0$, or underestimate the value of advance information and $E_t^i[\varepsilon_{t+j}^D] - E_t^u[\varepsilon_{t+j}^D] > 0$, the stock price does not immediately reflect the value of expected future dividends. Thus, uninformed investors' underestimation of the persistent component of dividends lowers the stock price by

$$w_t = -p_{u1}\left(F_t - \hat{F}_t^u\right) - \sum_{j=1}^{k}p_{u,k+3-j}\left(E_t^i\left[\varepsilon_{t+j}^D\right] - E_t^u\left[\varepsilon_{t+j}^D\right]\right). \tag{29}$$

Advance information permeates all three components of the stock price. The present value incorporates informed investors' forecast of future earnings innovations using the advance information signal. Specifically, earnings expected next period are discounted at rate R, earnings expected two periods later are discounted at rate R^2, and so on up to $t+k$, after which time no more advance information is available. Advance information also affects π_t. When the nontraded asset and the stock are substitutes, a good piece of advance information at date t raises $E_t^i[\varepsilon_{t+k}^q]$, which induces informed investors to sell the stock at time $t+k-1$, causing the current stock price to decline. Finally, because uninformed investors do not have any advance information, they make forecast errors relative to informed investors' information. These forecast errors show up in the information wedge, w_t.

In a non-revealing equilibrium, the information wedge is non-trivial because uninformed investors cannot distinguish between persistent shocks to earnings and persistent shocks to expected returns of the nontraded asset: Good news about future earnings (high F_t or high ε_{t+k}^D) or bad private nontraded asset returns (low Z_t) can both cause informed investors to buy the stock and the stock's price to rise. Therefore, observing price and earnings is insufficient for uninformed investors to identify these shocks. This implies that information asymmetry persists in

[10] In Appendix C, we prove that $e_{i2} > 0$ in the absence of advance information. In the general model, we are unable to prove $e_{i2} > 0$ because we are unable to show $Cov_t^i(Q_{t+1}, q_{t+1}) > 0$ analytically. As in the model without advance information, however, this positive covariance is intuitive as it reflects the fact that unexpected earnings and the unexpected private investment return are positively correlated, i.e., $\sigma_{Dq} > 0$. We verify this result numerically in all our examples below.

the equilibrium. However, we next show that the information wedge does not contribute directly to momentum.

To see this, we express excess stock returns as

$$Q_{t+1} = D_{t+1} + f_{t+1} - E_t^i(D_{t+1} + f_{t+1}) + \pi_{t+1} - R\pi_t + w_{t+1} - Rw_t. \tag{30}$$

Therefore, the conditional expected risk premium from the perspective of uninformed investors is given by

$$E_t^u[Q_{t+1}] = E_t^u[\pi_{t+1} - R\pi_t] = \mu_{Qt}^u,$$

since $E_t^u[w_t] = 0$. The law of iterated expectations implies

$$\begin{aligned} E[Q_{t+n}|Q_t] &= E\big[\mu^u_{Qt+n-1}|Q_t\big] = E\big[E_t^u[\pi_{t+n} - R\pi_{t+n-1}]|Q_t\big] \\ &= E[\pi_{t+n} - R\pi_{t+n-1}|Q_t] = E\big[E_t^i[\pi_{t+n} - R\pi_{t+n-1}]|Q_t\big], \end{aligned}$$

for any $n \geqslant 1$. Thus, the covariance between Q_{t+n} and Q_t is equal to the covariance between $\pi_{t+n} - R\pi_{t+n-1}$ and Q_t. This covariance can be computed by replacing $\pi_{t+n} - R\pi_{t+n-1}$ with either $E_t^i[\pi_{t+n} - R\pi_{t+n-1}]$ or $E_t^u[\pi_{t+n} - R\pi_{t+n-1}]$. Since these expected values reflect risk premium, momentum and reversal effects are driven by how expected future risk premia correlate with current excess returns.

4.2. Investors' forecasts

The presence of advance information implies that both the informed and uninformed investors must solve forecasting problems.[11] Informed investors use their private signals on k-period-ahead earnings innovations to learn about the growth potential in both the stock and private investment technology. Their information processing problem is straightforward because their information set includes uninformed investors' information set, which means that informed investors do not learn from the price level.

Using the projection theorem, we can easily derive informed investors' forecasts of future shocks:

$$E_t^i\big[\varepsilon^D_{t+k-j}\big] = \frac{\sigma_D^2}{\sigma_S^2 + \sigma_D^2} S_{t-j}, \quad 0 \leqslant j \leqslant k-1, \tag{31}$$

$$E_t^i\big[\varepsilon^q_{t+k-j}\big] = \bar{\sigma} S_{t-j}, \quad 0 \leqslant j \leqslant k-1, \tag{32}$$

where $\bar{\sigma}$ is given in (10).

At any date t, informed investor's advance information signals $S_t, S_{t-1}, \ldots, S_{t-k+1}$ are useful to forecast future transitory components of earnings $\varepsilon^D_{t+k}, \varepsilon^D_{t+k-1}, \ldots, \varepsilon^D_{t+1}$, respectively. Since ε^q_t and ε^D_t are correlated, these signals are also useful to forecast future transitory components of private investment returns. In addition, these forecasts are positively linearly related when $\sigma_{Dq} > 0$.

Turn to uninformed investors' forecasting problem. Because informed investors know more than uninformed investors, the most that uninformed investors can hope to learn is what informed investors know. This hierarchical information structure implies that there is no infinite regress

[11] Albuquerque et al. [2] generalize Wang [47] to an international economy and also require both informed and uninformed agents to solve forecasting problems.

problem (Wang [47]). Therefore, it is sufficient for uninformed investors to track the dynamics of the state vector $\hat{\mathbf{x}}_t^i$ estimated by the informed investors. In Appendix B, we present the filtering equations for the uninformed investors.

In the absence of advance information, only uninformed investors solve a filtering problem. In Appendix C, we show that the filtering equations are given by:

$$\begin{bmatrix} \hat{F}_t^u \\ \hat{Z}_t^u \end{bmatrix} = \begin{bmatrix} a_F \hat{F}_{t-1}^u \\ a_Z \hat{Z}_{t-1}^u \end{bmatrix} + \mathbf{K} \begin{bmatrix} D_t - E_{t-1}^u[D_t] \\ \Pi_t - E_{t-1}^u[\Pi_t] \end{bmatrix}, \tag{33}$$

where $\mathbf{K}$ is a 2×2 matrix with elements $k_{11}, k_{12}, k_{21} > 0$ and $k_{22} < 0$.[12]

The first term on the right hand side of (33) gives the expectation based on information prior to period t. The second term gives the update in expectations based on new information from unexpected fluctuations in earnings and the stock price. The sign restrictions on the elements of the Kalman gain matrix $\mathbf{K}$ reveal several properties. First, an unexpected increase in earnings D_t may indicate an increase in the persistent component of dividends, F_t, or an increase in the forecasting error $F_{t-1} - \hat{F}_{t-1}^u$. Not observing these components, uninformed investors may raise their estimate of F_t so that $k_{11} > 0$. Because forecasting errors of Z_t and F_t are positively correlated as shown in (26), uninformed investors also revise upwards their expectation of Z_t. Hence, $k_{21} > 0$. Second, an unexpected increase in $\Pi_t = p_{i1} F_t + p_{i2} Z_t$ may indicate an increase in F_t or a decrease in Z_t. Uninformed investors do not observe these two components separately, and thus raise their estimate of F_t and decrease their estimate of Z_t accordingly. This explains why $k_{12} > 0$ and $k_{22} < 0$.

In the presence of advance information we are unable to sign the elements of the Kalman gain matrix given in Appendix B, but we verify numerically that the effects described above carry through. In addition, there is a further confounding factor moving prices; advance information moves prices without affecting current earnings. Positive shocks to advance information thus mimic negative shocks to nontraded asset returns.

4.3. Optimal portfolios

It is straightforward to derive investors' optimal portfolios. For informed investors,

$$\theta_t^i = \frac{E_t^i[Q_{t+1}]}{\gamma(\sigma_Q^i)^2(1 - (\rho_{Qq}^i)^2)} - \frac{\rho_{Qq}^i E_t^i[q_{t+1}]}{\gamma \sigma_Q^i \sigma_q^i (1 - (\rho_{Qq}^i)^2)}, \tag{34}$$

$$\alpha_t^i = \frac{E_t^i[q_{t+1}]}{\gamma(\sigma_q^i)^2(1 - (\rho_{Qq}^i)^2)} - \frac{\rho_{Qq}^i E_t^i[Q_{t+1}]}{\gamma \sigma_Q^i \sigma_q^i (1 - (\rho_{Qq}^i)^2)}, \tag{35}$$

where $(\sigma_Q^i)^2$ is the conditional excess return variance as perceived by informed investors, $(\sigma_q^i)^2$ is the conditional variance of the private investment return as perceived by informed investors, and $\rho_{Qq}^i = Cov_t^i(Q_{t+1}, q_{t+1})/\sigma_q^i \sigma_Q^i$ is the conditional correlation between the private investment return and the excess return as perceived by informed investors. The preceding conditional vari-

[12] In our simplified version of Wang [47] with myopic investors, we are able to offer a proof of the signs of the coefficients from matrix $\mathbf{K}$. Wang [47] does not prove the signs of the coefficients of the Kalman gain matrix (his Eq. (10)) in his Theorem 1, which show up, and are critical, in all his subsequent results.

ances and correlation are constant over time due to the property of normal random variables. Similarly, the optimal portfolio for an uninformed investor is given by

$$\theta_t^u = \frac{1}{\gamma}\frac{E_t^u[Q_{t+1}]}{(\sigma_Q^u)^2}, \tag{36}$$

where $(\sigma_Q^u)^2$ is the conditional excess return variance as perceived by uninformed investors.

Eqs. (34), (35), and (36) reveal that the optimal portfolios are mean-variance efficient, trading-off expected return and risk. While investors display no dynamic hedging demand, informed investors have a static hedging demand (last terms on the right hand side of (34) and (35)) arising from the correlation between the stock return and the private investment return. This static hedging demand generates rebalancing trades.

Using (34), (35), and the law of iterated expectations, we can show that

$$E[Q_{t+1}|Q_t] = E\big[E_t^i[Q_{t+1}]|Q_t\big] = \gamma\big(\sigma_Q^i\big)^2 E\big[\theta_t^i|Q_t\big] + \gamma\, Cov_t^i(Q_{t+1}, q_{t+1})E\big[\alpha_t^i|Q_t\big]. \tag{37}$$

This equation is similar to (17) for the representative agent model. It shows that serial correlation in excess returns is explained by how investments in the stock, θ_t^i, and in the nontraded asset, α_t^i, correlate with the current excess stock return, Q_t. Momentum occurs when informed investors invest more in both the stock and the nontraded asset in response to a rise in current excess returns, provided $Cov_t^i(Q_{t+1}, q_{t+1}) > 0$.

Using (28), (29), (30), (31), and (32), we can derive informed investors' estimates of excess returns,

$$\begin{aligned} E_t^i[Q_{t+1}] = {} & e_0 + e_{i2}\big(Z_t + E_t^i\big[\varepsilon_{t+1}^q\big]\big) + (R - a_F)p_{u1}\big(F_t - \hat{F}_t^u\big) \\ & + \sum_{j=1}^k f_{Qj}^i\big(E_t^i\big[\varepsilon_{t+j}^D\big] - E_t^u\big[\varepsilon_{t+1}^D\big]\big), \end{aligned} \tag{38}$$

and uninformed investors' estimates of excess returns,

$$E_t^u[Q_{t+1}] = e_0 + e_{i2}\big(\hat{Z}_t^u + E_t^u\big[\varepsilon_{t+1}^q\big]\big), \tag{39}$$

where f_{Qj}^j is some constant for $j = 1, 2, \dots, k$.

Using these conditional expectations of excess stock returns, we can provide a sharper characterization of the optimal portfolios proven in Appendix B.

Proposition 2. *The equilibrium trading strategies satisfy*

$$\theta_t^i = f_0^i + f_Z^i\big(Z_t + E_t^i\big[\varepsilon_{t+1}^q\big]\big) + f_F^i\big(F_t - \hat{F}_t^u\big) + \sum_{j=1}^k f_{Dj}^i\big(E_t^i\big[\varepsilon_{t+j}^D\big] - E_t^u\big[\varepsilon_{t+j}^D\big]\big), \tag{40}$$

and

$$\theta_t^u = f_0^u + f_Z^u\big(\hat{Z}_t^u + E_t^u\big[\varepsilon_{t+1}^q\big]\big), \tag{41}$$

where f_0^i, $f_0^u > 0$, f_Z^i, f_F^i, f_Z^u, and f_{Dj}^i are constants. Also, $f_Z^i < 0$ and $f_Z^u > 0$ iff $Cov_t^i(Q_{t+1}, q_{t+1}) > 0$.

This proposition shows that informed investors trade for both speculative and rebalancing reasons. The expected return on the private investment determines their rebalancing trades. In addition, with advance information, informed investors' forecast of ε_{t+1}^q is not zero and hence it is included in the expected private investment return. When the stock and the nontraded asset are substitutes, i.e., $Cov_t^i(Q_{t+1}, q_{t+1}) > 0$, a high expected return on the private investment leads to rebalancing selling of the stock.

Speculative trading by informed investors arises from their knowledge of uninformed investors' forecast errors about F_t and $E_t^i[\varepsilon_{t+j}^D]$. The forecast errors $E_t^i[\varepsilon_{t+j}^D] - E_t^u[\varepsilon_{t+j}^D]$ show up only in the model with advance information, and, together with $F_t - \hat{F}_t^u$, constitute the private information of informed investors. This private information allows them to take speculative positions against expected future corrections of the uninformed investors' expectations. For example, when uninformed investors underestimate F_t, in the sense that $F_t > \hat{F}_t^u$, informed investors' speculative trading induces them to buy stocks in expectation of a high return in the future when high dividends are realized.

Uninformed investors trade for noninformational reasons. They are willing to trade only when they perceive to be accommodating informed investors' rebalancing trades because these trades always occur at favorable prices. As in the model without advance information, their trading is subject to adverse selection, because they do not know whether informed investors are trading in response to a change in the private investment return or to superior private information. Note that uninformed investors' forecast of the transitory component, ε_{t+1}^q, of future returns on the private investment is not zero and affects their trading strategies because they can use the stock price to make forecast and the stock price contains the advance information signal.

5. Model predictions

5.1. Stock return momentum and reversal effects

The following two theorems proven in Appendices B and C, respectively, are the main results of the paper, which focus on serial correlation of excess returns.

Theorem 1. *Given k-period-ahead advance information,*

$$E[Q_{t+n}|Q_t] = e_0 + e_{i2}\frac{a_Z^{n-1}\,Cov(Z_t, Q_t) + \mathbf{1}_{\{n\leqslant k\}}\bar{\sigma}\,Cov(S_{t+n-k}, Q_t)}{Var(Q_t)}Q_t, \tag{42}$$

for any $n \geqslant 1$, where $\bar{\sigma}$ is given in (10), $e_0 > 0$, and $e_{i2} > 0$ if and only if $Cov_t^i(Q_{t+1}, q_{t+1}) > 0$.[13]

For comparison, we also study the case without advance information.

Theorem 2. *In the case without advance information,*

$$E[Q_{t+n}|Q_t] = e_0 + e_{i2}a_Z^{n-1}\frac{Cov(Z_t, Q_t)}{Var(Q_t)}Q_t = e_0 + (Ra_Z - 1)a_Z^{n-1}f_Q Q_t, \tag{43}$$

where $e_0, e_{i2} > 0$ and $f_Q > 0$ are constants.

[13] The indicator function $\mathbf{1}_{\{n\leqslant k\}}$ equals 1 if $n \leqslant k$ and 0 otherwise.

Eq. (43) shows how current excess returns Q_t can forecast n-period-ahead, (one-period) excess returns. Empirical studies usually use cumulative n-period returns. Let $Q_{t,t+n} = \sum_{j=1}^{n} Q_{t+j}$ denote the cumulative n-period excess return. We can then derive

$$E[Q_{t,t+n}|Q_t] = ne_0 + (Ra_Z - 1) f_{Qn} Q_t,$$

where $f_{Qn} = (1 + a_Z + \cdots + a_Z^{n-1}) f_Q$. Thus, the properties of momentum and reversals follow from the properties of $E[Q_{t+n}|Q_t]$ given in Eq. (43). This equation demonstrates that the sign of the correlation between Q_{t+n} and Q_t is the same as the sign of $(Ra_Z - 1)$ for all $n \geqslant 1$. This means that the model without advance information cannot simultaneously predict both short-run momentum and long-run reversals in excess returns.[14]

To see the intuition, recall the discussion in the end of Section 4.1. Serial correlation in excess stock returns is driven by the covariance between the expected future risk premium and the current excess return. Without advance information, this covariance is determined exclusively by the covariance between Z_t and Q_t, using (39). The sign of this covariance is the same as that of $(Ra_Z - 1)$, just as in the representative-agent model studied in Section 2. The novelty relative to the model in Section 2 is that there is trading with investor heterogeneity. Following a positive shock to Z_t, informed investors sell the stock and invest in the private investment for rebalancing reasons. Uninformed investors buy the stock in expectation of high excess stock returns Q_{t+1} in the next period. Whether Q_t rises or falls depends on the persistence of Z_t. As in Section 2, we show in Appendix C that $Cov(Q_{t+1}, Q_t) > 0$ if and only if $Ra_Z > 1$. In addition, the covariance between Q_{t+n} and Q_t for $n > 1$ is determined by the covariance between Z_{t+n} and Q_t and hence follows the same sign as the covariance between Q_{t+1} and Q_t. The covariance between Q_{t+n} and Q_t decays at the rate a_Z as n increases as shown in Eq. (43).

The preceding analysis shows that the model without advance information cannot generate momentum and reversal effects simultaneously. We now turn to the model with advance information and Eq. (42). This equation decomposes the covariance between Q_{t+n} and Q_t into two components. The sign of the first component is determined by the sign of $Cov(Z_t, Q_t)$. This sign depends on the persistence of Z_t, as in the model without advance information analyzed earlier. The second component reflects the effect of advance information, which plays a role if and only if $n \leqslant k$, since after period $t + k$ all advance information up to date t loses its value. The sign of this component is determined by the sign of $Cov(S_{t+n-k}, Q_t)$.

From the discussion above, we deduce that our model must have two conditions to generate short-run momentum and long-run reversals simultaneously. First, the persistence a_Z must be sufficiently small. If it is too large, we cannot generate long-run reversals. Therefore, in the discussion below we assume that a_Z is sufficiently small. Second, given a small value of a_Z, we must have $Cov(S_{t+n-k}, Q_t) > 0$. Otherwise, we cannot generate short-run momentum.

The intuition behind the mechanism that generates momentum is similar to that described in Section 2 with the difference that we have to consider trading behavior given investor heterogeneity. Consider first the case of one-period-ahead advance information, i.e., $k = 1$. Suppose that informed investors receive a good signal S_t about future earnings ε_{t+1}^D at time t. This signal is partially incorporated in the stock price and hence raises the current excess return Q_t. Informed investors buy the stock for speculation and act as trend-chasers in response to good advance information. Uninformed investors sell the stock because they perceive the informed

[14] A similar condition is suggested in other papers with different models (e.g., Wang [46]). We note that a similar result applies when $\sigma_{Dq} < 0$. Its proof is available upon request. Also see footnote 5 in Section 2.

investors' speculation trades as rebalancing trades. They thus act as contrarian investors in response to advance information. The good advance information about future high earnings also signals future high returns on the private investment technology since earnings and private investment returns are positively correlated. Thus, informed investors also invest more in the private technology in response to a good piece of advance information. Because informed investors hold both more stocks and more private investments, they bear more aggregate risk, leading to higher expected excess returns. This generates one-period momentum. Momentum occurs because informed trend-chasers are able to hide their information under the guise of rebalancing trades.[15] Reversals occur once advance information materializes in period $t+1$.

We next consider the case of advance information about ε_{t+k}^{D} with $k>1$. We argue that serial correlation in one-period returns may display a cyclical pattern in single period returns at various horizons with negative serial correlations being followed by positive serial correlations and these followed by more negative serial correlations (see Jegadeesh and Titman [29] for such empirical evidence). To understand the intuition, consider the effects of a good signal S_t about $k=2$-period-ahead earnings at date t. This signal raises the stock price P_t and the excess return Q_t. But it is not useful for forecasting earnings or private investment returns at date $t+1$. Thus, informed investors have no incentives to hold more private investment returns for rebalancing. This implies that expected Q_{t+1} (risk premium) does not have to rise. The good signal gives high forecasts of earnings and private investment returns at date $t+2$, inducing investors to hold both more stocks and nontraded assets at time $t+1$. This leads to higher expected Q_{t+2}. After date $t+2$, the advance information signal S_t is useless and hence future excess returns fall.

Now, we conduct some numerical experiments. Table 1 shows the slope coefficients on the forecast of single period returns Q_{t+n} for $n \geqslant 1$, conditional on Q_t as well as the slope coefficient of the forecast of cumulative returns $Q_{t,t+n}$, conditional on Q_t. These equilibrium quantities, and those in the other tables below, can be computed directly from the model and do not require that the model be simulated. We find that when $k=1$, our model generates three-period momentum followed by a stock return reversal. The three-period cumulative return is positive because the first period positive return compensates for the negative return in the second and third periods. When $k=3$, cumulative returns are negative at all horizons.

5.2. *Return continuation and trading behavior*

Unlike the representative-agent model, our model can make predictions regarding investor trading behavior. Specifically, our model predicts that (i) informed investors follow trend-chasing strategies, also called positive feedback trades, and (ii) such trades are profitable. Consistent with these predictions is the finding by Moskowitz et al. [39] of significant time series momentum in various market indexes. Further, they document that the time series momentum can be linked to the trading behavior of speculators and benefits them at the expense of hedgers.

Further evidence can be obtained by inspecting the trading behavior of individual investors. It is common practice to identify uninformed investors with individual investors. There are two main findings about the trading of individual investors. First, individual investors act as contrarian investors, buying when the price is low and selling when the price is high. This finding is consistent with the model and our sources for momentum profits. Second, Kaniel et al. [31] show that return predictability at the individual investor portfolio level arises from own trading and not

[15] The patterns of momentum and reversal can be obtained in a cross section of firms when the firms have cash flows that are independently and identically distributed and comove with the private investment opportunity as described here.

Table 1
Momentum and reversal in the model with a single piece of advance information.

$n \setminus k$	1		2		3	
	One period	Cumulative	One period	Cumulative	One period	Cumulative
1	0.0026	0.0026	−0.0014	−0.0014	−0.0014	−0.0014
2	−0.0013	0.0013	0.0024	0.0010	−0.0013	−0.0028
3	−0.0012	0.0001	−0.0012	−0.0002	0.0022	−0.0006
4	−0.0011	−0.0010	−0.0011	−0.0012	−0.0011	−0.0017
5	−0.0010	−0.0020	−0.0010	−0.0023	−0.0010	−0.0027
6	−0.0009	−0.0028	−0.0009	−0.0032	−0.0009	−0.0036
7	−0.0008	−0.0036	−0.0008	−0.0040	−0.0008	−0.0044
8	−0.0007	−0.0043	−0.0007	−0.0047	−0.0007	−0.0051
9	−0.0006	−0.0050	−0.0006	−0.0054	−0.0007	−0.0058
10	−0.0006	−0.0055	−0.0006	−0.0060	−0.0006	−0.0064

The columns labeled "One period" display the slope coefficients of regressing single period returns, Q_{t+n}, on current returns:

$$Q_{t+n} = a_n + b_n Q_t + \varepsilon_{t,n}.$$

Likewise, the columns labeled "Cumulative" display the slope coefficients of regressing cumulative excess returns, $Q_{t,t+n}$, on current returns, Q_t. n is the holding period and k is the number of advance periods in the advance information. We set $\sigma_D = 1$, $\sigma_F = 0.5$, $\sigma_Z = 1$, $\sigma_q = 0.5$, $\sigma_S = 0.5$, $\sigma_{Dq} = 0.25$, $a_F = a_Z = 0.9$, $\lambda = 0.9$, $\gamma = 5$, and $r = 0.1$.

from past returns. In our paper too, a horse race between trades of uninformed investors and past returns reveals that return predictability arises from the former because we can show that

$$E\left[Q_{t+1}|\theta_t^u, Q_t\right] = E\left[E_t^u[Q_{t+1}]|\theta_t^u, Q_t\right] = E\left[E_t^u[Q_{t+1}]|\theta_t^u\right] = \gamma\left(\sigma_Q^u\right)^2\theta_t^u,$$

where we have used the law of iterated expectations and (36). Kaniel et al. [31] interpretation of their evidence is consistent with our story that individual investors supply the liquidity demanded by institutional investors.

In addition, our model allows us to trace out the implications of advance information for the serial correlation in returns and for mean volume.[16] The left panel in Fig. 1 depicts the effects on momentum and the right panel depicts the effects on volume of different levels of noise in advance information. Volume strictly increases with the amount of noise in advance information. This is intuitive because advance information is private to informed investors. As advance information becomes noisier, the information advantage of informed investors decreases, reducing uninformed investors' adverse selection problem, and resulting in an increase in trading volume.

The effect of noise σ_S in advance information about earnings on momentum is non-monotonic. When advance information is very noisy, it cannot be used by informed investors to forecast future investment opportunities, and the model reduces to the one without advance information. In this case, when a_Z is small, as assumed in the construction of the figure, the model generates reversals. As in Section 2, when advance information is very precise, the hedging incentive is reduced and there are few rebalancing trades in response to advance information. Hence, there is less trading and also momentum disappears. The effects of rebalancing and of speculative trades on momentum are large for intermediate levels of noise in advance information.

[16] Let volume be $vol_t = (1-\lambda)|\Delta\theta_t^u|$. Then $E[vol_t] = (1-\lambda)\sqrt{2var(\Delta\theta_t^u)/\pi}$.

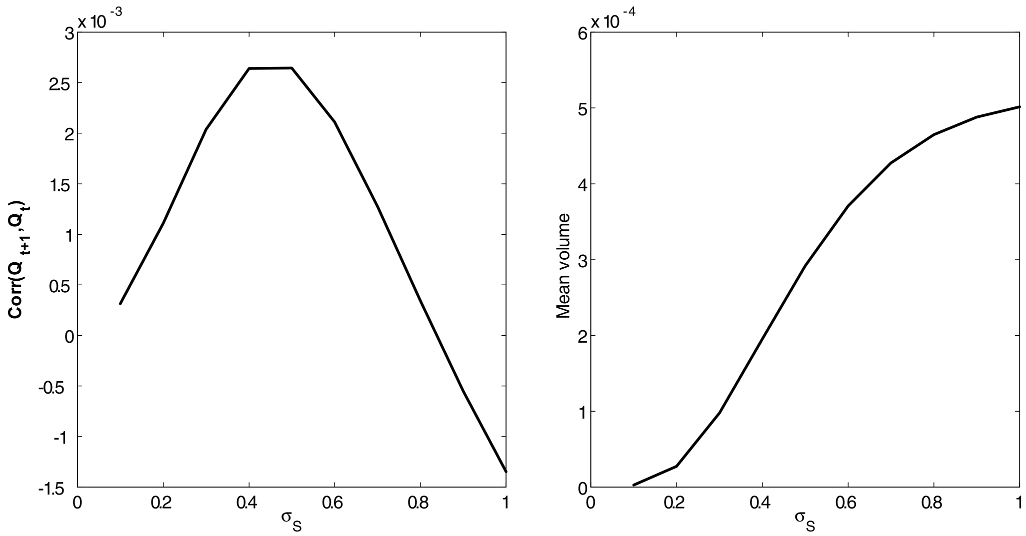

Fig. 1. One period serial correlation in returns (left panel) and trading volume (right panel) for various levels of noise in advance information, σ_S. Parameters are $k = 1$, $\sigma_D = 1$, $\sigma_F = 0.5$, $\sigma_Z = 1$, $\sigma_q = 0.5$, $\sigma_{Dq} = 0.25$, $a_F = a_Z = 0.9$, $\gamma = 5$, $\lambda = 0.9$, and $r = 0.1$.

Fig. 1 suggests that the relationship between momentum and volume may not be monotonic, especially if this relationship is driven by the variation in σ_S. When advance information is very precise, the two variables are positively correlated, but when advance information is very noisy, volume and momentum are negatively correlated. The relationship between volume and momentum when advance information is very precise is consistent with the evidence in Connolly and Stivers [15].

5.3. *Stock market profits*

An interesting question is whether our previously discussed trading strategies cause uninformed investors to systematically lose money. If the stock has zero net supply, then stock market trading is a zero sum game. When some investors make money, investors taking the other side of the trading positions will lose money. Our model does not result in a zero sum game, because the stock has positive net supply. In our model, both informed and uninformed investors can make positive profits from the stock market because there is a return to risk sharing and all investors are risk averse.

In addition, when informed investors trade for rebalancing reasons, uninformed investors trade at favorable prices and earn profits. On the flip side, when informed investors trade for speculative reasons—including after shocks to advance information that generate momentum—uninformed investors lose money. Formally, using (39) and (41), we show that uninformed investors' average profits are positive:

$$E\left[\theta_t^u Q_{t+1}\right] = f_0^u e_0 + e_{i2} f_Z^u \, Var\left(\hat{Z}_t^u + E_t^u\left[\varepsilon_{t+1}^q\right]\right) > 0,$$

where $e_{i2} f_Z^u > 0$. The first term on the right hand side of the above equation derives from a positive risk premium and the second term represents profits from accommodating rebalancing trades.

6. Extensions

6.1. Multiple pieces of advance information

The analysis thus far has shown that the model without advance information cannot generate momentum and reversal effects simultaneously. In addition, the model with one-period-ahead advance information can generate short-run momentum followed by reversals in stock returns. However, momentum only lasts for few periods.

In order to generate long-lived momentum followed by long-run reversals, we extend the model to incorporate multiple pieces of advance information. Specifically, we assume that at time t, informed investors receive a vector of signals $(S_t^k, \dots, S_t^1)$ about earnings innovations at $t+1$ through $t+k$:

$$S_t^k = \varepsilon_{t+k}^D + \varepsilon_t^{S_k}, \dots, S_t^1 = \varepsilon_{t+1}^D + \varepsilon_t^{S_1}. \tag{44}$$

Each $\varepsilon_t^{S_n}$ is assumed to be an i.i.d. normal random variable with mean zero, variance $\sigma_{S_n}^2$, and independent of any other shock. The informed investors' information set is thus given by

$$\mathcal{F}_t^i = \left\{ D_s, F_s, P_s, Z_s, \left(S_s^n\right)_{n=1,\dots,k}\colon s \leqslant t \right\}. \tag{45}$$

This assumption is quite natural as new information, say about end-of-quarter earnings, is likely to arrive at intermediate periods as the quarter nears its end. In addition, stale information is still useful for forecasting, and hence affects stock prices.

The intuition behind this modeling device is that the successive advance information news about the same future earnings can generate long-lived, large speculative trading effects and momentum. An important modeling issue is how to specify the quality of signals. Because up to period t informed investors will have received $k-1$ signals on ε_{t+1}^D already, the stock price increasingly reveals ε_{t+1}^D to the uninformed investors, reducing the motive for speculative trading by informed investors. It is therefore possible that, with too much information in prior periods, only the rebalancing trade motive is at work, generating negative serial correlation in returns. In order to obtain long-lived momentum when $k > 1$, the advance information needs to increase in quality as we approach the earnings realization, i.e., $\sigma_{S_k}^2 > \cdots > \sigma_{S_1}^2$. In this case, return reversals occur after at least k periods as the advance information effect dissipates and the stock price overshoots its long-run mean.

In Appendix D, we show how to solve informed investors' filtering problem. We can then use the previous solution method to show that the equilibrium in this section displays the same form as in Section 4. The only difference is that the informed investors' forecasting problem is different because they now have multiple pieces of advance information. We omit the detailed derivation here and turn to a numerical analysis.

For ease of exposition, we focus on the case with $k = 2$. We begin with a discussion of two limiting results. First, when the signal about two-period-ahead earnings is completely uninformative (i.e., $\sigma_{S_2} = \infty$), the model becomes that in Section 3 with $k = 1$. Consequently, our previous results in Section 3 apply here. Second, when the signal about the two-period-ahead earnings innovation is extremely precise (i.e., $\sigma_{S_2} \to 0$), we find numerically that asymmetric information increases so much, as the incentive to hedge is reduced, that there is no trading in equilibrium. As a result, stock returns are serially uncorrelated. The intuition is as follows. The stock price incorporates the information about the persistent and transitory components of earnings as well as the expected return on the private investment. Uninformed investors use the earnings realizations and the stock price to infer the value of these variables, but may attribute changes in

Table 2
Momentum and reversal in the model with multiple pieces of advance information.

$n \setminus \sigma_{S_1}$	0.1		0.4		0.80	
	One period	Cumulative	One period	Cumulative	One period	Cumulative
1	0.0152	0.0152	0.0738	0.0738	−0.1080	−0.1080
2	0.0141	0.0293	0.1057	0.1796	0.0526	−0.0553
3	−0.0004	0.0289	−0.0517	0.1279	−0.1969	−0.2522
4	−0.0004	0.0285	−0.0465	0.0814	−0.1772	−0.4294
5	−0.0003	0.0282	−0.0419	0.0395	−0.1595	−0.5888
6	−0.0003	0.0279	−0.0377	0.0018	−0.1435	−0.7323
7	−0.0003	0.0277	−0.0339	−0.0321	−0.1292	−0.8615
8	−0.0002	0.0275	−0.0305	−0.0626	−0.1162	−0.9778
9	−0.0002	0.0272	−0.0275	−0.0900	−0.1046	−1.0824
10	−0.0002	0.0271	−0.0247	−0.1148	−0.0942	−1.1765

The columns labeled "One period" display the slope coefficients of regressing single period returns, Q_{t+n}, on current returns:

$$Q_{t+n} = a_n + b_n Q_t + \varepsilon_{t,n}.$$

Likewise, the columns labeled "Cumulative" display the slope coefficients of regressing cumulative excess returns, $Q_{t,t+n}$, on current returns, Q_t. n is the holding period and σ_{S_1} is the variance of advance information about one-period ahead earnings. We set $k = 2$, $\sigma_D = 1$, $\sigma_F = 0.5$, $\sigma_Z = 1$, $\sigma_q = 0.5$, $\sigma_{S2} = 1$, $\sigma_{Dq} = 0.25$, $a_F = a_Z = 0.9$, $\lambda = 0.9$, $\gamma = 5$, and $r = 0.1$. Coefficients are multiplied by 100.

earnings innovations to changes in the various components of earnings or to changes in the private investment return. When informed investors receive very precise information about earnings innovations, they trade on this information more aggressively, and uninformed investors believe that informed investors' trading is generated by a speculative motivate rather than a rebalancing motive. Hence, uninformed investors refrain from trading.

We now turn to intermediate values of σ_{S_2}. Table 2 displays the slope coefficients of the forecast of single period returns Q_{t+n}, conditional on Q_t as well as the slope coefficients of the forecast of cumulative returns $Q_{t,t+n}$, conditional on Q_t for $\sigma_{S_2} = 1$ and various values of σ_{S_1}. This table reveals that our model with advance information about earnings innovations over two successive periods can generate momentum and reversal effects simultaneously. In addition, the duration of momentum depends on the precision of the advance information signals. In particular, when the signal about one-period-ahead earnings becomes more precise relative to the signal about two-period-ahead earnings, the momentum effect lasts longer. On the other hand, when this signal is sufficiently imprecise, the momentum effect disappears.

6.2. *Intertemporal consumption*

The assumption of myopic investors is important for tractability and allows us to derive analytically several equilibrium properties regarding the role of advance information. The main drawback is the omission of dynamic hedging demands. Dynamic hedging demands reflect a concern for stochastic changes in the investment opportunity set induced by changes in the state vector, and thus may be particularly relevant in a model with advance information where investors receive signals about k-period-ahead earnings.

In an online appendix, we solve a model where investors derive utility over infinite streams of consumption, keeping the rest of the structure as in the model of Section 3. To hedge against changes in investment opportunities, investors hold more of the stock if the stock pays out more in states where investment opportunities are bad. In particular, good advance information about future earnings on the stock implies that good investment opportunities are likely for both the stock and the private investment opportunity in the future and makes informed investors hold less of both assets.

We use numerical examples to evaluate the relative importance of the myopic demand studied earlier and the dynamic hedging demand. We find that the dynamic hedging demand is not important quantitatively. In response to a good signal about earnings innovation in the next period, the stock return increases, informed investors buy the stock for speculative reasons and uninformed investors sell the stock to accommodate these trades on impact. Informed investors also invest more in the nontraded asset, as they did in the myopic case, and thus bear greater risk. This leads stock returns to display short-run momentum.

Momentum is not monotonic with the precision of the advance information signal. As in Section 5.2, when advance information is very precise, there are few rebalancing trades in the nontraded asset, and momentum disappears. When advance information is very noisy, we are back in a model without advance information and with a_Z small, momentum also disappears. It is at intermediate levels of precision of advance information that the dynamic hedging demands become more important. Because dynamic hedging demands also allow informed investors to hide their speculative trades, momentum is stronger than in the myopic case.

7. Conclusion

In this paper, we study the implications of investors' trading behavior when some of the investors possess advance information. This information provides a new source of shocks that drives the serial correlation of excess returns. It helps generate the momentum and reversal effects in excess returns in a unified way. Our key insight is to recognize that advance information in the form of signals about future earnings that are uncorrelated with current earnings is informative about informed investors' other (private) investment opportunities. In response to a positive shock to advance information, the stock price rises and informed investors can profit by following trend-chasing strategies. They also invest more in private investment opportunities, leading to more aggregate risk to be borne so that the risk premium rises. This generates short-run momentum. When the advance information materializes, future excess returns fall generating long-run reversals.

We may extend our model in several directions. First, our model focuses on the implications of advance information for momentum and reversal effects. It would be interesting to further study the implications for trading volume, as in Wang [47] and LIorente et al. [35]. Second, we follow Wang [47] and assume a hierarchical information structure. Considering the case where information is symmetrically dispersed might be worthwhile (see, e.g., Bacchetta and van Wincoop [3,4] and Albagli et al. [1]). In this case, higher order expectations play an important role.

Appendix A. Derivation of equilibrium in Section 2

Substituting (7) into (1), we can derive the first-order conditions:

$$V_Q + \alpha_t V_{Qq} = \gamma^{-1}\mu_{Qt}, \tag{A.1}$$

$$V_{Qq} + \alpha_t V_q = \gamma^{-1}\mu_{qt}, \tag{A.2}$$

where we have imposed the market clearing condition $\theta_t = 1$. In these equations,

$$\mu_{Qt} = E_t[Q_{t+1}], \qquad \mu_{qt} = E_t[q_{t+1}], \qquad V_Q = Var_t(Q_{t+1}),$$
$$V_q = Var_t(q_{t+1}), \qquad V_{Qq} = Cov_t(Q_{t+1}, q_{t+1}).$$

We will show below that V_Q, V_q, and V_{Qq} are constant after we solve for the equilibrium price P_t. Using (A.1) yields:

$$E[Q_{t+1}|Q_t] = E[\mu_{Qt}|Q_t] = \gamma\big(V_Q + V_{Qq}E[\alpha_t|Q_t]\big).$$

Solving the system of Eqs. (A.1) and (A.2) yields:

$$\alpha_t = \gamma^{-1}\frac{\mu_{qt}}{V_q} - \frac{V_{Qq}}{V_q}, \tag{A.3}$$

$$\mu_{Qt} = \frac{V_{Qq}\mu_{qt}}{V_q} - \gamma\left(\frac{V_{Qq}^2}{V_q} - V_Q\right). \tag{A.4}$$

With advance information about next period's dividend innovations,

$$\mu_{qt} = Z_t + E\big[\varepsilon_{t+1}^q|S_t\big] = Z_t + \bar{\sigma}S_t,$$
$$V_q = Var\big[\varepsilon_{t+1}^q|S_t\big] = \sigma_q^2 - \frac{\sigma_{Dq}^2}{\sigma_S^2 + \sigma_D^2}, \tag{A.5}$$

where

$$\bar{\sigma} = \frac{\sigma_{Dq}}{\sigma_S^2 + \sigma_D^2}.$$

By the definition of μ_{Qt} and

$$E_t[D_{t+1}] = E_t\big[F_{t+1} + \varepsilon_{t+1}^D\big] = a_F F_t + \frac{\sigma_D^2}{\sigma_S^2 + \sigma_D^2}S_t,$$

we obtain

$$\mu_{Qt} = E_t[P_{t+1}] + E_t[D_{t+1}] - RP_t$$
$$= E_t[P_{t+1}] + a_F F_t + \frac{\sigma_D^2}{\sigma_S^2 + \sigma_D^2}S_t - RP_t.$$

Substituting (A.5) into (A.4) and using the above equation, we obtain a difference equation for P_t:

$$E_t[P_{t+1}] + a_F F_t + \frac{\sigma_D^2}{\sigma_S^2 + \sigma_D^2}S_t - RP_t = \frac{V_{Qq}[Z_t + \bar{\sigma}S_t]}{V_q} - \gamma\left(\frac{V_{Qq}^2}{V_q} - V_Q\right).$$

Solving this equation yields:

$$P_t = -\gamma\frac{V_Q V_q - V_{Qq}^2}{V_q}\frac{R^{-1}}{1 - R^{-1}} + \frac{R^{-1}a_F}{1 - R^{-1}a_F}F_t$$
$$- \frac{R^{-1}V_{Qq}}{V_q(1 - R^{-1}a_Z)}Z_t + R^{-1}\left[\frac{\sigma_D^2}{\sigma_S^2 + \sigma_D^2} - \frac{V_{Qq}\bar{\sigma}}{V_q}\right]S_t. \tag{A.6}$$

We can easily compute f_t given in (8). Using the above price equation, we then obtain Eq. (9).

Next, we can compute the excess stock return using (A.6):

$$Q_{t+1} = \gamma \frac{V_Q V_q - V_{Qq}^2}{V_q} + \frac{1}{1-R^{-1}a_F}\varepsilon_{t+1}^F + \left(\varepsilon_{t+1}^D - \frac{\sigma_D^2}{\sigma_S^2+\sigma_D^2} S_t\right) - \frac{R^{-1}V_{Qq}}{V_q(1-R^{-1}a_Z)}\varepsilon_{t+1}^Z + \frac{V_{Qq}}{V_q}(Z_t + \bar{\sigma} S_t) + R^{-1}\left[\frac{\sigma_D^2}{\sigma_S^2+\sigma_D^2} - \frac{V_{Qq}\bar{\sigma}}{V_q}\right] S_{t+1}.$$

Using this equation, we can derive that

$$V_Q = \frac{\sigma_F^2}{(1-R^{-1}a_F)^2} + \frac{\sigma_D^2\sigma_S^2}{\sigma_S^2+\sigma_D^2} + \left(\frac{R^{-1}V_{Qq}}{V_q(1-R^{-1}a_Z)}\right)^2 \sigma_Z^2 + R^{-2}\left[\frac{\sigma_D^2}{\sigma_S^2+\sigma_D^2} - \frac{V_{Qq}\bar{\sigma}}{V_q}\right]^2 (\sigma_S^2+\sigma_D^2),$$

$$V_{Qq} = E_t\left[\left(\varepsilon_{t+1}^D - E_t\varepsilon_{t+1}^D\right)\left(\varepsilon_{t+1}^q - E_t\varepsilon_{t+1}^q\right)\right] = \frac{\sigma_{Dq}\sigma_S^2}{\sigma_S^2+\sigma_D^2}.$$

Appendix B. Proofs for the model in Section 3

We first solve informed investors' filtering problem using the state space representation. This filtering problem can deliver (31) and (32). Though these equations can be easily derived using the projection theorem, we adopt the more complicated state space representation method because it is useful for solving uninformed investors' filtering problem. After, solving filtering problems, we prove Propositions 1–2 and Theorem 1.

Informed investors' filtering problem. We use the following state-space system representation

$$\mathbf{x}_t = \mathbf{A}_x \mathbf{x}_{t-1} + \mathbf{B}_x \boldsymbol{\varepsilon}_t,$$

where we write $\mathbf{A}_x$ and $\mathbf{B}_x$ as

$$\mathbf{A}_x = \begin{bmatrix} a_F & & & \\ & a_Z & & \\ & & 0 & \\ & & \mathbf{I}_k & 0 \end{bmatrix}_{k+3}, \qquad \mathbf{B}_x = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ \mathbf{0}_{k\times 5} & & \cdots & & \end{bmatrix}_{[k+3]\times 5}.$$

Note that $\mathbf{A}_x$ has one column with zeros only, column $k+3$ associated with ε_t^D. The informed investors' observable signals are summarized in the vector $\mathbf{y}_t^i = (D_t, F_t, Z_t, S_t)^\top$. This vector satisfies

$$\mathbf{y}_t^i = \mathbf{A}_{yi}\mathbf{x}_t + \mathbf{B}_{yi}\boldsymbol{\varepsilon}_t,$$

where we write $\mathbf{A}_{yi}$ as

$$\mathbf{A}_{yi} = \begin{bmatrix} 1 & 0 & \cdots & & 0 & 1 \\ 1 & 0 & & & & \\ 0 & 1 & 0 & & & \\ 0 & 0 & 1 & 0 & \cdots & 0 \end{bmatrix}_{4\times[k+3]}, \qquad \mathbf{B}_{yi} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{c}_5 \end{bmatrix}_{4\times 5}.$$

Note that the first two components of $\hat{\mathbf{x}}_t^i$ are given by F_t and Z_t since they are observable. Also, since D_t and F_t are observable, $\hat{\mathbf{x}}_t^i$ contains ε_t^D.

We can now derive the steady-state Kalman filters:

$$\hat{\mathbf{x}}_t^i = \mathbf{A}_x \hat{\mathbf{x}}_{t-1}^i + \mathbf{K}_i \hat{\boldsymbol{\varepsilon}}_t^i \tag{B.1}$$

and

$$\mathbf{y}_t^i = \mathbf{A}_{yi} \mathbf{A}_x \hat{\mathbf{x}}_{t-1}^i + \hat{\boldsymbol{\varepsilon}}_t^i, \tag{B.2}$$

where the innovation $\hat{\boldsymbol{\varepsilon}}_t^i = \mathbf{y}_t^i - E_{t-1}^i[\mathbf{y}_t^i]$ is normally distributed with mean zero and variance

$$\boldsymbol{\Sigma}_i = E\big[\hat{\boldsymbol{\varepsilon}}_t^i\big(\hat{\boldsymbol{\varepsilon}}_t^i\big)^\intercal\big] = \mathbf{A}_{yi}\mathbf{A}_x \boldsymbol{\Omega}_i \mathbf{A}_x^\intercal \mathbf{A}_{yi}^\intercal + (\mathbf{A}_{yi}\mathbf{B}_x + \mathbf{B}_{yi})\boldsymbol{\Sigma}(\mathbf{A}_{yi}\mathbf{B}_x + \mathbf{B}_{yi})^\intercal. \tag{B.3}$$

We focus on the steady-state Kalman filtering as in Wang [47]. The covariance matrix $\boldsymbol{\Omega}_i = E_t^i[(\mathbf{x}_t - \hat{\mathbf{x}}_t^i)(\mathbf{x}_t - \hat{\mathbf{x}}_t^i)^\intercal]$ and the Kalman gain matrix $\mathbf{K}_i$ satisfy

$$\boldsymbol{\Omega}_i = \big(\mathbf{A}_x \boldsymbol{\Omega}_i \mathbf{A}_x^\intercal + \boldsymbol{\Sigma}_{xx}\big) - \mathbf{K}_i \mathbf{A}_{yi}\big(\mathbf{A}_x \boldsymbol{\Omega}_i \mathbf{A}_x^\intercal + \boldsymbol{\Sigma}_{xx}\big), \tag{B.4}$$

and

$$\mathbf{K}_i = \big(\mathbf{A}_x \boldsymbol{\Omega}_i \mathbf{A}_x^\intercal + \boldsymbol{\Sigma}_{xx}\big)\mathbf{A}_{yi}^\intercal\big[\mathbf{A}_{yi}\big(\mathbf{A}_x \boldsymbol{\Omega}_i \mathbf{A}_x^\intercal + \boldsymbol{\Sigma}_{xx}\big)\mathbf{A}_{yi}^\intercal + \boldsymbol{\Sigma}_{yy}\big]^{-1}, \tag{B.5}$$

where we define

$$\boldsymbol{\Sigma}_{xx} = \mathbf{B}_x \boldsymbol{\Sigma} \mathbf{B}_x^\intercal, \qquad \boldsymbol{\Sigma}_{yy} = \mathbf{B}_{yi} \boldsymbol{\Sigma} \mathbf{B}_{yi}^\intercal. \tag{B.6}$$

Uninformed investors' filtering problem. Uninformed investors observe past dividends and prices. Recall the discussion in Section 4.1, observing the price P_t is equivalent to observing $\Pi_t = \mathbf{p}_i \hat{\mathbf{x}}_t^i$ for uninformed investors. Thus, we can write their observation system as

$$\mathbf{y}_t^u = \begin{bmatrix} \mathbf{p}_i \hat{\mathbf{x}}_t^i \\ D_t \end{bmatrix} = \mathbf{A}_{yu} \hat{\mathbf{x}}_t^i,$$

where

$$\mathbf{A}_{yu} = \begin{bmatrix} \mathbf{p}_i \\ \mathbf{c}_1 + \mathbf{c}_{k+3} \end{bmatrix}_{2\times(k+3)}.$$

Here $\mathbf{c}_j$ is the standard row vector with the jth element being 1 and the rest being zero. Unlike informed investors, uninformed investors face state dynamics given in (B.1). By the Kalman filtering theory (see, e.g., Wang [47]), uninformed investors' conditional forecast of the state vector is given by the steady-state Kalman filters:

$$\hat{\mathbf{x}}_t^u = \mathbf{A}_x \hat{\mathbf{x}}_{t-1}^u + \mathbf{K}_u \hat{\boldsymbol{\varepsilon}}_t^u, \tag{B.7}$$

and

$$\hat{\boldsymbol{\varepsilon}}_t^u \equiv \mathbf{y}_t^u - E_{t-1}^u\big[\mathbf{y}_t^u\big], \tag{B.8}$$

where $\hat{\boldsymbol{\varepsilon}}_t^u$ is normally distributed with mean zero and covariance matrix

$$\boldsymbol{\Sigma}_u = \mathbf{A}_{yu}\mathbf{A}_x \boldsymbol{\Omega}_u \mathbf{A}_x^\intercal \mathbf{A}_{yu}^\intercal + \mathbf{A}_{yu}\mathbf{K}_i \boldsymbol{\Sigma}_i \mathbf{K}_i^\intercal \mathbf{A}_{yu}^\intercal.$$

Moreover, the covariance matrix $\boldsymbol{\Omega}_u = E_t^u[(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u)(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u)^\intercal]$ and the Kalman gain matrix $\mathbf{K}_u$ satisfy

$$\boldsymbol{\Omega}_u = \left(\mathbf{A}_x \boldsymbol{\Omega}_u \mathbf{A}_x^\top + \boldsymbol{\Sigma}_{xx}\right) - \mathbf{K}_u \mathbf{A}_{yu}\left(\mathbf{A}_x \boldsymbol{\Omega}_u \mathbf{A}_x^\top + \boldsymbol{\Sigma}_{xx}\right), \tag{B.9}$$

where

$$\mathbf{K}_u = \left(\mathbf{A}_x \boldsymbol{\Omega}_u \mathbf{A}_x^\top + \boldsymbol{\Sigma}_{xx}\right)\mathbf{A}_{yu}^\top\left[\mathbf{A}_{yu}\left(\mathbf{A}_x \boldsymbol{\Omega}_u \mathbf{A}_x^\top + \boldsymbol{\Sigma}_{xx}\right)\mathbf{A}_{yu}^\top\right]^{-1}, \tag{B.10}$$

and $\boldsymbol{\Sigma}_{xx} = \mathbf{K}_i \boldsymbol{\Sigma}_i \mathbf{K}_i^\top$.

Proof of Proposition 1. We first use the conjectured price function (24) to derive

$$\begin{aligned} Q_{t+1} &= P_{t+1} + D_{t+1} - RP_t \\ &= -p_0 + \mathbf{p}_i \hat{\mathbf{x}}_{t+1}^i + \mathbf{p}_u \mathbf{I}_{-2} \hat{\mathbf{x}}_{t+1}^u + F_{t+1} + \varepsilon_{t+1}^D - R\left(-p_0 + \mathbf{p}_i \hat{\mathbf{x}}_t^i + \mathbf{p}_u \mathbf{I}_{-2} \hat{\mathbf{x}}_t^u\right) \\ &= e_0 + \mathbf{e}_i \hat{\mathbf{x}}_t^i + \mathbf{e}_u \hat{\mathbf{x}}_t^u + \mathbf{b}_Q \hat{\boldsymbol{\varepsilon}}_{t+1}^i, \end{aligned} \tag{B.11}$$

where

$$e_0 = rp_0, \tag{B.12}$$

$$\mathbf{e}_i = (\mathbf{p}_i + \mathbf{c}_1 + \mathbf{c}_{k+3})\mathbf{A}_x - R\mathbf{p}_i + \mathbf{p}_u \mathbf{I}_{-2} \mathbf{K}_u \mathbf{A}_{yu} \mathbf{A}_x \left(\mathbf{I}_{-2} - \mathbf{c}_2^\top \frac{1}{p_{i2}} \mathbf{p}_i \mathbf{I}_{-2}\right), \tag{B.13}$$

$$\mathbf{e}_u = \mathbf{p}_u \mathbf{I}_{-2} \mathbf{A}_x - R\mathbf{p}_u \mathbf{I}_{-2} - \mathbf{p}_u \mathbf{I}_{-2} \mathbf{K}_u \mathbf{A}_{yu} \mathbf{A}_x \left(\mathbf{I}_{-2} - \mathbf{c}_2^\top \frac{1}{p_{i2}} \mathbf{p}_i \mathbf{I}_{-2}\right), \tag{B.14}$$

$$\mathbf{b}_Q = (\mathbf{p}_i + \mathbf{c}_1 + \mathbf{c}_{k+3})\mathbf{K}_i + \mathbf{p}_u \mathbf{I}_{-2} \mathbf{K}_u \mathbf{A}_{yu} \mathbf{K}_i, \tag{B.15}$$

and where we have used

$$\begin{aligned} \hat{\boldsymbol{\varepsilon}}_{t+1}^u &= \mathbf{y}_{t+1}^u - \mathbf{A}_{yu} \mathbf{A}_x \hat{\mathbf{x}}_t^u \\ &= \mathbf{A}_{yu} \hat{\mathbf{x}}_{t+1}^i - \mathbf{A}_{yu} \mathbf{A}_x \hat{\mathbf{x}}_t^u \\ &= \mathbf{A}_{yu} \mathbf{A}_x \left(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u\right) + \mathbf{A}_{yu} \mathbf{K}_i \hat{\boldsymbol{\varepsilon}}_{t+1}^i \\ &= \mathbf{A}_{yu} \mathbf{A}_x \left(\mathbf{I}_{-2} - \mathbf{c}_2^\top \frac{1}{p_{i2}} \mathbf{p}_i \mathbf{I}_{-2}\right)\left(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u\right) + \mathbf{A}_{yu} \mathbf{K}_i \hat{\boldsymbol{\varepsilon}}_{t+1}^i, \end{aligned} \tag{B.16}$$

noting that ε_{t+1}^D—and not its expectation—is in $\hat{\mathbf{x}}_{t+1}^i$ at time $t+1$. The last equality follows from (26),

$$\hat{Z}_t^i - \hat{Z}_t^u = \frac{1}{p_{i2}} \mathbf{p}_i \mathbf{I}_{-2}\left(\hat{\mathbf{x}}_t^u - \hat{\mathbf{x}}_t^i\right),$$

and from

$$\begin{aligned} \hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u &= \mathbf{I}_{-2}\left(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u\right) + \mathbf{c}_2^\top\left(\hat{Z}_t^i - \hat{Z}_t^u\right) \\ &= \mathbf{I}_{-2}\left(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u\right) - \mathbf{c}_2^\top \frac{1}{p_{i2}} \mathbf{p}_i \mathbf{I}_{-2}\left(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u\right) \\ &= \left(\mathbf{I}_{-2} - \mathbf{c}_2^\top \frac{1}{p_{i2}} \mathbf{p}_i \mathbf{I}_{-2}\right)\left(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u\right). \end{aligned} \tag{B.17}$$

We next derive the conditional expectations:

$$E_t^i[Q_{t+1}] = e_0 + \mathbf{e}_i \hat{\mathbf{x}}_t^i + \mathbf{e}_u \hat{\mathbf{x}}_t^u, \tag{B.18}$$

$$E_t^u[Q_{t+1}] = e_0 + (\mathbf{e}_i + \mathbf{e}_u)\hat{\mathbf{x}}_t^u. \tag{B.19}$$

We use (26) to substitute out $\hat{Z}_t^u$ in $E_t^u[Q_{t+1}]$ (because likely $e_{i2} \neq 0$) and derive

$$\begin{aligned}
E_t^u[Q_{t+1}] &= e_0 + (\mathbf{e}_i + \mathbf{e}_u)\hat{\mathbf{x}}_t^u \\
&= e_0 + \mathbf{e}_i\big[\mathbf{I}_{-2}\hat{\mathbf{x}}_t^u + \mathbf{c}_2^{\mathsf{T}}\hat{Z}_t^u\big] + \mathbf{e}_u\hat{\mathbf{x}}_t^u \\
&= e_0 + \mathbf{e}_i\left[\mathbf{I}_{-2}\hat{\mathbf{x}}_t^u + \mathbf{c}_2^{\mathsf{T}}\hat{Z}_t^i - \frac{1}{p_{i2}}\mathbf{c}_2^{\mathsf{T}}\mathbf{p}_i\mathbf{I}_{-2}\big(\hat{\mathbf{x}}_t^u - \hat{\mathbf{x}}_t^i\big)\right] + \mathbf{e}_u\hat{\mathbf{x}}_t^u \\
&= e_0 + \mathbf{e}_i\left(\mathbf{c}_2^{\mathsf{T}}\mathbf{c}_2 + \frac{1}{p_{i2}}\mathbf{c}_2^{\mathsf{T}}\mathbf{p}_i\mathbf{I}_{-2}\right)\hat{\mathbf{x}}_t^i + \left[\mathbf{e}_i\left(\mathbf{I}_{-2} - \frac{1}{p_{i2}}\mathbf{c}_2^{\mathsf{T}}\mathbf{p}_i\mathbf{I}_{-2}\right) + \mathbf{e}_u\right]\hat{\mathbf{x}}_t^u \\
&= e_0 + \tilde{\mathbf{e}}_i\hat{\mathbf{x}}_t^i + \tilde{\mathbf{e}}_u\hat{\mathbf{x}}_t^u.
\end{aligned}$$

It is easy to derive

$$E_t^i[q_{t+1}] = E_t^i\big[Z_t + \varepsilon_{t+1}^q\big] = \left(\mathbf{c}_2 + \frac{\sigma_{Dq}}{\sigma_D^2}\mathbf{c}_{k+2}\right)\hat{\mathbf{x}}_t^i.$$

We can also derive the conditional variances

$$\begin{aligned}
Var_t^i(Q_{t+1}) &= \mathbf{b}_Q\boldsymbol{\Sigma}_i\mathbf{b}_Q^{\mathsf{T}}, \\
Var_t^i(q_{t+1}) &= Var_t^i\big(\varepsilon_{t+1}^q\big) = \sigma_q^2 - \frac{\sigma_{Dq}^2}{\sigma_D^2 + \sigma_S^2}, \\
Cov_t^i(Q_{t+1}, q_{t+1}) &= \mathbf{b}_Q E_t^i\big[\hat{\boldsymbol{\varepsilon}}_{t+1}^i\big(\varepsilon_{t+1}^q - E_t^i\big(\varepsilon_{t+1}^q\big)\big)\big] \\
&= \mathbf{b}_Q E_t^i\begin{bmatrix} \varepsilon_{t+1}^F + \varepsilon_{t+1}^D - E_t^i(\varepsilon_{t+1}^D) \\ \varepsilon_{t+1}^F \\ \varepsilon_{t+1}^Z \\ S_{t+1} \end{bmatrix}\big(\varepsilon_{t+1}^q - E_t^i\big(\varepsilon_{t+1}^q\big)\big), \\
&= \mathbf{b}_Q^{(1)} E_t^i\big[\big(\varepsilon_{t+1}^D - E_t^i\big(\varepsilon_{t+1}^D\big)\big)\big(\varepsilon_{t+1}^q - E_t^i\big(\varepsilon_{t+1}^q\big)\big)\big] \\
&= \mathbf{b}_Q^{(1)}\frac{\sigma_{Dq}\sigma_S^2}{\sigma_D^2 + \sigma_S^2},
\end{aligned}$$

where $\mathbf{b}_Q^{(1)}$ is the first element of the vector $\mathbf{b}_Q$.

Furthermore, we can derive the uninformed investors' conditional variance

$$\begin{aligned}
Var_t^u(Q_{t+1}) &= E_t^u\big[\big(\mathbf{e}_i\big(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u\big) + \mathbf{b}_Q\hat{\boldsymbol{\varepsilon}}_{t+1}^i\big)\big(\mathbf{e}_i\big(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u\big) + \mathbf{b}_Q\hat{\boldsymbol{\varepsilon}}_{t+1}^i\big)^{\mathsf{T}}\big] \\
&= \mathbf{e}_i\boldsymbol{\Omega}_u\mathbf{e}_i^{\mathsf{T}} + \mathbf{b}_Q\boldsymbol{\Sigma}_i\mathbf{b}_Q^{\mathsf{T}},
\end{aligned}$$

where we have used the following result:

$$E_t^u\big[\big(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u\big)\hat{\boldsymbol{\varepsilon}}_{t+1}^{i\mathsf{T}}\big] = E_t^u\big[E_t^i\big[\big(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u\big)\hat{\boldsymbol{\varepsilon}}_{t+1}^{i\mathsf{T}}\big]\big] = E_t^u\big[\big(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u\big)E_t^i\big[\hat{\boldsymbol{\varepsilon}}_{t+1}^{i\mathsf{T}}\big]\big] = 0.$$

Now, we use (34) and (36) to show that optimal stock holdings are linear functions of $\hat{\mathbf{x}}_t^i$ and $\hat{\mathbf{x}}_t^u$. Using the preceding conditional expectations we get

$$\theta_t^i = \frac{e_0 + \mathbf{e}_i\hat{\mathbf{x}}_t^i + \mathbf{e}_u\hat{\mathbf{x}}_t^u}{\gamma(\sigma_Q^i)^2(1 - (\rho_{Qq}^i)^2)} - \frac{\rho_{Qq}^i(\mathbf{c}_2 + \frac{\sigma_{Dq}}{\sigma_D^2}\mathbf{c}_{k+2})\hat{\mathbf{x}}_t^i}{\gamma\sigma_Q^i\sigma_q^i(1 - (\rho_{Qq}^i)^2)}, \tag{B.20}$$

$$\theta_t^u = \frac{e_0 + \tilde{\mathbf{e}}_i\hat{\mathbf{x}}_t^i + \tilde{\mathbf{e}}_u\hat{\mathbf{x}}_t^u}{\gamma(\sigma_Q^u)^2}. \tag{B.21}$$

We use the market clearing condition (22) to determine the coefficients in the price function. Substituting (B.20) and (B.21) into (22) yields

$$1=\frac{\lambda}{\gamma}\left[\frac{e_0+\mathbf{e}_i\hat{\mathbf{x}}_t^i+\mathbf{e}_u\hat{\mathbf{x}}_t^u}{(\sigma_Q^i)^2(1-\rho_{Qq}^2)}-\frac{\rho_{Qq}(\mathbf{c}_2+\frac{\sigma_{Dq}}{\sigma_D^2}\mathbf{c}_{k+2})\hat{\mathbf{x}}_t^i}{\sigma_Q^i\sigma_q^i(1-\rho_{Qq}^2)}\right]+(1-\lambda)\frac{e_0+\tilde{\mathbf{e}}_i\hat{\mathbf{x}}_t^i+\tilde{\mathbf{e}}_u\hat{\mathbf{x}}_t^u}{\gamma(\sigma_Q^u)^2}.$$

Matching coefficients on the constant, $\hat{\mathbf{x}}_t^i$, and $\hat{\mathbf{x}}_t^u$ we obtain

$$e_0=\gamma\frac{(\sigma_Q^i)^2(\sigma_Q^u)^2(1-\rho_{Qq}^2)}{\lambda(\sigma_Q^u)^2+(1-\lambda)(\sigma_Q^i)^2(1-\rho_{Qq}^2)}>0, \tag{B.22}$$

$$\frac{\lambda}{\gamma}\left[\frac{\mathbf{e}_i}{(\sigma_Q^i)^2(1-(\rho_{Qq}^i)^2)}-\frac{\rho_{Qq}^i(\mathbf{c}_2+\frac{\sigma_{Dq}}{\sigma_D^2}\mathbf{c}_{k+2})}{\sigma_Q^i\sigma_q^i(1-(\rho_{Qq}^i)^2)}\right]+\frac{1-\lambda}{\gamma}\frac{\mathbf{e}_i(\mathbf{c}_2^\intercal\mathbf{c}_2+\frac{1}{p_{i2}}\mathbf{c}_2^\intercal\mathbf{p}_i\mathbf{I}_{-2})}{(\sigma_Q^u)^2}=0, \tag{B.23}$$

$$0=\frac{\lambda}{\gamma}\frac{\mathbf{e}_u}{(\sigma_Q^i)^2(1-\rho_{Qq}^2)}+(1-\lambda)\frac{\mathbf{e}_i(\mathbf{I}_{-2}-\frac{1}{p_{i2}}\mathbf{c}_2^\intercal\mathbf{p}_i\mathbf{I}_{-2})+\mathbf{e}_u}{\gamma(\sigma_Q^u)^2}. \tag{B.24}$$

Eqs. (B.12) and (B.22) imply that $p_0>0$. Using

$$\mathbf{e}_i\left(\mathbf{c}_2^\intercal\mathbf{c}_2+\frac{1}{p_{i2}}\mathbf{c}_2^\intercal\mathbf{p}_i\mathbf{I}_{-2}\right)=e_{i2}\begin{bmatrix}\frac{p_{i1}}{p_{i2}} & 1 & \frac{p_{i3}}{p_{i2}} & \cdots & \frac{p_{i,k+3}}{p_{i2}}\end{bmatrix},$$

together with (B.23), we have

$$\begin{aligned}
&\frac{\lambda}{\gamma}\frac{e_{i1}}{(\sigma_Q^i)^2(1-(\rho_{Qq}^i)^2)}+\frac{1-\lambda}{\gamma}\frac{e_{i2}\frac{p_{i1}}{p_{i2}}}{(\sigma_Q^u)^2}=0,\\
&\frac{\lambda}{\gamma}\left[\frac{e_{i2}}{(\sigma_Q^i)^2(1-(\rho_{Qq}^i)^2)}-\frac{\rho_{Qq}^i}{\sigma_Q^i\sigma_q^i(1-(\rho_{Qq}^i)^2)}\right]+\frac{1-\lambda}{\gamma}\frac{e_{i2}}{(\sigma_Q^u)^2}=0,\\
&\frac{\lambda}{\gamma}\frac{e_{i3}}{(\sigma_Q^i)^2(1-(\rho_{Qq}^i)^2)}+\frac{1-\lambda}{\gamma}\frac{e_{i2}\frac{p_{i3}}{p_{i2}}}{(\sigma_Q^u)^2}=0,\\
&\quad\vdots\\
&\frac{\lambda}{\gamma}\left[\frac{e_{i,k+2}}{(\sigma_Q^i)^2(1-(\rho_{Qq}^i)^2)}-\frac{\frac{\sigma_{Dq}}{\sigma_D^2}\rho_{Qq}^i}{\sigma_Q^i\sigma_q^i(1-(\rho_{Qq}^i)^2)}\right]+\frac{1-\lambda}{\gamma}\frac{e_{i2}\frac{p_{i,k+2}}{p_{i2}}}{(\sigma_Q^u)^2}=0,\\
&\frac{\lambda}{\gamma}\frac{e_{ik+3}}{(\sigma_Q^i)^2(1-(\rho_{Qq}^i)^2)}+\frac{1-\lambda}{\gamma}\frac{e_{i2}\frac{p_{ik+3}}{p_{i2}}}{(\sigma_Q^u)^2}=0.
\end{aligned}$$

Solving these equation gives

$$e_{i2}=\frac{\lambda(\sigma_Q^u)^2}{\lambda(\sigma_Q^u)^2+(1-\lambda)(\sigma_Q^i)^2(1-(\rho_{Qq}^i)^2)}\frac{Cov_t^i(Q_{t+1},q_{t+1})}{(\sigma_q^i)^2}, \tag{B.25}$$

$$e_{i1}=-\frac{p_{i1}}{p_{i2}}\frac{(1-\lambda)(\sigma_Q^i)^2(1-(\rho_{Qq}^i)^2)}{\lambda(\sigma_Q^u)^2+(1-\lambda)(\sigma_Q^i)^2(1-(\rho_{Qq}^i)^2)}\frac{Cov_t^i(Q_{t+1},q_{t+1})}{(\sigma_q^i)^2}, \tag{B.26}$$

$$
\begin{aligned}
e_{i3} &= \frac{p_{i3}}{p_{i1}} e_{i1}, \\
&\vdots \\
e_{i,k+2} &= \left[\frac{\sigma_{Dq}}{\sigma_D^2} - \frac{p_{i,k+2}}{p_{i2}} \frac{(1-\lambda)(\sigma_Q^i)^2(1-(\rho_{Qq}^i)^2)}{\lambda(\sigma_Q^u)^2 + (1-\lambda)(\sigma_Q^i)^2(1-(\rho_{Qq}^i)^2)} \right] \frac{Cov_t^i(Q_{t+1}, q_{t+1})}{(\sigma_q^i)^2}, \\
e_{i,k+3} &= \frac{p_{i,k+3}}{p_{i1}} e_{i1}.
\end{aligned} \tag{B.27}
$$

Eq. (B.25) implies that $e_{i2} > 0$ if and only if $Cov_t^i(Q_{t+1}, q_{t+1}) > 0$.

Turning now to (B.24), we use

$$
\mathbf{e}_i \left(\mathbf{I}_{-2} - \frac{1}{p_{i2}} \mathbf{c}_2^\top \mathbf{p}_i \mathbf{I}_{-2} \right) = \left[e_{i1} - e_{i2} \frac{p_{i1}}{p_{i2}} \quad 0 \quad e_{i3} - e_{i2} \frac{p_{i3}}{p_{i2}} \quad \cdots \quad e_{i,k+3} - e_{i2} \frac{p_{i,k+3}}{p_{i2}} \right],
$$

to derive

$$
\begin{aligned}
0 &= \frac{\lambda}{\gamma} \frac{e_{u1}}{(\sigma_Q^i)^2(1-\rho_{Qq}^2)} + (1-\lambda) \frac{e_{i1} - e_{i2}\frac{p_{i1}}{p_{i2}} + e_{u1}}{\gamma(\sigma_Q^u)^2}, \\
0 &= \frac{\lambda}{\gamma} \frac{e_{u2}}{(\sigma_Q^i)^2(1-\rho_{Qq}^2)} + (1-\lambda) \frac{e_{u2}}{\gamma(\sigma_Q^u)^2}, \\
0 &= \frac{\lambda}{\gamma} \frac{e_{u3}}{(\sigma_Q^i)^2(1-\rho_{Qq}^2)} + (1-\lambda) \frac{e_{i3} - e_{i2}\frac{p_{i3}}{p_{i2}} + e_{u3}}{\gamma(\sigma_Q^u)^2}, \\
&\vdots \\
0 &= \frac{\lambda}{\gamma} \frac{e_{u,k+3}}{(\sigma_Q^i)^2(1-\rho_{Qq}^2)} + (1-\lambda) \frac{e_{i,k+3} - e_{i2}\frac{p_{i,k+3}}{p_{i2}} + e_{u,k+3}}{\gamma(\sigma_Q^u)^2}.
\end{aligned}
$$

Solving these equations yields

$$
e_{u1} = \frac{p_{i1}}{p_{i2}} \frac{(1-\lambda)(\sigma_Q^i)^2(1-\rho_{Qq}^2)}{\lambda(\sigma_Q^u)^2 + (1-\lambda)(\sigma_Q^i)^2(1-\rho_{Qq}^2)} \frac{Cov_t^i(Q_{t+1}, q_{t+1})}{(\sigma_q^i)^2}, \tag{B.28}
$$

$$
\begin{aligned}
e_{u2} &= 0, \\
e_{u3} &= \frac{p_{i3}}{p_{i2}} \frac{(1-\lambda)(\sigma_Q^i)^2(1-\rho_{Qq}^2)}{\lambda(\sigma_Q^u)^2 + (1-\lambda)(\sigma_Q^i)^2(1-\rho_{Qq}^2)} \frac{Cov_t^i(Q_{t+1}, q_{t+1})}{(\sigma_q^i)^2}, \\
&\vdots \\
e_{u,k+2} &= -\left(\frac{\sigma_{Dq}}{\sigma_D^2} - \frac{p_{i,k+2}}{p_{i2}} \right) \frac{(1-\lambda)(\sigma_Q^i)^2(1-\rho_{Qq}^2)}{\lambda(\sigma_Q^u)^2 + (1-\lambda)(\sigma_Q^i)^2(1-\rho_{Qq}^2)} \frac{Cov_t^i(Q_{t+1}, q_{t+1})}{(\sigma_q^i)^2}, \\
e_{u,k+3} &= \frac{p_{i,k+3}}{p_{i2}} \frac{(1-\lambda)(\sigma_Q^i)^2(1-\rho_{Qq}^2)}{\lambda(\sigma_Q^u)^2 + (1-\lambda)(\sigma_Q^i)^2(1-\rho_{Qq}^2)} \frac{Cov_t^i(Q_{t+1}, q_{t+1})}{(\sigma_q^i)^2}.
\end{aligned} \tag{B.29}
$$

To solve an equilibrium, we need to determine $1 + (k+3) + (k+2)$ price coefficients p_0, $\mathbf{p}_i$, and $\mathbf{p}_u$ (note that $p_{u2} = 0$). Eqs. (B.12) and (B.22) determine p_0. Equating (B.13) with (B.25), we obtain $(k+3)$ equations. Equating (B.14) with in (B.28), we obtain $(k+3)$ equations. Note that the second equation $e_{u2}=0$ is a redundant identity. Therefore, we essentially have $(k+2)$ equations. In summary, we obtain $(k+3)+(k+2)$ equations to solve for $(k+3)+(k+2)$ unknowns of $\mathbf{p}_i$, and $\mathbf{p}_u$. When solving these equations, we need to substitute in the preceding variances and covariances. If there is a solution, then we obtain a stationary equilibrium.

Now we derive restrictions on the coefficients in the price function. Adding equations in (B.25) and (B.28) yields

$$e_{i,j}+e_{u,j}=0, \quad \text{for all } j\neq 2, k+2, \tag{B.30}$$

$$e_{i2}+e_{u2}=e_{i2}, \tag{B.31}$$

$$e_{i,k+2}+e_{u,k+2}=e_{i2}\frac{\sigma_{Dq}}{\sigma_D^2}. \tag{B.32}$$

Adding Eqs. (B.13)–(B.14), we get

$$\mathbf{e}_i+\mathbf{e}_u=(\mathbf{p}_i+\mathbf{c}_1+\mathbf{c}_{k+3})\mathbf{A}_x-R\mathbf{p}_i+\mathbf{p}_u\mathbf{I}_{-2}\mathbf{A}_x-R\mathbf{p}_u\mathbf{I}_{-2}. \tag{B.33}$$

Simplifying yields:

$$p_{i1}+p_{u1}=a_F/(R-a_F), \tag{B.34}$$

$$p_{i2}=\frac{-e_{i2}}{R-a_Z}, \tag{B.35}$$

$$p_{ij}+p_{uj}=\frac{1-e_{i2}\frac{\sigma_{Dq}}{\sigma_D^2}}{R^{3+k-j}}, \quad \text{for } 3\leqslant j\leqslant k+2, \tag{B.36}$$

and

$$p_{i,k+3}+p_{u,k+3}=0. \tag{B.37}$$

We next show that $p_{u,k+3}=p_{i,k+3}=0$. We use Eq. (B.14). We can show that the last column of the row vector

$$\mathbf{p}_u\mathbf{I}_{-2}\mathbf{K}_u\mathbf{A}_{yu}\mathbf{A}_x\left(\mathbf{I}_{-2}-\mathbf{c}_2^{\intercal}\frac{1}{p_{i2}}\mathbf{p}_i\mathbf{I}_{-2}\right),$$

is equal to zero. Therefore, from Eq. (B.14):

$$e_{u,k+3}=-Rp_{u,k+3}.$$

Substituting for the equation for $e_{u,k+3}$ in (B.25) and $p_{i,k+3}+p_{u,k+3}=0$ yields:

$$\frac{p_{i,k+3}}{p_{i2}}\frac{(1-\lambda)(\sigma_Q^i)^2(1-\rho_{Qq}^2)}{\lambda(\sigma_Q^u)^2+(1-\lambda)(\sigma_Q^i)^2(1-\rho_{Qq}^2)}\frac{Cov_t^i(Q_{t+1},q_{t+1})}{(\sigma_q^i)^2}=Rp_{i,k+3}.$$

If we suppose $p_{i,k+3}\neq 0$,[17] then we write the above expression as

$$\frac{1}{p_{i2}}\frac{(1-\lambda)(\sigma_Q^i)^2(1-\rho_{Qq}^2)}{\lambda(\sigma_Q^u)^2+(1-\lambda)(\sigma_Q^i)^2(1-\rho_{Qq}^2)}\frac{Cov_t^i(Q_{t+1},q_{t+1})}{(\sigma_q^i)^2}=R. \tag{B.38}$$

[17] When $p_{i,k+3}\neq 0$, it is possible that $R+\mathbf{p}_u\mathbf{K}_{u,.2}a_Z=0$, in which case $Cov_t^i(Q_{t+1},q_{t+1})=0$. But then from (A.25) $e_{u,k+3}=0=p_{u,k+3}$, which gives a contradiction.

Substituting the value for p_{i2} from Eq. (B.35) obtains:

$$-(R-a_Z)\frac{(1-\lambda)(\sigma_Q^i)^2(1-\rho_{Qq}^2)}{\lambda(\sigma_Q^u)^2}=R,$$

which is impossible since $R>1>a_Z$. Therefore, $p_{i,k+3}=0=p_{u,k+3}$. Using (B.28) again we obtain $e_{u,k+3}=e_{i,k+3}=0$. Using the above coefficients restrictions we can derive the price function given in Proposition 1. □

Proof of Proposition 2. Plugging Eqs. (4) and (38) into (34), we obtain the expression in (40) by suitably defining coefficients in that equation. Plugging Eqs. (39) into (36), we can derive (41) by suitably defining coefficients in that equation. We now derive the signs of those coefficients. Since $e_0=rp_0>0$, it follows that $f_0^i>0$ and $f_0^u>0$. The sign of f_Z^u is identical to the sign of e_{i2} which is positive if and only if $Cov_t^i(Q_{t+1},q_{t+1})>0$. The sign of f_Z^i is identical to the sign of

$$\frac{e_{i2}}{\gamma(\sigma_Q^i)^2(1-(\rho_{Qq}^i)^2)}-\frac{\rho_{Qq}^i}{\gamma\sigma_Q^i\sigma_q^i(1-(\rho_{Qq}^i)^2)}.$$

Plugging Eq. (B.25) into this expression, we obtain:

$$\begin{aligned}&\frac{\alpha}{\gamma(\sigma_Q^i)^2(1-(\rho_{Qq}^i)^2)}\frac{Cov_t^i(Q_{t+1},q_{t+1})}{(\sigma_q^i)^2}-\frac{Cov_t^i(Q_{t+1},q_{t+1})}{\gamma(\sigma_Q^i\sigma_q^i)^2(1-(\rho_{Qq}^i)^2)}\\&\quad=-(1-\alpha)\frac{Cov_t^i(Q_{t+1},q_{t+1})}{\gamma(\sigma_Q^i\sigma_q^i)^2(1-(\rho_{Qq}^i)^2)},\end{aligned}$$

where

$$\alpha=\frac{\lambda(\sigma_Q^u)^2}{\lambda(\sigma_Q^u)^2+(1-\lambda)(\sigma_Q^i)^2(1-(\rho_{Qq}^i)^2)}\in(0,1).$$

Thus, $f_Z^i<0$ if and only if $Cov_t^i(Q_{t+1},q_{t+1})>0$. □

Proof of Theorem 1. Using (39) and the law of iterated expectations, we compute for general $n\geqslant 1$,

$$\begin{aligned}E[Q_{t+n}|Q_t]&=E\big[E_t^u[Q_{t+n}]|Q_t\big]=e_0+e_{i2}E\big[\hat{Z}_{t+n-1}^u+E_t^u\big[\varepsilon_{t+n}^q\big]|Q_t\big]\\&=e_0+e_{i2}E\big[Z_{t+n-1}+\varepsilon_{t+n}^q|Q_t\big]\\&=e_0+e_{i2}E\big[E_t^i\big(Z_{t+n-1}+\varepsilon_{t+n}^q\big)|Q_t\big]\\&=e_0+e_{i2}\frac{a_Z^{n-1}Cov(Z_t,Q_t)+\mathbf{1}_{\{n\leqslant k\}}Cov(E_t^i[\varepsilon_{t+n}^q],Q_t)}{Var(Q_t)}Q_t.\end{aligned}$$

Using (32), we obtain the desired result. □

Appendix C. Solution for the model without advance information

The solution for the model without advance information can be obtained as a special case of that for our general model. In addition, this model is a simplified version of Wang [47]. However, because of its simplicity, we are able to derive sharper results by signing many coefficients in the equilibrium equations. These results are absent in Wang [47]. Thus, we will focus on these extra results, without repeating the detailed derivations of equilibrium.

Uninformed investors' filtering problem. The state vector and the unforecastable shock vector are given by $\mathbf{x}_t = (F_t, Z_t)^{\mathsf{T}}$ and $\boldsymbol{\varepsilon}_t = (\varepsilon_t^D, \varepsilon_t^F, \varepsilon_t^Z, \varepsilon_t^q)^{\mathsf{T}}$, respectively. Let $\Sigma = E[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t^{\mathsf{T}}]$. We then have

$$\mathbf{x}_t = \mathbf{A}_x \mathbf{x}_{t-1} + \mathbf{B}_x \boldsymbol{\varepsilon}_t, \tag{C.1}$$

where

$$\mathbf{A}_x = \begin{bmatrix} a_F & 0 \\ 0 & a_Z \end{bmatrix}, \qquad \mathbf{B}_x = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \tag{C.2}$$

The uninformed investor has signals $\mathbf{y}_t = (D_t, \Pi_t)^{\mathsf{T}}$, which satisfies

$$\mathbf{y}_t = \mathbf{A}_y \mathbf{x}_t + \mathbf{B}_y \boldsymbol{\varepsilon}_t, \tag{C.3}$$

where

$$\mathbf{A}_y = \begin{bmatrix} 1 & 0 \\ p_{i1} & p_{i2} \end{bmatrix}, \qquad \mathbf{B}_y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{C.4}$$

Define

$$\boldsymbol{\Sigma}_{xx} = \mathbf{B}_x \boldsymbol{\Sigma} \mathbf{B}_x^{\mathsf{T}} = \begin{bmatrix} \sigma_F^2 & 0 \\ 0 & \sigma_Z^2 \end{bmatrix}, \qquad \boldsymbol{\Sigma}_{yy} = \mathbf{B}_y \boldsymbol{\Sigma} \mathbf{B}_y^{\mathsf{T}} = \begin{bmatrix} \sigma_D^2 & 0 \\ 0 & 0 \end{bmatrix} \tag{C.5}$$

and $\boldsymbol{\Omega}_t = E_t^u[(\mathbf{x}_t - \hat{\mathbf{x}}_t^u)(\mathbf{x}_t - \hat{\mathbf{x}}_t^u)^{\mathsf{T}}]$. As in Wang [47], we focus on the steady-state Kalman filtering. Let $\boldsymbol{\Omega}$ be the solution to the Riccati equation:

$$\boldsymbol{\Omega} = \left(\mathbf{A}_x \boldsymbol{\Omega} \mathbf{A}_x^{\mathsf{T}} + \boldsymbol{\Sigma}_{xx}\right) - \mathbf{K}\mathbf{A}_y\left(\mathbf{A}_x \boldsymbol{\Omega} \mathbf{A}_x^{\mathsf{T}} + \boldsymbol{\Sigma}_{xx}\right), \tag{C.6}$$

where

$$\mathbf{K} = \left(\mathbf{A}_x \boldsymbol{\Omega} \mathbf{A}_x^{\mathsf{T}} + \boldsymbol{\Sigma}_{xx}\right)\mathbf{A}_y^{\mathsf{T}}\left[\mathbf{A}_y\left(\mathbf{A}_x \boldsymbol{\Omega} \mathbf{A}_x^{\mathsf{T}} + \boldsymbol{\Sigma}_{xx}\right)\mathbf{A}_y^{\mathsf{T}} + \boldsymbol{\Sigma}_{yy}\right]^{-1}. \tag{C.7}$$

We then obtain the following steady-state filters:

$$\hat{\mathbf{x}}_t^u = \mathbf{A}_x \hat{\mathbf{x}}_{t-1}^u + \mathbf{K}\hat{\boldsymbol{\varepsilon}}_t^u, \tag{C.8}$$

and

$$\mathbf{y}_t = \mathbf{A}_y \mathbf{A}_x \hat{\mathbf{x}}_{t-1}^u + \hat{\boldsymbol{\varepsilon}}_t^u, \tag{C.9}$$

where $\hat{\boldsymbol{\varepsilon}}_t^u = \mathbf{y}_t - E_{t-1}^u[\mathbf{y}_t]$ is the innovation, which is normally distributed with mean of zero and covariance matrix

$$Var\left(\hat{\boldsymbol{\varepsilon}}_t^u\right) = \mathbf{A}_y \mathbf{A}_x \boldsymbol{\Omega} \mathbf{A}_x^{\mathsf{T}} \mathbf{A}_y^{\mathsf{T}} + (\mathbf{A}_y \mathbf{B}_x + \mathbf{B}_y)\boldsymbol{\Sigma}(\mathbf{A}_y \mathbf{B}_x + \mathbf{B}_y)^{\mathsf{T}}. \tag{C.10}$$

Post-multiplying both sides of (C.6) by $\mathbf{A}_y^{\mathsf{T}}$ and subtracting $\mathbf{K}\boldsymbol{\Sigma}_{yy}$ from both sides yields

$$\boldsymbol{\Omega}\mathbf{A}_y^{\mathsf{T}} - \mathbf{K}\boldsymbol{\Sigma}_{yy} = \left(\mathbf{A}_x \boldsymbol{\Omega} \mathbf{A}_x^{\mathsf{T}} + \boldsymbol{\Sigma}_{xx}\right)\mathbf{A}_y^{\mathsf{T}} - \mathbf{K}\left[\mathbf{A}_y\left(\mathbf{A}_x \boldsymbol{\Omega} \mathbf{A}_x^{\mathsf{T}} + \boldsymbol{\Sigma}_{xx}\right)\mathbf{A}_y^{\mathsf{T}} + \boldsymbol{\Sigma}_{yy}\right] = 0.$$

This equality can be written as

$$\begin{bmatrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{bmatrix} \begin{bmatrix} 1 & p_{i1} \\ 0 & p_{i2} \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} \sigma_D^2 & 0 \\ 0 & 0 \end{bmatrix},$$

where $\Omega = (\omega_{ij})$ with $\omega_{12} = \omega_{21}$. Therefore, we get the following 4 equations

$$\begin{bmatrix} \omega_{11} & \omega_{11} p_{i1} + \omega_{12} p_{i2} \\ \omega_{21} & \omega_{21} p_{i1} + \omega_{22} p_{i2} \end{bmatrix} = \sigma_D^2 \begin{bmatrix} k_{11} & 0 \\ k_{21} & 0 \end{bmatrix}.$$

Solving yields

$$k_{11} = \omega_{11}/\sigma_D^2 > 0, \qquad k_{21} = \omega_{21}/\sigma_D^2 > 0, \tag{C.11}$$

$$\omega_{12} = -p_{i1}/p_{i2}\omega_{11} > 0, \qquad \omega_{21} = -p_{i2}/p_{i1}\omega_{22} > 0, \tag{C.12}$$

where we have used the sign restrictions $p_{i1} > 0$ and $p_{i2} < 0$ proved below and the fact that the variances $\omega_{11}, \omega_{22} > 0$.

Now post-multiply both sides of (C.6) by $(\mathbf{A}_x \boldsymbol{\Omega} \mathbf{A}_x^\intercal + \boldsymbol{\Sigma}_{xx})^{-1}$ to get

$$\boldsymbol{\Omega}\left(\mathbf{A}_x \boldsymbol{\Omega} \mathbf{A}_x^\intercal + \boldsymbol{\Sigma}_{xx}\right)^{-1} = \mathbf{I} - \mathbf{K}\mathbf{A}_y$$

or

$$\omega_{11} \begin{bmatrix} 1 & -\frac{p_{i1}}{p_{i2}} \\ -\frac{p_{i1}}{p_{i2}} & (\frac{p_{i1}}{p_{i2}})^2 \end{bmatrix} \begin{bmatrix} a_F^2 \omega_{11} + \sigma_F^2 & -a_Z a_F \frac{p_{i1}}{p_{i2}} \omega_{11} \\ -a_F a_Z \frac{p_{i1}}{p_{i2}} \omega_{11} & a_Z^2 (\frac{p_{i1}}{p_{i2}})^2 \omega_{11} + \sigma_Z^2 \end{bmatrix}^{-1}$$
$$= \begin{bmatrix} 1 - k_{11} - k_{12} p_{i1} & -k_{12} p_{i2} \\ -k_{21} - k_{22} p_{i1} & 1 - k_{22} p_{i2} \end{bmatrix}.$$

Simplify this equation to obtain

$$\frac{\omega_{11}}{\Delta} \begin{bmatrix} a_Z(a_Z - a_F)(\frac{p_{i1}}{p_{i2}})^2 \omega_{11} + \sigma_Z^2 & \frac{p_{i1}}{p_{i2}}[a_F(a_Z - a_F)\omega_{11} - \sigma_F^2] \\ -\frac{p_{i1}}{p_{i2}}[a_Z(a_Z - a_F)(\frac{p_{i1}}{p_{i2}})^2 \omega_{11} + \sigma_Z^2] & -(\frac{p_{i1}}{p_{i2}})^2 [a_F(a_Z - a_F)\omega_{11} - \sigma_F^2] \end{bmatrix}$$
$$= \begin{bmatrix} 1 - k_{11} - k_{12} p_{i1} & -k_{12} p_{i2} \\ -k_{21} - k_{22} p_{i1} & 1 - k_{22} p_{i2} \end{bmatrix}, \tag{C.13}$$

with

$$\Delta = \left(a_F^2 \omega_{11} + \sigma_F^2\right)\left(a_Z^2 \left(\frac{p_{i1}}{p_{i2}}\right)^2 \omega_{11} + \sigma_Z^2\right) - \left(a_F a_Z \frac{p_{i1}}{p_{i2}} \omega_{11}\right)^2$$
$$= \sigma_F^2 a_Z^2 \left(\frac{p_{i1}}{p_{i2}}\right)^2 \omega_{11} + a_F^2 \sigma_Z^2 \omega_{11} + \sigma_F^2 \sigma_Z^2 > 0. \tag{C.14}$$

Now using the top right hand corner equation:

$$\frac{\omega_{11}}{\Delta} \frac{p_{i1}}{p_{i2}^2} \left[\sigma_F^2 - \left(1 - a_F^2\right)\omega_{11} + (1 - a_F a_Z)\omega_{11}\right] = k_{12}, \tag{C.15}$$

with $\sigma_F^2 - (1 - a_F^2)\omega_{11} > 0$ and $a_F, a_Z \in (0, 1)$, we obtain $k_{12} > 0$. Similarly, we have:

$$-\frac{\omega_{11}}{\Delta} \frac{p_{i1}}{p_{i2}} \left[a_Z(a_Z - a_F)\left(\frac{p_{i1}}{p_{i2}}\right)^2 \omega_{11} + \sigma_Z^2\right] = -k_{21} - k_{22} p_{i1} > 0. \tag{C.16}$$

Since

$$a_Z(a_Z-a_F)\left(\frac{p_{i1}}{p_{i2}}\right)^2\omega_{11}+\sigma_Z^2=\sigma_Z^2-\left(1-a_Z^2\right)\omega_{22}+(1-a_Fa_Z)\omega_{22}>0,$$

we obtain:

$$-p_{i1}k_{22}=k_{21}-\frac{\omega_{11}}{\Delta}\frac{p_{i1}}{p_{i2}}\left[a_Z(a_Z-a_F)\left(\frac{p_{i1}}{p_{i2}}\right)^2\omega_{11}+\sigma_Z^2\right]>0, \tag{C.17}$$

implying $k_{22}<0$. Note using the top left hand corner equation in (C.13),

$$1-k_{11}-k_{12}p_{i1}=\frac{\omega_{11}}{\Delta}\left[a_Z(a_Z-a_F)\left(\frac{p_{i1}}{p_{i2}}\right)^2\omega_{11}+\sigma_Z^2\right]>0.$$

Having shown that the right hand side of this expression is positive and that, under our guess, $p_{i1}>0$, we obtain $k_{11}<1$. □

Price function. As in the proof of Proposition 1, the price function is given by the first line of Eq. (27) but noting that the equilibrium parameters $\mathbf{p}_i$ and $\mathbf{p}_u$ will differ. The excess return Q_{t+1} satisfies

$$Q_{t+1}=e_0+\mathbf{e}_i\mathbf{x}_t+\mathbf{e}_u\hat{\mathbf{x}}_t^u+\mathbf{b}_Q\boldsymbol{\varepsilon}_{t+1}, \tag{C.18}$$

where $e_{u2}=0$,

$$e_{u1}=p_{u1}\left[a_F(1-k_{11}-p_{i1}k_{12})-R+a_Zp_{i1}k_{12}\right], \tag{C.19}$$

$$e_{i1}+e_{u1}=-(p_{i1}+p_{u1})(R-a_F)+a_F, \tag{C.20}$$

$$e_{i2}=-p_{i2}(R-a_Z). \tag{C.21}$$

In addition, e_{i1}, e_{i2} and e_{u1} satisfy Eqs. (B.25), (B.26), and (B.28). Note that the system of these three equations along with Eqs. (C.19)–(C.21) do not admit an analytical solution for p_{i1}, p_{i2}, and p_{u1}. An equilibrium exists if this system has a solution. A simple iterative numerical procedure can solve this system as in Appendix A.

Even though we cannot solve the equilibrium explicitly, we can derive restrictions on the coefficients in the price function, $p_{i2}<0$ and $0<p_{u1}<a_F/(R-a_F)$, as claimed in Section 3.1. First, adding Eqs. (B.26) and (B.28) yields $e_{i1}+e_{u1}=0$. Eq. (C.20) then implies

$$p_{i1}+p_{u1}=\frac{a_F}{R-a_F}. \tag{C.22}$$

We then take the ratio of (B.28) and (B.26), and substitute for e_{i2}, using (C.21) to derive

$$p_{i1}=-(R-a_Z)e_{u1}\frac{(1-\lambda)(\sigma_Q^i)^2(1-(\rho_{Qq}^i)^2)}{\lambda(\sigma_Q^u)^2}. \tag{C.23}$$

We wish to show that $p_{i1}>0$. Suppose to the contrary that $p_{i1}<0$. Then Eq. (C.23) implies that $e_{u1}>0$. We next use (C.19) to show that $p_{u1}<0$. To this end, we use the two equations implied in the top row of (C.13) to substitute for $1-k_{11}-p_{i1}k_{12}$ and $p_{i1}k_{12}$, respectively. We then obtain:

$$\begin{aligned}
&a_F(1-k_{11}-p_{i1}k_{12})-R+a_Z p_{i1}k_{12}\\
&\quad=\frac{\omega_{11}}{\Delta}a_F\left[a_Z(a_Z-a_F)\left(\frac{p_{i1}}{p_{i2}}\right)^2\omega_{11}+\sigma_Z^2\right]-R\\
&\qquad-\frac{\omega_{11}}{\Delta}a_Z\left(\frac{p_{i1}}{p_{i2}}\right)^2\left[a_F(a_Z-a_F)\omega_{11}-\sigma_F^2\right]\\
&\quad=\frac{\omega_{11}}{\Delta}\left(a_F\sigma_Z^2+a_Z\left(\frac{p_{i1}}{p_{i2}}\right)^2\sigma_F^2\right)-R\\
&\quad=\frac{a_Z(\frac{p_{i1}}{p_{i2}})^2\sigma_F^2\omega_{11}+a_F\sigma_Z^2\omega_{11}}{a_Z^2(\frac{p_{i1}}{p_{i2}})^2\sigma_F^2\omega_{11}+a_F^2\sigma_Z^2\omega_{11}+\sigma_F^2\sigma_Z^2}-R.
\end{aligned}$$

We now show this expression is negative and thus deduce $p_{u1}<0$. It suffices to show that

$$\frac{\omega_{11}}{\Delta}\left(a_F\sigma_Z^2+a_Z\left(\frac{p_{i1}}{p_{i2}}\right)^2\sigma_F^2\right)=\frac{a_Z(\frac{p_{i1}}{p_{i2}})^2\sigma_F^2\omega_{11}+a_F\sigma_Z^2\omega_{11}}{a_Z^2(\frac{p_{i1}}{p_{i2}})^2\sigma_F^2\omega_{11}+a_F^2\sigma_Z^2\omega_{11}+\sigma_F^2\sigma_Z^2}<1,$$

where we have substituted Eq. (C.14). This inequality is equivalent to

$$a_Z(1-a_Z)\left(\frac{p_{i1}}{p_{i2}}\right)^2\sigma_F^2\omega_{11}+(1-a_F)a_F\sigma_Z^2\omega_{11}<\sigma_F^2\sigma_Z^2.$$

This inequality is true since we can show that

$$\begin{aligned}
&a_Z(1-a_Z)\left(\frac{p_{i1}}{p_{i2}}\right)^2\sigma_F^2\omega_{11}+(1-a_F)a_F\sigma_Z^2\omega_{11}\\
&\quad=a_Z(1-a_Z)\sigma_F^2\omega_{22}+(1-a_F)a_F\sigma_Z^2\omega_{11}\\
&\quad<a_Z(1-a_Z)\frac{\sigma_F^2\sigma_Z^2}{(1-a_Z^2)}+(1-a_F)a_F\frac{\sigma_F^2\sigma_Z^2}{(1-a_F^2)}\\
&\quad=\frac{a_Z}{1+a_Z}\sigma_F^2\sigma_Z^2+\frac{a_F}{1+a_Z}\sigma_F^2\sigma_Z^2\\
&\quad<\sigma_F^2\sigma_Z^2,
\end{aligned}$$

where we have used (C.12) and the definition of Ω to derive

$$\begin{aligned}
&\left(\frac{p_{i1}}{p_{i2}}\right)^2\omega_{11}=\omega_{22}=Var(Z_t-\hat{Z}_t)=\frac{\sigma_Z^2}{1-a_Z^2}-Var(\hat{Z}_t),\\
&\omega_{11}=Var(F_t-\hat{F}_t)=\frac{\sigma_F^2}{1-a_F^2}-Var(\hat{F}_t),
\end{aligned}$$

and we note

$$a_Z,a_F\in(0,1)\quad\text{and}\quad\frac{a_Z}{1+a_Z}+\frac{a_F}{1+a_Z}<1.$$

Therefore, Eq. (C.19) and $e_{u1}>0$ imply that $p_{u1}<0$, which contradicts (C.22) because Eq. (C.22) implies that $p_{i1}+p_{u1}$ must be positive.

As a result, we must have $p_{i1}>0$. We then use (C.23) to deduce that $e_{u1}<0$. Since we have shown that the expression in the square bracket in Eq. (C.19) is negative, we conclude that $p_{u1}>0$. By (C.18), we can show that

$$Cov_t^i(Q_{t+1}, q_{t+1}) = \mathbf{b}_Q E_t^i\big[\boldsymbol{\varepsilon}_{t+1}\boldsymbol{\varepsilon}_{t+1}^{\top}\big]\mathbf{c}_4^{\top} = \mathbf{b}_Q \Sigma \mathbf{c}_4^{\top} = (1 + p_{u1}k_{11})\sigma_{Dq}, \tag{C.24}$$

which is positive by (C.11). Use this result and Eq. (B.26) to obtain $e_{i2} > 0$. By this result, Eq. (C.21), and $a_Z < R$, we obtain $p_{i2} < 0$. □

Proof of Theorem 2. Without advance information, Eq. (39) becomes

$$E_t^u[Q_{t+1}] = e_0 + e_{i2}E_t^u[Z_t].$$

It follows the law of iterated expectations that

$$\begin{aligned} E[Q_{t+1}|Q_t] &= E\big[E_t^u[Q_{t+1}]|Q_t\big] = e_0 + e_{i2}E\big[E_t^u[Z_t]|Q_t\big] \\ &= e_0 + e_{i2}E[Z_t|Q_t] = e_0 + e_{i2}\frac{Cov(Z_t, Q_t)}{Var(Q_t)}Q_t, \end{aligned}$$

where e_0 and $e_{i2} > 0$ are constants, as shown in the proof of Proposition 1.

Now we use (24) to compute the unconditional covariance $Cov(Z_t, Q_t)$:

$$\begin{aligned} Cov(Z_t, Q_t) &= E\big[Z_t\big(p_{i1}F_t + p_{u1}\hat{F}_t^u + p_{i2}Z_t + F_t + \varepsilon_t^D \\ &\quad - R\big(p_{i1}F_{t-1} + p_{u1}\hat{F}_{t-1}^u + p_{i2}Z_{t-1}\big)\big)\big] \\ &= E\big[p_{i2}Z_t^2 - p_{i2}RZ_tZ_{t-1} + p_{u1}Z_t\hat{F}_t^u - p_{u1}a_ZRZ_{t-1}\hat{F}_{t-1}^u\big] \\ &= p_{i2}(1 - Ra_Z)Var(Z_t) + p_{u1}(1 - Ra_Z)E\big(Z_t\hat{F}_t^u\big), \end{aligned}$$

where we use $E[Z_tF_t] = E[Z_tF_{t-1}] = E[Z_t\varepsilon_t^D] = 0$.

Multiplying by Z_t in both sides of (26) and taking expectations, we obtain

$$E\big[Z_t\hat{Z}_t^u\big] = Var(Z_t) - \frac{p_{i1}}{p_{i2}}E\big[Z_t\hat{F}_t^u\big].$$

We then derive

$$\frac{p_{i1}}{p_{i2}}E\big[Z_t\hat{F}_t^u\big] = Var(Z_t) - Var\big(\hat{Z}_t^u\big),$$

using

$$E\big(Z_t\hat{Z}_t^u\big) = E\big[E_t^u\big(Z_t\hat{Z}_t^u\big)\big] = E\big[\hat{Z}_t^u\hat{Z}_t^u\big] = Var\big(\hat{Z}_t^u\big).$$

Now, we obtain

$$Cov(Z_t, Q_t) = (1 - Ra_Z)p_{i2}\left\{Var(Z_t) + \frac{p_{u1}}{p_{i1}}\big[Var(Z_t) - Var\big(\hat{Z}_t^u\big)\big]\right\}.$$

Define

$$f_Q = -p_{i2}e_{i2}\frac{Var(Z_t) + \frac{p_{u1}}{p_{i1}}[Var(Z_t) - Var(\hat{Z}_t^u)]}{Var(Q_t)}.$$

To show this expression is positive, we only need to prove $Var(Z_t) > Var(\hat{Z}_t^u)$ because Proposition 1 shows that $p_{i2} < 0$, $e_{i2} > 0$, and $p_{i1}, p_{u1} > 0$. This fact follows from the decomposition of variance. Finally, for $n > 1$,

$$\begin{aligned} E[Q_{t+n}|Q_t] &= E\big[E_t^u[Q_{t+n}]|Q_t\big] = e_0 + e_{i2}E\big[E_t^u[Z_{t+n-1}]|Q_t\big] \\ &= e_0 + a_Z^{n-1}e_{i2}E[Z_t|Q_t] = e_0 + a_Z^{n-1}e_{i2}\frac{Cov(Z_t, Q_t)}{Var(Q_t)}Q_t. \end{aligned}$$

We can then use the previous analysis to obtain the desired result. □

Appendix D. Model with multiple pieces of advance information

To solve the model with multiple pieces of advance information, we define the state vector in Eq. (23), as in Section 3. Unlike Section 3, we define the unforecastable error term based on the period $t-1$ information as

$$\boldsymbol{\varepsilon}_t = \left(\varepsilon^D_{t+k}, \varepsilon^F_t, \varepsilon^Z_t, \varepsilon^q_{t+k}, \varepsilon^{S_k}_t, \dots, \varepsilon^{S_1}_t\right)^\mathsf{T}.$$

It is normally distributed with mean zero and covariance matrix $\Sigma = E[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t^\mathsf{T}]$. The only positive covariance that shows up in Σ is $E(\varepsilon^D_{t+k}\varepsilon^q_{t+k}) = \sigma_{Dq} > 0$. We can apply the same method in Section 3 to solve the model. The only modification lies in the inference problem for informed investors. This problem results in conditional expectations given by

$$E^i_t\left[\varepsilon^D_{t+k}\right] = \rho_k S^k_t,$$
$$E^i_t\left[\varepsilon^D_{t+j}\right] = E^i_{t-1}\left[\varepsilon^D_{t+j}\right] + \rho_j S^j_t, \quad 1 \leqslant j \leqslant k-1,$$

and

$$E^i_t\left[\varepsilon^q_{t+k}\right] = \frac{\sigma_{Dq}}{\sigma^2_D}\rho_k S^k_t,$$
$$E^i_t\left[\varepsilon^q_{t+j}\right] = E^i_{t-1}\left[\varepsilon^q_{t+j}\right] + \frac{\sigma_{Dq}}{\sigma^2_D}\rho_j S^j_t, \quad 1 \leqslant j \leqslant k-1,$$

where

$$\rho_k = \frac{\sigma^2_D}{\sigma^2_D + \sigma^2_{S_k}}, \quad \rho_{k-1} = \frac{(1-\rho_k)\sigma^2_D}{(1-\rho_k)\sigma^2_D + \sigma^2_{S_{k-1}}}, \quad \dots,$$
$$\rho_1 = \frac{(1-\rho_k)\cdots(1-\rho_2)\sigma^2_D}{(1-\rho_k)\cdots(1-\rho_2)\sigma^2_D + \sigma^2_{S_1}}.$$

Because the signals $S^k_{t+j-k}, \dots, S^1_{t+j-1}$ arrive sequentially and are correlated, each of them contributes to lowering the conditional variance of earnings innovations ε^D_{t+j} for $0 < j < k$, but, incrementally, the precision of each new signal changes with $\sigma^2_{S_j}$. Using these expressions we can construct a new Kalman gain matrix $\mathbf{K}_i$. Additional details are available upon request. □

Appendix E. Supplementary material

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.jet.2013.06.001.

References

[1] E. Albagli, C. Hellwig, A. Tsyvinski, A theory of asset pricing based on heterogeneous information, working paper, 2011.

[2] R. Albuquerque, G. Bauer, M. Schneider, International equity flows and returns: a quantitative equilibrium approach, Rev. Econ. Stud. 74 (2007) 1–30.

[3] P. Bacchetta, E. van Wincoop, Can information heterogeneity explain the exchange rate determination puzzle?, Amer. Econ. Rev. 96 (2006) 552–576.

[4] P. Bacchetta, E. van Wincoop, Higher order expectations in asset pricing, J. Money, Credit, Banking 40 (2008) 837–866.

[5] S. Banerjee, R. Kaniel, I. Kremer, Price drift as an outcome of differences in higher order beliefs, Rev. Finan. Stud. 22 (2009) 3707–3734.
[6] P. Beaudry, F. Portier, Exploring Pigou's theory of cycles, J. Monet. Econ. 51 (2004) 1183–1216.
[7] P. Beaudry, F. Portier, News, stock prices and economic fluctuations, Amer. Econ. Rev. 96 (2006) 1293–1307.
[8] J. Berk, R.C. Green, V. Naik, Optimal investment, growth options, and security returns, J. Finance 54 (1999) 1553–1607.
[9] V.L. Bernard, J.K. Thomas, Post-earnings-announcement drift: delayed price response or risk premium?, J. Acc. Res. 27 (1989) 1–36.
[10] D. Bernhardt, J. Miao, Informed trading when information becomes stale, J. Finance 59 (2004) 339–390.
[11] B. Biais, P. Bossaerts, C. Spatt, Equilibrium asset pricing and portfolio choice under asymmetric information, Rev. Finan. Stud. 23 (2010) 1503–1543.
[12] J. Campbell, S. Grossman, J. Wang, Trading volume and serial correlation in stock returns, Quart. J. Econ. 108 (1993) 905–939.
[13] G. Cespa, X. Vives, Dynamic trading and asset prices: Keynes vs. Hayek, Rev. Econ. Stud. 79 (2012) 539–580.
[14] L. Chan, N. Jegadeesh, J. Lakonishok, Momentum strategies, J. Finance 51 (1997) 1681–1713.
[15] R. Connolly, C. Stivers, Momentum and reversals in equity-index returns during periods of abnormal turnover and return dispersion, J. Finance 63 (2003) 1521–1555.
[16] D.M. Cutler, J.M. Poterba, L.H. Summers, Speculative dynamics, Rev. Econ. Stud. 58 (1991) 529–546.
[17] K. Daniel, S. Titman, Market reactions to tangible and intangible information, J. Finance 61 (2006) 1605–1643.
[18] W.F.M. De Bondt, R.H. Thaler, Does the stock market overreact?, J. Finance 40 (1985) 793–808.
[19] W.F.M. De Bondt, R.H. Thaler, Further evidence of investor overreaction and stock market seasonality, J. Finance 42 (1987) 557–581.
[20] E.F. Fama, Stock returns, expected returns, and real activity, J. Finance 45 (1990) 1089–1109.
[21] E.F. Fama, K.R. French, Permanent and temporary components of stock prices, J. Polit. Economy 96 (1988) 246–273.
[22] E.F. Fama, K.R. French, The cross-section of expected stock returns, J. Finance 47 (1992) 427–465.
[23] E.F. Fama, K.R. French, Common risk factors in the returns on stocks and bonds, J. Finan. Econ. 33 (1993) 3–56.
[24] E.F. Fama, K.R. French, Multifactor explanations of asset pricing anomalies, J. Finance 51 (1996) 55–83.
[25] E.F. Fama, K.R. French, Dissecting anomalies, J. Finance 63 (2008) 1653–1678.
[26] J. Gomes, L. Kogan, L. Zhang, Equilibrium cross section of returns, J. Polit. Economy 111 (2003) 693–732.
[27] C.W. Holden, A. Subrahmanyam, News events, information acquisition, and serial correlation, J. Bus. 75 (2002) 1–32.
[28] N. Jaimovich, S. Rebelo, Can news about the future drive the business cycle?, Amer. Econ. Rev. 99 (2009) 1097–1118.
[29] N. Jegadeesh, S. Titman, Returns to buying winners and selling losers: implications for stock market efficiency, J. Finance 48 (1993) 65–91.
[30] T.C. Johnson, Rational momentum effects, J. Finance 57 (2002) 585–608.
[31] R. Kaniel, G. Saar, S. Titman, Individual investor trading and stock returns, J. Finance 63 (2008) 273–310.
[32] J.M. Keynes, The General Theory of Employment, Interest and Money, Harcourt, Brace and Company, New York, 1936.
[33] R. La Porta, J. Lakonishok, A. Shleifer, R. Vishny, Good news for value stocks: further evidence on market efficiency, J. Finance 52 (1997) 859–874.
[34] J. Lewellen, J. Shanken, Learning, asset-pricing tests, and market efficiency, J. Finance 57 (2002) 1113–1145.
[35] G. LIorente, R. Michaely, G. Saar, J. Wang, Dynamic volume return relation of individual stocks, Rev. Finan. Stud. 15 (2002) 1005–1047.
[36] G. Lorenzoni, A theory of demand shocks, Amer. Econ. Rev. 99 (2009) 2050–2084.
[37] I. Makarov, O. Rytchkov, Forecasting the forecast of others: implications for asset pricing, J. Econ. Theory 147 (2012) 941–966.
[38] R. Merton, A simple model of capital market equilibrium with incomplete information, J. Finance 42 (1987) 483–510.
[39] T. Moskowitz, Y.H. Ooi, L.H. Pedersen, Time series momentum, J. Finan. Econ. 104 (2012) 228–250.
[40] A. Pigou, Industrial Fluctuations, MacMillan, London, 1926.
[41] J.M. Poterba, L.H. Summers, Mean reversion in stock prices: evidence and implications, J. Finan. Econ. 22 (1988) 27–59.
[42] K.G. Rouwenhorst, International momentum strategies, J. Finance 53 (1998) 267–284.
[43] G.W. Schwert, Stock returns and real activity: a century of evidence, J. Finance 45 (1990) 1237–1257.

[44] P.C. Tetlock, All the news that's fit to reprint: do investors react to stale information, Rev. Finan. Stud. 24 (2011) 1481–1512.
[45] D. Vayanos, P. Woolley, An institutional theory of momentum and reversal, working paper, 2008.
[46] J. Wang, A model of asset prices under asymmetric information, Rev. Econ. Stud. 60 (1993) 249–282.
[47] J. Wang, A model of competitive stock trading volume, J. Polit. Economy 102 (1994) 127–168.