Inflation and Debt Rollover under Low Interest Rates

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Abstract

We provide a new Keynesian model with overlapping generations to study the impact of temporary and permanent increases in fiscal deficits financed by debt rollover policy when interest rates are lower than economic growth rates. We show that the debt rollover policy is feasible in the monetary regime, but leads to very slow-moving debt. This policy generates persistent inflation for a temporary increase in fiscal deficits, but persistent disinflation for a permanent increase. In terms of social welfare, the debt rollover policy dominates the conventional fiscal rule to finance a temporary increase in fiscal deficits, but is dominated if the increase is permanent.

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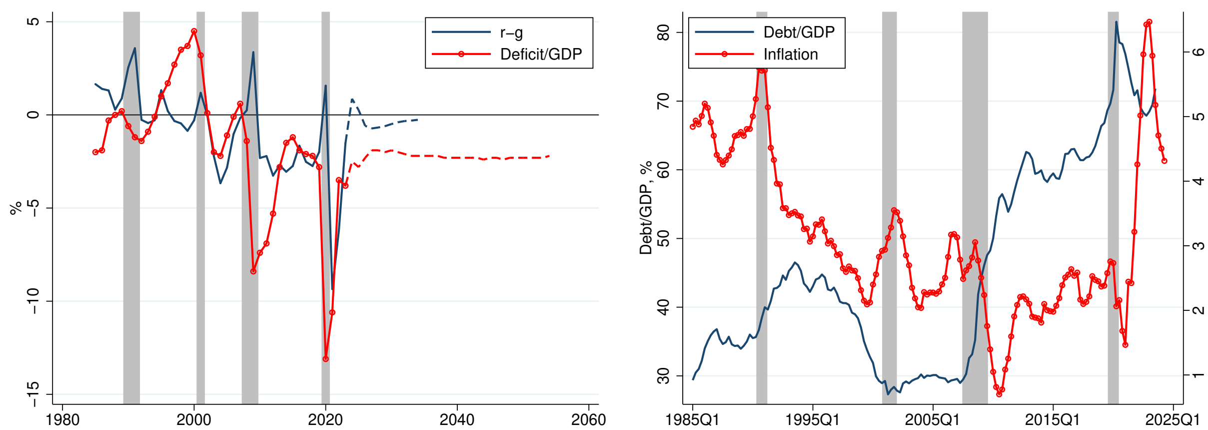
1 Introduction

As Panel (a) of Figure 1 shows, since 1985 nominal interest rates on the U.S. government bonds have generally fallen below nominal GDP growth rates (i.e., $r < g$). This remains true according to the projections by the U.S. Congressional Budget Office as of February 2024, as revealed by the dashed line. Panel (a) also shows that there have been frequent fiscal deficits over the years since 1985 until now and the deficits reached about 12% of GDP in 2020. As projected, the deficits will stay high far into the future. Panel (b) of Figure 1 shows that the U.S. debt/GDP ratios have surged to about 80% in 2020Q2 during the pandemic. At that time the inflation rate was 2.1% and then sharply increased to 6.4% in 2023Q1. Since then inflation rates declined.

Figure 1: Fiscal deficits, interest rates, public debt, and inflation rates.

(a) Fiscal deficits and low interest rates (b) Public debt and inflation



Notes: In Panel (a), the solid line with circles denotes the annual data on U.S. primary deficits-to-GDP ratio from 1985 to 2054. The solid line shows the annual data on the gap between the maturity-adjusted Treasury yield and the nominal GDP growth rate from 1985 to 2034. Solid lines trace the actual historical data, while the corresponding connected dashed lines depict projections by the U.S. Congressional Budget Office as of February 2024. The data for Panel (a) are from the U.S. Congressional Budget Office. We calculate the maturity-adjusted Treasury yield following Blanchard (2019). For Panel (b), we use the market value of U.S. public debt held by private investors from Hall, Payne, and Sargent (2018) to calculate the debt-to-GDP ratio. GDP and inflation (core CPI) data are extracted from FRED. The shaded regions in both panels indicate U.S. recession periods as determined by the NBER.

Given that $r < g$ may persist for a long period of time, a seemingly attractive policy–debt rollover policy–has drawn widespread attention since the seminal study by Blanchard (2019). This policy involves continually issuing new debt to repay old debt without a subsequent increase in taxes. While Blanchard (2019) has studied the impact of the debt rollover policy on the real activities and welfare in a stylized overlapping generations (OLG) model with two-period lived agents, his analysis abstracts away from monetary policy and inflation dynamics. Our goal is to study how debt rollover policy interacts with monetary policy in the presence of temporary and permanent fiscal deficits in a medium-scale new Keynesian (NK) model. We also use this model to quantify the impact on inflation, real activities, and welfare.

Our model builds on the OLG-NK model of Galí (2021) that features overlapping generations of the "perpetual youth" type as in Yaari (1965) and Blanchard (1985) and stochastic transitions to inactivity (retirement or unemployment). We extend Galí's (2021) model by introducing capital and fiscal policy. We show that there are two steady states with $r < g$ in the presence of permanent fiscal deficits. We focus on the steady state with a lower interest rate as it admits a more sensible comparative statics result. We study local equilibria around this steady state.

For the debt rollover policy to be feasible, we need to stabilize not only debt level, but also inflation. Thus, we need monetary policy to coordinate with fiscal policy such that local equilibrium is determinate. The difficulty is how to price public debt when $r < g$, as its fundamental value (the present value of future fiscal surpluses) may explode. We show that the value of public debt contains a bubble component, which makes debt value finite. The bubble component reflects the store of value of government bonds.

Following Leeper (1991), we consider an interest rate rule and a fiscal policy rule to model monetary and fiscal policies. For a calibrated model, we numerically compute the policy parameter space that delivers locally determinate equilibria. We find that the debt rollover policy is feasible, or delivers a unique equilibrium, when combined with an active monetary policy in the monetary regime. Within this regime, debt grows at an interest rate lower than the economic growth rate, generating persistently slow-moving debt levels. We also find the following main results:

1. A temporary fiscal transfer to all agents financed by debt rollover policy stimulates the economy in the short run but at the cost of persistent stagflation. Without endogenous tax adjustments, public debt stays persistently high under the debt rollover policy. This creates a compounded positive wealth effect that stimulates consumption and aggregate demand in the short run. During the transition to the original steady state, capital is persistently crowded out and output stays lower. Lower capital stock raises the firm marginal cost, reduces aggregate supply, and pushes up inflation persistently.

2. For a permanent increase in fiscal deficits caused by fiscal transfers to all agents, the debt rollover policy results in persistent disinflation and a slow transition to a new steady state with lower output and a higher real interest rate. Public debt slowly rises to a new higher steady-state level. As capital is gradually crowded out, aggregate investment declines, leading to a persistent decline of aggregate demand.

3. A hawkish monetary policy with strict inflation targeting can avoid the persistent inflation/disinflation dynamics when facing transitory/permanent fiscal deficits due to fiscal transfers. As a result, a temporary fiscal deficit will lose its power to stimulate the economy while a permanent increase in deficit will not lead to short-run recessions.

4. In terms of social welfare, the debt rollover policy dominates the conventional fiscal rule to

finance a temporary increase in fiscal deficits, but is dominated if the increase in deficits is permanent. The welfare gains mainly come from the reduction of consumption dispersion across cohorts.

Our paper is mainly related to three strands of the literature. First, our paper is related to the recent growing literature on debt sustainability in a low interest rate environment ($r < g$). Low interest rates can arise because public bonds are a store of value in OLG models or can provide liquidity benefits in infinite-horizon models. Based on the OLG model of Diamond (1965), Blanchard (2019) argues that the fiscal cost and the welfare cost of increasing public debt can be small given $r < g$ (also see Chalk, 2000). Kaas (2016) studies fiscal policy in an infinite-horizon model with financial frictions. Some recent papers study the feasibility of the debt rollover policy in the presence of aggregate risk (Mehrotra and Sergeyev, 2021; Kocherlakota, 2023a, 2023b; Aguiar, Amador, and Arellano, 2023; Brumm et al., 2021). Unlike our paper, all these papers do not consider the interaction with monetary policy and the impact on inflation dynamics.

Bassetto and Cui (2018), Brunnermeier, Merkel, and Sannikov (2020a), Brunnermeier, Merkel, and Sannikov (2020b), Reis (2021), Kaplan, Nikolakoudis, and Violante (2023), and Miao and Su (n.d.) analyze the interaction between monetary and fiscal policies with $r < g$. These papers typically study the determinacy of equilibria and the working of the fiscal theory of the price level (Leeper, 1991; Woodford, 1994, 1995; Sims, 1994; Cochrane, 1998). By contrast, we do not consider the fiscal regime and focus on the debt rollover policy in the monetary regime in the NK framework.

Kocherlakota (2022) and Mian, Straub, and Sufi (2022) also study the debt rollover policy in the NK framework. Using a tractable heterogeneous agent NK model, Kocherlakota (2022) shows that government debt bubbles can make fiscal policy more potent than monetary policy in stabilizing output and inflation. Mian, Straub, and Sufi (2022) use a bond-in-utility model to show that permanent deficit increases can be financed by a debt rollover policy at the zero lower bound (ZLB), because higher public debt reduces real interest rates by raising inflation. Unlike these papers, we study the debt rollover policy away from the ZLB using the OLG-NK framework of Galí (2021). We examine the welfare implications of the debt rollover policy for different generations, compared with other fiscal policy rules. We also analyze the impacts on real activities and inflation dynamics in response to temporary and permanent fiscal deficits.

Second, our paper is related to the literature on the OLG-NK models based on Blanchard (1985). Recent papers include Ascari and Rankin (2007), Leith and Von Thadden (2008), Ascari and Rankin (2013), Nistico (2016), Albonico, Ascari, and Gobbi (2021), Galí (2021), and Angeletos, Lian, and Wolf (2023), among many others. Except for Galí (2021), all these papers do not study $r < g$. Galí (2021) shows that introducing the retirement risk can generate $r < g$ and the existence of asset bubbles. Unlike our paper, he does not study fiscal policy and his focus is on the implications of asset bubbles for the design of monetary policy. Our paper is closely related to Angeletos, Lian, and Wolf (2023). They study how to finance fiscal deficits using the debt rollover policy through endogenous expansion of the tax base, without adjusting the tax rates. They do not consider $r < g$. Unlike their paper, the feasibility of the debt rollover policy in our paper does not rely on endogenous expansion of the tax base.

Third, our paper is related to the literature on asset bubbles surveyed by Miao (2014) and Martin and Ventura (2018). This literature typically does not study government debt bubbles and the interaction between fiscal and monetary policies. We borrow insights from this literature, especially Galí (2021), to generate $r < g$ in our model.

The rest of the paper is organized as follows: Section 2 describes the model setup. Section 3 analyzes the steady state with permanent deficits. Section 4 discusses the calibration of the model. Section 5 studies the feasibility of the debt rollover policy. Section 6 explores the impact of debt rollover policy on macroeconomic activities and inflation dynamics in response to temporary and permanent increases in fiscal deficits. Section 7 examines the welfare implications. Section 8 concludes the paper.

2 Model

The economy is populated by agents with identical preferences. The size of the population is constant and normalized to one. Each individual has a constant probability γ of surviving into the following period, independently of his age and economic status, while the remaining fraction $1-\gamma$ of the agents die. A new cohort of size $1-\gamma$ is born in each period to ensure the total population is constant.

Agents have two economic statuses: active and inactive. Active agents supply labor and manage firms. Each period an active agent has a constant probability $1-\nu$ of becoming inactive (retired or unemployed), i.e., of permanently losing his job and quitting his entrepreneurial activities. Let μ be the size of active agents. Assuming that newborns are all active, the constant size of active agents implies $\mu = 1-\gamma+\mu\gamma\nu$. Solving for μ yields $\mu = (1-\gamma)/(1-\nu\gamma) \in (0,1]$. The assumption of retirement (or, more generally, of declining labor incomes over the life cycle) can generate an equilibrium interest rate that is lower than the economic growth rate (Blanchard, 1985; Galí, 2021).

2.1 Consumers

A representative agent that is born in period s maximizes the following expected lifetime utility[1]

$$\mathbb{E}_s \sum_{t=s}^{\infty} (\beta\gamma)^{t-s} \ln\left(C_{t|s} - \Gamma^t V\left(L_{t|s}\right)\right), \tag{1}$$

1. The utility function (1) follows from Greenwood, Hercowitz, and Huffman (1988). Ascari and Rankin (2007) argue that this utility function helps avoid potential negative labor supply when agents accumulate sufficient wealth in the perpetual youth model. To avoid this issue, Galí (2021) assumes an exogenous labor supply function.

subject to the sequence of budget constraints

$$C_{t|s} + \mathbb{E}_t \left\{ \Lambda_{t,t+1} Z_{t+1|s} \right\} + B^n_{t|s}/P_t + Q_t K_{t|s} = A_{t|s} + W_t L_{t|s} + \Phi_{t|s} - T_{t|s},$$

for $t = s, s+1, s+2, ...$, where $C_{t|s}$ denotes the agent's real consumption at time t, $L_{t|s}$ denotes his labor supply, and Γ is the trend (gross) growth rate of the economy. As in Queralto (2020), the presence of Γ^t in the utility function ensures a balanced growth path with constant aggregate labor supply.

The function $V(\cdot)$ captures labor disutility in consumption units and takes the form:

$$V(L) = \frac{\eta}{1+\varphi} L^{1+\varphi},$$

where φ is the inverse of labor supply elasticity and η is a scaling parameter. If the agent is inactive, then $L_{t|s} = 0$ and $V(L_{t|s}) = 0$. In the above budget constraint, P_t denotes the aggregate price level, $A_{t|s}$ denotes the real financial wealth at the beginning of the period, W_t is the real wage rate, $\Phi_{t|s}$ denotes real profits from firms and capital producers, $T_{t|s}$ denotes the real lump-sum net taxes (taxes net of transfers).

Agents can trade three types of financial assets: nominal government bonds, capital, and a set of state-contingent securities. Let $B^n_{t|s}$ denote the principal value of nominal government bonds at time t that pay the nominal (gross) interest rate R^n_t at time $t+1$, $K_{t|s}$ the quantity of capital, and Q_t the real capital price. We assume complete markets for state-contingent securities in zero net supply. Let $Z_{t+1|s}$ denote the stochastic payoff at $t+1$ generated by the portfolio of securities purchased in period t and $\mathbb{E}_t \left\{ \Lambda_{t,t+1} Z_{t+1|s} \right\}$ is the market value of the portfolio, where $\Lambda_{t,t+1}$ is the real stochastic discount factor from t to $t+1$ (SDF) to be defined later. Only agents who are alive can trade in securities markets. The existence of complete securities markets allows agents to insure against the retirement risk.

At the beginning of period t, payoffs from holdings of government bonds, capital and state-contingent securities are $R^n_{t-1} B^n_{t-1|s}/P_t + ((1-\delta)Q_t + R^k_t) K_{t-1|s} + Z_{t|s}$. Here, δ is the depreciation rate of capital and R^k_t is the marginal product of capital or the real rental rate of capital. As standard in perpetual youth models, we assume that competitive insurance companies are operative and offer an annuity contract that collects financial wealth from the deceased agents and pays the survivors. Agents get payoffs from the annuity contract at the cost of giving up all their assets upon death. The zero-profit condition for the annuity contract implies that

$$A_{t|s} = \frac{1}{\gamma} \left(R^n_{t-1} B^n_{t-1|s}/P_t + \left((1-\delta) Q_t + R^k_t \right) K_{t-1|s} + Z_{t|s} \right). \tag{2}$$

The agent's first-order optimization conditions for government bonds and capital are

$$\mathbb{E}_t \Lambda_{t,t+1} \frac{R_t^n}{\Pi_{t+1}} = 1, \tag{3}$$

$$\mathbb{E}_t \Lambda_{t,t+1} \left(\frac{(1-\delta) Q_{t+1} + R_{t+1}^k}{Q_t} \right) = 1, \tag{4}$$

where the SDF from t to $t+j$ is

$$\Lambda_{t,t+j} = \beta^j \frac{C_{t|s} - \Gamma^t V(L_{t|s})}{C_{t+j|s} - \Gamma^{t+j} V(L_{t+j|s})}, \; j \geq 0.$$

The complete security market assumption ensures that agents of different cohorts and economic status have the same SDF $\Lambda_{t,t+j}$, which facilitates aggregation of individual consumption across households.

The first-order condition for labor supply is

$$\Gamma^t V'(L_{t|s}) = W_t. \tag{5}$$

This equation gives the same labor supply function as in Galí (2021) and thus the utility function (1) gives a microfoundation for his assumption. With an explicit microfoundation, this utility function allows us to conduct a more meaningful welfare analysis in Section 7.

In Appendix A, we derive the individual consumption rule as

$$C_{t|s} - \Gamma^t V(L_{t|s}) = (1 - \beta\gamma) \left(A_{t|s} + H_{t|s} \right), \tag{6}$$

where

$$H_{t|s} = \sum_{j=0}^{\infty} \gamma^j \mathbb{E}_t \Lambda_{t,t+j} \left[W_{t+j} L_{t+j|s} + \Phi_{t+j|s} - T_{t+j|s} - \Gamma^{t+j} V(L_{t+j|s}) \right]$$

is the human wealth of cohort s. Equation (6) shows that the labor-adjusted individual consumption is a fraction of the total wealth, i.e., the sum of financial wealth $A_{t|s}$ and human wealth $H_{t|s}$. Human wealth consists of present values of labor incomes and profits minus net taxes and disutility of labor in consumption units. The transversality condition must also be satisfied

$$\lim_{J \to \infty} \gamma^J \mathbb{E}_t \left[\Lambda_{t,t+J} A_{t+J|s} \right] = 0. \tag{7}$$

2.2 Firms

The economy has a continuum of monopolistically competitive intermediate goods producers. Each firm is managed by a specific active individual $i \in [0, \mu]$. Each firm hires labor $L_t(i)$ and rents capital

$K_t(i)$ to produce intermediate output $Y_t(i)$ with the following Cobb-Douglas production function

$$Y_t(i) = K_t^{\alpha}(i) \left(\Gamma^t L_t(i)\right)^{1-\alpha}, \tag{8}$$

where $\Gamma > 1$ denotes the gross rate of productivity growth, which is also the trend economic growth rate. The firm remains operative until its manager retires or dies, whatever comes first. If the manager retires or dies, one of the newly born individuals will take over the firm. The retired agents do not have dividend payout.

The cost minimization problem of the intermediate goods producers suggests that

$$\frac{K_t(i)}{L_t(i)} = \frac{\alpha}{1-\alpha}\frac{W_t}{R_t^k}. \tag{9}$$

All firms have the same capital-labor ratio. The intermediate goods firm's real marginal cost is

$$M_t = \left(\frac{R_t^k}{\alpha}\right)^{\alpha} \left(\frac{\Gamma^{-t} W_t}{1-\alpha}\right)^{1-\alpha}.$$

The intermediate goods are purchased by a final goods producer at the price of $P_t(i)$ to produce final goods using the production function as follows

$$Y_t \equiv \left(\mu^{-\frac{1}{\epsilon}} \int_0^{\mu} Y_t(i)^{1-\frac{1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}.$$

The cost-minimization problem of the final goods producer yields a demand schedule for each intermediate goods firm i:

$$Y_t(i) = \frac{1}{\mu}\left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t, \tag{10}$$

where $P_t \equiv \left(\mu^{-1} \int_0^{\mu} P_t(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$ denotes the aggregate price index.

We assume that in each period an intermediate goods firm has a probability $1-\theta$ to reset its price $P_t(i)$ (Calvo, 1983). Whenever the firm does not reset its price with probability θ, its price is assumed to be automatically increased at the steady-state inflation rate Π (Erceg, Henderson, and Levin, 2000). A firm adjusting its price in period t will choose the price P_t^* that maximizes

$$\max_{P_t^*} \sum_{j=0}^{\infty} (\nu\gamma\theta)^j \mathbb{E}_t \left\{\Lambda_{t,t+j} Y_{t+j|t} \left(\frac{\Pi^j P_t^*}{P_{t+j}} - M_{t+j}\right)\right\}$$

where $\Lambda_{t,t+j}$ is the real SDF and $Y_{t+j|t}$ is the firm demand in period $t+j$ when its last time to adjust its price is period t. Note that the term $(\nu\gamma)^j$ captures the probability of firm remaining operative, while θ^j is the probability that the reset price P_t^* remains effective.

2.3 Capital producers

Capital producers make new capital using final goods subject to adjustment costs (Christiano, Eichenbaum, and Evans, 2005). They then sell the new capital to consumers at the market price Q_t. The objective of a capital producer is to choose a sequence of investment $\{I_t\}$ to solve

$$\max \mathbb{E}_t \sum_{j=0}^{\infty} (\nu\gamma)^j \Lambda_{t,t+j} \left\{ Q_{t+j} \left[1 - f\left(\frac{I_{t+j}}{I_{t+j-1}} \right) \right] I_{t+j} - I_{t+j} \right\},$$

where the investment adjustment cost function takes the following form:

$$f\left(\frac{I_t}{I_{t-1}} \right) = \frac{\Omega_k}{2} \left(\frac{I_t}{I_{t-1}} - \Gamma \right)^2.$$

The capital producer's optimization condition is

$$\begin{aligned} 1 = Q_t \left[1 - \frac{\Omega_k}{2} \left(\frac{I_t}{I_{t-1}} - \Gamma \right)^2 \right] - Q_t \Omega_k \left(\frac{I_t}{I_{t-1}} - \Gamma \right) \frac{I_t}{I_{t-1}} \\ + \nu\gamma \mathbb{E}_t \Lambda_{t,t+1} Q_{t+1} \Omega_k \left(\frac{I_{t+1}}{I_t} - \Gamma \right) \left(\frac{I_{t+1}}{I_t} \right)^2. \end{aligned} \tag{11}$$

The law of motion for aggregate capital is

$$K_t = (1-\delta) K_{t-1} + \left[1 - f\left(\frac{I_t}{I_{t-1}} \right) \right] I_t. \tag{12}$$

2.4 Fiscal and Monetary Policies

The monetary authority sets the nominal interest rate according to the simple Taylor rule:

$$\ln\left(\frac{R_t^n}{R^n} \right) = \rho_R \ln\left(\frac{R_{t-1}^n}{R^n} \right) + (1-\rho_R)\phi_\pi \ln\left(\frac{\Pi_t}{\Pi} \right), \tag{13}$$

where $\Pi_t = P_t/P_{t-1}$ is the gross inflation rate, ρ_R is the inertia coefficient, and ϕ_π is the responsiveness coefficient. The variable R^n denotes the steady-state nominal interest rate and Π denotes the target inflation rate.

On the fiscal side, the government issues nominal bonds and collects lump-sum taxes to finance government expenditure. Let B_t^n denote the nominal principal value of government bonds with interest rate R_t^n, T_t the real aggregate net lump-sum tax (net of transfers), and G_t^e the real government expenditure. The government budget constraint is given by

$$P_t G_t^e + R_{t-1}^n B_{t-1}^n = P_t T_t + B_t^n.$$

In real terms, we have

$$G_t^e + \Pi_t^{-1} R_{t-1}^n B_{t-1} = T_t + B_t, \tag{14}$$

where $B_t \equiv B_t^n / P_t$ denotes the real principle value of government bonds.

The focus of our paper is to study the effects of temporary and permanent increases in fiscal deficit $-(T_t - G_t^e)$. For simplicity, we model increases in fiscal deficit as increases in fiscal transfer by fixing G_t^e. Specifically, the government provides lump-sum transfers $S_t \geq 0$ to all active and inactive agents and levies lump-sum taxes $T_t^a \geq 0$ only on active agents. We assume that both taxes and transfers do not depend on cohort, so the net tax for active and inactive agents of any cohort are

$$T_{t|s}^a = T_t^a - S_t, \; T_{t|s}^r = -S_t.$$

Since the size of active agents is μ, the net aggregate lump-sum tax is

$$T_t = \mu T_t^a - S_t. \tag{15}$$

The government sets its fiscal policies by specifying sequences of T_t^a, S_t, and G_t^e. We assume that the government keeps the (detrended) government expenditure constant $G_t^e / \Gamma^t = g^e$, where g^e is the detrended steady-state government expenditure. The government keeps the (detrended) transfer constant subject to deficit shocks:

$$S_t / \Gamma^t = s + z_{s,t}, \tag{16}$$

where s is the detrended steady-state transfer and $z_{s,t}$ is the deficit shock. Increases in fiscal transfer are thus driven by the deficit shock $z_{s,t}$. The government then adjusts the (detrended) tax according to the following tax rule (Leeper, 1991)

$$\tau_t^a - \tau^a = \phi_b (b_{t-1} - b), \tag{17}$$

where $\tau_t^a = T_t^a / \Gamma^t, b_t = B_t / \Gamma^t$ are the detrended tax and bond holdings and τ^a and b are their steady-state values.

In the above tax rule, the coefficient ϕ_b describes the responsiveness of taxes to government debt. The debt rollover policy corresponds to $\phi_b = 0$ as the government does not raise taxes in response to higher debt.

2.5 Aggregation and Equilibrium

Define aggregate consumption as the sum of consumption of all cohorts:

$$C_t = \sum_{s=-\infty}^{t} (1-\gamma)\gamma^{t-s} C_{t|s}.$$

In Appendix B, we derive the aggregate consumption function

$$C_t - \mu\Gamma^t V(L_t^a) = (1-\beta\gamma)(R_{t-1}^n B_{t-1}\Pi_t^{-1} + ((1-\delta)Q_t + R_t^k)K_{t-1} + H_t), \tag{18}$$

where H_t is the aggregate human wealth for all alive agents. The above equation shows that the real (maturity) value of public debt $R_{t-1}^n B_{t-1}\Pi_t^{-1}$ enters the aggregate consumption rule, which reflects the wealth effect of public debt.

From the consumer's optimal condition of labor supply (5), we see that the labor supply is independent of cohort. Hence, we use L_t^a to denote the amount of labor supplied by an active agent and write the optimality condition for labor supply as

$$\Gamma^t V'(L_t^a) = W_t. \tag{19}$$

The inactive agents do not supply any labor. Since the total size of the active population is μ, the aggregate labor supply $L_t = \mu L_t^a$. The market-clearing condition for labor is

$$L_t = \int_0^{\mu} L_t(i)di.$$

Aggregate capital satisfies $K_t = \sum_{s=-\infty}^{t}(1-\gamma)\gamma^{t-s}K_{t|s}$. The market-clearing condition for capital is

$$K_{t-1} = \int_0^{\mu} K_t(i)di.$$

From the firm's optimality condition (9), all firms have the same capital-labor input ratio, which also equals the ratio of aggregate capital over aggregate labor

$$\frac{K_{t-1}}{L_t} = \frac{K_t(i)}{L_t(i)}.$$

Integrating the firm's demand function (10) over all firms and using the production function (8), we obtain

$$Y_t \left[\frac{1}{\mu}\int_0^{\mu} \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} di\right] = \int_0^{\mu} Y_t(i)di = \Gamma^{t(1-\alpha)} \int_0^{\mu} \left(\frac{K_t(i)}{L_t(i)}\right)^{\alpha} L_t(i)di.$$

Since $K_{t-1}/L_t = K_t(i)/L_t(i)$, we derive the aggregate production function

$$Y_t = \Delta_t^{-1} K_{t-1}^{\alpha} \left(\Gamma^t L_t\right)^{1-\alpha}, \tag{20}$$

where

$$\Delta_t = \frac{1}{\mu}\int_0^{\mu} \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} di$$

denotes the price dispersion.

The market-clearing condition for government bonds is

$$\sum_{s=-\infty}^{t} (1-\gamma)\gamma^{t-s} B_{t|s}^n = B_t^n.$$

The resource constraint is

$$Y_t = C_t + I_t + G_t^e. \tag{21}$$

A competitive equilibrium is defined as the paths of aggregate variables such that all agents, intermediate goods firms, and capital goods producers optimize, all markets clear, and the government budget constraints hold. The full equilibrium system is shown in Appendix D and the detrended system is shown in Appendix E.

2.6 Public Debt Valuation

To understand the interaction between fiscal and monetary policies, it is important to understand how public debt is valued in the market.

In Appendix G, we derive that

$$\frac{R_{t-1}^n}{\Pi_t} B_{t-1} = \lim_{J\to\infty} \sum_{j=0}^{J} \mathbb{E}_t \Lambda_{t,t+j}(T_{t+j} - G_{t+j}^e) + \lim_{J\to\infty} \mathbb{E}_t \Lambda_{t,t+J+1} \frac{R_{t+J}^n}{\Pi_{t+J+1}} B_{t+J}. \tag{22}$$

This equation shows that the real value of public debt is equal to the fundamental value (i.e., expected present value of fiscal surpluses) plus a bubble component. As is well known, in a standard NK model with an infinitely-lived representative agent, the bubble component can be ruled out by the transversality condition. In our perpetual youth model, the transversality condition (7) cannot rule out bubbles as shown in Appendix G.[2]

When interest rates are lower than the economic growth rate, pricing the public debt as the fundamental value may be problematic. It is easier to see this point for the deterministic case

2. In that appendix, we also provide a different decomposition of the debt value into the sum of two finite components.

without trend economic growth (i.e., $\Gamma = 1$). By equation (3), the steady-state SDF satisfies

$$\Lambda_{t,t+1} = 1/R,\ \Lambda_{t,t+j} = 1/R^j,$$

where R is the steady-state value of the real interest rate: $R_t \equiv R^n_{t-1}/\Pi_t$. Let T and G^e denote the constant steady-state net taxes and government spending for $\Gamma = 1$. Then the fundamental value in the steady state becomes

$$\lim_{J\to\infty}\sum_{j=0}^{J}\Lambda_{t,t+j}(T_{t+j}-G^e_{t+j}) = \lim_{J\to\infty}\sum_{j=0}^{J}\frac{(T-G^e)}{R^j},$$

which explodes as $R < 1$. As demonstrated in the next section, we need the government to run permanent deficits in the steady state (i.e., $T < G^e$) for the steady-state interest R to be less than the economic growth rate Γ. Thus the steady-state fundamental value approaches negative infinity. By equation (22), we need the bubble component to approach positive infinity for the bond value to be finite. Bassetto and Cui (2018) and Brunnermeier, Merkel, and Sannikov (2020a) make a similar point.

3 Steady States

The model economy features a balanced growth path in the long run without aggregate uncertainty. That is, all aggregate quantities and the wage rate grow at the gross rate Γ, but the capital price, the interest rate, the inflation rate, and the capital return are constant. We use a lowercase letter to denote a detrended variable. That is, $x_t \equiv X_t/\Gamma^t$ for any variable X_t that grows at rate Γ on the balanced growth path. Let $\tau_t = T_t/\Gamma^t$ denote the detrended net taxes. We use a variable without a time subscript to denote the steady-state value of a detrended variable. In this section, we study the deterministic steady states of the detrended equilibrium system.

In Appendix F, we use the aggregate consumption rule (18) and other equilibrium conditions to derive the steady-state aggregate public bond demand equation:

$$\text{Demand:}\quad \frac{b}{y} = \frac{\beta\gamma - \gamma\nu R^{-1}\Gamma}{(1-\beta\gamma)(1-\gamma\nu)}\left[1 - \frac{g^e}{y} - \frac{(1-\alpha)M}{1+\varphi} - (1-(1-\delta)\Gamma^{-1})\frac{\alpha\Gamma M}{R-1+\delta}\right] - \frac{\alpha\Gamma M}{R-1+\delta} - \frac{\gamma\Gamma(1-\nu)}{(R-\gamma\Gamma)(1-\gamma\nu)}\frac{s}{y}, \tag{23}$$

where $R = R^n/\Pi$ is the steady-state real interest rate and $M \equiv (\epsilon-1)/\epsilon$ is the steady-state real marginal cost.

We use the government budget constraint (14) to derive the steady-state bond supply equation:

$$\text{Supply:} \quad \frac{b}{y}\left(R\Gamma^{-1} - 1\right) = \xi, \tag{24}$$

where $\xi \equiv (\tau - g^e)/y$ denotes the steady-state fiscal surplus-to-output ratio and $\tau = T_t/\Gamma^t$ is the steady-state net tax.

Given the fiscal targets ξ, s/y, and g^e/y, the market-clearing condition for the public bonds determines the equilibrium real interest rate R and the debt-to-output ratio b/y. That is, the two equations (23) and (24) jointly determine R and b/y. The rest steady-state variables can be computed accordingly as described in Appendix F. In that appendix, we also prove the following result.

Proposition 1 *Assume that the lump-sum transfer $s/y \geq 0$ is fixed. Then the bond demand function in (23) is monotonically increasing in the real interest rate R for sufficiently large R.*

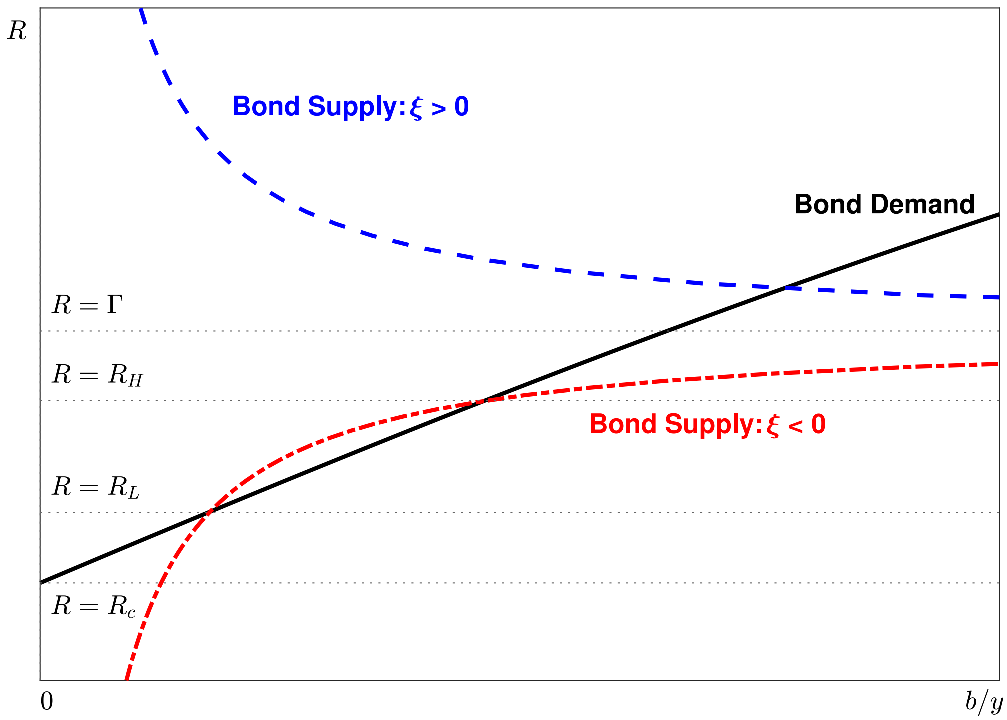
Intuitively, the bond demand increases in the real interest rate, since a higher interest rate makes it more attractive for agents to hold government bonds. We need a lower bound for R such that the bond demand function is well defined and an equilibrium exists. Figure 2 plots the bond demand function. The demand function crosses the vertical zero line at the interest rate R_c. The bond supply function is degenerate in the case of a balanced budget with zero deficit, i.e. $\xi = 0$. In this case, the government bonds become a pure bubble asset (Diamond, 1965). There is a steady state equilibrium at R_c and $b = 0$. There is another steady state with $b > 0$ if and only if $R_c < \Gamma$ as shown in the figure. This condition says that the interest rate in the steady-state equilibrium without a bubble is lower than the economic growth rate (Tirole, 1985). This condition also implies that the bubbleless economy is dynamically inefficient.

We can check that the bond demand curve shifts downward if the retirement risk $1-\nu$ is higher, the lump-sum transfer s/y is smaller, or the labor supply elasticity $1/\varphi$ is smaller. In this case, the bubble existence condition $R_c < \Gamma$ is more likely to be satisfied.

The shape of the bond supply curve (24) depends on the sign of the surplus-to-output ratio ξ. If the government maintains a permanent fiscal surplus $\xi > 0$, then the bond supply curve is downward sloping, as shown by the dashed line in Figure 2. In this case, the model has a unique steady state with $R > \Gamma$.

If the government runs a permanent fiscal deficit with $\xi < 0$, the bond supply curve is upward sloping and $R < \Gamma$ along the curve, as shown by the dash-dotted curve in Figure 2. The revenue to sustain fiscal deficits comes from the net proceeds from selling government bonds $(\Gamma - R)b$, which is positive if $R < \Gamma$. An increase in the real interest rate reduces the profit margin $\Gamma - R$. So the government needs to sell more bonds to finance a given level of fiscal deficit $\xi < 0$, resulting in an upward-sloping bond supply curve.

Figure 2: Demand and supply of government bonds



Note: The solid line is the bond demand curve. The two dashed lines are the bond supply curves for $\xi > 0$ and $\xi < 0$ respectively.

As Figure 2 shows, it is possible that the bond supply curve crosses the bond demand curve twice for a given level of permanent fiscal deficits, resulting in two steady-state equilibria with $R_L < R_H < \Gamma$. We call them steady state L and steady state H. An increase in fiscal deficits, or lower ξ, shifts the bond supply curve downward so that R_L and b_L/y_L rise, but R_H and b_H/y_H decline. We focus on the steady state L as it is consistent with the intuition that more deficits lead to more debt and higher interest rates.

When the fiscal deficit is sufficiently large, or ξ is sufficiently negative, the bond supply curve touches the demand curve just once. This level of deficit is the maximum sustainable deficit for a given $s/y \geq 0$. When the fiscal deficit is further increased, the supply curve does not cross the demand curve and thus there does not exist an equilibrium.

4 Calibration

As discussed in Section 3, for a permanent fiscal deficit $\xi < 0$, there may exist multiple steady states, with the corresponding real interest rates R_L and R_H satisfying $R_L < R_H < \Gamma$. We calibrate the model such that the steady state is associated with the lower interest rate R_L for a given permanent deficit. One period in the model corresponds to a quarter. There are 14 parameters in total as listed in Table 1.

Table 1: Calibrated parameters at quarterly frequency

Parameter	Values	Description	Target
α	0.33	Capital elasticity	Labor share of 0.67
δ	0.025	Capital depreciation rate	Capital depreciation rate of 10%
θ	0.75	Probability of not adjusting price	Ave. price duration of four quarters
ϵ	9	Elasticity of substitution	Ave. markup of 12.5%
Ω_k	2.48	Capital adjustment cost parameter	Price elasticity of investment of 0.40
Γ	1.0043	Gross growth rate of productivity	Ave. GDP pc. growth rate of 1.71%
γ	0.9959	Survival rate	Life expectancy at age 16 of 61.1 years
ν	0.9970	1- Retiring rate	Ave. employment ratio of 57.8%
φ	1.8519	Labor supply elasticity	Frisch elasticity of labor supply of 0.54
η	14.6	Labor disutility scaler	Ave. working hours of 0.33
β	0.9993	Discount factor	Real interest rate of 1.38%
Π	1.005	Steady-state inflation	Long-run inflation of 2%
g^e/y	0.21	Government spending/output ratio	Government spending/GDP ratio
τ/y	0.208	Lump-sum tax/output ratio	Public debt to GDP ratio of 43.7%

For the conventional parameters, we adopt values commonly used in the business cycle literature. Specifically, we set $\theta = 0.75$ to match an average duration of goods prices of approximately four quarters. The elasticity of substitution ϵ among goods is set to 9, corresponding to an average market markup of 12.5%. The capital depreciation rate δ is 0.025 so the annual depreciation rate is 10%. We set $\alpha = 0.33$ and $\varphi = 1.8519$ to obtain a Frisch elasticity of labor supply of 0.54 as suggested by Chetty et al. (2011). The parameter η is then chosen such that the average working hours of the employed amount to 0.33. According to Christiano, Eichenbaum, and Evans (2005), we set the capital adjustment cost parameter $\Omega_k = 2.48$ such that the price elasticity of capital investment is 0.40.

We calibrate the following parameters to match the data in the U.S. from 1985-2019 taken from FRED. The gross growth rate of productivity Γ is chosen to match an average annual real GDP per capita growth rate of 1.71%. We set the steady-state ratio of government expenditure over output g^e/y to match an average government spending-to-GDP ratio of 21.0%. We set the trend inflation $\Pi = 1.005$ to reflect the Fed's long-run inflation target of 2% annual rate.

The agent's survival probability γ and the probability of remaining active ν are important to our perpetual youth model. We set $\gamma = 0.9959$ so the expected lifetime $1/(4(1-\gamma))$ at age 16 is 61.1 years, which matches the average life expectancy in the U.S. of 77.1 years from the National Center for Health Statistics. We set $\nu = 0.997$ so that the share of employed agents $\mu = (1-\gamma)/(1-\nu\gamma)$ is 57.8%, which matches the average ratio of total non-farm employees over the working age population between 1985 and 2019. We calibrate the model without unemployment benefit, i.e., $s/y = 0$, for simplicity. We will consider cases with permanent increases in s/y in our later sections.

Lastly, we calibrate the discount factor β and steady-state tax-to-GDP ratio τ/y such that the model has two steady states. We pick the one with a lower real interest rate of 1.38% per annum, which matches the average maturity-adjusted real returns of US Treasury bonds between 1985 to 2019 (Blanchard, 2019). The debt-to-GDP ratio in this low-interest-rate steady state matches the average privately held public debt-to-GDP ratio of 43.7% between 1985 and 2019. We only consider the public debt held by private investors and exclude those held by government agencies and the Federal Reserve since in our model all the public debt is held by the households. The resulting steady-state deficit-to-output ratio is 0.14%, which is small relative to the forecasted deficit-to-GDP ratio around 2% in Figure 1. In our perpetual youth model with a surviving probability as high as $\gamma = 0.9969$, the demand for bonds is limited so that the implied maximum sustainable steady-state deficit-to-output ratio is only 0.23%.

We will consider different parameterizations of the monetary-fiscal policy mix (ϕ_b, ϕ_π). Our primary focus is a debt rollover policy with $\phi_b = 0$, implemented in the monetary regime. As a comparison, we consider an alternative value of $\phi_b = 0.04$, which is in the middle range of the annualized empirical estimates surveyed by Auclert, Rognlie, and Straub (2020). For the monetary policy parameters, we assume that $\phi_\pi = 1.5$ and $\rho_R = 0.8$, which are roughly consistent with the estimates in Clarida, Gali, and Gertler (1999) and Smets and Wouters (2007).

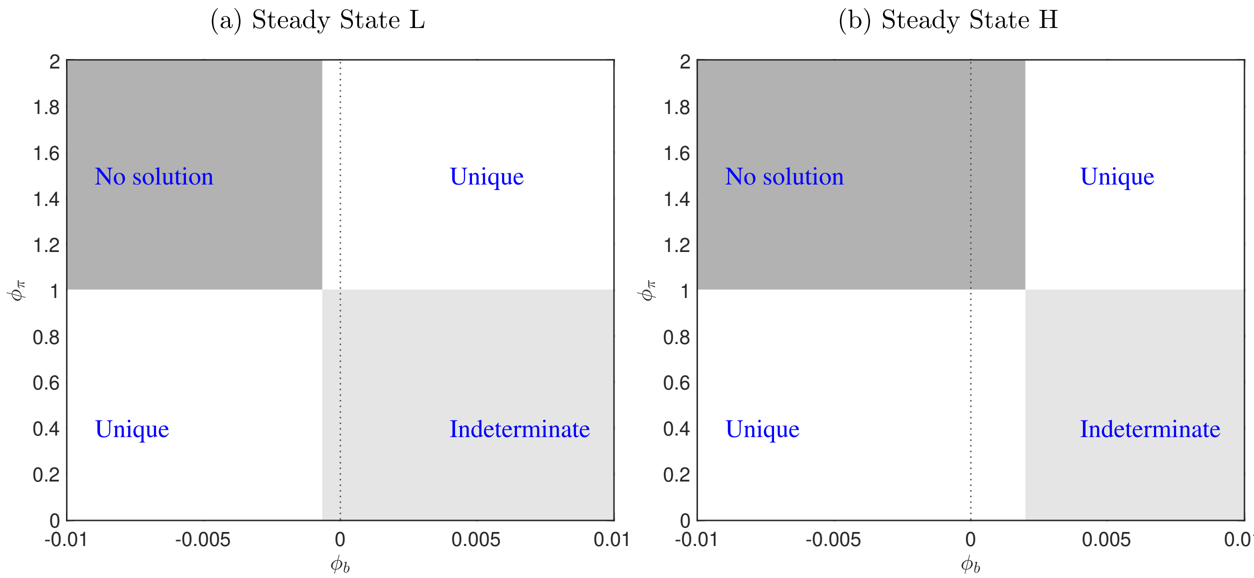
5 Feasibility of Debt Rollover Policy

Given the calibrated parameters in Table 1 and a fixed size of lump-sum transfer s/y, there are two steady states as discussed in Section 3. For debt rollover policy to be feasible, it is necessary that the equilibrium is locally determinate around a steady state. Due to the model's complexity, we rely on numerical solutions. More specifically, we first linearize the detrended system and then use Klein's (2000) method to analyze how the local determinacy is affected by the fiscal-monetary policy parameters (ϕ_b, ϕ_π). The results are shown in Figure 3.

Panel (a) of Figure 3 shows the local determinacy for policy mix (ϕ_b, ϕ_π) around the steady state L with the lower interest rate R_L. As in Leeper (1991), there are two policy regimes that ensure local determinacy of inflation and public debt, i.e. the monetary regime and the fiscal regime. In the monetary regime, public debt is repaid through tax revenue, and inflation is stabilized by an active monetary policy that adheres to the Taylor principle. In contrast, in the fiscal regime, public debt is managed through inflation revaluation, thus requiring a passive monetary policy to allow inflation to occur. In this paper we focus on the conventional monetary regime, leaving the analysis of the fiscal regime for future study.

As shown in Panel (a) of Figure 3, the monetary regime is around the upper-right corner, while the fiscal regime is around the lower-left corner. With low interest rates, panel (a) of Figure 3 suggests that debt rollover policy ($\phi_b = 0$) is feasible around the steady state L under the monetary regime with $\phi_\pi > 1$. However, as can be seen from Panel (b) of Figure 3, the same policy mix

Figure 3: Local determinacy regions for the policy mix parameters $\{\phi_b, \phi_\pi\}$



Notes: The figure shows the local determinacy property for fiscal-monetary policy mix (ϕ_b, ϕ_π) around the steady states with the baseline calibration in Table 1. Panel (a) corresponds to the steady state associated with the lower interest rate, i.e. $R = R_L < R_H$, while Panel (b) corresponds to the steady state associated with the higher interest rate, i.e. $R = R_H > R_L$. In both panels, the vertical dotted line represents the debt rollover policy, i.e. $\phi_b = 0$.

with debt rollover ($\phi_b = 0$) will lead to explosive dynamics under the monetary regime ($\phi_\pi > 1$). Even though $R = R_H < \Gamma$ still holds around the steady state H, the gap between the real interest rate R and the economic growth rate Γ is smaller due to the higher steady-state real interest rate. Additionally, an increase in public debt will push up real interest rates, leading to a further increase in interest expenses. The resulting larger interest expense makes the debt rollover policy infeasible around the steady state H.

6 Macroeconomic Effects of Debt Rollover Policy

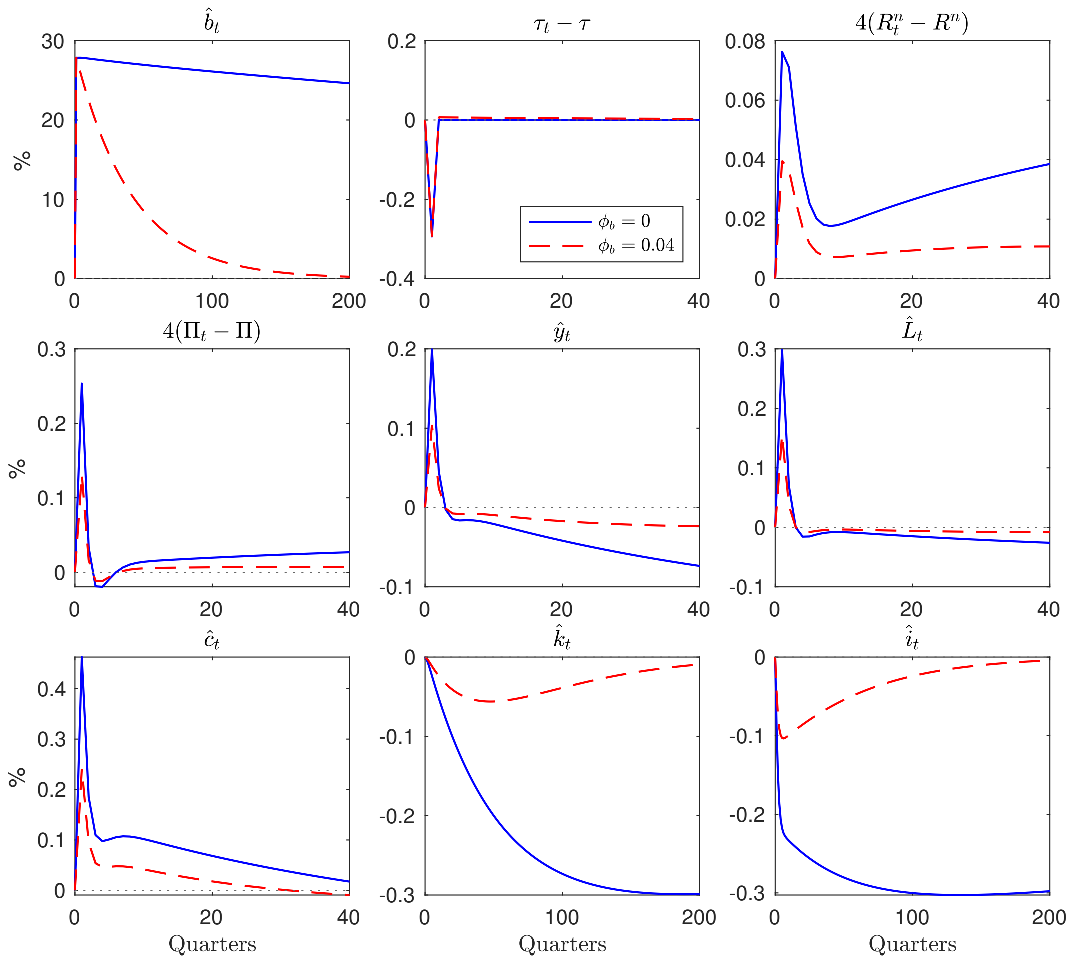
In this section, we use our calibrated model to study the macroeconomic effects of temporary and permanent increases in fiscal deficits financed by the debt rollover policy. Suppose that the economy initially stays in the steady state L. We then consider perfect foresight equilibria when the economy is hit by a temporary or permanent increase in fiscal deficits. We use nonlinear methods implemented by Dynare to solve the model (Adjemian et al., 2024).

6.1 Temporary Increase in Deficit

We first study the effects of a one-time increase in fiscal deficits induced by an increase in transfer s_1 from the initial steady state with $s = 0$ financed by the debt rollover policy. We set the magnitude of the deficit shock $z_{s,1}$ in (16) such that the deficit/GDP ratio $(\tau_1 - g^e)/(4y_1)$ increases by 12.21

percentage points on impact, which matches the size of the appropriations for key pandemic-related assistance programs in 2020 according to the Financial Report of the United States Government. We set $z_{s,t} = 0$ for $t \geq 2$. We compare the debt rollover policy ($\phi_b = 0$) with a conventional fiscal rule (17) ($\phi_b = 0.04$) that raises taxes in response to higher debt. Monetary policy follows the standard Taylor rule (13) with $\phi_\pi = 1.5$ and $\rho_R = 0.8$. Figure 4 shows the impulse responses.

Figure 4: Temporary increase in deficit



Note: This figure plots the impulse responses to a one-time increase in deficit in period 1. The nominal interest rate and inflation are in annualized percentage points deviation from their steady state. Net tax is in level deviation from its steady state. Other variables are in percentage change from their steady state, denoted by hat. For government bonds, capital, and investment, we plot 200 periods to show their slow dynamics under the debt rollover policy.

The temporary increase in deficit raises the debt level by almost 30% on impact. Due to the wealth effect of government debt in our OLG model, such an increase in government debt leads to a temporary boom with higher output, labor, inflation, and consumption. But investment declines on impact due to the crowding-out effect of debt. To understand the intuition, we derive the linearized aggregate Euler equation in Appendix C as follows:

$$\hat{\tilde{c}}_t = \frac{\nu\Gamma}{\beta R}\mathbb{E}_t\hat{\tilde{c}}_{t+1} - \frac{\nu\Gamma}{\beta R}\left(\hat{R}_t^n - \mathbb{E}_t\hat{\Pi}_{t+1}\right) + \frac{(1-\beta\gamma)(1-\gamma\nu)}{\beta\gamma}\frac{1}{\tilde{c}}\left(k(\hat{Q}_t + \hat{k}_t) + b\hat{b}_t\right), \tag{25}$$

where $\tilde{c}_t \equiv c_t - \mu V(L_t^a)$ is the aggregate consumption adjusted for labor disutility and the hatted variables denote their log deviations from the steady state. The last term in (25) captures the wealth effect of financial assets on consumption in the perpetual youth model. If $\gamma = \nu = 1$, the last term vanishes and the model is reduced to a representative agent model. With $\gamma < 1$ and $\nu < 1$, fluctuations in financial wealth affect aggregate demand. Intuitively, higher financial wealth means that the current cohorts have more resources to sell to future cohorts, thus raising their current consumption. Notice that labor also rises on impact as the rise of the aggregate demand raises the wage. Thus, both consumption c_t and adjusted consumption $\tilde{c}_t$ rise on impact.

Comparing the debt rollover policy with the convention fiscal rule, we find that the former generates a much larger temporary boom in output, consumption, labor, and inflation. This is because the magnitude of the wealth effect depends on the whole transition path of government debt due to the forward-looking nature of the Euler equation. We illustrate this point by iterating (25) forward:

$$\begin{aligned}\hat{\tilde{c}}_t =& -\mathbb{E}_t\sum_{j=0}^{\infty}\left(\frac{\nu\Gamma}{\beta R}\right)^{j+1}\left(\hat{R}_{t+j}^n - \hat{\Pi}_{t+1+j}\right)\\ &+ \frac{(1-\beta\gamma)(1-\gamma\nu)}{\beta\gamma}\frac{1}{\tilde{c}}\,\mathbb{E}_t\sum_{j=0}^{\infty}\left(\frac{\nu\Gamma}{\beta R}\right)^{j}\left(k(\hat{Q}_{t+j} + \hat{k}_{t+j}) + b\hat{b}_{t+j}\right).\end{aligned} \tag{26}$$

The last term captures the wealth effect of government bonds. The coefficient before this term $(1-\beta\gamma)(1-\nu\gamma)/(\beta\gamma)$ is very small (less than 0.0001 in our calibration), so any short-run increases in public debt have limited impacts on current consumption. However, if the increases in public debt are persistent, the effect on current consumption will be large.

Under the debt rollover policy, the speed of convergence of government debt is extremely slow. We can see this from the linearized law of motion of public debt:

$$\hat{b}_t = \left(\frac{R}{\Gamma} - \mu\phi_b\right)\hat{b}_{t-1} + \frac{R}{\Gamma}\hat{R}_t,\ t \geq 2.$$

When $\phi_b = 0$ and R is close to Γ as we calibrated to the data, the coefficient on the lagged debt level is equal to 0.9992, leading to extremely slow convergence of government debt. The half-life to convergence is about 300 years. As a result, the debt level remains high for a very long time as shown in Figure 4, generating a large compounded wealth effect on the current consumption. As a comparison, when $\phi_b = 0.04$, the initial rise of government debt is quickly repaid down by increases in tax revenue starting from period 2. The half-life to convergence is about 8 years. As a result, the wealth effect and the magnitude of the boom are much smaller (about a half of the case of $\phi_b = 0$).

Despite the initial temporary boom, the temporary increase in deficits leads to subsequent lower output, investment, and labor hours for a long period of time, especially under the debt rollover policy, until they slowly rise to their original steady-state levels in the long run. The heightened government debt persistently crowds out capital as long as the government debt is above its steady-state level. The combined effects of higher aggregate demand due to increased consumption and lower aggregate supply due to decreased capital generate higher inflation and lower output (i.e., stagflation) in the medium run. The central bank raises the nominal interest rate on impact until the inflation rate is below the target. Subsequently, the interest rate stays higher than the target for a long time to fight inflation during the transition to the steady state.

Due to the larger wealth effect caused by the slow-moving debt, the debt rollover policy leads to a larger temporary boom in output, consumption, and labor at the cost of more severe and persistent stagflation for a long period of time.

6.2 Permanent Increase in Deficit

The Congressional Budget Office projects that the U.S. government may have larger fiscal deficits in the upcoming thirty years. To understand the potential impacts, we study in this subsection a permanent increase in fiscal deficits and investigate the transition path from the initial low-deficit steady state to a new steady state with a higher level of deficits.

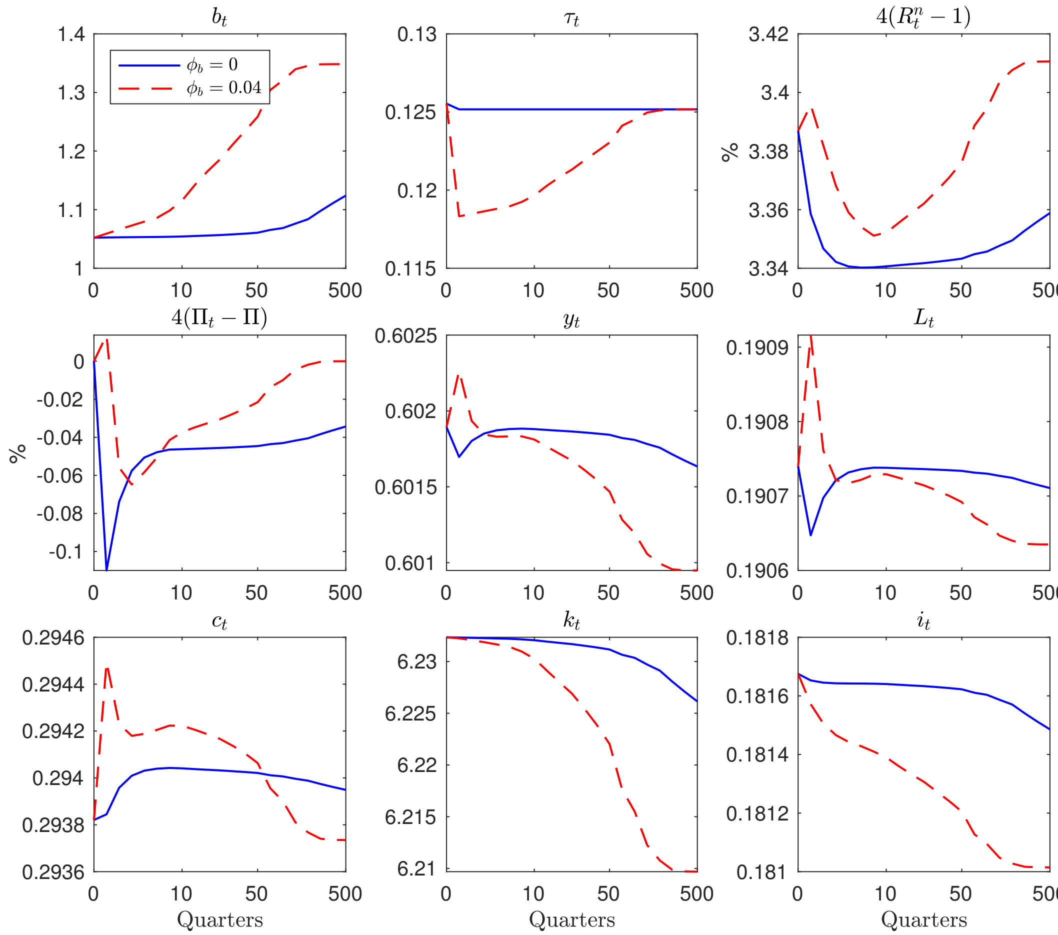
More specifically, we consider a permanent increase in fiscal deficits in the economy in period 1. We set the size of the permanent deficit increase to match the increase in the debt-to-output ratio around the pandemic period. The debt/GDP ratio increased sharply after the Great Recession and stabilized before the pandemic, averaging at 60.4% between 2010 and 2019. The debt/GDP increased sharply again during the pandemic and then fell a little bit, averaging at 72.8% between 2020 and 2023. We suppose that this increase of 12.4% is permanent by setting the magnitude of the permanent deficit shock $\{z_{s,t}\}_{t\geq 1}$ in (16) such that the debt/GDP ratio in the new steady state is 12.4 percentage points higher than the initial steady state.

In this case, the steady-state deficit/GDP ratio also increases. We pick the new steady state with a lower interest rate and denote it by R_{new}, which satisfies $R_L < R_{new} < \Gamma$ by Figure 2. To accommodate transition, we change the fiscal policy rule (17) to

$$\tau_t^a - \tau_{new}^a = \phi_b(b_{t-1} - b_{new}), \tag{27}$$

where τ_{new}^a and b_{new} are the new steady-state net taxes and debt level. We still adopt the same monetary policy rule as in the previous subsection so that the economy is in the monetary regime. Figure 5 compares the transition path under the debt rollover policy ($\phi_b = 0$) with that under the

Figure 5: Permanent increase in deficit



Note: This figure plots the transition path of a permanent increase in the deficit starting from period 1. The nominal interest rate is presented in annualized percentage points. Inflation is in annualized percentage points deviation from its steady state. The rest variables are in level.

conventional rule ($\phi_b = 0.04 > 0$).

After the permanent deficit shock, the economy transitions gradually to the new steady state characterized by higher deficit, public debt, and real interest rate, but lower output, consumption, capital, and labor (see Figure 2 for intuition). As highlighted in the previous subsection, the convergence of public debt to its new steady state is much slower under the debt rollover policy than under the conventional fiscal rule (27). Under the debt rollover policy, net taxes τ_t immediately declines to the new lower steady-state level τ_{new} because of a permanent increase in fiscal transfer. As debt gradually rises to its new steady-state level, capital is gradually crowded out and aggregate investment declines. As the net tax τ_t declines, aggregate consumption rises in the short run due to the wealth effect by (6). The net effect is that aggregate demand declines, leading to disinflation on impact. As aggregate demand stays low during the transition period, there is persistent disinflation.

By the Taylor rule, the central bank cuts the nominal interest rate for a long time until inflation rises to the target.

By contrast, under the conventional fiscal rule, τ_t immediately declines to a much smaller level and then gradually rises to τ_{new} in response to initial lower debt than the new steady-state level $b_{t-1} < b_{new}$. The initial large decline of τ_t generates a large wealth effect on consumption. This effect dominates that of the investment decline so that aggregate demand rises on impact, leading to higher inflation, output, and labor. In response, the central bank raises the nominal interest rate on impact and later cuts the interest rate to fight disinflation. Compared to the debt rollover policy, the transition to the new steady state is much faster.

6.3 Effects of Monetary Policy

In the previous two subsections, we have shown that, under the debt rollover policy, a temporary fiscal deficit shock leads to persistent stagflation, while a permanent fiscal deficit shock leads to persistent disinflation. Can monetary policy help? In this subsection, we study how monetary policy affects the dynamics of macroeconomic activities and inflation under the debt rollover policy.

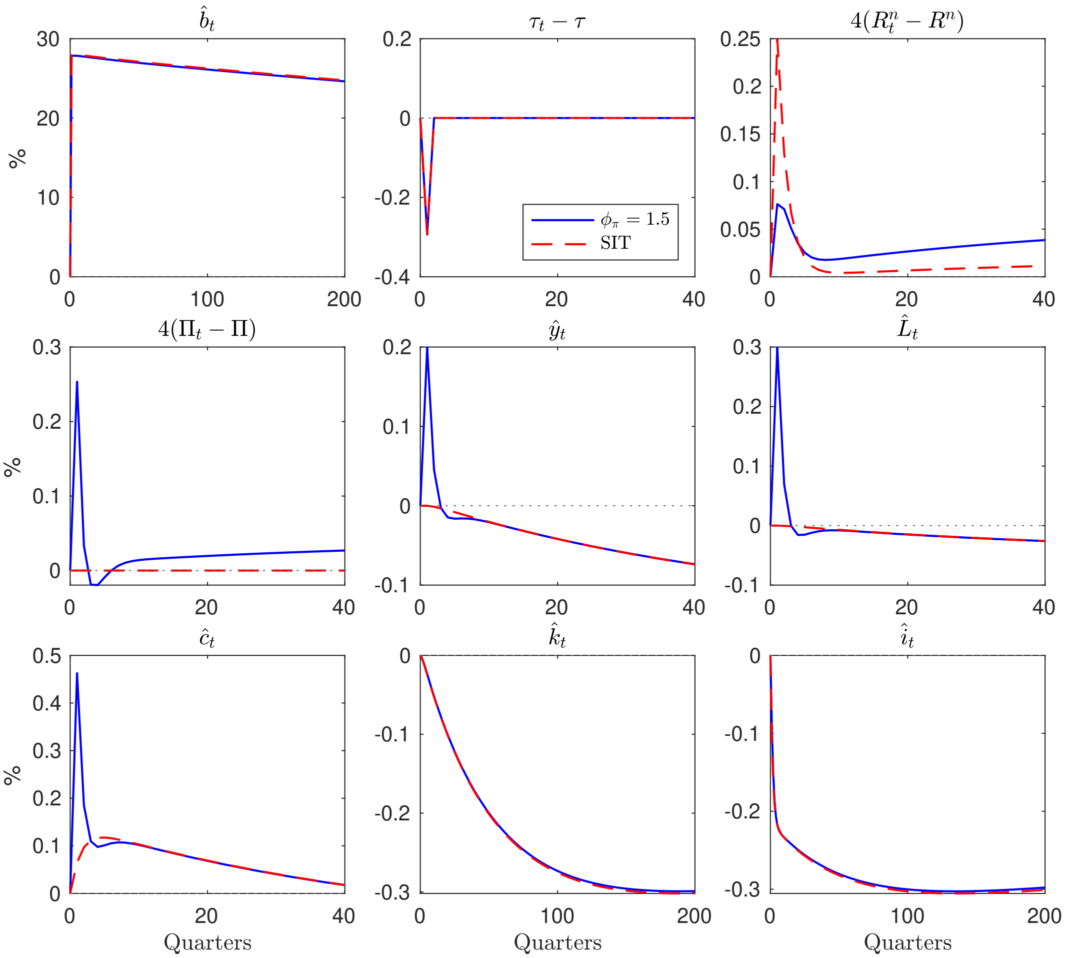
We are mainly interested in two specifications of monetary policy. The benchmark policy is the Taylor rule (13) with $\phi_\pi = 1.5$ and $\rho_R = 0.8$. The alternative monetary policy is the strict inflation targeting rule (SIT), where the monetary authority adjusts the nominal interest rate to fully stabilize inflation, i.e., $\Pi_t = \Pi$ for all t.

In Figure 6, we study how monetary policy affects the impacts of a transitory increase in deficits, financed by the debt rollover policy with $\phi_b = 0$. Under the SIT, the central bank raises the nominal interest rate aggressively so that inflation is perfectly stabilized. The side effect is that the short-run boom under the standard Taylor rule is now neutralized. In the medium run, the SIT cannot stabilize output even though inflation is perfectly stabilized. The persistent lower output is driven by the crowding-out effect of higher government debt discussed in Section 6.1. Therefore, a hawkish monetary policy helps stabilize inflation dynamics but at the expense of neutralizing the potential short-run stimulus effect of a temporary fiscal deficit shock.

In Figure 7, we consider how monetary policy can stabilize inflation dynamics with permanent deficit shocks under the debt rollover policy with $\phi_b = 0$. The permanent fiscal deficit shock leads to persistent disinflation along the transition path under the standard Taylor rule. The SIT can effectively correct the disinflation by keeping the real interest rate lower than in the case of the standard Taylor rule. As a result, the SIT leads to higher consumption and avoids the short-run recession that would happen under the standard Taylor rule.

To sum up, we find that monetary policy with strict inflation targeting can help to avoid the persistent inflation dynamics when the debt rollover policy is used to finance fiscal deficits. In the meantime, strict inflation targeting reduces the potential short-run stimulative effect of a temporary

Figure 6: Temporary increase in deficit: debt rollover with different monetary policies



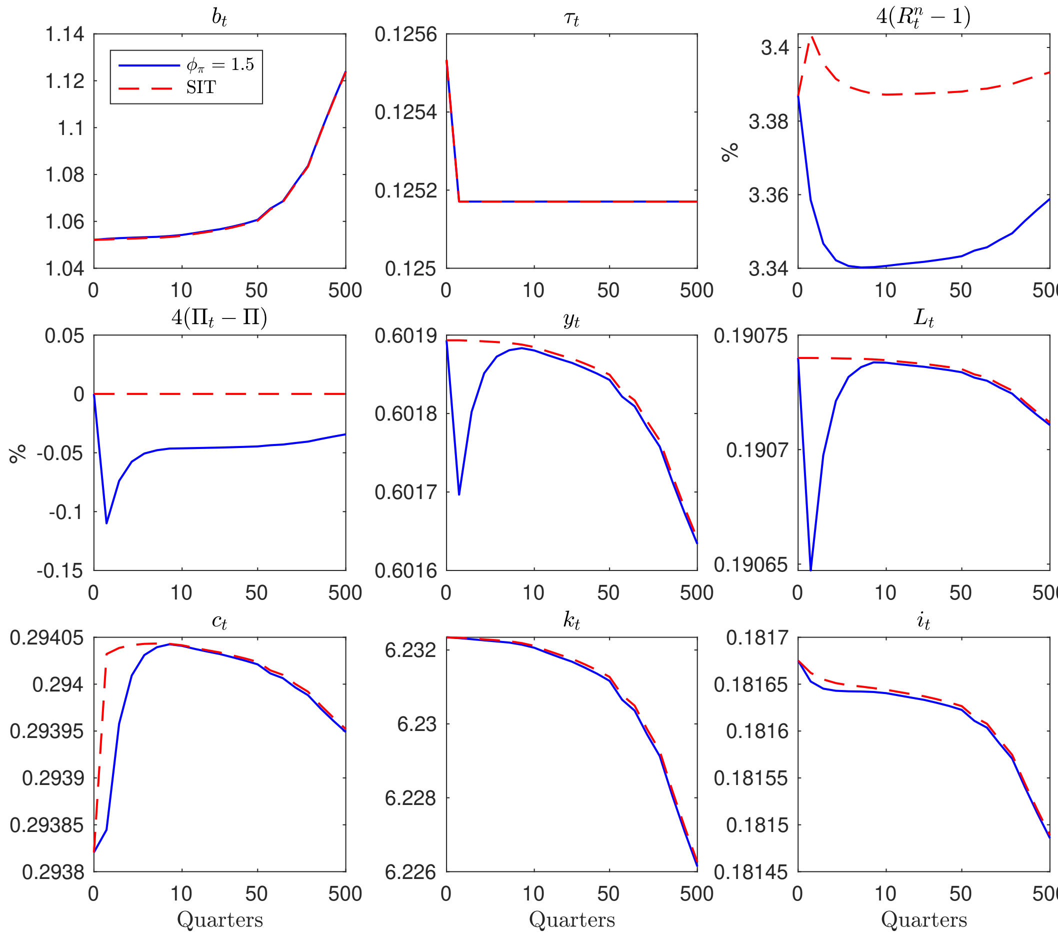
Note: This figure plots the impulse responses to a one-time increase in deficit in period 1. The nominal interest rate and inflation are in annualized percentage points deviation from their steady state. Net tax is in level deviation from its steady state. Other variables are in percentage change from their steady state, denoted by hat. For government bonds, capital, and investment, we plot 200 periods to show their slow dynamics under the debt rollover policy.

fiscal deficit shock but leads to higher short-run consumption under a permanent deficit shock.

7 Welfare Implications

Is debt rollover a desirable fiscal policy to finance temporary and permanent fiscal deficit shocks? In this section, we examine the welfare implications of deficits financed by debt rollover.

Figure 7: Permanent increase in deficit: debt rollover and monetary policies



Note: This figure plots the transition path of a permanent increase in the deficit starting from period 1. The nominal interest rate is presented in annualized percentage points. Inflation is in annualized percentage points deviation from its steady state. The rest variables are in level.

7.1 Social Welfare

Following Bonchi and Nisticò (2022), we first define (detrended) social welfare as the expected utility for all cohorts in the economy:

$$Wel_t^{soc} \equiv \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j u_{t+j}, \tag{28}$$

where $\mathbb{E}_t$ denotes the expectation operator for the retirement risk from time t on and u_{t+j} is the detrended period-utility for all cohorts alive at $t+j$,

$$u_{t+j} \equiv \sum_{s=-\infty}^{t+j} (1-\gamma)\gamma^{t+j-s} \ln\left(c_{t+j|s} - V\left(L_{t+j|s}\right)\right).$$

For the low-interest-rate steady state with $R < \Gamma$, the economy is dynamically inefficient and capital is over-accumulated. A permanent increase in fiscal deficits raises the real interest rate, as long as the deficit is sustainable (see Figure 2). As is well known, it leads to higher public debt, which crowds out capital and increases social welfare in the steady state.[3] For the numerical example in Section 6.2, the increase in social welfare is 0.45% measured by the consumption equivalent, i.e., the same percent compensation of the labor-adjusted consumption for each cohort from the initial steady state to achieve the social welfare in the new steady state.

Next, we examine how fiscal policies affect welfare gains of deficit shocks during the transition period and how these effects differ across different cohorts. We can rewrite the social welfare equation (28) as

$$Wel_t^{soc} = Wel_t^{ave} + (Wel_t^{soc} - Wel_t^{ave})$$

where

$$Wel_t^{ave} = \sum_{j=0}^{\infty} \beta^j \ln(c_{t+j} - \mu V(L_{t+j}^a)),$$

$$Wel_t^{soc} - Wel_t^{ave} = \sum_{j=0}^{\infty} \beta^j \Theta_{t+j},$$

and

$$\Theta_{t+j} \equiv (1-\gamma) \sum_{s=-\infty}^{t+j} \gamma^{t+j-s} \mathbb{E}_t \ln \left(\frac{c_{t+j|s} - V(L_{t+j|s})}{c_{t+j} - \mu V(L_{t+j}^a)} \right).$$

The term Wel_t^{ave} is the life-time *average utility* of a hypothetical infinitely-lived representative agent that consumes and works at the average level. The term $Wel_t^{soc} - Wel_t^{ave}$ is the difference between the total utility of our OLG model and the hypothetical average utility. This term measures *equality of welfare*. Due to the concavity of the utility function, this term or Θ_{t+j} is maximized if all cohorts have the same adjusted consumption.

Table 2: Social welfare of increase in deficit

	Temporary		Permanent	
	$\phi_b = 0$	$\phi_b = 0.04$	$\phi_b = 0$	$\phi_b = 0.04$
Social welfare	0.150	0.0089	0.250	0.397
Average utility	−0.020	−0.0005	0.099	0.078
Welfare equality	0.170	0.0094	0.151	0.319

Note: This table shows social welfare and its decomposition for the temporary and permanent increases in deficits studied in Section 6. Welfare and its components are presented in percentage change in labor-adjusted consumption equivalent units.

Table 2 shows the welfare gain and its decomposition for a temporary and a permanent increase

3. The proof is available upon request.

in the fiscal deficit. We compare the debt rollover policy with the conventional fiscal rule with $\phi_b = 0.04$. For a temporary increase in deficit, the debt rollover policy yields a larger welfare gain than the conventional fiscal rule. The temporary deficit shock raises the public debt level in the short run, which is beneficial as it increases the supply of the store of value. With the conventional fiscal rule, the debt level quickly falls back to the initial steady state level, so the benefit of more store of value is limited. On the other hand, the debt level remains high for a very long time under the debt rollover policy (recall Figure 4), so the benefit of more store of value is long-lasting. As a result, the debt rollover policy yields a larger overall welfare improvement.

The results are opposite when the increase in deficit is permanent. With a permanent deficit shock, the economy moves to a new steady state with higher public debt and steady-state social welfare. Now the conventional fiscal rule facilitates a faster transition and the economy can enjoy higher welfare in the new steady state earlier. The debt rollover policy, on the contrary, keeps public debt persistently below the new steady state level (recall Figure 5), and thus delays the benefit in the new steady state. As a result, the debt rollover policy yields a smaller overall welfare improvement when the increase in deficit is permanent.

7.2 Distributional Effects

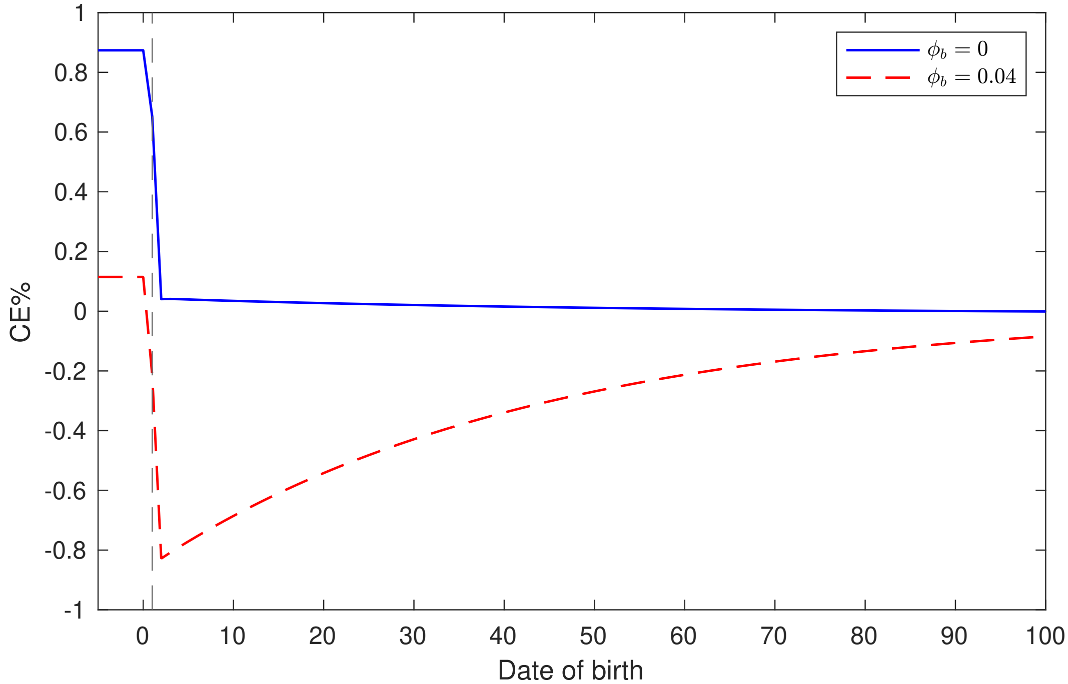
In terms of the distributional effect of deficit increases, we find that most of the welfare gain arises from the improved equality of welfare across cohorts as shown in Table 2. With $\nu = \gamma = 1$, the model is reduced to a representative household model without cross-sectional consumption dispersion. In the steady state, individuals maintain a flat lifetime consumption path with the real interest rate given by Γ/β. However, with $\nu < 1$ and $\gamma < 1$, individuals accept lower returns to save against the declining labor income path. The low-interest-rate environment reduces the benefit of investing in public bonds, leading to higher consumption inequality. This is because the saving costs accumulate as individuals age and hold more financial wealth so that old agents consume less than the young. Because increases in fiscal deficits raise the real interest rate as discussed in Section 6, they can reduce consumption inequality across cohorts by raising the benefit of saving in bonds.

To further investigate the distributional welfare consequence of increases in deficit, we calculate the expected (detrended) individual welfare for any cohort $s \leq t$ at time t:

$$Wel_{t|s} = \mathbb{E}_t \sum_{j=t}^{\infty} (\beta\gamma)^{j-t} \ln(c_{j|s} - V(L_{j|s})).$$

Similar to the welfare criterion in Hur (2018) and Heathcote et al. (2020), if $s < t$, it is the expected remaining lifetime utility of an incumbent agent; if $s = t$, we calculate the expected lifetime utility of a newborn agent.

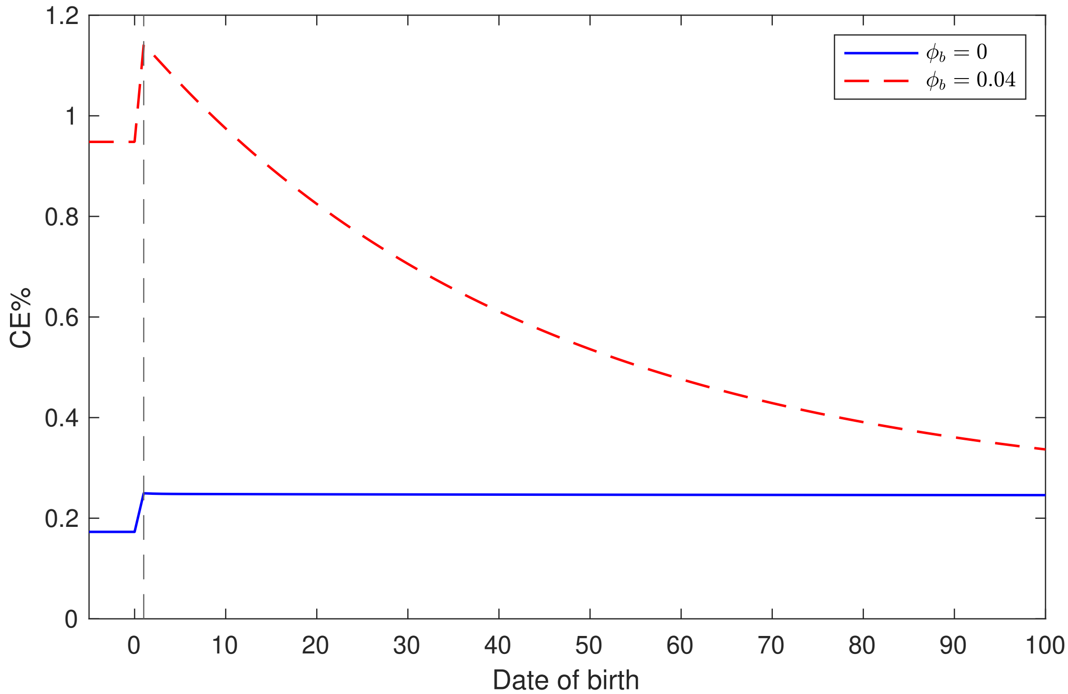
Figure 8: Welfare gain by cohort under a temporary increase in deficit



Note: This figure plots welfare gain for different cohorts under a temporary increase in deficit in period 1. Welfare gain is measured in percentage of labor-adjusted consumption equivalent units. For incumbent cohorts who are alive in period 1, we calculate their expected remaining lifetime utility in period 1. For future cohorts who are born in period $s \geq 2$, we calculate their expected lifetime utility in period s.

Figure 8 displays the welfare gain for different cohorts when there is a temporary increase in deficit in period 1. We compare the debt rollover policy with the tax rule with $\phi_b = 0.04$. We find several results. First, the debt rollover policy yields larger welfare gains for all cohorts, which is consistent with the larger social welfare gain in Table 2. Second, all incumbents (cohorts born before the shock in period 1, $s \leq 1$) have the same amount of welfare gain. This is due to the assumption of the complete set of state-contingent assets so that all incumbents have the same exposure to aggregate shocks. Third, the incumbents have higher welfare gains than the upcoming new generations. This is because the new generations born later than period 1 will not receive the one-time fiscal transfer that generates the increase in deficit in period 1. Lastly, future cohorts get welfare losses under the conventional tax rule with the cohort born in period 2 suffering the most. As a comparison, all future cohorts have similar welfare gains under the debt rollover policy. This is because the government raises taxes in response to the deficit shock under the conventional tax rule and higher taxes reduce welfare. Due to the assumption of the tax rule (17), cohorts born in the near future face larger increases in taxes and hence have lower welfare. On the contrary, the government does not need to raise any tax under the debt-rollover policy, causing no welfare costs to all future cohorts.

Figure 9: Welfare gain by cohort under a permanent increase in deficit



Note: This figure plots welfare gain for different cohorts under a permanent increase in deficit in period 1. Welfare gain is measured in percentage of labor-adjusted consumption equivalent units. For incumbent cohorts who are alive in period 1, we calculate their expected remaining lifetime utility in period 1. For future cohorts who are born in period $s \geq 2$, we calculate their expected lifetime utility in period s.

Figure 9 displays the welfare gain for different cohorts when there is a permanent increase in deficit from period 1 on. First, the permanent increase in deficit increases welfare for all cohorts under both the debt rollover policy and the conventional tax rule. This means that the transition to the new steady state with a higher permanent deficit is Pareto improving. Second, the debt rollover policy yields smaller welfare gains for all cohorts, which is consistent with the result for social welfare. Lastly, cohorts born in the near future have larger welfare gains under the conventional tax rule. This is because the government reduces taxes under this rule to build up public debt in response to the permanent increase in deficit. The cohorts born in the near future enjoy larger tax cuts. On the contrary, the debt rollover policy requires no tax cut and hence generates small but even benefits for all future cohorts. With financial wealth devalued by higher real interest rates, incumbents benefit slightly less from a permanent deficit shock compared to newly born individuals, who primarily hold human wealth.

In summary, we find that the debt rollover policy is better than the conventional fiscal rule in terms of social welfare if the increase in deficit is temporary, but is worse if the increase in deficit is permanent. There is a tradeoff between temporary vs. permanent deficit increases.

7.3 Optimal Policy Mix

Finally, we explore the optimal fiscal-monetary policy mix that maximizes social welfare under temporary and permanent increases in deficit. More specifically, we consider different combinations of fiscal-monetary policy parameters (ϕ_b, ϕ_π) and evaluate the corresponding welfare gain upon the same temporary and permanent fiscal deficit shock as in Section 6.1 and 6.2. Recall that ϕ_b controls the responsiveness of tax to debt level and the convergence speed of public debt, while ϕ_π controls the responsiveness of nominal interest rate to inflation fluctuations. Similar to Schmitt-Grohé and Uribe (2007), we consider a range for monetary policy coefficient of $\phi_\pi \in [1.2, 4]$. For the fiscal policy coefficient ϕ_b, we consider a range of $\phi_b \in [0, 0.1]$, which includes the range $(0.02, 0.08)$ of the empirical estimates as surveyed by Auclert, Rognlie, and Straub (2020).

Table 3: Social welfare improvement of temporary and permanent increase in deficit

	Temporary			Permanent		
	$\phi_\pi = 1.2$	$\phi_\pi = 1.5$	$\phi_\pi = 4$	$\phi_\pi = 1.2$	$\phi_\pi = 1.5$	$\phi_\pi = 4$
$\phi_b = 0$	0.149	0.150	0.150	0.249	0.250	0.250
$\phi_b = 0.01$	0.029	0.029	0.029	0.376	0.377	0.377
$\phi_b = 0.02$	0.016	0.016	0.016	0.389	0.389	0.389
$\phi_b = 0.04$	0.009	0.009	0.009	0.396	0.397	0.397
$\phi_b = 0.1$	0.004	0.004	0.004	0.402	0.402	0.402

Note: This table shows social welfare improvements of a temporary and a permanent increase in deficit under various monetary and fiscal policies. The size of the temporary and permanent increase in deficit is the same in Section 6.1 and 6.2. Welfare is presented in labor-adjusted consumption equivalent units.

Table 3 shows the social welfare improvements under various monetary and fiscal policies. We find that upon a temporary increase in deficit, the debt rollover policy ($\phi_b = 0$) is better than the tax rule ($\phi_b > 0$). The reason again is that higher public debt is beneficial and the debt rollover policy keeps the public debt high persistently. If the increase in the deficit is permanent, then debt rollover ($\phi_b = 0$) is worse than the tax rule ($\phi_b > 0$). In this case, the debt rollover policy keeps the public debt persistently lower than the new steady state level. Monetary policy does not affect welfare much. Still, it is better if the monetary policy responds to inflation (and disinflation) more aggressively.

8 Conclusion

In this paper, we study the impact of temporary and permanent increases in fiscal deficits financed by the debt rollover policy on the real activities, inflation dynamics, and welfare in a low interest rate environment. We find that the debt rollover policy is feasible under the monetary regime, in the sense that it can deliver a determinate equilibrium. This policy leads to slow-moving debt, thereby amplifying the wealth and crowding out effects, compared with conventional fiscal policies. As a result, the debt rollover policy can generate persistent inflation or disinflation during the transition period. This policy is better than the conventional fiscal rule in terms of social welfare if the increase in deficit is temporary, but is worse if the increase is permanent. The welfare gains mainly come from the reduction of the consumption dispersion across cohorts.

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Appendix

A Individual Consumption Function

In this appendix, we derive the individual consumption rule. Using (2), we can derive that

$$\begin{aligned}\gamma \mathbb{E}_t \Lambda_{t,t+1} A_{t+1|s} &= \mathbb{E}_t[\Lambda_{t,t+1} R_t^n B_{t|s}^n / P_{t+1} + \Lambda_{t,t+1}((1-\delta)Q_{t+1} + R_{t+1}^k) K_{t|s} + \Lambda_{t,t+1} Z_{t+1|s}] \\ &= B_{t|s}^n / P_t + Q_t K_{t|s} + \mathbb{E}_t \Lambda_{t,t+1} Z_{t+1|s},\end{aligned}$$

where the second equality follows from the asset pricing conditions for government bonds (3) and capital (4). Hence, we can rewrite the agent's budget constraint as

$$C_{t|s} + \gamma \mathbb{E}_t \left\{ \Lambda_{t,t+1} A_{t+1|s} \right\} = A_{t|s} + W_t L_{t|s} + \Phi_{t|s} - T_{t|s}.$$

Iterating the above equation forward and using the transversality condition, we have

$$\sum_{j=0}^{\infty} \gamma^j \mathbb{E}_t \Lambda_{t,t+j} C_{t+j|s} = A_{t|s} + \sum_{j=0}^{\infty} \gamma^j \mathbb{E}_t \Lambda_{t,t+j} \left[W_{t+j} L_{t+j|s} + \Phi_{t+j|s} - T_{t+j|s} \right]. \tag{A.1}$$

Reorganizing (A.1) yields

$$\begin{aligned}&\sum_{j=0}^{\infty} \gamma^j \mathbb{E}_t \Lambda_{t,t+j} \left(C_{t+j|s} - \Gamma^{t+j} V(L_{t+j|s}) \right) \\ =& A_{t|s} + \sum_{j=0}^{\infty} \gamma^j \mathbb{E}_t \Lambda_{t,t+j} \left[W_{t+j} L_{t+j|s} + \Phi_{t+j|s} - T_{t+j|s} - \Gamma^{t+j} V(L_{t+j|s}) \right].\end{aligned} \tag{A.2}$$

Note that the SDF is given by

$$\Lambda_{t,t+j} = \beta^j \frac{C_{t|s} - \Gamma^t V(L_{t|s})}{C_{t+j|s} - \Gamma^{t+j} V(L_{t+j|s})}.$$

Substituting the above expression into (A.2) yields

$$C_{t|s} - \Gamma^t V(L_{t|s}) = (1 - \beta\gamma) \left(A_{t|s} + H_{t|s} \right),$$

where

$$H_{t|s} = \sum_{j=0}^{\infty} \gamma^j \mathbb{E}_t \Lambda_{t,t+j} \left[W_{t+j} L_{t+j|s} + \Phi_{t+j|s} - T_{t+j|s} - \Gamma^{t+j} V(L_{t+j|s}) \right].$$

B Aggregation Consumption Function

In this appendix, we derive the aggregate consumption rule. We start with the individual consumption function:

$$C_{t|s} - \Gamma^t V(L_{t|s}) = (1-\beta\gamma)\left(A_{t|s} + H_{t|s}\right),$$

where

$$H_{t|s} = \sum_{j=0}^{\infty} \gamma^j \mathbb{E}_t \Lambda_{t,t+j}\left[W_{t+j}L_{t+j|s} + \Phi_{t+j|s} - T_{t+j|s} - \Gamma^{t+j}V(L_{t+j|s})\right].$$

We calculate the aggregate consumption function by adding up the individual consumption function for every cohort $s \leq t$.

Aggregating the left-hand-side of the individual consumption function yields

$$C_t - \mu\Gamma^t V(L_t^a),$$

where

$$C_t = \sum_{s=-\infty}^{t} (1-\gamma)\gamma^{t-s} C_{t|s}$$

is aggregate consumption and L_t^a is the labor supplied by an active agent in period t.

We next calculate the aggregate financial assets in period t. Assume that new born agents do not inherit any bonds and capital, i.e., $B_{t-1|t}^n = K_{t-|t} = 0$. The aggregate financial assets of all *alive* agents at the beginning of period t are

$$\begin{aligned}
&\sum_{s=-\infty}^{t} (1-\gamma)\gamma^{t-s} A_{t|s} \\
&= \sum_{s=-\infty}^{t} (1-\gamma)\gamma^{t-s}\frac{1}{\gamma}\left[Z_{t|s} + R_{t-1}^n B_{t-1|s}^n/P_t + ((1-\delta)Q_t + R_t^k)K_{t-1|s}\right] \\
&= \frac{1}{\gamma}Z_t + \sum_{s=-\infty}^{t-1} (1-\gamma)\gamma^{t-1-s}\left[R_{t-1}^n B_{t-1|s}^n/P_t + ((1-\delta)Q_t + R_t^k)K_{t-1|s}\right] \\
&= R_{t-1}^n B_{t-1}^n/P_t + ((1-\delta)Q_t + R_t^k)K_{t-1},
\end{aligned}$$

where we have used the assumption of zero net aggregate supply of state-contingent securities and the market clearing condition for government bonds $\sum_{s=-\infty}^{t-1}(1-\gamma)\gamma^{t-1-s}B_{t-1|s}^n = B_{t-1}^n$ and capital $\sum_{s=-\infty}^{t-1}(1-\gamma)\gamma^{t-1-s}K_{t-1|s} = K_{t-1}$.

We then calculate the real present value of aggregate human capital

$$H_t \equiv \sum_{s=-\infty}^{t} (1-\gamma)\gamma^{t-s} H_{t|s}.$$

We divide the aggregate human capital into two parts: the aggregate human capital for all agents that are active in period t, H_t^a, and the aggregate human capital for all agents that are inactive in period t, H_t^r. Then the aggregate human capital is just the sum of the two parts: $H_t = H_t^a + H_t^r$.

For an agent who is active in period t, regardless of his cohort, he has probability $(\gamma\nu)^j$ to remain active in period $t+j$, $j \geq 0$, so that $L_{t+j|s} = L_{t+j}^a, \Phi_{t+j|s} = \Phi_{t+j}^a, T_{t+j|s} = T_{t+j}^a - S_{t+j}$, and $\gamma^j - (\gamma\nu)^j$ probability to retire in period $t+j$ so that $N_{t+j|s} = 0, \Phi_{t+j|s} = 0, T_{t+j|s} = -S_{t+j}$. Here, Φ_{t+j}^a is the dividend received by an active agent in period $t+j$. Since the total mess of active agents is μ, the aggregate real present value of human capital for all active agents is

$$
\begin{aligned}
H_t^a = {} & \mu \mathbb{E}_t \sum_{j=0}^{\infty} (\gamma\nu)^j \Lambda_{t,t+j} \left(W_{t+j} L_{t+j}^a + \Phi_{t+j}^a - T_{t+j}^a + S_{t+j} - \Gamma^{t+j} V(L_{t+j}^a) \right) \\
& + \mu \mathbb{E}_t \sum_{j=0}^{\infty} \left(\gamma^j - (\gamma\nu)^j \right) \Lambda_{t,t+j} S_{t+j}.
\end{aligned}
$$

The real profits from all intermediate-good firms in period t is

$$
\frac{1}{P_t} \int_0^{\mu} \Big(Y_t(i) P_t(i) - W_t P_t L_t(i) - R_t^k P_t K_t(i) \Big) di.
$$

Using the fact that $\int_0^{\mu} Y_t(i) P_t(i) di = Y_t P_t$, the labor market clearing condition $\int_0^{\mu} L_t(i) di = L_t$, and $\int_0^{\mu} K_t(i) di = K_{t-1}$, the real profits from all intermediate-good firms can be written as

$$
Y_t - W_t L_t - R_t^k K_{t-1}.
$$

The dividends and profits from intermediate-good firms and capital producers received by an active agent are

$$
\Phi_t^a = \mu^{-1} [Y_t - W_t L_t - R_t^k K_{t-1} + \Phi_t^k],
$$

where Φ_t^k is the profits from the capital producers. Substituting the expression for Φ_t^a, we obtain

$$
\begin{aligned}
H_t^a = {} & \mathbb{E}_t \sum_{j=0}^{\infty} (\gamma\nu)^j \Lambda_{t,t+j} \Big(Y_{t+j} - R_{t+j}^k K_{t+j-1} + \Phi_{t+j}^k - \mu T_{t+j}^a + \mu S_{t+j} - \Gamma^{t+j} \mu V(L_{t+j}^a) \Big) \\
& + \mathbb{E}_t \sum_{j=0}^{\infty} \left(\gamma^j - (\gamma\nu)^j \right) \Lambda_{t,t+j} \mu S_{t+j}.
\end{aligned}
$$

Using $T_t = \mu T_t^a - S_{t+j}$, we have

$$
\begin{aligned}
H_t^a &= \mathbb{E}_t \sum_{j=0}^{\infty} (\gamma\nu)^j \Lambda_{t,t+j} \Big(Y_{t+j} - R_{t+j}^k K_{t+j-1} + \Phi_{t+j}^k - T_{t+j} - S_{t+j} - \Gamma^{t+j} \mu V(L_{t+j}^a) \Big) \\
&+ \mathbb{E}_t \sum_{j=0}^{\infty} \gamma^j \Lambda_{t,t+j} \mu S_{t+j}.
\end{aligned}
$$

For an agent who is inactive in period t, regardless of his cohort, he has probability $(\gamma)^j$ to remain inactive in period $t+j$ so that $N_{t+j|s} = 0, \Phi_{t+j|s} = 0, T_{t+j|s} = -S_{t+j}$. Since the total mess of inactive agents is $1-\mu$, the aggregate present value of human capital for all inactive agents is

$$
H_t^r = \mathbb{E}_t \sum_{j=0}^{\infty} \gamma^j \Lambda_{t,t+j} (1-\mu) S_{t+j}.
$$

Using all the results above, we obtain the aggregate consumption function

$$
C_t - \mu \Gamma^t V(L_t^a) = (1-\beta\gamma)(R_{t-1}^n B_{t-1} \Pi_t^{-1} + ((1-\delta)Q_t + R_t^k) K_{t-1} + H_t),
$$

where

$$
\begin{aligned}
H_t &= H_t^a + H_t^r, \\
H_t^a &= \mathbb{E}_t \sum_{j=0}^{\infty} (\gamma\nu)^j \Lambda_{t,t+j} \Big(Y_{t+j} - R_{t+j}^k K_{t+j-1} + \Phi_{t+j}^k - T_{t+j} - S_{t+j} - \Gamma^{t+j} \mu V(L_{t+j}^a) \Big) \\
&+ \mathbb{E}_t \sum_{j=0}^{\infty} \gamma^j \Lambda_{t,t+j} \mu S_{t+j}, \\
H_t^r &= \mathbb{E}_t \sum_{j=0}^{\infty} \gamma^j \Lambda_{t,t+j} (1-\mu) S_{t+j}.
\end{aligned}
$$

To facilitate numerical computations, we can write the above equations in a recursive form

$$
\begin{aligned}
H_t &= H_t^a + H_t^r, \\
H_t^a &= H_t^x + H_t^y, \\
H_t^x &= Y_t - R_t^k K_{t-1} + \Phi_t^k - T_t - S_t - \mu \Gamma^t V(L_t^a) + \gamma\nu \mathbb{E}_t \Lambda_{t,t+1} H_{t+1}^x, \\
H_t^y &= \mu S_t + \gamma \mathbb{E}_t \Lambda_{t,t+1} H_{t+1}^y, \\
H_t^r &= (1-\mu) S_t + \gamma \mathbb{E}_t \Lambda_{t,t+1} H_{t+1}^r.
\end{aligned}
$$

C Aggregate SDF and Euler Equation

In this appendix, we first derive the expression for aggregate stochastic discount factor $\Lambda_{t,t+1}$. We start with the aggregate consumption. By definition, the aggregate consumption in period $t+1$ is the sum of consumption of all cohorts:

$$C_{t+1} - \mu\Gamma^{t+1}V(L^a_{t+1}) = \sum_{s=-\infty}^{t+1} (1-\gamma)\gamma^{t+1-s}\left(C_{t+1|s} - \Gamma^{t+1}V(L_{t+1|s})\right).$$

With complete markets, all cohorts have the same SDF:

$$\Lambda_{t,t+1} = \beta \frac{C_{t|s} - \Gamma^t V(L_{t|s})}{C_{t+1|s} - \Gamma^{t+1}V(L_{t+1|s})}, \quad s \leq t.$$

Substituting the above expression for $\Lambda_{t,t+1}$ into the aggregate consumption equation, we have

$$\begin{aligned} C_{t+1} - \mu\Gamma^{t+1}V(L^a_{t+1}) &= (1-\gamma)\left[C_{t+1|t+1} - \Gamma^{t+1}V(L_{t+1|t+1})\right] \\ &\quad + \beta\Lambda^{-1}_{t,t+1} \sum_{s=-\infty}^{t} (1-\gamma)\gamma^{t+1-s}\left(C_{t|s} - \Gamma^t V(L_{t|s})\right) \\ &= (1-\gamma)\left[C_{t+1|t+1} - \Gamma^{t+1}V(L_{t+1|t+1})\right] + \beta\gamma\Lambda^{-1}_{t,t+1}\left(C_t - \mu\Gamma^t V(L^a_t)\right), \end{aligned}$$

where the second equality follows from the aggregation of consumption in period t. Rearranging yields

$$\Lambda_{t,t+1} = \beta \frac{C_t - \mu\Gamma^t V(L^a_t)}{C_{t+1} - \mu\Gamma^{t+1}V(L^a_{t+1})} \cdot \frac{\gamma}{1-(1-\gamma)\chi_{t+1}}, \tag{C.1}$$

where χ_t is the ratio of consumption of newly born agents relative to the aggregate consumption:

$$\chi_t \equiv \frac{C_{t|t} - \Gamma^t V(L^a_t)}{C_t - \mu\Gamma^t V(L^a_t)}.$$

Since newly born agents do not hold any financial assets, their aggregate consumption follows

$$C_{t|t} - \Gamma^t V(L^a_t) = \mu^{-1}(1-\beta\gamma)H^a_t.$$

Hence, we can rewrite χ_t as

$$\chi_t = \frac{\mu^{-1}H^a_t}{R^n_{t-1}B_{t-1}\Pi^{-1}_t + ((1-\delta)Q_t + R^k_t)K_{t-1} + H_t}.$$

We now derive an aggregate consumption Euler equation that is similar to the one in a representative household model. For simplicity, we focus on periods $t \geq 2$ after the initial lump-sum deficit shock and assume that $S_t = 0$ for $t \geq 2$.

With $S_t = 0$, aggregate human wealth can be simplified as

$$H_t = Y_t - R_t^k K_{t-1} + \Phi_t^k - T_t - \mu\Gamma^t V(L_t^a) + \gamma\nu\mathbb{E}_t\Lambda_{t,t+1}H_{t+1}. \tag{C.2}$$

We recall from Appendix B that the aggregate consumption rule is given by

$$C_t - \mu\Gamma^t V(L_t^a) = (1-\beta\gamma)\left(\frac{R_{t-1}^n}{\Pi_t}B_t + \left[R_t^k + (1-\delta)Q_t\right]K_{t-1} + H_t\right).$$

Solving for H_t and H_{t+1}, substituting into (C.2), and then reorganizing yield

$$C_t - \mu\Gamma^t V(L_t^a) = \frac{\nu}{\beta}\mathbb{E}_t\Lambda_{t,t+1}\left[C_{t+1} - \mu\Gamma^{t+1}V(L_{t+1}^a)\right] + \frac{(1-\beta\gamma)(1-\nu\gamma)}{\beta\gamma}\left(B_t + Q_t K_t\right), \tag{C.3}$$

where we have used the government budget constraint to eliminate T_t and the aggregate resource constraint to eliminate Y_t. Combining the bond pricing condition with the log-linearized (de-trended) version of (C.3), we can obtain the log-linearized Euler condition as in (25).

D Equilibrium System

Given the exogenous processes of $\{z_{s,t}\}_{t\geq 0}$ and the initial values $\{I_{-1}, K_{-1}, R_{-1}^n, B_{-1}, \Delta_{-1}, C_{-1}\}$, the equilibrium system consists of sequences of 16 variables, i.e. $\{C_t$, I_t, Y_t, L_t^a, L_t, K_t, Q_t, R_t^k, B_t, R_t^n, Π_t, W_t, Δ_t, p_t^*, T_t, $\Lambda_{t-1,t}\}$ for $t \geq 0$ that satisfy 16 conditions as follows:

1. Resource constraint,

$$C_t + I_t + G_t^e = Y_t. \tag{D.1}$$

2. Aggregate SDF,

$$\Lambda_{t-1,t} = \beta\frac{C_{t-1} - \mu\Gamma^{t-1}V(L_{t-1}^a)}{C_t - \mu\Gamma^t V(L_t^a)} \cdot \frac{\gamma}{1-(1-\gamma)\chi_t}, \tag{D.2}$$

where

$$\chi_t = \frac{\mu^{-1} H_t^a}{R_{t-1}^n B_{t-1} \Pi_t^{-1} + ((1-\delta) Q_t + R_t^k) K_{t-1} + H_t},$$
$$H_t = H_t^a + H_t^r,$$
$$H_t^a = H_t^x + H_t^y,$$
$$H_t^x = Y_t - R_t^k K_{t-1} + \Phi_t^k - T_t - S_t - \mu \Gamma^t V(L_t^a) + \gamma \nu \mathbb{E}_t \Lambda_{t,t+1} H_{t+1}^x,$$
$$H_t^y = \mu S_t + \gamma \mathbb{E}_t \Lambda_{t,t+1} H_{t+1}^y,$$
$$H_t^r = (1-\mu) S_t + \gamma \mathbb{E}_t \Lambda_{t,t+1} H_{t+1}^r,$$
$$\Phi_t^k = Q_t \left[1 - \frac{\Omega_k}{2} \left(\frac{I_t}{I_{t-1}} - \Gamma \right)^2 \right] I_t - I_t.$$

3. Labor supply,

$$W_t = \Gamma^t V' \left(L_t^a \right). \tag{D.3}$$

4. Asset pricing equation for government bonds,

$$\mathbb{E}_t \Lambda_{t,t+1} R_t^n \Pi_{t+1}^{-1} = 1. \tag{D.4}$$

5. Asset pricing equation for capital,

$$\mathbb{E}_t \Lambda_{t,t+1} \frac{(1-\delta) Q_{t+1} + R_{t+1}^k}{Q_t} = 1. \tag{D.5}$$

6. Aggregate labor,

$$\mu L_t^a = L_t. \tag{D.6}$$

7. Aggregate output,

$$Y_t = \Delta_t^{-1} K_{t-1}^{\alpha} \left(\Gamma^t L_t \right)^{1-\alpha}. \tag{D.7}$$

8. Optimal pricing rule,

$$p_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{F_t^a}{F_t^b}, \tag{D.8}$$

where

$$F_t^a = Y_t M_t + \nu \gamma \theta \mathbb{E}_t \Lambda_{t,t+1} \Pi^{-\varepsilon} \Pi_{t+1}^{\varepsilon} F_{t+1}^a,$$
$$F_t^b = Y_t + \nu \gamma \theta \mathbb{E}_t \Lambda_{t,t+1} \Pi^{1-\varepsilon} \Pi_{t+1}^{\varepsilon - 1} F_{t+1}^b,$$
$$M_t = \left(\frac{R_t^k}{\alpha} \right)^{\alpha} \left(\frac{\Gamma^{-t} W_t}{1-\alpha} \right)^{1-\alpha}.$$

9. Inflation,

$$1 = \left[\theta \left(\frac{\Pi}{\Pi_t} \right)^{1-\varepsilon} + (1-\theta) p_t^{*1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \tag{D.9}$$

10. Price dispersion,

$$\Delta_t = (1-\theta) p_t^{*-\varepsilon} + \theta \left(\frac{\Pi}{\Pi_t} \right)^{-\varepsilon} \Delta_{t-1}. \tag{D.10}$$

11. Monetary policy,

$$\ln \left(\frac{R_t^n}{R^n} \right) = \rho_R \ln \left(\frac{R_{t-1}^n}{R^n} \right) + (1-\rho_R) \phi_\pi \ln \left(\frac{\Pi_t}{\Pi} \right). \tag{D.11}$$

12. Fiscal policy,

$$\tau_t^a - \tau^a = \phi_b (b_{t-1} - b), \tag{D.12}$$

where

$$\begin{aligned} \tau_t^a &= T_t^a / \Gamma^t, \\ b_t &= B_t / \Gamma^t, \\ T_t &= \mu T_t^a - S_t, \\ S_t / \Gamma^t &= s + z_{s,t}. \end{aligned}$$

13. Government budget constraint,

$$\Pi_t^{-1} R_{t-1}^n B_{t-1} = T_t - G_t^e + B_t. \tag{D.13}$$

14. Optimality condition for capital producers,

$$\begin{aligned} 1 = Q_t &\left[1 - \frac{\Omega_k}{2} \left(\frac{I_t}{I_{t-1}} - \Gamma \right)^2 \right] - Q_t \Omega_k \left(\frac{I_t}{I_{t-1}} - \Gamma \right) \frac{I_t}{I_{t-1}} \\ &+ \nu \gamma \mathbb{E}_t \Lambda_{t,t+1} Q_{t+1} \Omega_k \left(\frac{I_{t+1}}{I_t} - \Gamma \right) \left(\frac{I_{t+1}}{I_t} \right)^2. \end{aligned} \tag{D.14}$$

15. Optimality condition for firms,

$$\frac{K_{t-1}}{L_t} = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t^k}. \tag{D.15}$$

16. Capital evolution,

$$K_t = (1-\delta) K_{t-1} + \left[1 - \frac{\Omega_k}{2} \left(\frac{I_t}{I_{t-1}} - \Gamma \right)^2 \right] I_t. \tag{D.16}$$

E Detrended System

For any variable X_t that grows with Γ^t, we denote its detrended variable as $x_t \equiv X_t/\Gamma^t$. Then given the exogenous processes of $\{z_{s,t}\}_{t\geq 0}$ and the initial values $\{i_{-1}, k_{-1}, R^n_{-1}, b_{-1}, \Delta_{-1}, c_{-1}\}$, the detrended equilibrium system consists of sequences of 16 variables, i.e. $\{c_t,\ i_t,\ y_t,\ L^a_t,\ L_t,\ Q_t,\ k_t,\ R^k_t,\ b_t,\ R^n_t,\ \Pi_t,\ w_t,\ \Delta_t,\ p^*_t,\ \tau_t,\ \Lambda_{t-1,t}\}$ for $t \geq 0$ that satisfy 16 conditions as follows:

1. Resource constraint,

$$c_t + i_t + g^e = y_t. \tag{E.1}$$

2. Aggregate SDF,

$$\Lambda_{t-1,t} = \beta\Gamma^{-1}\frac{c_{t-1} - \mu V(L^a_{t-1})}{c_t - \mu V(L^a_t)} \cdot \frac{\gamma}{1-(1-\gamma)\chi_t}, \tag{E.2}$$

where

$$\begin{aligned}
\chi_t &= \frac{\mu^{-1}h^a_t}{R^n_{t-1}b_{t-1}\Pi_t^{-1}\Gamma^{-1} + ((1-\delta)Q_t + R^k_t)k_{t-1}\Gamma^{-1} + h_t},\\
h_t &= h^a_t + h^r_t,\\
h^a_t &= h^x_t + h^y_t,\\
h^x_t &= y_t - R^k_t k_{t-1}\Gamma^{-1} + \phi^k_t - \tau_t - s_t - \mu V(L^a_t) + \gamma\nu\mathbb{E}_t\Lambda_{t,t+1}\Gamma h^x_{t+1},\\
h^y_t &= \mu s_t + \gamma\mathbb{E}_t\Lambda_{t,t+1}\Gamma h^y_{t+1},\\
h^r_t &= (1-\mu)s_t + \gamma\mathbb{E}_t\Lambda_{t,t+1}\Gamma h^r_{t+1},\\
\phi^k_t &= Q_t\left[1 - \frac{\Omega_k}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2\Gamma^2\right]i_t - i_t.
\end{aligned}$$

3. Labor supply,

$$w_t = V'\left(L^a_t\right). \tag{E.3}$$

4. Asset pricing equation for government bonds

$$\mathbb{E}_t\Lambda_{t,t+1}R^n_t\Pi^{-1}_{t+1} = 1. \tag{E.4}$$

5. Asset pricing equation for capital,

$$\mathbb{E}_t\Lambda_{t,t+1}\frac{(1-\delta)Q_{t+1} + R^k_{t+1}}{Q_t} = 1. \tag{E.5}$$

6. Aggregate labor,

$$\mu L^a_t = L_t. \tag{E.6}$$

7. Aggregate output,

$$y_t = \Delta_t^{-1} \left(k_{t-1}\Gamma^{-1}\right)^{\alpha} L_t^{1-\alpha}. \tag{E.7}$$

8. Optimal pricing rule,

$$p_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{f_t^a}{f_t^b}, \tag{E.8}$$

where

$$\begin{aligned} f_t^a =& y_t M_t + \nu\gamma\theta \mathbb{E}_t \Lambda_{t,t+1} \Pi^{-\varepsilon} \Pi_{t+1}^{\varepsilon} \Gamma f_{t+1}^a, \\ f_t^b =& y_t + \nu\gamma\theta \mathbb{E}_t \Lambda_{t,t+1} \Pi^{1-\varepsilon} \Pi_{t+1}^{\varepsilon-1} \Gamma f_{t+1}^b, \\ M_t =& \left(\frac{R_t^k}{\alpha}\right)^{\alpha} \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}. \end{aligned} \tag{E.9}$$

9. Inflation,

$$1 = \left[\theta \left(\frac{\Pi}{\Pi_t}\right)^{1-\varepsilon} + (1-\theta) p_t^{*1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{E.10}$$

10. Price dispersion,

$$\Delta_t = (1-\theta) p_t^{*-\varepsilon} + \theta \left(\frac{\Pi}{\Pi_t}\right)^{-\varepsilon} \Delta_{t-1}. \tag{E.11}$$

11. Monetary policy,

$$\ln\left(\frac{R_t^n}{R^n}\right) = \rho_R \ln\left(\frac{R_{t-1}^n}{R^n}\right) + (1-\rho_R)\phi_\pi \ln\left(\frac{\Pi_t}{\Pi}\right). \tag{E.12}$$

12. Fiscal policy,

$$\tau_t^a - \tau^a = \phi_b (b_{t-1} - b), \tag{E.13}$$

and

$$\begin{aligned} \tau_t &= \mu \tau_t^a - s_t, \\ s_t &= s + z_{s,t}. \end{aligned}$$

13. Government budget constraint,

$$\Pi_t^{-1} R_{t-1}^n b_{t-1} \Gamma^{-1} - b_t = \tau_t - g^e. \tag{E.14}$$

14. Optimality condition for capital producers,

$$1 = Q_t \left[1 - \frac{\Omega_k}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 \Gamma^2 \right] - Q_t \Omega_k \left(\frac{i_t}{i_{t-1}} - 1 \right) \frac{i_t}{i_{t-1}} \Gamma^2 + \nu \gamma \mathbb{E}_t \Lambda_{t,t+1} Q_{t+1} \Omega_k \left(\frac{i_{t+1}}{i_t} - 1 \right) \left(\frac{i_{t+1}}{i_t} \right)^2 \Gamma^3. \tag{E.15}$$

15. Optimality condition for firms,

$$\frac{k_{t-1}}{L_t \Gamma} = \frac{\alpha}{1-\alpha} \frac{w_t}{R_t^k}. \tag{E.16}$$

16. Capital evolution,

$$k_t = (1-\delta) k_{t-1} \Gamma^{-1} + \left[1 - \frac{\Omega_k}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 \Gamma^2 \right] i_t. \tag{E.17}$$

F Steady States

In this section, we derive the steady states of the detrended model. We use variables without time subscripts to denote their steady-state values.

We first calculate variables whose values are constant in any steady state. From (E.10) and (E.11), we have $p^* = \Delta = 1$. Using (E.8), we derive that $M = (\varepsilon - 1)/\varepsilon$. From the capital producer's optimization condition (E.15), we have $Q = 1$.

We define the real interest rate as $R_t = R_{t-1}^n \Pi_t^{-1}$. Using the consumer's optimization condition for government bonds, we have $R = 1/\Lambda$. Using the asset pricing equation for capital (E.5), we have $R^k = R - 1 + \delta$.

We then compute some expressions to facilitate the steady state calculation. We show that the rest variables can be written as functions of R^k, and hence are also functions of the real interest rate R.

First, Using the production function (E.7) and the firm's optimization condition (E.16) we derive that

$$\frac{y}{k} = \left(\frac{w}{R^k} \right)^{\alpha-1} \left(\frac{\alpha}{1-\alpha} \right)^{\alpha-1} \Gamma^{-1}.$$

Using the expression for the marginal cost (E.9), we can express y/k as a function of R:

$$\frac{y}{k} = \frac{1}{\alpha \Gamma M} (R - 1 + \delta). \tag{F.1}$$

Next, using the functional form of $V(\cdot)$, labor supply (E.3), the firm's optimization condition

(E.16) and the labor market clearing condition (E.6), we can express $\mu V(L^a)/k$ as a function of R:

$$\frac{\mu V(L^a)}{k} = \frac{1-\alpha}{(1+\varphi)\alpha} R^k \Gamma^{-1} \frac{\eta (L^a)^\varphi}{w}$$
$$= \frac{1-\alpha}{(1+\varphi)\alpha\Gamma}(R-1+\delta), \tag{F.2}$$

where the second equality follows from the optimal condition for labor supply (E.3).

Finally, using the production function and the expressions (F.1) and (F.2) we can write the steady state capital in terms of R

$$k = \mu\Gamma(1-\alpha)^{\frac{1}{\varphi}} \eta^{-\frac{1}{\varphi}} \alpha^{\frac{\alpha+\varphi}{(1-\alpha)\varphi}} M^{\frac{1+\varphi}{(1-\alpha)\varphi}} (R-1+\delta)^{\frac{-\alpha-\varphi}{(1-\alpha)\varphi}}. \tag{F.3}$$

We are now ready to solve the steady state by deriving a bond demand equation and a bond supply equation. Using the expression for human wealth, we derive that

$$h = \frac{1}{1-\gamma\nu R^{-1}\Gamma}\left(y - R^k k\Gamma^{-1} - \mu V(L^a) - \tau - s\right) + \frac{s}{1-\gamma R^{-1}\Gamma}.$$

Using the resource constraint and the law of motion for capital, we can rewrite the aggregate consumption rule as

$$y - g^e - k + (1-\delta)k\Gamma^{-1} - \mu V(L^a) = (1-\beta\gamma)[R\Gamma^{-1}(b+k) + h].$$

Substituting the expression for h, we derive that

$$\begin{aligned} b = &\frac{\beta\gamma - \gamma\nu R^{-1}\Gamma}{(1-\beta\gamma)R\Gamma^{-1}(1-\gamma\nu R^{-1}\Gamma)}(y - \mu V(L^a)) + \frac{(1-\beta\gamma)\gamma\nu - 1 + \gamma\nu R^{-1}\Gamma}{(1-\beta\gamma)R\Gamma^{-1}(1-\gamma\nu R^{-1}\Gamma)}k \\ &+ \frac{\beta\gamma - \gamma\nu R^{-1}\Gamma}{(1-\beta\gamma)R\Gamma^{-1}(1-\gamma\nu R^{-1}\Gamma)}(1-\delta)\Gamma^{-1}k + \frac{\gamma R^{-2}\Gamma^2(\nu-1)}{(1-\gamma\nu R^{-1}\Gamma)(1-\gamma R^{-1}\Gamma)}s \\ &+ \frac{R^{-1}\Gamma}{1-\gamma\nu R^{-1}\Gamma}\tau - \frac{R^{-1}\Gamma}{1-\beta\gamma}g^e. \end{aligned}$$

Using the government budget constraint to replace τ, using the expressions (F.1) and (F.2) to replace y and $\mu V(L^a)$, we can rewrite the above equation as

$$\begin{aligned} \frac{b}{y} = &\frac{\beta\gamma - \gamma\nu R^{-1}\Gamma}{(1-\beta\gamma)(1-\gamma\nu)}\left[1 - \frac{g^e}{y} - \frac{(1-\alpha)M}{1+\varphi} - (1-(1-\delta)\Gamma^{-1})\frac{\alpha\Gamma M}{R-1+\delta}\right] \\ &- \frac{\alpha\Gamma M}{R-1+\delta} - \frac{\gamma\Gamma(1-\nu)}{(R-\gamma\Gamma)(1-\gamma\nu)}\frac{s}{y} \equiv F(R). \end{aligned} \tag{F.4}$$

Given g^e/y and s/y, Equation (F.4) represents the government bond-to-output ratio as a function of the real interest rate. It can be interpreted as the bond demand equation since it is derived from the consumer's optimization condition.

On the other hand, we derive the bond supply equation from the government budget constraint (E.14):

$$\frac{b}{y} = \frac{1}{R\Gamma^{-1} - 1}\xi \tag{F.5}$$

where $\xi \equiv (\tau - g^e)/y$ is the steady state (negative) deficit-to-output ratio.

The steady state of the model is then determined by combining the bond demand and supply equation:

$$\begin{aligned}\frac{1}{R\Gamma^{-1} - 1}\xi = \frac{\beta\gamma - \gamma\nu R^{-1}\Gamma}{(1-\beta\gamma)(1-\gamma\nu)}\left[1 - \frac{g^e}{y} - \frac{(1-\alpha)M}{1+\varphi} - (1-(1-\delta)\Gamma^{-1})\frac{\alpha\Gamma M}{R-1+\delta}\right] \\ - \frac{\alpha\Gamma M}{R-1+\delta} - \frac{\gamma\Gamma(1-\nu)}{(R-\gamma\Gamma)(1-\gamma\nu)}\frac{s}{y}.\end{aligned} \tag{F.6}$$

Given the steady state deficit-to-output ratio ξ, Equation (F.6) determines the steady-state real interest rate R. The rest steady-state variables can be calculated accordingly. Specifically, given the steady-state real interest rate R, we have $R^k = R - 1 + \delta$,

$$\begin{aligned}k &= \mu\Gamma(1-\alpha)^{\frac{1}{\varphi}}\eta^{-\frac{1}{\varphi}}\alpha^{\frac{\alpha+\varphi}{(1-\alpha)\varphi}}M^{\frac{1+\varphi}{(1-\alpha)\varphi}}(R^k)^{\frac{-\alpha-\varphi}{(1-\alpha)\varphi}}, \\ w &= (1-\alpha)\left[\alpha^\alpha M\left(R^k\right)^{-\alpha}\right]^{\frac{1}{1-\alpha}}, \\ L^a &= \left[\frac{\frac{\mu V(L^a)}{k}k(1+\varphi)}{\mu\eta}\right]^{\frac{1}{1+\varphi}}, \\ y &= \frac{1}{\alpha\Gamma M}R^k k.\end{aligned}$$

For fiscal-policy-related variables, we need to make assumptions on how we reach different steady states indexed by R. We assume that the transfer is zero at the calibrated steady state. If the government increases deficit, it does so by increasing transfer and keeping taxes τ^a/y constant. If the government reduces the deficit, it does so by raising taxes and keeping transfers at zero. g^e/y keeps constant across steady states. Then, we can calculate τ^a/y and s/y from the following equation

$$\begin{aligned}\mu\frac{\tau^a}{y} - \frac{(R-\gamma\nu\Gamma)(1-\gamma)}{(R-\gamma\Gamma)(1-\gamma\nu)}\frac{s}{y} \quad &= \quad (R\Gamma^{-1}-1)A\left[((1-\delta)\Gamma^{-1}-1)B + C\right] \\ &\quad -(R\Gamma^{-1}-1)B + \frac{g^e}{y},\end{aligned} \tag{F.7}$$

where

$$A = \frac{\beta\gamma - \gamma\nu R^{-1}\Gamma}{(1-\beta\gamma)(1-\gamma\nu)},$$
$$B = \frac{\alpha\Gamma M}{R-1+\delta},$$
$$C = 1 - \frac{g^e}{y} - \frac{(1-\alpha)M}{1+\varphi}.$$

Equation (F.7) is derived by combining the bond demand and supply equation. Then $g^e = \frac{g^e}{y}y$, $s = \frac{s}{y}y$, $\tau^a = \frac{\tau^a}{y}y$, $\tau = \mu\tau^a - s$, $b = \frac{\tau - g^e}{R\Gamma^{-1}-1}$, and the steady-state human capital follows

$$h = \frac{1}{1-\gamma\nu R^{-1}\Gamma}\left(y - R^k k\Gamma^{-1} - \mu V(L^a) - \tau - s\right) + \frac{s}{1-\gamma R^{-1}\Gamma}.$$

Now we show the monotonicity of the bond demand function denoted by $b/y = F(R)$ in (F.4). Before that, we should notice some necessary conditions for the existence of equilibrium. First, we must have $R > 1-\delta$, otherwise the rental rate of capital $R^k = R - (1-\delta)$ is not positive. Second, we must have $R > \gamma\Gamma$. Otherwise, the individual transversality condition does not hold and the valuation of human wealth would be explosive. Third, the aggregate adjusted consumption to output ratio must be non-negative, i.e. $\tilde{c}/y \equiv (c - \mu V(L^a))/y > 0$, otherwise at least some individuals would have negative adjusted consumption in their utility function. Combining $\widetilde{c}/y > 0$, (E.1), (E.17), and (F.2), we obtain that

$$\frac{\widetilde{c}}{y} = 1 - \frac{g^e}{y} - \frac{(1-\alpha)M}{1+\varphi} - \left(1 - \frac{1-\delta}{\Gamma}\right)\frac{\alpha M\Gamma}{R-1+\delta}. \tag{F.8}$$

Therefore, the non-negative valuation of adjusted consumption $\tilde{c}/y \equiv (c - \mu V(L^a))/y > 0$ implies that

$$R > 1 - \delta + \left(1 - \frac{1-\delta}{\Gamma}\right)\frac{\alpha M\Gamma}{1 - \frac{g^e}{y} - \frac{(1-\alpha)M}{1+\varphi}} \equiv \underline{R}. \tag{F.9}$$

Therefore, for equilibrium to exist, it is necessary that $R > \max\{\gamma\Gamma, 1-\delta, \underline{R}\}$.

Assume that the lump-sum transfer $s/y \geq 0$ is fixed. Then the non-negative valuation of government bonds $F(R) \geq 0$ implies that

$$\begin{aligned}
&\frac{\beta\gamma}{(1-\beta\gamma)(1-\gamma\nu)}\left(1 - \frac{\nu}{\beta}\frac{\Gamma}{R}\right)\left[1 - \frac{g^e}{y} - \frac{(1-\alpha)M}{1+\varphi} - (1-(1-\delta)\Gamma^{-1})\frac{\alpha\Gamma M}{R-1+\delta}\right]\\
\geq&\frac{\alpha\Gamma M}{R-1+\delta} + \frac{\gamma\Gamma(1-\nu)}{(R-\gamma\Gamma)(1-\gamma\nu)}\frac{s}{y}\\
>&0,
\end{aligned}$$

where the second inequality follows from $R > 1-\delta$ and $R > \gamma\Gamma$. Combining with $R > \underline{R}$, we obtain from $F(R) \geq 0$ that $R \geq \nu\Gamma/\beta$. Given that $R > \max\{\gamma\Gamma, 1-\delta, \underline{R}\}$ and that $R > \nu\Gamma/\beta$, it is easy to verify that $F'(R) > 0$.

Therefore, for a given lump-sum transfer $s/y \geq 0$, the bond demand function $F(R)$ in (F.4) is monotonically increasing in R for $R > \max\{1-\delta, \gamma\Gamma, \underline{R}, \nu\Gamma/\beta\}$. The conditions for real interest rates, i.e. $R > \max\{1-\delta, \gamma\Gamma, \underline{R}, \nu\Gamma/\beta\}$ is necessary for (1) non-negative capital rental rate; (2) validity of household transversality condition; (3) non-negative household adjusted consumption; (4) non-negative demand for government bonds.

G Public Debt Valuation

In this appendix we derive equation (22). Substituting the bond pricing condition $\mathbb{E}_t\Lambda_{t,t+1}R^n_t/\Pi_{t+1} = 1$ into the government budget constraint, we obtain

$$\frac{R^n_{t-1}}{\Pi_t}B_{t-1} = (T_t - G^e_t) + \mathbb{E}_t\Lambda_{t,t+1}\frac{R^n_t}{\Pi_{t+1}}B_t. \tag{G.1}$$

Iterating the above condition forward, we obtain

$$\frac{R^n_{t-1}}{\Pi_t}B_{t-1} = \sum_{j=0}^{J}\mathbb{E}_t\Lambda_{t,t+j}(T_{t+j} - G^e_{t+j}) + \mathbb{E}_t\Lambda_{t,t+J+1}\frac{R^n_{t+J}}{\Pi_{t+J+1}}B_{t+J}, \tag{G.2}$$

where

$$\Lambda_{t,t+j} = \prod_{i=0}^{j}\Lambda_{t,t+i},\ \ \Lambda_{t,t} = 1.$$

In a representative household model, the transversality condition suggests that

$$\lim_{J\to\infty}\mathbb{E}_t\Lambda_{t,t+J+1}\frac{R^n_{t+J}}{\Pi_{t+J+1}}B_{t+J} = 0. \tag{G.3}$$

Combing the above transversality condition for government bonds with the limits of (G.2) implies that

$$\frac{R^n_{t-1}}{\Pi_t}B_{t-1} = \sum_{j=0}^{\infty}\mathbb{E}_t\Lambda_{t,t+j}(T_{t+j} - G^e_{t+j}), \tag{G.4}$$

with the tail bubble term in (G.2) vanished. This means that the transversality condition in the representative household model helps to rule out bubbles in government bonds, and bond value is fully determined by its fundamental value of primary surplus in (G.4).

However, the transversality condition in the perpetual youth model may not rule out government bond bubbles. The reason is that we are not able to derive a transversality condition at the aggregate level in the perpetual youth model, as assets can be passed on to future generations.

More specifically, the transversality condition for cohorts in period t is given by

$$\lim_{J\to\infty} \gamma^J \mathbb{E}_t \Lambda_{t,t+J} A_{t+J|s} = 0. \tag{G.5}$$

We notice that the individual's asset holdings are given by

$$A_{t|s} = \frac{1}{\gamma}\left(\frac{R^n_{t-1}B^n_{t-1|s}}{P_t} + \left[R^k_t + (1-\delta)Q_t\right] K_{t-1|s} + Z_{t|s}\right). \tag{G.6}$$

Substituting into (G.5) yields

$$\lim_{J\to\infty} \gamma^J \mathbb{E}_t \Lambda_{t,t+J}\left[\frac{R^n_{t+J-1}}{\Pi_{t+J}} B_{t+J-1|s} + \left[R^k_{t+J} + (1-\delta)Q_{t+J}\right] K_{t+J-1|s} + Z_{t+J|s}\right] = 0, \tag{G.7}$$

for any cohort s with $s \leq t$ at time t.

Aggregation across cohorts s for (G.7) yields

$$\lim_{J\to\infty} \gamma^J \mathbb{E}_t \Lambda_{t,t+J}\left[\frac{R^n_{t+J-1}}{\Pi_{t+J}} B_{t+J-1} + \left[R^k_{t+J} + (1-\delta)Q_{t+J}\right] K_{t+J-1} + Z_{t+J}\right] = 0. \tag{G.8}$$

By the market clearing condition $Z_{t+J} = 0$, we then have

$$\begin{aligned} & \lim_{J\to\infty} \gamma^J \mathbb{E}_t \Lambda_{t,t+J} \frac{R^n_{t+J-1}}{\Pi_{t+J}} B_{t+J-1} \\ = & \lim_{J\to\infty} \gamma^J \mathbb{E}_t \Lambda_{t,t+J} \frac{R^n_{t+J-1}}{\Pi_{t+J}} \left[R^k_{t+J} + (1-\delta)Q_{t+J}\right] K_{t+J-1} = 0. \end{aligned}$$

Because $\gamma \in (0,1)$, the above condition does not guarantee (G.3). Thus the individual transversality conditions in our OLG model cannot rule out bond bubbles.

In this case, taking limits of (G.2) yields

$$\frac{R^n_{t-1}}{\Pi_t} B_{t-1} = \lim_{J\to\infty} \sum_{j=0}^{J} \mathbb{E}_t \Lambda_{t,t+j}(T_{t+j} - G^e_{t+j}) + \lim_{J\to\infty} \mathbb{E}_t \Lambda_{t,t+J+1} \frac{R^n_{t+J}}{\Pi_{t+J+1}} B_{t+J}. \tag{G.9}$$

Noting that around the balanced growth path, we have B_t and $(T_t - G^e_t)$ growing at the rate of Γ, while $R = \Lambda^{-1} < \Gamma$ around the steady state. Dividing (G.9) by Γ^t, we can rewrite the condition in its detrended version on the balanced growth path:

$$\frac{Rb}{\Gamma} = (\tau - g^e) \lim_{J\to\infty} \sum_{j=0}^{J} \left(\frac{\Gamma}{R}\right)^j + b \cdot \lim_{J\to\infty} \left(\frac{\Gamma}{R}\right)^J. \tag{G.10}$$

Noting that $\tau - g^e < 0$ and $\Gamma/R > 1$ in the low interest rate environment, the first term on the right-hand side of (G.10), i.e. the fundamental value term, approaches negative infinity. In addition, the second term, i.e. the bubble term, approaches positive infinity. However, the sum of the two terms can lead to finite debt value.

We can offer an additional perspective on bond valuations by comparing them to a hypothetical representative household model without generational turnover. More specifically, in Appendix C, we show that the stochastic discount factor can be rewritten as

$$\Lambda_{t,t+1} = \beta \frac{C_t - \mu \Gamma^t V(L_t^a)}{C_{t+1} - \mu \Gamma^{t+1} V(L_{t+1}^a)} \cdot (1 + LIQ_{t+1}), \tag{G.11}$$

where

$$LIQ_t = \frac{(1-\gamma)(\chi_t - 1)}{1 - (1-\gamma)\chi_t}$$

captures the benefits of saving to insure against the declining lifetime labor income path, and

$$\chi_t \equiv \frac{C_{t|t} - \Gamma^t V(L_t^a)}{C_t - \mu \Gamma^t V(L_t^a)}$$

is the ratio of consumption of newborn agents relative to the aggregate consumption (mostly by the old). It should be noted that if this is a representative household model, we would have $\chi_{t+1} = 1$ as everyone consumes the same, and thus $LIQ_{t+1} = 0$. It follows that the SDF in (G.11) is reduced to the standard one in the representative household model. In the steady state, the SDF (G.11) suggests that the real interest rate $R = \Gamma/\beta$ and each individual would maintain a flat lifetime consumption.

However, if there are generational turnovers, young cohorts may have different consumption profiles when compared to the old. As the young cohorts age, they also become the old agents. If $\chi_{t+1} > 1$, newborns consume more than the old, suggesting it is quite beneficial to save for future consumption. In this case, we would have $LIQ_{t+1} > 0$. According to (G.11), this suggests that compared to a representative household model with SDF given by $\beta(C_t - \mu\Gamma^t V(L_t^a))/(C_{t+1} - \mu\Gamma^{t+1} V(L_{t+1}^a))$, there are additional benefits of saving due to declining lifetime consumption. In the steady state, this corresponds to the case where $R < \Gamma/\beta$. As a result, the real interest rate is low, leading agents to deplete their wealth and experience lower consumption as they age.

Denote $\widetilde{C}_t \equiv C_t - \Gamma^t V(L_t^a)$ the aggregate consumption adjusted for labor disutility. Then by the bond pricing condition $\mathbb{E}_t \Lambda_{t,t+1} R_t^n / \Pi_{t+1} = 1$ and condition (G.11), the government budget constraint can be rewritten as

$$\begin{aligned}\frac{R^n_{t-1}}{\Pi_t}B_{t-1} =&(T_t - G^e_t) + B_t \\ =&(T_t - G^e_t) + \mathbb{E}_t\Lambda_{t,t+1}\frac{R^n_t}{\Pi_{t+1}}B_t \\ =&(T_t - G^e_t) + \mathbb{E}_t\beta\frac{\widetilde{C}_t}{\widetilde{C}_{t+1}}\left(1 + LIQ_{t+1}\right)\frac{R^n_t}{\Pi_{t+1}}B_t \\ =&(T_t - G^e_t) + \mathbb{E}_t\beta\frac{\widetilde{C}_t}{\widetilde{C}_{t+1}}LIQ_{t+1}\frac{R^n_t}{\Pi_{t+1}}B_t \\ &+ \mathbb{E}_t\beta\frac{\widetilde{C}_t}{\widetilde{C}_{t+1}}\frac{R^n_t}{\Pi_{t+1}}B_t.\end{aligned}$$

Iterating the above condition forward, we obtain

$$\begin{aligned}\frac{R^n_{t-1}}{\Pi_t}B_{t-1} =&\sum_{j=0}^{J}\mathbb{E}_t\beta^j\frac{\widetilde{C}_t}{\widetilde{C}_{t+j}}\left(T_{t+j} - G^e_{t+j}\right) + \sum_{j=0}^{J}\mathbb{E}_t\beta^{j+1}\frac{\widetilde{C}_t}{\widetilde{C}_{t+j+1}}LIQ_{t+j+1}\frac{R^n_{t+j}}{\Pi_{t+j+1}}B_{t+j} \\ &+ \mathbb{E}_t\beta^{J+1}\frac{\widetilde{C}_t}{\widetilde{C}_{t+J+1}}\frac{R^n_{t+J}}{\Pi_{t+J+1}}B_{t+J}. \qquad \text{(G.12)}\end{aligned}$$

Around the balanced growth path, $\widetilde{C}_{t+J}$ grows at the same rate of B_{t+J}, while the real interest rate R^n_{t+J}/Π_{t+J+1} is constant. Therefore, we have

$$\lim_{J\to\infty}\mathbb{E}_t\beta^{J+1}\frac{\widetilde{C}_t}{\widetilde{C}_{t+J}}\frac{R^n_{t+J}}{\Pi_{t+J+1}}B_{t+J} = 0,$$

around the vicinity of the balanced growth path.

Taking limits of the above condition (G.12), we obtain

$$\frac{R^n_{t-1}}{\Pi_t}B_{t-1} = \underbrace{\sum_{j=0}^{\infty}\mathbb{E}_t\beta^j\frac{\widetilde{C}_t}{\widetilde{C}_{t+j}}\cdot\left(T_{t+j} - G^e_{t+j}\right)}_{\text{Valuation of Surplus in RANK}} + \underbrace{\sum_{j=0}^{\infty}\mathbb{E}_t\beta^{j+1}\frac{\widetilde{C}_t}{\widetilde{C}_{t+j}}\cdot LIQ_{t+j+1}\cdot\frac{R^n_{t+j}}{\Pi_{t+j+1}}B_{t+j}}_{\text{Store of Value Premium relative to RANK}}. \qquad \text{(G.13)}$$

By comparing to a representative agent New Keynesian model (RANK), where the SDF is given by $\beta\widetilde{C}_t/\widetilde{C}_{t+1}$, the valuation of public debt can be decomposed into two terms as shown in (G.13). The first term captures the present value of all current and future primary surplus $T_t - G^e_t$, with the discount factor being the SDF in a hypothetical RANK model. By dividing (G.13) by Γ^t, we can derive the detrended version of (G.13) on the balanced growth path:

$$\frac{Rb}{\Gamma} = \frac{\tau - g^e}{1-\beta} + \frac{\beta}{1-\beta}\cdot LIQ\cdot\frac{Rb}{\Gamma}. \qquad \text{(G.14)}$$

In the above condition, the first term on the right-hand side is the negative valuation of primary deficits if one uses the SDF in a hypothetical RANK model to evaluate the cash flow, while the second term is the store of value premium provided by government bonds relative to a RANK economy.