



Introduction to economic theory of bubbles[☆]



Jianjun Miao^{*}

Department of Economics, Boston University, 270 Bay State Road, Boston, MA 02215, USA
CEMA, Central University of Finance and Economics, China
AFR, Zhejiang University, China

ARTICLE INFO

Article history:

Received 28 May 2014

Accepted 3 June 2014

Available online 9 June 2014

Keywords:

Rational asset bubbles

Borrowing constraints

Arbitrage

Incomplete markets

Dynamic efficiency

Economic growth

ABSTRACT

This is an introduction to the special section on the economic theory of bubbles.

© 2014 Elsevier B.V. All rights reserved.

Asset markets around the world are very volatile. Two most important asset markets are the stock market and the housing market. Fig. 1 presents the US and the Japanese real stock market indices (S&P 500 and Nikkei 225).¹ This figure shows that the US stock market experienced a persistent boom from 1990 through 2000 and price indices more than doubled. The market went down by about a half from the peak of December 1999 to early 2003. It then came back reaching a peak in January 2007, followed by a crash to the bottom in March 2009. Between this short period, the stock market lost more than 50%. Since then the stock market gradually recovered. The Japan's stock market experienced a persistent boom until December 1989, rose by about 500% from early 1970. After that the stock market crashed and never came back with prices below a half of the peak.

Fig. 2 shows the US and the Japanese real housing prices, price–income ratios, and price–rent ratios.² This figure shows that the US

housing market experienced a persistent boom from early 1990s to December 2006 and increased by about 60% from the bottom to the peak in December 2006. It then dropped until September 2011 by about 40% from the peak. The Japan's housing market experienced a persistent boom from January 1980 until the peak of February 1991 and increased by about 60%. It then dropped continually until October 2011, lost more than 50%. The price–income ratios and the price–rent ratios tracked the housing prices closely both in the US and Japan, indicating that the fundamental factors alone cannot explain the housing price dynamics.

The boom and bust of the stock and housing markets have a large impact on the real economy. For example, it is widely believed that the recent Great Recession in the United States and the Japan's lost decade were caused by the crash of the housing market. Thus understanding the high asset market volatility is important for both economists and policymakers. It turns out that this is a challenging task (Shiller, 1981). While there are many studies based on new classical theory, an important line of research is based on the idea that asset prices contain a bubble component in addition to the fundamental component. The growth of the bubble can help explain the asset market boom and the collapse of the bubble can help explain the asset market crash.

deflator. The average real index in 2000 is normalized to 100. The price–income ratio is the ratio of the nominal house price index to the nominal per capita disposable income. The sample average is normalized to 100. The price–rental ratio is the ratio of the nominal house price index to the rent component of the consumer price index. The sample average is normalized to 100. All data are downloaded from [http://www.econ.queensu.ca/files/other/House_Price_indices%20\(OECD\).xls](http://www.econ.queensu.ca/files/other/House_Price_indices%20(OECD).xls). All series are quarterly and seasonally adjusted.

[☆] I would like to thank Atsushi Kajii for useful comments.

^{*} Correspondence to: Department of Economics, Boston University, 270 Bay State Road, Boston, MA 02215, USA. Tel.: +1 617 353 6675; fax: +1 617 353 4449.

E-mail addresses: miao@bu.edu, jianjunmiao9@gmail.com.

¹ The monthly real S&P 500 stock price index is downloaded from Robert Shiller's website: <http://www.econ.yale.edu/shiller/data.htm>. The monthly Nikkei 225 index is downloaded from Bloomberg. The real indices are deflated by the CPI data download from the OECD Main Economic Indicator Database.

² The US nominal house price index is the all-transaction index from Federal Housing Finance Agency. The Japan's nominal house price index is the nationwide urban land price index from the Japan Real Estate Institute. The real house price index is the nominal house price index deflated by the private consumption

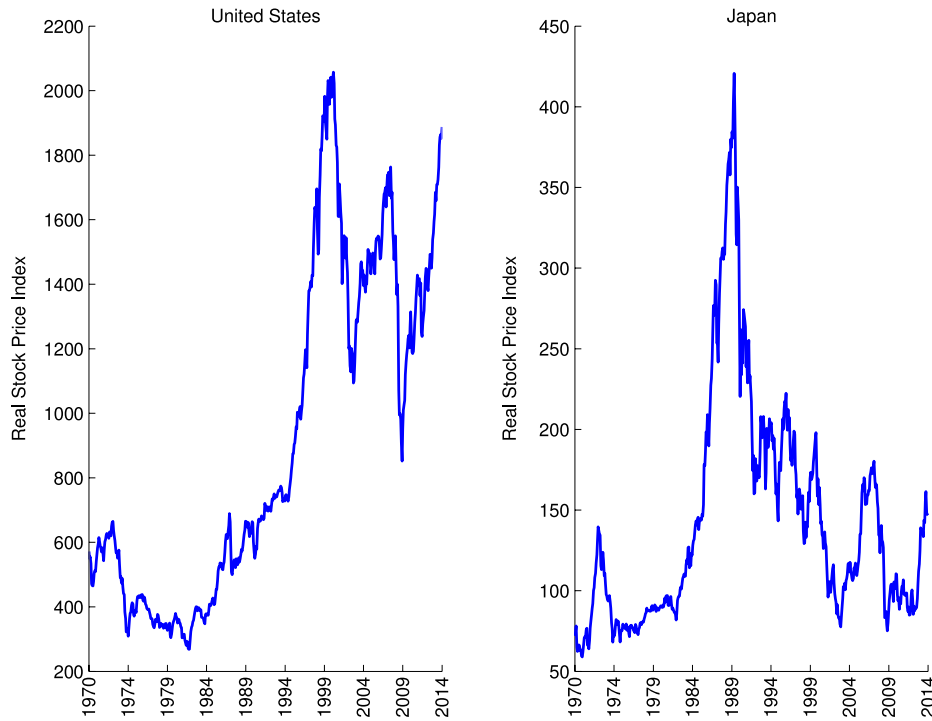


Fig. 1. The monthly real S&P 500 indices and the Japan's Nikkie 225 indices.

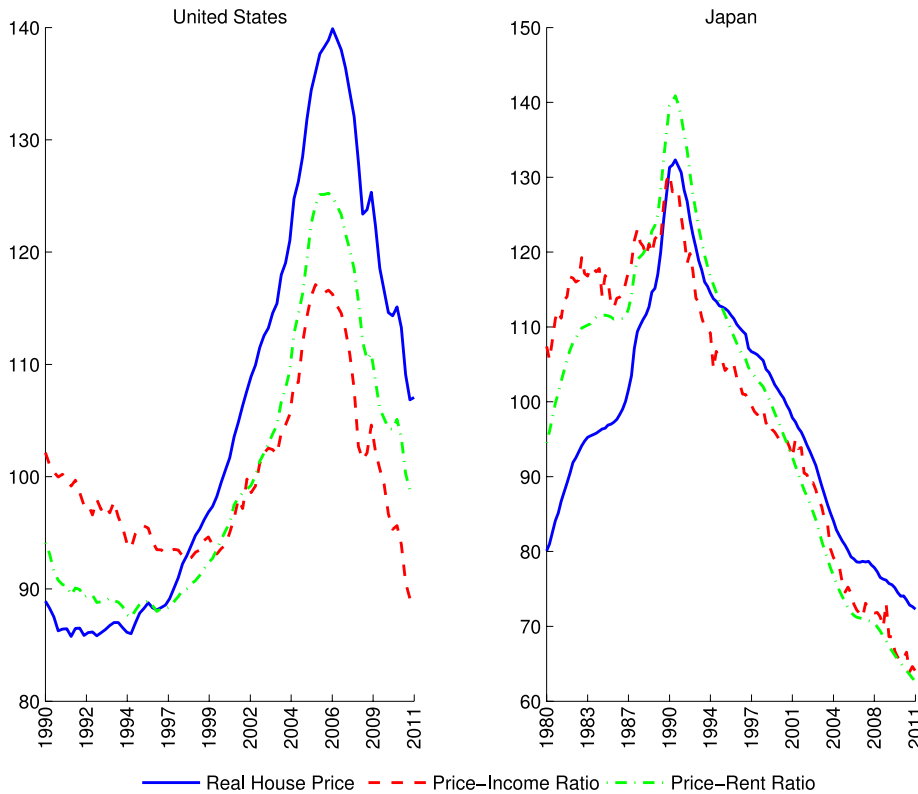


Fig. 2. The real house prices, price-income ratios, and price-rent ratios for the US and Japan.

This simple idea is intuitive and often talked about by the general public. It is also often mentioned by policymakers such as Ben Bernanke and Alan Greenspan.³ However, it has not entered

the mainstream economics. Based on my conversations with many economists, most of them resist to accept the idea of asset bubbles. I think there might be two reasons. First, the current core of macroeconomics is the dynamic stochastic general equilibrium approach and the idea of asset bubbles belongs to the periphery (Caballero, 2010). Research on asset bubbles is hard to get published, especially in top journals. This discourages researchers from

³ Bernanke wrote a research article with Mark Gertler on bubbles (Bernanke and Gertler, 1999).

working on this topic. Second, we do not know much about asset bubbles in economic theory. This topic is typically not taught in macroeconomics or microeconomics. There are many misunderstandings of some conceptual and theoretical issues. For example, some people often argue that the existence of bubbles implies that agents are irrational or there are arbitrage opportunities. There does exist a strand of literature on irrational bubbles (see, e.g., Shiller, 2005).⁴ However, I will confine my discussions on models of rational bubbles. By “rational” I mean economic agents have rational expectations and maximize their utility. Moreover, markets are competitive and clear in equilibrium.

The purpose of this special section is to promote theoretical research on rational asset bubbles. This section contains four papers that push the research frontier on this topic forward by studying infinite-horizon models. Before I discuss each of them, let me provide some background knowledge and a short review of the recent literature.

Let me start with a standard two-period asset valuation equation. Suppose that there is no uncertainty and the asset's gross required return is equal to a constant R . Let $\{D_t\}$ denote the stream of bounded asset payoffs. Then the (ex-payoffs) asset price P_t satisfies

$$P_t = \frac{D_{t+1} + P_{t+1}}{R}. \quad (1)$$

This two-period valuation equation can be derived from a no-arbitrage argument or intertemporal optimization. If the asset has a finite maturity date T , then $P_t = 0$ for $t \geq T$ since the asset will not be traded after date T . We can compute the asset price as

$$P_t = \sum_{j=1}^{T-t} \frac{D_{t+j}}{R^j}, \quad 0 \leq t < T.$$

In this case there is no asset bubble and the asset price is equal to the fundamental value, i.e., the discounted present value of future payoffs. If the asset has infinite maturity, the story is different. As $T \rightarrow \infty$, the fundamental value

$$P_t = f_t \equiv \sum_{j=1}^{\infty} \frac{D_{t+j}}{R^j}$$

is a solution to Eq. (1). Moreover, one can verify that the following expression is also a solution to (1):

$$P_t = f_t + B_t,$$

where $\{B_t\}$ satisfies

$$B_t = \frac{B_{t+1}}{R}. \quad (2)$$

If the asset can be freely disposed, then $B_t \geq 0$. The term $B_t > 0$ is called a rational bubble. In the deterministic setup the growth rate of the bubble is equal to the required return on the asset which is also equal to the interest rate.

Solving Eq. (1) forward yields

$$P_t = f_t + \lim_{T \rightarrow \infty} \frac{P_{t+T}}{R^T}.$$

This implies that

$$B_t = \lim_{T \rightarrow \infty} \frac{P_{t+T}}{R^T} = \lim_{T \rightarrow \infty} \frac{B_{t+T}}{R^T}.$$

If we assume that

$$\lim_{T \rightarrow \infty} \frac{P_{t+T}}{R^T} = 0 \quad \text{for all } t, \quad (3)$$

then $B_t = 0$ for all t . Eq. (3) is called a transversality condition. This condition is typically satisfied in a competitive equilibrium with infinitely lived agents.⁵ This is why bubbles can often be ruled out in standard models with infinitely lived agents. Intuitively bubbles represent an arbitrage opportunity for an infinitely lived agent; he can gain by permanently reducing his holdings of the bubble asset or by engaging in Ponzi schemes. Such an arbitrage opportunity cannot exist in a standard competitive equilibrium without market frictions.

The transversality condition is not needed in overlapping-generations (OLG) models with finitely lived agents. Thus a bubble can be easily generated in the OLG framework (Samuelson, 1958; Diamond, 1965; Tirole, 1985; Weil, 1987). A canonical example of a pure bubble is fiat money, which is an intrinsically useless asset. The OLG framework is often used as a foundation for the existence of fiat money and becomes the dominating framework to study bubbles.

Under what conditions can a bubble exist? To address this question, Santos and Woodford (1997) provide a general model of pure exchange economies. This model can incorporate incomplete markets with both infinitely lived and finitely lived agents. Santos and Woodford establish general conditions under which an asset bubble cannot exist. Their conditions can be informally described as follows: (1) Each agent is subject to borrowing constraints such that he cannot borrow more than the present value of his future endowments.⁶ (2) The present value of aggregate endowments is finite. (3) The asset is either of finite maturity or in positive net supply. If any of these conditions is violated, an asset bubble may exist.

We use a simple two-period OLG model of an endowment economy to illustrate the Santos–Woodford result. Suppose that there is no population growth and the only traded asset is fiat money. A young agent at generation t has utility $u(c_t^t) + \beta u(c_{t+1}^t)$, where $\beta \in (0, 1)$ and u satisfies the usual assumption. The endowment of a young agent and an old agent is given by w_1 and w_2 respectively. Clearly, autarky is an equilibrium in which fiat money has no value (the bubbleless equilibrium). When a bubble exists, Eq. (2) implies that $R = 1$ in the steady state. Thus the present value of aggregate endowments discounted by $R = 1$ is infinite, meaning that the existence of a bubble is associated with a low interest rate. In a steady state with bubble $B > 0$, the following Euler equation must hold:

$$u'(w_1 - B) = \beta u'(w_2 + B).$$

For a solution $B > 0$ to exist, we must have $u'(w_1) / [\beta u'(w_2)] < 1$. This means that the implicit interest rate in the bubbleless equilibrium is less than the rate of economic growth (which is zero), or the bubbleless equilibrium is dynamically inefficient (Gale, 1973). The presence of a bubble can solve the problem of asset shortage and achieve dynamic efficiency. The bubble plays the role of a store of value.

The result above can be carried over to production economies and economies with growth (Tirole, 1985). In particular, bubbles can occur only if the bubbleless economy is dynamically inefficient, i.e., the interest rate in the bubbleless steady state is below the rate of economic growth. Moreover, in a bubbly equilibrium, the growth rate of bubbles is equal to the growth rate of the economy, which is equal to the interest rate. Since Abel et al. (1989) do not find empirical support for dynamic inefficiency, people often argue that bubbles cannot occur in reality. In response, Farhi and Tirole (2012) and Martin and Ventura (2012) introduce financial frictions to the Tirole model and show that bubbles can exist

⁵ It is related to the transversality condition in infinite-horizon intertemporal optimization problem (e.g., Weitzman, 1973; Ekeland and Scheinkman, 1986).

⁶ These borrowing constraints will not bind with a positive probability at any date.

⁴ See Brunnermeier (2009) and Brunnermeier and Oehmke (2013) for surveys of models of rational and irrational bubbles.

even though the bubbleless economy is dynamically efficient. They show that low interest rates and dynamic efficiency are compatible in the presence of capital market imperfections.

Bubbles can crash. Based on Blanchard and Watson (1982), Weil (1987) introduce stochastic bubbles to OLG models. Martin and Ventura (2012) generalize Weil's (1987) approach by introducing investor sentiment shocks that can generate rich bubble dynamics. Other important contributions in the OLG framework include Caballero et al. (2006), Caballero and Krishnamurthy (2006), Arce and Lopez-Salido (2011), and Gali (2014).

While we have learned a lot of insights from the literature on OLG models of bubbles and significant progress has been made in this literature, infinite-horizon models with bubbles are underexplored. Given that many insights are available in OLG models, why do we need to study infinite-horizon models of bubbles, which seem to be more complicated? There are two reasons. First, there are some insights, specifically tied to infinitely lived agents, that are not available in OLG models. Exploring such insights can deliver useful new results. Second, the existing OLG models are very stylized in the sense that agents are typically assumed to live for two or three periods.⁷ It is impossible to interpret the period in the model in terms of the calendar time. Thus it is impossible to confront the model with the data. While developing intuition and insights is useful for understanding the economic phenomena, building models that have higher potential to be quantified is more important and more fruitful. Especially the large fluctuation in the asset prices is a quantitative observation used for the motivation of the theory of bubbles. The theory will be vacuous if it has no potential to explain the data.

Kocherlakota (1992, 2008) are two important seminal papers that study bubbles in infinite-horizon models.⁸ His idea is that one can introduce some sorts of portfolio constraints to limit the agents' arbitrage opportunities so that bubbles cannot be eliminated. Kocherlakota (1992) provides examples in which asset bubbles can exist in the presence of occasionally binding short sales constraints. Santos and Woodford (1997) provide an example based on Bewley (1980) in which a pure bubble exists when the borrowing constraints are occasionally binding.

Kocherlakota (2008) studies two complete markets economies with debt constraints. In the first economy the debt constraints are exogenously given. He shows that any discounted (by the pricing kernel) positive martingales can be introduced into the asset prices as bubbles, while leaving agents' consumption and the pricing kernel unchanged, as long as the debt limits are tightened by their initial endowment of the assets multiplied by the bubble term. Kocherlakota calls this result the bubble equivalence theorem. The intuition is simple. The introduction of a bubble gives agents a windfall, proportional to their initial holdings of the asset, which can be sterilized, leaving their budgets unaffected, by suitably tightening the debt limits. Kocherlakota (2008) also considers a second economy with endogenous debt constraints (Kehoe and Levine, 1993; Alvarez and Jermann, 2000). In this economy agents are punished by forcing into autarky if they choose to default on debt. There exist endogenous debt limits such that they are not too tight in the sense that agents will repay the debt and the continuation value of not defaulting is equal to the continuation value of defaulting. Kocherlakota shows that the equivalence theorem also holds for the second economy.

The four papers in this special section all work with infinite-horizon models. I now discuss them in turn. Werner (forthcoming) and Bejan and Bidian (forthcoming) are closely related and both

study endowment economies. Unlike Santos and Woodford (1997) who analyze the case of borrowing constraints, Werner (forthcoming) focuses on debt constraints with possibly endogenous debt bounds. He first establishes conditions such that asset bubbles cannot exist when agents face debt constraints. In addition to a uniform impatience condition on preferences, the other conditions are the same as conditions (2) and (3) for the Santos and Woodford result discussed above. The uniform impatience condition is typically satisfied by many standard utility functions such as the time-additive discounted expected utility with strictly increasing and continuous period utility functions. Thus for the existence of a bubble, either condition (2) or condition (3) must be violated.

Intuitively, for condition (2) to be violated, the interest rate must be low. A low interest rate can often occur in models with endogenous debt constraints as mentioned above. Both Bejan and Bidian (forthcoming) and Werner (forthcoming) generalize Kocherlakota's (2008) bubble equivalence theorem by allowing for incomplete financial markets and endogenous debt constraints with more general default penalties. In particular, following Hellwig and Lorenzoni (2009),⁹ they consider the penalty that allows the agents to save but not to borrow in the future. This generalization is important because one might infer from Kocherlakota (2008) that bubble injections are associated with knife-edge situations or an extreme punishment, and they might not apply to incomplete markets environments or even to economies with dynamically complete markets (rather than Arrow–Debreu complete). Bejan and Bidian (forthcoming) and Werner (forthcoming) show that bubbles can still be injected in a large class of economies. Unlike the complete markets model of Kocherlakota (2008), this result is not trivial because the pricing kernel is not unique under incomplete markets and this kernel may change once a bubble is injected.

Bejan and Bidian (forthcoming) show that if and only if a positive process can preserve the set of pricing kernels, then this process can be injected as bubbles without changing the equilibrium consumption allocation. They call such a process a kernel-preserving process and provide several equivalent characterizations of this process. They show that a kernel-preserving process can be injected as asset price bubbles in economies with both exogenous and endogenous debt constraints. Werner (forthcoming) shows that the most intuitive characterization of a kernel-preserving process by Bejan and Bidian (forthcoming) is that this process is a positive asset-span preserving discounted martingale. The property of a discounted martingale rules out arbitrage. The asset-span preserving property ensures that the set of pricing kernel will not change if a bubble is injected. Werner (forthcoming) provides a simple proof of the bubble equivalence theorem for the models with endogenous debt constraints. He also shows that bubbles can always exist in models with endogenous debt constraints if the asset is in zero net supply.

While models of endowment economies are useful for understanding the asset pricing phenomena, models of production economies are needed for understanding the long-run growth and the business cycles. Thus it is necessary to develop such models to understand the impact of bubbles on the macroeconomy, e.g., output, investment, and consumption. There has been a growing literature on infinite-horizon models of production economies with bubbles (e.g., Kiyotaki and Moore, 2008; Kocherlakota, 2009; Hirano and Yanagawa, 2011; Miao and Wang, 2011, 2012a,b, forthcoming; Wang and Wen, 2012; Miao et al., 2012, 2013, 2014). All these models, as well as Miao and Wang (forthcoming) in this special section, incorporate financial frictions in various forms of

⁷ While it is possible to build OLG models with realistic many-period lived agents, this will quickly complicate model analysis both analytically and numerically.

⁸ See Scheinkman and Weiss (1986) and Woodford (1990) for early contributions.

⁹ One may interpret the main result of Hellwig and Lorenzoni (2009) as a bubble equivalence theorem as in Kocherlakota (2008), but with a different default penalty.

borrowing or credit constraints. In these models bubbles can play other important roles in addition to a store of value. An important idea, shared in Miao and Wang (forthcoming) in this special section, is that bubbles can help relax credit constraints. However, understanding precisely how a bubble relaxes credit constraints is subtle as there is some confusion in the literature.

In order to appreciate this line of research properly, I take this opportunity to review the important elements starting with an example based on Kiyotaki and Moore (2008), Farhi and Tirole (2012), and Hirano and Yanagawa (2011). Suppose that an entrepreneur has an investment technology that produces one unit of output using one unit of investment. He can finance investment I_t by his endowment w_t , one-period debt b_t , and a bubble asset B_t so that $I_t = b_t + B_t + w_t$. Borrowing is subject to a credit constraint

$$(1 + r_{t+1}) b_t \leq \lambda I_t, \quad \lambda \in (0, 1), \quad (4)$$

where r_{t+1} is the interest rate from t to $t + 1$. This constraint says that the debt repayment in period $t + 1$ is limited by a fraction λ of the investment return I_t because the entrepreneur can only pledge this amount as collateral.¹⁰ In this case the investment satisfies

$$I_t = \frac{B_t + w_t}{1 - \lambda / (1 + r_{t+1})}.$$

This equation shows that the presence of a bubble $B_t > 0$ essentially raises the entrepreneur's net worth and hence investment. Thus bubbles can crowd in investment, rather than crowd out investment as in Diamond (1965) and Tirole (1985). But the role of bubbles in this type of borrowing constraints is still the store of value.

Kocherlakota (2009), Miao and Wang (2011), and Miao et al. (2014) introduce other types of borrowing constraints. Kocherlakota (2009) assumes that land is intrinsically useless, but used as collateral for borrowing so that the borrowing constraint is given by

$$(1 + r_{t+1}) b_t \leq P_{t+1} L_t, \quad (5)$$

where P_{t+1} is the land price in period $t + 1$ and L_t represents the land holdings chosen in period t . This borrowing constraint essentially follows from Kiyotaki and Moore (1997). Miao et al. (2014) consider a down-payment constraint

$$b_t \leq \theta P_t L_t, \quad \theta \in (0, 1) \quad (6)$$

where $P_t L_t$ represents the date- t purchase value of the land and the fraction $1 - \theta$ of the purchase value must be paid by the entrepreneur's net worth. The preceding borrowing constraints in (5) and (6) imply that when a land bubble does not exist (i.e., $P_t = 0$ for all t), there is no collateral for borrowing. The presence of a land bubble relaxes the borrowing constraints. Thus the land bubble can help solve the collateral shortage problem. The movements of the land bubble affect the borrowing capacity directly and hence investments. This role of bubbles is different from that discussed above.

Miao and Wang (2011) study stock market bubbles and suppose that the entrepreneur can pledge his reorganized firm value as collateral and the borrowing constraint becomes

$$b_t \leq V_t (\xi K_t), \quad \xi \in (0, 1), \quad (7)$$

where K_t is the capital stock, $1 - \xi$ represents the efficiency loss in case of default, and $V_t(\cdot)$ is the stock market value of the firm. They also consider several other types of borrowing constraints motivated from the optimal contracts with limited commitment (Alvarez and Jermann, 2000) and Albuquerque and Hopenhayn

(2004). They show that a bubble attached to firm value can exist under some conditions when the value function enters the borrowing constraint. In this case firm value takes the following form

$$V_t(K_t) = Q_t K_t + B_t, \quad (8)$$

where Q_t represents Tobin's marginal Q . The equation above implies that firm value consists of two components: the standard component $Q_t K_t$ as in Hayashi (1982) and a bubble component B_t .

More importantly, due to the borrowing constraints (4)–(7), the asset pricing equation for the bubble may be different from the standard equation in (2). In particular, Miao and his coauthors show that bubbles command a collateral yield or liquidity premium for (5)–(7). Formally, Miao and Wang (2011) show in a deterministic continuous-time model that the bubble attached to firm value satisfies

$$\dot{B}_t + \pi (Q_t - 1) B_t = r_t B_t, \quad (9)$$

where π represents the Poisson arrival rate of an investment opportunity. The collateral yield $\pi (Q_t - 1) B_t$ comes from the following intuition. Suppose that people believe that firm value contains a bubble. Then one dollar of bubble relaxes the borrowing constraint by one dollar by (7) and (8). This allows the firm to borrow one more dollar and invest one more dollar, thereby raising firm value by Q_t dollars. Subtracting one dollar investment cost and multiplying the arrival rate π , we obtain the extra profits generated by the bubble, i.e., collateral yield. The increased firm value justifies the initial belief about a bubble. The existence of a bubble is self-fulfilling. If no one believes in bubbles, then the above positive feedback loop mechanism will not work and no bubble can exist in equilibrium.

Unlike OLG models and many infinite-horizon models discussed above, the bubble component in firm value is not a directly traded asset. What is traded is the firm's stocks. One criticism of the literature on bubbles is that most papers discussed above assume that the traded bubble asset is intrinsically useless and hence a reinterpretation of money. But houses (land) and stocks in reality are not the same as money. This limits the application of the standard models of bubbles to quantitative explanations of the data. A notable feature of houses and stocks is that rents and dividends are endogenous in production economies and may be affected by bubbles. This feature is captured in the model of Miao and Wang (2011, forthcoming).

An important implication of (9) is that the growth rate of the bubble is less than the interest rate. This implies that the interest rate is higher than the rate of economic growth. This result is different from the conventional view about the existence condition for bubbles (Santos and Woodford, 1997) discussed above.

Miao and Wang (forthcoming) apply the borrowing constraints (7) as in Miao and Wang (2011) to study a two-sector model of endogenous growth. In the model of Miao and Wang (forthcoming), there are two types of capital goods produced separately in two sectors. One type of capital goods (produced in sector 1) has a positive externality to the productivity in producing aggregate output. This provides the engine of endogenous growth. The firms in both sectors face endogenous borrowing constraints as in (7). In this case firm bubbles can exist. There are several possibilities depending on self-fulfilling beliefs. First, bubbles can exist only in sector 1, but not in sector 2. Second, bubbles can exist only in sector 2, but not in sector 1. Finally, bubbles can exist in both the sectors. Bubbles have two effects. First, they help relax credit constraints and encourage investment. This is the credit easing effect. Second, bubbles have a capital reallocation effect. If bubbles occur in sector 1, then they enhance growth. But if bubbles occur in sector 2, then they retard growth because bubbles attract too many resources to the sector that does not help growth.

¹⁰ One may use capital (Kiyotaki and Moore, 1997) or output (Hirano and Yanagawa, 2011) as pledgeable collateral.

The impact of bubbles on growth has been studied in the OLG models with endogenous growth. As pointed out by Grossman and Yanagawa (1993), King and Ferguson (1993), and Saint-Paul (1992), bubbles have negative effects in dynamic models with externalities. In these models, too little capital is accumulated rather than too much, as in Tirole (1985). Thus, as bubbles crowd out investment, they lower growth and welfare. Using an R&D based model of endogenous growth (Romer, 1990), Olivier (2000) shows that when bubbles appear in the R&D firms, bubbles enhance growth. But when bubbles exist in nonproductive assets, bubbles retard growth. This result is similar to that in Miao and Wang (forthcoming). There is an important difference. Unlike in the model of Miao and Wang (forthcoming), in Olivier's OLG model (or any OLG model), the growth rate of the economy must be higher than the interest rate in the bubbleless equilibrium for a bubble to exist. In a deterministic model the interest rate is also equal to the capital return (or the marginal product of capital). Simple calculations show that the growth rate of the economy is typically lower than the capital return for many countries (Abel et al., 1989).¹¹ The model of Miao and Wang (forthcoming) does not suffer from this critique. As in the new classical growth model, the following consumption Euler equation holds in Miao and Wang (forthcoming):

$$\frac{\dot{C}_t}{C_t} = r_t - \rho, \quad (10)$$

where C_t represents aggregate consumption for the representative agent and ρ is the subjective discount rate. This equation shows that the rate of economic growth is lower than the interest rate. This is a robust prediction in the new classical growth framework. Eqs. (9) and (10) imply that when $\pi(Q_t - 1) = \rho$, a bubble can exist such that the growth rate of the bubble is equal to the growth rate of the economy.

Aoki et al. (forthcoming) also study an endogenous growth model. In their model each agent produces output using an AK technology subject to idiosyncratic productivity shocks. There is no asset for trading to insure idiosyncratic shock except for a safe bubble asset. The safe bubble asset is intrinsically useless, but can have a positive value in equilibrium because it can help insure against idiosyncratic shocks. This idea dates back to Bewley (1980) for the existence of value for fiat money. Unlike Bewley (1980), Aoki et al. (forthcoming) focus on the implications of bubbles for long-run growth in production economies. They also analyze the welfare implications of bubbles. They show that bubbles retard growth. The intuition is that bubbles crowd out investments. A surprising result is that bubbles improve welfare, unlike in the OLG models of endogenous growth discussed above. The intuition is that the safe bubble asset reduces consumption volatility and this positive effect dominates the former negative growth effect.

There is no credit constraint in the model of Aoki et al. (forthcoming). Such a constraint is a sufficient element to generate bubbles, but not necessary. Aoki et al. (forthcoming) show that a necessary and sufficient condition for the existence of a bubble is that the growth rate of the economy is higher than the interest rate in the bubbleless equilibrium, as in the OLG models discussed above. The low interest rate is generated by the uninsured idiosyncratic risk rather than credit constraints.

In summary, the four papers in this special section have contributed to the literature in various dimensions. They contain some insights that are absent from the OLG models. Many issues still

need further research. Some issues are related to those in monetary economics. For example, can bubbles and other types of assets coexist? What are the welfare effects of bubbles? What are the policy implications of bubbles? What are the quantitative effects of bubbles on business cycles and long-run growth? While OLG models can provide us some insights into these questions, I believe infinite-horizon models of production economies in the new classical growth framework are more promising. I hope this special section will stimulate more researchers to work in this exciting and underexplored area.

References

- Abel, Andrew B., Gregory Mankiw, N., Summers, Lawrence H., Zeckhauser, Richard J., 1989. Assessing dynamic efficiency: theory and evidence. *Rev. Econom. Stud.* 56, 1–19.
- Albuquerque, Rui, Hopenhayn, Hugo A., 2004. Optimal lending contracts and firm dynamics. *Rev. Econom. Stud.* 71, 285–315.
- Alvarez, Fernando, Jermann, Urban J., 2000. Efficiency, equilibrium, and asset pricing with risk of default. *Econometrica* 68, 775–798.
- Aoki, Kosuke, Nakajima, Tomoyuki, Nikolov, Kalin, 2014. Safe asset shortages and asset price bubbles. *J. Math. Econom.* (forthcoming).
- Arce, Oscar, Lopez-Salido, David, 2011. Housing bubbles. *Amer. Econ. J.: Macroeconomics* 3, 212–241.
- Bejan, Camelia, Bidian, Florin, 2014. Bubbles and trading in incomplete markets. *J. Math. Econom.* (forthcoming).
- Bernanke, Ben, Gertler, Mark, 1999. Monetary policy and asset price volatility. In: *Proceedings – Economic Policy Symposium – Jackson Hole*. Federal Reserve Bank of Kansas City, pp. 77–128.
- Bewley, Truman, 1980. The optimum quantity of money. In: Kareken, J.H., Wallace, N. (Eds.), *Models of Monetary Economies*. Minneapolis Federal Reserve Bank of Minneapolis, pp. 169–210.
- Blanchard, Camelia, Watson, Mark, 1982. Bubbles, rational expectations and financial markets, Harvard Institute of Economic Research Working Paper No. 945.
- Brunnermeier, Markus, 2009. In: Durlauf, Steven, Blume, Lawrence (Eds.), *Bubbles, Entry in: The New Palgrave Dictionary of Economics*, second ed.
- Brunnermeier, Markus K., Oehmke, Martin, 2013. Bubbles, financial crises, and systemic risk. In: *Handbook of the Economics of Finance*, Amsterdam.
- Caballero, Ricardo J., 2010. Macroeconomics after the crisis: time to deal with the pretense-of-knowledge syndrome. *J. Econ. Perspect.* 24, 85–102.
- Caballero, Ricardo J., Farhi, Emmanuel, Hammour, Mohamad, 2006. Speculative growth: hints from the US economy. *Amer. Econ. Rev.* 96, 1159–1192.
- Caballero, Ricardo J., Krishnamurthy, Arvind, 2006. Bubbles and capital flow volatility: causes and risk management. *J. Monetary Econ.* 53, 35–53.
- Diamond, Peter, 1965. National debt in a neoclassical growth model. *Amer. Econ. Rev.* 55, 1126–1150.
- Ekeland, Ivar, Scheinkman, José A., 1986. Transversality conditions for some infinite horizon discrete time optimization problems. *Math. Oper. Res.* 11, 216–229.
- Farhi, Emmanuel, Tirole, Jean, 2012. Bubbly liquidity. *Rev. Econ. Stud.* 79, 678–706.
- Gale, David, 1973. Pure exchange equilibrium of dynamic economic models. *J. Econom. Theory* 6, 12–36.
- Gali, Jordi, 2014. Monetary policy and rational asset price bubbles. *Amer. Econ. Rev.* 104, 721–752.
- Grossman, Gene, Yanagawa, Noriyuki, 1993. Asset bubbles and endogenous growth. *J. Monetary Econ.* 31, 3–19.
- Hayashi, Fumio, 1982. Tobin's marginal q and average q : a neoclassical interpretation. *Econometrica* 50, 213–224.
- Hellwig, Christian, Lorenzoni, Guido, 2009. Bubbles and self-enforcing debt. *Econometrica* 77, 1137–1164.
- Hirano, Tomohiro, Yanagawa, Noriyuki, 2011. Asset bubbles, endogenous growth and financial frictions, Working Paper, University of Tokyo.
- Keohoe, Timothy J., Levine, David K., 1993. Debt-constrained asset markets. *Rev. Econom. Stud.* 60, 865–888.
- King, Ian, Ferguson, Don, 1993. Dynamic inefficiency, endogenous growth, and ponzi games. *J. Monetary Econ.* 32, 79–104.
- Kiyotaki, Nobuhiro, Moore, John, 1997. Credit cycles. *J. Political Econ.* 105, 211–248.
- Kiyotaki, Nobuhiro, Moore, John, 2008. Liquidity, business cycles and monetary policy, Working Paper, Princeton University.
- Kocherlakota, Narayana, 1992. Bubbles and constraints on debt accumulation. *J. Econom. Theory* 57, 245–256.
- Kocherlakota, Narayana, 2008. Injecting rational bubbles. *J. Econom. Theory* 142, 218–232.
- Kocherlakota, Narayana, 2009. Bursting bubbles: consequences and cures, Working Paper, University of Minnesota.
- Martin, Alberto, Ventura, Jaume, 2012. Economic growth with bubbles. *Amer. Econ. Rev.* 102, 3033–3058.
- Miao, Jianjun, Wang, Pengfei, 2011. Bubbles and Credit Constraints, Working Paper, Boston University.
- Miao, Jianjun, Wang, Pengfei, 2012a. Bubbles and total factor productivity. *Amer. Econ. Rev. Pap. Proc.* 102, 82–87.
- Miao, Jianjun, Wang, Pengfei, 2012b. Banking bubbles and financial crises, Working Paper, Boston University.

¹¹ But one may argue that if the risk-free rate is used as the interest rate, then the growth rate of the economy is higher than the interest rate. Abel et al. (1989) argue that one has to introduce uncertainty to distinguish between returns on different assets.

- Miao, Jianjun, Wang, Pengfei, 2014. Sectoral bubbles, misallocation, and endogenous growth. *J. Math. Econom.* (forthcoming).
- Miao, Jianjun, Wang, Pengfei, Xu, Lifang, 2013. Bubbles and unemployment, Working Paper, Boston University.
- Miao, Jianjun, Wang, Pengfei, Xu, Zhiwei, 2012. A Bayesian DSGE model of stock market bubbles and business cycles, Working Paper, Boston University.
- Miao, Jianjun, Wang, Pengfei, Zhou, Jing, 2014. Housing bubbles and policy analysis, Working Paper, Boston University.
- Olivier, Jacques, 2000. Growth-enhancing bubbles. *Internat. Econom. Rev.* 41, 133–151.
- Romer, Paul, 1990. Endogenous technological change. *J. Political Econ.* 98, S71–S102.
- Saint-Paul, Gilles, 1992. Fiscal policy in an endogenous growth model. *Quart. J. Econ.* 107, 1243–1260.
- Samuelson, Paul A., 1958. An exact consumption-loan model of interest with or without the social contrivance of money. *J. Political Econ.* 66, 467–482.
- Santos, Manuel S., Woodford, Michael, 1997. Rational asset pricing bubbles. *Econometrica* 65, 19–58.
- Scheinkman, Jose A., Weiss, Laurence, 1986. Borrowing constraints and aggregate economic activity. *Econometrica* 54, 23–46.
- Shiller, Robert J., 1981. Do stock prices move too much to be justified by subsequent changes in dividends? *Amer. Econ. Rev.* 71, 421–436.
- Shiller, Robert, J., 2005. *Irrational Exuberance*, second ed.. Princeton University Press.
- Tirole, Jean, 1985. Asset bubbles and overlapping generations. *Econometrica* 53, 1499–1528.
- Wang, Pengfei, Wen, Yi, 2012. Speculative bubbles and financial crises. *Amer. Econ. J.: Macroeconomics* 4, 184–221.
- Weil, Philippe, 1987. Confidence and the real value of money in an overlapping generations economy. *Quart. J. Econ.* 102, 1–22.
- Weitzman, Martin L., 1973. Duality theory for infinite horizon convex models. *Manag. Sci.* 19, 783–789.
- Werner, Jan, 2014. Rational asset pricing bubbles and debt constraints. *J. Math. Econom.* (forthcoming).
- Woodford, Michael, 1990. Public debt as private liquidity. *Amer. Econ. Rev.* 80, 382–388.