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Editorial

Introduction to economic theory of bubbles II[☆]

In Volume 53 of 2014, this journal published a special section on economic theory of bubbles. Since then, there have been new developments in theory. The current special section contains two papers that reflect these developments.

Before discussing these two papers, let me provide some background. As is well known, an asset bubble cannot emerge in standard Arrow–Debreu economies. The basic intuition is that an asset bubble reflects mispricing and can be eliminated by arbitrage. Therefore, for a bubble to emerge, there must be some sorts of frictions to limit arbitrage.¹ One friction is the incomplete market participation as in the overlapping generations (OLG) model. This model has become the dominant framework for studying asset bubbles (Samuelson, 1958; Diamond, 1965; Tirole, 1985). Since the existing OLG models of bubbles are confined to two- or three-period lived agents, they cannot be quantified to confront with the data. Moreover, when agents are altruistic to their children, the model behaves like an infinite-horizon model with a dynastic family (Barro, 1974). Thus developing infinite-horizon models of bubbles is important for us to further understand asset bubbles both qualitatively and quantitatively.

For a bubble to emerge in an infinite-horizon model, typical frictions to limit arbitrage are missing markets and portfolio constraints (Kocherlakota, 1992; Santos and Woodford, 1997). Missing markets could be due to exogenous or endogenous market incompleteness. Portfolio constraints could be in the form of debt constraints, borrowing constraints, margin constraints, or short-sales constraints. The first special section in this journal contains some papers along this line.

One type of borrowing constraints has drawn wide attention recently. It is generated from optimal contracts with limited commitment. The basic premise is that contract enforcement is imperfect and agents have limited commitment in the contracts. As a result, they may renege on the contracts so that an optimal contract must ensure agents to have incentives to stay in the contract. The incentive constraints can deliver an endogenous borrowing constraint (Kehoe and Levine, 1993; Alvarez and Jermann, 2000, 2001).

In an important paper, Kocherlakota (2008) shows that the economy with the preceding endogenous borrowing constraints is equivalent to another economy with an asset bubble. The bubble

is like inside money and plays the same role as private debt. As in Kehoe and Levine (1993), Alvarez and Jermann (2000, 2001) and Kocherlakota (2008) assumes that the penalty for default is a permanent interdiction to trade. Hellwig and Lorenzoni (2009) show that a similar bubble equivalence theorem holds when the penalty for default is an interdiction to borrow.

In the first paper of this special section, Bidian (forthcoming) shows that the bubble equivalence theorem also holds for many other types of penalties, e.g., a temporary interdiction to trade for a finite and deterministic number of periods (Azariadis and Kaas, 2008) and an interdiction to trade for a random number of periods (Azariadis and Kaas, 2013).

Why do we care for different types of penalties? The reason is that they have drastically different welfare and policy implications. In the model of Hellwig and Lorenzoni (2009), due to the weaker penalty for default, trade can only be sustained under low interest rates to induce borrowers to repay debt. Bubbles make trade possible and larger bubbles lead to more risk-sharing and trading. Under the harsher penalties studied in Bidian (forthcoming), trade can occur without bubbles, in Pareto-dominating equilibria with high interest rates. Bubbles can lead to inefficiently low interest rates, deterring saving and risk-sharing. Their size is not monotonic with the amount of risk-sharing and trade in the economy.

To make the arguments concrete, Bidian (forthcoming) sets up a simple model with two agents and uncertainty is generated by a two-state Markov process. He characterizes both stationary and nonstationary equilibria in an explicit form. The transparent solution helps us understand economic intuition clearly.

One limitation of Bidian's (forthcoming) model and the other infinite-horizon models of bubbles discussed above is that these models focus only on endowment economies. To understand the macroeconomic implications of bubbles, one has to incorporate investment and production. Miao and Wang (2011) and their series of studies (Miao and Wang, 2012, 2014, 2015; Miao et al., 2015a,b) develop models of production economies in which stock price bubbles that are attached to productive assets (capital) can emerge. The key idea is based on optimal contracts with limited commitment for firms. Miao and Wang also show that a variety of types of punishment for default can generate a stock price bubble. Some types of punishment are similar to those in Bidian (forthcoming), while others are specific to production economies.

I now turn to the second paper by Doblas-Madrid (forthcoming) in this special section. This paper is based on Abreu and Brunnermeier (2003) and Doblas-Madrid (2012), both of which address the question of why a bubble can form when it will

[☆] I would like to thank Atsushi Kajii for useful comments.

¹ Irrationality can also generate a bubble. Here I focus on rational bubbles.

eventually crash in the future. In a fully rational model, an asset bubble must satisfy a one-period valuation equation which says that the asset value is equal to the sum of the discounted payoff and resale value. Since the pure bubble asset has no payoff, its value is fully determined by its future resale value. If everyone knows that the bubble will crash for sure in the future, then it should not have value today. This argument also works for any finite crash date in the future by backward induction. One natural solution to this problem is to assume that a bubble can crash with some probability, but it can also continue with a positive probability (Blanchard and Watson, 1982; Weil, 1987). Another solution is to introduce information asymmetry. If agents have imperfect information about the date after which a bubble emerges or about the crash date, then they may want to ride on the bubble and hope to sell to unlucky last-round buyers just before the bubble crashes. Abreu and Brunnermeier (2003) provide a model to formalize this intuition. They assume that some agents are rational and others are irrational. Moreover they assume that asset prices do not respond to demand and supply. Doblas-Madrid (2012) abandons these two unappealing assumptions by providing a fully rational version of the Abreu–Brunnermeier model in which asset prices are derived from market clearing.

One problem of the models of Abreu and Brunnermeier (2003) and Doblas-Madrid (2012) is that endowments must grow at an extraordinary rate in perpetuity. The reason is that asset booms can last for an unbounded number of periods in the models of Abreu and Brunnermeier (2003) and Doblas-Madrid (2012). This is feasible only if the funds agents bring to the market (i.e., endowments) also grow in perpetuity. But the assumption of perpetual high growth of endowments seems unrealistic because this implies that the fundamental value of the asset is infinite. The contribution of Doblas-Madrid (forthcoming) is to relax this assumption. His key idea is to consider asymmetric tapered strategies in which the bubble riding time can differ for different types of agents. This is different from the symmetric trigger strategies used in the previous papers. Under the tapered strategies, the bubble duration is shorter if the fundamental booms longer.

When restricting to tapered strategies, Doblas-Madrid (forthcoming) proves the existence of a perfect Bayesian equilibrium which features bubble riding. The key assumption for this result is that the random critical time for the bubble to emerge is bounded above by a finite number. Since the support for this critical time is finite, endowments do not have to grow too fast and the fundamental value of the asset is finite.

In summary, the two papers in this special section have contributed to the literature in various dimensions. They provide some new insights that are absent from the OLG models. From a macroeconomic point of view, developing models of production economies is important to understand the macroeconomic implications of asset bubbles. Further research along this line is of the foremost importance.

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