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RESEARCH ARTICLE

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Option exercise with temptation

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Abstract This paper adopts the Gul and Pensendorfer self-control utility model to analyze an agent's option exercise decision under uncertainty over an infinite horizon. The agent decides whether and when to do an irreversible activity. He is tempted by immediate gratification and suffers from self-control problems. The cost of self-control lowers the benefit from continuation or stopping and may erode the option value of waiting. When applied to the investment and exit problems, the model can generate the behavior of procrastination and preproperation. In addition, unlike the hyperbolic discounting model, the model here provides a unique prediction.

Keywords Time (in)consistency · Self-control · Temptation · Procrastination · Preproperation · Real options

JEL Classification Numbers D81 · D91 · G31

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1 Introduction

Suppose you have a referee report to write today. You feel writing the referee report is unpleasant and prefer to put off and do it tomorrow. But when tomorrow comes, you tend to delay again. This behavior is often referred to as procrastination—wait when you should do it. Suppose you have a coupon to see one movie over the next several weeks, and your allowance does not permit you to pay for a movie. You tend to see a movie in an earlier week even though there may be a better movie in a later week. This behavior is often referred to as preproperation—do it when you should wait.[1]

The procrastination and preproperation behavior is prevalent in many choice situations. Motivated by this behavior, this paper studies a general environment where an agent with *time-consistent preferences* makes irreversible binary choices under uncertainty over an infinite horizon. I adopt the Gul and Pesendorfer (2001, 2004) self-control utility model and interpret that behavior as an agent's struggling with temptations.[2] In this model, preferences are defined over a domain of sets of alternatives or decision problems. Utility depends on the decision problem from which current consumption is chosen. The interpretation is that temptation has to do with not just what the agent has consumed, but also what he could have consumed. The agent also seeks immediate gratification because an immediate benefit constitutes a temptation to the agent, but not because it has a higher relative weight. The agent may either succumb to temptations or exercise costly self-control to resist temptations.

The Gul-Pesendorfer model is time consistent because utility satisfies recursivity under the domain of decision problems. Thus, the standard recursive methods such as backward induction and dynamic programming can be applied. Importantly, the Gul-Pesendorfer model can explain time inconsistent behavior observed in some experiments as illustrated in Gul and Pesendorfer (2001, 2004). In addition to its tractability, the Gul-Pesendorfer model has clear welfare implications because it is based on the standard revealed preference principle. This is in contrast to the hyperbolic discounting model in which there is no generally agreed welfare criterion. In the hyperbolic discounting model, the agent at different dates is treated as a separate self. An alternative or a policy may be preferred by some selves, while it may make other selves worse off. The Pareto efficiency criterion and the long-run ex ante utility criterion are often adopted.

In Sect. 2, I model an agent's irreversible binary choice problem under uncertainty as an option exercise problem, or more technically, an optimal stopping problem. Irreversibility and uncertainty are important in many binary choice problems such as entry, exit, default, liquidation, project investment, and job search. According to the standard theory (see Dixit and Pindyck 1994), all these problems can be viewed as a problem where agents decide when to exercise an "option" analogous to a financial call option—it has the right but not the obligation to buy

[1] These examples and the term "preproperation" are borrowed from O'Donoghue and Rabin (1999a). As there, the comparison is based on the standard time consistent preferences benchmark.

[2] The Gul-Pesendorfer model has been applied to study taxation (Krusell et al. 2001), asset pricing (Krusell et al. 2002), (DeJong and Ripoll 2006), and nonlinear pricing (Esteban et al. 2006).

an asset at some future time of its choosing. This real options approach emphasizes the positive option value of waiting.

Unlike the standard theory, I make the distinction according to whether rewards and costs are immediate or delayed, as in O'Donoghue and Rabin (1999a) who first analyze procrastination and preproperation using a hyperbolic discounting model in a finite horizon setting.[3] This distinction is important to explain procrastination and preproperation in the hyperbolic discounting model since it makes present bias critical. This distinction is also important in the present model since it makes immediate temptation critical. After stating the model setup and assumptions, I present the self-control utility model developed by Gul and Pesendorfer (2001, 2004) and compare it with the hyperbolic discounting model. I then present propositions to characterize the optimal stopping rules for the general infinite-horizon model when the agent has self-control preferences. I describe the optimal stopping rules as a trigger policy whereby the agent stops the first time the state process hits a threshold value. I also explain the impact of temptation and self-control on the optimal stopping rules. In particular, I show that the cost of self-control may lower the benefit from both stopping and continuation and erode option value of waiting. Moreover, it may outweigh this option value if the level of self-control is sufficiently low.

In Sect. 3, I apply the results in Sect. 2 to investment and exit problems when the decision maker has self-control preferences. I also conduct welfare analysis. The investment and exit problems represent two different classes of option exercise problems. The project investment decision is an example in which an agent decides whether or not to exercise an option to pursue upside potential. Entry and job search are similar problems. I show the following: When the investment cost is immediate, the investor is tempted to delay investment. Thus, he procrastinates and the welfare loss is the forgone project value, which is equal to the cost of self-control. When the project value is immediate, the investor is tempted to invest early. Thus, he preproperates and the welfare loss is the forgone option value of waiting. If his level of self-control is sufficiently low, the investor may invest in negative net present value (NPV) projects. This reflects the trade-off between investing now but incurring financial losses and waiting but incurring self-control costs. When both the project value and investment cost are immediate, the investor also preproperates and the welfare loss is the forgone option value of waiting. In this case, he never invests in negative NPV projects. If his level of self-control is sufficiently low, he invests according to the myopic rule which compares the current period benefit and cost only.

After analyzing the investment problem, I turn to the exit problem, in which an owner/manager with self-control preferences decides when and if to terminate a project. This problem represents an example in which an agent decides whether or not to exercise an option to avoid downside losses. Other examples include default and liquidation decisions. I show the following: When the profits are immediate,

[3] Strotz (1956) first studies time-inconsistent preferences in economics. Akerlof (1991) analyzes procrastination, but frames his discussion very differently. The O'Donoghue and Rabin model has been generalized by a number of papers, e.g., O'Donoghue and Rabin (1999b, 2001), Brocas and Carrillo (2001, 2005). The hyperbolic discounting model has been applied to study consumption-saving (Laibson 1994, 1997), job search (DellaVigna and Paserman 2005), social security (Imrohoroglu et al. 2003), retirement (Diamond and Koszegi 2003), investment (Grenadier and Wang 2006), and general equilibrium (Herings and Rohde 2006).

the owner is tempted to continue the project even when he should terminate. Thus, he procrastinates. The welfare loss is equal to the cost of self-control. By contrast, when the fixed cost of continuing the project is immediate, the owner is tempted to avoid this cost and preproperates to terminate, even though he may make positive net profits. The welfare loss is the forgone current and future profit opportunities. When both the cost and profit are immediate, the owner also preproperates, but never terminates the project at a time when he makes a negative net profits. If the owner's level of self-control is sufficiently low, the owner terminates the project according to the myopic rule.

O'Donoghue and Rabin (2001) analyze a similar infinite horizon deterministic task choice problem using the hyperbolic discounting model. They show that their model typically has multiple equilibria using the "perception-perfect strategy" solution concept. They also show that some equilibria are cyclic, with some fixed intervals of length between action dates. I show that cyclic equilibria also arise in the problem under uncertainty analyzed here. These cyclic equilibria are counterintuitive and unappealing. By contrast, the Gul-Pesendorfer model admits a unique prediction. The importance of uniqueness is emphasized by Fudenberg and Levine (2006). Fudenberg and Levine provide a dual-self model which is also motivated by time inconsistency issues. They show that their reduced-form model is similar to the Gul-Pesendorfer model. They independently analyze an optimal stopping problem which is a special case of my general setup. As here, they also characterize the optimal stopping rule by a trigger policy and derive a unique solution. Their model differs from mine in that they assume the cost is stochastic and the reward is constant. Moreover, they consider only the case of immediate costs and future benefits.

I conclude the paper in Sect. 4 and relegate technical details to an appendix.

2 The model

I model an agent's option exercise decisions as an optimal stopping problem. Specifically, consider a discrete time and infinite horizon environment. In each period, the agent decides whether to stop a process and take a termination payoff, or to continue for one more period and make the same decision in the future. The decision is irreversible in the sense that if the agent chooses to stop, he makes no further choices. Formally, time is denoted by $t = 1, 2, \ldots,$ and uncertainty is generated by a state process $(x_t)_{t\geq 1}$. For simplicity, I assume that x_t is drawn identically and independently from a distribution F on $[a, A]$, where $A > a > 0$. Continuation at date t generates a payoff $\pi(x_t)$ and incurs a cost c_c, while stopping at date t yields a payoff $\Omega(x_t)$ and incurs a cost c_s, where π and Ω are continuous and increasing functions. I will provide applications in Sect. 3 to show that this simple model covers a wide rage of economic problems.

As in O'Donoghue and Rabin (1999a), I make an important distinction according to whether costs and rewards are obtained immediately or delayed. The term of immediate costs is used to refer to the situation where the cost is incurred immediately while the reward is delayed. The term of immediate rewards is used to refer to the situation where the reward is incurred immediately while the cost is delayed. For simplicity, I consider the case of one period delay only. In addition, I also consider the case where both costs and rewards are immediate. This case is not explicitly analyzed by O'Donoghue and Rabin (1999a). O'Donoghue and Rabin (1999a) give many examples to illustrate that the preceding distinction is meaningful in reality. Moreover, this distinction is important to generate procrastination and preproperation.

Unlike O'Donoghue and Rabin (1999a), I consider uncertainty and infinite horizon. Uncertainty is prevalent in intertemporal choices and infinite horizon is necessary to analyze long-run stationary decision problems. These two elements are building blocks in many economic models, especially in macroeconomics and finance. Incorporating them allows me to study some interesting applications in macroeconomics and finance, as illustrated in Sect. 3.

2.1 Self-control preferences

O'Donoghue and Rabin (1999a) explain procrastination and preproperation by adopting the time-inconsistent hyperbolic discounting model proposed by Phelps and Pollak (1968). This model can be described as follows. Let $U_t\left(c_t, ..., c_T\right)$ represent an agent's intertemporal preferences from a consumption stream $(c_t, ..., c_T)$ in period t. T could be finite or infinite. The hyperbolic discounting preferences are represented by

$$U_t\left(c_t, ..., c_T\right) = u_t\left(c_t\right) + \beta E\left[\sum_{k=1}^{T-t} \delta^k u_{t+k}\left(c_{t+k}\right)\right], \quad t \geq 1,$$

where $0 < \beta, \delta \leq 1$ and $u_{t+k}(\cdot)$ represents period $t + k$ utility function, $k = 0, \ldots, T - t$. In addition, δ represents long-run, time-consistent discounting and β represents a "bias for the present." The agent at each point in time is regarded as a separate "self" who is choosing his current behavior to maximize current preferences, while his future selves will control his future behavior. In this model, an agent must form expectation about his future selves' preferences. Two extreme assumptions are often made. In one extreme, the agent is naive and believes his future selves' preferences will be identical to her current self's, not realizing changing tastes. In the other extreme, the agent is sophisticated and knows exactly what his future selves' preferences will be. The solution concept of subgame perfect Nash equilibrium is often adopted. As typical in dynamic games, multiple equilibria may arise (see Proposition 6 below, Fudenberg and Levine 2006; Krusell and Smith 2003, and O'Donoghue and Rabin 2001).

The hyperbolic discounting model provides an intuitive explanation for procrastination and preproperation. The key intuition relies on the following feature of the hyperbolic discounting model. When $\beta < 1$, the agent gives more relative weight to period t when he makes a choice in period t than he does when he makes the choice in any period prior to period t. That is, the agent has a time-inconsistent taste for immediate gratification. There seems to be ample experimental evidence on the time-inconsistent behavior.[4] In a typical experiment, subjects choose between a smaller period t reward and a larger period $t + 1$ reward. If the choice is made in period t then the smaller earlier reward is chosen. If the choice is made earlier, then the larger later reward is chosen.

[4] See, for example, Thaler (1981), Ainslie and Haslam (1992), Kirby and Herrnstein (1995).

Gul and Pesendorfer (2001, 2004) propose an alternative interpretation of this behavior based on time-consistent preferences. Their key insight is that the agent finds immediate rewards tempting. When the agent makes the choice in period t, the period t reward constitutes a temptation to the agent. So he may choose a smaller period t reward rather than a larger period $t+1$ reward. However, if he makes a choice prior to period t, neither period t reward nor period $t+1$ reward can be consumed immediately and hence his decisions are unaffected by temptations.

To capture this intuition, Gul and Pesendorfer (2001, 2004) develop a model of self-control based on a choice theoretic axiomatic foundation.[5] They define self-control preferences over sets of alternative consumption levels or decision problems—a domain different from the usual one. The interpretation is that temptation has to do with not just what the agent chooses, but what he could have chosen. Specifically, let B_t be the agent's period t decision problem and W_t represent his intertemporal utility in period t. Then the self-control preferences are represented by

$$W_t\left(B_t\right) = \max_{c_t \in B_t} \left\{u_t\left(c_t\right) + v_t\left(c_t\right) + \delta E\left[W_{t+1}\left(B_{t+1}\right)\right]\right\} - \max_{c_t \in B_t} v_t\left(c_t\right), \quad t \geq 1. \tag{1}$$

If T is finite, since there is no continuation problem in period T,

$$W_T\left(B_T\right) = \max_{c_T \in B_T} \left\{u_T\left(c_T\right) + v_T\left(c_T\right)\right\} - \max_{c_T \in B_T} v_T\left(c_T\right). \tag{2}$$

Here $u_t + \delta W_{t+1}$ represents the commitment utility in period t and v_t is the temptation utility in period t. The expression $u_t\left(c_t\right) + v_t\left(c_t\right) + \delta E\left[W_{t+1}\left(B_{t+1}\right)\right]$ reflects the compromise between commitment and temptation. The agent's optimal choice in period t maximizes this expression. When this choice is identical to the temptation choice in the second maximum in (1) or (2), the agent succumbs to the temptation and there is no self-control cost. However, when the two choices do not coincide, the agent exercises costly self-control and $v_t\left(c_t\right) - \max_{c_t \in B_t} v_t\left(c_t\right)$ represents the cost of self-control. If $T = \infty$, I consider a stationary model and drop time subscripts,

$$W\left(B\right) = \max_{c \in B} \left\{u\left(c\right) + v\left(c\right) + \delta E\left[W\left(B'\right)\right]\right\} - \max_{c \in B} v\left(c\right). \tag{3}$$

Here B' denotes the choice problem in the next period and $E\left[\cdot\right]$ denotes the expectation operator.

An important feature of the Gul–Pesendorfer model is that it is time consistent since utility in (1)–(3) is defined recursively. Thus, the usual recursive method such as backward induction and dynamic programming can be applied. Importantly, in addition to this tractability, this model has clear welfare implications. That is, this model follows the revealed preference tradition of standard economic models: if the agent chooses one alternative over another, then he is better off with that choice. By contrast, time inconsistent models do not have a universally agreed welfare criterion. Some researchers such as Laibson (1994, 1997) adopt a Pareto efficiency criterion, requiring all period selves weakly prefer one strategy to another. Other

[5] See Gul and Pesendorfer (2001, 2004) for detailed axioms. The key axiom is set betweenness. Their model is more general than the one presented in this paper.

researchers such as O'Donoghue and Rabin (1999a) adopt an ex ante long-run utility criterion. The problem of the welfare analysis of the time inconsistent models is that the connection between choice and welfare is broken.

In the present paper, I adopt the Gul–Pesendorfer model to analyze the option exercise problem. In this problem, the set B consists of two elements representing the current period payoffs from stopping and continuation since the choice problems are binary. If the agent chooses to stop, then there is no continuation problem so that $B' = \emptyset$ and $W\left(B'\right) = 0$. If the agent chooses to continue, then he faces the same decision problem in the next period so that B' consists of two elements representing the payoffs from stopping and continuation in the next period. To simplify exposition, I assume risk neutrality throughout. That is, $u(c) = c$ and $v(c) = \lambda c$, $\lambda > 0$. Here λ is the self-control parameter. An increases in λ raises the weight on the temptation utility and leads to a decrease in the agent's (instantaneous) self-control.[6] When $\lambda = 0$, the model reduces to the standard time-additive expected utility model with exponential discounting.

2.2 Optimal stopping rules

I now adopt the Gul–Pesendorfer utility model (3) to solve the agent's option exercise problem by dynamic programming.[7] The key is to formulate Bellman equations. These Bellman equations are different for the cases of immediate costs, immediate rewards, and immediate costs and rewards. They are described as follows:

1. Immediate costs

$$W(x) = \max\left\{\delta\Omega(x) - (1+\lambda)\,c_s,\ \delta\pi(x) - (1+\lambda)\,c_c + \delta\int W\left(x'\right)\mathrm{d}F\left(x'\right)\right\} - \lambda\max\left\{-c_c, -c_s\right\}. \tag{4}$$

2. Immediate rewards

$$W(x) = \max\left\{(1+\lambda)\,\Omega(x) - \delta c_s,\ (1+\lambda)\,\pi(x) - \delta c_c + \delta\int W\left(x'\right)\mathrm{d}F\left(x'\right)\right\} - \lambda\max\left\{\pi(x), \Omega(x)\right\}. \tag{5}$$

3. Immediate costs and rewards

$$W(x) = \max\left\{(1+\lambda)\left(\Omega(x) - c_s\right),\ (1+\lambda)\left(\pi(x) - c_c\right) + \delta\int W\left(x'\right)\mathrm{d}F\left(x'\right)\right\} - \lambda\max\left\{\pi(x) - c_c, \Omega(x) - c_s\right\}. \tag{6}$$

[6] See Gul and Pesendorfer (2004) for the definition and characterization of measures of self-control. To distinguish between differences in impatience and differences in self-control, one should fix intertemporal choices and consider instantaneous self-control only.

[7] See Stokey and Lucas (1989) and Dixit and Pindyck (1994) for the theory of dynamic programming. The existence of a bounded and continuous value function is guaranteed by the contraction mapping theorem.

I explain (4) in some detail. The interpretations for the other two equations are similar. Suppose costs are immediate. In the current period, the agent faces the decision problem of whether to continue or to stop after observing the current state takes the value x. Stopping incurs an immediate cost c_s and yields the payoff $\Omega(x)$. However, the agent obtains this payoff in the next period so that he gets a discounted value $\delta\Omega(x)$. After stopping, the agent has no further choice, and hence the continuation value is zero. Because of the compromise between the temptation and the commitment utilities, the total payoff of stopping is $\delta\Omega(x) - (1+\lambda) c_s$, which is the first term in the first curly bracket in (4). Similarly, continuation incurs an immediate cost c_c and gets a discounted payoff from the next period $\delta\pi(x)$. The agent has to make the same choice of whether to stop or to continue in the next period, and hence gets continuation value $\delta \int W(x') \, \mathrm{d}F(x')$. Thus, we have the second term in the first curly bracket in (4). Finally, the agent is tempted by whether to stop now and avoid the cost of continuation c_c or to continue and avoid the cost of stopping c_s. Thus, the temptation choice is described by $\lambda \max\{-c_c, -c_s\}$, which is the last term in (4). Note that the rewards $\Omega(x)$ and $\pi(x)$ do not enter the temptation utility since they are obtained with a one period delay and hence do not tempt the agent.

Clearly, continuation is optimal for those values of x for which the maximum in the first line of (4) is attained at the second expression in the curly bracket. Immediate termination is optimal when the maximum is attained at the first expression. Call the corresponding divisions of the range of x the continuation region and the stopping region, respectively. A similar analysis applies to (5) and (6). In general, for arbitrary payoffs $\pi(x)$ and $\Omega(x)$, the continuation and stopping regions could be arbitrary. In most applications, these regions can be easily characterized. In particular, there is a threshold value such that it partitions the state space into a continuation region and a stopping region. Consequently, the optimal stopping rule is characterized by a trigger policy. That is, the agent stops the first time the process $(x_t)_{t\geq 1}$ hits the threshold value. Importantly, depending on the payoff structure, the stopping region could be above the threshold value or below it. The former case describes the problems of pursing upside potential such as investment and job search. The latter case describes the problems of avoiding downside losses such as exit and default. In the applications in Sect. 3, I will impose explicit assumptions and provide a more complete and transparent analysis of these problems.

Here I do not provide general conditions for the structure of the continuation and stopping regions.[8] Instead, I provide explicit characterizations of the threshold value for the case where the agent pursues upside potential only. That is, I consider that the continuation region is below the threshold value. I characterize the optimal stopping rule in the following proposition:

Proposition 1 *Consider each problem in* (4)–(6). *If* $\Omega(x) - \pi(x)$ *is strictly increasing in* x, *then the optimal stopping rule is described by one of the following cases: (a) The agent never stops. (b) The agent stops immediately. (c) There is a unique threshold value* $x^* \in [a, A]$ *such that the agent stops the first time the process* $(x_t)_{t\geq 1}$ *hits* x^* *from below.*

In what follows, I will focus on the third case since it is the most interesting case. To compare with the standard model, I denote by $\overline{x}$ the threshold value for an

[8] See Dixit and Pindyck (1994) [pp. 128–130], for discussions for standard preferences.

agent having standard preferences with $\lambda = 0$. Since the mean value of the option exercise time increases with the threshold value, comparative static analysis for the threshold value reveals properties of the average option exercise time.

Proposition 2 *Let the assumption in Proposition 1 hold. Suppose costs are immediate.*

(i) *The threshold value x^* satisfies the equation*

$$
\begin{aligned}
(1-\delta)\left[\delta\Omega(x^*) - c_s\right] &+ \lambda(1-\delta)\left[-c_s - \max\{-c_c, -c_s\}\right] \\
&= \delta\pi\left(x^*\right) - c_c + \delta\int_{x^*}^{A}\delta\left[\Omega(x') - \Omega(x^*)\right]\mathrm{d}F\left(x'\right) \\
&\quad + \delta\int_{a}^{x^*}\delta\left[\pi\left(x'\right) - \pi\left(x^*\right)\right]\mathrm{d}F\left(x'\right) \\
&\quad + \lambda\left[-c_c - \max\{-c_c, -c_s\}\right]. \qquad (7)
\end{aligned}
$$

(ii) *If $c_s \geq c_c$, then $x^* \geq \overline{x}$. If $c_s < c_c$, then $x^* < \overline{x}$.*

The interpretation of (7) is as follows. The expression on the left side of Eq. (7) describes the normalized per period benefit from stopping, while the expression on the right side describes the benefit from continuation or the opportunity cost of stopping. The agent optimally stops at the threshold value x^* such that he is indifferent between stopping and continuation.

Note that the benefit from continuation consists of not only the current period value but also an option value of waiting when the agent waits for one more period and gets a better draw $x' > x^*$ or a worse draw $x' < x^*$. The option value is represented by the integration terms on the right side of (7).

Importantly, all terms containing λ represent the cost of self-control. Specifically, the term on the first line of (7) represents the normalize per period cost of self-control if the agent chooses to stop. The term on the fourth line of (7) represents the cost of self-control if the agent chooses to continue. When $\lambda = 0$, the model reduces to the one with standard preferences.

For part (ii), if the cost of stopping is higher than the cost of continuation, i.e., $c_s \geq c_c$, then the agent is tempted to continue. Thus, if the agent chooses to continue, there is no self-control cost so that the term on the third line of (7) vanishes. By contrast, if the agent chooses to stop, then he has to exercise self-control and incurs a cost given in the second term in the first line of (7). Consequently, compared with the standard model, the benefit of stopping is lowered and the agent procrastinates to exercise the option. The interpretation of the other case ($c_s < c_c$) is similar.

Proposition 3 *Let the assumption in Proposition* 1 *hold. Suppose rewards are immediate.*

(i) *The threshold value x^* satisfies the equation*

$$
\begin{aligned}
&(1-\delta)\left[\Omega(x^*)-\delta c_s\right]+\lambda(1-\delta)\left[\Omega(x^*)-\max\left\{\pi\left(x^*\right),\Omega\left(x^*\right)\right\}\right] \\
&= \pi\left(x^*\right)-\delta c_c+\delta\int_a^{x^*}\left[\pi\left(x'\right)-\pi\left(x^*\right)\right]\mathrm{d}F\left(x'\right) \\
&\quad +\delta\int_{x^*}^{A}\left[\Omega(x')-\Omega(x^*)\right]\mathrm{d}F\left(x'\right) \\
&\quad +\lambda\left[\pi\left(x^*\right)-\max\left\{\pi\left(x^*\right),\Omega\left(x^*\right)\right\}\right] \\
&\quad +\lambda\delta\int_a^{x^*}\left[\pi\left(x'\right)\right. \\
&\quad -\left.\max\left\{\pi\left(x'\right),\Omega\left(x'\right)\right\}-\left(\pi\left(x^*\right)-\max\left\{\pi\left(x^*\right),\Omega\left(x^*\right)\right\}\right)\right]\mathrm{d}F\left(x'\right) \\
&\quad +\lambda\delta\int_{x^*}^{A}\left[\Omega(x')-\max\left\{\pi\left(x'\right),\Omega\left(x'\right)\right\}-\left(\Omega\left(x^*\right)\right.\right. \\
&\quad -\left.\left.\max\left\{\pi\left(x^*\right),\Omega\left(x^*\right)\right\}\right)\right]\mathrm{d}F\left(x'\right).
\end{aligned}
\tag{8}
$$

(ii) *If $\Omega(x)\geq\pi(x)$ for all x, then $x^*\leq\overline{x}$.*

The interpretation of (8) is similar to that of (7). Unlike in the case of immediate costs, the agent is tempted by stochastic rewards in each period. Thus, self-control incurs not only a current period cost but also a future period cost. The former is represented by the fourth line of (8). The latter is represented by the last two lines of Eq. (8). In particular, in the next period the agent is tempted to stop or continue depending on whether the state in the next period is better than x^* $(x' > x^*)$ or worse than x^* $(x' < x^*)$. To resist this temptation, the agent must incur a future self-control cost.

Consider part (ii). When the rewards from stopping are always higher than the rewards from continuation, $\Omega(x)\geq\pi(x)$, the agent is tempted to stop. Stopping at x^* means the agent succumbs to the temptation and hence there is no cost of self-control. Thus, the second term in the first line of (8) vanishes. If the agent decides to continue at x^*, he has to resist the temptation to stop and hence incurs a cost of self-control represented by the third line of (8). Consider next the future cost of self-control. In the next period, if $x' > x^*$, the agent stops and succumbs to the temptation. There is no cost of self-control and hence the term in the last line of (8) vanishes. If $x' < x^*$, the agent should continue and incur a cost of self-control in the next period. This implies that the benefit from continuation is lowered, compared with the standard model. Consequently, the agent preproperates to exercise the option.

Proposition 4 *Let the assumption in Proposition* 1 *hold. Suppose both costs and rewards are immediate.*

(i) *The threshold value x^* satisfies the equation*

$$
\begin{aligned}
&(1-\delta)\left[\Omega(x^*)-c_s\right]+(1-\delta)\lambda\left[\Omega(x^*)-c_s\right.\\
&\quad\left.-\max\left\{\pi\left(x^*\right)-c_c,\Omega\left(x^*\right)-c_s\right\}\right]\\
&=\left(\pi\left(x^*\right)-c_c\right)+\delta\int_a^{x^*}\left[\pi\left(x'\right)-\pi\left(x^*\right)\right]\mathrm{d}F\left(x'\right)\\
&\quad+\delta\int_{x^*}^{A}\left[\Omega(x')-\Omega(x^*)\right]\mathrm{d}F\left(x'\right)\\
&\quad+\lambda\left(\pi\left(x^*\right)-c_c-\max\left\{\pi\left(x^*\right)-c_c,\Omega\left(x^*\right)-c_s\right\}\right)\\
&\quad+\lambda\delta\int_a^{x^*}\left[\pi\left(x'\right)-c_c-\max\left\{\pi\left(x'\right)-c_c,\Omega\left(x\right)-c_s\right\}\right.\\
&\quad-\left.\left(\pi\left(x^*\right)-c_c-\max\left\{\pi\left(x^*\right)-c_c,\Omega\left(x^*\right)-c_s\right\}\right)\right]\mathrm{d}F\left(x'\right)\\
&\quad+\lambda\delta\int_{x^*}^{A}\left[\Omega(x')-c_s-\max\left\{\pi\left(x'\right)-c_c,\Omega\left(x'\right)-c_s\right\}\right.\\
&\quad-\left.\left(\Omega\left(x^*\right)-c_s-\max\left\{\pi\left(x^*\right)-c_c,\Omega\left(x^*\right)-c_s\right\}\right)\right]\mathrm{d}F\left(x'\right).
\end{aligned}
\tag{9}
$$

(ii) *If $\Omega(x)-c_s \geq \pi(x)-c_c$ for all x, then $x^* \leq \overline{x}$.*

The interpretation of this proposition is similar to that of Proposition 3. So I omit it.

Finally, when the continuation region is above the threshold value, the agent tries to avoid downside losses. This happens in the exit problem as described in the next section. One can provide characterizations for the threshold value similar to Propositions 1–4.

3 Applications

This section applies the setup and results in Sect. 2 to study investment and exit problems.

3.1 Investment

An important type of option exercise problems is the irreversible investment problem.[9] Consider that a risk-neutral investor decides whether and when to invest in a project with stochastic values x_t in period t. Investment incurs a lump sum cost I at

[9] The standard real options approach assumes that investment payoffs can be spanned by traded securities so that preferences do not matter for investment timing (Dixit and Pindyck 1994). Here we assume that these securities are not available to the decision maker.

the time of the investment. This investment problem can be cast into the framework laid out in Sect. 2 by setting[10]

$$\Omega(x) = x,\ c_s = I, \quad \pi(x) = c_c = 0.$$

In standard investment problems, costs and benefits come at the same time. In reality, there are many instances where costs and benefits do not arrive at the same time. For example, an important feature of real investment is time to build. It is often the case that it takes time to complete a factory or develop a new product. This is an instance of immediate costs and delayed rewards. As a different example, some firms start investing in a project financed by borrowing. Debts may be gradually repaid after the firms earn profits. This is an instance of immediate rewards and delayed costs.

I now analyze these different cases by rewriting the Bellman equations (4)–(6) as follows:

1. Immediate costs

$$W(x) = \max\left\{\delta x - (1+\lambda) I, \delta \int W(x')\, \mathrm{d}F(x')\right\} - \lambda \max\{0, -I\}. \quad (10)$$

2. Immediate rewards

$$W(x) = \max\left\{(1+\lambda) x - \delta I, \delta \int W(x')\, \mathrm{d}F(x')\right\} - \lambda \max\{x, 0\}. \quad (11)$$

3. Immediate costs and rewards

$$W(x) = \max\left\{(1+\lambda)(x - I), \delta \int W(x')\, \mathrm{d}F(x')\right\} - \lambda \max\{x - I, 0\}. \quad (12)$$

From the above equations, the effect of self-control is transparent. When rewards are immediate, the investor is tempted to invest now. He may either succumb to temptation or exercise costly self-control. Self-control acts as if the benefit of waiting is lowered by λx in utility value. Thus, the investor has an incentive to preproperate. By contrast, when costs are immediate, the investor is tempted to wait. Self-control acts as if the cost of investment is increased by an amount of λI. This causes the investor to procrastinate. The interesting case is when both costs and rewards are immediate. When $x > I$, the investor is tempted to invest earlier. But when $x < I$, the investor is tempted to wait. Thus, the result seems to be ambiguous. Using Propositions 1–4, I formalize the preceding intuition and characterize the optimal investment rule for each case in the following:

Proposition 5 *Under the conditions given in the appendix, there is a unique threshold value* $x^* \in [a, A]$ ($\overline{x} \in [a, A]$) *such that the investor with self-control preferences (standard preferences) invests the first time the process* $(x_t)_{t\geq 1}$ *reaches this value.*

[10] The stopping problem analyzed in Fudenberg and Levine (2006) is a special case of my general framework by setting $\pi(x) = x$, $c_c = c_s = 0$, and $\Omega(x) = \frac{\delta}{1-\delta} v$ for some constant $v > 0$. They analyze the case with immediate costs and future rewards only.

(i) *If costs are immediate, then* x^* *satisfies*

$$\left(\delta x^* - I\right)(1-\delta) - \lambda I(1-\delta) = \delta \int_{x^*}^{A} \delta\left(x - x^*\right) \mathrm{d}F(x), \tag{13}$$

and $\overline{x}$ *is the solution for* $\lambda = 0$. *Moreover,* $x^* > \overline{x} > I/\delta$ *and* x^* *increases with* λ.

(ii) *If rewards are immediate, then* x^* *satisfies*

$$\left(x^* - \delta I\right)(1-\delta) = \delta \int_{x^*}^{A} \left(x - x^*\right) \mathrm{d}F(x) + \lambda\delta \int_{a}^{x^*} \left(x^* - x\right) \mathrm{d}F(x) - \lambda x^*, \tag{14}$$

and $\overline{x}$ *is the solution for* $\lambda = 0$. *Moreover,* $x^* < \overline{x}$, $\overline{x} > \delta I$ *and* x^* *decreases with* λ.

(iii) *If both costs and rewards are immediate then* x^* *satisfies*

$$\begin{aligned}\left(x^* - I\right)(1-\delta) = \delta \int_{x^*}^{A} \left(x - x^*\right) \mathrm{d}F(x) - \lambda\left(x^* - I\right) \\ + \lambda\delta \int_{a}^{x^*} \left[\left(x^* - I\right) - \max(0, x - I)\right] \mathrm{d}F(x),\end{aligned} \tag{15}$$

and $\overline{x}$ *is the solution for* $\lambda = 0$. *Moreover,* $I \leq x^* < \overline{x}$ *and* x^* *decreases with* λ.

The left and right sides of Eq. (13)–(15) describe the utility benefits from investment and waiting, respectively. At the investment threshold x^*, the investor is indifferent between investing and waiting. Before analyzing the impact of self-control, I first discuss briefly the solution for the standard model corresponding to $\lambda = 0$. As is well known, because of irreversibility and uncertainty, waiting has positive option value. The option value in each case is represented by the first term on the right side of the corresponding equations (13)–(15). Due to this option value, the investor with standard preferences invests at the time when the threshold value is higher than the cost (e.g., in part (i) of Proposition 5, $\delta\bar{x} > I$). Thus, the standard net present value (NPV) rule leads to a non-optimal early investment time. This result is well known in the finance literature (e.g., Dixit and Pindyck 1994).

I next turn to the case with self-control. Consider part (i) of Proposition 5. If costs are immediate, the investor is tempted to wait. To resist this temptation, investing now must incur a self-control cost $\lambda I(1-\delta)$, this lowers the benefit from investment as revealed on the left side of (13). Thus, the investor chooses to procrastinate in the sense that he invests at a time later than that when he has standard preferences. Since x^* increases with the self-control parameter λ, the agent delays further as the self-control parameter becomes larger.[11] When λ is

[11] The mean value of the waiting time is given by $(1 - F(x^*))^{-1}$. It is increasing in the threshold value x^*.

sufficiently large, x^* may exceed the upper bound A so that the investor never undertakes the investment project.

By contrast, if rewards are immediate, then the investor is tempted to invest now. This case is analyzed in part (ii) of Proposition 5. Waiting incurs a direct current period self-control cost λx^*, which lowers the option value of waiting. Importantly, self-control adds a positive option value of waiting to invest, $\lambda\delta \int_a^{x^*} (x^* - x)\, \mathrm{d}F(x)$. This is because, when the investor waits for one more period to invest at x^* and gets a worse draw $x < x^*$, the cost of self control is less than the project value measured by the temptation utility.[12] One can show that this positive value is dominated by the current self-control cost. Thus, compared with the standard model with $\lambda = 0$, the benefit from waiting is lowered and the investor chooses to preproperate.

Part (ii) of Proposition 5 also shows that x^* decreases with the self-control parameter λ. Thus, as λ gets larger and larger, the investor invests sooner and sooner. When λ is sufficiently large, the investor invests at a threshold value lower than that prescribed by the NPV rule. Under this rule, the threshold value is δI.[13] This result implies that the investor may obtain negative NPV at the time of investment. This result seems counterintuitive. In fact, tempted by investing now, the investor may reason, "If I invest now, I get a reward and incur a cost in the future. If I do not invest now, I have to exercise costly self-control. The cost of self-control may outweigh the option value of waiting. Thus, I prefer to invest now even though I get negative NPV." In reality, we do observe the phenomena that investors rush to embark on investments with negative NPV. For example, Rook (1987) finds empirical evidence that the presence of credit opportunities results in present-oriented, unplanned, and impulse buying.

I now consider part (iii) of Proposition 5 where both costs and rewards are immediate. It is important to observe that the investor would never invest at a project value less than the cost; that is, x^* cannot be less than I. This is because when $x^* < I$, the investor has no temptation to invest and can choose costlessly not to invest, thereby obtaining the outside value zero. Given $x^* \geq I$, at the threshold value x^* the investor is tempted to invest. Thus, there is no self-control cost of investing at x^*. Consider now the self-control cost of waiting. Waiting incurs a current period self-control cost $\lambda(x^* - I)$. Waiting also has an option value (measured by the temptation utility) represented by the last term in (15) when the investor gets a worse draw $x < x^*$. One can show that the current self-control cost dominates so that the benefit from waiting is lowered. Thus, compared with the standard model, the investor preproperates. Note that as in the case of immediate rewards, x^* decreases with the self-control parameter λ. As λ is sufficiently large, the threshold value converges to the value I under the myopic rule.

Since O'Donoghue and Rabin (1999a) seminal work, the behavior of procrastination and preproperation has been often analyzed using the hyperbolic discounting

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[12] When he gets a better draw $x > x^*$, the investor succumbs to the temptation of investing so that there is no self-control cost.

[13] Note that the risk-neutral agent discounts future cash flows according to the long-run discount factor δ.

model. To compare with this model, I consider only procrastination when costs are immediate.[14]

Proposition 6 *Suppose costs are immediate and the agent has the sophisticated hyperbolic discounting preferences.*

(i) *If* $\beta\delta\,(a - \delta E\,[x])\,/\,(1 - \beta\delta) < I < \beta\delta A$, *then there is a stationary equilibrium where each self invests when* x_t *is bigger than some threshold* $x^{**} \in (a, A)$.
(ii) *There is an open set of parameter values for which there are other equilibria. In particular, there is a "2-cycle equilibrium" where the odd-numbered selves never invest and even ones always invest.*

This proposition demonstrates that the hyperbolic discounting model has multiple equilibria. One equilibrium has a similar feature to Proposition 5. However, there is another equilibrium having cycles. Similar results are obtained by Fudenberg and Levine (2006) and O'Donoghue and Rabin (2001) for the task choice problem different from the one analyzed here. As in Fudenberg and Levine (2006), I view that the cyclic equilibrium is counterintuitive and the multiplicity of equilibrium is unappealing.

I now turn to welfare implications. I ask the question: How severely does the self-control problem hurt a person? I compute the utility loss from investment for an investor with self-control preferences, compared with an investor with standard preferences. Let $V\,(x)$ be the value function for the investor with standard preferences corresponding to $\lambda = 0$. The utility loss from self-control problems could be measured as $V\,(x) - W\,(x)$. One can interpret $V\,(x)$ as the commitment preference as in Gul and Pesendorfer (2001, 2004). Then $V\,(x) - W\,(x)$ measures the utility loss if the agent cannot precommit and suffers from self-control problems. I evaluate this measure at the time when the agent with self-control preferences invests. That is, this value is given by $V\,(x^*) - W\,(x^*)$.

The following proposition gives the utility loss.

Proposition 7 *Let* x^* *and* $\overline{x}$ *be given in Proposition* 5. *When costs are immediate, the utility loss from investment is given by* λI. *When rewards are immediate or both costs and rewards are immediate, the utility loss from investment is given by* $\overline{x} - x^*$.

By Proposition 5, when costs are immediate, the investor with self-control preferences procrastinates – he waits when he should invest if he had standard preferences. The utility loss is the forgone project value. This loss is increasing in the self-control parameter λ. However, it does not increase with λ without bound. This is because as λ approaches the value $\delta A/I - 1$, the investment threshold x^* approaches the upper bound of the project value A. When λ is increased further, no investment is ever made and the agent gets zero. Thus, the upper bound of the utility loss is $\delta A - I$. When rewards are immediate or both costs and reward are immediate, the investor with self-control preferences preproperates — he invests

[14] I also consider a sophisticated agent's behavior only. A complete characterization for the naive or partially sophisticated agent is not my focus and is beyond the scope of the present paper.

Table 1 This table presents solutions for the investment threshold values, the mean waiting time until investment and the utility loss. The utility loss is measured as $(V(x^*) - W(x^*))/V(x^*)$

	λ	Threshold	Waiting time	Utility loss
	0	0.96	22.5	
Immediate costs	0.3	0.97	36.0	0.40
	0.6	0.99	90.0	0.77
	0	0.74	4.0	
Immediate rewards	0.3	0.49	2.0	0.87
	0.6	0.32	1.5	1.44
Immediate costs and rewards	0	0.76	4.2	
	0.3	0.69	3.2	0.29
	0.6	0.65	2.8	0.44

when he should wait if he had standard preferences. The utility loss is then the forgone option value of waiting.

The following example illustrates Propositions 5 and 7 numerically.

Example 1 Let $a = 0$, $A = 1$, $\delta = 0.9$, $I = 0.5$, and $F(x) = x$. Table 1 reports the solution. It reveals that even for small self-control problems, i.e. small λ, the utility loss could be quite large. For example, when costs are immediate and $\lambda = 0.6$, the investor procrastinates about 70 periods to invest. The utility loss accounts for 77% of the project value. When rewards are immediate and $\lambda = 0.6$, the investor preproperates to invest in negative NPV projects since the investment threshold 0.32 is less than $\delta I = 0.45$ according to the NPV rule. The utility loss accounts for 144% of the option value. When both costs and rewards are immediate, the investor also preproperates. But the utility loss is less than that in the case of immediate rewards.

3.2 Exit

Some researchers have found experimental evidence that people procrastinate to terminate projects (see, for example, Staw 1976; Staw and McClane 1984, and Statman and Caldwell 1987). While several explanations are available in the literature, the interpretation suggested here is simple. If owners/managers perceive that the rewards of the projects are immediate, but the costs of continuation come with delay, then they are tempted by the immediate benefits. To resist this temptation, they must suffer from self-control costs. These self-control costs lower the benefit from termination. Thus the owners/managers prefer to delay termination. While my model can explain this procrastination behavior, it can also generate the behavior of preproperation if the costs of continuation come earlier than the benefits from the projects.

I now apply the general setup laid out in Sect. 2 to the project termination problem or exit problem.[15] I interpret the process $(x_t)_{t\geq 1}$ as the stochastic profit

[15] The problem can be reinterpreted as one in which an owner/manager decides when to shut down a firm.

flows from a project. It incurs a fixed cost $c_f > 0$ to continue the project. Normalize the scrapping value of the project to zero. A risk-neutral owner/manager with self-control preferences decides when and if to terminate the project. This problem fits into our framework by setting

$$\Omega(x) = 0, \quad c_s = 0, \quad \pi(x) = x, \quad c_c = c_f.$$

As in the investment problem described in the preceding subsection, there are many instances that profits and costs may not come at the same time. Thus, I consider three cases and rewrite the Bellman equations (4)–(6) as follows:

1. Immediate costs

$$W(x) = \max\left\{0, \delta x - (1+\lambda) c_f + \delta \int W(x') \,\mathrm{d}F(x')\right\} - \lambda \max(0, -c_f). \tag{16}$$

2. Immediate rewards

$$W(x) = \max\left\{0, (1+\lambda) x - \delta c_f + \delta \int W(x') \,\mathrm{d}F(x')\right\} - \lambda \max(x, 0). \tag{17}$$

3. Immediate costs and rewards

$$W(x) = \max\left\{0, (1+\lambda)(x - c_f) + \delta \int W(x') \,\mathrm{d}F(x')\right\} - \lambda \max\{x - c_f, 0\}. \tag{18}$$

The following proposition characterizes the solution.

Proposition 8 *Under the conditions given in the appendix, there is a unique threshold value $x^* \in [a, A]$ ($\overline{x} \in [a, A]$) such that the owner with self-control preferences (standard preferences) terminates the project the first time the process $(x_t)_{t \geq 1}$ falls below this value.*

(i) *If costs are immediate, then x^* satisfies*

$$0 = \delta x^* - c_f + \delta \int_{x^*}^{A} \delta (x - x^*) \,\mathrm{d}F(x) - \lambda c_f, \tag{19}$$

and $\overline{x}$ is the solution for $\lambda = 0$. Moreover, $x^* > \overline{x}$, $\overline{x} < c_f/\delta$ and x^* increases with λ.

(ii) *If rewards are immediate, then x^* satisfies*

$$-\lambda x^* (1 - \delta) - \lambda \delta \int_{a}^{x^*} (x^* - x) \,\mathrm{d}F(x) = x^* - \delta c_f + \delta \int_{x^*}^{A} (x - x^*) \,\mathrm{d}F(x), \tag{20}$$

and $\overline{x}$ is the solution for $\lambda = 0$. Moreover, $x^* < \overline{x} < \delta c_f$ and x^* decreases with λ.

(iii) *If both costs and rewards are immediate, then* x^* *satisfies*

$$0 = x^* - c_f + \delta \int_{x^*}^{A} \left(x - x^*\right) \mathrm{d}F\left(x\right)$$

$$+ \lambda\left(x^* - c_f\right) + \lambda\delta \int_{x^*}^{A} \left(x - x^* - \max\left\{x - c_f, 0\right\}\right) \mathrm{d}F\left(x\right), \qquad (21)$$

and $\overline{x}$ *is the solution for* $\lambda = 0$. *Moreover,* $c_f \geq x^* > \overline{x}$ *and* x^* *increases with* λ.

One can interpret equations (19)–(21) as follows. Their left and right sides represent the utility benefits from termination and continuation of the project, respectively. At the threshold value x^*, the owner is indifferent between termination and continuation. In the standard model with $\lambda = 0$, because of irreversibility and uncertainty, there is a positive option value of waiting in the hope of getting better shocks. The owner will not terminate the project as soon as he incurs losses since keeping it alive has an option value. The option value of waiting for each case is represented by the third term on the corresponding right side of equations (19)–(21). Only when the loss is large enough, will the owner terminate the project.

I next turn to the case where the owner has self-control preferences. When costs are immediate, the owner is tempted to terminate the project. Exercising self-control is costly. The cost of self-control is represented by the last term in (19). It lowers the benefit from continuation of the project by eroding the option value of waiting. Thus, he preproperates to terminate early. Note that the termination threshold x^* is increasing in the self-control parameter λ.[16] When λ is large enough, x^* approaches the upper bound of profits A. In this case, the owner succumbs to temptation and terminates the project immediately even if the project can still make positive net profits.

When rewards are immediate, the owner is tempted to continue the project even though he may suffer from losses. To resist this temptation, he incurs a current period self-control cost represented by the first term on the left side of equation (20). He also incurs a future self-control cost represented by the second term on the left side of (20). The latter cost arises when the value of future profits is less then x^*. These two components of self-control cost lower the benefit from termination. Thus, the owner procrastinates to terminate the project. In particular, the termination threshold x^* is lower than the value $\overline{x}$ when the owner has standard preferences. When λ is large enough, x^* approaches zero and the owner will always keep the project alive even though he makes no profits.

Consider the case where both costs and rewards are immediate. Since profits are stochastic, the owner is tempted to continue the project if its profits are higher than the fixed cost and is tempted to terminate the project if its profits are lower than the fixed cost. It seems that there is no unambiguous conclusion. However, it is important to note that the owner will never terminate the project at the profit level higher than the fixed cost. Otherwise, at that profit level the owner has no temptation to exit. Thus, stay for one more period incurs no self-control cost and

[16] The mean value of the exit time is given by $F\left(x^*\right)^{-1}$, which decreases with the threshold value.

the owner can still make positive profits. Because of this fact, at the termination threshold, the owner cannot make positive profits and has a temptation to terminate the project. Exercising self-control is costly, which lowers the benefit from continuation of the project. The cost of self-control is represented by the last two terms in equation (21). Thus, the owner preproperates to terminate the project at a time earlier than that in the model with standard preferences. Part (iii) of Proposition 8 also implies that the termination threshold x^* increases with λ. In particular, when λ is sufficiently large, x^* approaches the fixed cost c_f so that the project is terminated according to the myopic rule. In this case, the cost of self-control erodes completely away the option value of waiting.

I finally analyze welfare implications. Similarly to Proposition 7, the following proposition gives the utility loss due to self-control problems.

Proposition 9 *Let x^* and $\overline{x}$ be given in Proposition 7. When costs are immediate, the utility loss is given by $\delta x^* - \delta \overline{x}$. When rewards are immediate, the utility loss is given by λx^*. When both rewards and costs are immediate, the utility loss is given by $x^* - \overline{x}$.*

By Proposition 8, when costs are immediate or both rewards and costs are immediate, the owner preproperates — he terminates the project when he should continue if he had standard preferences. The utility loss is the forgone profit opportunities. When rewards are immediate, the owner procrastinates — he continues the project when he should terminate it if he had standard preferences. The utility loss is the cost of self-control incurred from resisting the temptation to stay. As in the investment model, this cost does not increases with λ without bound. When λ is sufficiently large, the termination threshold approaches zero and the owner never terminates the project. The maximal utility loss from keeping the project alive is $\delta \left(c_f - Ex\right) / (1 - \delta)$, which is the absolute value of the NPV of profits and is positive by the assumption in the appendix.

The following example illustrates Propositions 8–9 numerically.

Example 2 Let $a = 0$, $A = 1$, $\delta = 0.9$, $c_f = 0.6$, and $F(x) = x$. Table 2 reports the solution. It reveals the following: When costs are immediate, the owner termi-

Table 2 This table presents solutions for the exit threshold values, the mean waiting time until exit and the utility loss. The utility loss is measured as the fraction of the profits at exit

	λ	Threshold	Waiting time	Utility loss
	0	0.59	1.7	
Immediate costs	0.2	0.78	1.3	0.24
	0.4	0.93	1.1	0.36
	0	0.35	2.9	
Immediate rewards	0.2	0.31	3.2	0.2
	0.4	0.28	3.5	0.4
Immediate costs	0	0.48	2.1	
and rewards	0.2	0.50	2.0	0.04
	0.4	0.51	1.9	0.07

nates the project too early even if he suffers from very small self-control problems, i.e., $\lambda = 0.2$. The project is terminated at the profit level 0.78, which is bigger than the fixed cost 0.6. If the owner had standard preferences, he should terminate the project at the value 0.59 less than the fixed cost because of the option value of waiting. The utility loss accounts for 24% of the profits. When rewards are immediate, the owner procrastinates. He suffers from a larger loss if he has a lower level of self-control since the profit level at termination becomes smaller. The utility loss is proportional to λ since it is equal to $\lambda x^*/x^* = \lambda$. When both costs and rewards are immediate, the owner preproperates. However, the utility loss is less than that when costs are immediate.

4 Conclusion

This paper adopts the Gul–Pesendorfer self-control utility model to analyze an option exercise problem under uncertainty over an infinite horizon for an agent who is tempted by immediate gratification and suffers from self-control problems. Unlike the time-inconsistency approach which depends on the expectations about future selves' preferences, there is no multiplicity of predictions. When applied to the investment and exit problems, the present model has a number of testable implications. For example, the present model implies that overinvestment, excess entry, procrastination to terminate a project or shut down a firm may be the rational choices of those investors/managers/entrepreneurs having self-control preferences, who are tempted by immediate profit opportunities. On the other hand, the opposite phenomena can be caused by such decision makers who are tempted to avoid immediate costs. Further, when both costs and rewards are immediate, the myopic option exercise rule may be optimal for such decision makers, who have sufficiently low levels of self-control.

Appendix: Proofs

Proof of Proposition 1 I provide the proof for the problem in (4) only. The proof for other cases is similar. Subtract $\delta\pi(x)$ from the two sides in Eq. (4) to obtain

$$\begin{aligned} W(x) &- \delta\pi(x) \\ &= \max\left\{\delta\Omega(x) - \delta\pi(x) - (1+\lambda)c_s, -(1+\lambda)c_c + \delta\int W(x')\,\mathrm{d}F(x')\right\} \\ &\quad - \lambda\max\{-c_c, -c_s\}. \end{aligned} \tag{A.1}$$

Note that the agent's optimal choice is determined by the first max operator in (A.1). Since the second term in this max operator is a constant independent of x and since $\Omega(x) - \pi(x)$ is strictly increasing in $x \in [a, A]$, the following three cases may arise. (a) The first term is larger than the second term for all $x \in [a, A]$. In this case the agent stops immediately. (b) The first term is less than the second term for all $x \in [a, A]$. In this case, the agent never stops. (c) There is a unique threshold value $x^* \in [a, A]$ such that the first term is equal to the second term. In this case, the agent stops when $x \geq x^*$. That is, he stops the first time the process $(x)_{t\geq 1}$ hits x^* from below. □

Proof of Proposition 2 (i) The value function W satisfies

$$W(x) = \begin{cases} \delta\Omega(x) - (1+\lambda)\,c_s - \lambda \max\{-c_c, -c_s\} & \text{if } x \geq x^*, \\ \delta\pi(x) - (1+\lambda)\,c_c + \delta \int W(x')\,\mathrm{d}F(x') - \lambda \max\{-c_c, -c_s\} & \text{if } x < x^*. \end{cases} \tag{A.2}$$

Since $W(x)$ is continuous at the threshold value x^*,

$$\begin{aligned} \delta\Omega(x^*) - (1+\lambda)\,c_s &= \delta \int W(x')\,\mathrm{d}F(x') + \delta\pi(x^*) - (1+\lambda)\,c_c \\ &= \delta \int_a^{x^*} W(x')\,\mathrm{d}F(x') + \delta \int_{x^*}^{A} W(x')\,\mathrm{d}F(x') + \delta\pi(x^*) - (1+\lambda)\,c_c. \end{aligned} \tag{A.3}$$

Substituting (A.2) into this equation yields

$$\begin{aligned} &\delta\Omega(x^*) - (1+\lambda)\,c_s \\ &= \delta \int_a^{x^*} \left\{ \delta \int W(x)\,\mathrm{d}F(x) + \delta\pi(x') - (1+\lambda)\,c_c - \lambda \max\{-c_c, -c_s\} \right\} \\ &\quad \times \mathrm{d}F(x') \\ &\quad + \delta \int_{x^*}^{A} \left[\delta\Omega(x') - (1+\lambda)\,c_s - \lambda \max\{-c_c, -c_s\} \right] \\ &\quad \times \mathrm{d}F(x') + \delta\pi(x^*) - (1+\lambda)\,c_c. \end{aligned}$$

Using (A.3) to substitute $\delta \int W(x)\,dF(x)$ delivers

$$\begin{aligned} &\delta\Omega(x^*) - (1+\lambda)\,c_s \\ &= \delta \int_a^{x^*} \left[\delta\Omega(x^*) - (1+\lambda)\,c_s - \delta\pi(x^*) + \delta\pi(x') - \lambda \max\{-c_c, -c_s\} \right] \mathrm{d}F(x') \\ &\quad + \delta \int_{x^*}^{A} \left[\delta\Omega(x') - (1+\lambda)\,c_s - \lambda \max\{-c_c, -c_s\} \right] \mathrm{d}F(x') \\ &\quad + \delta\pi(x^*) - (1+\lambda)\,c_c. \end{aligned}$$

Subtracting $\left[\delta\Omega(x^*) - (1+\lambda)c_s\right]\delta F(x^*)$ on each side of the above equation yields

$$\begin{aligned}
&\left[\delta\Omega(x^*) - (1+\lambda)c_s\right]\left[1 - \delta F\left(x^*\right)\right] \\
&\quad = \delta \int_a^{x^*} \left[\delta\pi\left(x'\right) - \delta\pi\left(x^*\right) - \lambda \max\left\{-c_c, -c_s\right\}\right] \mathrm{d}F\left(x'\right) \\
&\qquad + \delta \int_{x^*}^{A} \left[\delta\Omega(x') - (1+\lambda)c_s - \lambda \max\left\{-c_c, -c_s\right\}\right] \mathrm{d}F\left(x'\right) \\
&\qquad + \delta\pi\left(x^*\right) - (1+\lambda)c_c.
\end{aligned}$$

Subtracting $\left[\delta\Omega(x^*) - (1+\lambda)c_s\right]\delta\left[1 - F(x^*)\right]$ on each side of the above equation and simplifying yield the desired result.

(ii) If $c_s \geq c_c$, then

$$\lambda\left[c_s(1-\delta) - \delta\max\left\{-c_c, -c_s\right\} - c_c\right] = \lambda(1-\delta)(c_s - c_c) \geq 0.$$

If $c_s < c_c$, then

$$\lambda\left[c_s(1-\delta) - \delta\max\left\{-c_c, -c_s\right\} - c_c\right] = \lambda(c_s - c_c) < 0.$$

□

Proof of Proposition 3 (i) The value function W satisfies

$$W(x) = \begin{cases} (1+\lambda)\,\Omega(x) - \delta c_s - \lambda\max\left\{\pi(x), \Omega(x)\right\} & \text{if } x \geq x^*, \\ (1+\lambda)\,\pi(x) - \delta c_c + \delta\int W\left(x'\right)\mathrm{d}F\left(x'\right) - \lambda\max\left\{\pi(x), \Omega(x)\right\} & \text{if } x < x^*. \end{cases} \tag{A.4}$$

Since $W(x)$ is continuous at the threshold value x^*, it follows that

$$\begin{aligned}
(1+\lambda)\,\Omega(x^*) - \delta c_s &= (1+\lambda)\,\pi\left(x^*\right) - \delta c_c + \delta\int W\left(x'\right)\mathrm{d}F\left(x'\right) \\
&= \delta\int_a^{x^*} W\left(x'\right)\mathrm{d}F\left(x'\right) + \delta\int_{x^*}^{A} W\left(x'\right)\mathrm{d}F\left(x'\right) \\
&\quad + (1+\lambda)\,\pi\left(x^*\right) - \delta c_c.
\end{aligned} \tag{A.5}$$

Substitute (A.4) into this equation to deduce

$$(1+\lambda)\,\Omega(x^*) - \delta c_s$$
$$= \delta \int_a^{x^*} \left\{(1+\lambda)\,\pi\,(x) - \delta c_c + \delta \int W\left(x'\right) \mathrm{d}F\left(x'\right) - \lambda \max\left\{\pi\,(x)\,,\,\Omega\,(x)\right\}\right\}$$
$$\times\, \mathrm{d}F\,(x)$$
$$+\, \delta \int_{x^*}^{A} \left[(1+\lambda)\,\Omega(x) - \delta c_s - \lambda \max\left\{\pi\,(x)\,,\,\Omega\,(x)\right\}\right]$$
$$\times\, dF\,(x) + (1+\lambda)\,\pi\left(x^*\right) - \delta c_c.$$

Using (A.5) to substitute $\delta \int W\left(x'\right) dF\left(x'\right)$ yields

$$(1+\lambda)\,\Omega(x^*) - \delta c_s$$
$$= \delta \int_a^{x^*} \left\{(1+\lambda)\,\pi\,(x) + (1+\lambda)\,\Omega(x^*) - \delta c_s - (1+\lambda)\,\pi\left(x^*\right)\right.$$
$$\left. -\, \lambda \max\left\{\pi\,(x)\,,\,\Omega\,(x)\right\}\right\} \mathrm{d}F\,(x)$$
$$+\, \delta \int_{x^*}^{A} \left\{(1+\lambda)\,\Omega(x) - \delta c_s - \lambda \max\left\{\pi\,(x)\,,\,\Omega\,(x)\right\}\right\}$$
$$\times\, \mathrm{d}F\,(x) + (1+\lambda)\,\pi\left(x^*\right) - \delta c_c.$$

Subtract $\left[(1+\lambda)\,\Omega(x^*) - \delta c_s\right] \delta F\,(x^*)$ on each side of the above equation to derive

$$\left[(1+\lambda)\,\Omega(x^*) - \delta c_s\right]\left[1 - \delta F\left(x^*\right)\right] = \delta \int_a^{x^*} \left\{(1+\lambda)\,\pi\,(x) - (1+\lambda)\,\pi\left(x^*\right)\right\}$$
$$\mathrm{d}F\left(x'\right) - \lambda\delta \int \max\left\{\pi\,(x)\,,\,\Omega\,(x)\right\} \mathrm{d}F\,(x) + \delta \int_{x^*}^{A} \left\{(1+\lambda)\,\Omega(x') - \delta c_s\right\}$$
$$\mathrm{d}F\left(x'\right) + (1+\lambda)\,\pi\left(x^*\right) - \delta c_c.$$

Finally, subtract $\left[(1+\lambda)\,\Omega(x^*)-\delta c_s\right]\delta\left[1-F(x^*)\right]$ on each side of the above equation and rearrange to deduce

$$\begin{aligned}\left[(1+\lambda)\,\Omega(x^*)-\delta c_s\right](1-\delta) &= \delta\int_a^{x^*}(1+\lambda)\left[\pi(x)-\pi\left(x^*\right)\right]\mathrm{d}F\left(x'\right)\\&+\delta\int_{x^*}^{A}(1+\lambda)\left[\Omega(x')-\Omega(x^*)\right]\mathrm{d}F\left(x'\right)+\pi\left(x^*\right)-\delta c_c+\lambda\pi\left(x^*\right)-\lambda\delta\\&\times\int\max\{\pi(x),\Omega(x)\}\,\mathrm{d}F(x).\end{aligned}$$

Rearranging yields the desired result.

(ii) If $\Omega(x)\geq\pi(x)$ for all x, then

$$\begin{aligned}&\lambda\delta\int_a^{x^*}[\pi(x)-\max\{\pi(x),\Omega(x)\}]\,\mathrm{d}F(x)\\&\quad+\lambda\delta\int_{x^*}^{A}[\Omega(x)-\max\{\pi(x),\Omega(x)\}]\,\mathrm{d}F(x)\\&\quad-\lambda\left[1-\delta F\left(x^*\right)\right]\left[\Omega(x^*)-\pi\left(x^*\right)\right]\\&=\lambda\delta\int_a^{x^*}[\pi(x)-\Omega(x)]\,\mathrm{d}F(x)-\lambda\left[1-\delta F\left(x^*\right)\right]\left[\Omega(x^*)-\pi\left(x^*\right)\right]<0.\end{aligned}$$

Thus, $x^*\leq\overline{x}$. □

Proof of Proposition 4 (i) Rewrite the value function as follows

$$\begin{aligned}&W(x)\\&=\begin{cases}(1+\lambda)\,(\Omega(x)-c_s)-\lambda\max\{\pi(x)-c_c,\Omega(x)-c_s\} & \text{if } x\geq x^*,\\(1+\lambda)\,(\pi(x)-c_c)+\delta\int W\left(x'\right)\mathrm{d}F\left(x'\right)-\lambda\max\{\pi(x)-c_c,\Omega(x)-c_s\} & \text{if } x<x^*.\end{cases}\end{aligned}\tag{A.6}$$

Since $W(x)$ is continuous at the threshold value x^*, it follows that

$$\begin{aligned}&(1+\lambda)\left(\Omega(x^*)-c_s\right)\\&\quad=(1+\lambda)\left(\pi\left(x^*\right)-c_c\right)+\delta\int W\left(x'\right)\mathrm{d}F\left(x'\right)\\&\quad=\delta\int_a^{x^*}W\left(x'\right)\mathrm{d}F\left(x'\right)+\delta\int_{x^*}^{A}W\left(x'\right)\mathrm{d}F\left(x'\right)+(1+\lambda)\left(\pi\left(x^*\right)-c_c\right).\end{aligned}\tag{A.7}$$

Substitute (A.6) into this equation to deduce

$$\begin{aligned}
&(1+\lambda)\left(\Omega(x^*)-c_s\right)\\
&= \delta \int_a^{x^*} \Big\{(1+\lambda)(\pi(x)-c_c)+\delta\int W\left(x'\right)\mathrm{d}F\left(x'\right)\\
&\qquad - \lambda \max\{\pi(x)-c_c,\Omega(x)-c_s\}\Big\}\,\mathrm{d}F(x)\\
&\quad + \delta\int_{x^*}^{A}\{(1+\lambda)(\Omega(x)-c_s)-\lambda\max\{\pi(x)-c_c,\Omega(x)-c_s\}\}\\
&\quad \times \mathrm{d}F(x)+(1+\lambda)\left(\pi\left(x^*\right)-c_c\right).
\end{aligned}$$

Use (A.7) to substitute $\delta\int W\left(x'\right)\mathrm{d}F\left(x'\right)$ to deduce

$$\begin{aligned}
&(1+\lambda)\left(\Omega(x^*)-c_s\right)\\
&= \delta \int_a^{x^*} \{(1+\lambda)\left(\pi(x)-\pi\left(x^*\right)\right)+(1+\lambda)\left(\Omega(x^*)-c_s\right)\\
&\quad - \lambda \max\{\pi(x)-c_c,\Omega(x)-c_s\}\}\,\mathrm{d}F\left(x'\right)\\
&\quad + \delta\int_{x^*}^{A}\{(1+\lambda)(\Omega(x)-c_s)-\lambda\max\{\pi(x)-c_c,\Omega(x)-c_s\}\}\\
&\quad \times \mathrm{d}F(x)+(1+\lambda)\left(\pi\left(x^*\right)-c_c\right).
\end{aligned}$$

Subtract $(1+\lambda)(\Omega(x^*)-c_s)\,\delta F(x^*)$ on each side of the above equation to derive

$$\begin{aligned}
&(1+\lambda)\left(\Omega(x^*)-c_s\right)\left[1-\delta F\left(x^*\right)\right]\\
&= \delta \int_a^{x^*} \left\{(1+\lambda)\left(\pi(x)-\pi\left(x^*\right)\right)\right\}\mathrm{d}F(x)-\lambda\delta\\
&\qquad \int \max\{\pi(x)-c_c,\Omega(x)-c_s\}\,\mathrm{d}F(x)\\
&\quad + \delta\int_{x^*}^{A}\{(1+\lambda)(\Omega(x)-c_s)\}\,\mathrm{d}F(x)+(1+\lambda)\left(\pi\left(x^*\right)-c_c\right).
\end{aligned}$$

Finally, subtract $\left[(1+\lambda)\,\Omega(x^*) - c_s\right]\delta\left[1 - F(x^*)\right]$ on each side of the above equation and rearrange to deduce

$$\begin{aligned}
&(1+\lambda)\left(\Omega(x^*) - c_s\right)(1-\delta) \\
&\quad = \delta\int_a^{x^*}(1+\lambda)\left[\pi(x) - \pi\left(x^*\right)\right]\mathrm{d}F(x) + \delta\int_{x^*}^{A}(1+\lambda)\left[\Omega(x) - \Omega(x^*)\right]\mathrm{d}F(x) \\
&\qquad + (1+\lambda)\left(\pi\left(x^*\right) - c_c\right) - \lambda\delta\int \max\left\{\pi(x) - c_c, \Omega(x) - c_s\right\}\mathrm{d}F(x).
\end{aligned}$$

Rearranging yields the desired result.

(ii) The proof is similar to that of Proposition 3. □

Proof of Proposition 5 The Eqs. (13)–(15) determining x^* are derived from Propositions 2–4. One can verify that the left sides of these equations are increasing functions of x^*, while the right sides are decreasing functions of x^*. To show that there is a unique interior solution to these equations, one need only show that there is a unique intersection point for each pair of curves implied by those functions. To guarantee this, the following conditions are necessary and sufficient so that one can apply the intermediate value theorem:

- For part (i),

$$\frac{\delta\left(a - \delta E[x]\right)}{(1-\delta)(1+\lambda)} \le I \le \frac{\delta A}{(1+\lambda)}.$$

- For part (ii),

$$\frac{a(1+\lambda) - \delta E[x]}{1-\delta} \le \delta I \le A(1+\lambda) + \frac{\lambda\delta E[x]}{1-\delta}.$$

- For part (iii),

$$\frac{a(1+\lambda) - \delta E[x]}{1-\delta+\lambda} \le I \le A + \frac{\lambda\delta E\left[\max(0, x - I)\right]}{(1-\delta)(1+\lambda)}.$$

Figure 1 illustrates part (i). As λ is increased, the curve implied by the left side of (13) shifts down so that x^* increases. Thus, $x^* > \bar{x}$, where $\bar{x}$ corresponds to the solution for $\lambda = 0$. Parts (ii) and (iii) are proved similarly. □

Proof of Proposition 6 The proof adapts that of Theorem 4 in Fudenberg and Levine (2006).

(i) Consider a dynamic game between selves. The date t-self has payoff $\delta\beta x_t - I$ if he invests at date t, payoff $\beta\delta^{\tau+1}E[x] - \beta\delta^{\tau}I$ if a future self invests at date $t+\tau$, and payoff 0 if no self ever invests.[17] To construct the stationary equilibrium, I will solve for the threshold value $x^{**} \in (a, A)$. Let W^{**} be the expected discounted sum of payoffs from tomorrow on if the agent does not invest. Then by dynamic programming,

$$W^{**} = F\left(x^{**}\right)\delta W^{**} + \left(1 - F\left(x^{**}\right)\right)\left[\delta E\left(x | x > x^{**}\right) - I\right].$$

[17] Note that rewards are obtained in the next period.

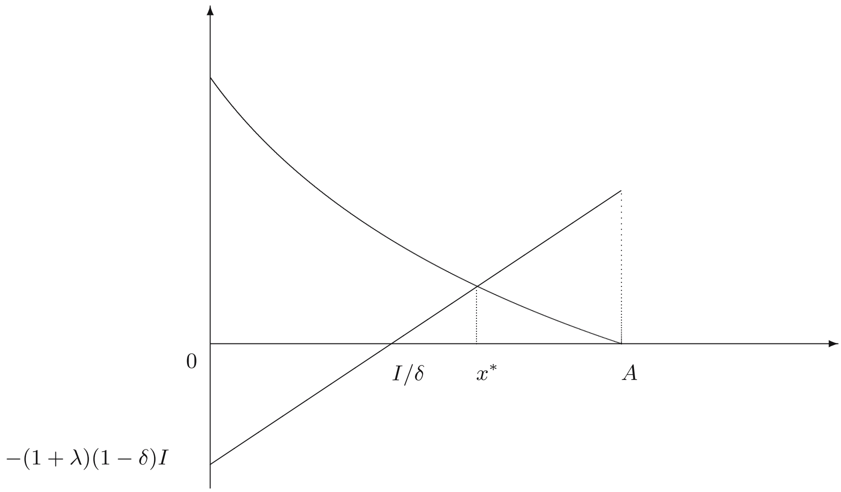

Fig. 1 The determination of the threshold value x^*. The *upward sloping line* and the *downward sloping curve* represent the left and right sides of Eq. (13), respectively, as functions of x^*. The intersection point determines the optimal threshold value

A self is indifferent between investing and waiting at x^{**} if

$$\delta\beta x^{**} - I = \delta\beta W^{**}.$$

Using the preceding two equations, one can show that x^{**} satisfies

$$\delta\beta x^{**} - I - \frac{\beta\delta\left(1 - F\left(x^{**}\right)\right)\left(\delta E\left[x|x > x^{**}\right] - I\right)}{1 - F\left(x^{**}\right)\delta} = 0.$$

Let $g\left(x^{**}\right)$ be the expression on the left side of this equation. Then it suffices to show that there is an x^{**} such that $g\left(x^{**}\right) = 0$. In fact, by assumption, $g\left(a\right) = \delta\beta a - I\left(1 - \beta\delta\right) - \beta\delta^2 E\left[x\right] < 0$ and $g\left(A\right) = \delta\beta A - I > 0$. The intermediate value theorem yields the desired result.

(ii) I construct an equilibrium where the odd-numbered selves never invest and even ones always invest. The equilibrium payoff of an even-numbered self is $\delta\beta x_t - I$, and his payoff if he waits is $\delta^3\beta E\left[x\right] - \delta^2\beta I$. The even self's strategy is a best response for all x_t if

$$\delta\beta a - I > \delta^3\beta E\left[x\right] - \delta^2\beta I.$$

The equilibrium payoff of an odd-numbered self is $\delta^2\beta E\left[x\right] - \beta\delta I$ and his payoff if he deviates to invest is $\delta\beta x_t - I$. Thus, waiting is a best response for all x_t if

$$\delta^2\beta E\left[x\right] - \beta\delta I > \delta\beta A - I.$$

Simplifying the preceding two equations yields the condition

$$\frac{\delta\beta\left(A - \delta E\left[x\right]\right)}{1 - \beta\delta} < I < \frac{\delta\beta\left(a - \delta^2 E\left[x\right]\right)}{1 - \beta\delta^2}.$$

Clearly there is an open set of parameter values satisfying this condition. □

Proof of Proposition 7 When costs are immediate, the agent with standard preferences has already made the investment at x^* since his investment threshold $\overline{x} < x^*$. Thus, $V(x^*) = \delta x^* - I$ and the welfare loss is $V(x^*) - W(x^*) = (\delta x^* - I) - (\delta x^* - (1+\lambda) I) = \lambda I$. When rewards are immediate, the agent with standard preferences does not invest at x^* since his investment threshold $\overline{x} > x^*$. Thus, $V(x^*) = \delta \int V(x') \mathrm{d}F(x') = \overline{x} - \delta I$ and the welfare loss is $V(x^*) - W(x^*) = (\overline{x} - \delta I) - (x^* - \delta I) = \overline{x} - x^*$. The case with both immediate costs and rewards is similar to the case with immediate rewards. □

Proof of Proposition 8 The proof is similar to that of Propositions 1–5. In particular, one can show that the expressions on the right sides of equations (19)–(21) are increasing functions of x^* and the expressions on left sides of these equations are decreasing functions of x^*. I omit the detailed argument. The conditions for the existence and uniqueness of the threshold value are given below:

- For part (i),

$$\delta A \geq (1+\lambda) c_f \geq \delta (1-\delta) a + \delta^2 E[x].$$

- For part (ii),

$$\delta E[x] + a(1-\delta)(1+\lambda) \leq \delta c_f \leq A(1+\lambda) - \lambda \delta E[x].$$

- For part (iii),

$$a(1-\delta) + \frac{\lambda\delta\left(E[x] - E\left[\max\left(x - c_f, 0\right)\right]\right)}{1+\lambda} \leq c_f \leq A.$$

□

Proof of Proposition 9 The proof is similar to that of Proposition 7. When costs are immediate, since $x^* > \overline{x}$, an agent with standard preferences will not exit at x^*. $V(x^*) = \delta x^* - c_f + \delta \int V(x') \mathrm{d}F(x') = \delta x^* - c_f - (\delta \overline{x} - c_f) = \delta x^* - \delta \overline{x}$. Since $W(x^*) = 0$, the utility loss is $V(x^*) - W(x^*) = \delta x^* - \delta \overline{x}$. When rewards are immediate, since $x^* < \overline{x}$, $V(x^*) = 0$. Since $W(x^*) = -\lambda x^*$, $V(x^*) - W(x^*) = \lambda x^*$. Finally, when both costs and rewards are immediate, $V(x^*) = x^* - c_f + \delta \int V(x') \mathrm{d}F(x') = x^* - c_f - (\overline{x} - c_f) = x^* - \overline{x}$. Since $W(x^*) = 0$, $V(x^*) - W(x^*) = x^* - \overline{x}$. □

References

Ainslie, G., Haslam, N.: Hyperbolic discounting. In: Loewenstein, G., Elster, J. (eds.) Choice over time. New York: Russell Sage Foundation (1992)

Akerlof, G.: Procrastination and obedience. Am. Econ. Rev. **81**, 1–19 (1991)

Brocas, I., Carrillo, J.D.: Rush and procrastination under hyperbolic discounting and interdependent activities. J Risk Uncertain **22**, 141–164 (2001)

Brocas, I., Carrillo, J.D.: A theory of haste. J Econ Behav Organ **56**, 1–23 (2005)

DeJong, D., Ripoll, M.: Do self-control preferences help explain the puzzling behavior of asset prices? J Monetary Econ (forthcoming 2006)

DellaVigna, S., Paserman, M.D.: Job search and impatience. J Labor Econ **23**, 527–588 (2005)

Diamond, P., Koszegi, B.: Quasi-hyperbolic discounting and retirement. J Public Econ **87**, 1839–1872 (2003)

Dixit, A., Pindyck, R.: Investment under uncertainty. Princeton: Princeton University Press, (1994)
Esteban, S., Miyagawa, E., Shum, M.: Nonlinear pricing with self-control preferences, J Econ Theory (forthcoming 2006)
Fudenberg, D., Levine, D.K.: A dual self model of impulse control. Am Econ Rev (forthcoming 2006)
Grenadier, S., Wang, N.: Investment under uncertainty and time-inconsistent preferences. J Financ Econ (forthcoming 2006)
Gul, F., Pesendorfer, W.: Temptation and self-control. Econometrica **69**, 1403–1436 (2001)
Gul, F., Pesendorfer, W.: Self-control and the theory of consumption. Econometrica **72**, 119–158 (2004)
Herings, P., Rohde, K.: Time-inconsistent preferences in a general equilibrium model. Econ Theory **29**, 591–619 (2006)
Imrohoroglu, A., Imrohoroglu, S., Joines, D.: Time inconsistent preferences and social security. Q J Econ **118**, 745–784 (2003)
Kirby, K., Herrnstein, R.J.: Preference reversal due to myopia of delayed reward. Psychol Sci **6**, 83–89 (1995)
Krusell, P., Smith, A.: Consumption-savings decisions with quasi-geometric discounting. Econometrica **71**, 365–375 (2003)
Krusell, P., Kuruscu, B., Smith, A.: Temptation and taxation. working paper, University of Rochester (2001)
Krusell, P., Kuruscu, B., Smith, A.: Time orientation and asset prices. J Monetary Econ **49**, 107–135 (2002)
Laibson, D.: Self-control and saving, working paper, Harvard University (1994)
Laibson, D.: Golden eggs and hyperbolic discounting. Q J Econ **112**, 443–447 (1997)
O'Donoghue, T., Rabin, M.: Doing it now or later. Am Econ Rev **89**, 103–124 (1999a)
O'Donoghue, T., Rabin, M.: Incentives for procrastinators. Q J Econ **114**, 769–816 (1999b)
O'Donoghue, T., Rabin, M.: Choice and procrastination. Q J Econ **116**, 121–160 (2001)
Phelps, E.S., Pollak, R.A.: On second-best national saving and game-equilibrium growth. Rev Econ Stud **35**, 201–208 (1968)
Rook, D.W.: The buying impulse. J Consumer Res **14**, 189–199 (1987)
Statman, M., Caldwell, D.: Applying behavioral finance to capital budgeting: project terminations. Financ Manag **16**, 7–15 (1987)
Staw, B.M.: Knee-deep in the big buddy: a study of escalating commitment to a chosen course of action. Organ Behav Hum Perform, 27–44 (1976)
Staw, B.M., McClane, S.: Throwing good money after bad: escalation in a banking context. Symposium presentation, Academy of Management, 44th Annual Meeting in Boston (1984)
Stokey, N., Lucas, R.E. with Prescott, E.C.: Recursive methods in economic dynamics. Cambridge, MA: Harvard University Press (1989)
Strotz, R.H.: Myopia and inconsistency in dynamic utility maximization. Rev. Econ. Stud. **23**, 165–180 (1956)
Thaler, R.: Some empirical evidence on dynamic inconsistency. Econ Lett **8**, 201–207 (1981)