Does Calvo Meet Rotemberg at the Zero Lower Bound?*

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Abstract

This paper compares the conventional Calvo and Rotemberg price adjustments at the zero lower bound (ZLB) on nominal interest rates. Although the two pricing mechanisms are equivalent to a first-order approximation around the zero inflation steady state, they produce very different results, based on a fully-nonlinear method. Specifically, the nominal interest rate hits the ZLB more frequently in the Calvo model than in the Rotemberg model. At the ZLB, deflation is larger and recessions are more severe in the Calvo model. The main reason for the difference in results is that price adjustment costs show up in the resource constraints in the Rotemberg. When they are rebated to the household, the two models behave similarly.

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1 Introduction

The recent experiences of Japan, the United States, and the Eurozone with (nearly) zero nominal interest rates have raised some important questions to macroeconomists: How does the economy behave when it faces a zero lower bound (ZLB) on nominal interest rates? What are the implications of the ZLB for monetary policy? To address these questions, researchers typically adopt the dynamic new Keynesian (DNK) framework with the price setting mechanism as one of its most important building blocks (Woodford (2003) and Gali (2008)).

The two most popular modeling approaches to price setting used in the ZLB literature are due to Calvo (1983) and Rotemberg (1982). According to the Calvo approach, firms face an exogenously fixed probability of adjusting their prices each period (so the relative price dispersion becomes a state variable), while, according to the Rotemberg approach, firms pay quadratic adjustment costs to adjust their prices. It is well known that the two approaches are equivalent up to a first-order approximation around the zero-inflation steady state in the absence of the ZLB, given an assumption such that the price stickiness parameters in the two models imply the same log-linearized Phillips curve.

However, whether or not these two approaches generate the same results in a fully nonlinear dynamic stochastic general equilibrium (DSGE) framework with an occasionally binding ZLB constraint is not well understood.[1] Understanding this issue is important not only for academic researchers but also for policymakers because one may provide different answers to the questions raised above and draw different policy implications if the two approaches generate very different results.

Our main contribution is to shed light on this issue by developing a result on model mechanics which are obscured by comparisons of non-nested estimated models. Our goal is to compare the quantitative predictions of the Calvo-pricing model and the Rotemberg-pricing model in the presence of the ZLB on nominal interest rates using a global nonlinear numerical method. We focus on the following specific questions: (1) Do the two models generate the same policy functions and business cycle dynamics? (2) How often does the ZLB bind in each model? and (3) Under what condition do these two models

[1]Braun and Waki (2010) compare the Calvo and Rotemberg models at the ZLB using a nonlinear shooting method in a deterministic perfect foresight framework. Ascari and Rossi (2012), Ascari et al. (2011), Khan (2005), and Leith and Liu (2016) compare the Calvo and Rotemberg models without a ZLB constraint based on linear approximation. Ngo (2018) compares inflation costs at steady state for the Calvo and the Rotemberg price adjustments.

produce similar results?

Based on a conventional calibration such that the log-linearized Calvo and Rotemberg models are equivalent, we find the following main results using nonlinear solution methods. First, far above the ZLB, the two models produce very similar results: almost identical policy functions and generalized impulse response functions. Second, under an adverse preference shock driving the economy close to the ZLB, the interest rate cut is greater in the Calvo model than in the Rotemberg model. As a result, the ZLB binds more frequently in the Calvo model. Specifically, the unconditional probability of hitting the ZLB is around 5.5% in the Calvo model, while it is only 2.7% in the Rotemberg model. Third, at the ZLB, the difference between the two models is magnified substantially. Conditional on the ZLB binding at least one period, inflation and output decrease substantially more in the Calvo model than in the Rotemberg model. Finally, rebating the quadratic adjustment costs to the household makes the Calvo and Rotemberg models behave similarly at the ZLB.

To understand the difference in the results of the two models, we have to understand the nature of the price rigidities underlying these models. In the Calvo model each firm adjusts prices with a fixed probability in each period, generating price dispersion. Price dispersion results in an inefficiency loss in aggregate production and acts as a negative productivity shift in the production function. Moreover, price dispersion is a backward-looking state variable and cannot change suddenly under an adverse shock, introducing an inertial component into the model (Ascari and Rossi (2012)). By contrast, the Rotemberg model assumes a quadratic cost of adjusting prices, which depends on inflation or deflation and can change suddenly under an adverse shock. In the standard Rotemberg model, aggregate output is used for consumption, government spending, and the price adjustment costs. Under an adverse shock that causes the ZLB to bind, large deflation leads the price adjustment costs to rise. This effect mitigates the decline of aggregate demand and inflation (less deflation). As a result, the interest rate cut is larger in the Calvo model when there is an adverse shock that drives the economy near the ZLB, and the ZLB binds more frequently in the Calvo model.

Notice that the price dispersion effect in the Calvo model and the mitigating effect of price adjustment costs in the Rotemberg model vanish in the log-linearized equilibrium system around the zero-inflation steady state because the relative price dispersion and the price adjustment costs are equal to zero up to a first-order approximation. This is why the log-linearized Calvo and Rotemberg models are equivalent under the assumption such that the price stickiness parameters in the two models imply the same Phillips curve, but they behave very differently based on nonlinear solutions.

When the price adjustment costs are rebated or paid to the household so that these costs do not show up in the aggregate resource constraints as in Ascari and Rossi (2012) and Eggertsson and Singh (2016), the previous effect of mitigating the decline of aggregate demand also vanishes.[2] We find that the Rotemberg model with rebates and the Calvo model generate very similar results based on nonlinear solutions. In addition, we find that most of the difference in results comes from the mitigating effect of price adjustment costs, not from the price dispersion effect. Therefore, we suggest that future ZLB research should use the Calvo model or the Rotemberg model with rebates. These two models not only generate comparable results but also avoid astronomical price adjustment costs at ZLB in the conventional Rotemberg model as noted by Eggertsson and Singh (2016). Our results complement theirs with a complete stochastic and nonlinear characterization instead of a simple "one-transition" model of the ZLB.

2 Models

We present two otherwise identical standard DNK models with different pricing mechanisms. Both models consist of a continuum of identical households, a continuum of identical competitive final good producers, a continuum of monopolistically competitive intermediate goods producers, and a government (monetary and fiscal authorities). Here we briefly describe the setups and leave the detailed description to Appendix A.

The representative household maximizes his expected discounted utility subject to the budget constraint. The flow utility/disutility comes from working and consuming final goods that are produced by the final good producers, who buy and aggregate a variety of intermediate goods using a CES technology. There is a unit mass of intermediate goods producers on $[0, 1]$ that are monopolistic competitors. Each intermediate good $i \in [0, 1]$ is produced by one producer using a linear technology that transforms one unit of labor input into one unit of output.

Intermediate good producers have market power and set prices to maximize discounted profits. They face frictions to adjust prices and thus price adjustments are

[2]We assume an economy-wide labor market in the Calvo model so that price dispersion becomes a state variable. Eggertsson and Singh (2016) assume industry-specific labor so that price dispersion is not a state variable.

sticky. According to Calvo (1983), in each period an intermediate goods firm i keeps its previous price with probability θ and resets its price with probability $(1-\theta)$. The Calvo model uses an economy-wide labor market so price dispersion becomes a state variable. Alternatively, Rotemberg (1982) assumes that each intermediate goods firm i faces costs of adjusting prices in terms of final goods. We adopt a quadratic adjustment cost function proposed by Ireland (1997), which is commonly used in the ZLB literature,

$$\frac{\varphi}{2}\left(\frac{P_t\left(i\right)}{P_{t-1}\left(i\right)}-1\right)^2 Y_t,$$

where φ is the adjustment cost parameter which determines the degree of nominal price rigidity, $P_t\left(i\right)$ denotes the price of intermediate good i, and Y_t denotes the aggregate output.

The central bank conducts monetary policy by setting the interest rate using a simple Taylor rule subject to the ZLB condition. The government runs a balanced budget and raises lump-sum taxes to finance government spending. The detailed specification of the Taylor rule and fiscal rule are presented in Appendix A. The log-linearized system is presented in Appendix B.

3 Calibration and solution method

Most of the parameters used in this paper are conventional. Table 1 shows the values of the parameters. Appendix C discusses the sources and meanings of these values in detail. The probability of keeping prices unchanged in the Calvo model is calibrated to be $\theta = 0.75$, resulting in the average duration of four quarters with prices being kept unchanged. We set the price adjustment cost parameter in the Rotemberg model $\varphi = 78$ to ensure that the two models are equivalent to the first-order approximation around the steady state with zero inflation.

We set the persistence of the preference shock, which is the only shock that drives the nominal interest rate to the ZLB, to the conventional value of 0.8. We then choose the standard deviation of the preference shock innovations so that the unconditional probability of hitting the ZLB is around 5.5% in the Calvo model.

We use nonlinear solution methods to solve and simulate the two models. Our projection method is close to, but slightly different from, the one used in Fernandez-Villaverde et al. (2015). Similar to their method, we do not approximate the policy function for

Table 1: Calibration

Symbol	Description	Values
β	Quarterly discount factor	0.99
γ	CRRA parameter	1
η	Inverse labor supply elasticity	1
ε	Monopoly power	7.66
θ	Probability of keeping prices unchanged in the Calvo model	0.75
φ	Price adjustment cost parameter in the Rotemberg model	78
π	Inflation target	0
ϕ_π	Weight of inflation target in the Taylor rule	1.5
ϕ_y	Weight of output target in the Taylor rule	0.25
S_g	Share of the government spending at the steady state	0.2
σ_β	Standard deviation of the innovation of preference shocks (%)	0.19
ρ_β	AR-coefficient of preference shocks	0.8
σ_g	Standard deviation of the innovation of government spending shocks (%)	0.25
ρ_g	AR-coefficient of government spending shocks	0.8

the nominal interest rate. Instead, the nominal interest rate is always determined by the Taylor rule at every state, in or out of the set of collocation nodes. However, different from them, we approximate the expectations as a function of state variables using a finite element method called the cubic spline interpolation (Judd (1998) and Miranda and Fackler (2002)). The main advantage of this approach is that we do not have to worry about the kink of policy function when the ZLB starts binding. Furthermore, expectations can smooth out the kink. The detailed algorithm and computation errors can be found in Appendix D.

4 Results

4.1 Policy functions

The state variables for the Calvo model include the relative price dispersion (Δ_{t-1}), the preference shock (β_t), and the government spending shock (g_t). By contrast, the relative price dispersion Δ_{t-1} does not appear in the Rotemberg model and its state variables are only β_t and g_t. We express equilibrium variables as policy functions of state variables.

To illustrate the difference between the two models, we first show their policy functions for various positive values of the preference shock holding other state variables at their steady state levels. As indicated in Figure 1, the solid blue lines represent the results from the Calvo model with the ZLB, while the dashed green lines represent those from the Calvo model without the ZLB. The dot-dashed red lines show the results from the Rotemberg model with the ZLB. Without the ZLB, the Rotemberg model produces results similar to those of the Calvo model.

When there is a positive preference shock, households value their future consumption more, so they tend to save more and consume less today, putting downward pressure on output and the price level. To restore consumption and output, the real interest rate must fall. If the central bank were not restrained by the ZLB, it could adjust the nominal interest rate so that the actual real interest rate would be the same as the natural real rate, as indicated by the dashed green lines in Panels A and C of Figure 1.

However, because the ZLB is imposed, a large positive preference shock causes the ZLB to bind. As a result, the actual real interest rate will be larger than the natural real interest rate because the nominal interest rate cannot be negative. In general, the results from the Calvo and Rotemberg models have similar features to those in Adam and Billi (2007) and Ngo (2014b). We summarize these features below.

First, in the absence of the ZLB, the central bank can somewhat stabilize the economy by adjusting the nominal interest rate using the simple Taylor rule in both the Calvo model and the Rotemberg model. Note that the bank cannot completely offset the shocks to obtain the target output and inflation as it could using optimal monetary policy.

Second, when the ZLB is present, the central bank cannot stabilize output and inflation under shocks that cause the ZLB to bind. As seen in Panels A and C of Figure 1, when nominal interest rates hit the ZLB, the real interest rates in both models are higher than the real rate in the Calvo model without the ZLB. As a result, consumption, GDP, and labor fall substantially, as shown in Panels D, E, and F of Figure 1.

Now let us turn our attention to analyzing the differences between the Calvo model and the Rotemberg model in the fully nonlinear framework, outside and at the ZLB. It is worth reminding the reader that the parameters in the two models are calibrated such that they generate the same results to a first-order approximation, even in the presence of the ZLB. In addition, the deterministic steady state is the same in the two models. As Figure 1 shows, when the ZLB is not binding or the nominal interest rate is positive, these models generate very similar policy functions for consumption, GDP, and labor.

With the presence of the ZLB, interest rate policy is more aggressive in the Calvo

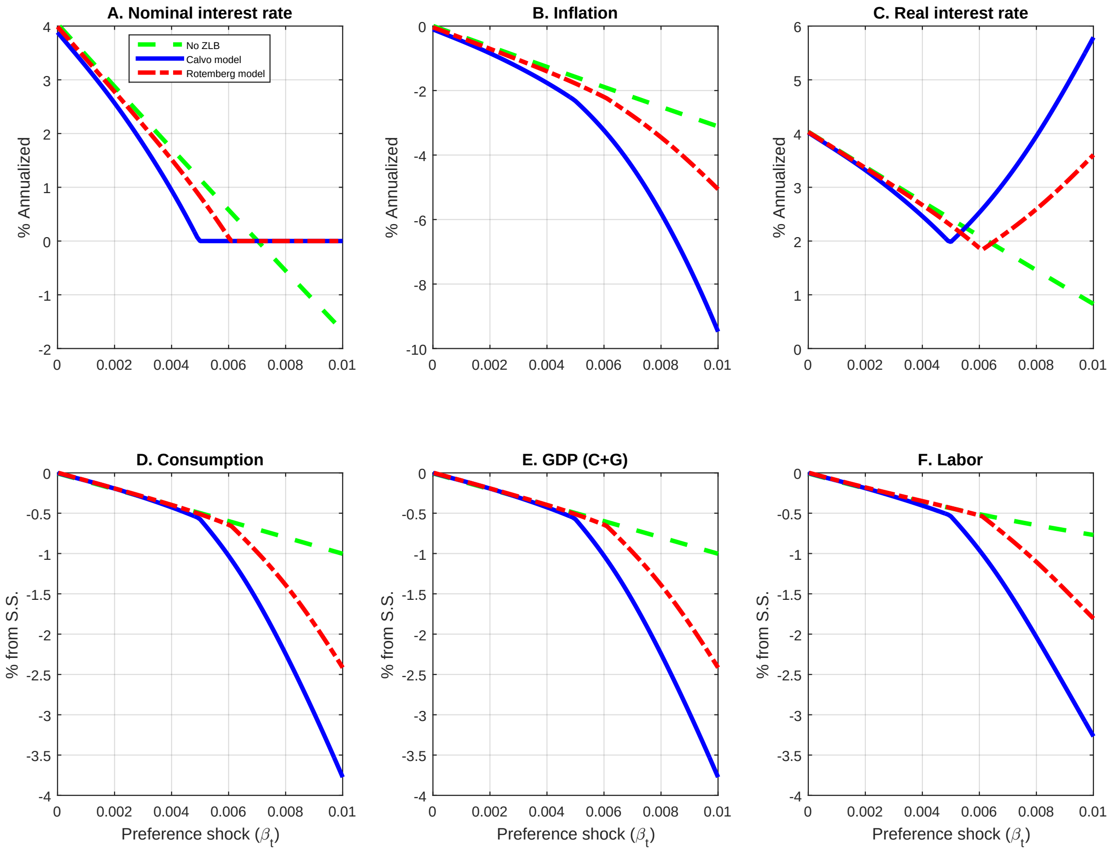

Figure 1: Policy functions when the relative price dispersion and the government spending shock are held at their steady-state levels ($\Delta_{t-1} = g_t = 1$). Note that the nominal variables including inflation and the nominal and real interest rates are expressed in annualized percentage terms; the real variables including consumption, labor, and GDP are expressed in percentage deviations from the deterministic steady state.

model than in the Rotemberg model. Specifically, given the same adverse preference shock, the central bank cuts the policy rate more in the Calvo model, as seen in Panel A of Figure 1. Thus, the ZLB binds more frequently in the Calvo model than in the Rotemberg model.

It is more interesting that, when the ZLB binds in the two models, the Calvo model generates a more severe recession. Given the same size of the preference shock that drives the ZLB to bind in the two models, the declines in GDP and inflation are larger in the Calvo model than in the Rotemberg model, as seen in Panels B, D, E, and F of Figure 1. These results are associated with the fact that the real interest rate is much higher in the Calvo model than in the Rotemberg model (Panel C of Figure 1), leading to a higher incentive for households to save and a sharper decline in consumption and output in the Calvo model than in the Rotemberg model.

4.2 Generalized impulse response functions

Although the policy function is useful in providing a complete picture of the solution, the endogenous state variable, which is the relative price dispersion (Δ_{t-1}), is kept at the deterministic steady state value.[3] In this subsection, we compare the dynamics of the two models by computing generalized impulse response functions (GIRFs) based on Monte Carlo simulations, as described in Koop et al. (1996). Specifically, suppose that the economy is hit by a one-time one-standard-deviation shock to the subjective discount factor at time t, the generalized impulse response of variable Y after n periods is defined as

$$GI_Y^n\left(\sigma_\beta, g_{t-1}, \beta_{t-1}, \Delta_{t-1}\right) = E\left[Y_{t+n} | \left(\varepsilon_{\beta t}, \varepsilon_{gt}\right) = \left(\sigma_\beta, 0\right), g_{t-1}, \beta_{t-1,} \Delta_{t-1}\right] - E\left[Y_{t+n} | g_{t-1}, \beta_{t-1,} \Delta_{t-1}\right],$$

where $\left(g_{t-1}, \beta_{t-1}, \Delta_{t-1}\right)$ is the state of the economy.[4] Given any triple $\left(g_{t-1}, \beta_{t-1}, \Delta_{t-1}\right)$, we compute $GI_Y^n\left(\sigma_\beta, g_{t-1}, \beta_{t-1}, \Delta_{t-1}\right)$ using Monte Carlo simulations.

Before showing the result regarding GIRFs, we would like to describe how to obtain our simulated distribution of the state (g, β, Δ). We first simulate the model 299,999 periods by drawing government spending and time discount factor innovations, starting from the deterministic steady state. We then discard the first 999 periods to avoid the dependence of the initial state. We now obtain a sample of 299,000 points for

[3]Based on our simulation, the steady state value of the price dispersion is very close to the mean value. So, the policy function is very informative.

[4]Since government spending and the time discount factor are stochastic, Y_{t+n} is a random variable.

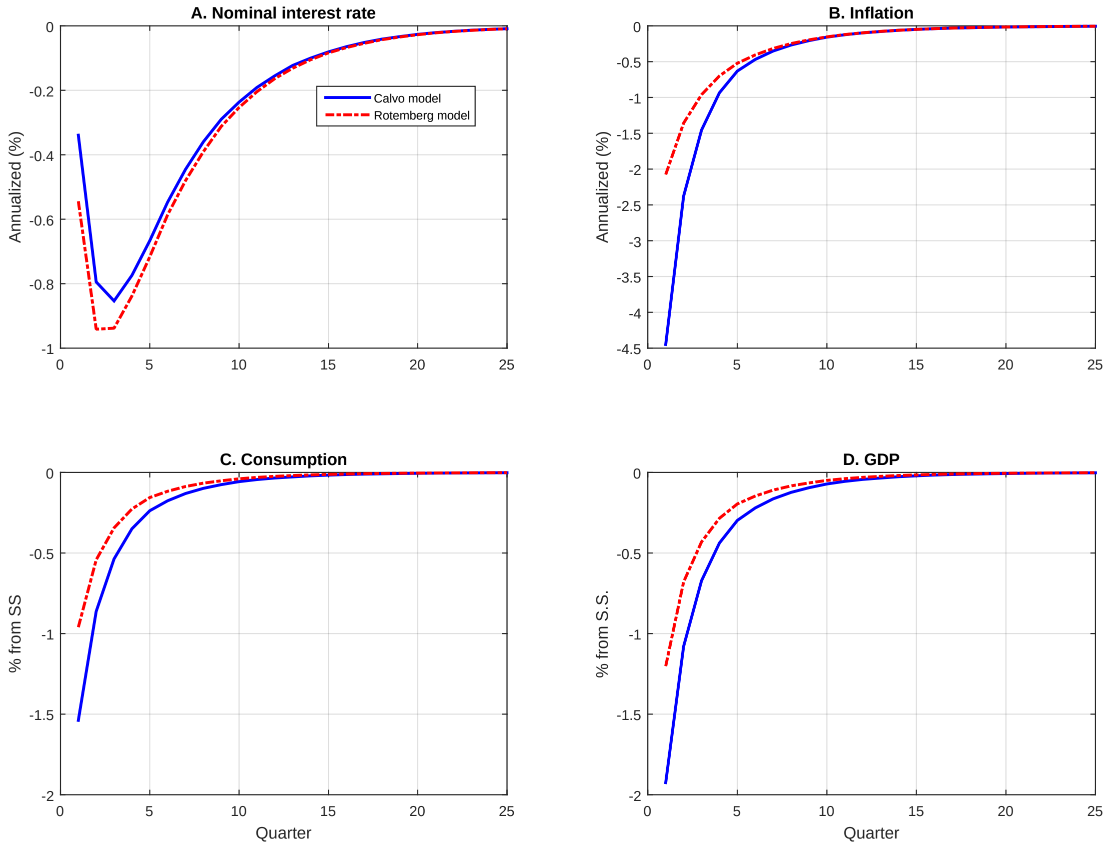

Figure 2: GIRFs under a standard-deviation shock to β_t at the ZLB. The nominal variables including inflation and the nominal interest rate are expressed in annualized percentage terms, while the real variables including consumption and GDP are expressed in percentage deviations relative to the deterministic steady state.

(g, β, Δ), which can also be considered as the ergodic distribution of (g, β, Δ). A triple $\left(g_{t-1}, \beta_{t-1}, \Delta_{t-1}\right)$ can be randomly drawn from this distribution.

Figure 2 shows the average GIRFs at the ZLB under a one-standard deviation shock to β_t. To compute these GIRFs, we first draw a random sample of 1,000 initial states (g, β, Δ) from the ergodic distribution (which has 299,000 elements as mentioned above), such that starting from each of these initial states the ZLB binds even without any further adverse preference shocks. Given each initial state from this sample, we compute GIRFs, $GI_Y^n\left(\sigma_\beta, g_{t-1}, \beta_{t-1}, \Delta_{t-1}\right)$, using Monte Carlo simulations. Finally, we report the average of GIRFs based on this sample. We find that the GIRFs are very different for the Calvo and Rotemberg models. For example, inflation, consumption, and GDP decline more on impact in the Calvo model than in the Rotemberg model. The GIRFs reported in Figures 2 support our finding in the previous subsection that the two models produce quite different results at the ZLB. Moreover, the Calvo model generates a more severe recession at the ZLB than the Rotemberg model does.[5]

Richter and Throckmorton (2016) estimate both the Calvo and Rotemberg models in the presence of the ZLB. They find that real GDP and inflation decline by more in the Rotemberg model than in the Calvo model, a result different from ours. The reason is that they follow a different strategy to assign parameter values. We show that the Calvo and Rotemberg models with a ZLB constraint imply the same log-linearized equilibrium system around the zero-inflation steady state, after imposing an assumption that links the parameters of price stickiness in the two models. Based on Bayesian estimations, Richter and Throckmorton (2016) find that this assumption roughly holds for their estimated parameter values. However, since their estimated steady-state inflation rate is positive, their estimated Calvo and Rotemberg models are not identical up to a first-order approximation.

4.3 Simulations

In this subsection, we compare the two models using very long time series simulations. In particular, we created a simulated dataset of 290,000 periods and report the key statistics for macro variables in the two models.

Panel A of Table 2 shows that the unconditional probability of hitting the ZLB in the Calvo model is 5.6% while it is only 2.7% in the Rotemberg model. This result supports our analysis using policy functions in the previous subsection that the ZLB binds more often in the Calvo model than in the Rotemberg. In addition, even though the average ZLB duration is only about 2 quarters in the two models, there are some recessions in which the nominal interest rate can stay at the ZLB as long as 17 periods using the Calvo model. The longest ZLB duration is only 13 periods in the Rotemberg model based on the simulated data,.

Columns 2 and 3 of Panel B of Table 2 shows that, at the ZLB, the Calvo model generates more severe recession. On average, at the ZLB output declines about 1.27% in

[5]We also compute the GIRFs under a standard-deviation shock to β_t outside the ZLB. The results are very similar between the Calvo model and the Rotemberg model. To save space, we do not report them here. However, they are available upon request.

Table 2: Key statistics

	Calvo model Benchmark	Rotemberg model Benchmark	Calvo model ($\Delta = 1$)	Rotemberg with Rebates
A. Unconditional ZLB probability (%) and duration (quarters)				
ZLB probability	5.6	2.7	5.7	5.7
Average ZLB duration	2.0	1.8	2.0	2.0
B. Average change in macroeconomic variables from SS (%), conditional on binding ZLB				
Ex ante real interest rate	0.69	0.58	0.69	0.69
Inflation rate	-0.92	-0.75	-0.93	-0.89
Consumption	-1.27	-1.12	-1.25	-1.18
Hours worked	-1.07	-0.89	-1.15	-1.18
GDP	-1.27	-1.12	-1.25	-1.18

Note: The statistics are computed based on a 299,9999 random draws starting from the deterministic steady state (SS); the first 999 observations are dropped to eliminate the effect of initial values. The statistics for ZLB durations and other macroeconomic variables are computed conditional on binding ZLB. In this table, ZLB durations are single completed spells.

the Calvo model and 1.12% in the Rotemberg model. In addition, the Calvo model generates 0.17% more deflation. Moreover, based on the simulated data the worst recession in the Calvo model is much worse than that in the Rotemberg model. Specifically, in the worst recession real GDP declines about 7.23% in the Calvo model while it declines only 4.63% in the Rotemberg model. This comparison still understates the relative severity because the ZLB occurs more frequently in the Calvo model.

4.4 Why do the two models produce different results?

To understand why the two models produce different results, we have to understand the nature of the price rigidities underlying these models. In the Calvo model with an economy-wide labor market as considered in this paper, each firm adjusts prices with a fixed probability in each period, generating price dispersion. Price dispersion results in an inefficiency loss in aggregate production and acts as a negative productivity shift in the production function, as in Ascari and Rossi (2012). Moreover, price dispersion is a backward-looking state variable and cannot change suddenly under an adverse shock, introducing an inertial component to the model. Because of the interaction of deflation (at the ZLB) and price dispersion, the Calvo model may produce more severe recessions with longer ZLB spells.[6]

By contrast, the Rotemberg model assumes a quadratic cost of adjusting prices, which depends on inflation or deflation and can change suddenly under an adverse shock. In the standard Rotemberg model aggregate output is used for consumption, government spending, and the price adjustment costs. Under an adverse shock that causes the ZLB to bind, large deflation leads the price adjustment costs to rise. This effect mitigates the decline of aggregate demand. Hence deflation and recessions are more severe in the Calvo model than in the Rotemberg model. As a result, the interest rate cut is larger in the Calvo model when there is an adverse shock that drives the economy near the ZLB, and the ZLB binds more frequently in the Calvo model.

Notice that the price persistence effect and the mitigating effect of price adjustment costs vanish in the log-linearized equilibrium system around the zero-inflation steady state because both the price persistence and the price adjustment costs equal zero up

[6]Eggertsson and Krugman (2012) show that a rise in productivity is contractionary at the ZLB due to the Fisher effect. If price dispersion is inertial in the Calvo model, it would not drop immediately under an adverse shock, causing a relative improvement in productivity and a worse recession at the ZLB compared to the case without the inertia.

to a first-order approximation. This is why the log-linearized Calvo and Rotemberg models are equivalent under the assumption that the price stickiness parameters in the two models imply the same Phillips curve, but they behave very differently based on nonlinear solutions.

As explained above, due to the price persistence effect and the mitigating effect of price adjustment costs, the Calvo and the Rotemberg models generate different results. To isolate these two effects and to see which one dominates, we implement two experiments: (1) we shut down the price persistence dynamics by setting the relative price persistence to be one (the steady state value); this experiment has the flavor of Eggertsson and Singh (2016), where they assume industry-specific labor in the Calvo model so that the relative price dispersion ceases to be a state variable; and (2) we modify the original Rotemberg model by rebating the price adjustment costs to the household so that these costs do not show up in the aggregate resource constraints, as in Ascari and Rossi (2012) and Eggertsson and Singh (2016).

The simulation results for experiments 1 and 2 are presented in columns 4 and 5 of table 2. We can see that shutting down the price persistence effect in the Calvo model does not significantly change the results. The results from the Calvo model with constant price dispersion are still very different from those in the benchmark Rotemberg model. However, rebating the price adjustment costs to the household makes the Calvo and the Rotemberg models behave very similarly. Specifically, the unconditional probability of hitting the ZLB is almost the same in the Calvo model and the Rotemberg model with rebates, around 5.5%. In addition, the severity of recessions at the ZLB are quite similar in both the Calvo model and the Rotemberg model with rebates. Our results are consistent with Eggertsson and Singh (2016). They find that the nonlinear Calvo model with industry-specific labor and the nonlinear Rotemberg model behave very differently, but these two models give similar results when price adjustment costs are rebated to the household.

5 Conclusion

This paper explores the difference between the Calvo price setting and the Rotemberg counterpart in a fully nonlinear DNK framework with an occasionally binding ZLB constraint on nominal interest rates. We find that under a preference shock driving the economy close to the ZLB, the central bank cuts the nominal interest rate more aggressively in the Calvo model than in the Rotemberg model. As a result, the nominal interest hits the ZLB more frequently in the Calvo model. In particular, the unconditional probability of hitting the ZLB is 5.5% in the Calvo model, while it is only 2.7% in the Rotemberg model. We also find that, conditional on binding ZLB, recessions are more severe with larger deflation in the Calvo model than in the Rotemberg model.

The main reason for the difference in results is that price adjustment costs show up in the resource constraints in the Rotemberg, making output and price level in the Rotemberg decline less under an adverse shock that makes the ZLB bind. When the price adjustment cost is rebated to household, the two models behave very similarly. Therefore, we suggest that future research on the ZLB should use the Calvo model or the Rotemberg model with rebates. These two models not only generate comparable results but also avoid astronomical price adjustment costs at the ZLB in the conventional Rotemberg model as noted by Eggertsson and Singh (2016).

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Appendix

A Models

We present two otherwise identical DNK models with different pricing mechanisms. Both models consist of a continuum of identical households, a continuum of identical competitive final good producers, a continuum of monopolistically competitive intermediate goods producers, and a government (monetary and fiscal authorities).

Households

The representative household maximizes his expected discounted utility

$$E_1 \left\{ \sum_{t=1}^{\infty} \left(\Pi_{j=0}^{t-1} \beta_j \right) \left(\frac{C_t^{1-\gamma}}{1-\gamma} - \chi \frac{N_t^{1+\eta}}{1+\eta} \right) \right\} \tag{A.1}$$

subject to the budget constraint

$$P_t C_t + (1+i_t)^{-1} B_t = W_t N_t + B_{t-1} + \Pi_t + T_t, \tag{A.2}$$

where C_t is consumption of final goods, i_t is the nominal interest rate, B_t denotes one-period bond holdings, N_t is labor, W_t is the nominal wage rate, Π_t is the profit income, T_t is the lump-sum tax, and β_t denotes the preference shock. We assume that β_t follows an AR(1) process

$$\ln(\beta_t) = (1-\rho_\beta) \ln \beta + \rho_\beta \ln \left(\beta_{t-1} \right) + \varepsilon_{\beta t}, \ \ \beta_0 = 1 \tag{A.3}$$

where $\rho_\beta \in (0,1)$ is the persistence of the preference shock and $\varepsilon_{\beta t}$ is the innovation of the preference shock with mean 0 and variance σ_β^2. The preference shock is a reduced form of more realistic forces that can drive the nominal interest rate to the ZLB.[7]

The first-order conditions for the household optimization problem are given by

$$\chi N_t^\eta C_t^\gamma = w_t, \tag{A.4}$$

and

$$E_t \left[M_{t,t+1} \left(\frac{1+i_t}{1+\pi_{t+1}} \right) \right] = 1, \tag{A.5}$$

[7] This setting is very common in the ZLB literature, for example see Nakata (2011) and Ngo (2014b) among others. Another way to make the ZLB binding is to introduce a deleveraging shock as in Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2011).

where $w_t = W_t/P_t$ is the real wage, $\pi_t = P_t/P_{t-1} - 1$ is the inflation rate, and the stochastic discount factor is given by

$$M_{t,t+1} = \beta_t \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma}. \tag{A.6}$$

Final good producers

To produce the final good, the final good producers buy and aggregate a variety of intermediate goods indexed by $i \in [0,1]$ using a CES technology

$$Y_t = \left(\int_0^1 Y_t\left(i\right)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}},$$

where ε is the elasticity of substitution among intermediate goods. The profit maximization problem is given by

$$\max \; P_t Y_t - \int_0^1 P_t\left(i\right) Y_t\left(i\right) di,$$

where $P_t\left(i\right)$ and $Y_t\left(i\right)$ are the price and quantity of intermediate good i. Profit maximization and the zero-profit condition give the demand for intermediate good i

$$Y_t\left(i\right) = \left(\frac{P_t\left(i\right)}{P_t} \right)^{-\epsilon} Y_t, \tag{A.7}$$

and the aggregate price level

$$P_t = \left(\int P_t\left(i\right)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}. \tag{A.8}$$

Intermediate goods producers

There is a unit mass of intermediate goods producers on $[0,1]$ that are monopolistic competitors. Suppose that each intermediate good $i \in [0,1]$ is produced by one producer using the linear technology

$$Y_t\left(i\right) = N_t\left(i\right), \tag{A.9}$$

where $N_t(i)$ is labor input. Cost minimization implies that each firm faces the same real marginal cost

$$mc_t = mc_t\left(i\right) = w_t. \tag{A.10}$$

Price setting mechanisms

Intermediate good producers have market power and set prices to maximize discounted profits. They face frictions to adjust prices and thus price adjustments are sticky. We follow Calvo (1983) and Rotemberg (1982) to model sticky prices.

Calvo pricing

According to Calvo (1983), in each period an intermediate goods firm i keeps its previous price with probability θ and resets its price with probability $(1-\theta)$. The price setting problem is given by

$$\max_{P_t(i)} E_t \sum_{j=0}^{\infty} \left\{ \theta^j M_{t,t+j} \left[\frac{P_t(i)}{P_{t+j}} - mc_{t+j} \right] Y_{t+j}(i) \right\} \tag{A.11}$$

subject to its demand (A.7), where $M_{t,t+j}$ is the stochastic discount factor defined as

$$M_{t,t} = 1,\ M_{t,t+j} = \prod_{s=0}^{j-1} M_{t+s,t+s+1} \text{ for } j \geq 1.$$

The optimal relative price $p_t^* = P_t^*(i)/P_t$ is the same for all firms that have a chance to reset their prices today and is given by

$$p_t^* = \frac{P_t^*(i)}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{j=0}^{\infty} \left\{ \theta^j M_{t,t+j} \left(\frac{P_{t+j}}{P_t} \right)^{\varepsilon} Y_{t+j} mc_{t+j} \right\}}{E_t \sum_{j=0}^{\infty} \left\{ \theta^j M_{t,t+j} \left(\frac{P_{t+j}}{P_t} \right)^{\varepsilon - 1} Y_{t+j} \right\}}. \tag{A.12}$$

After some manipulation, we can rewrite the optimal pricing rule as

$$p_t^* = \frac{S_t}{F_t}, \tag{A.13}$$

where S_t and F_t satisfy the following recursive equations:

$$S_t = \frac{\varepsilon}{\varepsilon - 1} C_t^{-\gamma} Y_t w_t + \theta \beta E_t \left[\beta_t (1 + \pi_{t+1})^{\varepsilon} S_{t+1} \right], \tag{A.14}$$

$$F_t = C_t^{-\gamma} Y_t + \theta \beta E_t \left[\beta_t (1 + \pi_{t+1})^{\varepsilon - 1} F_{t+1} \right]. \tag{A.15}$$

Rotemberg pricing

Rotemberg (1982) assumes that each intermediate goods firm i faces costs of adjusting prices in terms of final goods. In this paper, we use a quadratic adjustment cost function, which is proposed by Ireland (1997) and which is one of the most common functions used in the ZLB literature:

$$\frac{\varphi}{2}\left(\frac{P_t(i)}{P_{t-1}(i)}-1\right)^2 Y_t,$$

where φ is the adjustment cost parameter which determines the degree of nominal price rigidity.[8] The problem of firm i is given by

$$\max_{\{P_{i,t}\}} E_t \sum_{j=0}^{\infty}\left\{M_{t,t+j}\left[\left(\frac{P_{t+j}(i)}{P_{t+j}}-mc_t\right)Y_{t+j}(i)-\frac{\varphi}{2}\left(\frac{P_{t+j}(i)}{P_{t+j-1}(i)}-1\right)^2 Y_{t+j}\right]\right\} \tag{A.16}$$

subject to its demand (A.7). In a symmetric equilibrium, all firms will choose the same price and produce the same quantity, i.e., $P_t(i)=P_t$ and $Y_t(i)=Y_t$. The optimal pricing rule then implies that

$$(1-\varepsilon+\varepsilon w_t-\varphi\pi_t(1+\pi_t))Y_t+\varphi E_t\left[M_{t,t+1}\pi_{t+1}(1+\pi_{t+1})Y_{t+1}\right]=0. \tag{A.17}$$

Monetary and fiscal policies

The central bank conducts monetary policy by setting the interest rate using a simple Taylor rule subject to the ZLB condition:

$$\frac{1+i_t}{1+i}=\max\left\{\left(\frac{GDP_t}{GDP}\right)^{\phi_y}\left(\frac{1+\pi_t}{1+\pi}\right)^{\phi_\pi},\frac{1}{1+i}\right\} \tag{A.18}$$

where $GDP_t \equiv C_t+G_t$ denotes the gross domestic product (GDP) and GDP, π, and i denote the steady state GDP level, the targeted inflation rate, and the steady-state nominal interest rate, respectively.[9]

Following Fernandez-Villaverde et al. (2015) and Aruoba and Schorfheide (2013), we assume that the government runs a balanced budget and raises lump-sum taxes to finance

[8] For example see Nakata (2011) and Aruoba and Schorfheide (2013) among others. It would also be interesting to compare the time-dependent Calvo price setting to another state-dependent price setting as in Dotsey et al. (1999) and Ngo (2014a) at the ZLB.

[9] It would be interesting to compare the Calvo and Rotemberg price adjustments at the ZLB under a different monetary policy regime such as nominal GDP-level targeting in Billi (2017) or under a more realistic framework with financial frictions as in Girdenas (2018).

government spending, which is given by

$$\frac{G_t}{GDP_t} = S_g g_t,$$

where S_g denotes the steady state share of the government spending and g_t denotes the government spending shock that follows an AR(1) process

$$\ln g_t = \rho_g \ln g_{t-1} + \varepsilon_{gt},$$

where $\rho_g \in (0, 1)$ is the persistence parameter and ε_{gt} is the innovation with mean 0 and variance σ_g^2.

Equilibrium systems

In the Calvo model we define aggregate labor as $N_t = \int N_t(i)\, di$. By equations (A.7) and (A.9),

$$N_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t.$$

This implies that aggregate output is given by

$$Y_t = \frac{N_t}{\Delta_t}, \tag{A.19}$$

where Δ_t is called the relative price dispersion and is defined as

$$\Delta_t = \int \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} di. \tag{A.20}$$

Equation (A.19) shows that the relative price dispersion Δ_t acts as a negative technology shock. An increase in the relative price dispersion reduces aggregate output.

By (A.20), Δ_t satisfies the recursive equation

$$\Delta_t = \theta \Pi_t^{\varepsilon} \Delta_{t-1} + (1-\theta)\, (p_t^*)^{-\varepsilon}. \tag{A.21}$$

By (A.8),

$$p_t^* = \left(\frac{1 - \theta \Pi_t{}^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{1-\varepsilon}}. \tag{A.22}$$

Combining these two equations yields

$$\Delta_t = (1-\theta)\left(\frac{1 - \theta \Pi_t^{\varepsilon-1}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}} + \theta \Pi_t^{\varepsilon} \Delta_{t-1}. \tag{A.23}$$

The resource constraint for the Calvo model is given by

$$C_t + G_t = Y_t. \tag{A.24}$$

The equilibrium system for the Calvo model consists of a system of ten nonlinear difference equations (A.4), (A.5), (A.13), (A.14), (A.15), (A.18), (A.19), (A.22), (A.23) (A.24) for ten variables w_t, C_t, p_t^*, S_t, F_t, i_t, N_t, π_t, Δ_t, and Y_t.

In the Rotemberg model the relative price dispersion is always equal to one. Aggregate output satisfies

$$Y_t = N_t, \tag{A.25}$$

and the resource constraint is given by

$$C_t + G_t + \frac{\varphi}{2}\pi_t^2 Y_t = Y_t. \tag{A.26}$$

The equilibrium system for the Rotemberg model consists of a system of six nonlinear difference equations (A.4), (A.5), (A.17), (A.18), (A.25), (A.26) for six variables w_t, C_t, i_t, π_t, N_t, and Y_t.

B Log-linearized systems

By a standard procedure (e.g., Woodford (2003), Gali (2008), and Miao (2014)), we can derive the log-linearized approximations around the non-stochastic steady state with zero inflation. We omit the detailed derivations and present the solutions directly. The log-linearized system for the Calvo model is given by

$$\begin{aligned}
i_t &= \max\left\{0, \phi_y x_t + \phi_\pi \pi_t + \ln(1/\beta) + \phi_y \ln\left(Y_t^f\right)\right\}, \\
x_t &= E_t x_{t+1} - \frac{1}{\gamma}\left(i_t - E_t \pi_{t+1}\right) + \frac{1}{\gamma} r_t^n, \\
\pi_t &= \beta E_t \pi_{t+1} + \kappa x_t,
\end{aligned}$$

where κ is defined as

$$\kappa = \frac{(\gamma+\eta)(1-\theta)(1-\theta\beta)}{\theta},$$

Y_t^f is the flexible price equilibrium output

$$Y_t^f = \left(\frac{\varepsilon - 1}{\varepsilon\chi}\frac{1}{(1 - S_g g_t)^\gamma}\right)^{\frac{1}{\eta+\gamma}},$$

$x_t = \ln Y_t - \ln Y_t^f$ denotes the output gap, and r_t^n is the real natural rate of interest

$$r_t^n = -\ln\beta - \ln\beta_t + \frac{\gamma\eta}{\eta+\gamma}\frac{S_g}{1-S_g}\left(1-\rho_g\right)\ln g_t.$$

The log-linearized system for the Rotemberg model is given by

$$\begin{aligned} i_t &= \max\left\{0, \phi_y x_t + \phi_\pi \pi_t + \ln(1/\beta) + \phi_y \ln\left(Y_t^f\right)\right\}, \\ x_t &= E_t x_{t+1} - \frac{1}{\gamma}\left(i_t - E_t\pi_{t+1}\right) + \frac{1}{\gamma} r_t^n, \\ \pi_t &= \beta E_t \pi_{t+1} + \widetilde{\kappa} x_t, \end{aligned}$$

where $\widetilde{\kappa}$ is given by

$$\widetilde{\kappa} = \frac{(\gamma+\eta)(\varepsilon-1)}{\varphi}.$$

From the equations above, we deduce that by choosing the price adjustment cost parameter $\varphi = (\varepsilon-1)\theta/\left[(1-\theta)(1-\theta\beta)\right]$, we have $\kappa = \widetilde{\kappa}$ and the two models give the same equilibrium system to the first-order approximation. Therefore, results from the two models are the same even in the presence of the ZLB. We emphasize that this result holds true only when we approximate around the zero-inflation steady state. Ascari and Rossi (2012) show that the two models imply different results to the first-order approximation when the approximation is around a non-zero-inflation steady state.

C Calibration discussion

We calibrate the parameters using conventional values. The quarterly subjective discount factor β is set at 0.99 such that the annual real interest rate is 4%, as in Woodford (2003), Gali (2008) and Christiano et al. (2011). The constant relative risk aversion parameter γ is 1, corresponding to a log utility function with respect to consumption. This utility function is commonly used in the business cycles literature. The labor supply elasticity with respect to wages is set at 1, or $\eta = 1$, as in Christiano et al. (2011). The value of χ is calibrated to obtain the steady state faction of working hours of 1/3. The elasticity of substitution among differentiated intermediate goods ϵ is 7.66, corresponding to a 15% net markup. This value is popular in the literature, for example Adam and Billi (2007).

The probability of keeping prices unchanged in the Calvo model is calibrated to be $\theta = 0.75$, resulting in the average duration of four quarters with prices being kept unchanged. The price stickiness parameter is in line with the empirical evidence reported by Nakamura and Steinsson (2008). We then set the price adjustment cost parameter in the Rotemberg model $\varphi = 78$ to ensure that the two models are equivalent to the first-order approximation around the steady state with zero inflation. This value also implies that the average duration of keeping prices unchanged is four quarters, as in the Calvo model.

The inflation target is set at zero. We choose this value to make sure that, together with $\theta = 0.75$ and $\varphi = 78$, the Calvo model and the Rotemberg model are equivalent to the first-order approximation. The main purpose of the paper is to compare the two models using a global nonlinear method. Hence, it is crucial to make sure that they are the same to the first-order approximation. We set the parameters in the Taylor rule $\phi_\pi = 1.5$ and $\phi_y = 0.25$, as in Fernandez-Villaverde et al. (2015), which are in the range of the empirical studies. The share of the government spending in output is $S_g = 0.20$, as in Christiano et al. (2011).

We set the persistence of the preference shock and the government spending shock $\rho_\beta = \rho_g = 0.8$, as in Adam and Billi (2007) and Fernandez-Villaverde et al. (2015). Following Fernandez-Villaverde et al. (2012) we set the standard deviation of the innovation of government spending shocks $\sigma_g = 0.25\%$. The most important parameter left to calibrate is the standard deviation of the innovation of preference shocks, σ_β. We decide to set $\sigma_\beta = 0.19\%$ so that the unconditional probability of hitting the ZLB is 5.5% in the Calvo model, which is consistent with that of Fernandez-Villaverde et al. (2015) and the empirical literature before the Great Recession.

D Solution method

Our solution method is close to, but slightly different from, the one used in Fernandez-Villaverde et al. (2015). Similar to their method, we do not approximate the policy function for the nominal interest rate. Instead, the nominal interest rate is always determined by equation (A.18) at every state, in or out of the set of collocation nodes. However, different from them, we approximate the expectations as function of state using a finite element method called the cubic spline interpolation; see Judd (1998) and Miranda and Fackler (2002) for more details. The main advantage of this approach is we do not have to worry about the kink when the ZLB starts binding.

This appendix shows the solution method used to solve the Calvo model. The Rotemberg model can be solved in the same way. Following Miranda and Fackler (2002), we rewrite the functional equations governing the equilibrium in the Calvo model in a more compact form:

$$f\left(s, X\left(s\right), E\left[Z(X\left(s'\right))\right]\right) = 0. \tag{D.1}$$

where

- $f : R^{3+7+3} \to R^7$ is the equilibrium relationship;
- $s = (\Delta, \beta, g)$ is the current state of the economy;
- $X(s) = (R(s), C(s), N(s), S(s), F(s), \Pi(s), Y(s))'$, and $X : R^3 \to R^8$ is the policy function;
- s' is the next period's state that evolves according to the following motion equation:

  $$s^{'} = g(s, X(s), \varepsilon) = \begin{bmatrix} \Delta' = (1-\theta)\left(\frac{1-\theta\Pi(s)^{\varepsilon-1}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}} + \theta\Pi\left(s\right)^{\varepsilon}\Delta \\ \beta' = \beta^{\rho_\beta}\exp(\varepsilon_\beta) \\ g' = g^{\rho_g}\exp(\varepsilon_g) \end{bmatrix},$$

  where ε_β and ε_g are the innovations of the preference and the government spending shocks;
- $Z(X\left(s'\right)) = \begin{pmatrix} Z_1\left(X\left(s'\right)\right) = \frac{C(s')^{-\gamma}}{\Pi(s')} \\ Z_2\left(X\left(s'\right)\right) = \Pi\left(s'\right)^{\varepsilon-1}F\left(s'\right) \\ Z_3\left(X\left(s'\right)\right) = \Pi\left(s'\right)^{\varepsilon}S\left(s'\right) \end{pmatrix}.$

Instead of solving policy function, we actually solve the expectations as functions of state using a finite element method called the cubic spline interpolation. Define $h(s) = E\left[Z(X\left(s'\right))|s\right]$, below is the simplified algorithm:

- *Step 1: Define the space of the approximating functions and collocation nodes* $S = (S_1, ..., S_N)$, where $N = N_\Delta \times N_\beta \times N_g$, and N_Δ, N_β, N_g are the numbers of grid points along each dimension of the state space. In this paper, we approximate the expectations:

  $h(s) = (\phi(s)\theta_{h_1}, \phi(s)\theta_{h_2}, \phi(s)\theta_{h_3})'$ or

  $h(s) = \phi(s)\Theta,$

  where

– $\phi(s)$ is a $1 \times N$ matrix of cubic spline basis functions evaluated at state $s \in S = (S_1, ..., S_N)$.

– $\Theta = (\theta_{h_1}; \theta_{h_2}; \theta_{h_3})$ is a $N \times 3$ coefficient matrix that we want to approximate.

- *Step 2: Initialize the coefficient matrix Θ^0 and set up stopping rules.*

- *Step 3: At each iteration j given the corresponding Θ^j, we implement the following sub-steps:*

  1. At each collocation node s_i, $s_i \in \{S_1..S_N\}$, compute $h(s_i)$ using the approximating functions for the expectations.
  2. Solve for $X(s_i)$ such that $f(s_i, X(s_i), h(s_i)) = 0$. We solve this complementarity problem using the Newton method.

- *Step 4: Update h using the following sub-steps:*

  1. Approximate policy functions for C, Π, F, S, Δ using the cubic spline interpolation.
  2. At each collocation node s_i, $s_i \in \{S_1..S_N\}$, update $h(s_i) = (h_1(s_i), h_2(s_i), h_3(s_i))$ using

$$h_1(s_i) = \sum_j w_j \left[\frac{C(s')^{-\gamma}}{\Pi(s')} \right] \tag{D.2}$$

$$h_2(s_i) = \sum_j w_j \left[\Pi(s')^{\varepsilon-1} F(s') \right] \tag{D.3}$$

$$h_3(s_i) = \sum_j w_j \left[Pi(s')^{\varepsilon} S(s') \right] \tag{D.4}$$

  where the innovations for the preference and government spending shocks are discretized using the Tauchen and Hussey (1991) method with 25 nodes.

- *Step 5: Update $\Theta^{j+1} = \Phi^{-1}\Theta^j$, where $\Phi = (\phi(s_1), ..., \phi(s_N))'$.*[10]

- *Step 6: Check the stopping rules. If not satisfied go to Step 3; otherwise go to Step 7.*

[10]We also keep track of the convergence of the policy functions for C, Π, S, F, Δ. They always converge to the fixed point much faster than the expectations functions do. Note that the expectations functions are quite smooth by nature, and so do the policy functions for C, Π, S, F, Δ.

- *Step 7: Report results.* We use the approximated expectation functions to solve for the equilibrium value at any state. So, we are able to find almost exactly the kink for the nominal interest rate.

In addition, we write our code using a parallel computing method that allows us to split up a large number of collocation nodes into smaller groups assigned to different processors to be solved simultaneously. This procedure reduces computation time significantly. We obtain the maximal absolute residual across the equilibrium conditions of the order of 10^{-8} for almost all states off the collocation nodes. For a few states when the ZLB becomes binding, the maximal absolute residual is of the order of 10^{-5}. This is quite standard given the kink in the interest rate policy function; see Miranda and Fackler (2002) and Judd et al. (2011) for more information.