Supplementary Material for "Advance Information and Asset Prices"

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Abstract

In this supplementary material, we provide additional proofs omitted in Section 2 of the paper and also provide additional analysis for two variations of the representative agent model studied in Section 2 of the paper.

1. Proofs for Section 2

This section derives the equilibrium details for the representative agent model studied in Section 2 of the paper. The representative investor solves the problem:

$$\max_{\theta_t,\alpha_t} E_t\left\{-e^{-\gamma W_{t+1}}\right\}, \tag{1}$$

subject to the budget constraint

$$W_{t+1} = \theta_t Q_{t+1} + \alpha_t q_{t+1} + W_t R. \tag{2}$$

Here E_t denotes the conditional expectation operator given the information up to date t. The information includes realization of dividends and returns up to date t and the advance information signal:

$$S_t = \varepsilon^D_{t+1} + \varepsilon^S_t. \tag{3}$$

Conjecture that the joint distribution of Q_{t+1} and q_{t+1} conditional on date t information is normal:

$$\begin{bmatrix} Q_{t+1} \\ q_{t+1} \end{bmatrix} |t \sim N\left(\mu_t, V\right), \tag{4}$$

where

$$\mu_t = \begin{bmatrix} \mu_{Qt} \\ \mu_{qt} \end{bmatrix}, \quad V = \begin{bmatrix} V_Q & V_{Qq} \\ V_{Qq} & V_q \end{bmatrix}.$$

We can solve for V_q :

$$V_q = Var_t\left(Z_t + \varepsilon^q_{t+1}\right) = Var\left(\varepsilon^q_{t+1}|S_t\right) = \sigma_q^2 - \frac{\sigma^2_{Dq}}{\sigma_S^2 + \sigma_D^2}.$$

We will show below that other elements in V are also constant.

We can then substitute the budget constraint (2) into (1) to derive the first-order conditions:

$$V_Q + \alpha_t V_{Qq} = \gamma^{-1}\mu_{Qt}, \tag{5}$$

$$V_{Qq} + \alpha_t V_q = \gamma^{-1}\mu_{qt}, \tag{6}$$

where we have imposed the market clearing condition $\theta_t = 1$.

Using the law of iterated expectations and (5), we can show that

$$E\left[Q_{t+1}|Q_t\right] = E\left[\mu_{Qt}|Q_t\right] = \gamma\left(V_Q + V_{Qq}E\left[\alpha_t|Q_t\right]\right).$$

Solving equations (5) and (6) yields:

$$\alpha_t = \frac{\mu_{qt}}{\gamma V_q} - \frac{V_{Qq}}{V_q}, \tag{7}$$

$$\mu_{Qt} = \frac{\mu_{qt}V_{Qq}}{V_q} + \gamma\frac{V_Q V_q - V_{Qq}^2}{V_q}. \tag{8}$$

Now, we compute

$$\begin{aligned} \mu_{Qt} &= E_t\left[P_{t+1}\right] + E_t\left[D_{t+1}\right] - RP_t \\ &= E_t\left[P_{t+1}\right] + E_t\left[F_{t+1} + \varepsilon_{t+1}^D\right] - RP_t \\ &= E_t\left[P_{t+1}\right] + a_F F_t + E_t\left[\varepsilon_{t+1}^D\right] - RP_t \\ &= E_t\left[P_{t+1}\right] + a_F F_t + \frac{\sigma_D^2}{\sigma_S^2 + \sigma_D^2}S_t - RP_t, \end{aligned}$$

and

$$\mu_{qt} = E_t\left[Z_t + \varepsilon_{t+1}^q\right] = Z_t + E_t\left[\varepsilon_{t+1}^q\right] = Z_t + \bar{\sigma}S_t, \tag{9}$$

where we have used the projection theorem and defined

$$\bar{\sigma} \equiv \frac{\sigma_{Dq}}{\sigma_S^2 + \sigma_D^2}.$$

Substituting these two equations into (8) yields:

$$P_t = R^{-1}\left\{a_F F_t + \frac{\sigma_D^2}{\sigma_S^2 + \sigma_D^2}S_t - \frac{V_{Qq}\left[Z_t + \bar{\sigma}S_t\right]}{V_q} + \gamma\left(\frac{V_{Qq}^2}{V_q} - V_Q\right)\right\} + \frac{E_t\left[P_{t+1}\right]}{R}.$$

Solving this first-order stochastic difference equation for P_t by repeated substitution yields:

$$\begin{aligned} P_t = &-\frac{\gamma\left(V_Q V_q - V_{Qq}^2\right)}{V_q}\frac{R^{-1}}{1 - R^{-1}} + \frac{R^{-1}a_F}{1 - R^{-1}a_F}F_t \\ &-\frac{V_{Qq}R^{-1}}{V_q\left(1 - R^{-1}a_Z\right)}Z_t + R^{-1}\left[\frac{\sigma_D^2}{\sigma_S^2 + \sigma_D^2} - \frac{V_{Qq}\bar{\sigma}}{V_q}\right]S_t, \end{aligned} \tag{10}$$

where we have imposed the transversality (or no-bubble) condition $\lim_{T\to\infty} E_t\left[P_{T+1}/R^T\right] = 0$.

We can decompose the stock price into two components:

$$P_t = f_t + \pi_t, \quad \text{where } f_t = E_t\left[\sum_{s=1}^{\infty} R^{-s} D_{t+s}\right].$$

We then compute

$$\begin{aligned}
f_t &= E_t\left(R^{-1}D_{t+1} + R^{-2}D_{t+2} + ...\right) \\
&= E_t\left[R^{-1}\left(a_F F_t + \varepsilon^F_{t+1} + \varepsilon^D_{t+1}\right) + R^{-2}\left(a_F F_{t+1} + \varepsilon^F_{t+2} + \varepsilon^D_{t+2}\right) + ...\right] \\
&= R^{-1}a_F F_t + R^{-1}E\left[\varepsilon^D_{t+1}|S_t\right] + R^{-2}a_F^2 F_t + ... \\
&= \frac{R^{-1}a_F}{1-R^{-1}a_F}F_t + R^{-1}E\left[\varepsilon^D_{t+1}|S_t\right] \\
&= \frac{R^{-1}a_F}{1-R^{-1}a_F}F_t + R^{-1}\frac{\sigma_D^2}{\sigma_S^2+\sigma_D^2}S_t.
\end{aligned}$$

Using this equation and (10), we can derive

$$\pi_t = -\frac{\gamma\left(V_Q V_q - V_{Qq}^2\right)}{V_q}\frac{R^{-1}}{1-R^{-1}} - \frac{R^{-1}V_{Qq}}{V_q}\left(\frac{Z_t}{1-R^{-1}a_Z} + \bar{\sigma}S_t\right).$$

Equation (10) reveals that the equilibrium price is linear in normally distributed state variables, F_t, Z_t, and S_t. We deduce that the conjecture in (4) is correct. To complete the derivation of equilibrium, it remains to derive the covariance matrix V.

Having solved for the price function, we can write the excess stock return as

$$\begin{aligned}
Q_{t+1} &= \frac{\gamma\left(V_Q V_q - V_{Qq}^2\right)}{V_q} + \frac{1}{1-R^{-1}a_F}\varepsilon^F_{t+1} + \varepsilon^D_{t+1} - \frac{\sigma_D^2}{\sigma_S^2+\sigma_D^2}S_t \\
&\quad - \frac{R^{-1}V_{Qq}}{V_q\left(1-R^{-1}a_Z\right)}\varepsilon^Z_{t+1} + \frac{V_{Qq}}{V_q}\left[Z_t + \bar{\sigma}S_t\right] \\
&\quad + R^{-1}\left[\frac{\sigma_D^2}{\sigma_S^2+\sigma_D^2} - \frac{V_{Qq}\bar{\sigma}}{V_q}\right]S_{t+1}.
\end{aligned} \tag{11}$$

Using this equation, we can derive

$$
\begin{aligned}
V_Q &= \frac{\sigma_F^2}{\left(1-R^{-1}a_F\right)^2} + \frac{\sigma_D^2\sigma_S^2}{\sigma_S^2+\sigma_D^2} + \left(\frac{R^{-1}V_{Qq}}{V_q\left(1-R^{-1}a_Z\right)}\right)^2 \sigma_Z^2 \\
&\quad + R^{-2}\left[\frac{\sigma_D^2}{\sigma_S^2+\sigma_D^2} - \frac{V_{Qq}\bar{\sigma}}{V_q}\right]^2 \left(\sigma_S^2+\sigma_D^2\right), \\
V_{Qq} &= E_t\left[\left(\varepsilon_{t+1}^D - E_t\varepsilon_{t+1}^D\right)\left(\varepsilon_{t+1}^q - E_t\varepsilon_{t+1}^q\right)\right] = \frac{\sigma_{Dq}\sigma_S^2}{\sigma_S^2+\sigma_D^2}. \qquad (12)
\end{aligned}
$$

These two equations can be solved for V_Q and V_{Qq}, completing the derivation of the equilibrium.

Turn to the analysis of momentum and reversals. Using the law of iterated expectations and equations (8) and (9), we can derive

$$
\begin{aligned}
E\left[Q_{t+1}|Q_t\right] &= E\left[\mu_{Qt}|Q_t\right] \\
&= \frac{\gamma\left(V_QV_q - V_{Qq}^2\right)}{V_q} + \frac{V_{Qq}}{V_q}E\left[Z_t+\bar{\sigma}S_t|Q_t\right] \\
&= \frac{\gamma\left(V_QV_q - V_{Qq}^2\right)}{V_q} + \frac{V_{Qq}}{V_q}\frac{Cov\left[Z_t+\bar{\sigma}S_t, Q_t\right]}{Var\left(Q_t\right)}Q_t.
\end{aligned}
$$

Since $V_{Qq} > 0$ if and only if $\sigma_{Dq} > 0$ by (12), we obtain that

$$
Cov\left(Q_{t+1}, Q_t\right) > 0 \text{ if and only if } Cov\left[Z_t+\bar{\sigma}S_t, Q_t\right] > 0. \qquad (13)
$$

Using (11), we can compute

$$
\begin{aligned}
Cov\left(Z_t, Q_t\right) &= E\left[\left(a_ZZ_{t-1}+\varepsilon_t^Z\right)\left(-\frac{R^{-1}V_{Qq}}{V_q\left(1-R^{-1}a_Z\right)}\varepsilon_t^Z + \frac{V_{Qq}}{V_q}Z_{t-1}\right)\right] \\
&= -\frac{R^{-1}V_{Qq}}{V_q\left(1-R^{-1}a_Z\right)}\sigma_Z^2 + \frac{V_{Qq}}{V_q}a_ZE\left[Z_{t-1}^2\right] \\
&= \frac{\sigma_{Dq}\sigma_S^2}{\sigma_S^2+\sigma_D^2}\frac{\sigma_Z^2}{V_q}\frac{a_ZR-1}{\left(R-a_Z\right)\left(1-a_Z^2\right)},
\end{aligned}
$$

and

$$
\begin{aligned}
Cov\left(S_t, Q_t\right) &= R^{-1}\left[\frac{\sigma_D^2}{\sigma_S^2+\sigma_D^2} - \frac{V_{Qq}\bar{\sigma}}{V_q}\right] E\left[S_t^2\right] \\
&= R^{-1}\left[\frac{\sigma_D^2}{\sigma_S^2+\sigma_D^2} - \frac{V_{Qq}\bar{\sigma}}{V_q}\right]\left(\sigma_S^2+\sigma_D^2\right) \\
&= R^{-1}\left[\sigma_D^2 - \frac{\sigma_{Dq}^2\sigma_S^2}{\sigma_q^2\sigma_S^2+\sigma_q^2\sigma_D^2\left(1-\rho_{Dq}^2\right)}\right] \\
&= R^{-1}\frac{\left(\sigma_S^2+\sigma_D^2\right)\sigma_D^2\sigma_q^2\left(1-\rho_{Dq}^2\right)}{\sigma_q^2\sigma_S^2+\sigma_q^2\sigma_D^2\left(1-\rho_{Dq}^2\right)},
\end{aligned}
$$

where $\rho_{Dq} \in (0,1)$ is the correlation coefficient between ε_{t+1}^D and ε_{t+1}^q. Combining the above two equations yields:

$$
\begin{aligned}
& Cov\left[Z_t+\bar{\sigma}S_t, Q_t\right] \\
=\ & \frac{\sigma_{Dq}\sigma_S^2}{\sigma_S^2+\sigma_D^2}\frac{\sigma_Z^2}{V_q}\frac{a_Z R-1}{\left(R-a_Z\right)\left(1-a_Z^2\right)} + R^{-1}\frac{\left(\sigma_S^2+\sigma_D^2\right)\sigma_D^2\sigma_q^2\left(1-\rho_{Dq}^2\right)}{\sigma_q^2\sigma_S^2+\sigma_q^2\sigma_D^2\left(1-\rho_{Dq}^2\right)}\bar{\sigma} \\
=\ & \frac{\sigma_{Dq}}{\sigma_q^2\sigma_S^2+\sigma_q^2\sigma_D^2\left(1-\rho_{Dq}^2\right)}\left[\frac{\sigma_S^2\sigma_Z^2\left(a_Z R-1\right)}{\left(R-a_Z\right)\left(1-a_Z^2\right)} + \frac{\sigma_D^2\sigma_q^2}{R}\left(1-\rho_{Dq}^2\right)\right]. \qquad (14)
\end{aligned}
$$

Next, we derive that

$$
\begin{aligned}
E\left[Q_{t+2}|Q_t\right] &= E\left[\mu_{Qt+1}|Q_t\right] \\
&= \frac{\gamma\left(V_Q V_q - V_{Qq}^2\right)}{V_q} + \frac{V_{Qq}}{V_q}E\left[Z_{t+1}+\bar{\sigma}S_{t+1}|Q_t\right] \\
&= \frac{\gamma\left(V_Q V_q - V_{Qq}^2\right)}{V_q} + \frac{V_{Qq}}{V_q}\frac{Cov\left[Z_{t+1}+\bar{\sigma}S_{t+1}, Q_t\right]}{Var\left(Q_t\right)}Q_t,
\end{aligned}
$$

where we use (11) to show that

$$
\begin{aligned}
Cov\left(Z_{t+1}, Q_t\right) &= E\left[\left(a_Z\left(a_Z Z_{t-1}+\varepsilon_t^Z\right)+\varepsilon_{t+1}^Z\right)\left(\frac{-R^{-1}V_{Qq}}{V_q\left(1-R^{-1}a_Z\right)}\varepsilon_t^Z+\frac{V_{Qq}}{V_q}Z_{t-1}\right)\right] \\
&= E\left[a_Z^2 Z_{t-1}\left(\frac{V_{Qq}}{V_q}Z_{t-1}\right)+a_Z\varepsilon_t^Z\left(-\frac{R^{-1}V_{Qq}}{V_q\left(1-R^{-1}a_Z\right)}\varepsilon_t^Z\right)\right] \\
&= -\frac{R^{-1}V_{Qq}}{V_q\left(1-R^{-1}a_Z\right)}a_Z\sigma_Z^2+\frac{V_{Qq}}{V_q}a_Z^2 E\left[Z_{t-1}^2\right] \\
&= a_Z\frac{\sigma_{Dq}\sigma_S^2}{\sigma_S^2+\sigma_D^2}\frac{\sigma_Z^2}{V_q}\left[\frac{a_Z R-1}{\left(R-a_Z\right)\left(1-a_Z^2\right)}\right],
\end{aligned}
$$

and

$$Cov\left(S_{t+1}, Q_t\right)=0.$$

Therefore,

$$E\left[Q_{t+2}|Q_t\right]=\frac{\gamma\left(V_Q V_q-V_{Qq}^2\right)}{V_q}+a_Z\frac{V_{Qq}}{V_q}\frac{\frac{\sigma_{Dq}\sigma_S^2}{\sigma_S^2+\sigma_D^2}\sigma_Z^2\left(a_Z R-1\right)}{V_q Var\left(Q_t\right)\left(R-a_Z\right)\left(1-a_Z^2\right)}Q_t.$$

Similarly, we can show that

$$E\left[Q_{t+n}|Q_t\right]=\frac{\gamma\left(V_Q V_q-V_{Qq}^2\right)}{V_q}+a_Z^{n-1}\frac{V_{Qq}}{V_q}\frac{\frac{\sigma_{Dq}\sigma_S^2}{\sigma_S^2+\sigma_D^2}\sigma_Z^2\left(a_Z R-1\right)}{V_q\left(R-a_Z\right)\left(1-a_Z^2\right)Var\left(Q_t\right)}Q_t,$$

for any $n \geq 2$. Thus,

$$Cov\left(Q_{t+n}, Q_t\right)>0 \text{ if and only if } a_Z R>1 \text{ for } n \geq 2. \tag{15}$$

By (13), (14), and (15), we deduce that if $a_Z R > 1$, then we get momentum for all horizons in that $Cov\left(Q_{t+n}, Q_t\right) > 0$, for all $n \geq 1$. Now, suppose that $a_Z R < 1$. Then we get reversals after one period in that $Cov\left(Q_{t+n}, Q_t\right) < 0$ for all $n \geq 2$. To get one period momentum, i.e., $Cov\left(Q_{t+1}, Q_t\right) > 0$, we need

$$\frac{\sigma_S^2\sigma_Z^2\left(a_Z R-1\right)}{\left(R-a_Z\right)\left(1-a_Z^2\right)}+\frac{\sigma_D^2\sigma_q^2}{R}\left(1-\rho_{Dq}^2\right)>0.$$

This condition is satisfied when a_Z is sufficiently close to $1/R$.

To see what will happen if we drop the advance information, we set $\sigma_S^2 \to \infty$ so that advance information is useless. Then we can compute that

$$E\left[Q_{t+n}|Q_t\right] = \frac{\gamma\left(V_Q V_q - V_{Qq}^2\right)}{V_q} + a_Z^{n-1}\frac{V_{Qq}}{V_q}\frac{\sigma_{Dq}\sigma_Z^2\left(a_Z R - 1\right)}{V_q Var\left(Q_t\right)\left(R - a_Z\right)\left(1 - a_Z^2\right)}Q_t,$$

for all $n \geq 1$, where

$$\begin{aligned} V_{Qq} &= \sigma_{Dq},\ V_q = \sigma_q^2, \\ V_Q &= \frac{\sigma_F^2}{\left(1 - R^{-1}a_F\right)^2} + \sigma_D^2 + \frac{R^{-2}\sigma_Z^2}{\left(1 - R^{-1}a_Z\right)^2}\left(\frac{\sigma_{Dq}}{\sigma_q^2}\right)^2. \end{aligned}$$

We can see that

$$Cov\left(Q_{t+n}, Q_t\right) > 0 \text{ if and only if } a_Z R > 1 \text{ for all } n \geq 1.$$

Thus, we cannot generate momentum in the short run and subsequent reversals in the long run for the model without advance information.

2. New Correlation: $E\left[\varepsilon_t^F \varepsilon_t^Z\right] = \sigma_{FZ} > 0$

To see whether the assumption of advance information is needed in our model, we drop the advance information signal from the information set and consider a different information structure by assuming that

$$E\left[\varepsilon_t^F \varepsilon_t^Z\right] = \sigma_{FZ} > 0. \tag{16}$$

We still maintain the assumption that $E\left[\varepsilon_t^D \varepsilon_t^q\right] = \sigma_{Dq} > 0$. These two assumptions imply that both the persistent and transitory components of the dividend process and the nontraded asset return process are positively correlated. We have shown in Section 1 that if $\sigma_{FZ} = 0$, the model without advance information cannot generate momentum in the short run and subsequent reversals in the long run. Using the same representative agent model, we shall demonstrate below that the assumption of $\sigma_{FZ} > 0$ cannot deliver momentum in the short run and subsequent reversals in the long run.

As in Section 1, we conjecture that the joint distribution of Q_{t+1} and q_{t+1} conditional on date t information follows a normal distribution given in (4). The equilibrium conditions are still given by (5) and (6). Since $\mu_{qt} = E_t\left[q_{t+1}\right] = Z_t$, $V_q = Var_t\left(q_{t+1}\right) = \sigma_q^2$, we can rewrite these conditions as follows:

$$\begin{aligned} V_Q + \alpha_t V_{Qq} &= \gamma^{-1} \mu_{Qt}, \\ V_{Qq} + \alpha_t \sigma_q^2 &= \gamma^{-1} Z_t. \end{aligned}$$

Solving this system of equations gives:

$$\alpha_t = \frac{Z_t}{\gamma V_q} - \frac{V_{Qq}}{\sigma_q^2}, \tag{17}$$

$$\mu_{Qt} = \frac{V_{Qq} Z_t}{\sigma_q^2} + \gamma \frac{V_Q \sigma_q^2 - V_{Qq}^2}{\sigma_q^2}. \tag{18}$$

By definition,

$$\begin{aligned} \mu_{Qt} &= E_t \left[P_{t+1} \right] + E_t \left[D_{t+1} \right] - R P_t \\ &= E_t \left[P_{t+1} \right] + a_F F_t - R P_t. \end{aligned}$$

Substituting this equation into (17) yields:

$$P_t = R^{-1} \left\{ a_F F_t - \frac{V_{Qq}}{\sigma_q^2} Z_t + \gamma \left(\frac{V_{Qq}^2}{\sigma_q^2} - V_Q \right) \right\} + R^{-1} E_t \left[P_{t+1} \right].$$

Repeating substitution yields:

$$\begin{aligned} P_t &= R^{-1} \left\{ a_F F_t - \frac{V_{Qq}}{\sigma_q^2} Z_t + \gamma \left(\frac{V_{Qq}^2}{\sigma_q^2} - V_Q \right) \right\} \\ &\quad + R^{-2} \left\{ a_F^2 F_t - \frac{V_{Qq}}{\sigma_q^2} a_Z Z_t + \gamma \left(\frac{V_{Qq}^2}{\sigma_q^2} - V_Q \right) \right\} + \ldots \\ &= -\frac{\gamma \left(V_Q \sigma_q^2 - V_{Qq}^2 \right)}{\sigma_q^2} \frac{R^{-1}}{1 - R^{-1}} + \frac{R^{-1} a_F}{1 - R^{-1} a_F} F_t - \frac{V_{Qq}}{\sigma_q^2} \frac{R^{-1}}{1 - R^{-1} a_Z} Z_t, \end{aligned}$$

where we have imposed the transversality (or no-bubble) condition $\lim_{T \to \infty} E_t \left[P_{T+1} / R^T \right] = 0$.

Having solved for the price function, we can get the excess return:

$$
\begin{aligned}
Q_{t+1} &= \frac{\gamma\left(V_Q\sigma_q^2 - V_{Qq}^2\right)}{\sigma_q^2}\frac{R^{-1}}{1-R^{-1}} + \frac{R^{-1}a_F}{1-R^{-1}a_F}F_{t+1} - \frac{V_{Qq}}{\sigma_q^2}\frac{R^{-1}}{1-R^{-1}a_Z}Z_{t+1} + D_{t+1} \\
&\quad -R\left[\gamma\frac{V_{Qq}^2 - V_Q\sigma_q^2}{\sigma_q^2}\frac{R^{-1}}{1-R^{-1}} + \frac{R^{-1}a_F}{1-R^{-1}a_F}F_t - \frac{V_{Qq}}{\sigma_q^2}\frac{R^{-1}}{1-R^{-1}a_Z}Z_t\right],
\end{aligned}
$$

or, simplifying,

$$
Q_{t+1} = \frac{\gamma\left(V_Q\sigma_q^2 - V_{Qq}^2\right)}{\sigma_q^2} + \frac{1}{1-R^{-1}a_F}\varepsilon_{t+1}^F + \frac{V_{Qq}}{\sigma_q^2}Z_t - \frac{V_{Qq}}{\sigma_q^2}\frac{R^{-1}}{1-R^{-1}a_Z}\varepsilon_{t+1}^Z + \varepsilon_{t+1}^D. \tag{19}
$$

This leads to

$$
\begin{aligned}
V_Q &= \frac{1}{\left(1-R^{-1}a_F\right)^2}\sigma_F^2 + \left(\frac{V_{Qq}}{\sigma_q^2}\frac{R^{-1}}{1-R^{-1}a_Z}\right)^2\sigma_Z^2 \\
&\quad -2\frac{1}{1-R^{-1}a_F}\frac{V_{Qq}}{\sigma_q^2}\frac{R^{-1}}{1-R^{-1}a_Z}\sigma_{FZ} + \sigma_D^2 \\
V_{Qq} &= E_t\left[\left(\frac{1}{1-R^{-1}a_F}\varepsilon_{t+1}^F - \frac{V_{Qq}}{\sigma_q^2}\frac{R^{-1}}{1-R^{-1}a_Z}\varepsilon_{t+1}^Z + \varepsilon_{t+1}^D\right)\varepsilon_{t+1}^q\right] = \sigma_{Dq}.
\end{aligned}
$$

We then complete the equilibrium solution.

To study momentum and reversals, we use (18) to show that

$$
\begin{aligned}
E\left[Q_{t+1}|Q_t\right] &= E\left[\mu_{Qt}|Q_t\right] \\
&= \gamma\frac{V_Q\sigma_q^2 - V_{Qq}^2}{\sigma_q^2} + \frac{V_{Qq}}{\sigma_q^2}E\left(Z_t|Q_t\right) \\
&= \gamma\frac{V_Q\sigma_q^2 - \sigma_{Dq}^2}{\sigma_q^2} + \frac{\sigma_{Dq}}{\sigma_q^2}\frac{Cov\left[Z_t, Q_t\right]}{Var\left(Q_t\right)}Q_t,
\end{aligned}
$$

and, for $n \geq 2$,

$$\begin{aligned}
E\left[Q_{t+n}|Q_t\right] &= E\left[\gamma\frac{V_Q\sigma_q^2 - V_{Qq}^2}{\sigma_q^2} + \frac{1}{1-R^{-1}a_F}\varepsilon_{t+n}^F + \frac{V_{Qq}}{\sigma_q^2}Z_{t+n} - \frac{V_{Qq}}{\sigma_q^2}\frac{R^{-1}}{1-R^{-1}a_Z}\varepsilon_{t+n}^Z + \varepsilon_{t+n}^D|Q_t\right] \\
&= \gamma\frac{V_Q\sigma_q^2 - V_{Qq}^2}{\sigma_q^2} + \frac{V_{Qq}}{\sigma_q^2}E\left[Z_{t+n}|Q_t\right] \\
&= \gamma\frac{V_Q\sigma_q^2 - V_{Qq}^2}{\sigma_q^2} + \frac{V_{Qq}}{\sigma_q^2}a_Z^{n-1}E\left[Z_t|Q_t\right] \\
&= \gamma\frac{V_Q\sigma_q^2 - \sigma_{Dq}^2}{\sigma_q^2} + \frac{\sigma_{Dq}}{\sigma_q^2}a_Z^{n-1}\frac{Cov\left[Z_t, Q_t\right]}{Var\left(Q_t\right)}Q_t,
\end{aligned}$$

Thus,

$$Cov\left(Q_{t+n}, Q_t\right) > 0 \text{ if and only if } Cov\left[Z_t, Q_t\right] > 0,$$

for all $n \geq 1$ under the assumption $\sigma_{Dq} > 0$. We can use (19) to explicitly compute

$$\begin{aligned}
Cov\left[Z_t, Q_t\right] &= E\left[\left(a_Z Z_{t-1} + \varepsilon_t^Z\right)\left(\frac{1}{1-R^{-1}a_F}\varepsilon_t^F + \frac{V_{Qq}}{\sigma_q^2}Z_{t-1} - \frac{V_{Qq}}{\sigma_q^2}\frac{R^{-1}}{1-R^{-1}a_Z}\varepsilon_t^Z + \varepsilon_t^D\right)\right] \\
&= \frac{1}{1-R^{-1}a_F}\sigma_{FZ} + \frac{V_{Qq}}{\sigma_q^2}a_Z\frac{\sigma_Z^2}{1-a_Z^2} - \frac{V_{Qq}}{\sigma_q^2}\frac{R^{-1}}{1-R^{-1}a_Z}\sigma_Z^2 \\
&= \frac{1}{1-R^{-1}a_F}\sigma_{FZ} + \frac{V_{Qq}}{\sigma_q^2}\sigma_Z^2\frac{a_Z\left(1-R^{-1}a_Z\right) - R^{-1}\left(1-a_Z^2\right)}{\left(1-a_Z^2\right)\left(1-R^{-1}a_Z\right)} \\
&= \frac{1}{1-R^{-1}a_F}\sigma_{FZ} + \frac{\sigma_{Dq}}{\sigma_q^2}\sigma_Z^2 R^{-1}\frac{Ra_Z - 1}{\left(1-a_Z^2\right)\left(1-R^{-1}a_Z\right)}.
\end{aligned}$$

This implies that the model without advance information cannot generate momentum in the short run and subsequent reversals in the long run no matter whether we impose assumption (16) or not. We can get short-run momentum by assuming $\sigma_{FZ} > 0$ and a_Z sufficiently large so that $Cov\left[Z_t, Q_t\right] > 0$. But in this case, one will get momentum for all horizons and cannot generate subsequent reversals.

3. Advance Information about ε_{t+1}^F and $E\left(\varepsilon_{t+1}^F\varepsilon_{t+1}^q\right) > 0$

In Section 2 of the paper, we have shown that the assumption that the investor has advance information about the transitory component of earnings and that this component is positively correlated with the nontraded asset return can generate short-run momentum and long-run reversals. In this section, we use a representative agent model to show that the assumption that the investor has advance information about the persistent component of earnings and that this component is positively correlated with the nontraded asset return can also generate short-run momentum and long-run reversals.

Assume that the advance information signal is given by

$$S_t = \varepsilon_{t+1}^F + \varepsilon_t^S.$$

In addition, assume that $E\left(\varepsilon_{t+1}^F \varepsilon_{t+1}^q\right) = \sigma_{Fq} > 0$, but $E\left(\varepsilon_{t+1}^D \varepsilon_{t+1}^q\right) = \sigma_{Dq} = 0$. We still maintain other correlation assumptions as in Section 2 of the paper.

Conjecture that the joint distribution of Q_{t+1} and q_{t+1} conditional on date t information follows a normal distribution given in (4). We can solve for V_q :

$$V_q = Var_t\left(Z_t + \varepsilon_{t+1}^q\right) = Var\left(\varepsilon_{t+1}^q | S_t\right) = \sigma_q^2 - \frac{\sigma_{Fq}^2}{\sigma_S^2 + \sigma_F^2}. \tag{20}$$

We will show below that other elements in V are also constant.

The equilibrium conditions are still given by (7) and (8). We then compute:

$$\begin{aligned}
\mu_{Qt} &= E_t\left[P_{t+1}\right] + E_t\left[D_{t+1}\right] - RP_t \\
&= E_t\left[P_{t+1}\right] + E_t\left[F_{t+1} + \varepsilon_{t+1}^D\right] - RP_t \\
&= E_t\left[P_{t+1}\right] + a_F F_t + E\left[\varepsilon_{t+1}^D | S_t\right] - RP_t \\
&= E_t\left[P_{t+1}\right] + a_F F_t + \bar{\sigma}_F S_t - RP_t,
\end{aligned}$$

and

$$\mu_{qt} = E_t\left[Z_t + \varepsilon_{t+1}^q\right] = Z_t + E\left[\varepsilon_{t+1}^q | S_t\right] = Z_t + \bar{\sigma}_q S_t, \tag{21}$$

where

$$\bar{\sigma}_q \equiv \frac{\sigma_{Fq}}{\sigma_F^2 + \sigma_S^2}, \quad \bar{\sigma}_F \equiv \frac{\sigma_F^2}{\sigma_S^2 + \sigma_F^2}.$$

Substituting the above expressions for μ_{Qt} and μ_{qt} into (8) yields a difference equation for P_t :

$$P_t = R^{-1}\left\{a_F F_t + \bar{\sigma}_F S_t - \frac{V_{Qq}}{V_q}\left(Z_t + \bar{\sigma}_q S_t\right) + \gamma\left(\frac{V_{Qq}^2}{V_q} - V_Q\right)\right\} + R^{-1} E_t\left[P_{t+1}\right].$$

Solving this equation by repeated substitution yields:

$$\begin{aligned} P_t &= -\frac{\gamma\left(V_Q V_q - V_{Qq}^2\right)}{V_q}\frac{R^{-1}}{1-R^{-1}} + \frac{R^{-1}}{1-R^{-1}a_F}\left(a_F F_t + \bar{\sigma}_F S_t\right) \\ &\quad -\frac{R^{-1}V_{Qq}}{V_q\left(1-R^{-1}a_Z\right)}Z_t - R^{-1}\frac{V_{Qq}\bar{\sigma}_q}{V_q}S_t, \end{aligned}$$

where we have imposed the transversality (or no-bubble) condition $\lim_{T\to\infty} E_t\left[P_{T+1}/R^T\right] = 0$.

Using this equation, we can derive the excess return:

$$\begin{aligned} Q_{t+1} &= \frac{\gamma\left(V_Q V_q - V_{Qq}^2\right)}{V_q} + \frac{1}{1-R^{-1}a_F}\varepsilon_{t+1}^F + \varepsilon_{t+1}^D - \frac{R^{-1}V_{Qq}}{V_q\left(1-R^{-1}a_Z\right)}\varepsilon_{t+1}^Z + \frac{V_{Qq}}{V_q}Z_t \\ &\quad + \left(\frac{1}{1-R^{-1}a_F}\bar{\sigma}_F - \frac{V_{Qq}\bar{\sigma}_q}{V_q}\right)\left(R^{-1}S_{t+1} - S_t\right). \end{aligned}$$

From this equation, we can see that $V_Q = Cov_t\left(Q_{t+1}\right)$ is constant and

$$\begin{aligned} V_{Qq} &= Cov_t\left(Q_{t+1}, q_{t+1}\right) \\ &= E_t\left[\left(Q_{t+1} - \mu_{Qt}\right)\left(q_{t+1} - \left(Z_t + \bar{\sigma}_q S_t\right)\right)\right] \\ &= \frac{1}{1-R^{-1}a_F}\frac{\sigma_S^2\sigma_{Fq}}{\sigma_S^2 + \sigma_F^2}. \end{aligned} \tag{22}$$

Thus, $V_{Qq} > 0$ if and only if $\sigma_{Fq} > 0$. This completes the derivation of equilibrium.

Turn to the study of momentum and reversals. Using (8) and (21), we can compute

$$\mu_{Qt} = \frac{V_{Qq}}{V_q}\left(Z_t + \bar{\sigma}_q S_t\right) - \gamma\left(\frac{V_{Qq}^2}{V_q} - V_Q\right).$$

It follows from the law of iterated expectations that

$$\begin{aligned} E\left[Q_{t+1}|Q_t\right] &= E\left[\mu_{Qt}|Q_t\right] \\ &= \frac{V_{Qq}}{V_q}E\left[Z_t + \bar{\sigma}_q S_t|Q_t\right] - \gamma\left(\frac{V_{Qq}^2}{V_q} - V_Q\right) \\ &= \frac{\gamma\left(V_Q V_q - V_{Qq}^2\right)}{V_q} + \frac{V_{Qq}}{V_q}\frac{Cov\left[Z_t + \bar{\sigma}_q S_t, Q_t\right]}{Var\left(Q_t\right)}Q_t. \end{aligned}$$

Thus,

$$Cov\left(Q_{t+1}, Q_t\right) > 0 \text{ if and only if } Cov\left[Z_t + \bar{\sigma}_q S_t, Q_t\right] > 0.$$

For $n \geq 2$, we can compute that

$$\begin{aligned}
E\left[Q_{t+n}|Q_t\right] &= E\left[\mu_{Qt+n-1}|Q_t\right] \\
&= \frac{\gamma\left(V_Q V_q - V_{Qq}^2\right)}{V_q} + \frac{V_{Qq}}{V_q}E\left[Z_{t+n-1} + \bar{\sigma}_q S_{t+n-1}|Q_t\right] + \\
&= \frac{\gamma\left(V_Q V_q - V_{Qq}^2\right)}{V_q} + \frac{V_{Qq}}{V_q}\frac{Cov\left[Z_{t+n-1} + \bar{\sigma}_q S_{t+n-1}, Q_t\right]}{Var\left(Q_t\right)}Q_t \\
&= \frac{\gamma\left(V_Q V_q - V_{Qq}^2\right)}{V_q} + a_Z^{n-1}\frac{V_{Qq}}{V_q}\frac{Cov\left[Z_t, Q_t\right]}{Var\left(Q_t\right)}Q_t,
\end{aligned}$$

where we have used the fact that $Cov\left(S_{t+n-1}, Q_t\right) = 0$ for $n \geq 2$. Thus, for all $n \geq 2$,

$$Cov\left(Q_{t+n}, Q_t\right) > 0 \text{ if and only if } Cov\left[Z_t, Q_t\right] > 0.$$

It remains to compute that

$$\begin{aligned}
Cov\left(Z_t, Q_t\right) &= E\left[\left(a_Z Z_{t-1} + \varepsilon_t^Z\right)\left(-\frac{R^{-1}V_{Qq}}{V_q\left(1 - R^{-1}a_Z\right)}\varepsilon_t^Z + \frac{V_{Qq}}{V_q}Z_{t-1}\right)\right] \\
&= \frac{V_{Qq}}{\bar{v}_q}\left(a_Z\frac{\sigma_Z^2}{1 - a_Z^2} - \frac{R^{-1}}{1 - R^{-1}a_Z}\sigma_Z^2\right) \\
&= \frac{V_{Qq}}{V_q}\frac{\sigma_Z^2}{R}\left(\frac{Ra_Z - 1}{\left(1 - a_Z^2\right)\left(1 - R^{-1}a_Z\right)}\right).
\end{aligned}$$

Thus, to obtain long-run reversals, we need to assume that $a_Z R < 1$.

We also compute that

$$\begin{aligned}
Cov\left(S_t, Q_t\right) &= E\left[S_t\left(\frac{1}{1 - R^{-1}a_F}\bar{\sigma}_F - \frac{V_{Qq}\bar{\sigma}_q}{V_q}\right)R^{-1}S_t\right] \\
&= \left(\frac{1}{1 - R^{-1}a_F}\bar{\sigma}_F - \frac{V_{Qq}\bar{\sigma}_q}{V_q}\right)R^{-1}\left(\sigma_F^2 + \sigma_S^2\right) \\
&= \left(\frac{1}{1 - R^{-1}a_F}\frac{\sigma_F^2}{\sigma_S^2 + \sigma_F^2} - \frac{V_{Qq}}{V_q}\frac{\sigma_{Fq}}{\sigma_F^2 + \sigma_S^2}\right)R^{-1}\left(\sigma_F^2 + \sigma_S^2\right) \\
&= \left(\frac{1}{1 - R^{-1}a_F}\sigma_F^2 - \frac{V_{Qq}}{V_q}\sigma_{Fq}\right)R^{-1}.
\end{aligned}$$

Substituting (20) and (22) into the above equation, we obtain:

$$
\begin{aligned}
Cov\left(S_t, Q_t\right) &= \left(\sigma_F^2 - \frac{1}{V_q}\sigma_{Fq}^2\frac{\sigma_S^2}{\sigma_S^2+\sigma_F^2}\right)\frac{R^{-1}}{1-R^{-1}a_F} \\
&= \left(\sigma_F^2 - \frac{\sigma_{Fq}^2\sigma_S^2}{\sigma_q^2\left(\sigma_S^2+\sigma_F^2\right)-\sigma_{Fq}^2}\right)\frac{R^{-1}}{1-R^{-1}a_F} \\
&= \frac{\sigma_F^2\sigma_q^2-\sigma_{Fq}^2}{\sigma_q^2\left(\sigma_S^2+\sigma_F^2\right)-\sigma_{Fq}^2}\frac{\left(\sigma_F^2+\sigma_S^2\right)R^{-1}}{1-R^{-1}a_F} \\
&= \frac{1-\rho_{Fq}^2}{\sigma_q^2\sigma_S^2+\sigma_q^2\sigma_F^2\left(1-\rho_{Fq}^2\right)}\frac{\sigma_F^2\sigma_q^2\left(\sigma_F^2+\sigma_S^2\right)R^{-1}}{1-R^{-1}a_F},
\end{aligned}
$$

where $\rho_{Fq} \in (0,1)$ is the correlation coefficient between ε_{t+1}^F and ε_{t+1}^q. It follows that $Cov\left(S_t, Q_t\right) > 0$. A sufficient condition to generate one-period momentum is that a_Z is sufficiently close to $1/R$ so that $Cov\left[Z_t + \bar{\sigma}_q S_t, Q_t\right] > 0$. If $a_Z < 1/R$, then we obtain reversals from period 2 on.

4. Main Model with Intertemporal consumption

The assumption of myopic investors made in the paper is important for tractability and allows us to derive analytically several equilibrium properties regarding the role of advance information. The main drawback is the omission of dynamic hedging demands. Dynamic hedging demands introduce a concern for stochastic changes in the investment opportunity set, as given by changes in the state vector, and thus may be particularly relevant in a model with advance information where investors get signals about k-period ahead earnings.

Here, we solve a model identical to the main model in the paper, but where investors have the intertemporal preferences ($l = i, u$):

$$
-E_t^l\left[\sum_{j=0}^{\infty}\beta^j \exp\left(-\gamma c_{t+j}^l\right)\right],
$$

where $\beta \in (0,1)$. They face intertemporal budget constraints:

$$
W_{t+1}^l = R\left(W_t^l - c_t^l\right) + \boldsymbol{\psi}_t^{l\intercal}\mathbf{R}_{t+1}^l.
$$

The vectors $\boldsymbol{\psi}_t^l$ and $\mathbf{R}_t^l$ denote asset holdings and per share excess returns to assets that are available to investor l. In particular, for informed investors, $\boldsymbol{\psi}_t^i = \left(\theta_t^i, \alpha_t^i\right)^\intercal$ and $\mathbf{R}_t^i = \left(Q_t, q_t\right)^\intercal$, and for uninformed investors, $\boldsymbol{\psi}_t^u = \theta_t^u$ and $\mathbf{R}_t^u = Q_t$.

Let $V^l\left(W_t^l, \boldsymbol{\phi}_t^l\right)$ denote the value function of investor l, where $\boldsymbol{\phi}_t^l$ is a transformation of the estimated state vector $\hat{\mathbf{x}}_t^l$. Below, we show how to set up the vector $\boldsymbol{\phi}_t^l$ and prove that (dropping superscripts for simplicity):

$$V\left(W_t, \boldsymbol{\phi}_t\right) = -\exp\left[-\kappa - \tilde{\gamma} W_t - \mathbf{u}^\intercal \boldsymbol{\phi}_t - \frac{1}{2}\boldsymbol{\phi}_t^\intercal \mathbf{U} \boldsymbol{\phi}_t\right],$$

where $\tilde{\gamma} = \gamma\frac{R-1}{R}$, and κ, $\mathbf{u}$ and $\mathbf{U}$ are constants. The upshot of the analysis is investors' asset holdings. Asset holdings can be broken down into two parts. One is the myopic asset demand analyzed in the paper and given by:

$$\tilde{\gamma}^{-1}\left[\mathbf{B}_R \Gamma \mathbf{B}_R^\intercal\right]^{-1} E_t\left(\mathbf{R}_{t+1}\right),$$

and the other is the dynamic hedging demand,

$$-\tilde{\gamma}^{-1}\left[\mathbf{B}_R \Gamma \mathbf{B}_R^\intercal\right]^{-1} Cov_t\left(\left(\mathbf{u}^\intercal + E_t^l\left[\boldsymbol{\phi}_{t+1}\right]^\intercal \mathbf{U}\right)\boldsymbol{\phi}_{t+1}, \mathbf{R}_{t+1}\right).$$

The matrix $\mathbf{B}_R \Gamma \mathbf{B}_R^\intercal$ is a transformed covariance matrix of asset returns, adjusted for the risk profile of each investor. Note that uninformed investors hold only one risky asset and thus $\mathbf{B}_R \Gamma \mathbf{B}_R^\intercal$ is a scalar whereas informed investors hold two risky assets and thus $\mathbf{B}_R \Gamma \mathbf{B}_R^\intercal$ is a 2×2 matrix. As in the main model, the myopic demands for the informed and uninformed investor depend crucially on the expected asset returns $E_t^i\left(\mathbf{R}_{t+1}\right)$ and $E_t^u\left(\mathbf{R}_{t+1}\right)$, respectively. The analysis in the paper applies here, though we are unable to determine the signs analytically as we did there.

The dynamic hedging demand reflects a concern for changing investment opportunities: Investors hold more of the stock if the stock pays out more in states where investment opportunities are bad, i.e., when $\left(\mathbf{u}^\intercal + E_t^l\left[\boldsymbol{\phi}_{t+1}\right]^\intercal \mathbf{U}\right)\boldsymbol{\phi}_{t+1}$ is low. In particular, good advance information about future earnings on the stock received at t implies that good investment opportunities are likely for both the stock and the private investment opportunity in the future and makes informed investors hold less of both assets.

We use numerical examples to evaluate the relative importance of the above myopic and dynamic hedging demands. We find that the dynamic hedging demand is not important quantitatively. In particular, in response to a good signal about earnings innovation in the next period, the stock return increases, informed investors buy the stock for speculative reasons and uninformed investors sell the stock to accommodate these trades on impact. Informed investors also invest more in the nontraded asset, as they did in the myopic case, and thus bear greater risk. This leads stock returns to display short term momentum.

The table below presents some numerical examples and simple comparative statics on how the informativeness of advance information affects momentum. Momentum is stronger when advance information is less precise. As in the main model, when advance information is very precise, there are few rebalancing trades in the nontraded asset, and momentum disappears. When advance information is very noisy, we are back in a model without advance information and with a_Z small, momentum also disappears. It is at intermediate levels of precision of advance information that the dynamic hedging demands become more important. Because dynamic hedging demands also allow informed investors to hide their speculative trades, momentum is stronger than in the myopic case.

The solution method is similar to that for Proposition 1 and that in Wang (1994). We sketch it here. We conjecture that the equilibrium price function takes the same form as in the myopic investor case

$$P_t = -p_0 + \mathbf{p}_i \hat{\mathbf{x}}_t^i + \mathbf{p}_u \hat{\mathbf{x}}_t^u.$$

We shall verify that this conjecture is correct and derive the equilibrium system of equations for the coefficients in the price function. Given the conjectured price function, we write the excess stock returns Q_{t+1} as in equation (B.11) with the coefficients restrictions:

$$e_{i,j} + e_{u,j} = 0, \text{ for all } j \neq 2, k+2, \tag{23}$$

$$e_{i2} + e_{u2} = e_{i2}, \tag{24}$$

$$e_{i,k+2} + e_{u,k+2} = e_{i2} \frac{\sigma_{Dq}}{\sigma_D^2}. \tag{25}$$

We note that the informed and uninformed investors solve similar filtering problems to those in the myopic investor case. To solve the investors' consumption and portfolio choice problems,

Table 1.

MOMENTUM AND REVERSAL

IN THE MODEL WITH INTERTEMPORAL CONSUMPTION AND ADVANCE INFORMATION

$n \setminus \sigma_S$	0.25		0.5		0.75	
	Intertemporal	Myopic	Intertemporal	Myopic	Intertemporal	Myopic
1	-0.0035	0.0016	0.0027	0.0026	0.0024	0.0008
2	-0.0028	-0.0001	-0.0015	-0.0013	-0.0017	-0.0030
3	-0.0025	-0.0001	-0.0014	-0.0012	-0.0016	-0.0027
4	-0.0022	-0.0001	-0.0013	-0.0011	-0.0014	-0.0024
5	-0.0020	-0.0001	-0.0011	-0.0010	-0.0013	-0.0022
6	-0.0018	-0.0001	-0.0010	-0.0009	-0.0011	-0.0019
7	-0.0016	-0.0001	-0.0009	-0.0008	-0.0010	-0.0018
8	-0.0015	-0.0001	-0.0008	-0.0007	-0.0009	-0.0016
9	-0.0013	-0.0001	-0.0007	-0.0006	-0.0008	-0.0014
10	-0.0012	-0.0001	-0.0007	-0.0006	-0.0007	-0.0013

The table displays the slope coefficients of regressing single period returns, Q_{t+n}, on current returns:

$$Q_{t+n} = a_n + b_n Q_t + \varepsilon_{t,n}.$$

The columns labeled "Intertemporal" refer to the model with intertemporal consumption and the columns labeled "Myopic" refer to the model in Section 3 of the paper. We set $k = 1$, $\sigma_D = 1$, $\sigma_F = 0.5$, $\sigma_Z = 1$, $\sigma_q = 0.5$, $\sigma_{Dq} = 0.25$, $a_F = a_Z = 0.9$, $\lambda = 0.9$, $\gamma = 5$, and $r = 0.1$.

we use dynamic programming and define the state vectors as follows:

$$\begin{aligned}\boldsymbol{\phi}_t^u &= \left[\begin{array}{cccc} \hat{Z}_t^u & E_t^u\left[\varepsilon_{t+k}^D\right] & \ldots & E_t^u\left[\varepsilon_{t+1}^D\right] \end{array}\right]^\intercal, \\ \boldsymbol{\phi}_t^i &= \left[\begin{array}{ccccc} Z_t & E_t^i\left[\varepsilon_{t+k}^D\right] & \ldots & E_t^i\left[\varepsilon_{t+1}^D\right] & \zeta_t^{u\intercal} \end{array}\right]^\intercal,\end{aligned}$$

where

$$\zeta_t^u = \left[\begin{array}{cccc} F_t - \hat{F}_t & \hat{\varepsilon}_{t+k}^{Di} - \hat{\varepsilon}_{t+k}^{Du} & \ldots & \hat{\varepsilon}_{t+1}^{Di} - \hat{\varepsilon}_{t+1}^{Du} \end{array}\right]^\intercal.$$

Start with uninformed investors' optimization problem. Noting that equations (B.12)-(B.15) still hold, we can derive:

$$\mathbf{R}_{t+1}^u = \bar{\mathbf{R}}^u + \mathbf{A}_R\boldsymbol{\phi}_t^u + \mathbf{B}_R\mathbf{v}_{t+1}^u,$$

where $\bar{\mathbf{R}}^u = e_0$,

$$\mathbf{A}_R = \left[\begin{array}{ccc} e_{i2} & 0...0 & e_{i2}\frac{\sigma_{Dq}}{\sigma_D^2} \end{array}\right],\ \mathbf{B}_R = \left[\begin{array}{cc} \mathbf{e}_i\mathbf{S}_x & \mathbf{b}_Q \end{array}\right],\ \mathbf{v}_{t+1}^u = \left[\begin{array}{c} \boldsymbol{\zeta}_t^u \\ \hat{\boldsymbol{\varepsilon}}_{t+1}^i \end{array}\right],$$

and the matrix $\mathbf{S}_x$ is such that $\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u = \mathbf{S}_x\boldsymbol{\zeta}_t^u$. It is then easy to derive $Var_t^u\left[\mathbf{v}_{t+1}^u\right]$. To determine the process for $\boldsymbol{\phi}_t^u$, use equation (B.16) to get:

$$\begin{aligned}\hat{\mathbf{x}}_{t+1}^u &= \mathbf{A}_x\hat{\mathbf{x}}_t^u + \mathbf{K}_u\hat{\boldsymbol{\varepsilon}}_{t+1}^u \\ &= \mathbf{A}_x\hat{\mathbf{x}}_t^u + \mathbf{K}_u\left[\mathbf{A}_{yu}\mathbf{A}_x\left(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u\right) + \mathbf{A}_{yu}\mathbf{K}_i\hat{\boldsymbol{\varepsilon}}_{t+1}^i\right] \\ &= \mathbf{A}_x\hat{\mathbf{x}}_t^u + \mathbf{K}_u\mathbf{A}_{yu}\mathbf{A}_x\mathbf{S}_x\boldsymbol{\zeta}_t^u + \mathbf{K}_u\mathbf{A}_{yu}\mathbf{K}_i\hat{\boldsymbol{\varepsilon}}_{t+1}^i. \end{aligned} \tag{26}$$

By selecting the appropriate rows and columns from the last equation, we arrive at

$$\boldsymbol{\phi}_{t+1}^u = \mathbf{A}_\phi^u\boldsymbol{\phi}_t^u + \mathbf{B}_\phi^u\mathbf{v}_{t+1}^u.$$

For informed investors, note that the conditional expected returns and variances of stock returns and private investment obey the same restrictions as for the myopic case. We can thus write:

$$\mathbf{R}_{t+1}^i = \bar{\mathbf{R}}^i + \mathbf{A}_R^i\boldsymbol{\phi}_t^i + \mathbf{B}_R^i\mathbf{v}_{t+1}^i,$$

where $\bar{\mathbf{R}}^i = \begin{bmatrix} e_0 & 0 \end{bmatrix}^\intercal$,

$$\begin{aligned}
\mathbf{A}_R &= \begin{bmatrix} e_{i2} & 0_{1x(k-1)} & e_{i2}\sigma_{Dq}/\sigma_D^2 & e_{i1} & e_{i,3} & \ldots & e_{ik+1} & -e_{u,k+2} \\ 1 & 0_{1x(k-1)} & \sigma_{Dq}/\sigma_D^2 & 0 & 0 & \ldots & & \end{bmatrix}, \\
\mathbf{B}_R &= \begin{bmatrix} \mathbf{b}_Q & 0 \\ 0_{1x4} & 1 \end{bmatrix}, \mathbf{v}_{t+1}^i = \begin{bmatrix} \hat{\boldsymbol{\varepsilon}}_{t+1}^i \\ \varepsilon_{t+1}^q - E_t^i\left(\varepsilon_{t+1}^q\right) \end{bmatrix}.
\end{aligned}$$

To derive the process for $\boldsymbol{\phi}_t^i$, we use the filtering equation:

$$\hat{\mathbf{x}}_{t+1}^i = \mathbf{A}_x\hat{\mathbf{x}}_t^i + \mathbf{K}_i\hat{\boldsymbol{\varepsilon}}_{t+1}^i,$$

and (26) to get:

$$\hat{\mathbf{x}}_{t+1}^i - \hat{\mathbf{x}}_{t+1}^u = \left(\mathbf{I} - \mathbf{K}_u\mathbf{A}_{yu}\right)\mathbf{A}_x\left(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u\right) + \left(\mathbf{I} - \mathbf{K}_u\mathbf{A}_{yu}\right)\mathbf{K}_i\hat{\boldsymbol{\varepsilon}}_{t+1}^i.$$

We then obtain:

$$\boldsymbol{\phi}_{t+1}^i = \mathbf{A}_\phi^i\boldsymbol{\phi}_t^i + \mathbf{B}_\phi^i\mathbf{v}_{t+1}^i.$$

After expressing returns as functions of the appropriate state vectors and unforecastable errors for the informed and uninformed investors, we solve their optimization problem. The algebra is messy and the derivations take quite long and we omit the results from this paper but keep them available upon request. The end result is a set of conditions that can be solved for the constants κ, $\mathbf{u}$, and $\mathbf{U}$ for each investor type, and the first order conditions that yield the asset demands:

$$\boldsymbol{\psi}_t = \tilde{\gamma}^{-1}\left[\mathbf{B}_R\Gamma\mathbf{B}_R^\intercal\right]^{-1}\left[\bar{\mathbf{R}} - \mathbf{B}_R\Gamma\mathbf{B}_\phi^\intercal\mathbf{u} + \left(\mathbf{A}_R - \mathbf{B_R}\boldsymbol{\Gamma}\mathbf{B}_{\boldsymbol{\phi}}^\intercal\mathbf{U}^\intercal\mathbf{A}_\phi\right)\boldsymbol{\phi}_t\right],$$

where $\Gamma = \mathbf{B}_{\boldsymbol{\phi}}^\intercal\mathbf{U}\mathbf{B}_{\boldsymbol{\phi}} + Var_t^{-1}\left(\mathbf{v_{t+1}}\right)$. The term,

$$\tilde{\gamma}^{-1}\left[\mathbf{B}_R\Gamma\mathbf{B}_R^\intercal\right]^{-1}\left[\bar{\mathbf{R}} + \mathbf{A}_R\boldsymbol{\phi}_t\right] = \tilde{\gamma}^{-1}\left[\mathbf{B}_R\Gamma\mathbf{B}_R^\intercal\right]^{-1}E_t\left(\mathbf{R}_{t+1}^l\right),$$

gives the myopic demand whereas the term,

$$\begin{aligned}
& -\tilde{\gamma}^{-1}\left[\mathbf{B}_R\Gamma\mathbf{B}_R^\intercal\right]^{-1}\left[\mathbf{B}_R\Gamma\mathbf{B}_\phi^\intercal\mathbf{u} + \mathbf{B}_R\boldsymbol{\Gamma}\mathbf{B}_{\boldsymbol{\phi}}^\intercal\mathbf{U}^\intercal\mathbf{A}_\phi\boldsymbol{\phi}_t\right] \\
= & -\tilde{\gamma}^{-1}\left[\mathbf{B}_R\Gamma\mathbf{B}_R^\intercal\right]^{-1}Cov_t^l\left(\left(\mathbf{u}^\intercal + E_t^l\left[\boldsymbol{\phi}_{t+1}^l|\boldsymbol{\phi}_t^l\right]^\intercal\mathbf{U}\right)\boldsymbol{\phi}_{t+1}^l, \mathbf{R}_{t+1}^l\right),
\end{aligned}$$

gives the intertemporal hedging demands. The covariance above is adjusted for investors' risk preferences. We use the asset demands and the market clearing condition to solve for the coefficients in the price function.