Credit Risk and Business Cycles*

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Abstract

We incorporate long-term defaultable corporate bonds and credit risk in a dynamic stochastic general equilibrium business cycle model. Credit risk amplifies aggregate technology shocks. The debt-capital ratio is a new state variable and its endogenous movements provide a propagation mechanism. The model can match the persistence and volatility of output growth as well as the mean equity premium and the mean risk-free rate as in the data. The model implied credit spreads are countercyclical and forecast future economic activities because they affect firm investment through Tobin's Q. They also forecast future stock returns through changes in the market price of risk. Finally, we show that financial shocks to the credit markets are transmitted to the real economy through Tobin's Q.

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1 Introduction

The recent financial crisis indicates that credit risk impacts the macroeconomy in a significant way. This fact leads to a number of recent empirical studies documenting the relationship between credit spreads–the difference between yields on corporate bonds and Treasury securities with comparable maturity–and economic activities (see, e.g., Gilchrist, Yankov and Zakrajsek (2009) and references cited therein). In addition, it also spurs renewed interest in developing theoretical models to understand how credit risk affects business cycles.

Most extant business cycle models do not incorporate defaultable corporate bonds and credit risk. According to the real business cycle (RBC) theory, the driving force of business cycles is the shock to aggregate productivity. The pioneering studies of Kydland and Prescott (1982) and Long and Plosser (1983) show that technology shocks built in a dynamic general equilibrium framework can explain business cycles reasonably well, but financial and monetary factors do not play an important role. The RBC approach has developed rapidly and incorporated various forms of market failure (see King and Rebelo (1999) for a survey). Building on the dynamic stochastic general equilibrium framework of the RBC theory, the New Keynesian approach incorporates the monetary sector and shocks to the demand side of the economy (e.g., Dotsey, King, and Wolman (1999) and Christiano, Eichenbaum and Evans (2005)). For both approaches, there is no generally accepted model that explicitly incorporates credit markets for long-term corporate bonds to study the impact of credit risk on economic fluctuations.

In this paper, we attempt to develop such a model in a simplest possible setting. Our model builds on the basic model of Jermann (1998). The Jermann model features habit formation preferences and capital adjustment costs. Both elements are useful to explain asset pricing puzzles in a production economy. We depart from Jermann (1998) in three dimensions. First, we allow firms to issue defaultable corporate bonds to finance investment. The choice between debt and equity reflects the tradeoff between the tax advantage of debt and the associated bankruptcy and agency costs. This choice matters for real investment and production at the firm level and hence the real economy at the aggregate level, because the assumptions of the Modigliani and Miller Theorem are violated in our model. Second, we introduce firm heterogeneity. Firms are ex ante identical, but differ ex post because they may experience idiosyncratic liquidity shocks. A bad liquidity shock to firm profits reduces cash flows and may trigger firms to default on debt. Third, we introduce endogenous leisure, which is necessary for a business cycle analysis. The most widely adopted approach to modelling endogenous labor supply is based on King, Plosser and Rebelo (1988). While this approach has proven useful to understand real quantities, it does not perform well in explaining asset pricing puzzles compared with a model with exogenous leisure (see, e.g., Jermann (1998) and Boldrin, Christiano and Fisher (2001)). We adopt an alternative approach based on Greenwood, Hercowitz and Huffman (1998). Guvenen (1999) shows that this utility performs well for asset pricing and business cycle analysis.

Our calibrated model performs reasonably well in matching both key business cycle moments and financial moments such as the mean equity premium and the mean risk-free rate observed in the data. Traditional production-based dynamic general equilibrium models have difficulties in performing well in these two dimensions simultaneously, especially when leisure is endogenous. Production-based equilibrium models with financial leverage possibly combined with nonstandard preferences such as habit formation or Epstein-Zin utility are promising (e.g., Jermann (1998) and Guvenen (2009)). However, these models typically consider risk-free debt and ignore credit risk.

Our model with credit risk delivers a number of interesting results that are in line with the empirical evidence (e.g., Covas and den Haan (2010) and Gilchrist, Yankov and Zakrajsek (2009)). We show that default rates and credit spreads are countercyclical and debt issuance is procyclical. In addition, credit spreads forecast future growth of output, employment, investment, and consumption as well as future stock returns. In particular, widening of credit spreads signals a future recession. The key intuition is that credit spreads are linked to corporate bond prices which are forward looking variables and affect firm investment through Tobin's Q. This channel is also found theoretically and tested empirically by Philippon (2009) in a partial equilibrium framework. Credit spreads are countercyclical due to both the cash flow effect and the discount rate effect in that payoffs to bond holders decline and marginal utility is high in a recession. A rise in credit spreads signals a rise in future default rates after a negative technology shock. In response, firms reduce debt and investment. In addition, an increase in default rates raise expected default costs, leading to a loss of firm assets and the aggregate capital stock. These effects together signals a future recession. In a recession, marginal utility and hence the market price of risk are high. Thus, expected future stock returns are also high.

Unlike standard RBC models, our model with credit risk provides a mechanism to amplify and propagate technology shocks. The debt-capital ratio (or leverage) is a new endogenous state variable in our model. A negative technology shock leads to a decrease in the debt-capital ratio and an increase in the default rate. This contributes to a further decrease in investment and hence output, in addition to the usual effect of the decrease in the return to capital (i.e., the marginal product of capital). Furthermore, as discussed above, the rise in default costs lowers the aggregate capital stock, which further contributes to the decline in output. As a result, a negative technology shock is amplified. This shock is also propagated in our model because the debt-capital ratio is an endogenously moving state variable. The impulse response function of output is hump shaped, generating a persistent economic fluctuation. Quantitatively, our model can match the volatility of output growth and the relative volatility of consumption growth, employment growth, and consumption growth, as in the data. It can also match the autocorrelation of output growth, as in the data.

We also show that financial shocks to the credit markets are transmitted to the real economy through Tobin's Q. A negative shock to the recovery rates of corporate bonds reduces debt value and hence real investment through Tobin's Q. The impulse response properties of the real quantities to this shock are similar to those in the case of technology shocks except for one difference. A negative financial shock reduces output and hours, while a negative technology shock leads to a decrease in output, but an increase in hours.

Our paper makes a methodological contribution by incorporating long-term defaultable corporate bonds in a dynamic stochastic general equilibrium model with firm heterogeneity. There are two major difficulties. First, the usual way of modelling long-term debt with finite maturity in the corporate finance literature (e.g., Merton (1974)) makes the model nonstationary. When solving bond prices, one has to track the time to maturity. Computing a dynamic general equilibrium model is even more difficult because one has to track the whole history of non-matured debts as new debt is issued each period. To deal with this difficulty, we adapt the continuous-time modelling of Leland (1998) and Hackbarth, Miao and Morellec (2006) to our discrete-time framework.[1] Specifically, we assume that each period each unit of bonds retires with some positive probability, and with the remaining probability this unit does not retire and pays out a coupon rate. The inverse of the retiring probability gives the maturity of the bonds. This modelling allows us to take the total number of bonds as a state variable, which

[1]Philippon (2009) also adopts this modelling approach in his discrete time model. Unlike our model, the Modigliani and Miller theorem holds in his model. His study focuses on how to empirically test the q-theory of investment using bond prices, instead of equity prices.

simplifies a firm's dynamic programming problem.[2]

Second, the discrete nature of default decision makes the firm's dynamic programming problem nonconvex. In addition, it makes the aggregation of individual firm decisions difficult. To deal with this difficulty, we adapt the generalized (s,S) approach of Caballero and Engel (1999) to our model with default. Specifically, we introduce a liquidity shock that is identically and independent distributed across firms and over time. A bad liquidity shock reduces firm profits and may trigger default. The default decision is characterized by a cutoff value of the liquidity shock. All defaulting firms choose the same cutoff.[3] To make aggregation easy, we try to build our model so that it has a linear homogeneity property. A key assumption is that firms have a constant-returns-to-scale technology.[4] With the homogeneity property, our model permits exact aggregation in that only mean matters for the aggregate economy. We do not need to track firm distributions to solve the model numerically. Instead, our exact aggregation allows us to use the standard log-linear or second-order approximation method in the business cycle literature instead of the Krusell and Smith (1998) method.

Our paper contributes to the recent literature that tries to incorporate financial factors in business cycle analysis. Important seminal contributions include Bernanke, Gertler and Gilchrist (1999), Carlstrom and Fuerst (1997) and Kiyotaki and Moore (1997).[5] All these papers provide an amplification and propagation mechanism stemming from the credit markets. Both Bernanke, Gertler and Gilchrist (1999) and Carlstrom and Fuerst (1997) consider one-period defaultable debt contracts between risk-neutral entrepreneurs and lenders. They do not study long-term defaultable corporate bonds. The Carlstrom and Fuerst model generates counterfactual procyclical credit spreads. Kiyotaki and Moore (1997) focus on risk-free debts with collateral constraints. Default never occurs in equilibrium in their model. Along the lines of Kiyotaki and Moore (1997) and Albuquerque and Hopenhayn (2004), Cooley, Marimon and Quadrini (2005) study the aggregate implications of debt contracts with limited enforcement, Jermann and Quadrini (2009) examine how financial shocks affect the real economy, and Liu, Wang and Zha (2009) propose a quantitatively significant mechanism that amplifies and

[2] A similar modelling approach is also applied in the international finance literature on sovereign bonds, although these bonds are different from corporate bonds. See Chatterjee and Eyigungor (2010) and references cited therein.

[3] This modelling is also similar to that in the state-dependent pricing literature (e.g., Dotsey et al. (1999)).

[4] Bernanke, Gertler and Gilchrist (1999) and Carlstrom and Fuerst (1997) make a similar assumption to facilitate aggregation.

[5] Recent papers include Christiano, Motto, and Rostagno (2009), Faia and Monacelli (2007) and Gourio (2010). See Gertler and Kiyotaki (2010) for a survey and additional references cited therein.

propagates shocks through credit constraints.

Our paper is also related to Gomes and Schmid (2010) who focus on asset pricing implications of credit risk by introducing aggregate shocks and Epstein and Zin preferences into the model of Miao (2005). Miao (2005) studies the interaction between investment and financing decisions in a stationary equilibrium by introducing endogenous debt-equity choice into the industry dynamics model of Hopenhayn (1992) and Hopenhayn and Rogerson (1993).[6] Both Gomes and Schmid (2010) and Miao (2005) consider defaultable debt contracts with infinite maturity, as in Leland (1994). As in Miao (2005), Gomes and Schmid (2010) assume that firms choose optimal capital structure only in the initial entry stage. After entry, firms do not adjust capital structure. By contrast, in the present paper, capital structure choice is dynamic in that firms can adjust capital structure at any point in time. In addition, unlike the present paper, Gomes and Schmid (2010) assume exogenous leisure. Their model is not suitable for a full business cycle analysis.

Our modelling of corporate bonds is related to the corporate finance literature on the pricing of corporate securities (see, e.g., Merton (1974), Leland (1994, 1998), Golstein, Ju, and Leland (2001), Hackbarth, Miao and Morellec (2006), Chen, Collin-Dufresne and Goldstein (2009), Bhamra, Kuhn and Strebulaev (2008, 2009), and Chen (2010)). This literature typically considers a pure exchange economy or simply assumes exogenous pricing kernel and takes a firm's cash flows as exogenously given. It typically does not study firms' endogenous production and investment decisions in a general equilibrium framework.

The remainder of the paper proceeds as follows. Section 2 presents the model. Section 3 provides a characterization of equilibrium. Section 4 calibrates the model. Section 5 provides numerical results. Section 6 presents additional quantitative experiments. Section 7 concludes. An appendix collects technical details.

2 The Model

We consider a discrete-time and infinite-horizon economy consisting of a continuum of identical households, a continuum of heterogeneous firms and a government. Households trade firms' shares and corporate bonds. Firms make investment and financing decisions by issuing debt or equity. There is a benefit to use debt financing because firms pay corporate income taxes

[6] Also see Gomes (2001) and Cooley and Quadrini (2001) for related extensions of the Hopenhayn (1992) model.

and interest payments on debt are tax deductible. For simplicity, we assume that the only role of the government in the model is to collect corporate income taxes and to transfer these tax revenues to households.

2.1 Households

All households are identical with a unit mass. The representative household derives utility from consumption C_t and labor N_t according to the following habit formation utility function:

$$\max E\left\{\sum_{t=0}^{\infty}\beta^t\frac{1}{1-\gamma}\left(C_t-hC_{t-1}-\eta\frac{N_t^{1+\varsigma}}{1+\varsigma}\right)^{1-\gamma}\right\},$$

where $\beta\in(0,1)$ is the subjective discount factor, $h\in(0,1)$ is the habit persistence parameter, $\gamma>0,\neq 1$ is the risk aversion parameter, η is the weight on labor, and $1/\varsigma$ is the Frisch elasticity of labor supply. Alternatively, one may introduce leisure into utility as in King, Plosser and Rebelo (1988). Boldrin, Christiano and Fisher (2001) and Jermann (1998) show that this way of introducing leisure in a standard RBC model cannot generate high equity premium as in the data under reasonable calibration. We introduce this preference in our model with credit risk in Section 6.3 and obtain the same finding. We thus adapt the formulation of Greenwood, Hercowitz and Huffman (1988) to our habit formation preferences.

Households choose equity and (corporate and riskless) bond holdings. The net supply of riskless bonds is zero. For simplicity of asset pricing, we assume that the households can also trade complete contingent claims. As a result, we can use the pricing kernel $\beta\Lambda_{t+1}/\Lambda_t$ to price any assets in the economy, where

$$\Lambda_t=\left[C_t-hC_{t-1}-\eta\frac{N_t^{1+\varsigma}}{1+\varsigma}\right]^{-\gamma}-\beta hE_t\left[C_{t+1}-hC_t-\eta\frac{N_{t+1}^{1+\varsigma}}{1+\varsigma}\right]^{-\gamma},\tag{1}$$

is the marginal utility of consumption. The first-order condition for labor is given by:

$$w_t\Lambda_t=\left[C_t-hC_{t-1}-\eta\frac{N_t^{1+\varsigma}}{1+\varsigma}\right]^{-\gamma}\eta N_t^{\varsigma}.\tag{2}$$

where w_t is the wage rate in period t. Without habit formation (i.e., $h=0$), the above two equations imply that labor supply is determined by the wage rate only and there is no wealth effect.

2.2 Firms

There is a continuum of firms indexed by $j \in [0,1]$. These firms are ex ante identical. They differ ex post because they face idiosyncratic liquidity shocks. We focus on a single firm j's behavior. In period t, firm j combines its existing capital stock k_t^j and hires labor n_t^j to produce output y_t^j according to the production function:

$$y_t^j = A_t F(k_t^j, n_t^j),$$

where A_t denotes the aggregate productivity shock. Assume that A_t follows the process:

$$\ln A_t = \rho \ln A_{t-1} + \sigma \varepsilon_t,$$

where ε_t is an identically and independently distribution normal random variable. We assume that the production function has constant returns to scale and takes the Cobb-Douglas form $F(k,n) = k^\alpha n^{1-\alpha}$, where $\alpha \in (0,1)$.

After solving the static labor choice problem, we use the constant-return-to-scale property of F to derive firm j's operating profits as:

$$R_t k_t^j = \max_{n_t^j} A_t F\left(k_t^j, n_t^j\right) - w_t n_t^j,$$

where R_t is the rental rate of capital.

Firm j is subject to additive liquidity shocks, which change its operating profits by the amount $z_t^j k_t^j$. The scaling by k_t^j introduces a linear homogeneity property of our model, which is important for aggregation and for reducing state variables. For simplicity, we assume that z_t^j is identically and independently distributed across firms and over time. It has mean zero and draws from a distribution with the cumulative distribution function Φ over $[z_{\min}, z_{\max}]$. Let the density function be ϕ.

Capital adjustment is costly. If firm j makes investment I_t^j, its capital stock satisfies the law of motion:

$$k_{t+1}^j = (1-\delta)k_t^j + \Psi(I_t^j / k_t^j)k_t^j, \tag{3}$$

where the function Ψ represents adjustment costs as in Hayashi (1982) and Jermann (1998). We assume that Ψ is concave. In addition, $\Psi(x) = x$ and $\Psi'(x) = 1$ where x represents the deterministic steady state investment rate. This assumption implies that there is no adjustment cost in the deterministic steady state.

Since interest payments on debt are tax deductible, firms have an incentive to issue debt. To study the impact of credit risk on business cycles, we consider defaultable corporate bonds. In the beginning of period t, firm j has the capital stock k_t^j and the amount of outstanding defaultable corporate bonds b_t^j. All corporate bonds have finite maturity. To stay in a stationary environment, we adapt the continuous time model of Leland (1998) and Hackbarth, Miao and Morellec (2006) to our discrete time framework. Specifically, we assume that with probability λ, bonds retire and each unit of outstanding bonds promises to pay one unit of the consumption good in the next period. And with probability $1-\lambda$, this unit of bonds rolls over and the firm promises to pay a coupon rate c.

Denote the corporate bond price in period $t-1$ by p_{t-1}^j. The interest rate on the corporate bond between periods $t-1$ and t is given by $\tilde{r}_t^j = 1/p_{t-1}^j - 1$ if the bonds retire in period t, and the interest rate on bonds which do not retire is given by $\tilde{r}_t^j = c/p_{t-1}^j$. We can compute firm j's tax shields in period t as:

$$\tau \tilde{r}_t^j p_{t-1}^j b_t^j = \tau \left[\lambda \left(1/p_{t-1}^j - 1\right) + (1-\lambda) c/p_{t-1}^j\right] p_{t-1}^j b_t^j = \tau \left(\lambda \left(1 - p_{t-1}^j\right) + (1-\lambda)\, c\right) b_t^j, \quad (4)$$

where τ is the corporate income tax rate.

Firm j's after-tax profits are given by:

$$\pi_t^j \equiv (1-\tau)(R_t k_t^j - z_t^j k_t^j) + \tau \left(\lambda \left(1 - p_{t-1}^j\right) + (1-\lambda)\, c\right) b_t^j + \tau \delta k_t^j, \quad (5)$$

where δ is the depreciation rate of capital. The last term in the preceding equation is the depreciation allowance.

A firm may default on its debt. Default is triggered endogenously by the inability of the firm to raise sufficient equity capital to meet its debt obligations. This case applies to debt with no protective debt covenants, as discussed in Leland (1994). Some debt contracts have a protective covenant stipulating that the firm must always have sufficient cash to pay its interest payments or stipulating that the asset value of the firm must always exceed the principal value of the debt: a positive net-worth requirement. In this paper, we focus on the case with unprotected debt as it is more common for long-term bonds (see Leland (1994)).

Debt is costly because there are bankruptcy costs and agency costs. If firm j issues new debt b_{t+1}^j, then it incurs agency costs $\varphi\left(b_{t+1}^j / k_{t+1}^j\right) k_{t+1}^j$, where φ is an increasing and convex function. In addition, we assume that φ and its derivative are equal to zero in the deterministic steady state. We also assume that if firm j defaults in period t, bond holders collect the firm's after-tax profits π_t^j and a fraction $\xi \in (0,1)$ of firm j's assets k_t^j. The remaining fraction represents bankruptcy costs. The bond holders then reorganize the firm so that it starts operation again with the new initial assets ξk_t^j.[7] We assume that bankruptcy costs are shared by all existing creditors so that the reorganized firm has initial debt ξb_t^j. By limited liability, the equity holders get nothing in the event of default.

The bond value satisfies the no arbitrage condition:

$$\begin{aligned} b_{t+1}^j p_t^j &= E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \chi \left[\lambda + (1-\lambda)(c + p_{t+1}^j) \right] b_{t+1}^j \\ &\quad + E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} (1-\chi) \left(\pi_{t+1}^j + \xi J_{t+1}(k_{t+1}^j, b_{t+1}^j) \right), \end{aligned} \tag{6}$$

where χ is an indicator function that equals one in the event of default and zero in the event of no default. This equation says that the bond value is equal to the risk-adjusted discounted present value of future payoffs. The future payoffs are contingent on the default and non-default events.

The aggregate state of the economy is the aggregate distribution of firms over capital and debt, together with the aggregate productivity shock A_t. Firm j's individual state is given by (k_t^j, b_t^j, z_t^j). This state space is large and hard to manage for numerical computations. We will reduce this state space later. Firm j takes the aggregate and individual states as given and solves a dynamic programming problem to choose the optimal default, investment, and financing decisions. To describe this problem, we let $V_t(k_t^j, b_t^j, z_t^j)$ denote firm j's cum-dividend equity value in period t. It satisfies the following Bellman equation:

$$V_t(k_t^j, b_t^j, z_t^j) = \max \left\{ 0, \pi_t^j - (\lambda + (1-\lambda)\, c)\, b_t^j + J_t(k_t^j, b_t^j) \right\}, \tag{7}$$

where $J_t(b_t^j, k_t^j)$ is the period-t value of equity in the non-default states, excluding the current after-tax profits π_t^j as well as the matured debt payments λb_t^j and coupon payments $(1-\lambda)\, c b_t^j$. It satisfies the following equation:

$$J_t(k_t^j, b_t^j) = \max_{b_{t+1}^j, I_t^j} p_t^j \left(b_{t+1}^j - (1-\lambda)\, b_t^j \right) - I_t^j - \varphi \left(\frac{b_{t+1}^j}{k_{t+1}^j} \right) k_{t+1}^j + E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \bar{V}_{t+1}(k_{t+1}^j, b_{t+1}^j), \tag{8}$$

subject to the law of motion for capital (3), and the bond value equation (6), where $\bar{V}_t(k_t^j, b_t^j) = \int V_t(k_t^j, b_t^j, z) d\Phi(z)$ is the expected value of equity before the realization of idiosyncratic shocks. In addition, Λ_t is the marginal utility of consumption derived in Section 2.2.

[7] We do not consider entry and exit. Cooley and Quadrini (2001), Miao (2005), and Gomes and Schmid (2010) study this issue.

Equation (7) says that if firm j chooses to default, equity holders get nothing. If the firm chooses to continue operation, then it produces after-tax profits π_t^j and pays debt λb_t^j and coupon payment $(1-\lambda)\, cb_t^j$. It also raises new additional debt $p_t^j b_{t+1}^j - (1-\lambda)\, b_t^j$ and makes investment I_t^j. The optimal choice solves problem (8).

2.3 Competitive Equilibrium

A competitive equilibrium can be defined in a standard way. For simplicity, we describe a competitive equilibrium by sequences of aggregate quantities, $(Y_t, C_t, K_{t+1}, I_t, N_t)$, individual firms' choices, $\left(n_t^j, I_t^j, k_{t+1}^j, y_t^j\right)$, and prices, $\left(w_t, V_t^j, p_t^j\right)$, such that each firm and each household solve their optimal decision problems, and markets clear in the sense that $N_t = \int n_t^j dj$, $Y_t = \int y_t^j dj$, $K_t = \int k_t^j dj$, $I_t = \int I_t^j dj$, and

$$Y_t = C_t + I_t + \int \varphi(b_{t+1}^j / k_{t+1}^j) k_{t+1}^j dj, \tag{9}$$

$$K_{t+1} = \int_{\mathcal{B}_1} \left((1-\delta)k_t^j + \Psi(I_t^j, k_t^j)\right) dj + \int_{\mathcal{B}_2} \left((1-\delta)\xi k_t^j + \Psi(I_t^j, \xi k_t^j)\right) dj, \tag{10}$$

where $\mathcal{B}_1 \in [0,1]$ is the set of non-defaulting firms and $\mathcal{B}_2 = [0,1] \backslash \mathcal{B}_1$ is the set of default firms. Equation (9) is the aggregate resource constraint and equation (10) is the law of motion for aggregate capital.

3 Characterization of Equilibrium

We first derive a single firm's decision problem taking prices as given. We then aggregate individual choices and characterize the competitive equilibrium by a system of nonlinear difference equations. Finally, we analyze the deterministic steady state.

3.1 A Single Firm's Decision Problem

From equation (7), we deduce that firm j defaults if and only if $\pi_t^j - (\lambda + (1-\lambda)\, c)\, b_t^j + J_t(k_t^j, b_t^j) \leq 0$. It follows from equation (5) that firm j defaults if and only if $z_t^j \geq z_t^{*j}$ where z_t^{*j} is the default trigger determined by the equation: $\pi_t^j - (\lambda + (1-\lambda)\, c)\, b_t^j + J_t(k_t^j, b_t^j) = 0$. Substituting π_t^j given in (5) into this equation yields:

$$\begin{aligned} (\lambda + (1-\lambda)\, c)\, b_t^j &= (1-\tau)(R_t k_t^j - z_t^{*j} k_t^j) + J_t(k_t^j, b_t^j) \\ &\quad + \tau \left[\lambda \left(1 - p_{t-1}^j\right) + (1-\lambda)\, c\right] b_t^j + \tau \delta k_t^j. \end{aligned} \tag{11}$$

Note that z_t^{*j} may exceed $z_{\max}$, the upper support of z_t^j. In the analysis below, we will focus on the interior case: $z_t^{*j} \in (z_{\min}, z_{\max})$.

Conditional on not defaulting on debt, firm j solves problem (8). Defaulting but reorganized firms solve a similar problem, but starting with an initial capital stock $\xi_t k_t^j$ and initial debt $\xi_t b_t^j$. To analyze problem (8), we first use equation (7) and integrate out idiosyncratic shocks to derive:

$$\begin{aligned} \bar{V}_{t+1}(k_{t+1}^j, b_{t+1}^j) &= \int V_{t+1}(k_{t+1}^j, b_{t+1}^j, z_{t+1}^j) d\Phi(z_{t+1}^j) \\ &= \int_{z_{\min}}^{z_{t+1}^{*j}} \left[\pi_{t+1}^j - (\lambda + (1-\lambda)c) b_{t+1}^j + J_{t+1}\left(k_{t+1}^j, b_{t+1}^j\right)\right] d\Phi(z_{t+1}^j). \end{aligned} \tag{12}$$

Using equations (5) and (11), we rewrite the above equation as:

$$\bar{V}_{t+1}(k_{t+1}^j, b_{t+1}^j) = (1-\tau)\, k_{t+1}^j \int_{z_{\min}}^{z_{t+1}^{*j}} \left(z_{t+1}^{*j} - z\right) d\Phi(z). \tag{13}$$

Exploiting the linear homogeneity property of our model structure, we conjecture that:

$$J_t\left(k_t^j, b_t^j\right) = J_t(1, \omega_t^j) k_t^j, \tag{14}$$

where $\omega_t^j \equiv b_t^j / k_t^j$ is the debt-capital ratio. Without risk of confusion, we simply use $J_t(\omega_t^j)$ to denote $J_t(1, \omega_t^j)$. In addition, we let $i_t^j \equiv I_t^j / k_t^j$ denote firm j's investment rate. We can then rewrite equation (3) as:

$$k_{t+1}^j = g(i_t^j) k_t^j, \text{ where } g(i) = 1 - \delta + \Psi(i). \tag{15}$$

We now use equations (13) and (15) to rewrite problem (8) as:

$$\begin{aligned} J_t(\omega_t^j) = & \max_{\omega_{t+1}^j, i_t^j} p_t^j \left[\omega_{t+1}^j g(i_t^j) - (1-\lambda)\omega_t^j\right] - i_t^j - \varphi(\omega_{t+1}^j) g(i_t^j) \\ & + g(i_t^j)(1-\tau) E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \int_{z_{\min}}^{z_{t+1}^{*j}} \left(z_{t+1}^{*j} - z\right) d\Phi(z), \end{aligned} \tag{16}$$

where z_{t+1}^{*j} satisfies equation (11). In addition, this problem is subject to the constraint (6), which requires that debt value be fair priced in a competitive market. To solve this problem, we note that the bond price p_t^j and the default trigger z_{t+1}^{*j} depend implicitly on the debt-capital ratio ω_{t+1}^j.

The following proposition characterizes the solution to problem (16). Appendix A1 presents the proof.

Proposition 1 *The optimal investment rate i_t^j, debt-capital ratio ω_{t+1}^j and default trigger z_t^j are time-invariant functions of aggregate states (K_t, A_t) and individual states ω_t^j. These policy functions are not firm specific. Dropping index j, the following first-order conditions hold:*

$$1/g'\left(i_t\right) = p_t \omega_{t+1} + (1-\tau) E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \int_{z_{\min}}^{z_{t+1}^*} (z_{t+1}^* - z) \phi(z) dz - \varphi(\omega_{t+1}), \tag{17}$$

and

$$\begin{aligned} & \frac{\partial p_t}{\partial \omega_{t+1}} \left[\omega_{t+1} g(i_t) - (1-\lambda)\,\omega_t\right] + p_t g(i_t) \\ = \quad & -g(i_t)(1-\tau) E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \Phi(z_{t+1}^*) \frac{\partial z_{t+1}^*}{\partial \omega_{t+1}} + \varphi'(\omega_{t+1}) g(i_t). \end{aligned} \tag{18}$$

Equation (17) reflects a modified q-theory of investment. To see this, observe that the first two terms on the right-hand side of this equation is Tobin's average Q (adjusted for taxes and leverage). More specifically, the first term is equal to the ratio of debt value to capital $\left(p_t b_{t+1}\right)/k_{t+1}$ and the second term is equal to the ratio of equity value to capital $\overline{V}_{t+1}/k_{t+1}$ by equation (13). The sum of these two terms is the average Q (the ratio of firm value to capital) because firm value is equal to the sum of debt value $p_t b_{t+1}$ and equity value $\overline{V}_{t+1}$. Because of linear homogeneity, the marginal Q is equal to the average Q in our model (Hayashi (1982)). The last term on the right side of equation (17) reflects agency costs. High leverage induces high agency costs which in term reduces investment.

Empirical studies typically uses equation (17) to test the q-theory of investment. The empirical difficulty is how to measure Tobin's Q or the right-hand side of this equation. Based on an equation similar to (17), Philippon (2009) argues that bond prices and credit spreads provide information about Q and hence can predict investment. An increase in credit spreads reduces Tobin's Q and hence investment. Equation (17) is key to understanding the linkage between the credit markets and the real economy.

The left side of equation (18) represents the marginal benefits of debt. Note that changes in the debt-capital ratio today also affect the bond price indirectly because the bond price reflects future contingent claims to debt holders. These claims are determined by future default probabilities (default triggers), which are affected by today's debt choice. The right side of equation (18) represents the marginal costs of debt which consist of bankruptcy costs and agency costs. At an optimum, they must be identical.

Note that equations (17) and (18) are first-order necessary conditions. They may not be sufficient for optimality. Since it is nontrivial to establish sufficient conditions analytically for the general case of finite maturity debt, we give these conditions for the special deterministic case with one-period debt (i.e., $\lambda = 1$) in Appendix A3.[8]

In equation (18), we need to determine $\frac{\partial p_t}{\partial \omega_{t+1}}$ and $\frac{\partial z^*_{t+1}}{\partial \omega_{t+1}}$, denoted by π^p_t and π^z_{t+1}, respectively. In doing so, we use the following two equations. Dropping the index j, we use equations (5), (11) and (14) to show that z^*_{t+1} satisfies:

$$(1-\tau)(R_{t+1} - z^*_{t+1}) + \tau\left(\lambda\left(1-p_t\right) + \left(1-\lambda\right)c\right)\omega_{t+1} + \tau\delta + J_{t+1}(\omega_{t+1}) = \left(\lambda + \left(1-\lambda\right)c\right)\omega_{t+1}. \tag{19}$$

Dropping the index j, we use equations (5), (14), and (19) to rewrite equation (6) as:

$$\begin{aligned} p_t\omega_{t+1} \;\; = \;\; & \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t}\Big\{\left[\left(1-\lambda\right)\Phi(z^*_{t+1})p_{t+1} + \lambda + (1-\lambda)c\right]\omega_{t+1} \\ & - \int_{z^*_{t+1}}^{z_{\max}}\left[(1-\tau)\left(z - z^*_{t+1}\right) + (1-\xi)J_{t+1}(\omega_{t+1})\right]d\Phi(z)\Big\}. \end{aligned} \tag{20}$$

Differentiating the two sides of the above two equations with respect to ω_{t+1}, we obtain:

$$-(1-\tau)\pi^z_{t+1} + \tau\left(\lambda\left(1-p_t\right) + \left(1-\lambda\right)c\right) - \tau\lambda\omega_{t+1}\pi^p_t + \frac{\partial J_{t+1}}{\partial \omega_{t+1}} = \lambda + \left(1-\lambda\right)c, \tag{21}$$

and

$$\begin{aligned} & p_t + \omega_{t+1}\pi^p_t \\ = \;\; & \beta E_t\frac{\Lambda_{t+1}}{\Lambda_t}\left[\left(\lambda + \left(1-\lambda\right)c\right) + (1-\lambda)\Phi(z^*_{t+1})p_{t+1} + (1-\lambda)\phi(z^*_{t+1})\pi^z_{t+1}\omega_{t+1}p_{t+1}\right] \\ & + \beta E_t\frac{\Lambda_{t+1}}{\Lambda_t}\left\{(1-\tau)[1-\Phi(z^*_{t+1})]\pi^z_{t+1} + (1-\xi)J_{t+1}\phi(z^*_{t+1})\pi^z_{t+1}\right\} \\ & - \beta E_t\frac{\Lambda_{t+1}}{\Lambda_t}[1-\Phi(z^*_{t+1})]\frac{\partial J_{t+1}}{\partial \omega_{t+1}}. \end{aligned} \tag{22}$$

Applying the envelope condition to (16), we obtain:

$$\frac{\partial J_t}{\partial \omega_t} = -p_t\left(1-\lambda\right). \tag{23}$$

We use this equation to substitute out $\frac{\partial J_{t+1}}{\partial \omega_{t+1}}$ in equations (21) and (22). The resulting two equations determine π^p_t and π^z_{t+1}.

[8] We check the second-order condition numerically in the deterministic steady state for the general finite maturity debt case in Section 5. Since our numerical method is based on a local approximation around the deterministic steady state, we expect that it also holds for small shocks by continuity.

3.2 Aggregation and Equilibrium System

We now aggregate individual behavior and derive the equilibrium system. We will show that we do not need to track firm distribution as a state variable. Under our model assumptions, only mean matters for aggregation. First, we use the constant-returns-to-scale property of the production function to derive the aggregate output as:

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}. \tag{24}$$

Using this aggregate production function, we can show that the rental rate and the wage rate satisfy:

$$R_t = \alpha A_t K_t^{\alpha-1} N_t^{1-\alpha}, \tag{25}$$

$$w_t = (1-\alpha) A_t K_t^{\alpha} N_t^{-\alpha}. \tag{26}$$

Next, we suppose all firms start with the same initial debt-capital ratio ω_0. By Proposition 1, the optimal investment rate i_t^j, debt-capital ratio ω_{t+1}^j, and the default trigger z_t^j are independent of firm identity j for any time t. This feature allows us to aggregate firm behavior tractability. We drop the firm index j. In any period t, there is a mass $\Phi(z_t^*)$ of firms that do not default on debt. The remaining firms with mass $[1-\Phi(z_t^*)]$ default on debt and are reorganized to continue operation in the next period with recovered assets. Thus, the law of motion for aggregate capital stock (10) satisfies:

$$K_{t+1} = g(i_t)\left\{\Phi(z_t^*) + [1-\Phi(z_t^*)]\,\xi_t\right\} K_t. \tag{27}$$

For a non-defaulting firm j, its investment satisfies $I_t^j = k_t^j i_t$, and for a defaulting and reorganized firm j, $I_t^j = \xi_t k_t^j i_t$. Aggregating across all these firms, we obtain:

$$I_t = i_t\left\{\Phi(z_t^*) + [1-\Phi(z_t^*)]\,\xi_t\right\} K_t. \tag{28}$$

Finally, we characterize a competitive equilibrium by 13 variables $\{Y_t, K_{t+1}, I_t, C_t, N_t, \Lambda_t, i_t, \omega_{t+1}, z_t^*, p_t, \pi_t^z, \pi_t^p, J_t\}$ that solve a system of 13 nonlinear difference equations (24), (27), (28), (9), (2), (1), (17), (18), (19), (20), (21), (22), and an equation for J_t. We obtain the last equation by removing the superscript j and the max operator in equation (16):

$$J_t = p_t\left[\omega_{t+1} g(i_t) - (1-\lambda)\,\omega_t\right] - i_t - \varphi(\omega_{t+1}) g(i_t) + g(i_t)(1-\tau) E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \int_0^{z_{t+1}^*} (z_{t+1}^* - z)\, d\Phi(z). \tag{29}$$

We also use (25) and (26) to substitute out R_t and w_t in the preceding equations. We apply the log-linear approximation method to solve this nonlinear system. Following Jermann (1998), we combine the log-linear approximation method with the nonlinear asset pricing formula to compute asset returns.[9] The predetermined endogenous state variables are $\{K_t, C_{t-1}, \omega_t, p_{t-1}, \pi^p_{t-1}\}$ and the exogenous state variable is A_t. Other variables are jump variables.

3.3 Steady State

To apply the log-linear approximation method, we need to solve for the deterministic steady state. In a deterministic steady state, there is no aggregate shock, but firms are still subject to idiosyncratic liquidity shocks. By a law of large numbers, all aggregate variables are constant over time in a deterministic steady state. We use variables without time subscripts to denote their steady-state values. We adopt the following solution procedure.

First, we solve for the steady-state default trigger z^*. In doing so, we write the steady state versions of equations (27), (17), (29), and (20) as:

$$1 = (1-\delta+i)\left\{\Phi(z^*) + [1-\Phi(z^*)]\,\xi\right\}, \tag{30}$$

$$1 = \omega p + \beta(1-\tau)\int_0^{z^*}[z^*-z]d\Phi(z), \tag{31}$$

$$J = [1-\delta+i-(1-\lambda)]\,\omega p - i + \beta\,(1-\delta+i)\,(1-\tau)\int_0^{z^*}(z^*-z)d\Phi(z), \tag{32}$$

$$\frac{1}{\beta} = (1-\lambda)\Phi(z^*) + \frac{\lambda+(1-\lambda)\,c}{p} - \frac{1}{\omega p}\int_{z^*}^{z_{\max}}\left\{(1-\tau)\,(z-z^*) + (1-\xi)J\right\}d\Phi(z), \tag{33}$$

where we have used the assumptions in Section 2 to substitute out: $g\,(i) = 1-\delta+i$, $g'\,(i) = 1$ and $\varphi'\,(\omega) = 0$.

From equation (30), we can solve for i as a function of z^*, denoted by $i\,(z^*)$. Equation (31) implies that ωp is a function of z^* denoted by $\omega p\,(z^*)$. Substituting $i\,(z^*)$ and $\omega p\,(z^*)$ into equation (32), we can show that J is also a function z^*, denoted by $J\,(z^*)$.

We now solve for $(\lambda+(1-\lambda)\,c)\,/p$ as a function of z^*. We write the steady-state version of equations (21), (22), and manipulate these equations to derive:

$$\frac{\pi^z}{p} = \frac{1}{1-\tau}\left[(\tau-1)\frac{\lambda+(1-\lambda)c}{p} - (1-\lambda) - \lambda\tau\right] - \frac{\lambda\tau}{1-\tau}\frac{\omega\pi^p}{p}, \tag{34}$$

[9] We also use the second-order approximation method to solve the model and find the solutions are similar.

and

$$
\begin{aligned}
\frac{1}{\beta}\left[1+\frac{\omega}{p}\pi^{p}\right] \quad = \quad & \frac{\lambda+(1-\lambda)\,c}{p}+(1-\lambda)\Phi(z^{*})+(1-\lambda)\phi(z^{*})\omega p\,(z^{*})\,\frac{\pi^{z}}{p} \\
& +\{(1-\tau)[1-\Phi(z^{*})]+(1-\xi)J(z^{*})\phi(z^{*})\}\,\frac{\pi^{z}}{p} \\
& +\beta\,(1-\lambda)\,[1-\Phi(z^{*})]. \qquad (35)
\end{aligned}
$$

Similarly, we write the steady-state version of equation (18) to obtain:

$$
0=\pi^{p}\frac{\omega}{p}\,[g(i)-(1-\lambda)]+g(i)+\beta g(i)(1-\tau)\Phi(z^{*})\frac{\pi^{z}}{p}. \qquad (36)
$$

We then substitute $i\,(z^{*})$ and $\omega p\,(z^{*})$ into equations (34), (35), and (36), and use the resulting equations to solve for $(\lambda+(1-\lambda)\,c)\,/p$, π^{z}/p and $\pi^{p}\omega/p$ as functions of z^{*}. Note that these three equations form a system of linear equations that can be easily solved. Finally, we substitute the solution for $(\lambda+(1-\lambda)\,c)\,/p$ together with $J\,(z^{*})$ and $\omega p\,(z^{*})$ into equation (33). The resulting equation determines the steady-state default trigger z^{*}. We focus on the case with an interior solution, $z^{*}\in(z_{\min},z_{\max})$.

After we obtain z^{*}, we can then immediately obtain the solution for i, p, ω, J, π^{p} and π^{z}. In addition, we can also derive other steady-state values by the following procedure. First, we use $R=\alpha Y/K$ and the steady-state version of equation (19) to solve for Y/K. Second, we use the steady-state version of (28) to derive the steady-state aggregate investment rate:

$$
\frac{I}{K}=i\,(z^{*})\,\{\Phi(z^{*})+[1-\Phi(z^{*})]\,\xi\}\,. \qquad (37)
$$

This equation determines I/K. Hence, we can solve for $I/Y=I/K\cdot K/Y$. Third, we use the resource constraint (9) to derive:

$$
1=\frac{C}{Y}+\frac{I}{Y}, \qquad (38)
$$

where we have used the assumption that the steady-state agency costs equal zero. This equation determines the consumption-output ratio C/Y. Finally, the steady-state output Y is derived by the production function:

$$
Y=\left(\frac{1}{Y/K}\right)^{\frac{\alpha}{1-\alpha}}N\ \text{ with } N=1, \qquad (39)
$$

where we normalize the steady-state labor to one. After determining the above steady-state values, we can solve for all other equilibrium steady-state values easily.

A useful result for our later numerical analysis is the following:

Proposition 2 *In a deterministic steady state, the default trigger z^* and real allocation are independent of the coupon rate c.*

The intuition for this result is as follows. In general, default trigger depends on the bond price, the coupon rate, the leverage ratio and default costs. By the steady-state bond valuation equation (33) and our previous analysis, the dependence of the default trigger on the coupon rate is through the current yield $(\lambda + (1-\lambda)\, c)\,/p$ only. But the current yield is an implicit function of the default trigger, independent of the coupon rate. Changes in the coupon rate lead to changes in the bond price, leaving the current yield unaffected. Thus, the default trigger is independent of the coupon rate. It follows from this result and equation (31) that the steady-state ωp is also independent of the coupon rate. We then deduce that the steady-state debt-capital ratio ω decreases with the coupon rate because the steady-state bond price p increases with the coupon rate.

The dependence of the real allocation on the coupon rate is through the default trigger only. In a deterministic steady state, the coupon rate does not affect the equilibrium allocation because the default trigger is independent of the coupon rate,

4 Calibration

In order to obtain quantitative results, we need to parameterize our model. We choose the following functional forms. First, following Jermann (1998), we specify the adjustment cost function as:

$$\Psi\left(I/K\right) = \frac{i^{\theta}}{1-\theta}\left(\frac{I}{K}\right)^{1-\theta} - \frac{\theta i}{1-\theta} \tag{40}$$

where $\theta > 0$ is the adjustment cost parameter. This functional form implies that in the deterministic steady state all equilibrium variables are independent of the adjustment cost parameter θ. That is, the models with and without capital adjustment costs yield an identical deterministic steady state.

Second, for simplicity, we specify the agency cost function as:

$$\varphi\left(x\right) = \frac{1}{2}\left(x-\omega\right)^{2},$$

where ω is the steady state debt-capital ratio. This functional form satisfies our assumption in Section 2 since $\varphi(\omega) = \varphi'(\omega) = 0$ in the steady-state. This functional form ensures that agency costs do not play a role in the deterministic steady state.

Third, to facilitate integration, we specify a power function distribution for the idiosyncratic liquidity shock with distribution function: $\Phi(z) = \left(z + \frac{\kappa}{\kappa+1}\right)^{\kappa}$ with support $\left(-\frac{\kappa}{\kappa+1}, \frac{1}{\kappa+1}\right)$, where $\kappa > 0$ is the shape parameter. This distribution has a zero mean.

Now, we follow the standard methodology in the business cycle literature to assign parameter values. A period in the model corresponds to one quarter. We set $\alpha = 0.33$, implying that the labor income share is about 2/3 of output. We set the quarterly depreciate rate $\delta = 0.025$. Taking into account of personal and corporate income taxes, we set the effective corporate income tax rate $\tau = 0.2$. We set $\varsigma = 0.4$ as in Jaimovich and Rebelo (2009), which implies the Frisch labor elasticity is equal to 2.5. We choose the weight on labor η so that the steady-state labor supply is normalized to one. Following King, Plosser, and Rebelo (1988) and Dotsey, King, and Wolman (1999), we set the discount factor $\beta = 0.9855$ to yield a steady-state return to capital of about 6 percent per annum, which is the average real return to equity from 1948 to 1981. We set $\lambda = 0.025$, implying the average maturity of the bond is 10 years, as in Philippon (2009). Since the coupon rate c does not affect the steady-state default trigger and the real allocation by Proposition 2, we set $c = 0.01$.

From the analysis in Section 3.3, we deduce that given values for (β, δ, τ, c), we can solve for the steady-state values of z^* and $p(z^*)$ for each pair of values (κ, ξ), independent of other parameter values in the model. We choose the values of κ and ξ so that the model implied steady state matches the following two credit market statistics. First, using the Moody's 2005 annual report, Chen, Collin-Dufresne and Goldstein (2009) find that the average 4-year future cumulative default rate for Baa-rated bonds is 1.55 percent. Thus, we require that the steady-state 4 year default rate in the model, $[1 - \Phi(z^*)] \times 16$, stay close to 1.55 percent. Second, using data from the Federal Reserve, Chen, Collin-Dufresne and Goldstein (2009) report that the annual average composite Baa-Aaa spread is 109 basis points. In our model, the steady-state annual credit spread between the 10-year corporate bond and the riskfree rate is equal to $(1 + r_d - 1/\beta) \times 4$, where the steady-state bond yield r_d and bond price $p(z^*)$ satisfy $p(z^*) = (\lambda + (1 - \lambda) c) / (\lambda + r_d)$. We choose parameter values such that this spread stays close to 109 basis points.

It remains to pin down values for $(\rho, \sigma, h, \theta, \gamma)$. We try to match the following five moments: (i) the mean risk-free rate, (ii) the mean equity premium, (iii) the volatility of consumption growth relative to output growth, (iv) the volatility of investment growth relative to output growth, and (v) the volatility of output growth.

We use the sample period from 1947:1 to 2007:4 to compute moments for financial data. For stock returns, we compute the returns (cum-dividend) of the CRSP value-weighted market portfolio (including the NYSE, AMEX and NASDAQ). We roll over the 90 Day T-Bill return series from the CRSP Fama Risk-Free Rate file to compute the annual riskfree rates. All nominal quantities are deflated using the Consumer Price Index, taken from the Bureau of Labor Statistics. We find that the annualized mean real risk-free rate is equal to 1.51 percent and the annualized mean equity premium is equal to 7.23 percent.

We use the same sample period from 1947:1 to 2007:4 to compute statistics for macroeconomic quantities. We construct data from BEA as in Liu, Wang and Zha (2009). Aggregate consumption is the per capita consumption of non-housing services and nondurable goods. Aggregate investment is the per capita investment, which includes durable good consumption and private nonresidential investment. Output is the sum of per capita consumption and per capita investment. We find that the quarterly volatility of output growth rate is 1.08 percent, the volatility of consumption growth relative to that of output growth is 0.51 percent, and the volatility of investment relative to that of output growth is 3.96 percent.

In summary, we choose values for 7 parameters $x = (\rho, \sigma, h, \theta, \gamma, \kappa, \xi)$ to match the above five moments together with the two credit market statistics by minimizing the following criterion:

$$\mathcal{L}(x) = [\hat{\mathbf{m}} - \mathbf{f}(x)]' \hat{\mathbf{V}}^{-1} [\hat{\mathbf{m}} - \mathbf{f}(x)],$$

where $\hat{\mathbf{m}}$ is a column vector consisting of data moments and the vector $\mathbf{f}(x)$ is the model implied corresponding moments for a given parameter value. The matrix $\hat{\mathbf{V}}$ is a weighting matrix. For simplicity, we take $\hat{\mathbf{V}}$ as an identity matrix. We put the following restrictions on parameter values: $\rho \in (0,1)$, $\sigma \in (0, 0.02]$, $h \in (0,1)$, $\gamma \in (0,10)$, and $\xi \in (0,1)$. In addition, κ must satisfy the assumption in Appendix A3.

We list our calibrated parameter values in Table 1. Our calibrated values for ρ and σ are within the range used in the RBC literature, though the value for σ is a little high. The calibrated habit persistence parameter is close to that in Boldrin, Christiano, and Fisher (2001) and Christiano, Eichenbaum and Evans (2005). The calibrated value for θ implies that the Q-elasticity of investment is 0.25, which is close to the one used in Jermann (1998) and Boldrin, Christiano, and Fisher (2001). The calibrated recovery rate ξ is within the range in the data used by Chen (2010).

[Insert Table 1 Here.]

Table 2 presents the data moments and our model implied moments. This table indicates that our calibrated model matches data moments reasonably well.

[Insert Table 2 Here.]

5 Quantitative Results

We assess the model's quantitative performance by examining its equilibrium predictions for the impact of credit risk on business cycles. We first examine the model's predictions of unconditional moments of aggregate quantities and asset returns and compare those with the data. We then investigate the role of credit spreads in forecasting future movements in economic activities and stock returns. For comparison, we also study a benchmark real business cycle (RBC) model without the credit market. It is similar to the baseline model of Jermann (1998) with two differences: (i) there is leisure in the habit formation preference and (ii) there is corporate tax.

5.1 Unconditional Moments

We first examine our model's asset pricing implications. Table 3 presents the unconditional moments for the risk-free rate and the equity return. The risk-free rate from period t to $t+1$ on the one-period risk-free bond is given by $R_{f,t+1} = 1/E_t\left[\beta\Lambda_{t+1}/\Lambda_t\right]$. We define the (gross) aggregate equity return from period t to period $t+1$ as $R_{e,t+1} = \left(P_{e,t+1} + D_{t+1}\right)/P_{e,t}$, where $P_{e,t}$ is the period-t equity price of the stream of aggregate dividends $\{D_s\}_{s\geq t+1}$. Aggregate dividends D_t consist of dividends from both non-defaulting firms and defaulting but reorganized firms. In our complete markets model, the aggregate equity price $P_{e,t}$ satisfies the asset pricing equation:

$$P_{e,t} = E_t \frac{\beta\Lambda_{t+1}}{\Lambda_t}\left(P_{e,t+1} + D_{t+1}\right).$$

Although our focus is not on asset pricing, our model does a good job in matching the mean risk-free rate and the mean equity premium as in the data. Compared to the benchmark RBC model without a credit market, our model delivers a lower mean risk-free rate and a higher mean equity premium. The intuition is that due to the presence of credit risk, our model generates a larger precautionary saving effect, which lowers the risk-free rate. In addition, due to the presence of endogenous financial leverage, dividends in our model are more volatile and hence both the mean equity premium and equity volatility are higher than in the benchmark model.

The mean equity premium in our model is closer to that in the data, while our model generated equity volatility is too high relative to the data. Another limitation of our model is that it delivers a counterfactually high volatility of the risk-free rate. This problem is well known and is common for the habit formation preference (e.g., see Jermann (1998) for a discussion).

[Insert Table 3 Here.]

Next, we consider our model's predictions for some key credit market statistics. We define the corporate credit spread in our model as the yield spread between long-term corporate bond and the long-term riskless discount bond with identical maturity. We model the long-term riskless discount bond in a way similar to the modelling of the long-term defaultable corporate bonds. Thus, the price q_t of the riskless discount bond with maturity $1/\lambda$ in our complete markets model satisfies:

$$q_t = E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left[\lambda + (1 - \lambda) q_{t+1} \right].$$

The credit spread CS_t is given by:

$$CS_t = \frac{E_t \left[\lambda + (1 - \lambda)(c + p_{t+1}) \right]}{p_t} - \frac{E_t \left[\lambda + (1 - \lambda) q_{t+1} \right]}{q_t},$$

where the bond maturity is $1/(4\lambda)$ years, and the corporate bond price p_t is given by (20).

In our model, credit spreads depend on not only firm specific characteristics, but also the macroeconomic conditions for three reasons. First, the stochastic discount factor $\beta \Lambda_{t+1}/\Lambda_t$ depends on the macroeconomic conditions because it is related to marginal utility of consumption which responds to aggregate productivity shocks. In a recession, consumption is low and marginal utility is high so that bond price is low. Second, the default trigger z_t^* depends on the macroeconomic conditions because aggregate productivity shocks affect firm profits and hence the default probability. Third, the recovery value of bonds depends on the macroeconomic conditions.

What is the cyclical behavior of the credit markets? Table 4 shows that default rates and credit spreads are countercyclical, while debt issuance is procyclical. This result is consistent with the empirical evidence documented in the literature (e.g., Covas and den Haan (2010)). The intuition is as follows. In a recession, firms perceive high default risk and thus issue less debt. Furthermore, credit spreads widen because of both the cash flow effect and the discount rate (or the risk premium) effect. The cash flow effect means that the expected payoffs to debt holders fall in a recession. The discount rate effect means that consumption is low and marginal utility is high in a recession. Both effects contribute to the countercyclical movements of credit spreads.

[Insert Table 4 Here.]

Now, we turn to business cycle statistics implied by our model. Table 5 presents some selected simulated statistics from our model with credit risk and the benchmark RBC model as well as the corresponding statistics in the data. Panel A indicates that our model with credit risk matches reasonably well the volatility of output growth, the volatility of consumption growth relative to output growth, the volatility of investment growth relative to output growth, and the volatility of total hours growth relative to output growth, observed in the data. By contrast, the benchmark RBC model generates a much lower volatility of output growth and a much lower relative volatility of consumption growth, while this model implied relative volatilities of investment growth and hours growth are too volatile. The key intuition for this result is that credit risk in our model amplifies the effect of technology shocks. In addition to a direct negative effect on output, a negative technology shock may trigger firms to default on debt. Asset values in defaulting firms are discounted because of costs involved in fire sales or reorganization. As a result, the aggregate capital stock in the economy is reduced, leading to a further contraction in output.

Panel B of Table 5 presents the contemporaneous correlations of consumption, investment and hours with output. This panel indicates that both the benchmark RBC model and our model with credit risk generate qualitatively correct predictions about the correlation between consumption growth and output growth. In addition, both models generate a negative correlation between output and hours. This result is due to the presence of capital adjustment costs and habit formation. The intuition is similar to that in Francis and Ramey (2005). A positive aggregate productivity shock generates a positive wealth effect. This wealth effect raises households' consumption. Due to habit formation, consumption rises slowly. Thus, households put the extra resources to investment. But high adjustments costs on investment prevent the households from increasing investment by too much. The only alternative for the household to "spend" the new wealth is to increase leisure.

[Insert Table 5 Here.]

Recent empirical studies show that positive technology shocks lead to short-run declines in hours (Shea (1998), Gali (1999), Basu et al. (2004) and Francis and Ramey (2005)). The positive correlation between output growth and hours growth in the data could be due to other types of shocks. Although this issue is highly debated in the literature, our model can generate the negative correlation between technology shocks and hours growth. In Section 6.2, we will show that financial shocks may lead to a positive correlation between output growth and hours growth.

Panel C of Table 5 presents the autocorrelations of output growth, consumption growth, investment growth and hours growth. This panel shows that all four variables exhibit a strong positive autocorrelation as in the data. Cogley and Nason (1995) show that many standard RBC models cannot replicate the positive autocorrelation of output growth. This difficulty also appears in our benchmark RBC model with habit formation and capital adjustment costs. By contrast, our model with credit risk can successfully replicate the positive autocorrelation of output growth. However, our model implied autocorrelation of consumption growth is too high relative to the data. In addition, our model cannot generate the large autocorrelations of investment growth and hours growth as in the data.

5.2 Amplification and Propagation of Shocks

To understand the moments presented in the previous tables and the mechanics of our model, we study the economy's responses to a negative one-standard-deviation shock to the aggregate productivity. Figure 1 plots the impulse response functions for output, consumption, investment, hours, capital, and the debt-capital ratio. The solid and dashed lines represent these functions implied by our model with credit risk and the benchmark RBC model, respectively. Most RBC models are criticized on the ground that they lack an amplification and propagation mechanism. As the dashed lines in Figure 1 show, on impact, output, consumption, and investment decrease by about 0.6, 0.1, and 2.7 percent, respectively, but hours increase by about 1.5 percent. After the initial impact period, output and investment increase monotonically to their steady state levels, while hours decrease monotonically. Consumption first decreases modestly and then gradually rises to its steady-state level, reflecting households' habit formation preferences. Capital follows a similar pattern, except that it is predetermined.

[Insert Figure 1 Here.]

We now turn to our model with credit risk. The solid lines in Figure 1 illustrates that the impact effect on output, consumption, investment and hours in our model are much larger than the benchmark RBC model. In addition, output, consumption, investment, hours, and capital all follow hump-shaped paths toward their steady state values. As Cogley and Nason (1995) argue, the hump-shaped output response is important for the propagation of technology shocks. This hump-shaped response explains why our model with credit risk can match the autocorrelation of output as in the data, while the lack of the hump shape explains why the benchmark RBC model cannot match it. Because the humps of the investment and hours paths are not significant enough, our model cannot match their autocorrelations as in the data.

In standard business cycle models, capital stock is the only endogenous state variable. It evolves slowly through investment. The dynamics of the capital stock govern the equilibrium dynamics of the economy. By contrast, our model with endogenous capital structure has a new endogenous state variable–the debt-capital ratio. This state variable helps magnify and propagate exogenous shocks. The last panel of Figure 1 presents the response of the debt-capital ratio to the negative technology shock. This ratio is predetermined and falls by about 1.8 percent in the second period. Later on, it rises over time until overshooting its steady state value, and then gradually falls to its steady state. Following a negative technology shock, firms profits fall and, hence, are more likely to default on debt. Facing this high credit risk, firms reduce borrowing by lowering the debt-capital ratio. This leads firms to reduce investment. The initial drop in investment is much larger than the benchmark RBC model without credit risk. As the technology shock diminishes over time, firms start to increase debt and investment.

Besides the presence of a new state variable, capital in our model with credit risk also follows different dynamics than that in the benchmark model. As equation (27) shows, the possibility of default affects the accumulation of aggregate capital. Defaulting and reorganized firms incur default costs in that they lose part of their capital stock. The loss of capital contributes further to the drop in output in the economy following a negative technology shock.

[Insert Figure 2 Here.]

Figure 2 presents the responses of the 4-year default rate, the credit spread, the stock return, and the riskfree rate to a negative one-standard-deviation shock to aggregate productivity. The first panel shows that the 4-year default rate in our model rises by about 12 percentage point on impact. As the negative technology shock diminishes over time, the 4-year default rate eventually decreases and returns to its steady-state value. The default rate in our model is determined by the default trigger z_t^* which is state dependent. A negative technology shock reduces firm profits and hence the default trigger. As a result, firms are more likely to default. This default risk or credit risk is the key to understanding our model mechanism.

As the default risk rises, the payout to debt holders falls and hence the bond price falls, leading to a rise in the bond yield and the credit spread. The credit spread rises by 250 basis points on impact following a negative one-standard-deviation technology shock. Figure 2 also shows that the one-period risk-free rate (or interest rate) $R_{f,t+1}$ in both the benchmark RBC and our model with credit risk rises on impact and then decreases to its steady-state level. The intuition can be gained from the log-linearized equation:

$$\hat{R}_{f,t+1} = E_t \left[\hat{\Lambda}_t - \hat{\Lambda}_{t+1} \right],$$

where a variable with a hat denotes the deviation relative to its steady-state value. Following a negative technology shock, consumption falls and hence marginal utility falls on impact. As the shock dies out, marginal utility decreases over time due to the habit formation preferences (not shown in Figure 2), i.e., $\hat{\Lambda}_t > E_t \left[\hat{\Lambda}_{t+1} \right]$. By contrast, in the standard RBC models with time-additive utility, the interest rate falls on impact and then gradually rises to its steady-state level. For this utility, marginal utility is determined by current consumption, while for the habit formation preferences it is determined by consumption relative to habits. Another way to understand the different interest rate response in the standard RBC model is to note that the interest rate in that model is also tied to the return on capital or the marginal product of capital if there is no adjustment cost. The marginal product of capital falls following a negative technology shock so that the interest rate falls too.

Figure 2 also shows that the realized stock return in both the benchmark RBC model and the model with credit risk falls on impact in response to a negative technology shock. This reflects the fact that firm profits and dividends fall on impact.

5.3 Forecasting Properties

The impulse response functions displayed in Figures 1 and 2 indicate that asset prices are linked to economic activity. This is intuitive because asset prices are forward-looking variables and their movements may signal future economic conditions. Recent empirical research has emphasized the role of corporate credit spreads in forecasting economic activity (see, e.g., Gilchrist, Yankov and Zakrajsek (2009)).

To examine this issue in our model with credit risk, we first calculate some simple correlations between credit spreads and macroeconomic quantities. Table 6 presents the correlations between credit spreads and the h-period-ahead growth in output, consumption, investment, and hours. This table shows that credit spreads are negatively correlated with all these growth rates. Thus widening of credit spreads is positively related to a future recession.

[Insert Table 6 Here.]

Next, we conduct a regression analysis. Gilchrist, Yankov and Zakrajsek (2009) find that corporate credit spreads can forecast future output and employment growth, based on the US data. Using simulated data implied by our model with credit risk, we conduct a similar analysis. In addition, we also examine whether credit spreads can predict future consumption growth and investment growth. Table 7 presents the result for the forecasting horizon $h = 1, 2, 4$, and 8. The panel titled "Simple" means that credit spreads are the only independent variable. The panel titled "Augmented" means that we include lagged dependent variables as additional explanatory variables.

[Insert Table 7 Here.]

Table 7 shows that, for both the simple and augmented regressions, the coefficients on credit spreads have a negative sign, indicating that a rise in credit spreads signal a future recession. The coefficients increase with the forecasting horizon and are significant at the 5 percent level. When including lagged dependent variables, the coefficients on credit spreads fall, but are still significant. The adjusted R^2s increase with the forecasting horizon initially and eventually decrease with the forecasting horizon. The adjusted R^2s are large for the regressions on output, hours and consumption. The regressions on consumption generally have the largest adjusted R^2s. Even though the regressions on investment have small R^2s, the coefficients on credits spreads are still significant.

To help understand the economic intuition behind the above regressions, we follow Chen, Collin-Dufresne and Goldstein (2009) and regress four-year cumulative default rate on credit spreads. Table 8 presents the result and shows that the estimated coefficient on credit spreads is equal to 2.139 and significant at the 5 percent level. Thus, an increase in credit spreads signals an increase in the future default rates. In response, firms reduce debt issuance and investment. In addition, more firms default and incur large default costs, leading to a decline in the aggregate capital stock. These two factors cause the future recession.

[Insert Table 8 Here.]

Finally, we consider stock returns. Early empirical studies such as Fama and French (1989) find that credit spreads can forecast stock returns. We now use model simulated data to run regressions. Table 9 presents the result. This table shows that all the regression coefficients on the credit spreads are positive, meaning that widening of credit spreads signals future high stock returns. The intuition is the following. High credit spreads signal future recessions, times with low consumption and high marginal utility. Thus, the market price of risk and expected stock returns are high in recessions.

[Insert Table 9 Here.]

6 Additional Experiments

To examine the robustness of our results and to extend our model, we conduct some additional experiments by studying three issues: (i) the role of debt maturity, (ii) the effects of financial shocks, and (iii) alternative modelling of endogenous labor supply.

6.1 Debt Maturity

One contribution of our paper is the modelling of the long-term defaultable corporate bonds in a general equilibrium framework. In our model, when λ decreases from 1 to 0, debt maturity changes from one period to infinitely many periods. We fix all other parameter values and solve the model numerically again by varying λ. In experiments not reported in this paper, we find that the default rate and the debt-capital ratio increase with debt maturity while the dividend volatility decreases with it. The intuition can be best understood by considering the two extreme cases with one-period debt and infinite-maturity debt. For a given leverage ratio, a firm has to pay back debt each period if the debt has one-period maturity, while it pays only a fraction of debt (coupon rate) if the debt has infinite maturity. Thus, *ceteris paribus*, a bad liquidity shock is more likely to trigger default for one-period debt. Thus, a firm with one-period debt prefers to issue less debt in order to reduce default rates. In addition, a firm with longer maturity debt prefers to issue more debt because it exploits more tax advantages.

Why dividends are more volatile for longer maturity debt? For one-period debt, firms must go to the credit market each period to raise new debt. Thus, equity holders are exposed more to the bond price risk. For infinite-maturity debt, firms pay a fixed coupon rate each period and do not need to issue new debt each period. Thus, dividends are relatively more stable. In a model with long-term risk-free discount bonds and with exogenous leverage, Jermann (1998) also finds that dividend volatility is very high for short-term bonds and that incorporating long-term bonds is important to fit data.

We also find that debt maturity does not affect our qualitative results on macroeconomic variables and key intuition, e.g., the amplification and propagation mechanism and the cyclical and forecasting properties. Its effect is mainly on quantitative predictions.

6.2 Financial Shocks

So far, we have focused on the impact of technology shocks. Our model framework can easily incorporate other types of aggregate shocks. In this section, we introduce shocks to the credit markets. Specifically, we suppose that the recovery rate fluctuates over time due to aggregate shocks to the liquidation values.[10] We specify the recovery rates as the following AR(1) process: $\xi_t = \xi\zeta_t$ and

$$\ln \zeta_t = \tilde{\rho} \ln \zeta_{t-1} + e_t,$$

where e_t is an identically and independently distributed normal random variable. We use the data described in Chen (2010) to estimate this process and set $\tilde{\rho}$ as the point estimate, 0.89.

We now consider the impulse responses of the economy to a 10-percentage-point decrease in the recovery rate (from 27.09 percent in Table 1 to 17.09 percent). Figures 3 and 4 present the results. These figures reveal that the impact of the financial shocks are generally small compared to the impact of the real technology shock. The impacts of these two types of shocks on output, consumption, investment, capital, debt-capital ratios, and credit spreads are qualitatively identical. However, the impacts on hours, the default rates, stock returns and risk-free rates are qualitatively different. The intuition is the following. A decrease in the recovery rate means that bankruptcy costs rise. This induces firms to reduce debt, leading to a decrease in the default rates on impact, rather than an increase in it in the case of a negative technology shock. The payoffs to bond holders still fall because liquidation values fall. Thus, the bond value falls and credit spreads rise. The rise in credit spreads reduce Tobin's Q and hence investment (see equation (17)). Thus, shocks to the credit markets are transmitted to the real side of the economy in a way similar to the technology shocks. Based on these impulse

[10] Jermann and Quadrini (2009) and Gomes and Schmid (2010) also study similar types of shocks.

responses, we deduce that changes in credit spreads caused by financial shocks can also forecast future real economic activities.

[Insert Figures 3-4 Here.]

Unlike a negative technology shock, a negative financial shock to the recovery rate of bonds reduces hours on impact. The intuition is the following. As we discuss above, consumption and investment fall on impact. Since capital is predetermined, hours must also fall on impact to preserve the aggregate resource constraint if there is no technology shock to output. Our analysis suggests that the positive correlation between output growth and hours growth observed in the data could be due to financial shocks to the credit markets rather than technology shocks alone.

We also find a negative financial shock reduces the risk-free rate on impact, rather than raising it. The intuition is that a negative shock to the credit markets induces investors to demand more risk-free bonds, instead of risky corporate bonds. Thus, the risk-free bond price rises and hence the risk-free rate falls.

6.3 Alternative Modelling of Endogenous Leisure

We have adopted a utility function that combines habit formation in consumption with endogenous leisure modelled in a way similar to that in Greenwood, Hercowitz and Huffman (1988). We now consider the more commonly adopted modelling approach of King, Plosser and Rebelo (1988). Specifically, we suppose that the utility function is given by:

$$E\left\{\sum_{t=0}^{\infty}\beta^t\left[\frac{(C_t-hC_{t-1})^{1-\gamma}}{1-\gamma}-\eta\frac{N_t^{1+\varsigma}}{1+\varsigma}\right]\right\}. \tag{41}$$

We calibrate our model with this preference as in Section 4. By this procedure the parameter values for α, β, δ, τ, κ, ξ, λ and η are identical to those listed in Table 1. We also consider three values for $\zeta \in \{1, 0.4, 0\}$, implying that the Frisch elasticity of labor supply is equal to 1, 2.5 and infinity. The last case corresponds to indivisible labor. Other calibrated parameter values are listed in Table 10. We also list our targeted moments in the data and model implied moments in Table 11. We find that the model with credit risk and with the preference given in (41) performs much worse than that described in Section 2. In particular, we find that the estimated volatility of the technology shock is too high and is always at the upper bound. In addition, the mean equity premium is below 1 percent for all three commonly adopted values of Frisch elasticity of labor supply. Thus, the model with credit risk and with the preference given in (41) has difficulty in matching equity premium as in the data.

[Insert Table 10 and 11 Here.]

The intuition for this finding is similar to that described in Boldrin, Christiano and Fisher (2001) for the benchmark RBC model without credit markets. Specifically, fluctuations in labor hours do not directly affect marginal utility of consumption according to the preference given in (41). Thus, it is relatively less costly to smooth consumption by adjusting labor supply. There is less need to vary capital accumulation, and hence there is not much variation in the demand for capital. Without much variation in the demand for capital, there cannot be large fluctuations in its price and hence capital gains and equity returns.

7 Conclusion

In this paper, we incorporate long-term defaultable corporate bonds and credit risk in a dynamic stochastic general equilibrium business cycle model. Our model links the bond and stock markets to the real economy. The Modigliani and Miller Theorem does not hold true in our model because of the presence of corporate taxes, bankruptcy costs and agency costs. We show that credit risk amplifies aggregate technology shocks. The debt-capital ratio provides a new state variable and its endogenous movements provide a propagation mechanism. The model can match the persistence and volatility of output growth as well as the mean equity premium and the mean risk-free rate as in the data. The model implied credit spreads are countercyclical and forecast future real economic activities and stock returns. We also show that financial shocks are transmitted to the real economy through Tobin's Q. A negative financial shock to the recovery rate of bonds reduces both hours and output on impact, while a negative technology shock leads to a decline in output but an increase in hours.

Our model has some limitations. For example, compared to the data, the model implied persistence of investment growth and hours growth is too low, while the model implied persistence of consumption is too high. In addition, the volatility of the risk-free rate is too high, compared to the data. Some of these limitations are inherited from the habit formation preference studied in this paper. For future research, it would be interesting to explore the implications of Epstein-Zin preferences. In addition, it would be also interesting to consider alternative modelling of adjustment costs of capital. For example, one may introduce adjustment costs in the gross rate of change in investment, as in Christiano, Eichenbaum and Evans (2005).

A Appendix

A.1 Proof of Proposition 1

Factoring out k_t^j on the two sides of equation (11) and using equation (14) and the definition of $J_{t+1}(\omega_t^j)$ and ω_t^j, we derive that z_{t+1}^{j*} satisfies:

$$\begin{aligned}(\lambda + (1-\lambda)\,c)\,\omega_{t+1}^j &= (1-\tau)(R_{t+1} - z_{t+1}^{*j}) + J_{t+1}(\omega_{t+1}^j) + \tau\delta \\ &\quad + \tau\left(\lambda\left(1-p_t^j\right) + (1-\lambda)\,c\right)\omega_{t+1}^j.\end{aligned} \tag{42}$$

Similarly, using equations (5), (14), and (42), we rewrite equation (6) as:

$$\begin{aligned}p_t^j\omega_{t+1}^j &= \beta E_t\frac{\Lambda_{t+1}}{\Lambda_t}\left\{\left[(1-\lambda)\,\Phi(z_{t+1}^{*j})p_{t+1}^j + \lambda + (1-\lambda)c\right]\omega_{t+1}^j\right. \\ &\quad \left. - \int_{z_{t+1}^{*j}}^{z_{\max}}\left[(1-\tau)\left(z - z_{t+1}^{*j}\right) + (1-\xi)J_{t+1}(\omega_{t+1}^j)\right]d\Phi(z)\right\}.\end{aligned} \tag{43}$$

The above two equations show that z_{t+1}^{*j} and p_t^j depend on ω_{t+1}^j and aggregate states. Given these two constraints, we observe that the dynamic programming problem (16) does not depend on firm-specific state z_t^j. Thus, the optimal policy functions does not depend on firm identity. The first-order conditions follow from simple algebra. Q.E.D.

A.2 Proof of Proposition 2

Equations (35)-(36) form a system of three linear equations for $\frac{\lambda+(1-\lambda)c}{p}$, $\frac{\pi^z}{p}$ and $\frac{\omega}{p}\pi^p$. These equations are independent of the coupon rate c. We can solve for $\frac{\lambda+(1-\lambda)c}{p}$ as an implicit function of z^*. As analyzed in Section 3.3, ωp and J are implicit functions of z^* independent of c. Substituting these functions into equation (33), we can solve for z^*, which is independent of c. Finally, by the analysis in Section 3.3, the real allocation depends on z^*, which is also independent of c. Q.E.D.

A.3 Second-Order Condition

We derive the second-order condition for the choice of optimal debt-capital ratio in problem (16) for the case of deterministic steady state with one-period debt. In this case, $\lambda = 1$, $\Lambda_{t+1} = \Lambda_t$. It follows from equations (31) and (32) that $J = 1-\delta$. Importantly, the choices of debt and investment are separable in problem (16). Thus, the firm's optimal debt choice problem becomes:

$$\max_{\omega_{t+1}} p_t\omega_{t+1} + (1-\tau)\beta\int_0^{z_{t+1}^*}\left(z_{t+1}^* - z\right)d\Phi\left(z\right). \tag{44}$$

The two constraints (11) and (6) become:

$$\left(z_{t+1}^{*}-R\right)(1-\tau)=(\tau-1)\omega_{t+1}-\tau p_t\omega_{t+1}+\delta\tau+J, \tag{45}$$

$$p_t\omega_{t+1}=\beta\left\{\omega_{t+1}-\int_{z_{t+1}^{*}}^{z_{\max}}\left[(1-\tau)\left(z-z_{t+1}^{*}\right)+(1-\xi)J\right]d\Phi(z)\right\}. \tag{46}$$

Substituting the expression for $p_t\omega_{t+1}$ given in (46) into (44), we rewrite the objective function as:

$$\beta\{\omega_{t+1}+(1-\tau)(z_{t+1}^{*}-\bar{z})-[1-\Phi(z_{t+1}^{*})](1-\xi)J\}, \tag{47}$$

where $\bar{z}$ is the mean of z_t.

Substituting the expression for $p_t\omega_{t+1}$ given in (46) into (45) and simplifying yields the expression of ω_{t+1}:

$$\begin{aligned}\omega_{t+1} &= \frac{\tau\beta}{(1+\tau\beta-\tau)}\left\{(1-\tau)\int_{z_{t+1}^{*}}^{z_{\max}}\left(z-z_{t+1}^{*}\right)d\Phi(z)+[1-\Phi(z_{t+1}^{*})](1-\xi)J\right\}\\ &\quad+\frac{J-\left(z_{t+1}^{*}-R\right)(1-\tau)+\delta\tau}{(1+\tau\beta-\tau)}.\end{aligned} \tag{48}$$

Substituting this expression for ω_{t+1} into (47), we find that the firm's optimization problem is simplified to choose z_{t+1}^{*} only. We can then derive the first-order condition as:

$$\begin{aligned}0 &= \frac{\tau\beta(1-\tau)}{(1+\tau\beta-\tau)}\{-[1-\Phi(z_{t+1}^{*})](1-\tau)-\phi(z_{t+1}^{*})(1-\xi)J\}\\ &\quad-\frac{(1-\tau)}{(1+\tau\beta-\tau)}+(1-\tau)+\phi(z_{t+1}^{*})(1-\xi)J,\end{aligned}$$

where ϕ is the density function for z_t. The second-order condition is given by:

$$\frac{\tau\beta(1-\tau)}{(1+\tau\beta-\tau)}[\phi(z_{t+1}^{*})(1-\tau)-\phi'(z_{t+1}^{*})(1-\xi)J]+\phi'(z_{t+1}^{*})(1-\xi)J<0.$$

Simplifying yields:

$$\tau\beta\phi(z_{t+1}^{*})+\phi'(z_{t+1}^{*})(1-\xi)J<0. \tag{49}$$

Notice that $\phi(z_{t+1}^{*})\geq 0$. So a necessary condition for the second-order condition to hold is that $\phi'(z_{t+1}^{*})<0$.

For the power function distribution used in our model, we have:

$$\phi(z_{t+1}^{*})=\kappa\left(z_{t+1}^{*}+\frac{\kappa}{\kappa+1}\right)^{\kappa-1}, \tag{50}$$

and

$$\phi'(z^*_{t+1}) = \kappa(\kappa-1)\left(z^*_{t+1} + \frac{\kappa}{\kappa+1}\right)^{\kappa-2}. \tag{51}$$

Since $J = 1-\delta$, the second-order condition (49) becomes:

$$\tau\beta\kappa(z^*_{t+1} + \frac{\kappa}{\kappa+1}) + \kappa(\kappa-1)(1-\xi)(1-\delta) < 0. \tag{52}$$

Since $z^*_{t+1} + \frac{\kappa}{\kappa+1} \in (0,1)$, A sufficient condition for the second-order condition to hold is given by:

$$\tau\beta + (\kappa-1)(1-\xi)(1-\delta) < 0, \tag{53}$$

or

$$\kappa < 1 - \frac{\tau\beta}{(1-\xi)(1-\delta)}. \tag{54}$$

References

Albuquerque, Rui, and Hugo A. Hopenhayn, 2004, Optimal Lending Contracts and Firm Dynamics, *Review of Economic Studies* 71, 285-315.

Basu, Susanto, John G. Fernald and Miles S. Kimball, 2006, Are Technology Improvements Contractionary?, *American Economic Review* 96, 1418-1448.

Bernanke, Ben, Mark Gertler and Simon Gilchrist, 1999, Financial Accelerator in a Quantitative Business Cycle Framework, In: J. Taylor and M. Woodford (eds.), *Handbook of Macroeconomics*, Elsevier Science, Amsterdam.

Bhamra, Harjoat, Lars-Alexander Kuhn and Ilya Strebulaev, 2008, The Aggregate Dynamics of Capital Structure and Macroeconomic Risk, forthcoming in *Review of Financial Studies.*

Bhamra, Harjoat, Lars-Alexander Kuhn and Ilya Strebulaev, 2009, The Levered Equity Risk Premium and Credit Spreads: A Unified Framework, forthcoming in *Review of Financial Studies.*

Boldrin, Michele, Lawrence J. Christiano, and Jonas D. M. Fisher, 2001, Habit Persistence, Asset Returns and the Business Cycle, *American Economic Review* 91, 149-166.

Caballero, Ricardo and Eduardo Engel, 1999, Explaining Investment Dynamics in U.S. Manufacturing: A Generalized (S,s) Approach, *Econometrica* 67, 783-826.

Carlstrom, Charles T. and Timothy S. Fuerst (1997), Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis, *American Economic Review 87,* 893-910.

Chatterjee, Satyajit and Burcu Eyigungor, 2010, Maturity, Indebtedness, and Default Risk, working paper, Federal Reserve Bank of Philadelphia.

Chen, Hui, 2010, Macroeconomic Conditions and the Puzzles of Credit Spreads and Capital Structure, forthcoming in *Journal of Finance.*

Chen, Long, Pierre Collin-Dufresne, and Robert Goldstein, 2009, On the Relation between Credit Spread Puzzles and the Equity Premium Puzzle, *Review of Financial Studies* 22, 3367-3409.

Christiano, Lawrence, J., Martin Eichenbaum, and Charles L. Evans, 2005, Nominal Rigidities, and the Dynamic Effects of a Shock to Monetary Policy, *Journal of Political Economy* 113, 1-46.

Christiano, Lawrence, J., Roberto Motto and Massimo Rostagno, 2009, Financial Factors in Business Fluctuations, working paper, Northwestern University.

Cogley, Timothy, and James M. Nason, 1995, Output Dynamics in Real-Business-Cycle Models, *American Economic Review* 85, 492-511.

Cooley, Thomas, Ramon Marimon and Vincenzo Quadrini, 2004, Aggregate Consequences of Aggregate Contract Enforceability, *Journal of Political Economy* 112, 817-847.

Cooley, Thomas F., and Vincenzo Quadrini, 2001, Financial Markets and Firm Dynamics, *American Economic Review* 91, 1286-1310.

Covas, Francisco and Wouter den Haan, 2010, The Cyclical Behavior of Debt and Equity Finance, forthcoming in *American Economic Review.*

Dotsey, Michael, Robert G. King, and Alexander Wolman, 1999, State Dependent Pricing and the General Equilibrium Dynamics of Money and Output, *Quarterly Journal of Economics*, 104, 655-690.

Faia, Ester and Tommaso Monacelli, 2007, Optimal Interest Rate Rules, Asset Prices and Credit Frictions, *Journal of Economic Dynamics and Control* 31, 3228-3254.

Fama, Eugene F., and Kenneth R. French, 1989, Business Conditions and Expected Returns on Stocks and Bonds, *Journal of Financial Economics* 25, 23-49.

Francis, Neville and Valerie A. Ramey, 2005, Is the Technology-Driven Real Business Cycle Hypothesis Dead? Shocks and Aggregate Fluctuations Revisited, *Journal of Monetary Economics* 52, 1379-1399.

Gali, Jordi, 1999. Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations, *American Economic Review* 89, 249–271.

Gertler, Mark, and Nobuhiro Kiyotaki, 2010, Financial Intermediation and Credit Policy in Business Cycle Analysis, working paper, New York University.

Gilchrist, Simon, Vladimir Yankov, and Egon Zakrajsek, Credit Market Shocks: Evidence from Corporate Bond and Stock Markets, forthcoming in *Journal of Monetary Economics.*

Goldstein, Robert, Nengjiu Ju, and Hayne Leland, 2001, An EBIT Based Model of Dynamic Capital Structure, *Journal of Business* 74, 483-512.

Gomes, Joao F., 2001, Financing Investment, *American Economic Review* 91, 1263-1285.

Gomes, Joao F. and Lukas Schmid, 2010, Equilibrium Credit Spreads and the Macroeconomy, working paper, University of Pennsylvania.

Gourio, Francois, 2010, Credit Risk and Disaster Risk, work in progress, Boston University.

Greenwood, Jeremy, Zvi Hercowitz, and Gregory W. Huffman, 1988, Investment, Capacity Utilization, and the Real Business Cycles, *American Economic Review* 78, 402-417.

Guvenen, Fatih, 2009, A Parsimonious Macroeconomic Model for Asset Pricing, *Econometrica* 77, 1711-1750.

Hackbarth, Dirk, Erwan Morellec, and Jianjun Miao, 2006, Capital Structure, Credit Risk, and Macroeconomic Conditions, *Journal of Financial Economics* 82, 519-550.

Hayashi, Fumio, 1982, Tobin's Marginal q and Average q: A Neoclassical Interpretation, *Econometrica* 50, 213-224.

Hopenhayn, Hugo A., 1992, Entry, Exit, and Firm Dynamics in Long Run Equilibrium, *Econometrica* 60, 1127-1150.

Hopenhayn, Hugo A., and Richard Rogerson, 1993, Job Turnover and Policy Evaluation: A General Equilibrium Analysis, *Journal of Political Economy* 101, 915-938.

Jaimovich, Nir, and Sergio T. Rebelo, 2009, Can News about the Future Drive the Business Cycle?, *American Economic Review* 99, 1097-1118.

Jermann, Urban, 1998, Asset Pricing in Production Economies, *Journal of Monetary Economics* 41, 257-275.

Jermann, Urban and Vincenzo Quadrini, 2009, Macroeconomic Effects of Financial Shocks, working paper, University of Southern California.

King, Robert G., Charles I. Plosser, and Sergio T. Rebelo, 1988, Production, Growth and Business Cycles: I. The Neoclassical Model, *Journal of Monetary Economics* 21, 195-232.

King, Robert G. and Sergio T. Rebelo, 1999, Resuscitating Real Business Cycles, in J. B. Taylor & M. Woodford (ed.), *Handbook of Macroeconomics*, Edition 1, Volume 1, Chapter 14, 927-1007, Elsevier.

Kiyotaki, Nobuhiro and John Moore, 1997, Credit Cycles, *Journal of Political Economy* 105, 211-248.

Krusell, Per, and Anthony Smith, 1998, Income and Wealth Heterogeneity in the Macroeconomy, *Journal of Political Economy* 106, 867-898.

Kydland, Finn E. and Edward C. Prescott, 1982, Time to Build and Aggregate Fluctuations, *Econometrica* 50, 1345-70.

Leland, Hayne E., 1994, Corporate Debt Value, Bond Covenants, and Optimal Capital Structure, *Journal of Finance* 49, 1213-1252.

Leland, Hayne E., 1998, Agency Costs, Risk Management, and Capital Structure, *Journal of Finance* 53, 1213-1243.

Liu Zheng, Pengfei Wang, and Tao Zha, 2009, Do Credit Constraints Amplify Macroeconomic Fluctuations?, working paper, Hong Kong University of Science and Technology.

Long, John B, Jr and Charles I. Plosser, 1983, Real Business Cycles, *Journal of Political Economy* 91, 39-69,

Merton, Robert C., 1974, On the Pricing of Corporate Debt: The Risk Structure of Interest Rates, *Journal of Finance* 29, 449-470.

Miao, Jianjun, 2005, Optimal Capital Structure and Industry Dynamics, *Journal of Finance* 6, 2621-2659.

Philippon, Thomas, 2009, The Bond Market's q, *Quarterly Journal of Economics* 124, 1011-1056.

Shea, John, 1998, What Do Technology Shocks Do? NBER Macroeconomics Annual, 275–310.

Table 1. Parameter Values

	Parameters	Values
α	capital share	0.3300
β	discount factor	0.9855
δ	depreciation rate	0.0250
τ	effective corporate tax	0.2000
ρ	persistence of technology shock	0.8808
σ	volatility of technology shock	0.0159
κ	shape parameter	0.0892
ξ	bond recovery rate	0.2709
$1/(4\lambda)$	bond maturity (years)	10
θ	adjustment cost parameter	4.0558
γ	coefficient of relative risk aversion	2.4133
ς	labor elasticity	0.4000
η	preference weight on leisure	0.5667
h	habit persistence	0.6999

Table 2. Target Moments

	Data	Model
mean riskfree rate	1.51	1.45
mean risk premium	7.23	6.76
credit spreads	109	102
4-year default rate	1.55	1.48
output volatility	1.08	1.08
relative consumption volatility	0.51	0.62
relative investment volatility	3.96	3.13

Note: The data are described in Section 4. Except for credit spreads, all numbers are in percentage. Credit spreads are measured in basis point. Financial returns are annualized. The model implied moments are obtained by simulating the models for 20,000 periods.

Table 3. Asset Return Moments

	$E[R_f]$	$\sigma(R_f)$	$E[R_e - R_f]$	$\sigma(R_e)$
Data	1.51	2.25	7.23	16.54
Benchmark	2.88	11.62	5.47	19.59
Our model	1.45	10.83	6.76	21.11

Note: The data are described in Section 4. Financial returns are annualized. The model implied moments are obtained by simulating the models for 20,000 periods.

Table 4. Credit Market Statistics

Correlation with $\Delta \ln Y_t$	Default Rate	Credit Spread	Debt issuance
Model	-0.73	-0.49	0.21
Data	-0.33	-0.36	0.46

Note: The correlations of default rates and credit spreads with output growth in the data are taken from Gomes and Schmid (2010). The correlation of debt issuance with output growth in the data is taken from Covas and den Haan (2010). The corresponding model implied correlations are obtained by simulating the model for 20,000 periods.

Table 5. Selected Business Cycle Moments

	Data	Benchmark	Our model
A: Volatility			
$\sigma(\Delta \ln Y_t)$	1.09	0.59	1.07
$\sigma(\Delta \ln C_t)/\sigma(\Delta \ln Y_t)$	0.51	0.25	0.62
$\sigma(\Delta \ln I_t)/\sigma(\Delta \ln Y_t)$	3.96	4.66	3.13
$\sigma(\Delta \ln N_t)/\sigma(\Delta \ln Y_t)$	0.90	2.68	1.12
B: Correlation with GDP			
$corr(\Delta \ln C_t, \Delta \ln Y_t)$	0.48	0.63	0.87
$corr(\Delta \ln I_t, \Delta \ln Y_t)$	0.88	0.98	0.92
$corr(\Delta \ln N_t, \Delta \ln Y_t)$	0.75	-0.96	-0.44
C: Autocorrelation			
$\rho(\Delta \ln Y_t)$	0.35	0.02	0.40
$\rho(\Delta \ln C_t)$	0.17	0.62	0.83
$\rho(\Delta \ln I_t)$	0.30	-0.06	0.05
$\rho(\Delta \ln N_t)$	0.61	-0.07	0.07

Note: The data are described in Section 4. The model implied moments are obtained by simulating the models for 20,000 periods.

Table 6. Correlations of Credit Spread with Economic Activities

X	$\text{Corr}(\Delta \ln X_{t+1}, CS_t)$	$\text{Corr}(\Delta \ln X_{t+2}, CS_t)$	$\text{Corr}(\Delta \ln X_{t+4}, CS_t)$
Output	-0.36	-0.41	-0.43
Consumption	-0.58	-0.58	-0.56
Investment	-0.11	-0.15	-0.19
Hours	-0.34	-0.44	-0.54

Note: All these correlations are obtained by simulating the models for 20,000 periods.

Table 7. Regression Results

	Simple		Augmented	
Horizon: h	Coefficient	Adjusted R^2	Coefficient	Adjusted R^2
		Output		
1	-0.394 (0.007)	0.133	-0.101 (0.009)	0.230
2	-0.751 (0.012)	0.171	-0.231 (0.015)	0.279
4	-1.372 (0.020)	0.190	-0.561 (0.025)	0.277
8	-2.335 (0.035)	0.186	-1.402 (0.005)	0.225
		Consumption		
1	-0.397 (0.004)	0.342	-0.039 (0.003)	0.714
2	-0.755 (0.008)	0.335	-0.111 (0.007)	0.662
4	-1.374 (0.014)	0.312	-0.333 (0.015)	0.551
8	-2.315 (0.027)	0.271	-0.986 (0.033)	0.390
		Hours		
1	-0.405(0.008)	0.115	-0.406 (0.008)	0.127
2	-0.774 (0.011)	0.194	-0.776 (0.011)	0.213
4	-1.414(0.016)	0.288	-1.415(0.015)	0.310
8	-2.373(0.023)	0.356	-2.376(0.022)	0.374
		Investment		
1	-0.380 (0.024)	0.0128	-0.350 (0.030)	0.0127
2	-0.733 (0.034)	0.0227	-0.705 (0.046)	0.0226
4	-1.364 (0.050)	0.0361	-1.437 (0.064)	0.0362
8	-2.414 (0.073)	0.0512	-2.882(0.094)	0.0542

Note: We simulate the model with credit risk by 20,000 periods to obtain the simulated data. The dependent variable is $\ln(X_{t+h}/X_t)$ where X denotes the variable in the title of each panel. "Simple" means regressions with credit spreads as the single independent variable. "Augmented" means regressions with up to 4-period lagged dependent variables as additional independent variables. All the estimated coefficients are for the credit spreads. Numbers in parenthesis are the standard errors of these estimates. All regressions coefficients are obtained by the OLS method.

Table 8. Regression of Default Rate on Credit Spreads

coefficient	Adjusted R^2
2.14 (0.025)	0.265

Note: We simulate the model with credit risk by 20,000 periods to obtain the simulated data. The dependent variable is the 4-year default rate. The independent variable is credit spreads. The coefficient on credit spreads is estimated by the OLS method. The number in parenthesis is the standard error of this estimate.

Table 9. Asset Return Regressions

	Simple		Augmented	
Horizon	Coefficient	adjusted R^2	Coefficient	adjusted R^2
1	0.565 (0.084)	0.041	0.869 (0.092)	0.045
2	1.204 (0.112)	0.026	1.665 (0.121)	0.031
4	1.961 (0.164)	0.015	3.077 (0.178)	0.028
8	3.069 (0.224)	0.012	5.046(0.242)	0.032

Note: We simulate the model with credit risk by 20,000 periods to obtain the simulated data. The dependent variables are the long-horizon cumulative stock return, with the horizon $h = 1, 2, 4, 8$. For the panel titled "Simple," the independent variable is credit spreads. For the panel titled "Augmented," the independent variables also include dividend yields. We use the OLS estimation method. The numbers in parenthesis are the standard errors of the estimates.

Table 10. Parameter Values for KPR Preferences

Parameters	$\varsigma = 1$	$\varsigma = 0.4$	$\varsigma = 0$
ρ	0.9651	0.9284	0.9146
σ	0.0200	0.0200	0.0200
θ	6.2500	1.1605	0.6635
γ	2.9718	5.0000	4.2732
h	0.9500	0.3973	0.100

Table 11. Target Moments for KPR Preferences

	Data	Model		
		$\varsigma = 1$	$\varsigma = 0.4$	$\varsigma = 0$
mean risk-free rate	1.51	5.26	5.66	5.76
mean equity premium	7.23	0.76	0.30	0.17
credit spreads	109	102	102	102
4-year default rate	1.55	1.48	1.48	1.48
output volatility	1.08	0.26	0.74	0.96
relative consumption volatility	0.51	0.62	0.57	0.56
relative investment volatility	3.96	2.98	2.87	2.82

Note: The data are described in Section 4. Except for credit spreads, all numbers are in percentage. Credit spreads are measured in basis point. Financial returns are annualized. The model implied moments are obtained by simulating the model for 20,000 periods. The preference is given by equation (41).

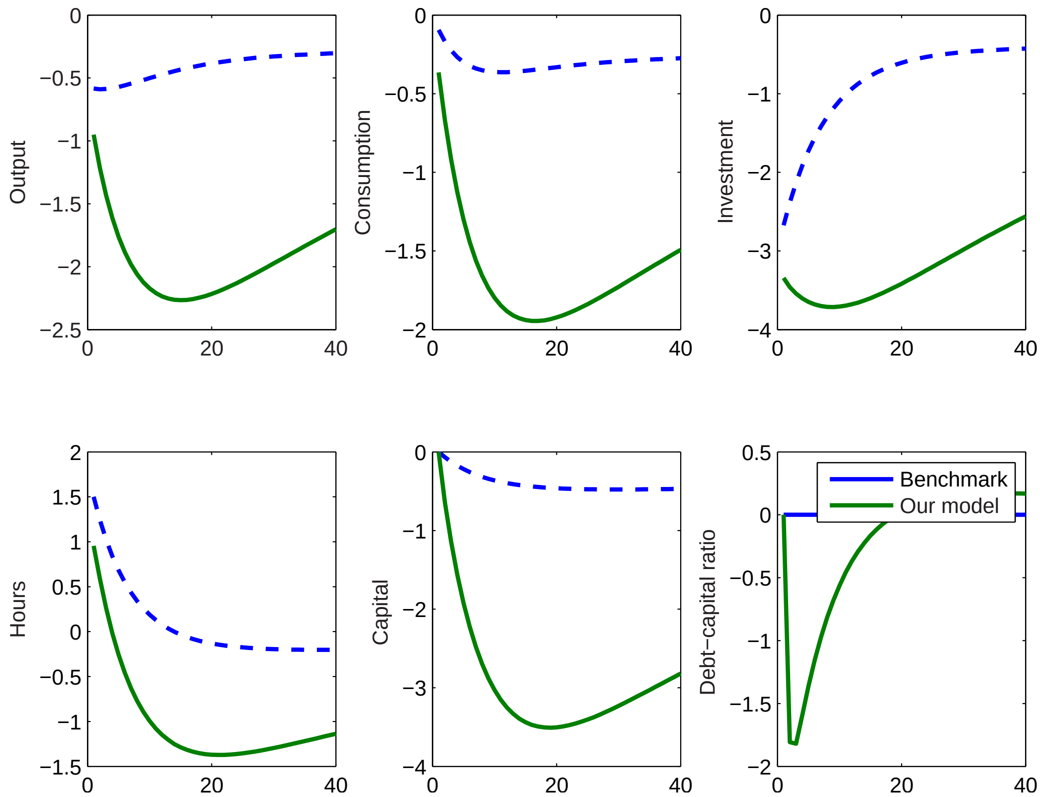

Figure 1: **Impulse responses to a negative technology shock.** This figure plots impulse responses of output, consumption, investment hours, capital, and the debt-capital ratio (measured in percentage deviation from the steady state) to a negative one-standard-deviation shock to aggregate productivity.

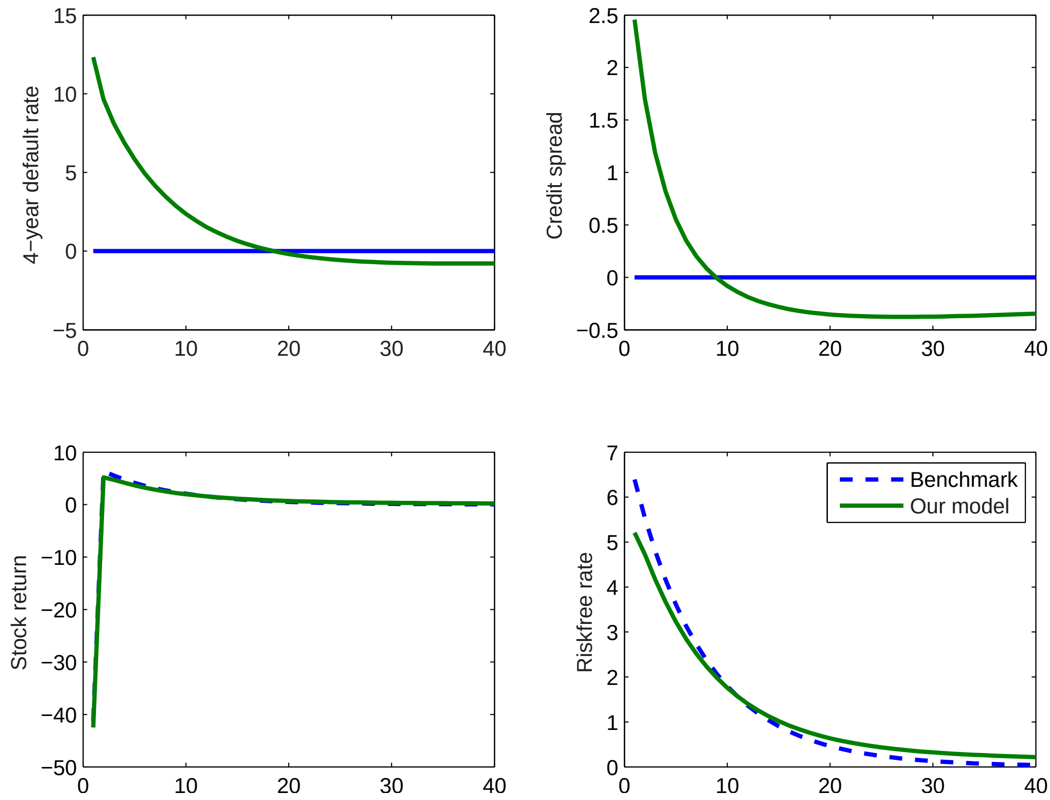

Figure 2: **Impulse responses to a negative technology shock.** This figure plots impulse responses of the 4-year cumulative default rate, credit spreads, stock return, and the riskfree rate to a negative one-standard-deviation shock to aggregate productivity. All vertical axes are measured in percentage and describe level changes from the steady-state values.

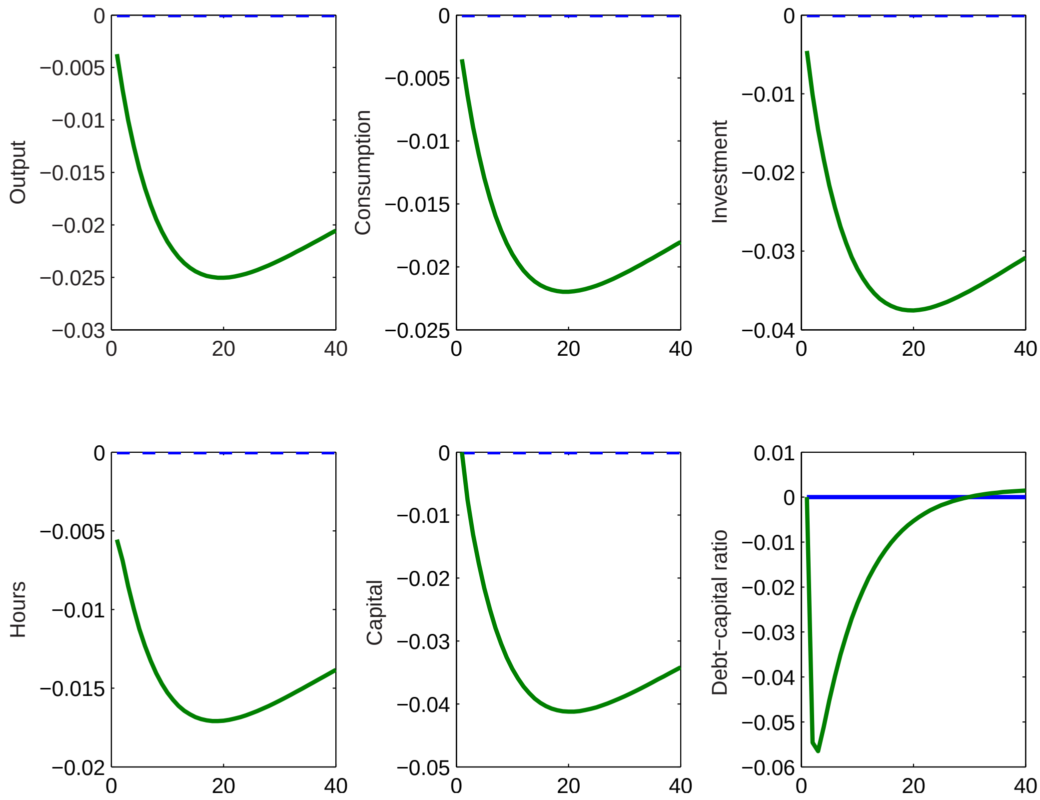

Figure 3: **Impulse responses to a negative financial shock.** This figure plots impulse responses of output, consumption, investment hours, capital, and the debt-capital ratio (measured in percentage deviation from the steady state) to a 10-percentage-point decrease in the recovery rate.

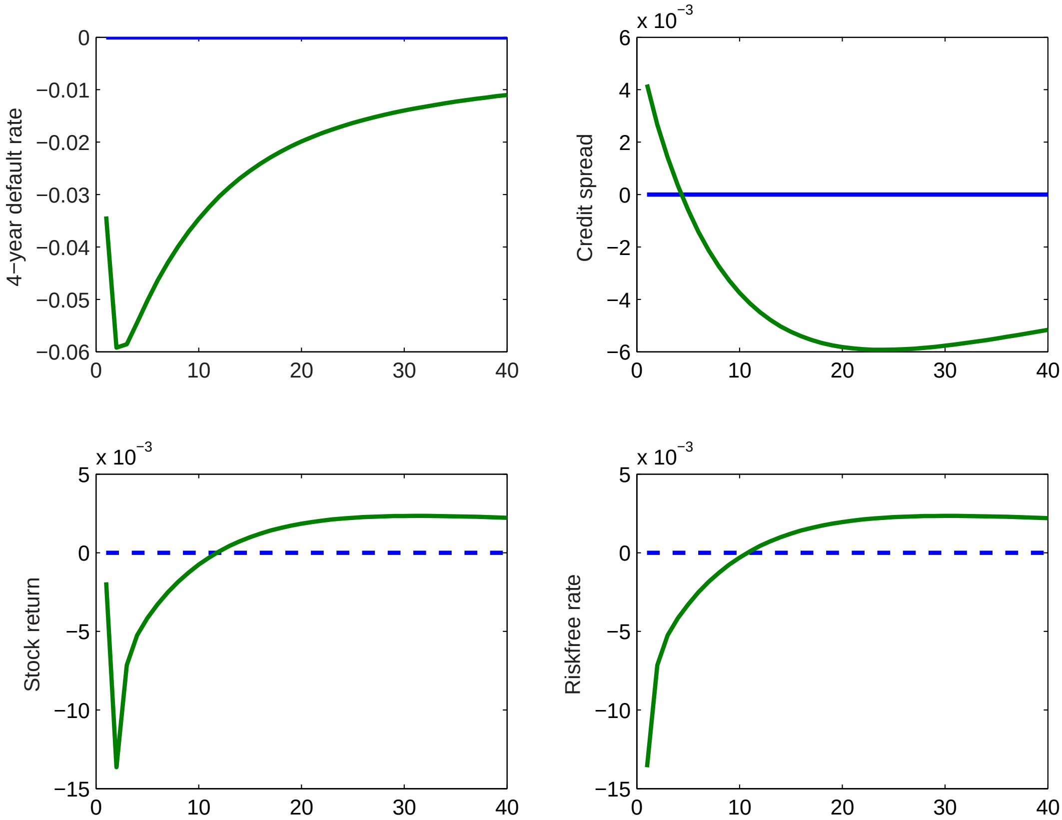

Figure 4: **Impulse responses to a negative financial shock.** This figure plots impulse responses of the 4-year cumulative default rate, credit spreads, stock return, and the riskfree rate to a 10-percentage-point decrease in the recovery rate. All vertical axes are measured in percentage and describe level changes from the steady-state values.