Asset Bubbles and Foreign Interest Rate Shocks*

Jianjun Miao† Pengfei Wang‡ Jing Zhou§

This version: March 2021

Abstract

We provide an estimated DSGE model of a small open economy with both domestic and international financial market frictions. Firms face credit constraints and trade an intrinsically useless asset. Low foreign interest rates are conducive to bubble formation. An asset bubble provides liquidity and relaxes credit constraints. It provides a powerful amplification and propagation mechanism. Our estimated model based on Bayesian methods explains the high volatilities of consumption and stock prices relative to output, countercyclical trade balance, and procyclical stock prices observed in the Mexican data over the period 1990Q1-2011Q4.

JEL Classification: E32; E44; F41

Keywords: Asset bubbles, business cycles, small open economy, sudden stop, liquidity, DSGE, Bayesian estimation

---

*We are grateful for the comments from the editor and two anonymous referees, with which we improve our paper significantly. We also thank Fernando Broner, Yan Bai, Qingyuan Du, Haizhou Huang, Ji Huang, Yi Huang, Kang Shi, Jenny Xu, Eric Young, Shangjin Wei for helpful discussions. We have also benefited from comments from seminar and conference participants at Hong Kong Baptist University, Tsinghua University, Peking University, Shanghai Jiaotong University, Fudan University, LAEF Conference on Bubbles at Santa Barbara, 2017 Meetings of the Society for Economic Dynamics, Spring 2017 Mid-west Macroeconomics Meeting, 2017 Asian Meeting of Econometric Society, 2017 China Meeting of the Econometric Society, 2017 China International Conference in Finance, and 2017 Tsinghua Workshop in International Finance. Pengfei Wang acknowledges the financial support from RGC for #16515216 GRF grant. Jing Zhou acknowledges the financial support from Shanghai Institute of International Finance and Economics and Shanghai Pujiang Program #17PJC020.

†Department of Economics, Boston University, 270 Bay State Road, Boston, MA 02215. Tel:(+1)617-353-6675. Email: miaoj@bu.edu. Homepage: http://people.bu.edu/miaoj.

‡Peking University HSBC Business School, University Town, Nanshan District, Shenzhen, China 518055. Tel: (+86)755-2603-5380. Email: pfwang@phbs.pku.edu.cn

§School of Economics, Fudan University, 600 Guoquan Road, Shanghai, China 200433. Tel: (+86)21-6465-2989. Email: jingzhou@fudan.edu.cn

1 Introduction

The 2008 financial crisis generated renewed interest in asset bubbles (Martin and Ventura, 2012; Miao and Wang, 2018). It is often argued that the Federal Reserve's excessively low interest rate policy in the early 2000s helped cause a housing bubble in the United States and that the collapse of the bubble led to an unprecedented global financial crisis. Emerging economies seem particularly vulnerable to this type of "bubble-driven" crisis. When a bubble emerges, asset prices, investment, and capital inflows all enjoy booms and then suddenly suffer remarkable reversals. Examples include Chile and Mexico in the early 1980s, Argentina and Mexico in the 1990s, and Southeast Asia, namely, Malaysia, Indonesia, Thailand, and South Korea, in the 1990s. Since interest rates in emerging economies are strongly driven by interest rates in advanced economies (Neumeyer and Perri, 2005; Uribe and Yue, 2006; and Maćkowiak, 2007), some observers have argued that strong expansionary monetary policies implemented by major advanced economies in the aftermath of the 2008 financial crisis would cause bubbles and inevitable crises in emerging countries. Given the prevalence of "bubble-driven" crises in emerging countries, understanding and quantifying the role of foreign interest rates on bubbles is therefore very important in accounting for the business cycle fluctuations in these countries.

The goal of our paper is to develop and estimate a dynamic stochastic general equilibrium (DSGE) model of asset bubbles to understand the impact of foreign interest rate shocks on macroeconomic fluctuations and asset prices in emerging economies. We extend the recent development of theories of asset bubbles in infinite-horizon models with idiosyncratic shocks and credit frictions to a small open economy model. In addition to one-period bonds, firms can also trade an intrinsically useless asset. If all agents believe that the intrinsically useless asset has value, it serves as self-insurance against idiosyncratic shocks by providing liquidity for firms to finance real investment.[1] This belief can be self-fulfilling and results in a bubbly equilibrium with stock prices of firms being inflated by the bubble. On the other hand, when no one believes that the intrinsically useless asset has value, this belief can also be self-fulfilling and results in a bubbleless equilibrium.

We first provide a steady-state analysis of the impact of the foreign interest rate on the bubble in an infinite-horizon small open economy. We prove that a low foreign interest rate causes capital inflow and a low domestic interest rate, thereby fueling a domestic asset bubble.

[1] We follow Wang and Wen (2012) and Miao and Wang (2018) and model firms' idiosyncratic shocks as idiosyncratic shocks to investment efficiency.

The size of the bubble decreases with the foreign interest rate. When the foreign interest rate is sufficiently high, the asset bubble can burst. This result extends the existing results of Tirole (1985), Santos and Woodford (1997), Miao et al. (2015b), and Miao and Wang (2018) for closed economies.

Our most important contribution is to quantitatively evaluate the impacts of foreign interest rate shocks by taking our model to the Mexican data over the period 1990Q1-2011Q4 using Bayesian estimation. To the best of our knowledge, our paper provides the first estimated DSGE model with asset bubbles in a small open economy. To provide a fair evaluation of foreign interest rate shocks, we also include four other types of exogenous shocks often used in the literature (long- and short-run productivity shocks, preference shocks, and foreign demand shocks) to fit five time series of the demeaned foreign interest rate, the growth rate of the sum of US and Canadian real GDP, and the growth rates of Mexican real GDP, real investment, and real consumption. We find that our estimated model matches the Mexican business cycle data including stock prices reasonably well. In particular, our estimated model matches the salient features of high volatilities of consumption and stock prices relative to output, strong countercyclical trade balance, and procyclical stock prices. In addition, our estimated impulse responses to a foreign interest rate shock from the model match those from the data remarkably well. Following a rise in the foreign interest rate, real stock prices decline substantially, while GDP, investment and consumption all decrease significantly at the same time. Accompanied by significant real depreciation, the trade balance increases significantly. The magnitude of these responses in the model is very close to the VAR evidence in the data. As we only use a subset of data for our Bayesian estimation, fitting all VAR responses provides a very convincing post-estimation check for the model.

Our estimation suggests that foreign interest rate shocks play a nontrivial role, particularly in explaining the variation in the net exports/GDP ratio (33.9% in the impact period), stock prices (27.9% in the impact period), and investment (23.6% in the impact period). The trend shocks to productivity are also important. They explain the bulk of the variation in the net exports/GDP ratio (43.0% in the impact period), stock prices (39.7% in the impact period), and investment (16.0% in the impact period). Our key insight is that asset bubbles provide a powerful amplification and propagation mechanism, which is essential for these results. When the bubble channel is shut down, the correlation between the net exports/GDP ratio and GDP reduces to -0.14 from -0.53, too weak compared to the data (-0.50). The correlation between stock prices and consumption becomes -0.46, producing an incorrect sign that is in contrast to those in the model with bubbles (0.37) and in the data (0.53). Furthermore, the correlation between the net exports/GDP ratio and stock prices increases to 0.71 from -0.66, clearly in conflict with the data (-0.45). Moreover, the relative volatility of stock prices decreases from 4.82 to 2.25, significantly below the 6.39 observed in the data. In addition, in this case, both foreign interest rate shocks and trend productivity shocks play essentially no role in explaining the variation in the net exports/GDP ratio, stock prices, and investment.

To further investigate the importance of the amplification and propagation mechanism generated by bubbles, we also estimate a model without asset bubbles. We find that this model performs much worse. In particular, it cannot match the high volatilities of consumption and stock prices or the significant countercyclicality of trade balance. The model without asset bubbles also produces an incorrect sign of the correlation between stock prices and real consumption. Furthermore, the estimated volatilities of the long- and short-run productivity shocks and preference shocks are several times larger than their counterparts in the model with bubbles, highlighting the necessary role of bubbles in amplifying and propagating exogenous shocks to macroeconomic fluctuations. After a rise in the foreign interest rate, the responses of stock prices, GDP, consumption, investment, net exports and the real exchange rate show the same signs as the data, but their magnitudes are too small. Note that we have excluded stock prices from both estimations to avoid the natural disadvantage of the bubbleless model to produce volatile asset prices. Nevertheless, the data still favor the model with asset bubbles based on the marginal density of macroeconomic data alone.

Our paper is related to three strands of the literature. First, our paper builds on the literature on international real business cycles (RBCs).[2] Aguiar and Gopinath (2007) argue that the long-run productivity shock in a standard RBC model is important to explain the high consumption volatility and the countercyclical trade balance in emerging markets. By contrast, García-Cicco et al. (2010) find that when estimated over a long sample, the RBC model driven by permanent and transitory productivity shocks performs poorly at explaining observed business cycles in Argentina and Mexico along a number of dimensions. Neumeyer and Perri (2005) and Uribe and Yue (2006) argue instead that the introduction of foreign interest rate shocks coupled with financial frictions is important to explain the empirical regularities of emerging economies. Chang and Fernández (2013) estimate a model with financial frictions that

[2] This literature is too large for us to discuss all of the related studies. Important related early contributions include Mendoza (1991) and Backus et al. (1992).

includes all three types of shocks using Bayesian methods. They find that financial frictions play a dominant role in amplifying conventional productivity shocks and, less markedly, interest rate shocks; trend shocks, in contrast, play a very minor role. This literature typically ignores asset prices and does not study the high stock market volatility and the comovement between stock prices and the real economy. We contribute to this literature by showing that asset bubbles are important to explain these facts. This contribution is important because neoclassical models do not provide a fully adequate explanation of asset price movements (Schmitt-Grohé and Uribe, 2012). Our estimated model shows that both long-run productivity shocks and interest rate shocks are important drivers of business cycle fluctuations in emerging economies.

Second, our paper is related to the recent literature on sudden stops (e.g., Calvo, 1998; Gopinath, 2004; Martin and Rey, 2006; Gertler et al. 2007; Mendoza, 2010; Fernández and Gulan, 2015; Aoki et al. 2016; and Perri and Quadrini, 2018).[3] This literature views credit frictions as the central feature of the transmission mechanism that drives sudden stops. Mendoza (2010) also emphasizes the amplification and asymmetry of macroeconomic fluctuations that result from the debt-deflation transmission mechanism. Asset prices in this literature typically refer to capital prices or Tobin's Q. By contrast, firms can own both capital and bubble assets in our model. The stock market value of firms contains a fundamental component, equal to Tobin's Q multiplied by the capital stock, and a bubble component, equal to the value of the bubble asset. The movement of both Tobin's Q and asset bubbles contributes to stock market fluctuations. Our model can hence produce the aforementioned stylized facts of a "sudden stop" when foreign interest rates suddenly change from low to high, causing a boom and bust in the real economy through the expansion and contraction of bubbles.

Third, our paper is related to the recent literature on asset bubbles in open economies (e.g., Caballero and Krishnamurthy, 2006; Ventura, 2012; Basco, 2014; and Martin and Ventura, 2015a,b). This literature typically adopts the overlapping-generations (OLG) framework. Like our paper, this literature emphasizes the importance of credit constraints. Martin and Ventura (2012, 2015a,b) also discuss the crowding-in and crowding-out effects of asset bubbles, similar to those in our paper. Our paper differs from this literature in the addressed questions and modeling details. More importantly, unlike the OLG models, our model is in the DSGE framework, which can confront data using Bayesian estimation. In our infinite-horizon framework, credit constraints are essential for the emergence of asset bubbles, as in Kiyotaki and Moore

[3]See Korinek and Mendoza (2014) for a survey of this literature.

(2019), in the sense that a bubble could not emerge without credit constraints. By contrast, a bubble can still emerge in OLG models without credit constraints, and their presence allows bubbles to emerge in dynamically efficient economies (Farhi and Tirole, 2012; and Martin and Ventura, 2012). Our infinite-horizon DSGE model complements the existing OLG models.

Finally, our paper is related to studies that quantitatively investigate the importance of asset bubbles. A notable example is the paper by Miao et al. (2015a), who show that asset bubbles driven by sentiment shocks help explain the high stock price volatility and the comovement between the stock market and the real economy in a closed economy model. Our study differs from their work in three respects. First, the international real business cycle patterns we attempt to account for in this paper are completely absent in their closed economy setup. Second, the foreign interest rate affects the existence condition of asset bubbles, which is different from that in the closed economy model of Miao et al. (2015a). Third, they emphasize the role of the sentiment shock in stock markets, whereas we focus on the role of the foreign interest rate shock in this paper.

2 Empirical Evidence

In this section, we first demonstrate the relevance of stock markets in emerging markets and the related business cycle properties. Then, we empirically examine the impacts of foreign interest rate shocks on the macroeconomy in emerging markets, especially in the presence of stock market prices.

2.1 Stock Markets in Emerging Market Economies

Following Fernández and Gulan (2015) and Aguiar and Gopinath (2007), we consider 13 emerging market economies and 13 developed economies.[4] Figure 1 presents a comparison of stock market capitalization as a percentage of GDP between emerging markets and advanced economies. Since the emerging markets in our sample do not have sufficient information about stock market capitalization until 1995, we consider the period since 1995 using data from the World Bank.

[4]The emerging markets are Argentina, Brazil, Chile, Colombia, Ecuador, Korea, Malaysia, Mexico, Peru, Philippines, South Africa, Thailand, Turkey. We consider them because information is available for these countries on national account components, stock markets, and foreign interest rates. The developed economies are Australia, Austria, Belgium, Canada, Denmark, Finland, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden and Switzerland.

[Insert Figure 1 Here]

Compared to those in advanced economies, stock markets in emerging markets were smaller at the beginning of the sample period. However, they grew rapidly and demonstrated their importance, similar to their counterparts in developed markets in later periods. This justifies the necessity of paying attention to stock markets in emerging economies.

[Insert Table 1 Here]

The business cycle properties related to real stock prices in emerging markets are also comparable to their counterparts in developed markets, as reported in Table 1. We find the following patterns: (i) real stock prices exhibit high volatilities relative to real GDP in emerging markets, which is also repeatedly observed in developed markets; (ii) the correlation of real stock prices with GDP, consumption, and investment in emerging markets are quite close to their counterparts in developed markets; and (iii) the correlation between real stock prices and trade balance is slightly negative, with stronger significance in emerging markets than that in developed markets. These patterns suggest some similarities between stock market fluctuations in both emerging and developed markets. On the other hand, the other business cycle moments in Table 1 exhibit the well-known distinguishing patterns of emerging markets documented in the literature, for instance, a high GDP volatility, high relative volatilities of consumption, investment, and trade balance, and significant countercyclicality in the trade balance.

2.2 Impacts of Foreign Interest Rate Shocks on Emerging Market Economies

Next, we investigate how real aggregate activities and stock markets in emerging economies respond to foreign interest rate shocks. We estimate a quarterly first-order panel VAR model. The following observables are included in order: (i) foreign interest rates, (ii) real stock prices, (iii) real GDP, (iv) real investment, (v) real consumption, (vi) trade balance as a percentage of GDP, (vii) real exchange rates, and (viii) the relative price of investment. This ordering supposes that foreign interest rates are exogenous to local markets and contemporaneously affect domestic aggregates. Moreover, we allow real stock prices to contemporaneously affect domestic real variables, to be consistent with our theory of asset bubbles that are contained in stock prices.

Following a standard convention in the literature (e.g., Uribe and Yue, 2006; Fernández-Villaverde et al. 2011, and Fernández and Gulan, 2015), the foreign interest rate, measuring the interest rate faced by an individual emerging market in international markets, is constructed as the sum of the international risk-free interest rate and a country-specific spread. The international risk-free interest rate is measured by the US T-bill rate. The country-specific spread is measured by the Emerging Markets Bond Index Plus (EMBI) from J.P. Morgan. This index provides secondary market prices of emerging market bonds that are actively traded and denominated in US dollars in a few emerging markets. Our sample countries are as before, and due to data availability, our sample period is 1994Q1-2011Q4.

The real stock price index is the nominal stock price index divided by the CPI. The nominal stock price index is from the Global Database in CEIC. The CPI is from the IMF. The real exchange rate is measured by the terms of trade of each country. Following Restuccia and Urrutia (2011) and Karabarbounis and Neiman (2014), the relative price of investment is from Penn World Table 9.1.[5] The data for the other variables are taken from Fernández and Gulan (2015). The foreign interest rates and trade balance/GDP ratio are in levels, and the other variables are in growth rates.

Our ordering strategy of treating the foreign interest rate as exogenous can be empirically supported by Uribe and Yue (2006) and Longstaff et al. (2011). Uribe and Yue (2006) find that at least two-thirds of the fluctuations in country spreads are explained by innovations that are exogenous to domestic conditions. Longstaff et al. (2011) report that the country spreads for sovereign debt of 26 open economies are driven much more by global factors than by local forces. The evidence in these two papers suggests that a substantial part of the changes in the foreign interest rate is exogenous to the country. With our sample used in Section 4, we test this assumption by ordering the foreign interest rate last in the VAR to allow the domestic factors to have contemporaneous impacts on it. We find that the movements of all domestic factors account for only approximately 33% of the instantaneous response in the foreign interest rate, which accords with the findings in the aforementioned studies.

[**Insert Figure 2 Here**]

Figure 2 displays the impulse response functions for a positive one-standard-deviation innovation to the foreign interest rate, with 95% confidence bands. All entries are in percent. In the impact period, following a rise in the foreign interest rate, the real stock price declines

[5] As described in Feenstra et al. (2015), the construction method for Penn World Table 9.1 considers different types of capital and asset-specific depreciation rates and investment deflators, making the relative price of investment provided in this table comparable across countries and periods. We use Denton's (1971) method to interpolate the annual series at a quarterly frequency.

substantially. GDP, investment, consumption and the relative price of investment all decrease significantly at the same time. Accompanied by significant real depreciation, the trade balance increases significantly. The real depreciation will be key in the transmission mechanism to be discussed in Section 4.

We perform some sensitivity analyses. The first exercise is to change the measure of the real exchange rate from the terms of trade to the real effective exchange rate. The second exercise is to change the measure of the foreign interest rate from the country-specific EMBI index to the average EMBI in emerging markets. Our results are robust to these changes. The third exercise is to consider different orderings of the stock prices in the VAR. We check the other six cases, as we move the stock price from position three to eight (that is, to the last position). We also conduct the same exercise for the real exchange rates as the fourth exercise. Although not shown here, the associated impulse response functions for the third and fourth exercises are very similar to Figure 2. Regardless of the ordering of the real stock prices or the real exchange rates, the key macro aggregates decline, the real exchange rate depreciates, and the trade balance increases, all of which are significant at the 95% confidence level.

3 The Model

Motivated by the evidence presented in the previous section, we consider a discrete-time (real) DSGE model of a small open economy populated by a representative household, a continuum of identical capital producers with a unit measure, and a continuum of ex ante identical but ex post heterogeneous firms with a unit measure. There is no government or monetary authority. The household consists of two types of members: workers and bankers. Workers supply labor to firms and trade firm shares. Financial transactions between domestic and foreign residents are intermediated by domestic financial institutions or simply bankers. Firms buy capital goods from capital producers and trade a bubble asset. Each firm is subject to idiosyncratic investment efficiency shocks. Suppose that the law of large numbers holds for idiosyncratic shocks. Since our main objective is to quantitatively analyze the business cycle properties of our model, we also introduce several commonly used reduced-form real frictions such as capital adjustment costs, habits in consumption, and portfolio adjustment costs in estimated DSGE models.[6]

[6]See An and Schorfheide (2007) for a survey of Bayesian methods to estimate DSGE models.

3.1 Firms

There are three types of goods: domestic consumption goods, domestic capital goods, and foreign goods. Firms in the small open economy use foreign goods as an input factor to produce domestic consumption goods. As Mendoza (2010) emphasizes, imported inputs are important for the initial decrease in output during a sudden stop. In each period $t = 0, 1, ...$, one unit of foreign goods can be exchanged for e_t units of domestic consumption goods (e_t is called the real exchange rate). The total demand of the rest of the world for domestic consumption goods is exogenously given by

$$X_t = e_t^{\sigma} Y_t^*, \tag{1}$$

where $\sigma > 0$ and Y_t^* denotes an exogenous component of foreign demand. When e_t is larger, the domestic consumption goods are cheaper, and hence foreign demand is higher.

A domestic firm indexed by $j \in [0, 1]$ uses a constant returns-to-scale technology to produce output Y_{jt} according to

$$Y_{jt} = K_{jt-1}^{\alpha} (A_t N_{jt})^{1-\alpha-\gamma} M_{jt}^{\gamma}, \quad \alpha \in (0,1), \quad \gamma \in (0,1), \quad \alpha + \gamma \in (0,1), \tag{2}$$

where A_t, K_{jt-1}, N_{jt}, and M_{jt}, represent aggregate productivity, capital input, labor input, and imported material input, respectively. Let $A_t = A_t^g \exp(a_t)$ where A_t^g is the trend productivity and a_t is the transitory productivity (Aguiar and Gopinath, 2007; and García-Cicco et al. 2010). Assume that $A_t^g = A_{t-1}^g \exp(g_t)$,

$$\begin{aligned} g_t &= (1 - \rho_g) g + \rho_g g_{t-1} + \sigma_g \varepsilon_{gt}, \\ a_t &= \rho_a a_{t-1} + \sigma_a \varepsilon_{at}, \end{aligned}$$

where $g > 0$, $\rho_a, \rho_g \in (-1, 1)$, and $\sigma_a, \sigma_g > 0$. The positive growth rate g ensures a positive steady-state net interest rate. For balanced growth, suppose that Y_t^* grows at the same rate g on average. Let $Y_t^* = \exp(gt) y_t^*$, where y_t^* follows an AR(1) process

$$\ln(y_t^*) = \rho_{y^*} \ln\left(y_{t-1}^*\right) + \sigma_{y^*} \varepsilon_{y^* t}.$$

Here, $\rho_{y^*} \in (-1, 1)$ and $\sigma_{y^*} > 0$. Assume that all innovations ε_{at}, ε_{gt}, and ε_{y^*t} are IID standard normal random variables and independent of each other.

Firm j solves the following static labor and material input choice problem:

$$\max_{N_{jt}, M_{jt}} \; K_{jt-1}^{\alpha} (A_t N_{jt})^{1-\alpha-\gamma} M_{jt}^{\gamma} - W_t N_{jt} - e_t M_{jt}, \tag{3}$$

where W_t denotes the wage rate. It is straightforward to show that the maximized objective is equal to $R_{kt}K_{jt-1}$, where R_{kt} satisfies

$$R_{kt} = \alpha A_t^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{W_t}\right)^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{\gamma}{e_t}\right)^{\frac{\gamma}{\alpha}}. \tag{4}$$

We will show later that R_{kt} is equal to the marginal product of capital.

To make investment in period t, firm j purchases I_{jt} units of new capital goods from domestic capital producers at price P_{kt}. One unit of newly purchased capital is transformed into ε_{jt} units of installed capital, so the law of motion for capital follows

$$K_{jt} = (1-\delta)K_{jt-1} + \varepsilon_{jt}I_{jt}, \tag{5}$$

where $\delta \in (0,1)$ represents the depreciation rate. Here, ε_{jt} represents a firm-specific investment efficiency shock that is assumed to be drawn independently and identically across firms and over time from the cumulative distribution function F (the density function is f) on $[\varepsilon_{\min}, \varepsilon_{\max}] \subset [0,\infty)$. Assume that there is no insurance market against the idiosyncratic investment efficiency shock and that investment is irreversible at the firm level such that $I_{jt} \geq 0$.

Firms can trade two types of assets: a one-period risk-free bond and a bubble asset. One unit of the bond delivers one unit of domestic consumption goods in the next period. Let R_{ft} denote the domestic market interest rate between periods t and $t+1$. When firm j's bond holdings B_{jt} in period t satisfy $B_{jt} < (\geq) 0$, B_{jt} is interpreted as borrowing (saving). Firms can borrow or lend abroad, but this can only be intermediated by the bankers. Firms face borrowing constraints and use their physical capital as collateral. The credit constraint is given by

$$\frac{B_{jt}}{R_{ft}} \geq -\mu K_{jt-1}, \tag{6}$$

where $\mu \in (0,1)$ is a pledgeability parameter and reflects frictions in the domestic financial market. The credit constraint can be justified by an optimal contract with limited enforcement. The firm cannot commit to repay its debt and hence needs to use its capital as collateral. For convenience, we assume that only the existing capital can be used as collateral.[7] In the event of default, each unit of capital is assumed to have $\mu \in (0,1)$ units of liquidated value. This yields

[7]We do not use the bubble asset as collateral and use the current value of capital as collateral only to simplify the algebra. This can be justified by a particular debt contract form. As Caballero and Krishnamurthy (2006), Miao et al. (2015b), and Miao and Wang (2018) show, using the future capital value as collateral, as in Kiyotaki and Moore (1997), will complicate the algebra without changing any key insights.

(6). Similar borrowing constraints for firms are commonly used in the literature on models with financial frictions (see Gertler and Kiyotaki, 2010; and Jermann and Quadrini, 2012).

The bubble asset is intrinsically useless and can be viewed as uncultivated land or some toxic asset. Normalize its supply to one.[8] Let H_{jt} denote firm j's holdings of the bubble asset chosen in period t. Assume that firms cannot short the bubble asset so that $H_{jt} \geq 0$. Short-sale constraints are widely adopted in the finance literature (Woodford, 1986; Kocherlakota, 1992; Scheinkman and Xiong, 2003). They reflect institutional features such as direct transaction costs and default risk associated with short sales or Securities and Exchange Commission rules. Without this assumption, a bubble cannot emerge by arbitrage (e.g., Kocherlakota, 1992).

The flow-of-funds constraint for firm j is given by

$$D_{jt} = R_{kt}K_{jt-1} - P_{kt}I_{jt} - \frac{B_{jt}}{R_{ft}} + B_{jt-1} + P_t \left(H_{jt-1} - H_{jt} \right), \tag{7}$$

where D_{jt} and P_t denote dividends and the price of the bubble asset, respectively.

For simplicity, we assume that equity financing is too costly for firms to raise new funds. Then, the firm faces the following equity constraint

$$D_{jt} \geq 0. \tag{8}$$

There is substantial empirical evidence that equity issuance as a source of external financing is limited relative to debt issuance (see, e.g., Jermann and Quadrini, 2012; Frank and Goyal, 2008; Hennessy and Whited, 2007). In terms of theory, Myers (1984) argues that firms prefer internal to external financing and debt to equity if external financing is used because of adverse selection. Issuing new equity may signal bad news to outside shareholders when there is information asymmetry between managers and outside shareholders.

Now, we describe firm j's decision problem by dynamic programming. Let $V_t(\varepsilon_{jt}, K_{jt-1}, H_{jt-1}, B_{jt-1})$ denote firm j's value function, where we suppress aggregate state variables as arguments. The dynamic programming problem is given by

$$V_t(\varepsilon_{jt}, K_{jt-1}, H_{jt-1}, B_{jt-1}) = \max_{H_{jt}, I_{jt} \geq 0,\ B_{jt}} D_{jt} + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}(\varepsilon_{jt+1}, K_{jt}, H_{jt}, B_{jt}),$$

subject to (5), (6), (7), and (8). Here, $\beta \in (0,1)$ denotes the subjective discount factor, and $\mathbb{E}_t$ represents the conditional expectation operator. Assume that firms are owned by house-

[8] In Appendix B, we show that with the aggregate supply of the bubble asset normalized to one, the bubble price grows at the rate g along the balanced growth path. We can show that if the aggregate supply of the bubble asset grows at an exogenous rate g_H, then the price of the bubble asset grows at the rate $g - g_H$ along the balanced growth path, and the bubbly model still holds without essential changes.

holds, and thus, we use the representative household's marginal utility Λ_{t+1} to discount future dividends.

3.2 Capital Producers

A representative capital producer creates domestic capital goods using domestic consumption goods as its input subject to flow adjustment costs. The capital producer sells new capital goods to investing firms at price P_{kt} in period t. The objective function of a capital producer is to choose $\{I_t\}$ to solve

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t}{\Lambda_0} D_t^k,$$

where

$$D_t^k = P_{kt} I_t - \left[1 + \frac{\Omega_k}{2} \left(\frac{I_t}{I_{t-1}} - \exp(g) \right)^2 \right] I_t \tag{9}$$

represents dividends and $\Omega_k > 0$ is an adjustment cost parameter. The adjustment cost vanishes on the deterministic balanced growth path.

3.3 Bankers

Bankers internationally intermediate financial transactions. The international financial transactions are subject to quadratic adjustment costs that are rebated to households in a lump-sum manner. Introducing such costs is a flexible approach to model the apparent deviation of uncovered interest rate parity (UIP) in emerging countries. These adjustment costs reflect transaction costs associated with cross-border capital flows. Lothian and Wu (2011) find supporting evidence that small deviations from UIP for a relatively long period of time are due to market frictions such as transaction costs. This assumption is also used in studies of emerging countries (see, e.g., Chang et al. 2015).[9] Let R_{ft}^* denote the exogenous foreign interest rate between periods t and $t+1$. Assume that R_{ft}^* follows an AR(1) process

$$\ln\left(R_{ft}^*\right) = \left(1 - \rho_{R_f^*}\right) \ln\left(R_f^*\right) + \rho_{R_f^*} \ln\left(R_{ft-1}^*\right) + \sigma_{R_f^*} \varepsilon_{R_f^* t},$$

where $\rho_{R_f^*} \in (-1, 1)$, $\sigma_{R_f^*} > 0$, and the innovation $\varepsilon_{R_f^* t}$ is an IID standard normal random variable that is independent of all other innovations in the model.

[9] See Aoki et al. (2016) for related modeling. Our modeling of bankers is similar to that described in Chapter 4 of Uribe and Schmitt-Grohé (2017). One difference is that we allow for an interest rate differential in the steady state.

The flow-of-funds constraint of a representative banker is given by

$$D_t^b = \frac{B_{t-1}}{R_{ft-1} e_{t-1}} R_{ft-1}^* e_t - B_{t-1} - \left[\frac{\Omega}{2Y_t} \left(B_t - A_{t-1} b \right)^2 - \Omega_0 A_t \right], \tag{10}$$

where D_t^b is the profit that the banker gives to his/her family in period t, $B_t \geq (<) 0$ is the bond supply (demand) from the banker, and the expression in square brackets represents the portfolio adjustment costs. Along the deterministic balanced growth path, A_t, Y_t, B_t and the adjustment costs all grow at rate g. Along this path, we let $b = B_t / A_t$ and choose Ω_0 such that the adjustment costs vanish.

The interpretation of (10) is as follows. The banker sells B_{t-1} units of bonds to domestic residents in period $t-1$ at price $1/R_{ft-1}$ and converts the proceed into units of foreign goods at rate $1/e_{t-1}$. The banker then saves in a foreign bank and obtains interest at the gross rate R_{ft-1}^* in period t. After converting into units of domestic consumption goods at rate e_t, the banker obtains returns $\frac{B_{t-1}}{R_{ft-1} e_{t-1}} R_{ft-1}^* e_t$. After subtracting debt repayment B_{t-1} and adjustment costs incurred for the choice of B_t in period t, we obtain the profit D_t^b given in equation (10).

The banker chooses $\{B_t\}$ to maximize the expected present value of profits:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t}{\Lambda_0} D_t^b.$$

3.4 Households

Each household is an extended family consisting of a continuum of identical workers of unit mass and a continuum of identical bankers of unit mass. Each worker in the family supplies labor to firms and delivers the wage income to his/her family. The family pools the labor income from the workers and dividends from bankers, firms, and capital goods producers and distributes them equally among family members. A representative household chooses shareholding $\{\psi_{j,t+1}\}$ and family consumption and labor supply, $\{C_t\}$ and $\{N_t\}$, to maximize its lifetime utility,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \xi_t \beta^t \left[\ln(C_t - hC_{t-1}) - \nu \frac{N_t^{1+\varphi}}{1+\varphi} \right],$$

subject to the budget constraints

$$\begin{aligned} \int \psi_{j,t+1} \left(V_{jt} - D_{jt} \right) dj + C_t &= W_t N_t + \int \psi_{jt} V_{jt} dj \\ &\quad + D_t^b + D_t^k + \left[\frac{\Omega}{2Y_t} \left(B_t - A_{t-1} b \right)^2 - \Omega_0 A_t \right], \end{aligned} \tag{11}$$

where V_{jt} denotes firm j's stock market value. The parameter h governs the strength of consumption habit formation, $1/\varphi$ is the Frisch elasticity of labor supply, and ν is a weight on labor disutility. The variable ξ_t represents an exogenous preference shock that follows an AR(1) process

$$\ln(\xi_t) = \rho_\xi \ln(\xi_{t-1}) + \sigma_\xi \varepsilon_{\xi t},$$

where $\rho_\xi \in (-1, 1)$, $\sigma_\xi > 0$ and $\varepsilon_{\xi t}$ is an IID standard normal random variable. Given the utility function, the marginal utility is

$$\Lambda_t = \frac{\xi_t}{C_t - hC_{t-1}} - \beta h \mathbb{E}_t \frac{\xi_{t+1}}{C_{t+1} - hC_t}, \tag{12}$$

and the stochastic discount factor for asset pricing is $\beta\Lambda_{t+1}/\Lambda_t$.

Note that we have assumed that households do not trade bonds or the bubble asset. Allowing them to trade these assets will not affect our results. In this case households will choose not to hold any assets because their equilibrium returns are too low (see Kiyotaki and Moore, 2019 and Miao et al., 2015b for a similar result). We will illustrate this point in (20) below.

3.5 Competitive Equilibrium

Denote $K_t = \int K_{jt}dj$, $M_t = \int_0^1 M_{jt}dj$, and $Y_t = \int Y_{jt}dj$. A competitive equilibrium consists of sequences of aggregate quantities $\{C_t, K_t, I_t, Y_t, B_t, H_t, M_t\}$, shareholdings $\{\psi_{j,t+1}\}$, and prices $\{W_t, P_t, R_{kt}, R_{ft}, e_t, P_{kt}\}$ such that:

(i) Households, capital producers, firms, workers, and bankers optimize.

(ii) The markets for labor, the bubble asset, domestic capital goods, bonds, stocks, and domestic consumption goods all clear so that

$$N_t = \int_0^1 N_{jt}dj, \; H_t = \int_0^1 H_{jt}dj = 1, \; I_t = \int_0^1 I_{jt}dj, \tag{13}$$

$$B_t = \int_0^1 B_{jt}dj, \; \psi_{j,t+1} = 1, \tag{14}$$

$$Y_t = C_t + \left[1 + \frac{\Omega_k}{2}\left(\frac{I_t}{I_{t-1}} - \exp(g)\right)^2\right] I_t + X_t. \tag{15}$$

(iii) The law of motion of aggregate capital follows

$$K_t = (1-\delta)K_{t-1} + \int_0^1 \varepsilon_{jt} I_{jt} dj.$$

Appendix B provides a full characterization of the equilibrium system. One of the key steps is to derive the solution to the firm's decision problem. Define Tobin's (marginal) Q as

$$Q_t = \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \frac{\partial V_{t+1}(\varepsilon_{jt+1}, K_{jt}, H_{jt}, B_{jt})}{\partial K_{jt}},$$

Note that the price of new capital goods P_{kt} is not equal to Q_t in general due to idiosyncratic investment efficiency shocks. The following proposition characterizes firm j's optimal policies. Its proof is relegated to Appendix A.

Proposition 1 *Denote* $\bar{\varepsilon}_t = P_{kt}/Q_t \in (\varepsilon_{\min}, \varepsilon_{\max})$.

(i) When $\varepsilon_{jt} \geq \bar{\varepsilon}_t$, *firm* j *makes real investment,*

$$I_{jt} = \frac{1}{P_{kt}} \left(R_{kt} K_{jt-1} + \mu K_{jt-1} + B_{jt-1} + P_t H_{jt-1} \right), \tag{16}$$

sells all of its bubble asset holdings, i.e., $H_{jt} = 0$, *and exhausts its borrowing limit.*

(ii) When $\varepsilon_{jt} < \bar{\varepsilon}_t$, *firm* j *makes no real investment and is willing to hold any amount of the bubble asset and bonds as long as condition (6) holds.*

(iii) Tobin's Q, the bubble asset price, and the domestic interest rate satisfy

$$Q_t = \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left[R_{kt+1} + (1-\delta) Q_{t+1} + (R_{kt+1} + \mu) \int_{\bar{\varepsilon}_{t+1}}^{\varepsilon_{\max}} \left(\varepsilon \frac{Q_{t+1}}{P_{kt+1}} - 1 \right) f(\varepsilon) d\varepsilon \right], \tag{17}$$

$$P_t = \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} P_{t+1} \left[1 + \int_{\bar{\varepsilon}_{t+1}}^{\varepsilon_{\max}} \left(\varepsilon \frac{Q_{t+1}}{P_{kt+1}} - 1 \right) f(\varepsilon) d\varepsilon \right], \tag{18}$$

$$\frac{1}{R_{ft}} = \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left[1 + \int_{\bar{\varepsilon}_{t+1}}^{\varepsilon_{\max}} \left(\varepsilon \frac{Q_{t+1}}{P_{kt+1}} - 1 \right) f(\varepsilon) d\varepsilon \right], \tag{19}$$

and the usual transversality conditions.

Equation (16) shows that investment by efficient firms is financed by four sources: internal funds $R_{kt}K_{jt-1}$, debt collateralized by capital μK_{jt-1}, payoffs from bond holdings B_{jt-1}, and sales of the bubble asset $P_t H_{jt-1}$. Equations (17), (18), and (19) are the asset-pricing equations for capital, the bubble asset, and the bond, respectively. The integral terms in these three equations capture the liquidity premium because capital, the bubble asset (if its price is positive), and the bond can raise the firm's net worth. We focus on the integral term in equation (18). At time $t+1$, when its investment efficiency shock $\varepsilon_{jt+1} \geq \bar{\varepsilon}_{t+1}$, firm j sells its bubble asset holdings to finance real investment. Each dollar of the payoff can generate $(\varepsilon_{jt+1} Q_{t+1}/P_{kt+1} - 1) > 0$

units of profits. Then the credit constraint (6), the short-sale constraint and the equity constraint (8) all bind. When $\varepsilon_{jt+1} < \bar{\varepsilon}_{t+1}$, firm j finds it optimal to make no investment, and these three constraints do not bind. The interpretations of the liquidity premiums provided by capital and bonds are similar. The bubble asset and bonds are perfect substitutes. This insight was first developed by Kiyotaki and Moore (2019) when regarding the bubble asset as fiat money. Note that the investment cutoff $\bar{\varepsilon}_{t+1}$ is identical for all firms, a feature useful for aggregation.

To illustrate the importance of the liquidity premium for the emergence of a bubble, we write the asset-pricing equation for the bubble without the liquidity premium as follows:

$$P_t = \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} P_{t+1}.$$

This equation cannot hold in a deterministic steady state with a positive bubble price for $\beta \in (0,1)$. In other words, the transversality condition will rule out bubbles.

We emphasize that limited external financing (e.g., the borrowing constraint, equity constraint, and short-sale constraint) is needed for the emergence of a bubble. If external financing is unlimited, then in equilibrium, only firms with the highest investment efficiency shock will invest, which will drive down Tobin's Q_t to $P_{kt}/\varepsilon_{\max}$ and hence the liquidity premium to zero. As a result, a bubble will be ruled out by the standard transversality condition.

By equations (18) and (19), we have

$$P_t > \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} P_{t+1} \text{ and } \frac{1}{R_{ft}} > \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t}. \tag{20}$$

These inequalities imply that households will not hold any of the bubble asset or bonds in equilibrium because their returns are too low.

We can show that the stock market value of all firms is given by[10]

$$V_t = \int \mathbb{E}_t \left[\frac{\beta \Lambda_{t+1}}{\Lambda_t} V_{jt+1} \right] dj = Q_t K_t + P_t + \frac{B_t}{R_{ft}}. \tag{21}$$

It consists of three components: capital value $Q_t K_t$, the bubble P_t, and bond value B_t/R_{ft}, of which $Q_t K_t$ and B_t/R_{ft} represent the fundamental components and P_t represents the bubble component. The asset bubble contributes to the stock market fluctuations in two channels: Through the direct channel, the asset bubble P_t affects the stock market value V_t. Through the indirect channel, the bubble raises a firm's investment capacity, as shown in equation (16).

[10]Use equations (A.3)-(A.6) in Appendix A, and apply the market-clearing conditions.

Thus, the asset bubble also amplifies the movements of the fundamental value $Q_t K_t$ through firms' investment.

Note that $P_t = 0$ for all t always satisfies (18). We call such an equilibrium a bubbleless equilibrium. If there is an equilibrium such that $P_t > 0$ for all t and asymptotically $\lim_{t\to\infty} P_t > 0$, we call it a bubbly equilibrium. We will focus on these two types of equilibria in this paper.

3.6 Deterministic Steady-State Equilibria

The economy features a stochastic trend. In Appendix B, we present the detrended equilibrium system. To solve for an equilibrium, we need to understand deterministic steady states, in which all detrended variables are constant over time. We use a lower-case variable to denote the detrended value and a variable without the time subscript t to denote its steady-state value. Our model features two types of steady states: bubbly and bubbleless. We use the subscript f to denote any variable in a bubbleless (fundamental) steady state, while we use the subscript b to denote a variable in a bubbly steady state.

In Appendix C, we provide a detailed steady-state analysis. Here, we informally present key results. We find that the critical condition for the existence of a bubbly steady state is

$$1 < \beta \left[1 + \int_{\bar{\varepsilon}_f}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\bar{\varepsilon}_f} - 1 \right) f(\varepsilon) d\varepsilon \right]. \tag{22}$$

The interpretation is as follows: Suppose that the economy is initially in the bubbleless steady state. The right-hand side of (22) represents the steady-state benefit of purchasing one unit of the bubble asset, while the left-hand side is the associated cost. The benefit comes from the liquidity role of the bubble discussed earlier for (18). If condition (22) holds, then the benefit is larger than the associated cost. Thus, the firm is willing to pay a positive price for the bubble asset. This condition is similar to that in Miao et al. (2015b). The main difference is that here, the foreign interest rate R_f^* affects (22) because it affects the bubbleless steady-state investment threshold $\bar{\varepsilon}_f$.

How is condition (22) related to the traditional bubble existence condition that the interest rate in the bubbleless economy must be lower than the rate of economic growth (Tirole, 1985)? In Appendix C, we show that the bubbleless steady-state domestic interest rate R_{ff} is given by

$$R_{ff} = R_f(\bar{\varepsilon}_f) \equiv \frac{\exp(g)}{\beta \left[1 + \int_{\bar{\varepsilon}_f}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\bar{\varepsilon}_f} - 1 \right) f(\varepsilon) d\varepsilon \right]}. \tag{23}$$

It follows that condition (22) is equivalent to $R_{ff} < \exp(g)$, which is consistent with Tirole's (1985) condition. However, unlike his OLG model, our infinite-horizon economy is not dynamically inefficient since there is no overaccumulation of capital.

In Appendix C, we also show that the smaller the foreign interest rate R_f^*, the more likely a domestic bubble is to emerge in the sense that condition (22) is more likely to hold. When R_f^* is sufficiently high, a bubble cannot emerge. Intuitively, as R_f^* declines, there are capital inflows accompanied by real exchange rate appreciation, lending booms, and investment booms. The increased demand for domestic bonds reduces the domestic interest rate, thereby fueling a bubble. On the other hand, as R_f^* rises, we have reversals. When R_f^* is sufficiently high, the net international investment position is sufficiently high that firms have sufficient liquidity to finance investment. Then, they have no demand for the bubble asset, and a bubble cannot emerge.

4 Quantitative Results

In this section, we analyze the quantitative effects of foreign interest rate shocks on the business cycle properties of a small open economy. Models with rational bubbles typically have a continuum of equilibria. Our model has two deterministic steady states, as discussed in the previous section. As shown in Tirole (1985) and Miao and Wang (2018), the bubbly steady state is a saddle. This implies that there is a unique local equilibrium around the bubbly steady state that will converge to the bubbly steady state in the absence of shocks. The bubbleless steady state is a sink and has indeterminacy of degree one. Namely, there is a continuum of equilibrium paths that converge to the bubbleless steady state. There can also be other types of equilibria such as stochastic bubble equilibria as in Weil (1987), in which bubbles will burst in the next period with some probability. In this paper, we take the unique bubbly equilibrium around the bubbly steady state as a benchmark. We first estimate the log-linearized version of the detrended bubbly equilibrium.[11] Using the calibrated and estimated parameters, we simulate the model and compare the model-generated business cycle moments with those in the data. We then examine the variance decomposition of all shocks and study impulse response functions to illustrate the model mechanism.

[11] Despite the heterogeneous binding patterns of the constraints among individual firms, our model permits exact aggregation by the law of large numbers. The aggregate equilibrium system can be numerically solved by log-linear approximation as in the standard DSGE literature (see Appendix D).

4.1 Parameter Estimates

To make our results comparable with the literature on the business cycles of emerging markets (e.g., Aguiar and Gopinath, 2007, and García-Cicco et al. 2010), we choose Mexico as the domestic country and take Mexican data from Fernández and Gulan (2015).[12] The sample period is from 1990Q1 to 2011Q4, except for foreign interest rates, which are available only from 1994Q1 to 2011Q4. One period in the model corresponds to a quarter. We group all model parameters into two categories.

Calibration The parameters in the first category are either calibrated or taken from the literature. We set the unconditional mean of the domestic growth rate $g = 0.01$, consistent with the annual growth rate of Mexico in the sample period before the financial crisis. As in the literature (e.g., Uribe and Yue, 2006; Aguiar and Gopinath, 2007; and Chang and Fernández, 2013), we set the subjective discount factor $\beta = 0.98$, the capital share $\alpha = 0.34$, and the capital depreciation rate $\delta = 0.05$. We set $\varphi = 2$ so that the Frisch elasticity is 0.5 and set the labor weight parameter ν so that the steady-state labor hours equal 1/3. We set the share of imported goods $\gamma = 0.22$, which is consistent with the data in our sample period. We set the price elasticity of exported domestic goods $\sigma = 1.0$, which lies in the range of the empirical estimates by Feenstra et al. (2014). Suppose that the idiosyncratic investment efficiency shock is drawn from a Pareto distribution $F(\varepsilon) = 1 - \left(\frac{\varepsilon_{min}}{\varepsilon}\right)^{\eta}$. We set $\eta = 8$ and $\varepsilon_{\min} = (\eta - 1)/\eta$ so that the mean of ε is 1, and the investment-to-GDP ratio is approximately 0.2 in the bubbly steady state. This value is in line with the average investment-to-GDP ratio of Mexico in the sample period.

We set the steady-state quarterly (gross) foreign interest rate $R_f^* = 1.007$, which is the median of foreign interest rates faced by Mexican corporations. These interest rates are constructed using the Corporate Emerging Market Bond Index (CEMBI) spreads over the period 2001Q4-2011Q4 (Fernández and Gulan, 2015). We set the parameter for the adjustment costs of capital flow $\Omega = 0.13$ such that the net exports/GDP ratio in the bubbly steady state is -1.2%, as observed on average in Mexico over the sample period.

[Insert Table 2 Here]

[12]The median of the stock market capitalization to GDP ratio is 31% in Mexico in our sample period. This ratio is comparable to a few developed small open economies. For example, in the same sample period, the median of the stock market capitalization to GDP ratio is 34% in Portugal, 37% in New Zealand, and 44% in Norway.

The calibrated parameters are listed in Table 2. All the other parameters are estimated by Bayesian methods. They include parameters governing all of the shock processes, the habit formation parameter h, the aggregate capital adjustment cost parameter Ω_k and the financial friction parameter μ.

Shocks and Data Our model includes five exogenous shocks. The parameters governing these shocks are $\rho_g, \sigma_g, \rho_a, \sigma_a, \rho_\xi, \sigma_\xi, \rho_{y^*}, \sigma_{y^*}, \rho_{R_f^*}$ and $\sigma_{R_f^*}$. We use five quarterly time series of data in our estimation: the demeaned foreign interest rate, the growth rate of the sum of US and Canadian real GDP, and the growth rates of Mexican real GDP, real investment and real consumption. The foreign interest rate of Mexico is constructed using the EMBI index in the same manner as in Section 2. Fernández and Gulan (2015) show that the EMBI index and the CEMBI index are highly correlated. Thus, we use the EMBI index instead of the CEMBI index in our Bayesian estimation because the former has a much larger sample size.

Following Adolfson et al. (2007), Lubik and Schorfheide (2007), and Justiniano and Preston (2010), we use the growth rate of the sum of US and Canadian real GDP to estimate foreign demand shocks, as exports to US and Canada constitute roughly 80% of Mexican total exports.[13] The growth rates of Mexican real GDP, real investment, and real consumption data are informative to estimate the underlying long- and short-run productivity shocks as well as preference shocks. In particular, Aguiar and Gopinath (2007) argue that the long-run productivity shock is the driving force generating a high consumption volatility relative to output, which is one of the defining characteristics of emerging markets.

Notice that to avoid the generic disadvantage of the bubbleless model of generating a large stock price volatility and to allow for a fair comparison between the bubbly and bubbleless models, we do not include asset prices in the data for estimation. All the results on asset prices are endogenously determined in the bubbly and bubbleless models.

Priors and Posteriors We choose the standard prior distributions for h and Ω_k as in the literature. The prior of the collateral parameter μ follows a Beta distribution with mean 0.3 and standard deviation 0.05, similar to Miao et al. (2015a).

Assume that the prior distribution for the persistence parameter for the foreign interest rate shock follows a Beta distribution with mean 0.85 and standard deviation 0.05. The 90 percent

[13] We assume that the foreign demand shock is independent of the foreign interest rate shock. To verify its validity of this assumption in our sample, we extend the baseline VAR exercise with the foreign demand proxy ordered last and find that the response of foreign demand to a foreign interest rate shock is insignificant.

interval of this prior distribution covers the values used in the related literature (e.g. Neumeyer and Perri, 2005; Uribe and Yue, 2006; and Chang and Fernández, 2013). Following Smets and Wouters (2007) and Liu et al. (2013), we assume that the prior distributions of the persistence parameters for all other shocks follow a Beta distribution with mean 0.5 and standard deviation 0.2. The prior distributions of the standard deviation of innovations for all shocks follow an inverse Gamma distribution with a mean 1 percent and standard deviation ∞. We find that our estimates of these parameters are quite robust and not sensitive to the prior distribution.

Information of the prior and posterior distributions of the estimated parameters is listed in Table 3. The modes, means, and 5th and 95th percentiles of the posterior distributions for the estimated parameters are computed using the Metropolis-Hastings algorithm with 200,000 draws.[14]

[Insert Table 3 Here]

We take the posterior modes as the parameter estimates. We find that the estimate for h is 0.56, close to the value of 0.50 estimated by Liu et al. (2013) and the value of 0.61 estimated by Jermann and Quadrini (2012). Our estimate for Ω_k is 2.18, which is in line with the values estimated in other studies (e.g., 2.48 in Christiano et al. 2005 and 2.0 in Liu et al., 2011). Our estimate for the financial market friction parameter μ is 0.30, lying in the range estimated by Covas and Den Hann (2011).

Our estimated foreign interest rate shock is similar to those reported in Neumeyer and Perri (2005) ($\rho_{R_f^*} = 0.81, \sigma_{R_f^*} = 0.63\%$), Uribe and Yue (2006) ($\rho_{R_f^*} = 0.83, \sigma_{R_f^*} = 0.7\%$), and Chang and Fernández (2013) ($\rho_{R_f^*} = 0.81, \sigma_{R_f^*} = 0.42\%$). The estimated foreign demand shock is quite persistent (0.997) and not volatile (0.79%). This is not surprising because Mexico's two largest trade partners – the US and Canada – are two advanced economies and have relatively stable business cycles. As in Liu et al. (2013) and García-Cicco et al. (2010), the estimated preference shock is quite volatile.

The estimates for the long- and short-run productivity shocks are debated in the literature. For example, Aguiar and Gopinath (2007) use Generalized Method of Moments (GMM) based on Mexican quarterly data during 1980-2003 to estimate the role of trend productivity shocks in business cycles. They find that there is little persistence but a high volatility in the long-run productivity shock. Using Mexican annual data for the period 1900-2005, García-Cicco et al.

[14]The Markov Chain Monte Carlo (MCMC) univariate convergence diagnostic shows that our posterior distribution of each parameter constructed from random draws converges to a stationary distribution.

(2010) find that the quarterly persistence of the long-run productivity shock is 0.71 and the quarterly volatility is 0.85%. Using Bayesian methods based on the same sample in Aguiar and Gopinath (2007), Chang and Fernández (2013) find that the persistence of long- and short-run productivity shocks are 0.72 and 0.89, and the standard deviations of innovations are 0.12% and 0.66%. While the unconditional volatility of our estimated long-run productivity shock is at the upper end of the range reported by these studies, the relative size of these two types of shocks lies within the range implied by these studies.

4.2 Estimation of the Bubbleless Model

To compare the bubbly economy with the bubbleless economy, we fix the calibrated parameter values as in the previous section and re-estimate the other parameters based on the bubbleless equilibrium. Note that since the bubble price is endogenous in our model, the two versions of models are estimated with the same degrees of freedom. The estimation results are also given in Table 3. We find that the log marginal densities of data for the baseline bubbly model and the bubbleless model are 1118.2 and 1074.0, respectively, and the log posterior likelihood at the posterior modes for the baseline bubbly model and the bubbleless model are 1167.2 and 1124.6, respectively. Both measures suggest that the data favor the bubbly model given the same prior information.

Now, we compare the posterior modes for the shocks in the two models. We find that the estimates for the volatilities and persistence of the foreign demand shock and the foreign interest rate shock are similar across the two models, indicating that these two shocks are primarily pinned down by the exogenous data series rather than model properties. Notably, the estimates of the other three shocks have large differences: The estimated shocks in the bubbleless model are much larger than those in the bubbly model. Specifically, the unconditional volatilities of the long-run productivity shock, short-run productivity shock and preference shock in the bubbleless model are 4.0 times, 2.5 times and 14.7 times larger than their counterparts in the bubbly model, respectively.

The key reason is that the bubbleless model does not have a large amplification mechanism generated by asset bubbles. Thus it needs large shocks, together with smaller capital adjustment costs (0.09 in the bubbleless model vs. 2.18 in the bubbly model), to match the high volatility of investment relative to output in the data.

4.3 Business Cycle Moments

In this section, we simulate the benchmark bubbly model with the calibrated and estimated parameters and then compare the model-generated real business cycle moments with those observed in the data. We also compare our benchmark model with two bubbleless models. Bubbleless model 1 uses the same parameter values as the benchmark model but focuses on the bubbleless equilibrium. Bubbleless model 2 uses the re-estimated parameter values as in Section 4.2. We report the results in Table 4. All variables except the net exports/GDP ratio are in logs and Hodrick-Prescott (HP) filtered. The net exports/GDP ratio is directly HP filtered.

[**Insert Table 4 Here**]

We find that our benchmark bubbly model matches the business cycle volatility, comovement and autocorrelation in the data quite well. One weakness is that our bubbly model overpredicts the volatility of the net exports/GDP ratio (2.76% vs. 1.61%) and underpredicts the stock price volatility relative to GDP (4.82 vs. 6.39) given that these data are out of sample for our estimation. By contrast, both bubbleless models perform much worse, especially along the dimension related to stock prices. The key reason is that both of these models lack the propagation and amplification mechanisms generated by the asset bubble. To understand the role of the asset bubble, we first consider bubbleless model 1 by shutting down the bubble channel in the benchmark model using the same parameter values. We find that bubbleless model 1 explains less investment volatility and only 35% of stock market volatility in the data.

Strong countercyclicality of the trade balance is a striking feature that distinguishes emerging economies from developed economies, which show mild countercyclical or procyclical trade balance. For example, according to Neumeyer and Perri (2005), the correlation between the net exports/GDP ratio and GDP is -0.61 on average in emerging markets, while the correlation is -0.23 on average in developed economies. Similarly, Aguiar and Gopinath (2007) report that the average correlation for emerging markets is -0.51, while the average correlation for developed economies is only -0.17. Furthermore, Fernández and Gulan (2015) document that the average correlation between the net exports/GDP ratio and GDP is -0.40 in emerging economies and 0.33 in developed economies. The correlation between the net exports/GDP ratio and GDP is -0.53 in our estimated bubbly model and -0.50 in the data. The correlation between the net exports/GDP ratio and stock prices is -0.66 in the model, which is also comparable to the value of -0.46 in the data. In addition, the autocorrelation of the net exports/GDP generated by the bubbly model is 0.66, lying in the reasonable range suggested by García-Cicco et al. (2010).

It is evident from the fourth column of Table 4 that asset bubbles are the key mechanism for the success of the bubbly model. When the bubble channel is shut down, the correlation between the net exports/GDP ratio and GDP becomes −0.14, too weak to properly reflect the data. The correlation between stock prices and consumption becomes −0.46, producing an incorrect sign relative to those in the bubbly model and the data. Furthermore, the correlation between the net exports/GDP ratio and stock prices becomes 0.71, which is clearly in conflict with the data.

To give the bubbleless model a fairer chance to match the data, we consider bubbleless model 2 with re-estimated parameter values. Because of the lack of the amplification and propagation mechanism generated by the asset bubble, as discussed in Section 4.2, the estimated shocks of productivity and intertemporal preference are much larger than those in the benchmark model. As a result, the model-implied GDP volatility is too high, while the model-implied stock market volatility is too low, compared to the data.

4.4 Variance Decomposition

In this section, we study the relative importance of the five shocks in our models using variance decomposition. Specifically, we calculate the fractions of the movements in the impulse responses generated by each shock, with one-standard-deviation innovation each. The results for the bubbly economy are reported in Table 5.

[Insert Table 5 Here]

Consistent with the literature (e.g., Aguiar and Gopinath, 2007; Chang and Fernández, 2013; and Fernández and Gulan, 2015), long-run productivity shocks play an important role in explaining variations in output, investment, consumption, the net exports/GDP ratio, and stock prices. Short-run productivity shocks are also important to explain variations in output, investment, and stock prices but not as important for variations in consumption and the net exports/GDP ratio. Preference shocks explain a large fraction of consumption movements in the short and medium run. Since the estimated foreign demand shocks are small, they do not contribute substantially to Mexican business cycles, generally less than 4%.

Notably, foreign interest rate shocks play a nontrivial role, particularly in explaining the variations in the net exports/GDP ratio (33.9% in the impact period), stock prices (27.9% in the impact period), and investment (23.6% in the impact period). Foreign interest rate shocks are transmitted through the asset bubble channel in our model. Our finding is consistent with previous studies on the importance of foreign interest rate shocks. For example, Neumeyer and Perri (2005) report that output volatility declines by about 30% when eliminating foreign interest rate shocks. By applying a VAR analysis to seven emerging markets, Martin and Yue (2006) document that foreign interest rate shocks explain about one-third of movements in aggregate activity and about 43% of movements in the trade balance/output ratio. Based on a Bayesian estimation of an annual sample of Argentine data, García-Cicco et al. (2010) find that country premium shocks are responsible for substantial fractions of investment growth (62.4%) and variations in the trade balance/GDP ratio (78.9%) but only small fractions of output growth (2.9%) and consumption growth (5.2%). Chang and Fernández (2013) report that in Mexico foreign interest rate shocks are responsible for 5.1% of output growth variation, 9.2% of consumption growth variation, 22.2% of investment growth variation and 41.2% of the change in the trade balance/GDP ratio.

[Insert Tables 6-7 Here]

For comparison, we report the variance decomposition for bubbleless model 1 in Table 6 and for bubbleless model 2 in Table 7. Obviously, the contribution of the foreign interest rate shocks in bubbleless model 1 is remarkably smaller than that in the bubbly model for each key macro variable. Instead, short-run productivity shocks and preference shocks in this bubbleless model contribute more to explaining the variation in all variables. In particular, the preference shock dictates the movements of consumption and explains the negative correlation between consumption and stock prices generated by bubbleless model 1, a prediction inconsistent with the data (Table 4). In bubbleless model 2, given the absence of asset bubbles and the larger estimated size of other shocks, the contribution of foreign interest rate shocks is also generally negligible, as shown in Table 7.

What is the contribution of foreign interest rate shocks to the movement of the historical paths of Mexican aggregates? To answer this question, we compute the historical paths of GDP, consumption, investment, and the net exports/GDP ratio implied by our bubbly model when all shocks are turned on and when the foreign interest rate shock alone is turned off.[15] We find that the foreign interest rate shocks explain about 9.7% of movements in GDP, 16.5%

[15]Note that when all shocks are turned on, our estimated paths fit the actual data almost exactly.

of movements in consumption, 10.6% of movements in investment and 33.1% of movements in the net exports/GDP ratio, which are roughly consistent with our forecasting variance decomposition in Table 5. For the real stock prices which are not included in the estimation, our model-generated path of the bubbly economy with all shocks turned on is correlated with the actual data at 0.84 (after taking logarithm), while the path with the foreign interest rate shock alone turned off is correlated with the actual data at 0.56.[16]

4.5 Impulse Responses

We now present impulse response functions for the long- and short-run productivity shocks and foreign interest rate shocks to understand our model mechanism because these are the most important shocks for explaining Mexican business cycles as shown in the previous section. Figures 3 through 5 present the results for the benchmark bubbly model (solid lines) and for bubbleless model 1 (dashed lines). All variables are expressed as the percent deviation from their deterministic balanced growth values, except for the interest rate and net exports/GDP ratio, which are in level deviation.

[Insert Figures 3-4 Here]

Figures 3 and 4 show that the long- and short-run productivity shocks have qualitatively similar impulse response properties. The main difference is that there is a trend effect for the long-run productivity shock. In response to a positive shock to the long-run productivity, the demand for investment increases, and the asset bubble expands on impact, which raises firms' net worth and allows them to make more investment. This generates a large amplification effect, in contrast to the bubbleless economy, which displays much weaker responses of consumption, investment, output, and stock prices.

Moreover, the real exchange rate and net exports exhibit different responses in the bubbly and bubbleless economies, which explains the stronger countercyclicality in the bubbly model than in the bubbleless model. In particular, for the bubbleless economy, in response to a positive productivity shock, output rises on impact, thereby raising the supply of domestic goods in the international market and leading to a real depreciation and greater exports. Then

[16]For bubbleless model 2, the correlation between the model-generated real stock prices with all shocks turned on and the actual data is 0.56, indicating weaker fitness than the bubbly model. The correlation between the model-generated stock prices with the foreign interest rate shock alone turned off and the actual data is 0.55. The limited difference between these two correlations implies a negligible role of foreign interest rate shocks in bubbleless model 2. This is not surprising given the large sizes of other shocks, as discussed in Section 4.2.

the net exports/GDP ratio becomes larger, implying a positive correlation between the net exports/GDP ratio and GDP. By contrast, due to the amplification and propagation effects of the asset bubble, both domestic investment and consumption in the bubbly economy increase much more than those in the bubbleless economy in response to a positive shock to long- or short-run productivity, implying greater domestic demand for domestic goods and hence real appreciation (to a long-run productivity shock) or relatively weaker depreciation (to a short-run productivity shock). Thus, the net exports/GDP ratio decreases while domestic output increases, generating a negative correlation between the net exports/GDP ratio and output.

[**Insert Figure 5 Here**]

Figure 5 presents the impulse responses to a positive foreign interest rate shock. In response to this shock, bankers tend to borrow from domestic firms by selling bonds to invest in the international financial market, meaning that capital begins to flow out of the domestic economy. The rise of the foreign interest rate reduces the demand for the domestic bond, leading the domestic interest rate to rise. This in turn reduces the demand for the bubble asset. Capital outflows reduce the resources available for domestic firms to invest in the bubble asset, which further reduces the demand for the bubble asset. As a result, the asset bubble dampens substantially, leading to a decline in net worth, investment, consumption, output, and stock prices.

Moreover, a higher foreign interest rate also implies greater demand for foreign goods to make financial investments, causing real depreciation. Real depreciation increases exports and decreases imports. As a result, the trade balance increases.

We emphasize that the interactions of capital flows, the domestic interest rate, and the real exchange rate are reinforced by asset bubbles. Capital outflows cause real exchange rate depreciation, which causes domestic output and consumption to decline. At the same time, the domestic interest rate has to rise. When the asset bubble exists, shrinking of the asset bubble reduces the net worth of firms, and hence investment and consumption substantially decline. This exaggerates the real exchange rate depreciation, causing domestic output, investment, and consumption to decline further. As a result, the domestic interest rate has to rise further to induce savings. This further rise in the domestic interest rate causes the price of the bubble asset to decline further. This loop continues, given the amplification effect of bubbles in an open economy, and explains why consumption falls more than output in the bubbly economy.

Such strong interactions among foreign interest rates, real exchange rates, and the domestic macroeconomy in emerging markets are also found in empirical studies. Calvo et al. (1993) show that foreign interest rate shocks are a major source of capital inflows and real exchange rate appreciation in Latin America. They also find that during a period of low US interest rates, an important part of capital inflows to Mexico financed increased private investment. Canova (2005) documents that a US contractionary monetary shock significantly and simultaneously increases Latin American interest rates and US monetary policy shocks cause strong fluctuations in Latin American output. Maćkowiak (2007) reports that domestic interest rates increase and real exchange rates depreciate in multiple emerging markets following a US monetary policy contraction, a pattern that is economically significant and robust.

Consistent with the stylized facts of "sudden stops" in emerging markets, Tobin's Q and stock prices fall on impact in response to a positive foreign interest rate shock. This pattern can hardly be generated in typical real business cycle models, in which the capital stock falls in recession periods, yielding an increase in the marginal product of capital. Since Tobin's Q reflects the present value of the marginal product of capital, it must rise. In our model, however, the marginal product of capital falls due to real depreciation (see equation (4)). Mendoza (2010) also generates this result in a small open economy with an occasionally binding borrowing constraint. Unlike in his model, the stock market value of firms in our model contains the market value of both capital and the bubble, as shown in equation (21). It decreases sharply on impact due to the collapse of the asset bubble and decreased Tobin's Q.

The responses of the bubbleless economy shown in Figure 5 are much weaker than those of the bubbly economy in terms of magnitude, although they are all in the same directions as those in the bubbly economy. This is because without asset bubbles, the negative net worth effect is much weaker. In other words, the amplification effect on the interactions among capital flows, the domestic interest rate and the real exchange rate analyzed above is absent here. As a result, the increase in the domestic interest rate and the real depreciation are much milder in the bubbleless model than in the bubbly model, which explains why the countercyclicality of the trade balance/GDP ratio delivered by the bubbleless model is much weaker than that delivered by the bubbly model.

We now investigate whether the impulse responses of our two estimated models to a rise in the foreign interest rate are consistent with those in the data through VAR estimation. Figure 6 plots the data-implied impulse responses using our Mexican sample, along with its

95% confidence intervals, versus the impulse responses from our models. The variables with trends are reported in growth rates, and variables without trends are in levels. The figure illustrates that the bubbly model-implied impulse responses track the data-implied impulse responses well and almost lie inside the 95% confidence intervals. One exception is that the contemporaneous response of the net exports/GDP ratio is larger than that in the data, which is a limitation also suggested by the business cycle moments in Table 4. In contrast, the immediate responses from our bubbleless model 2 are significantly smaller and often outside the confidence intervals. Considering that only a subset of the variables reported here is included in our estimation (recall that the data on stock prices, the net exports/GDP ratio, the real exchange rate and the relative price of investment are not used in the estimation), this post-estimation check convincingly suggests that the bubbly model outperforms its bubbleless counterpart in quantitatively capturing the impact of a foreign interest rate shock.

[**Insert Figure 6 Here**]

4.6 Introducing Sentiment Shocks

As shown in Table 4, the benchmark model accounts for approximately 75% of stock market volatility. This is reasonably good since we do not include any data related to stock prices in the estimation. However, this fit can be further improved by introducing sentiment shocks into the bubbly model as in Martin and Ventura (2012), Galí (2014), Miao et al. (2015a), and Dong et al. (2020). Instead of providing a detailed derivation, we directly present the equilibrium system. In particular, the asset pricing equation (18) for the bubble becomes

$$P_t = \beta \chi_t \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} P_{t+1} \left[1 + \int_{\bar{\varepsilon}_{t+1}}^{\varepsilon_{max}} \left(\varepsilon \frac{Q_{t+1}}{P_{kt+1}} - 1 \right) f(\varepsilon) d\varepsilon \right],$$

where χ_t represents a sentiment shock, which is assumed to follow an AR(1) stochastic process $\ln(\chi_t) = \rho_\chi \ln(\chi_{t-1}) + (1-\rho_\chi)\ln(\chi) + \sigma_\chi \varepsilon_{\chi t}$, $\rho_\chi \in (-1, 1)$, $\sigma_\chi > 0$, and $\varepsilon_{\chi t}$ is an IID standard normal random variable. All other equilibrium equations are unchanged.

We then re-estimate this version of the bubbly model by including the growth rate of Mexican real stock prices in the data. We find that the log marginal densities of data increases from 1118.2 for the benchmark model to 1141.4, and the log posterior likelihood at the posterior modes increases from 1167.2 for the benchmark model to 1217.6, suggesting an obvious improvement in the model fit to the data. We also find that the estimated model with sentiment shocks can generate a higher stock market volatility because sentiment shocks directly affect stock prices, while other business cycle moments are similar to the model without sentiment shocks. We present the detailed results in online Appendix E.

5 Conclusion

We present a DSGE model of a small open economy with asset bubbles. We find that asset bubbles provide a powerful amplification and propagation mechanism. They are important in explaining the high stock market volatility and the comovement between the stock market and the real economy. In addition to the long-run productivity shock found to be important in the literature, our Bayesian estimation shows that the foreign interest rate shock is important in driving the movements of stock prices and macro quantities in the Mexican economy. The foreign interest rate shock explains approximately 20-30% of the variation in the stock market in Mexican data for the period 1990Q1-2011Q4.

Appendix

A Proof of Proposition 1

We consider firm j's decision problem. Solving the static labor and imported material choice problem in (3) gives demand for labor and imported material

$$N_{jt} = A_t^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{W_t}\right)^{\frac{1-\gamma}{\alpha}} \left(\frac{\gamma}{e_t}\right)^{\frac{\gamma}{\alpha}} K_{jt-1}, \tag{A.1}$$

$$M_{jt} = A_t^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{W_t}\right)^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{\gamma}{e_t}\right)^{\frac{\alpha+\gamma}{\alpha}} K_{jt-1}. \tag{A.2}$$

Substituting these equations into (3) yields (4).

Now we solve the firm's dynamic problem. Conjecture that the value function takes the following form

$$V_t(\varepsilon_{jt}, K_{jt-1}, H_{jt-1}, B_{jt-1}) = \phi_t^K(\varepsilon_{jt})K_{jt-1} + \phi_t^H(\varepsilon_{jt})H_{jt-1} + \phi_t^B(\varepsilon_{jt})B_{jt-1}, \tag{A.3}$$

where $\phi_t^K(\varepsilon_{jt})$, $\phi_t^H(\varepsilon_{jt})$ and $\phi_t^B(\varepsilon_{jt})$ are to be determined. In this case Tobin's Q satisfies

$$Q_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \int \phi_{t+1}^K(\varepsilon) dF(\varepsilon). \tag{A.4}$$

Conjecture that

$$P_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \int \phi_{t+1}^H(\varepsilon) dF(\varepsilon), \tag{A.5}$$

$$\frac{1}{R_{ft}} = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \int \phi_{t+1}^B(\varepsilon) dF(\varepsilon). \tag{A.6}$$

Substituting the flow-of-funds constraint and the conjectured value function into the right-hand side of the Bellman equation, we obtain

$$\begin{aligned} & D_{jt} + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}\left(\varepsilon_{jt+1}, K_{jt}, H_{jt}, B_{jt}\right) \\ = \; & R_{kt}K_{jt-1} - P_{kt}I_{jt} - \frac{B_{jt}}{R_{ft}} + B_{jt-1} + P_t\left(H_{jt-1} - H_{jt}\right) + (1-\delta)Q_tK_{jt-1} + \varepsilon_{jt}Q_tI_{jt} + \frac{B_{jt}}{R_{ft}} + P_tH_{jt} \\ = \; & R_{kt}K_{jt-1} + (1-\delta)Q_tK_{jt-1} + B_{jt-1} + P_tH_{jt-1} + \left(\varepsilon_{jt}Q_t - P_{kt}\right)I_{jt}. \end{aligned} \tag{A.7}$$

If $\varepsilon_{jt} < P_{kt}/Q_t$, the firm will not invest, i.e. $I_{jt} = 0$. And the firm is indifferent between saving and borrowing, and is indifferent between purchasing and selling the bubble. If $\varepsilon_{jt} \geq P_{kt}/Q_t$, the firm makes real investment as much as possible. Thus it exhausts the borrowing limit and sells the bubble to finance investment. By (6), (7), (8) and $H_{jt} \geq 0$, we have

$$\begin{aligned} P_{kt}I_{jt} & \leq R_{kt}K_{jt-1} - \frac{B_{jt}}{R_{ft}} + B_{jt-1} + P_t\left(H_{jt-1} - H_{jt}\right) \\ & \leq R_{kt}K_{jt-1} + \mu K_{jt-1} + B_{jt-1} + P_tH_{jt-1}. \end{aligned}$$

We then obtain (16).

Plugging the decision rules in the Bellman equation, we obtain

$$V_t(\varepsilon_{jt}, K_{jt-1}, H_{jt-1}, B_{jt-1}) = \begin{cases} R_{kt}K_{jt-1} + (1-\delta)K_{jt-1} + B_{jt-1} + P_tH_{jt-1} \\ +(\varepsilon_{jt}Q_t/P_{kt} - 1)\left(R_{kt}K_{jt-1} + \mu K_{jt-1} + B_{jt-1} + P_tH_{jt-1}\right), & \text{if } \varepsilon_{jt} \geq \bar{\varepsilon}_t; \\ R_{kt}K_{jt-1} + (1-\delta)K_{jt-1} + B_{jt-1} + P_tH_{jt-1}, & \text{if } \varepsilon_{jt} < \bar{\varepsilon}_t. \end{cases}$$

Matching coefficients in the preceding equation and equation (A.3) and making use of equation (A.4), (A.5) and (A.6) yield the equations in Proposition 1. Q.E.D.

B Equilibrium System

After aggregating individual decision rules and imposing market-clearing conditions, we obtain the equilibrium system shown by the following proposition.

Proposition 2 *The equilibrium system is given by the following equations: (1), (12), (15), (17), (18), (19),* $\bar{\varepsilon}_t Q_t = P_{kt}$,

$$\begin{aligned} P_{kt} &= 1 + \frac{\Omega_k}{2}\left(\frac{I_t}{I_{t-1}} - \exp(g)\right)^2 + \Omega_k\left(\frac{I_t}{I_{t-1}} - \exp(g)\right)\frac{I_t}{I_{t-1}} \\ &\quad -\beta\Omega_k\mathbb{E}_t\frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{I_{t+1}}{I_t} - \exp(g)\right)\left(\frac{I_{t+1}}{I_t}\right)^2, & \text{(B.1)} \\ 0 &= \beta\mathbb{E}_t\frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{R^*_{ft}}{R_{ft}}\frac{e_{t+1}}{e_t} - 1\right) - \frac{\Omega}{Y_t}\left(B_t - A_{t-1}b\right), & \text{(B.2)} \\ I_t &= \frac{1}{P_{kt}}\left(R_{kt}K_{t-1} + \mu K_{t-1} + B_{t-1} + P_t\right)\int_{\bar{\varepsilon}_t}^{\varepsilon_{\max}} f(\varepsilon)d\varepsilon, & \text{(B.3)} \\ K_t &= (1-\delta)K_{t-1} + \frac{1}{P_{kt}}\left(R_{kt}K_{t-1} + \mu K_{t-1} + B_{t-1} + P_t\right)\int_{\bar{\varepsilon}_t}^{\varepsilon_{\max}} \varepsilon f(\varepsilon)d\varepsilon, & \text{(B.4)} \\ Y_t &= K_{t-1}^{\alpha}M_t^{\gamma}\left(A_tN_t\right)^{1-\alpha-\gamma}, & \text{(B.5)} \\ e_tM_t &= \gamma Y_t, & \text{(B.6)} \\ N_t &= A_t^{\frac{1-\alpha-\gamma}{\alpha}}\left(\frac{1-\alpha-\gamma}{W_t}\right)^{\frac{1-\gamma}{\alpha}}\left(\frac{\gamma}{e_t}\right)^{\frac{\gamma}{\alpha}}K_{t-1}, & \text{(B.7)} \\ R_{kt} &= \alpha K_{t-1}^{\alpha-1}M_t^{\gamma}\left(A_tN_t\right)^{1-\alpha-\gamma}, & \text{(B.8)} \\ \nu\xi_tN_t^{\varphi} &= W_t\Lambda_t, & \text{(B.9)} \\ X_t - e_tM_t &= \frac{B_t}{R_{ft}} - B_{t-1}\frac{R^*_{ft-1}}{R_{ft-1}}\frac{e_t}{e_{t-1}}. & \text{(B.10)} \end{aligned}$$

for the endogenous variables $\{Q_t, \bar{\varepsilon}_t, P_t, R_{ft}, B_t, I_t, K_t, Y_t, M_t, C_t, N_t, X_t, e_t, W_t, R_{kt}, \Lambda_t, P_{kt}\}$. *The usual transversality conditions hold.*

Equation (B.3) gives aggregate investment, which is financed by internal funds $R_{kt}K_{t-1}$, debt collateralized by physical capital μK_{t-1}, sales of the bubble asset P_t, and the repayment from bonds B_{t-1}. The integral term in this equation reflects the fact that only firms with efficiency levels higher than $\bar{\varepsilon}_t$ make investment. If there is a bubble $P_t > 0$, then firms with their efficiency levels higher than $\bar{\varepsilon}_t$ will make more investment. This is the crowd-in effect of the asset bubble. On the other hand, firms with their efficient levels lower than $\bar{\varepsilon}_t$ have to buy the bubble from efficient firms and do not make investment. This is the crowd-out effect. The net effect of an asset bubble on aggregate investment depends on the relative strength of the crowd-in and crowd-out effects.

Equation (B.4) gives the law of motion for aggregate capital. Equation (B.5) gives aggregate output. Equation (B.6) shows that the imports/output ratio is equal to γ due to the Cobb-Douglas production function. Thus GDP in our model is given by

$$GDP_t = C_t + \left[1 + \frac{\Omega_k}{2}\left(\frac{I_t}{I_{t-1}} - \exp(g)\right)^2\right] I_t + X_t - e_t M_t = (1-\gamma)Y_t.$$

Equations (B.7) and (B.9) give the labor demand and supply relations. Equation (B.8) shows that R_{kt} is equal to the marginal product of capital or the rental rate. Equation (B.10) endogenously determines the real exchange rate e_t. $NX_t = X_t - e_t M_t$ represents the trade balance or net exports.

Proof of Proposition 2 Equation (B.1) follows from the first-order condition for the capital goods producer's decision problem. Equation (B.2) follows from the first-order condition for the banker's decision problem. We use the decision rule in Proposition 1 and the Law of Large Numbers to derive aggregate investment in equation (B.3)

$$\begin{aligned} I_t &= \int_{\varepsilon_{jt}<\bar{\varepsilon}_t} 0 \cdot dj + \int_{\varepsilon_{jt}\geq\bar{\varepsilon}_t} \frac{1}{P_{kt}} \left(R_{kt}K_{jt-1} + \mu K_{jt-1} + B_{jt-1} + P_t H_{jt-1}\right) dj \\ &= \frac{1}{P_{kt}} \left(R_{kt}K_{t-1} + \mu K_{t-1} + B_{t-1} + P_t\right) \left(\int_{\bar{\varepsilon}_t}^{\varepsilon_{max}} f(\varepsilon) d\varepsilon\right), \end{aligned}$$

where the last equality is due to the fact that ε is IID. Equation (B.4) follows from aggregating equation (5).

Substituting (A.1) into the expression of aggregate labor, we have

$$\begin{aligned} N_t &= \int_0^1 N_{jt} dj = \int_0^1 A_t^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{W_t}\right)^{\frac{1-\gamma}{\alpha}} \left(\frac{\gamma}{e_t}\right)^{\frac{\gamma}{\alpha}} K_{jt-1} dj \\ &= A_t^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{W_t}\right)^{\frac{1-\gamma}{\alpha}} \left(\frac{\gamma}{e_t}\right)^{\frac{\gamma}{\alpha}} K_{t-1}. \end{aligned}$$

Equation (B.7) follows from the preceding equation.

Substituting (A.2) into the expression of aggregate imported material, we have

$$\begin{aligned} M_t &= \int_0^1 M_{jt} dj = \int_0^1 A_t^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{W_t} \right)^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{\gamma}{e_t} \right)^{\frac{\alpha+\gamma}{\alpha}} K_{jt-1} dj \\ &= A_t^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{W_t} \right)^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{\gamma}{e_t} \right)^{\frac{\alpha+\gamma}{\alpha}} K_{t-1}. \end{aligned} \tag{B.11}$$

Substituting (A.1) and (A.2) into the expression of the aggregate output Y_t, we can derive

$$\begin{aligned} Y_t &= \int_0^1 K_{jt-1}^{\alpha} (A_t N_{jt})^{1-\alpha-\gamma} M_{jt}^{\gamma} dj \\ &= \int_0^1 K_{jt-1}^{\alpha} A_t^{1-\alpha-\gamma} A_t^{\frac{1-\alpha-\gamma}{\alpha}(1-\alpha-\gamma)} \left(\frac{1-\alpha-\gamma}{W_t} \right)^{\frac{1-\gamma}{\alpha}(1-\alpha-\gamma)} \left(\frac{\gamma}{e_t} \right)^{\frac{\gamma}{\alpha}(1-\alpha-\gamma)} K_{jt-1}^{1-\alpha-\gamma} \\ &\quad A_t^{\frac{1-\alpha-\gamma}{\alpha}\gamma} \left(\frac{1-\alpha-\gamma}{W_t} \right)^{\frac{1-\alpha-\gamma}{\alpha}\gamma} \left(\frac{\gamma}{e_t} \right)^{\frac{\alpha+\gamma}{\alpha}\gamma} K_{jt-1}^{\gamma} dj \\ &= A_t^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{W_t} \right)^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{\gamma}{e_t} \right)^{\frac{\gamma}{\alpha}} K_{t-1}. \end{aligned}$$

Using the last equation and the preceding expressions of N_t and M_t, we can derive equations (B.5) and (B.6).

We substitute the flow-of-funds constraints for the firms, capital goods producers and bankers, (7), (9) and (10), into the budget constraint of the households (11), and obtain

$$C_t = Y_t - e_t M_t - \left[1 + \frac{\Omega_k}{2} \left(\frac{I_t}{I_{t-1}} - \exp(g) \right)^2 \right] I_t - \frac{B_t}{R_{ft}} + B_{t-1} \frac{R_{ft-1}^*}{R_{ft-1}} \frac{e_t}{e_{t-1}}.$$

Then comparing it with the resource constraint (15) will directly lead to (B.10). Finally, we use (4), (B.7), and (B.11) to derive (B.8). Q.E.D.

Detrended Equilibrium System We can verify that the equilibrium variables N_t, e_t, $\bar{\varepsilon}_t$, R_{kt}, R_{ft}, Q_t, and P_{kt} do not have a trend, while all other equilibrium variables grow at the stochastic rate of productivity A_t except for Λ_t and Y_t^*. The variable Λ_t declines with A_t, and Y_t^* grows at the constant rate g. We thus denote $\lambda_t = \Lambda_t A_t$ and normalize any other growing variable, say Z_t, by A_t and use a lower case variable $z_t = Z_t / A_t$ to denote the detrended value. We then obtain the detrended equilibrium system:

$$Q_t = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\exp(g_{at+1})} \left[R_{kt+1} + (1-\delta) Q_{t+1} + (R_{kt+1} + \mu) \int_{\bar{\varepsilon}_{t+1}}^{\varepsilon_{\max}} \left(\varepsilon \frac{Q_{t+1}}{P_{kt+1}} - 1 \right) f(\varepsilon) d\varepsilon \right], \tag{B.12}$$

$$p_t = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} p_{t+1} \left[1 + \int_{\bar{\varepsilon}_{t+1}}^{\varepsilon_{\max}} \left(\varepsilon \frac{Q_{t+1}}{P_{kt+1}} - 1 \right) f(\varepsilon) d\varepsilon \right], \tag{B.13}$$

$$\frac{1}{R_{ft}} = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\exp(g_{at+1})} \left[1 + \int_{\bar{\varepsilon}_{t+1}}^{\varepsilon_{\max}} \left(\varepsilon \frac{Q_{t+1}}{P_{kt+1}} - 1 \right) f(\varepsilon) d\varepsilon \right], \tag{B.14}$$

$$0 = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\exp(g_{at+1})} \left(\frac{R^*_{ft}}{R_{ft}} \frac{e_{t+1}}{e_t} - 1 \right) - \frac{\Omega}{y_t} \left(b_t - \frac{b}{\exp(g_{at})} \right), \tag{B.15}$$

$$i_t = \frac{1}{P_{kt}} \left[R_{kt} \frac{k_{t-1}}{\exp(g_{at})} + \mu \frac{k_{t-1}}{\exp(g_{at})} + \frac{b_{t-1}}{\exp(g_{at})} + p_t \right] \left(\int_{\bar{\varepsilon}_t}^{\varepsilon_{\max}} f(\varepsilon) d\varepsilon \right), \tag{B.16}$$

$$k_t = \frac{1-\delta}{\exp(g_{at})} k_{t-1} + \frac{1}{P_{kt}} \left[R_{kt} \frac{k_{t-1}}{\exp(g_{at})} + \mu \frac{k_{t-1}}{\exp(g_{at})} + \frac{b_{t-1}}{\exp(g_{at})} + p_t \right] \left(\int_{\bar{\varepsilon}_t}^{\varepsilon_{max}} \varepsilon f(\varepsilon) d\varepsilon \right), \tag{B.17}$$

$$\begin{aligned} P_{kt} = {} & 1 + \frac{\Omega_k}{2} \left(\exp(g_{at}) \frac{i_t}{i_{t-1}} - \exp(g) \right)^2 + \Omega_k \left(\exp(g_{at}) \frac{i_t}{i_{t-1}} - \exp(g) \right) \exp(g_{at}) \frac{i_t}{i_{t-1}} \\ & - \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \Omega_k \left(\exp(g_{at+1}) \frac{i_{t+1}}{i_t} - \exp(g) \right) \exp(g_{at+1}) \left(\frac{i_{t+1}}{i_t} \right)^2 \end{aligned} \tag{B.18}$$

$$y_t = \left(\frac{k_{t-1}}{\exp(g_{at})} \right)^{\alpha} m_t^{\gamma} N_t^{1-\alpha-\gamma}, \tag{B.19}$$

$$e_t m_t = \gamma y_t, \tag{B.20}$$

$$y_t = c_t + \left[1 + \frac{\Omega_k}{2} \left(\exp(g_{at}) \frac{i_t}{i_{t-1}} - \exp(g) \right)^2 \right] i_t + x_t, \tag{B.21}$$

$$x_t = e_t^{\sigma} \frac{\exp(gt)}{A_t} y_t^*, \tag{B.22}$$

$$\bar{\varepsilon}_t Q_t = P_{kt}, \tag{B.23}$$

$$\nu \xi_t N_t^{\varphi} = w_t \lambda_t, \tag{B.24}$$

$$\lambda_t = \frac{\xi_t}{c_t - h \frac{c_{t-1}}{\exp(g_{at})}} - \beta h \mathbb{E}_t \frac{\xi_{t+1}}{\exp(g_{at+1}) c_{t+1} - h c_t} \tag{B.25}$$

$$x_t - e_t m_t = \frac{b_t}{R_{ft}} - \frac{b_{t-1}}{\exp(g_{at})} \frac{R^*_{ft-1}}{R_{ft-1}} \frac{e_t}{e_{t-1}}, \tag{B.26}$$

$$N_t = \left(\frac{1-\alpha-\gamma}{w_t} \right)^{\frac{1-\gamma}{\alpha}} \left(\frac{\gamma}{e_t} \right)^{\frac{\gamma}{\alpha}} \frac{k_{t-1}}{\exp(g_{at})}, \tag{B.27}$$

$$R_{kt} = \alpha \exp(g_{at}) \frac{y_t}{k_{t-1}}, \tag{B.28}$$

for the endogenous variables $\{Q_t, \bar{\varepsilon}_t, p_t, R_{ft}, b_t, i_t, k_t, y_t, m_t, c_t, x_t, e_t, w_t, R_{kt}, P_{kt}, N_t, \lambda_t\}$. The growth of the aggregate productivity is

$$A_t / A_{t-1} \equiv \exp(g_{at}) = \exp(g_t) \exp(a_t) / \exp(a_{t-1}). \tag{B.29}$$

In the deterministic steady state $g_{at} = g$.

C Steady State

We first derive some common equations in both types of steady state. We use equation (17) and $Q = P_k/\bar{\varepsilon}$ where $P_k = 1$ to derive the steady-state marginal product of capital

$$R_k = \frac{\exp(g) - \beta + \beta\delta - \beta\mu\left(\int_{\bar{\varepsilon}}^{\varepsilon_{\max}} (\varepsilon - \bar{\varepsilon}) f(\varepsilon)d\varepsilon\right)}{\beta\left[\bar{\varepsilon} + \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} (\varepsilon - \bar{\varepsilon}) f(\varepsilon)d\varepsilon\right]}. \tag{C.1}$$

We then use (19) to derive the steady-state domestic interest rate

$$R_f = \frac{\exp(g)}{\beta\left[1 + \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\bar{\varepsilon}} - 1\right) f(\varepsilon)d\varepsilon\right]}.$$

It is determined by the domestic economic growth rate, subjective discount factor, and the liquidity premium. Both R_k and R_f are functions of $\bar{\varepsilon}$, which are denoted by $R_k(\bar{\varepsilon})$ and $R_f(\bar{\varepsilon})$. The two types of steady state differ in the investment threshold $\bar{\varepsilon}$. We derive this threshold and other equilibrium variables later.

The following lemma is useful for the analysis.

Lemma 1 *The asset-to-output ratio in a steady state is*

$$\frac{b}{y} = \frac{\beta\left(R_f^*/R_f(\bar{\varepsilon}) - 1\right)}{(\exp(g) - 1)\,\Omega}, \tag{C.2}$$

where R_f is given by (23).

Proof of Lemma 1 By (19), we could derive the steady-state domestic interest rate (23). We use the detrended equilibrium system presented in Appendix B to study steady state. By (B.2), we derive the steady-state condition

$$0 = \frac{\beta}{\exp(g)}\left(\frac{R_f^*}{R_f} - 1\right) - \frac{\Omega}{y}\left(b - \frac{b}{\exp(g)}\right).$$

We then obtain (C.2). Q.E.D.

This lemma says that the steady-state asset-to-output ratio is determined by the interest rate differential. When the foreign interest rate R_f^* is higher (lower) than the domestic interest rate R_f, there is a capital account surplus (deficit) or capital outflow (inflow). When the foreign interest rate rises, capital flow reverses, that is, capital inflow decreases and capital outflow increases.

C. 1 Bubbleless Steady State

Equation (C.2) shows that b/y depends on $\bar{\varepsilon}$. We thus simply write it as a function of $\bar{\varepsilon}$, $b/y\left(\bar{\varepsilon}\right)$. We impose the following assumption to ensure the existence of a bubbleless steady state.

Assumption 1 *Assume that*

$$\delta + \exp(g) - 1 < \left(R_k\left(\varepsilon_{\min}\right) + \mu + \frac{b}{y}\left(\varepsilon_{\min}\right)\frac{R_k\left(\varepsilon_{\min}\right)}{\alpha\exp\left(g\right)}\right)\mathbb{E}\left[\varepsilon\right],$$

where R_k is given by (C.1) and b/y is given by (C.2).

The following proposition characterizes the existence condition for the bubbleless steady state.

Proposition 3 *Let assumption 1 hold. The equation*

$$\delta + \exp(g) - 1 = \left(R_k\left(\bar{\varepsilon}\right) + \mu + \frac{b}{y}\left(\bar{\varepsilon}\right)\frac{R_k\left(\bar{\varepsilon}\right)}{\alpha\exp\left(g\right)}\right)\int_{\bar{\varepsilon}}^{\varepsilon_{max}}\varepsilon f(\varepsilon)d\varepsilon \tag{C.3}$$

has a unique solution for $\bar{\varepsilon} \in \left(\varepsilon_{\min}, \varepsilon_{\max}\right)$, denoted by $\bar{\varepsilon}_f$. If further

$$1 - \left(\alpha + \frac{\alpha\mu}{R_k\left(\bar{\varepsilon}_f\right)} + \frac{1}{\exp(g)}\frac{b}{y}\left(\bar{\varepsilon}_f\right)\right)\int_{\bar{\varepsilon}_f}^{\varepsilon_{\max}} f(\varepsilon)d\varepsilon > \gamma + \frac{1}{\exp(g)}\frac{b}{y}\left(\bar{\varepsilon}_f\right)\frac{\exp(g) - R_f^*}{R_f\left(\bar{\varepsilon}_f\right)} > 0, \tag{C.4}$$

then there is a unique bubbleless steady state.

Assumption 1 allows us to use the intermediate value theorem to derive a solution to equation (C.3). A sufficiently small μ ensures the uniqueness of the solution to (C.3). After obtaining $\bar{\varepsilon}_f$, we then solve for the equilibrium real exchange rate e_f using equations (1), (B.5), (B.6) and (B.10). Then the other equilibrium variables can be easily determined using Proposition 2. We impose the two inequalities in (C.4) to ensure exports and consumption are positive in the bubbleless steady state.

Lemma 2 *When $\mu > 0$ is sufficiently small, $\frac{\partial R_k(\bar{\varepsilon})}{\partial\bar{\varepsilon}} < 0$.*

Proof. We first derive some common equations in both types of steady state. We use equation (17) and $Q = P_k/\bar{\varepsilon}$ where $P_k = 1$ to derive the steady-state marginal product of capital in equation (C.1).

Substituting $Q = 1/\bar{\varepsilon}$ into the steady-state version of (B.12), we can derive

$$\begin{aligned} R_k &= \frac{\left[\exp(g) - \beta + \beta\delta\right]/\bar{\varepsilon} - \beta\mu\int_{\bar{\varepsilon}}^{\varepsilon_{\max}}\left(\frac{\varepsilon}{\bar{\varepsilon}} - 1\right)f(\varepsilon)d\varepsilon}{\beta\left[1 + \int_{\bar{\varepsilon}}^{\varepsilon_{\max}}\left(\frac{\varepsilon}{\bar{\varepsilon}} - 1\right)f(\varepsilon)d\varepsilon\right]} \\ &= \frac{\left[\exp(g) - \beta + \beta\delta\right] - \beta\mu\int_{\bar{\varepsilon}}^{\varepsilon_{\max}}\left(\varepsilon - \bar{\varepsilon}\right)f(\varepsilon)d\varepsilon}{\beta\left[\bar{\varepsilon} + \int_{\bar{\varepsilon}}^{\varepsilon_{\max}}\left(\varepsilon - \bar{\varepsilon}\right)f(\varepsilon)d\varepsilon\right]}, \end{aligned}$$

which is equation (C.1).

The partial derivative of R_k with respect to $\bar{\varepsilon}$ is

$$\frac{\partial R_k}{\partial \bar{\varepsilon}} = \frac{\mu\beta \int_{\bar{\varepsilon}}^{\varepsilon_{max}} \varepsilon f(\varepsilon) d\varepsilon - [\exp(g) - \beta + \beta\delta] F(\bar{\varepsilon})}{\beta \left[\bar{\varepsilon} + \int_{\bar{\varepsilon}}^{\varepsilon_{max}} (\varepsilon - \bar{\varepsilon}) f(\varepsilon) d\varepsilon\right]^2}.$$

When $\mu = 0$, it is negative. So when μ is sufficiently small, it is also negative. ■

Proof of Proposition 3: We use the detrended equilibrium system presented in Appendix B to study the bubbleless steady state. In the bubbleless steady state with $p = 0$, we divide both sides of the steady-state version of equation (B.17) by k to derive

$$\delta + \exp(g) - 1 = \left(R_k + \mu + \frac{b}{k}\right) \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} \varepsilon f(\varepsilon) d\varepsilon.$$

By equation (B.28), $R_k = \alpha \exp(g) y/k$, we then obtain (C.3). In that equation, R_k is a decreasing function of $\bar{\varepsilon}$. By (23), R_f is an increasing function of $\bar{\varepsilon}$. Thus, the expression on the right-hand side of equation (C.3) is a decreasing function of $\bar{\varepsilon}$. This expression takes value 0 when $\bar{\varepsilon} = \varepsilon_{\max}$ and is larger than $\delta + \exp(g) - 1$ when $\bar{\varepsilon} = \varepsilon_{\min}$ by Assumption 1. Thus it follows from the intermediate value theorem that there is a unique solution for $\bar{\varepsilon} \in (\varepsilon_{\min}, \varepsilon_{\max})$, denoted by $\bar{\varepsilon}_f$, to the equation (C.3).

According to (B.18), $P_{kf} = 1$ in the steady state. After determining $\bar{\varepsilon}_f$, we can derive $Q_f = P_{kf}/\bar{\varepsilon}_f = 1/\bar{\varepsilon}_f$, $R_{kf} = R_k(\bar{\varepsilon}_f)$, and $R_{ff} = R_f(\bar{\varepsilon}_f)$ by definition and equations (23) and (C.1). By Lemma 1, $b_f/y_f = b/y(\bar{\varepsilon}_f)$. By (B.16), we can derive

$$\frac{i_f}{y_f} = \frac{1}{\exp(g)} \left(R_{kf}\frac{k_f}{y_f} + \mu\frac{k_f}{y_f} + \frac{b_f}{y_f}\right) \int_{\bar{\varepsilon}_f}^{\varepsilon_{max}} f(\varepsilon) d\varepsilon.$$

Since $R_{kf} = \alpha \exp(g) y_f/k_f$, we have

$$\frac{i_f}{y_f} = \left(\alpha + \frac{\alpha\mu}{R_{kf}} + \frac{1}{\exp(g)}\frac{b_f}{y_f}\right) \int_{\bar{\varepsilon}_f}^{\varepsilon_{max}} f(\varepsilon) d\varepsilon.$$

Substituting (B.20) into equation (B.26) and dividing both sides by y_t, we obtain

$$\frac{x_t}{y_t} = \gamma + \frac{1}{R_{ft}}\frac{b_t}{y_t} - \frac{1}{\exp(g_{at})}\frac{b_{t-1}}{y_t}\frac{R^*_{ft-1}}{R_{ft-1}}\frac{e_t}{e_{t-1}}. \quad \text{(C.5)}$$

In the bubbleless steady state, the equation above becomes

$$\frac{x_f}{y_f} = \gamma + \frac{1}{\exp(g)}\frac{b_f}{y_f}\frac{\exp(g) - R^*_f}{R_{ff}}. \quad \text{(C.6)}$$

Since exports are positive by equation (1), we need the expression above to be positive, which gives the second inequality in condition (C.4).

We use the resource constraint to derive

$$\begin{aligned}\frac{c_f}{y_f} &= 1-\frac{i_f}{y_f}-\frac{x_f}{y_f}\\ &= 1-\left(\alpha+\frac{\alpha\mu}{R_{kf}}+\frac{1}{\exp(g)}\frac{b_f}{y_f}\right)\left(\int_{\bar{\varepsilon}_f}^{\varepsilon_{\max}} f(\varepsilon)d\varepsilon\right)-\gamma-\frac{1}{\exp(g)}\frac{b_f}{y_f}\frac{\exp(g)-R_f^*}{R_{ff}}. \quad \text{(C.7)}\end{aligned}$$

To ensure $c_f > 0$, we impose the first inequality in condition (C.4).

Given $\bar{\varepsilon}_f$ determined earlier, we next determine the steady-state labor. By equation (B.25),

$$\lambda = \frac{\exp(g)-\beta h}{[\exp(g)-h]\,c}. \quad \text{(C.8)}$$

According to the proof of Proposition 2, it is easy to see that $w_t N_t = (1-\alpha-\gamma)y_t$. Using this equation, (B.9) and (C.8), we obtain

$$\nu N_f^{1+\varphi} = w_f N_f \lambda_f = (1-\alpha-\gamma)y_f\lambda_f = (1-\alpha-\gamma)\frac{\exp(g)-\beta h}{\exp(g)-h}\frac{y_f}{c_f},$$

which pins down N_f.

Now we determine the real exchange rate e_f. Substituting (B.20) and (B.28) into (B.19) yields

$$y_t = \left(\frac{\alpha y_t}{R_{kt}}\right)^{\alpha}\left(\frac{\gamma y_t}{e_t}\right)^{\gamma} N_t^{1-\alpha-\gamma}, \quad \text{(C.9)}$$

or

$$y_t = \left(\frac{\alpha}{R_{kt}}\right)^{\frac{\alpha}{1-\alpha-\gamma}}\left(\frac{\gamma}{e_t}\right)^{\frac{\gamma}{1-\alpha-\gamma}} N_t. \quad \text{(C.10)}$$

Therefore, combining equations (B.19) and (B.22) generates

$$\frac{x_f}{y_f} = \frac{e_f^{\sigma} y^*}{\left(\frac{\alpha}{R_{kf}}\right)^{\frac{\alpha}{1-\alpha-\gamma}}\left(\frac{\gamma}{e_f}\right)^{\frac{\gamma}{1-\alpha-\gamma}} N_f},$$

which leads to

$$e_f = \left[\frac{x_f}{y_f}\frac{\gamma^{\frac{\gamma}{1-\alpha-\gamma}}}{y^*}\left(\frac{\alpha}{R_{kf}}\right)^{\frac{\alpha}{1-\alpha-\gamma}} N_f\right]^{\frac{1}{\sigma+\frac{\gamma}{1-\alpha-\gamma}}}. \quad \text{(C.11)}$$

This equation can be used to solve for a closed-form expression for e_f given that R_{kf}, R_{ff}, and b_f/y_f are determined by $\bar{\varepsilon}_f$.

After e_f is determined, we use equation (B.22) to solve for x_f. Then k_f, m_f and y_f can be derived directly using equations (B.28), (B.20), and (C.10). Since we have already solved b_f/y_f, i_f/y_f, c_f/y_f, x_f/y_f, and y_f, we can derive b_f, i_f, c_f, and x_f. Finally, we use (B.27) to solve for w_f and (B.25) to solve for λ_f. Q.E.D.

C. 2 Bubbly Steady State

The following proposition characterizes the existence of the bubbly steady state.

Proposition 4 *Suppose that the assumptions in Proposition 3 hold so that there exists a unique bubbleless steady state with the investment threshold given by $\bar{\varepsilon}_f$. If there is a bubbly steady state, then the following condition holds*

$$1 < \beta \left[1 + \int_{\bar{\varepsilon}_f}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\bar{\varepsilon}_f} - 1 \right) f(\varepsilon) d\varepsilon \right].$$

Conversely, if this condition holds, then the equation

$$1 = \beta \left[1 + \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\bar{\varepsilon}} - 1 \right) f(\varepsilon) d\varepsilon \right] \tag{C.12}$$

has a unique solution for $\bar{\varepsilon} \in (\bar{\varepsilon}_f, \varepsilon_{\max})$, denoted by $\bar{\varepsilon}_b$, and if further

$$\begin{aligned} & 1 - \left(\alpha + \frac{\alpha\mu}{R_k(\bar{\varepsilon}_b)} + \frac{1}{\exp(g)} \frac{b}{y}(\bar{\varepsilon}_b) + \frac{p}{y_b} \right) \int_{\bar{\varepsilon}_b}^{\varepsilon_{\max}} f(\varepsilon) d\varepsilon \\ > \quad & \gamma + \frac{1}{\exp(g)} \frac{b}{y}(\bar{\varepsilon}_b) \frac{\exp(g) - R_f^*}{R_f(\bar{\varepsilon}_b)} > 0, \end{aligned} \tag{C.13}$$

where

$$\frac{p}{y_b} = \frac{\alpha}{R_k(\bar{\varepsilon}_b)} \left(\frac{\delta + \exp(g) - 1}{\int_{\bar{\varepsilon}_b}^{\varepsilon_{\max}} \varepsilon f(\varepsilon) d\varepsilon} - \mu \right) - \alpha - \frac{1}{\exp(g)} \frac{b}{y}(\bar{\varepsilon}_b), \tag{C.14}$$

then there exists a unique bubbly steady state with the investment threshold given by $\bar{\varepsilon}_b$ and the bubble-to-output ratio p/y_b given above.

Proof of Proposition 4 We use the detrended equilibrium system presented in Appendix B to study the bubbly steady state. Suppose that a bubbly steady state exists. We want to derive condition (22). In the bubbly steady state with $p > 0$, we use the steady-state version of (B.13) to derive (C.12). We use (B.17) to derive

$$\begin{aligned} \frac{p}{y_b} &= \frac{\alpha}{R_{kb}} \left(\frac{\delta + \exp(g) - 1}{\int_{\bar{\varepsilon}_b}^{\varepsilon_{\max}} \varepsilon f(\varepsilon) d\varepsilon} - \mu \right) - \alpha - \frac{1}{\exp(g)} \frac{b_b}{y_b} \\ &= \frac{\alpha}{R_{kb}} \left(\frac{\delta + \exp(g) - 1}{\int_{\bar{\varepsilon}_b}^{\varepsilon_{max}} \varepsilon f(\varepsilon) d\varepsilon} - \mu \right) - \alpha - \frac{1}{\exp(g)} \frac{\beta}{(\exp(g) - 1)\Omega} \left(\frac{R_f^*}{R_{fb}} - 1 \right) > 0. \end{aligned} \tag{C.15}$$

In the bubbleless steady state, we use (B.17) to derive

$$0 = \frac{\alpha}{R_{kf}} \left(\frac{\delta + \exp(g) - 1}{\int_{\bar{\varepsilon}_f}^{\varepsilon_{\max}} \varepsilon f(\varepsilon) d\varepsilon} - \mu \right) - \alpha - \frac{1}{\exp(g)} \frac{\beta}{(\exp(g) - 1)\Omega} \left(\frac{R_f^*}{R_{ff}} - 1 \right). \tag{C.16}$$

Since R_k decreases in $\bar{\varepsilon}$ by Lemma 2 and R_f increases in $\bar{\varepsilon}$ by (23), we can deduce that the expression

$$\frac{\alpha}{R_k}\left(\frac{\delta+\exp(g)-1}{\int_{\bar{\varepsilon}}^{\varepsilon_{\max}}\varepsilon f(\varepsilon)d\varepsilon}-\mu\right)-\alpha-\frac{1}{\exp(g)}\frac{\beta}{(\exp(g)-1)\Omega}\left(\frac{R_f^*}{R_f}-1\right)$$

increases in $\bar{\varepsilon}$. It follows from (C.15) and (C.16) that $\bar{\varepsilon}_b > \bar{\varepsilon}_f$. Thus we use (C.12) to deduce condition (22).

Now suppose that (22) holds. We want to derive a bubbly steady state. The expression on the right-hand side of equation (C.12) is a decreasing function of $\bar{\varepsilon}$. It takes the value $\beta < 1$ when $\bar{\varepsilon} = \varepsilon_{\max}$ and a value larger than 1 when $\bar{\varepsilon} = \bar{\varepsilon}_f$ by (22). By the intermediate value theorem, there is a unique solution, denoted by $\bar{\varepsilon}_b$, to equation (C.12).

By equation (B.1), we have $P_{kb} = 1$. After determining $\bar{\varepsilon}_b$, we can derive $Q_b = P_{kb}/\bar{\varepsilon}_b = 1/\bar{\varepsilon}_b$, $R_{kb} = R_k(\bar{\varepsilon}_b)$, and $R_{fb} = R_f(\bar{\varepsilon}_b)$ by definition and equations (23) and (C.1). By Lemma 1, $b_b/y_b = b/y(\bar{\varepsilon}_b)$. We then use (B.17) to derive

$$\begin{aligned}\frac{p}{y_b} &= \frac{\alpha}{R_{kb}}\left(\frac{\delta+\exp(g)-1}{\int_{\bar{\varepsilon}_b}^{\varepsilon_{\max}}\varepsilon f(\varepsilon)d\varepsilon}-\mu\right)-\alpha-\frac{1}{\exp(g)}\frac{b}{y}(\bar{\varepsilon}_b) \\ &> \frac{\alpha}{R_{kf}}\left(\frac{\delta+\exp(g)-1}{\int_{\bar{\varepsilon}_f}^{\varepsilon_{\max}}\varepsilon f(\varepsilon)d\varepsilon}-\mu\right)-\alpha-\frac{1}{\exp(g)}\frac{b}{y}(\bar{\varepsilon}_f) \\ &= 0,\end{aligned} \tag{C.17}$$

where the inequality is due to the fact that R_k and $\int_{\bar{\varepsilon}}^{\varepsilon_{max}}\varepsilon f(\varepsilon)d\varepsilon$ decrease in $\bar{\varepsilon}$ and R_f increases in $\bar{\varepsilon}$. The last equality follows from (C.3) and $R_{kf} = R_k(\bar{\varepsilon}_f)$. Thus $p > 0$.

We use a similar procedure in the proof of Proposition 3 to solve for the bubbly steady-state labor:

$$\nu N_b^{1+\varphi} = (1-\alpha-\gamma)\frac{\exp(g)-\beta h}{\exp(g)-h}\frac{y_b}{c_b}.$$

When the second inequality in (C.13) holds, the right-hand side of the above equation is positive so that this equation gives a unique positive solution for N_b.

Next we derive the bubbly steady-state real exchange rate e_b. Again, similar to the proof of Proposition 3, equations (C.5) and (C.10) imply

$$\frac{1}{N_b}e_b^{\sigma}y^*\left(\frac{R_{kb}}{\alpha}\right)^{\frac{\alpha}{1-\alpha-\gamma}}\left(\frac{e_b}{\gamma}\right)^{\frac{\gamma}{1-\alpha-\gamma}} = \frac{x_b}{y_b} = \gamma+\frac{1}{\exp(g)}\frac{b_b}{y_b}\frac{\exp(g)-R_f^*}{R_{fb}}.$$

Given that the second inequality in (C.13) holds and steady-state labor N_b is solved, the equation above gives a unique solution for $e_b > 0$.

We can then solve for x_b using equation (B.22), and solve for k_b, m_b and y_b using equations (B.20), (B.28), and (C.10). Since we have solved for b_b/y_b and y_b, we can derive b_b. We solve for p using (C.17) and solve for i_b and w_b using equations (B.16) and (B.27). Finally, we solve for c_b using equation (B.21) and λ_b using equation (B.25). We need the first inequality in (C.13) to ensure $c_b > 0$. Q.E.D.

The following proposition studies the impact of the foreign interest rate.

Proposition 5 *The smaller the foreign interest rate R_f^*, the more likely a domestic bubble can emerge in the sense that condition (22) is more likely to hold. When R_f^* is sufficiently high, a bubble cannot emerge. When a bubbly steady state exists, the capital rental rate R_{kb}, the domestic interest rate R_{fb}, and the investment threshold $\bar{\varepsilon}_b$ do not change with the foreign interest rate R_f^*, but the bubble-to-output ratio $\frac{p}{y_b}$ decreases with R_f^*.*

Proof of Proposition 5 It follows from Lemma 1 that b/y increases with R_f^*. We deduce that the expression on the right-hand side of equation (C.3) increases in R_f^* and decreases in $\bar{\varepsilon}_f$. Thus the solution $\bar{\varepsilon}_f$ to this equation must increase with R_f^*. When R_f^* is smaller, $\bar{\varepsilon}_f$ is smaller and hence the condition (22) is more likely to hold, that is, an asset bubble is more likely to emerge. But when R_f^* is sufficiently high, $\bar{\varepsilon}_f$ approaches $\varepsilon_{\max}$. In this case condition (22) cannot hold and hence a bubble cannot emerge.

When a bubble emerges, $\bar{\varepsilon}_b$ is determined by equation (C.12) which does not depend on R_f^*. Thus Q_b, R_{kb} and R_{fb} do not depend on R_f^* either. They all remain constant when R_f^* varies.

It follows from (C.14) that p/y_b decreases with R_f^*. Q.E.D.

D Log-linearized System

We present the log-linearized system for the bubbly equilibrium. The log-linearized system for the bubbleless equilibrium system is similar except that $P_t = 0$. Let $\hat{b}_t$, $\hat{g}_{at}$, and $\hat{g}_t$ denote level deviations from their steady-state values, and $\hat{z}_t$ denotes the log deviation from its steady-state value for any other variable z_t.

(1) The log-linearized version of equation (B.12):

$$\begin{aligned}\hat{Q}_t &= \mathbb{E}_t\hat{\lambda}_{t+1} - \hat{\lambda}_t - \mathbb{E}_t\hat{g}_{at+1} + \frac{\beta}{\exp(g)}\frac{R_k}{Q}\mathbb{E}_t\hat{R}_{kt+1} + \frac{\beta}{\exp(g)}(1-\delta)\mathbb{E}_t\hat{Q}_{t+1} \\ &\quad + \frac{\beta}{\exp(g)}\frac{R_k\frac{1}{\eta-1}\left(\frac{\varepsilon_{min}}{\bar{\varepsilon}}\right)^{\eta}}{Q}\mathbb{E}_t\hat{R}_{kt+1} - \eta\frac{\beta}{\exp(g)}\frac{(R_k+\mu)\frac{1}{\eta-1}\left(\frac{\varepsilon_{min}}{\bar{\varepsilon}}\right)^{\eta}}{Q}\mathbb{E}_t\hat{\bar{\varepsilon}}_{t+1}. \quad \text{(D.1)}\end{aligned}$$

(2) The log-linearized version of equation (B.13):

$$\hat{p}_t = \mathbb{E}_t\hat{\lambda}_{t+1} - \hat{\lambda}_t + \mathbb{E}_t\hat{p}_{t+1} - \eta(1-\beta)\mathbb{E}_t\hat{\bar{\varepsilon}}_{t+1}. \tag{D.2}$$

(3) The log-linearized version of equation (B.14):

$$-\hat{R}_{ft} = \mathbb{E}_t\hat{\lambda}_{t+1} - \hat{\lambda}_t - \mathbb{E}_t\hat{g}_{at+1} - \eta(1-\beta)\mathbb{E}_t\hat{\bar{\varepsilon}}_{t+1}. \tag{D.3}$$

(4) The log-linearized version of equation (B.15):

$$\begin{aligned} 0 \;=\; & \frac{\beta}{\exp(g)}\left(\frac{R_f^*}{R_f} - 1\right)\left(\mathbb{E}_t\hat{\lambda}_{t+1} - \hat{\lambda}_t - \mathbb{E}_t\hat{g}_{at+1}\right) + \frac{\beta}{\exp(g)}\frac{R_f^*}{R_f}\left(\hat{R}_{ft}^* - \hat{R}_{ft} + \mathbb{E}_t\hat{e}_{t+1} - \hat{e}_t\right) \\ & -\frac{\Omega}{y}\hat{b}_t + \Omega\frac{\exp(g)-1}{\exp(g)}\frac{b}{y}\hat{y}_t - \frac{\Omega}{\exp(g)}\frac{b}{y}\hat{g}_{at}. \end{aligned} \tag{D.4}$$

(5) The log-linearized version of equation (B.16):

$$\begin{aligned} \hat{\imath}_t + \hat{P}_{kt} \;=\; & \left(\frac{\varepsilon_{min}}{\bar{\varepsilon}}\right)^{\eta}\frac{R_k k}{\exp(g)i}\left(\hat{R}_{kt} + \hat{k}_{t-1} - \hat{g}_{at}\right) + \left(\frac{\varepsilon_{min}}{\bar{\varepsilon}}\right)^{\eta}\frac{\mu k}{\exp(g)i}\left(\hat{k}_{t-1} - \hat{g}_{at}\right) \\ & + \left(\frac{\varepsilon_{min}}{\bar{\varepsilon}}\right)^{\eta}\frac{1}{\exp(g)i}\hat{b}_{t-1} - \left(\frac{\varepsilon_{min}}{\bar{\varepsilon}}\right)^{\eta}\frac{b}{\exp(g)i}\hat{g}_{at} \\ & + \left(\frac{\varepsilon_{min}}{\bar{\varepsilon}}\right)^{\eta}\frac{p}{i}\hat{p}_t - \eta\hat{\bar{\varepsilon}}_t. \end{aligned} \tag{D.5}$$

(6) The log-linearized version of equation (B.17):

$$\hat{k}_t = \frac{1-\delta}{\exp(g)}\left(\hat{k}_{t-1} - \hat{g}_{at}\right) + \frac{\delta + \exp(g) - 1}{\exp(g)}\left(\hat{\imath}_t + \hat{\bar{\varepsilon}}_t\right). \tag{D.6}$$

(7) The log-linearized version of equation (B.18):

$$\hat{P}_{kt} = \Omega_k \exp(2g)\left[\hat{g}_{at} + (1+\beta)\hat{\imath}_t - \hat{\imath}_{t-1} - \beta\mathbb{E}_t\hat{g}_{at+1} - \beta\mathbb{E}_t\hat{\imath}_{t+1}\right]. \tag{D.7}$$

(8) The log-linearized version of equation (B.19):

$$\hat{y}_t = \alpha\hat{k}_{t-1} - \alpha\hat{g}_{at} + \gamma\hat{m}_t + (1-\alpha-\gamma)\hat{N}_t. \tag{D.8}$$

(9) The log-linearized version of equation (B.20):

$$\hat{y}_t = \hat{e}_t + \hat{m}_t. \tag{D.9}$$

(10) Log-linearized version of equation (B.21):

$$\hat{y}_t = \frac{c}{y}\hat{c}_t + \frac{i}{y}\hat{\imath}_t + \frac{x}{y}\hat{x}_t. \tag{D.10}$$

(11) The log-linearized version of equation (B.22):

$$\hat{x}_t = \sigma \hat{e}_t + \hat{y}_t^* + \widehat{\zeta}_t. \tag{D.11}$$

where $\zeta_t \equiv \frac{\exp(gt)}{A_t}$ and

$$\widehat{\zeta}_t = \widehat{\zeta}_{t-1} + \hat{g}_t + \hat{a}_t - \hat{a}_{t-1}. \tag{D.12}$$

(12) The log-linearized version of equation (B.23):

$$\hat{\bar{\varepsilon}}_t + \hat{Q}_t = \hat{P}_{kt}. \tag{D.13}$$

(13) The log-linearized version of equation (B.24):

$$\hat{\xi}_t + \varphi \hat{N}_t = \hat{w}_t + \hat{\lambda}_t. \tag{D.14}$$

(14) The log-linearized version of equation (B.25):

$$\begin{aligned} \hat{\lambda}_t \quad = \quad & -\frac{\exp(g)}{\exp(g) - \beta h} \left[\frac{\exp(g)}{\exp(g) - h} \hat{c}_t - \frac{h}{\exp(g) - h} \left(\hat{c}_{t-1} - \hat{g}_{at} \right) - \hat{\xi}_t \right] \\ & + \frac{\beta h}{\exp(g) - \beta h} \left[\frac{\exp(g)}{\exp(g) - h} \left(\mathbb{E}_t \hat{g}_{at+1} + \mathbb{E}_t \hat{c}_{t+1} \right) - \frac{h}{\exp(g) - h} \hat{c}_t - \mathbb{E}_t \hat{\xi}_{t+1} \right]. \end{aligned} \tag{D.15}$$

(15) The log-linearized version of equation (B.26):

$$\begin{aligned} 0 \quad = \quad & \frac{x}{y} \hat{x}_t - \gamma \left(\hat{e}_t + \hat{m}_t \right) - \frac{1}{R_f y} \hat{b}_t + \frac{b}{R_f y} \hat{R}_{ft} + \frac{R_f^*}{\exp(g) R_f y} \hat{b}_{t-1} \\ & + \frac{b}{\exp(g) y} \frac{R_f^*}{R_f} \left(-\hat{g}_{at} + \hat{R}_{ft-1}^* - \hat{R}_{ft-1} + \hat{e}_t - \hat{e}_{t-1} \right). \end{aligned} \tag{D.16}$$

(16) The log-linearized version of equation (B.27):

$$\hat{N}_t = -\frac{1-\gamma}{\alpha} \hat{w}_t - \frac{\gamma}{\alpha} \hat{e}_t + \hat{k}_{t-1} - \hat{g}_{at}. \tag{D.17}$$

(17) The log-linearized version of equation (B.28):

$$\hat{R}_{kt} = \hat{g}_{at} + \hat{y}_t - \hat{k}_{t-1}. \tag{D.18}$$

The log-linearized shock processes are given by

$$\begin{aligned} \hat{g}_t &= \rho_g \hat{g}_{t-1} + \sigma_g \varepsilon_{gt}, \\ \hat{a}_t &= \rho_a \hat{a}_{t-1} + \sigma_a \varepsilon_{at}, \\ \hat{\xi}_t &= \rho_\xi \hat{\xi}_{t-1} + \sigma_\xi \varepsilon_{\xi t}, \\ \hat{y}_t^* &= \rho_{y^*} \hat{y}_{t-1}^* + \sigma_{y^*} \varepsilon_{y^* t}, \\ \hat{R}_{ft}^* &= \rho_{R_f^*} \hat{R}_{ft-1}^* + \sigma_{R_f^*} \varepsilon_{R_f^* t}. \end{aligned}$$

E Introducing Sentiment Shocks

The equilibrium system with sentiment shocks is characterized by the following equations: (1), (12), (15), (17), (19), (B.1), (B.2), (B.3), (B.4), (B.5), (B.6), (B.7), (B.8), (B.9), (B.10), $\bar{\varepsilon}_t Q_t = P_{kt}$, and

$$P_t = \beta \chi_t \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} P_{t+1} \left[1 + \int_{\bar{\varepsilon}_{t+1}}^{\varepsilon_{max}} \left(\varepsilon \frac{Q_{t+1}}{P_{kt+1}} - 1 \right) f(\varepsilon) d\varepsilon \right],$$

for the endogenous variables $\{Q_t, \bar{\varepsilon}_t, P_t, R_{ft}, B_t, I_t, K_t, Y_t, M_t, C_t, N_t, X_t, e_t, W_t, R_{kt}, \Lambda_t, P_{kt}\}$. The usual transversality conditions hold.

Since the sentiment shock affects the asset bubble directly, it is an additional channel through which the bubbly model is able to generate a higher volatility of stock prices than the bubbleless model. Therefore, it is expected that the bubbly model with the sentiment shock fits the stock price data better.

We set $\chi = 0.9975$, following Miao et al. (2015a). We adopt parameter values in Table 2 and estimate other parameters with the sentiment shock included. The results are as follows.

Table E.1: Estimated Parameters with Sentiment Shocks

Parameter	Prior Distribution			Posterior Distribution			
	Distr.	Mean	S.D.	Mode	Mean	5%	95%
				Bubbly model			
h	Beta	0.5	0.2	0.40	0.41	0.28	0.53
Ω_k	Gamma	3	2	0.78	0.89	0.57	1.30
μ	Beta	0.3	0.05	0.24	0.23	0.18	0.29
ρ_g	Beta	0.5	0.2	0.995	0.991	0.981	0.998
ρ_a	Beta	0.5	0.2	0.85	0.85	0.76	0.93
ρ_ξ	Beta	0.5	0.2	0.60	0.61	0.45	0.76
ρ_{y^*}	Beta	0.5	0.2	0.997	0.995	0.990	0.999
$\rho_{R_f^*}$	Beta	0.85	0.05	0.79	0.80	0.76	0.84
ρ_χ	Uniform	0.5	0.3	0.999	0.999	0.999	0.999
σ_g (%)	Inv. Gamma	0.01	Inf	0.42	0.48	0.37	0.63
σ_a (%)	Inv. Gamma	0.01	Inf	4.11	4.32	3.73	5.00
σ_ξ (%)	Inv. Gamma	0.01	Inf	6.68	7.18	5.83	8.79
σ_{y^*} (%)	Inv. Gamma	0.01	Inf	0.79	0.81	0.71	0.92
$\sigma_{R_f^*}$ (%)	Inv. Gamma	0.01	Inf	0.45	0.46	0.40	0.53
σ_χ (%)	Inv. Gamma	0.01	Inf	0.14	0.14	0.12	0.17

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm.

Note that the sentiment shock does not enter the bubbleless model so we omit the estimation of the bubbleless model here.

Introducing sentiment shocks slightly changes the other estimated parameters. The only significant changes are capital adjustment cost parameter Ω_k changing to 2.18 to 0.78 and the standard deviation of preference shock, changing from 8.77% to 6.68%. The others are all in the neighborhood of the previously estimated values without sentiment shocks.

The introduction of sentiment shocks leads to an improvement in the model fit in terms of the log densities of data and the log posterior likelihood at the posterior mode. The log marginal densities of data and the log posterior likelihood at the posterior modes for this model are 1141.4 and 1217.6, respectively, while their counterparts for the benchmark model are 1118.2 and 1167.2, respectively.

We simulate the bubbly and bubbleless model 1 with the parameters obtained in Table E.1 and present the real-business-cycle moments in Table E.2. For comparison, we also repeat the moments generated from the bubbly and bubbleless model 1 in the benchmark case.

Comparing columns 3 and 5 in Table E.2, we can see that, adding the sentiment shocks to the model significantly raises the volatility of stock prices, making it quantitatively close to the data. Moreover, the contrast between the bubbly and the bubbleless models in this aspect is even sharper, whereas the patterns in the other moments are not essentially affected, supporting the amplification and propagation mechanism of our bubbly model. The impulse responses from this model are qualitatively similar to Figures 3 through 5 from the benchmark model. For space limitation, we omit them here.

Table E.2: Real Business Cycle Moments with the Sentiment Shock

Moment	Data	Bubbly (Benchmark)	Bubbleless 1 (Benchmark)	Bubbly (Sentiment)	Bubbleless 1 (Sentiment)
			Standard Deviation		
$\mathrm{std(gdp_t)}$ (%)	2.68	3.18	2.33	3.31	2.49
$\mathrm{std(c_t)/std(gdp_t)}$	1.32	1.61	1.63	1.59	1.54
$\mathrm{std(i_t)/std(gdp_t)}$	2.89	2.84	2.25	2.99	2.87
$\mathrm{std(nx_t/gdp_t)}$ (%)	1.61	2.76	1.21	2.52	0.97
$\mathrm{std(sp_t)/std(gdp_t)}$	6.39	4.82	2.25	7.08	1.34
			Correlation with GDP		
$\mathrm{corr(c_t, gdp_t)}$	0.73	0.72	0.58	0.64	0.46
$\mathrm{corr(i_t, gdp_t)}$	0.79	0.74	0.59	0.75	0.64
$\mathrm{corr(nx_t/gdp_t, gdp_t)}$	-0.50	-0.53	-0.14	-0.54	-0.18
$\mathrm{corr(sp_t, gdp_t)}$	0.64	0.48	0.38	0.33	0.50
			Correlation with Stock Prices		
$\mathrm{corr(c_t, sp_t)}$	0.53	0.37	-0.46	0.27	-0.43
$\mathrm{corr(i_t, sp_t)}$	0.52	0.70	0.71	0.41	0.83
$\mathrm{corr(nx_t/gdp_t, sp_t)}$	-0.45	-0.66	0.71	-0.45	0.48
			Autocorrelation		
$\mathrm{corr(y_t, y_{t-1})}$	0.78	0.79	0.76	0.81	0.78
$\mathrm{corr(c_t, c_{t-1})}$	0.75	0.70	0.68	0.75	0.72
$\mathrm{corr(i_t, i_{t-1})}$	0.78	0.81	0.85	0.85	0.84
$\mathrm{corr(nx_t/gdp_t, nx_{t-1}/gdp_{t-1})}$	0.82	0.66	0.41	0.74	0.53
$\mathrm{corr(sp_t, sp_{t-1})}$	0.74	0.49	0.43	0.69	0.51

Note: Columns 3 and 4 display the real-business-cycle moments for the bubbly and bubbleless economies based on the same parameter values estimated using the benchmark bubbly model. Columns 5 and 6 display the moments for the models with the sentiment shocks.

Reference

Adolfson, M., Laséen, S., Lindé, J., Villani, M., 2007. Bayesian estimation of an open economy DSGE model with incomplete pass-through. Journal of International Economics 72, 481-511.

Aguiar, M., Gopinath, G., 2007. Emerging market business cycles: The cycle is the trend. Journal of Political Economy 115, 69-102.

An, S., Schorfheide, F., 2007. Bayesian analysis of dynamic stochastic general equilibrium models, Econometric Reviews 26(2-4), 113-172.

Aoki, K., Benigno, G., Kiyotaki, N., 2016. Monetary and financial policies in emerging markets. Working paper. Princeton University.

Backus, D.K., Kehoe, P.J., Kydland, F.E., 1992. International real business cycles. Journal of Political Economy 100(4), 745-775.

Basco, S., 2014. Globalization and financial development: A model of the Dot-Com and the housing bubbles. Journal of International Economics 92,78–94.

Blanchard, O. J., Watson, M. W., 1982. Bubbles, rational expectations, and financial markets, in: Wachtel, P. (Eds.), Crisis in the Economic and Financial Structure, Lexington, pp. 295-315.

Caballero, R.J., Krishnamurthy, A., 2006. Bubbles and capital flow volatility: Causes and risk management. Journal of Monetary Economics 53, 35-53.

Calvo, G.A., 1998. Capital flows and capital-market crises: The simple economics of Sudden Stops. Journal of Applied Economics 1, 35-54.

Calvo, G.A., Leiderman, L., Reinhart, C.M., 1993. Capital inflows and real exchange rate appreciation in Latin America: The role of external factors. IMF Staff Papers 40, 108-151.

Canova, F., 2005. The transmission of US shocks to Latin America. Journal of Applied Econometrics 20, 229-251.

Chang, C., Liu, Z., Spiegel, M.M., 2015. Capital controls and optimal Chinese monetary policy. Journal of Monetary Economics 74, 1-15.

Chang, R., Fernández, A., 2013. On the sources of aggregate fluctuations in emerging economies. International Economic Review 54, 1265-1293.

Christiano, L.J., Eichenbaum, M., Evans, C.L., 2005. Nominal rigidities and the dynamic effects of a shock to monetary policy. Journal of Political Economy 113, 1-45.

Covas, F., Den Hann W.J., 2011. The cyclical behavior of debt and equity finance. American Economic Review 101, 877-899.

Denton, F.T., 1971. Adjustment of monthly or quarterly series to annual totals: An approach based on quadratic minimization. Journal of the American Statistical Association 66, 99-102.

Dong, F., Miao, J., Wang, P., 2020. Asset bubbles and monetary policy. Review of Economic Dynamics 37, S68-S98.

Farhi, E., Tirole, J., 2012. Bubbly liquidity. Review of Economic Studies 79, 678-706.

Feenstra, R.C., Luck, P., Obsteld, M., Russ, K.N., 2014. In search of the Armington elasticity. NBER Working Paper No. 20063.

Feenstra, R.C., Inklaar, R., Timmer, M.P., 2015. The next generation of the Penn World Table. American Economic Review 105, 3150-3182.

Fernández-Villaverde, J., Guerrón-Quintana, P., Rubio-Ramírez, J.F., Uribe, M., 2011. Risk matters: The real effects of volatility shocks. American Economic Review 101, 2530-2561.

Fernández, A., Gulan, A., 2015. Interest rate, leverage, and business cycles in emerging economies: The role of financial frictions. American Economic Journal: Macroeconomics 7, 153-188.

Frank, M. Z., Goyal, V. K., 2008. Trade-off and pecking order theories of debt. In Handbook of Empirical Corporate Finance, Vol. 2, edited by B. Espen Eckbo, 135-202. Amsterdam: North Holland.

Galí, J., 2014. Monetary policy and rational asset price bubbles. American Economic Review 104(3), 721-752.

García-Cicco, J., Pancrazi, R., Uribe, M., 2010. Real business cycles in emerging countries? American Economic Review 100, 2510-2531.

Gertler, M., Gilchrist, S., Natalucci, F.M., 2007. External constraints on monetary policy and the financial accelerator. Journal of Money, Credit, and Banking 39, 295-330.

Gertler, M., Kiyotaki, N., 2010. Financial intermediation and credit policy in business cycle analysis, in: Friedman, B. M., Woodford, M. (Eds.), Handbook of Monetary Economics, Elsevier, pp. 547-599.

Gopinath, G., 2004. Lending booms, sharp reversals and real exchange rate dynamics. Journal of International Economics 62, 1-23.

Hennessy, C. A., Whited, T. M., 2007. How costly is external financing? Evidence from a structural estimation. Journal of Finance 62, 1705-1745.

Jermann, U., Quadrini, V., 2012. Macroeconomic effects of financial shock. American Economic Review 102, 238-271.

Justiniano, A., Preston, B., 2010. Can structural small open-economy models account for the influence of foreign disturbances? Journal of International Economics 81, 61-74.

Karabarbounis, L., Neiman, B., 2014. The global decline of the labor share. The Quarterly Journal of Economics 129(1), 61-103.

Kocherlakota, N. R., 1992. Bubbles and constraints on debt accumulation. Journal of Economic Theory 57, 245-256.

Kiyotaki, N., Moore, J., 1997. Credit cycles. Journal of Political Economy 105, 211-48.

Kiyotaki, N., Moore, J., 2019. Liquidity, business cycles and monetary policy. Journal of Political Economy 127(6), 2926-2966.

Korinek, A., Mendoza, E.G., 2014. From Sudden Stops to Fisherian deflation: Quantitative theory and policy. Annual Review of Economics 6, 299-332.

Liu, Z., Wang P., Zha, T., 2013. Land-price dynamics and macroeconomic fluctuations. Econometrica 81, 1147-1184.

Liu, Z., Waggoner, D.F., Zha, T., 2011. Sources of macroeconomic fluctuations: A regime-switching DSGE approach. Quantitative Economics 2, 251-301.

Longstaff, F.A., Pan, J., Pedersen, L.H., Singleton, K.J., 2011. How sovereign is sovereign credit risk? American Economic Journal: Macroeconomics 3, 75-103.

Lubik, T.A., Schorfheide, F., 2007. Do central banks respond to exchange rate movements? A structural investigation. Journal of Monetary Economics 54, 1069-1087.

Lothian, J. R., Wu, L., 2011. Uncovered interest-rate parity over the past two centuries. Journal of International Money and Finance 30, 448-473.

Maćkowiak, B., 2007. External shocks, U.S. monetary policy and macroeconomic fluctuations in emerging markets. Journal of Monetary Economics 54, 2512–2520.

Martin, P., Rey, H., 2006. Globalization and emerging markets: With or without crash? American Economic Review 96, 1631-51.

Martin, A., Ventura, J. 2012. Economic growth with bubbles. American Economic Review

102, 3033-3058.

Martin, A., Ventura, J., 2015a, Asset bubbles and Sudden Stops in a small open economy, In: C. Raddatz, D. Saravia and J. Ventura (Eds.), Global Liquidity, Spillovers to Emerging Markets, and Policy Responses, Central Bank of Chile, 315-341.

Martin, A., Ventura, J., 2015b, The international transmission of credit bubbles: Theory and policy. Journal of Monetary Economics 76, S37-S56.

Mendoza, E.G., 1991. Real Business Cycles in a Small Open Economy. American Economic Review 81(4), 797-818.

Mendoza, E.G., 2010. Sudden Stops, Financial crises, and leverage. American Economic Review 100, 1941-1966.

Myers, S. C., 1984. The capital structure puzzle. Journal of Finance 39 (3), 575-592.

Miao, J., Wang, P., 2018. Asset bubbles and credit constraints. American Economic Review 108, 2590-2628.

Miao, J., Wang, P., Xu, Z. 2015a. A Bayesian DSGE model of stock market bubbles and business cycles. Quantitative Economics 6, 599-635.

Miao, J., Wang, P., Zhou, J., 2015b. Asset bubbles, collateral, and policy analysis. Journal of Monetary Economics 76, S57-S70.

Neumeyer, P.A., Perri, F., 2005. Business cycles in emerging economies: The role of interest rates. Journal of Monetary Economics 52, 345–380.

Perri, F., Quadrini, V., 2018. International recessions. American Economic Review 108, 935-984.

Restuccia, D., Urrutia, C., 2011. Relative prices and investment rate. Journal of Monetary

Economics 47(1), 93-121.

Santos, M.S., Woodford, M., 1997. Rational asset pricing bubbles. Econometrica 65, 19-58.

Scheinkman, J. A., Xiong, W., 2003. Overconfidence and speculative bubbles. Journal of Political Economy 111, 1183-1220.

Schmitt-Grohé, S., Uribe, M., 2012. What's news in business cycles? Econometrica 80(6), 2733-2764.

Smets, F., Wouters, R., 2007. Shocks and frictions in US business cycles: A Bayesian DSGE approach. American Economic Review 97, 586-606.

Tirole, J., 1985. Asset bubbles and overlapping generations. Econometrica 53, 1499-1528.

Uribe, M., Schmitt-Grohé, S., 2017. Open Economy Macroeconomics, New Jersey: Princeton University Press.

Uribe, M., Yue, V.Z., 2006. Country spreads and emerging countries: Who drives whom? Journal of International Economics 69, 6-36.

Ventura, J., 2012. Bubbles and capital flows. Journal of Economic Theory 147, 738-758.

Wang, P., Wen, Y., 2012. Speculative bubbles and financial crises. American Economic Journal: Macroeconomics 4, 184-221.

Weil, P., 1987. Confidence and the real value of money in an overlapping generations economy, The Quarterly Journal of Economics 102, 1-22.

Woodford, M., 1986. Stationary sunspot equilibria in a finance constrained economy. Journal of Economic Theory 40, 128-137.

Table 1: Business Cycle Moments: Emerging vs. Developed Markets

Moments	Emerging markets		Developed markets	
$std(gdp_t)$	2.95	(0.16)	1.84	(0.13)
$std(c_t)/std(gdp_t)$	1.04	(0.04)	0.71	(0.06)
$std(i_t)/std(gdp_t)$	3.68	(0.16)	2.47	(0.14)
$std(nx_t/gdp_t)$	2.72	(0.17)	1.36	(0.07)
$std(sp_t)/std(gdp_t)$	6.77	(0.27)	9.83	(0.77)
$corr(nx_t/gdp_t, gdp_t)$	-0.32	(0.06)	0.14	(0.12)
$corr(c_t, gdp_t)$	0.71	(0.03)	0.65	(0.04)
$corr(i_t, gdp_t)$	0.73	(0.03)	0.75	(0.03)
$corr(sp_t, gdp_t)$	0.52	(0.04)	0.51	(0.04)
$corr(sp_t, c_t)$	0.40	(0.05)	0.48	(0.06)
$corr(sp_t, i_t)$	0.39	(0.04)	0.42	(0.04)
$corr(sp_t, nx_t/gdp_t)$	-0.18	(0.04)	-0.06	(0.03)

Note: (i) The sample of emerging markets is the same as described in Footnote 4 but excludes Ecuador for which the data of macroeconomic and stock prices do not have overlapped periods.

(ii) The variables gdp_t, c_t, i_t and nx_t/gdp_t denote real GDP, real consumption, real investment and the net exports/GDP ratio, respectively. The real stock price sp_t is nominal stock price index divided by the CPI. The nominal stock price index is from Global Database in CEIC. The CPI is from the IMF. All the other data come from Fernández and Gulan (2015).

(iii) All series are seasonally adjusted if necessary. All series except nx_t/gdp_t are logged and then HP filtered with the multiplier set at 1600.

(iv) We compute the moments with the cyclical parts of each variable using GMM. The group average of each moment is the weighted average of the unbalanced panels. Standard errors are reported in brackets.

Table 2: Calibrated Parameters

Parameter	Value	Description
g	0.01	Average quarterly growth rate of long-run productivity
β	0.98	Discount factor
α	0.34	Capital share in output
δ	0.05	Depreciation rate
φ	2	Inverse of Frisch elasticity
γ	0.22	Imported good share in output
σ	1.0	Price elasticity of export
η	8	Shape parameter of Pareto distribution
R_f^*	1.007	Average quarterly gross foreign interest rate
Ω	0.13	Capital flow adjustment cost

Table 3: Estimated Parameters

Parameter	Prior Distribution			Posterior Distribution			
	Distr.	Mean	S.D.	Mode	Mean	5%	95%
			Bubbly model				
h	Beta	0.5	0.2	0.56	0.57	0.45	0.67
Ω_k	Gamma	3	2	2.18	2.51	1.57	3.74
μ	Beta	0.3	0.05	0.30	0.30	0.22	0.37
ρ_g	Beta	0.5	0.2	0.99	0.99	0.97	0.996
ρ_a	Beta	0.5	0.2	0.74	0.75	0.64	0.84
ρ_ξ	Beta	0.5	0.2	0.38	0.38	0.21	0.55
ρ_{y^*}	Beta	0.5	0.2	0.997	0.995	0.989	0.999
$\rho_{R_f^*}$	Beta	0.85	0.05	0.75	0.75	0.70	0.80
σ_g (%)	Inv. Gamma	0.01	Inf	0.39	0.44	0.34	0.58
σ_a (%)	Inv. Gamma	0.01	Inf	4.60	4.88	4.03	5.91
σ_ξ (%)	Inv. Gamma	0.01	Inf	8.77	9.30	7.57	11.29
σ_{y^*} (%)	Inv. Gamma	0.01	Inf	0.79	0.81	0.71	0.92
$\sigma_{R_f^*}$ (%)	Inv. Gamma	0.01	Inf	0.47	0.48	0.41	0.55
			Bubbleless model				
h	Beta	0.5	0.2	0.53	0.51	0.37	0.63
Ω_k	Gamma	3	2	0.09	0.11	0.05	0.18
μ	Beta	0.3	0.05	0.30	0.29	0.26	0.33
ρ_g	Beta	0.5	0.2	0.96	0.95	0.92	0.97
ρ_a	Beta	0.5	0.2	0.93	0.93	0.88	0.97
ρ_ξ	Beta	0.5	0.2	0.98	0.97	0.94	0.99
ρ_{y^*}	Beta	0.5	0.2	0.997	0.995	0.989	0.999
$\rho_{R_f^*}$	Beta	0.85	0.05	0.89	0.88	0.83	0.93
σ_g (%)	Inv. Gamma	0.01	Inf	3.06	3.24	2.69	3.89
σ_a (%)	Inv. Gamma	0.01	Inf	6.20	6.51	5.57	7.57
σ_ξ (%)	Inv. Gamma	0.01	Inf	27.70	30.64	14.39	59.08
σ_{y^*} (%)	Inv. Gamma	0.01	Inf	0.79	0.81	0.71	0.92
$\sigma_{R_f^*}$ (%)	Inv. Gamma	0.01	Inf	0.45	0.46	0.40	0.53

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm.

Table 4: Real Business Cycle Moments

Moment	Data	Bubbly	Bubbleless 1	Bubbleless 2
Standard Deviation				
$std(gdp_t)$ (%)	2.68	3.18	2.33	8.21
$std(c_t)/std(gdp_t)$	1.32	1.61	1.63	1.01
$std(i_t)/std(gdp_t)$	2.89	2.84	2.25	2.57
$std(nx_t/gdp_t)$ (%)	1.61	2.76	1.21	1.40
$std(sp_t)/std(gdp_t)$	6.39	4.82	2.25	0.79
Correlation with GDP				
$corr(c_t, gdp_t)$	0.73	0.72	0.58	0.61
$corr(i_t, gdp_t)$	0.79	0.74	0.59	0.79
$corr(nx_t/gdp_t, gdp_t)$	-0.50	-0.53	-0.14	-0.43
$corr(sp_t, gdp_t)$	0.64	0.48	0.38	0.55
Correlation with Stock Prices				
$corr(c_t, sp_t)$	0.53	0.37	-0.46	-0.04
$corr(i_t, sp_t)$	0.52	0.72	0.71	0.75
$corr(nx_t/gdp_t, sp_t)$	-0.45	-0.66	0.71	-0.31
Autocorrelation				
$corr(y_t, y_{t-1})$	0.78	0.79	0.76	0.92
$corr(c_t, c_{t-1})$	0.75	0.70	0.68	0.90
$corr(i_t, i_{t-1})$	0.78	0.81	0.85	0.87
$corr(nx_t/gdp_t, nx_{t-1}/gdp_{t-1})$	0.82	0.66	0.41	0.71
$corr(sp_t, sp_{t-1})$	0.74	0.49	0.43	0.89

Note: Columns 3 and 4 display the real business cycle moments for the bubbly and bubbleless economies based on the same parameter values estimated using the bubbly model. Column 5 displays the real business cycle moments for bubbleless model 2 using re-estimated parameter values.

Table 5: Variance Decomposition of Bubbly Economy (%)

Period	Long-run Prod.	Short-run Prod.	Preference	Foreign Demand	Foreign Interest Rate
			Output		
Q1	8.9	84.1	0.5	1.9	4.6
Q4	32.9	52.7	0.6	3.1	10.7
Q8	55.5	31.4	1.5	2.9	8.6
Q16	82.6	11.1	0.9	1.7	3.6
			Consumption		
Q1	25.2	13.9	48.5	1.5	10.8
Q4	47.8	13.6	26.2	1.6	10.8
Q8	67.2	8.5	15.1	1.4	7.8
Q16	84.8	3.7	6.6	0.9	3.9
			Investment		
Q1	16.0	34.0	23.1	3.3	23.6
Q4	18.5	30.5	22.2	3.9	24.9
Q8	23.1	26.2	21.9	4.2	24.6
Q16	40.1	20.3	17.0	3.6	19.1
			Net exports/GDP		
Q1	43.0	0.3	20.2	2.7	33.9
Q4	51.5	3.3	8.1	2.8	34.3
Q8	55.9	3.3	7.1	2.4	31.2
Q16	60.4	3.3	6.3	2.2	27.7
			Stock prices		
Q1	39.7	18.7	9.6	4.0	27.9
Q4	43.3	18.6	10.4	3.5	24.2
Q8	48.8	16.3	9.7	3.2	22.0
Q16	59.4	12.6	7.9	2.6	17.5

Table 6: Variance Decomposition of Bubbleless Model 1 (%)

Period	Long-run Prod.	Short-run Prod.	Preference	Foreign Demand	Foreign Interest Rate
			Output		
Q1	0.7	97.4	1.0	0.8	0.2
Q4	8.2	88.4	1.2	1.6	0.6
Q8	35.2	59.2	3.0	2.1	0.5
Q16	80.5	16.6	1.6	1.3	0.1
			Consumption		
Q1	1.1	11.6	86.5	0.2	0.7
Q4	8.9	19.8	69.8	0.5	1.0
Q8	31.3	17.7	49.3	0.9	0.8
Q16	70.7	7.8	20.4	0.8	0.3
			Investment		
Q1	0.7	39.4	57.2	0.4	2.2
Q4	0.4	42.6	54.2	0.8	2.0
Q8	3.8	42.0	51.1	1.4	1.7
Q16	47.7	22.5	27.4	1.4	0.9
			Net exports/GDP		
Q1	0.002	11.9	83.7	0.01	4.4
Q4	0.1	11.3	78.9	0.1	9.6
Q8	0.1	11.7	78.7	0.2	9.4
Q16	0.4	11.4	78.8	0.2	9.2
			Stock prices		
Q1	1.1	33.7	62.4	0.3	2.5
Q4	1.4	34.8	61.1	0.4	2.2
Q8	1.4	34.9	61.0	0.6	2.1
Q16	10.0	31.6	55.5	1.1	1.8

Table 7: Variance Decomposition of Bubbleless Model 2 (%)

Period	Long-run Prod.	Short-run Prod.	Preference	Foreign Demand	Foreign Interest Rate
			Output		
Q1	2.1	94.8	1.6	0.4	1.2
Q4	50.1	47.8	1.3	0.2	0.5
Q8	81.7	16.5	1.5	0.1	0.1
Q16	94.7	3.9	1.3	0.04	0.02
			Consumption		
Q1	37.3	7.8	54.4	0.1	0.4
Q4	55.4	6.5	37.8	0.1	0.2
Q8	74.3	5.0	20.6	0.05	0.1
Q16	91.9	2.4	5.7	0.03	0.03
			Investment		
Q1	28.0	44.6	23.4	0.2	3.7
Q4	10.5	36.5	50.8	0.2	2.1
Q8	29.3	20.2	49.5	0.1	0.9
Q16	71.1	5.6	23.0	0.04	0.2
			Net exports/GDP		
Q1	37.7	0.2	40.9	0.01	21.1
Q4	28.1	2.8	48.9	0.1	20.2
Q8	25.5	2.9	52.3	0.1	19.2
Q16	35.3	2.5	45.6	0.1	16.5
			Stock prices		
Q1	41.7	37.1	16.4	0.3	4.4
Q4	14.9	30.7	52.3	0.2	1.9
Q8	12.0	20.9	66.3	0.2	0.7
Q16	49.9	7.4	42.5	0.1	0.1

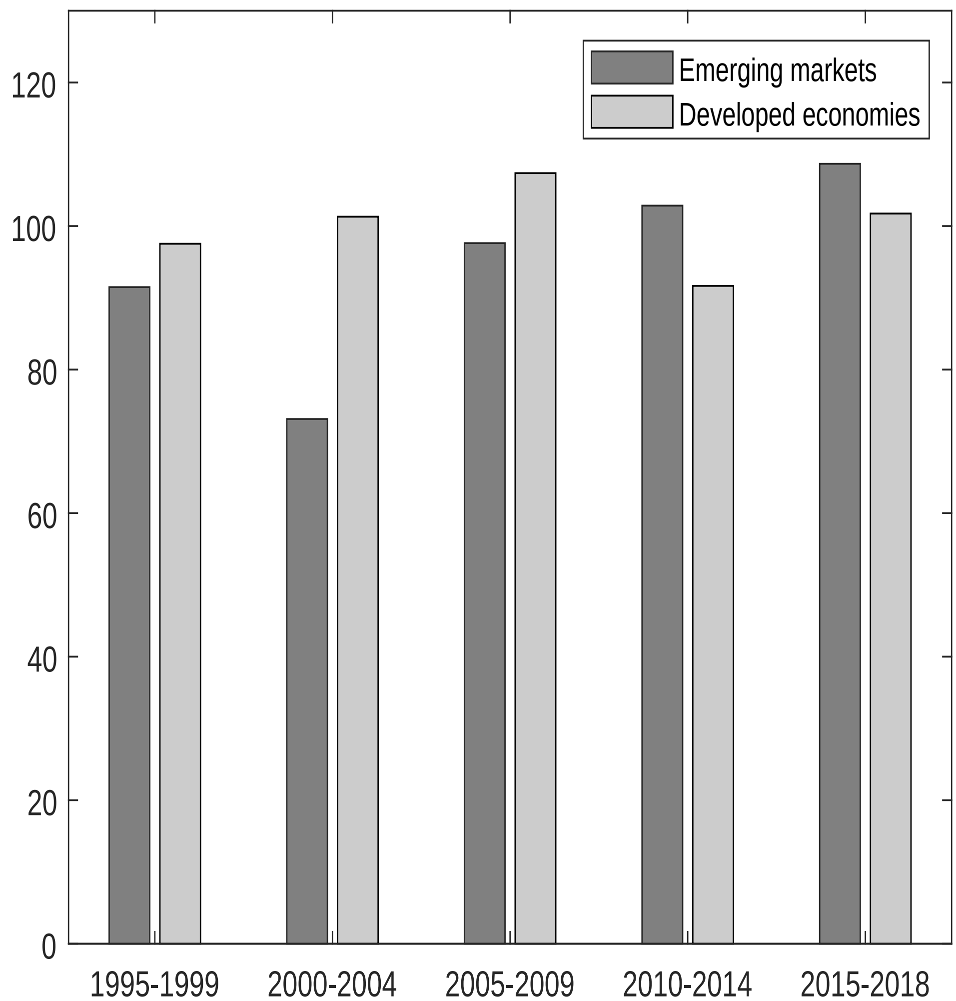

Figure 1: Stock market capitalization (in the percentage of GDP).

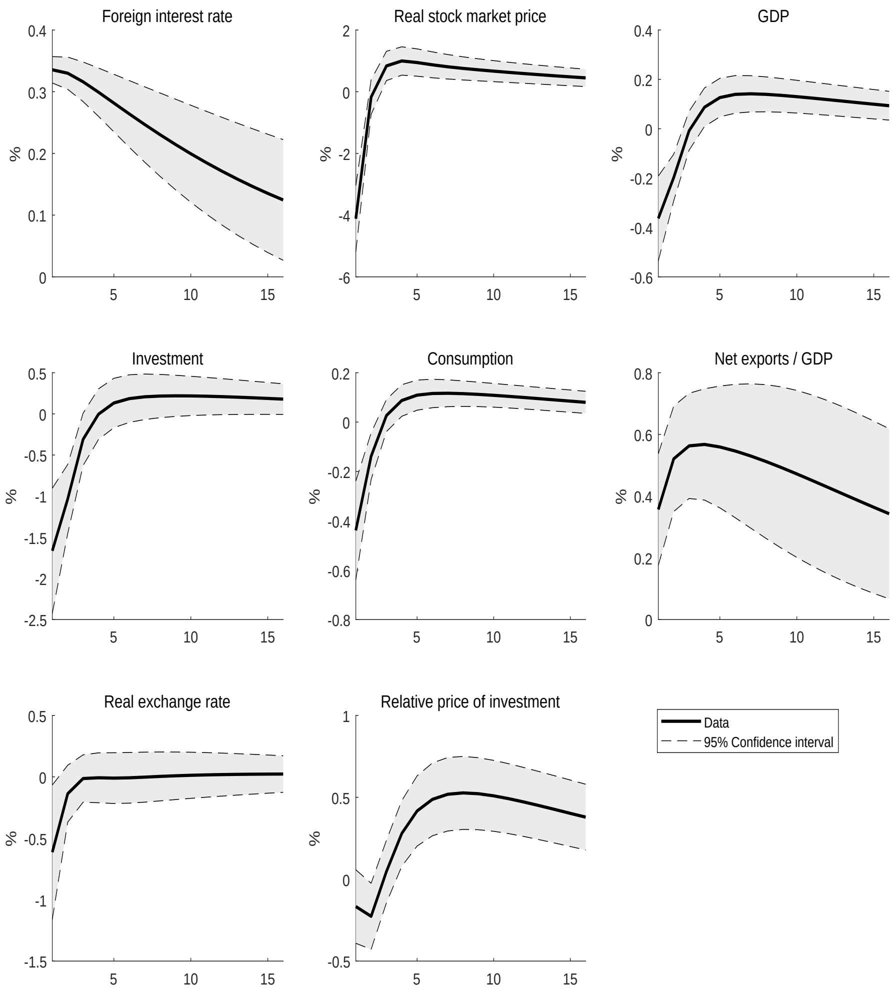

Figure 2: Impulse responses to a positive one-standard-deviation innovation in the foreign interest rate generated by the panel VAR estimation.

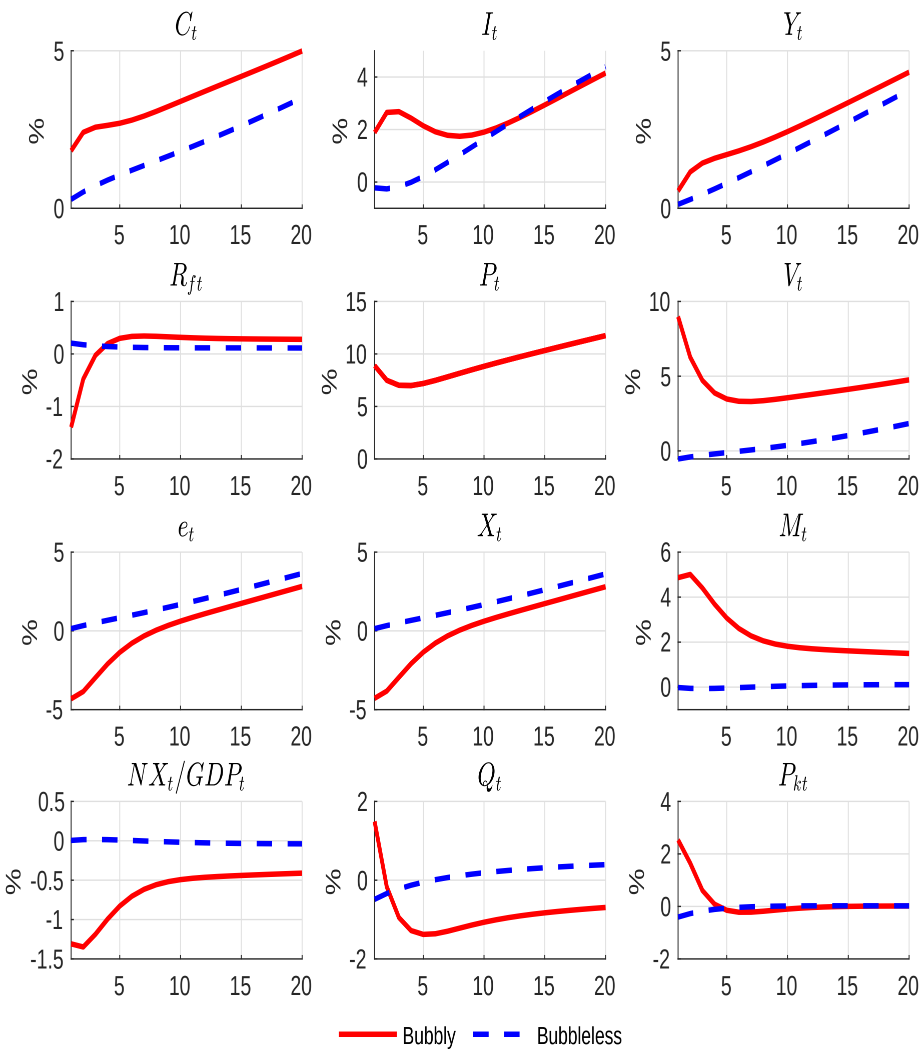

Figure 3: Impulse responses to a positive one-standard-deviation shock to the long-run productivity for the domestic economy. The solid lines describe the bubbly economy based on the bubbly model estimation. The dashed lines describe the bubbleless economy with the same parameters. The real interest rate R_{ft} and the net exports/GDP ratio NX_t/GDP_t are expressed as level deviations from their nonstochastic steady state values. The real exchange rate e_t, Tobin's Q_t, and the capital good price P_{kt} are expressed as percentage deviations from their nonstochastic steady state values. Other variables (consumption C_t, investment I_t, output Y_t, asset bubble P_t, stock price V_t, exports X_t, and imports M_t) are detrended and expressed as percentage deviations from their corresponding nonstochastic steady state values.

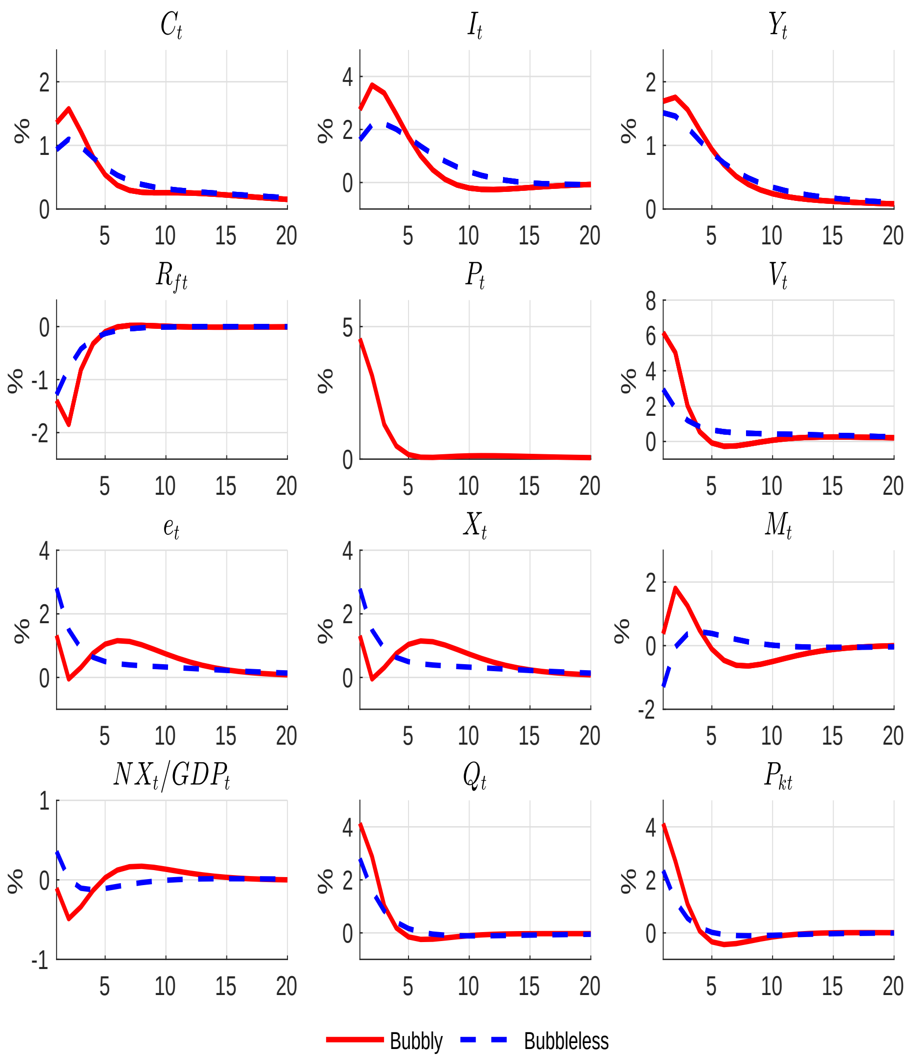

Figure 4: Impulse responses to a positive one-standard-deviation shock to the short-run productivity for the domestic economy. The solid lines describe the bubbly economy based on the bubbly model estimation. The dashed lines describe the bubbleless economy with the same parameters. The real interest rate R_{ft} and the net exports/GDP ratio NX_t/GDP_t are expressed as level deviations from their nonstochastic steady state values. The real exchange rate e_t, Tobin's Q_t, and the capital good price P_{kt} are expressed as percentage deviations from their nonstochastic steady state values. Other variables (consumption C_t, investment I_t, output Y_t, asset bubble P_t, stock price V_t, exports X_t, and imports M_t) are detrended and expressed as percentage deviations from their corresponding nonstochastic steady state values.

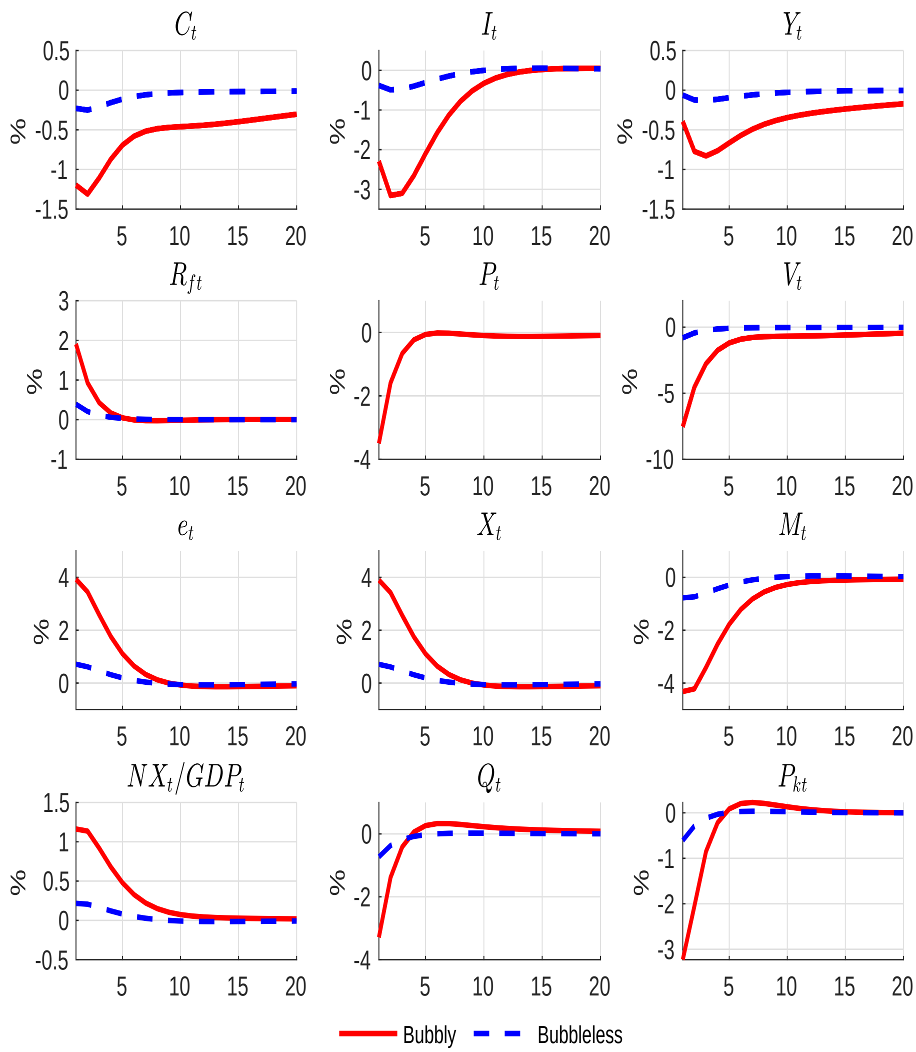

Figure 5: Impulse responses to a positive one-standard-deviation shock to the foreign interest rate for the domestic economy. The solid lines describe the bubbly economy based on the bubbly model estimation. The dashed lines describe the bubbleless economy with the same parameters. The real interest rate R_{ft} and the net exports/GDP ratio NX_t/GDP_t are expressed as level deviations from their nonstochastic steady state values. The real exchange rate e_t, Tobin's Q_t, and the capital good price P_{kt} are expressed as percentage deviations from their nonstochastic steady state values. Other variables (consumption C_t, investment I_t, output Y_t, asset bubble P_t, stock price V_t, exports X_t, and imports M_t) are detrended and expressed as percentage deviations from their corresponding nonstochastic steady state values.

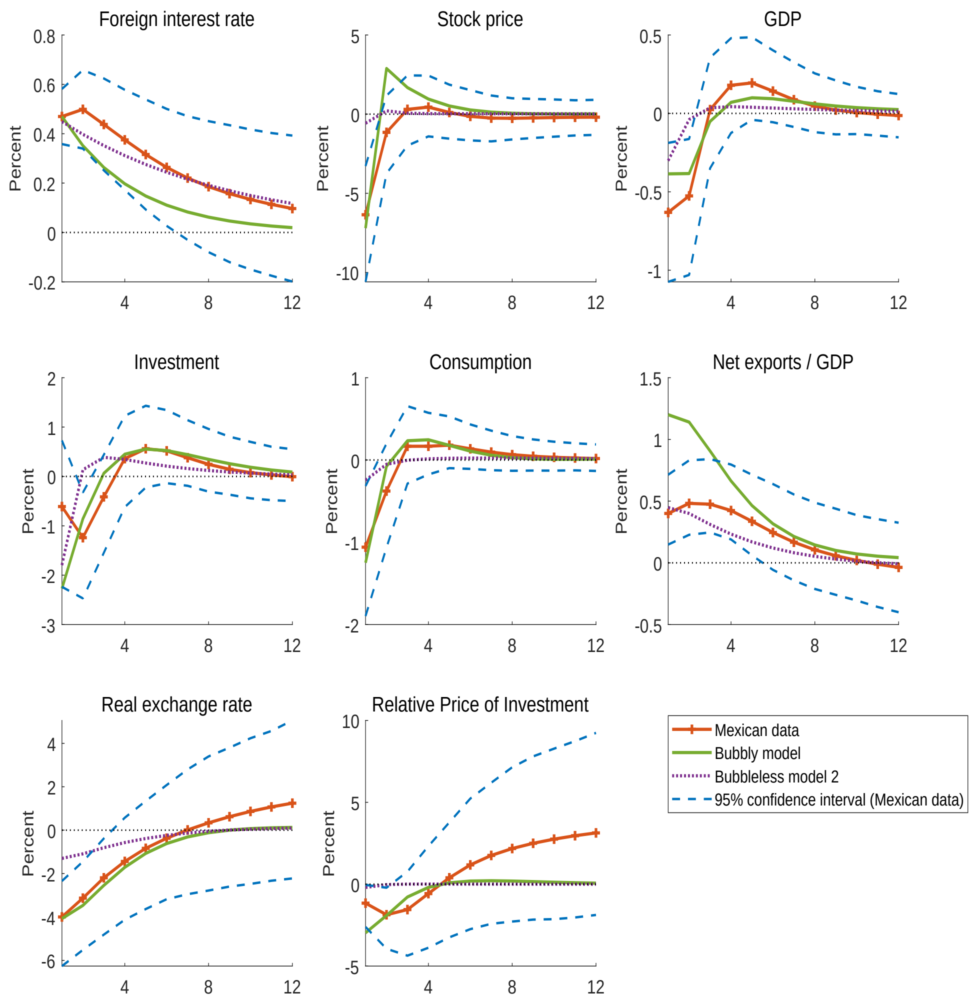

Figure 6: Data and model-implied impulse responses to a positive one-standard-deviation innovation in the foreign interest rate.