China's Housing Bubble, Infrastructure Investment, and Economic Growth *

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Abstract

China's housing prices have been growing rapidly over the past few decades, despite low growth in rents. We study the impact of housing bubbles on China's economy, based on the understanding that local governments use land-sale revenue to fuel infrastructure investment. We calibrate our model to the Chinese data over the period 2003-2013 and find that our calibrated model can match the declining capital return and GDP growth, the average housing price growth, and the rising infrastructure to GDP ratio in the data. We conduct two counterfactual experiments to estimate the impact of a bubble collapse and a property tax.

Keywords: Housing Bubble, Infrastructure, Economic Growth, Chinese Economy, Property Tax.
JEL codes: O11, O16, O18, P24, R21, R31.

Shortened Title: Housing Bubble and Infrastructure

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1 Introduction

China implemented a series of market-oriented housing reforms in the 1990s. Since then, the Chinese real estate market has experienced a dramatic and long-lasting boom. This boom has an important impact on the Chinese macroeconomy. Based on annual data during the period 2003-2013, we find the following stylized facts as detailed in Section 2:

- The growth rates of GDP were high on average (10 percent) and declined over time.
- The growth rates of housing prices were high on average (10 percent) and the growth rates of rents were low on average (0.5 percent).
- The residential investment to GDP ratios were high on average (8.6 percent) and increased over time.[2]
- The government land-sale revenue to GDP ratios increased over time.
- The infrastructure investment to GDP ratios increased over time.[3]
- The returns to capital were high on average (10 percent) and declined over time.

In this paper we propose a two-sector overlapping-generations (OLG) model to explain these facts. The model features a housing sector that produces houses using land, capital, and labor as inputs, and a nonhousing sector that produces a final nonhousing good using capital and labor as inputs. There are two key ingredients in our model. First, rational expectations of lower returns to capital in the long run can induce current generations of entrepreneurs to seek alternative stores of value for their rapidly growing wealth. In a financially underdeveloped economy with a shortage of financial assets, housing becomes a natural investment option for current entrepreneurs, who rationally anticipate a strong demand for such assets by future generations. Speculation and low growth rates of housing rents together sustain a self-fulfilling growing housing bubble.

Second, we incorporate China's institutional feature of land policies. In China land is owned by the state and local governments collect land-transferring fees through land sales. High land prices associated with high housing prices generate a large revenue for local governments, and Chinese law requires that a certain fraction of the land-sale revenue be used toward infrastructure investment.[4] Thus a housing bubble can lead to increased infrastructure investment, which in turn facilitates

[2] In contrast, the U.S. average ratio was 4.2 percent and the highest was 6.7 percent according to the U.S. quarterly data over 2003Q1-2013Q4.

[3] Based on the IMF Investment and Capital Stock data set, China had the highest ratio of the infrastructure stock to the capital stock among the 15 largest economies in 2015.

[4] See the Measures for the Management of Income and Expenditure from the Assignment of the Right to Use State-Owned Land (2006), issued by the Chinese State Council.

production and raises nonhousing firms' productivity. This crowding-in effect of a housing bubble can raise GDP and economic growth.

On the other hand, a housing bubble can harm economic growth, because of the resource reallocation effect and the traditional crowding-out effect on capital accumulation (Tirole 1985). In particular, purchases of the housing asset crowd out entrepreneurs' resources for capital investment. This crowding-out effect lowers GDP growth. Moreover, in our two-sector model, capital and labor flow into the housing sector from the nonhousing sector in the wake of rising housing prices. This resource reallocation effect lowers nonhousing sector output and raises residential investment. When the housing sector accounts for a small share of the economy, a housing bubble causes GDP to decline in the long run.

We empirically test the above crowding-in and -out effects in Section 2.2. To address the endogeneity issue in regressions, we adopt the instrument in Guren et al. (2021). We find that higher growth of housing prices increases infrastructure investment, but decreases capital investment in the nonhousing sector, and increases labor in the housing sector.

Theoretically, we build a simplified model in Section 3 to illustrate our story, and then calibrate an extended model to confront the Chinese data during the period 2003-2013.[5] We find that our quantitative model can explain the stylized facts described earlier. We also conduct a growth accounting exercise to understand how housing bubbles affect economic growth. We find that the decline of GDP growth over 2003-2013 is due to the decline of nonhousing sector growth, which is driven mainly by the decline of capital growth, as capital flows from the nonhousing sector into the bubbly housing sector. Importantly, we also find that the deregulation on local government debt in 2009 plays a minor role in explaining the surge of the ratio of infrastructure to GDP during 2009-2013.

A standard model without housing bubbles has difficulty explaining the above stylized facts. Such a model implies that the housing price and rents grow at the same rate in the long run, so the model cannot generate a long-lasting boom of housing prices given the very low rent growth in China. As a result, the standard model also has difficulty explaining the rapid growth of land-sale revenue, residential investment, and infrastructure investment.

We next consider two counterfactual experiments. There have been substantial concerns in China's academic and policy circles that rising housing prices might have developed into a gigantic housing bubble, which might eventually burst and damage China's economy. To control housing prices, the Chinese government has considered a property tax for more than a decade, but has not implemented it so far. We use our calibrated model to answer two counterfactual questions: (1) what would the consequence of a housing bubble crash be? and (2) how would adopting a property

[5] We focus on the period 2003-2013 because land-sale was institutionalized by the Chinese government in 2003, and the data set on which our calibration is based (i.e., the proprietary housing-price data provided by Fang et al. (2015)) ended in 2013.

tax affect the economy?

For the first question, we suppose that the housing bubble collapses in 2025, and then simulate the equilibrium dynamics afterwards. Unsurprisingly, the market prices of all existing houses would take a big hit. Since newly built houses enter GDP, China's GDP growth rate would decrease from 5.3 percent to 3 percent in 2025. After a few years, GDP would gradually recover and rise above the level in the case with no bubble collapse. The reason is that collapsed housing prices reduce the government's land-sale revenue, and consequently fewer resources are invested in infrastructure and housing assets. With more capital invested in the nonhousing sector and with labor flowing back into the nonhousing sector, increased output from this sector would make up for the lost value of new houses.

To answer the second question, we suppose that the Chinese government imposes a permanent 1.5 percent property tax on all homes starting in 2025 and uses the property tax revenue to finance infrastructure investment.[6] This policy would immediately reduce the bubble size in 2025 because the after-tax return of owning a home would be lower. GDP also would decline in 2025. However, after 30 years, GDP would be 18.5 percent higher due to higher accumulation of capital because entrepreneurs would have invested more in capital and capital also would have flowed from the housing sector into the nonhousing sector.

Related literature. Our paper is related to three strands of the literature and contributes to the literature by providing the first quantitative study of the impact of Chinese housing bubbles on infrastructure investment and economic growth.

The first strand is the recent literature on housing bubbles, e.g., Arce and López-Salido (2011), Zhao (2015), Chen and Wen (2017), and Dong et al. (2019).[7] Arce and López-Salido (2011) and Zhao (2015) consider endowment economies in the OLG framework, while Dong et al. (2019) introduce production in an infinite-horizon growth model. Both Chen and Wen (2017) and our paper adopt the OLG rational-bubble model in Tirole (1985) to study the Chinese housing market. To form growing housing bubbles, both papers rely on the crucial feature that the capital return is low in the long run, even if it is high during the initial stage of transitional dynamics. There are three main differences between Chen and Wen (2017) and our paper:

- Chen and Wen (2017) only focus on the crowding-out effect of the housing bubble on capital

[6] The average property tax rate in the U.S. across all states is around 1.4 percent.

[7] There is also a literature that studies housing prices without bubbles using DSGE models. This "fundamental perspective" of housing prices argues that house price fluctuations are driven mostly by housing demand shocks. Important papers include Davis and Heathcote (2005), Davis and Heathcote (2007), Iacoviello (2005), Iacoviello and Neri (2010), Liu et al. (2013), and Miao et al. (2020) among others. Liu et al. (2021) provide a microeconomic foundation for housing demand shocks in a framework with heterogeneous beliefs. Kaplan et al. (2020) emphasize the shift in beliefs about future housing demand as a main driver of the U.S. housing price fluctuations around the Great Recession. Han et al. (2018) study housing prices in Beijing through fundamental economic variables such as income growth and demographics in an OLG model. They find that the fundamental housing value in their model is substantially lower than the data.

investment as in Tirole (1985).[8] They do not study the Chinese institutional feature that the government uses land sales to finance infrastructure investment and the associated crowding-in effect. We introduce housing production and land as input into the model in order to build the linkage between housing investment and the infrastructure investment, an issue silent in Chen and Wen (2017) but the key of our paper. Our key insight is that an increase in housing price, by increasing the demand for land and thus the land sales revenue, crowds in infrastructure investment. Thus our model predicts a negative correlation between private investment and infrastructure investment (or local government debt). Huang et al. (2018) confirm this prediction using firm-level and industry-level data.

- Chen and Wen (2017) do not distinguish between land and housing and assume that they are a pure bubble asset without fundamentals. They also assume that the housing bubble never bursts. In our model, houses pay rents and thus have a positive fundamental value. Moreover, agents have rational beliefs that a housing bubble can burst randomly. After the housing bubble bursts, houses are still valuable instead of valueless. Our model helps us understand better how bubble burst would affect the Chinese economy.

- In Chen and Wen (2017), the stock of land/housing is exogenously fixed, while in our paper local governments supply new land to housing firms (i.e., homebuilders) every period. This sequential land supply generates land-sale revenue for local governments, which is the key for the aforementioned crowding-in effect on infrastructure. To study the land purchase by housing firms, we model a housing sector producing houses in addition to the traditional nonhousing sector producing consumption goods, while Chen and Wen (2017) consider two sectors with state-owned and private firms producing the same final good.

Our paper is also related to a second strand of literature that studies how asset bubbles crowd in capital through the collateral channel as in Kiyotaki and Moore (1997), including Martin and Ventura (2012), Farhi and Tirole (2012), Miao et al. (2015), Hirano and Yanagawa (2016), and Miao and Wang (2014, 2018). In these papers bubbly assets can be used as collateral or simply raise net worth. The bursting of a bubble tightens the firms' credit constraints, forcing them to cut back investment. Although the collateral channel of the housing bubble is essential in understanding the impact of crashes in the 1989 Japanese housing market and in the 2007 U.S. housing market, its importance in China is still under debate.

In an empirical study, Wu et al. (2015) find that the collateral channel effect in China does not exist either for firms overall or for private firms, while Chen et al. (2015) provide empirical evidence that this effect is significant for private firms, but not significant for state-owned enterprises. As

[8] In an empirical study, Chen et al. (2017) find evidence that a higher land price crowds out firms' investment unrelated to acquiring commercial land.

Song et al. (2011) point out, most of private firms' investment comes from self-financing and only 10 percent comes from bank loans. Since private firms' investment accounted for around 25 percent of total investment during the period 2006-2013, even if we assume that all loans to private firms require residential housing as collateral, only 2.5 percent of total investment would be affected by the real estate price through the collateral channel. This number can be smaller in the data because it is typically small private firms that use housing as collateral. For all these reasons, neither Chen and Wen (2017) nor our model considers the collateral channel of housing prices.

Miao and Wang (2014) study a two-sector infinite-horizon model of stock price bubbles based on the collateral channel. They show that the emergence of a bubble in one sector may misallocate resources and retard economic growth. They do not focus on housing bubbles and their quantitative implications as we do in this paper.

Finally, our paper is related to a large literature on the role of government spending in economic growth, e.g., Barro (1990), Baxter and King (1993), Glomm and Ravikumar (1994), and Bassetto and Sargent (2006). As in the literature, efficiency in our model requires a good balance between infrastructure investment and private capital investment. This literature does not study housing bubbles and their impact on infrastructure investment. While infrastructure is purely funded by tax revenue in the literature, here it is also funded by the government sale of land to the housing sector.

Chen et al. (2020) study the impact of the 2009 Chinese monetary stimulus and its interaction with infrastructure spending on credit allocation. Xiong (2019) uses a tournament model to show that Chinese local governments have a strong incentive to invest in infrastructure. Unlike these papers, we emphasize the channel of the land-sale revenue, which accounts for more than half of local governments' revenue and is the collateral for more than half of their debt. Supporting our model, Mo (2018) finds evidence that Chinese local governments tend to increase investment in infrastructure when holding a large share of land-sale revenue in the total government revenue.

2 Stylized Facts

In this section we first describe some stylized facts based on China's aggregate annual data over 2003-2013 and then provide empirical evidence that supports our model mechanism based on China's province-level data.

2.1 Aggregate Evidence

As Wu et al. (2014), Chen and Wen (2017), and Fang et al. (2015) point out, the official national housing price indices published by the Chinese government tend to underestimate housing price growth due to measurement problems and the failure of controlling for housing quality. To correct these issues, Wu et al. (2014) and Fang et al. (2015) propose new methods to construct Chinese housing price indices. We adopt the aggregate price index data of Fang et al. (2015) because their data are based on housing prices of 120 major cities over 2003-2013, while the data of Wu et al. (2014) cover only 35 major cities over 2006-2010. As a result, we focus all our (annual) data on the period over 2003-2013 and find the following stylized facts (see Figure 1):

- High and declining GDP growth. The average growth rate of GDP was 10 percent based on the data from the China Statistical Yearbook. The GDP growth rate decreased from more than 14 percent to 7 percent during the period 2003-2013.

- High growth rates of housing prices and low growth rates of rents. After adjusting for inflation, we find that the average growth rate of real housing prices was 10 percent. By contrast, the average growth rate of housing rents was only 0.5 percent. The housing rents correspond to the urban household renting price indices taken from the National Bureau of Statistics of China (NBSC).

- Increasing residential investment to GDP ratios. The residential investment to GDP ratio increased from 6 percent in 2003 to 11 percent in 2013 based on data from the NBSC. The average ratio during this period was 8.6 percent.

- Increasing land-sale revenue to GDP ratios. The land-sale revenue data are taken from the Finance Yearbook of China issued by the Chinese Ministry of Finance. The land-sale revenue increased from 4 percent of GDP to 7 percent of GDP, and became the most important source of income for Chinese local governments. The average ratio during the period 2003-2013 was 4.9 percent. The land-sale revenue accounted for 25 percent of total fiscal income on average over 2003-2013, and the share increased to more than 30 percent after 2009.

- Increasing infrastructure investment to GDP ratios. China does not directly report public investment data. Following Jin (2016) and Wu et al. (2021), we define infrastructure investment as the total investment across four industries: (1) production and supply of electricity, gas, and water; (2) transport, storage, and post; (3) information transmission, computer services, and software (or telecommunications and other information transmission services); and (4) management of water conservancy, environment and public facilities. The infrastructure investment to GDP ratio increased dramatically from 6.5 percent to 9.7 percent during the period 2003-2013 based on data from the NBSC. The average ratio during this period was 7.5 percent.

- High average and declining capital returns. Bai et al. (2006) construct post-tax capital return data for 1978-2005 and Bai and Zhang (2014) extend these data to 2013. Based on their data, we find that the average capital return was 10 percent during the period 2003-2013. The capital return dropped dramatically from 15 percent in 2003 to 5 percent in 2013.

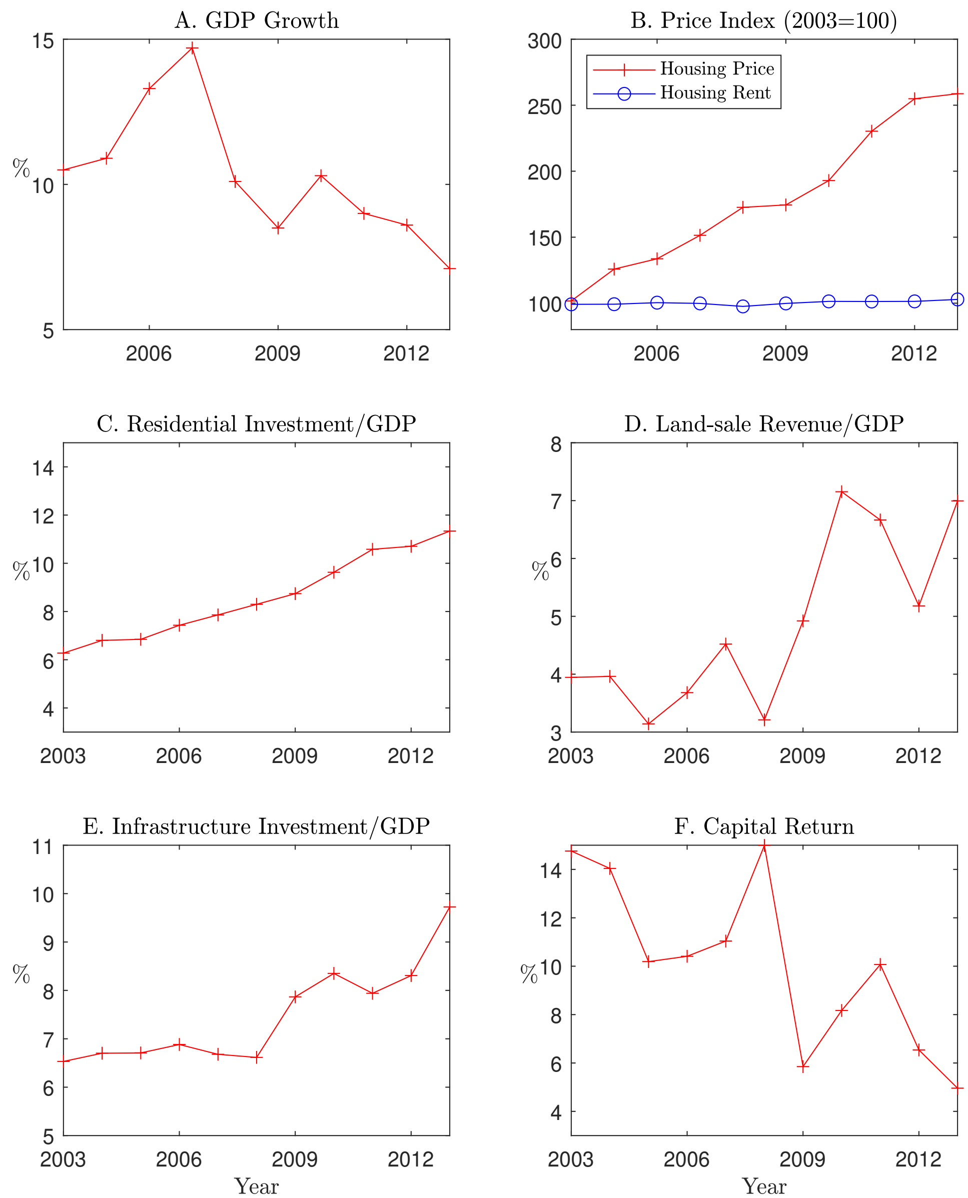

Figure 1: Stylized facts.

2.2 Cross-province Evidence

In this subsection we provide micro-level evidence to support our key model mechanism: high housing price growth stimulates infrastructure investment, but crowds out capital investment and labor in the nonhousing sector. We also provide evidence that infrastructure growth raises firm productivity in the nonhousing sector, which is the key channel for the crowding-in effect of housing prices.

We adopt annual province-level data from the NBSC, which reports annual GDP, fixed asset investment in different sectors, and average newly built housing prices for 31 provinces.[9] We do not use more micro city-level data because categorical investment data are unavailable in China at the city level. Infrastructure investment is measured by the province-level fixed asset investment in infrastructure. Capital investment is computed as the gross fixed asset investment minus infrastructure and residential investments. Housing prices and GDP are deflated by the GDP deflator, while all investment data are deflated by the investment goods price index. We use the population working in the real-estate sector as a proxy for employment (labor) in the housing sector. All data cover 2003-2016.

The regressions are specified as follows:

$$\Delta y_{i,t} = \alpha_i + \gamma_t + \beta * \Delta Q_{i,t} + \Gamma * X_{i,t} + \epsilon_{i,t}, \tag{1}$$

where α_i is a province fixed effect, γ_t is a year fixed effect, $\Delta Q_{i,t}$ is the log annual change in housing prices in province i at year t, $X_{i,t}$ is a set of controls, and $\epsilon_{i,t}$ is the error term. The variable $\Delta y_{i,t}$ represents log annual change in infrastructure investment ($\Delta infra$), capital investment ($\Delta capital$), and labor ($\Delta labor$) in the housing sector, respectively.

In equation (1), the error term and the change of housing prices might be correlated because causation runs both ways between $\Delta y_{i,t}$ and housing prices. For example, better infrastructure increases total factor productivity, which leads to higher income of households; with stronger demand of housing, housing prices will increase. To mitigate such endogeneity concerns, the literature has used housing supply elasticity as an instrument for the change of housing prices, e.g., Mian et al. (2013) and Mian and Sufi (2014). More recently, Guren et al. (2021) construct a sensitivity instrument as an improved proxy for (the inverse of) housing supply elasticity (See Appendix A for more details). In this paper, we adopt the sensitivity instrument from Guren et al. (2021), by regressing local (i.e., province-level) housing prices on regional housing prices. We also include a set of controls $X_{i,t}$ (log change in provincial GDP, land supply, and foreign direct investment, etc) to control for differential sensitivity of housing price to regional business cycles, changes in land supply and investment inflow. The regression results are reported in Table 1.

[9] Bai et al. (2006) argues that in China *capital formation* is a more accurate measure of investment activity than *fixed-asset investment*. Here we use the latter because capital formation by different sectors is not available at the province level.

Table 1: Cross-province panel regressions

VARIABLES	(1) $\Delta infra$	(2) $\Delta capital$	(3) $\Delta labor$
ΔQ	0.127^{**}	-0.203^{**}	0.447^{**}
	(0.049)	(0.080)	(0.188)
Observations	403	403	403
Province	Yes	Yes	Yes
Year	Yes	Yes	Yes

Note: Robust standard errors in parentheses $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table 2: Cross-province panel regression of TFP on Infrastructure

METHODS	Olley and Pakes (1996)	Levinsohn and Petrin (2003)
$\Delta infra$	0.19^{**}	0.12^{*}
	(0.056)	(0.039)
Observations	124	124
Province	Yes	Yes
Year	Yes	Yes

Note: Robust standard errors in parentheses $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

We find that all slope coefficients of housing price growth are significant. The coefficient is positive for infrastructure investment and housing-sector labor, but negative for capital investment. These results show that high price growth contributes to increased infrastructure investment growth but decreased capital investment growth in the nonhousing sector, and increased labor growth in the housing sector.

Lastly, we run a panel regression of the growth of province-level manufacturing productivity on the growth of infrastructure investment. The regression is specified as

$$\Delta TFP_{i,t} = \alpha_i + \gamma_t + \beta * \Delta infra_{i,t} + \Gamma * X_{i,t} + \epsilon_{i,t},$$

where α_i and γ_t are province and year fixed effects, and $\Delta TFP_{i,t}$ and $\Delta infra_{i,t}$ are respectively the log annual changes in manufacturing TFP and infrastructure investment in province i at year t. We adopt the firm-level data during 2003-2007 from China Industry Business Performance Database constructed by NBSC. This data set includes all manufacturing firms with sales above 5 million RMB. We end the period at 2007 because firms' value added, which is essential to estimate firms' TFP, was not reported after 2007. To estimate each manufacturing firm's TFP, we apply two standard methods from Olley and Pakes (1996) and Levinsohn and Petrin (2003). To obtain province-level productivity, we take employment-weighted average across firms. The regression result in Table 2 shows that the growth of infrastructure investment contributes positively to manufacturing TFP.

3 Basic Model

In this section we provide a small open economy two-sector OLG model of housing bubbles based on Tirole (1985) and Chen and Wen (2017). We make the following simplifying assumptions to illustrate the main model mechanism: (1) there is no population growth or technical progress; (2) the housing asset does not pay any rents and therefore is a pure bubble; (3) capital depreciates fully; and (4) the government runs a balanced budget. We will relax these assumptions in Section 4 to confront the data.

3.1 Households

As in Song et al. (2011) and Chen and Wen (2017), there are two types of households in our small open economy: workers and entrepreneurs. They both live for two periods. Time runs forever and is denoted by $t = 0, 1, 2, \cdots$. At the initial time $t = 0$, there is an old worker who is endowed with bonds b_0, and there is an old entrepreneur who is endowed with k_0 units of capital and h_0 units of a housing asset. In each period $t \geq 0$, a young worker and a young entrepreneur are born to replace the old. Each young worker supplies one unit of labor inelastically. After receiving their wage income, young workers choose consumption and savings. Because they are assumed to be out of the domestic capital and housing markets, they save only through risk-free bonds. The optimization problem for a newborn worker of age 1 is given by

$$\begin{aligned} \max \quad & \log(c_{1,t}^{w}) + \beta \log(c_{2,t+1}^{w}) \\ s.t. \quad & c_{1,t}^{w} + b_{t+1} = w_t, \\ & c_{2,t+1}^{w} = R^f b_{t+1}, \end{aligned}$$

where $c_{1,t}^{w}$ and $c_{2,t+1}^{w}$ are their consumption when young and old, $\beta \in (0, 1)$ is the discount factor, w_t is the wage rate, b_{t+1} is the holding of the risk-free bond, and R^f is the exogenous interest rate in the international financial market.

Entrepreneurs have the same preferences as workers. After inheriting an initial wealth level m_t from an old entrepreneur, a young entrepreneur of age 1 in period t can invest in both capital and housing to solve the following problem:

$$\begin{aligned} \max \quad & \log(c_{1,t}^{e}) + \beta \log(c_{2,t+1}^{e}) \\ s.t. \quad & c_{1,t}^{e} + k_{t+1} + Q_t h_{t+1} = m_t, \\ & c_{2,t+1}^{e} = R_{t+1} k_{t+1} + Q_{t+1}(1 - \delta_h) h_{t+1}, \end{aligned}$$

where $c_{1,t}^{e}$ and $c_{2,t+1}^{e}$ are their consumption when young and old, $h_{t+1} \geq 0$ is their holdings of the housing asset, $Q_t \geq 0$ is the price of housing, $k_{t+1} \geq 0$ is their holdings of capital, R_{t+1} is the capital return between periods t and $t+1$, and δ_h is the depreciation rate of housing. In the simple model housing is a pure bubble asset without any fundamentals and hence its fundamental value is zero. Entrepreneurs trade houses for speculation. Assume that entrepreneurs cannot borrow and $R_f < R_{t+1}$.[10] Then entrepreneurs will not hold any bond in equilibrium. We will relax the no-borrowing assumption in Section 4.

Since the utility function is logarithmic, the entrepreneur's optimal saving is given by

$$k_{t+1} + Q_t h_{t+1} = \frac{\beta}{1+\beta} m_t. \tag{2}$$

They will invest in both capital and housing only if the following no-arbitrage condition is satisfied:

$$R_{t+1} = \frac{Q_{t+1}(1-\delta_h)}{Q_t} \text{ for } Q_t > 0. \tag{3}$$

That is, the returns on housing and capital are the same. In Section 4 we will show that they are not the same when the housing bubble can burst stochastically.

3.2 Nonhousing Sector

Each old entrepreneur owns a firm that produces the final consumption good using capital and labor as inputs. After investment of k_{t+1} at time t, each old entrepreneur at $t+1$ receives output given by

$$y_{t+1} \equiv \hat{A}_{t+1}^{\theta} k_{t+1}^{\alpha} n_{c,t+1}^{1-\alpha}, \quad \alpha \in (0,1),$$

where k_{t+1} and $n_{c,t+1}$ are the firm's capital and labor, $\hat{A}_{t+1}$ is the firm's productivity, and $\theta > 0$ is an elasticity parameter. Following Glomm and Ravikumar (1994), we assume that the firm's productivity depends on infrastructure in the following way:

$$\hat{A}_{t+1} \equiv A_{t+1}/(K_{t+1}^{\rho} N_{c,t+1}^{1-\rho}),$$

where A_{t+1} is the aggregate infrastructure stock and $\rho \in (0,1)$ is a parameter. We normalize A_{t+1} by aggregate capital K_{t+1} and aggregate labor $N_{c,t+1}$ for two reasons. First, in many cases, such as highways, utilities, and bridges, the productivity of infrastructure is indeed diluted by congestion when more people or firms use the same piece of infrastructure. Second, as Glomm and Ravikumar (1994) point out, in a model with endogenous infrastructure, the steady-state or balanced-growth path is not guaranteed if A_{t+1} is not normalized.

The old entrepreneur in period $t+1$ pays the government output tax at the rate τ, pays the worker $w_{t+1} n_{c,t+1}$ as wage, and pays the young entrepreneur a fraction ψ of after-wage income $(1-\tau) y_{t+1} - w_{t+1} n_{c,t+1}$ as initial wealth, i.e.,

$$m_{t+1} = \psi \left((1-\tau) y_{t+1} - w_{t+1} n_{c,t+1} \right). \tag{4}$$

[10] We will verify this assumption in equilibrium. During the period 2003-2013, China's average capital return was above 10 percent while its deposit rate was around zero.

The remainder is $R_{t+1}k_{t+1}$, which can be written as

$$R_{t+1}k_{t+1} \equiv \max_{n_{c,t+1}} \quad (1-\psi)\left[(1-\tau)\hat{A}_{t+1}^{\theta}k_{t+1}^{\alpha}n_{c,t+1}^{1-\alpha} - w_{t+1}n_{c,t+1}\right].$$

We make the following assumption on parameters:

Assumption 1 *$\alpha - \rho\theta > 0$ and $\alpha + (1-\rho)\theta < 1$.*

The first inequality in this assumption guarantees the marginal return to capital is positive. The second inequality guarantees the return on the whole reproducible part (infrastructure and capital) is weakly decreasing.

Let

$$\phi_t \equiv \frac{Q_t h_{t+1}}{k_{t+1} + Q_t h_{t+1}}$$

denote the fraction of housing investment in a young entrepreneur's saving. By (2) and the definition of m_t, we can derive

$$k_{t+1} = (1-\phi_t)\frac{\beta}{1+\beta}m_t = (1-\phi_t)\frac{\beta}{1+\beta}\psi\alpha(1-\tau)y_t. \tag{5}$$

Clearly, the emergence of a housing bubble in the sense that $Q_t > 0$ crowds out capital investment as $\phi_t \in (0,1)$.

3.3 Housing Sector

There is a continuum of competitive firms producing houses using labor and land as inputs. We do not consider capital input in the basic model for simplicity. We will relax this assumption in our quantitative model of Section 4. Each housing firm purchases land from the government that is the sole supplier at the price p_{Lt} at time t. Workers are freely mobile across the housing and nonhousing sectors. Each housing firm sells newly built houses to entrepreneurs at the price Q_t. Its profit maximization problem is given by

$$\max_{l_t, n_{h,t}} \quad Q_t l_t^{\alpha_l} n_{h,t}^{1-\alpha_l} - p_{Lt}l_t - w_t n_{h,t},$$

where $\alpha_l \in (0,1)$ is an elasticity parameter, and l_t and $n_{h,t}$ are the demand for land and labor, respectively.

Due to the constant-returns-to-scale technology, aggregation implies that total newly built houses $Y_{h,t}$ satisfy

$$Y_{h,t} = L_t^{\alpha_l} N_{h,t}^{1-\alpha_l},$$

where L_t is the aggregate land supply set exogenously by the government and $N_{h,t}$ is the aggregate labor hired by the housing sector. The total housing stock H_t evolves according to

$$H_{t+1} = (1-\delta_h)H_t + Y_{h,t}.$$

3.4 Government and Infrastructure

The government supplies L_t units of land to the market exogenously in period t.[11] To guarantee the existence of a bubbly steady state, we assume that $\lim_{t\to\infty} L_t = L^* > 0$, where L^* is the land supply in the long run.

For simplicity suppose that the government runs a balanced budget without issuing bonds. Its only spending is infrastructure investment. Thus the government infrastructure expenditure is equal to its total revenue $\tau Y_t + p_{Lt}L_t$, where Y_t denotes aggregate final good output. The stock of infrastructure evolves as

$$A_{t+1} = (1-\delta_a)A_t + \tau Y_t + p_{Lt}L_t, \tag{6}$$

where δ_a is the depreciation rate of infrastructure.

3.5 Resource Constraint

The budget constraints of workers imply

$$c^w_{1,t} + c^w_{2,t} + b_{t+1} = w_t + R^f b_t, \tag{7}$$

where $c^w_{1,t} + c^w_{2,t}$ is the sum of the consumption of old workers of generation-$(t-1)$ and young workers of generation-t. In the domestic market the resource constraint is

$$A_{t+1} - (1-\delta_a)A_t + c^e_{1,t} + c^e_{2,t} + K_{t+1} + w_t = Y_t. \tag{8}$$

Because of constant-returns-to-scale technology, aggregate nonhousing output satisfies

$$Y_t = \hat{A}_t^{\theta} K_t^{\alpha} N_{c,t}^{1-\alpha},$$

where $N_{c,t} = 1 - N_{h,t}$ denotes aggregate labor in the nonhousing sector. In the simple model GDP is defined as the sum of nonhousing output Y_t and residential investment $Q_t Y_{h,t}$.

Summing up (7) and (8) yields

$$A_{t+1} - (1-\delta_a)A_t + K_{t+1} + c^e_{1,t} + c^e_{2,t} + c^w_{1,t} + c^w_{2,t} + b_{t+1} - R^f b_t = Y_t. \tag{9}$$

On the left side of equation (9), $A_{t+1} - (1-\delta_a)A_t$ is the infrastructure investment, K_{t+1} is the private capital investment, $b_{t+1} - R^f b_t$ is the surplus in the current account, and the rest is aggregate consumption.

[11] Implicitly, there is a pool of residential land maintained by the Chinese government. While L_t is the outflow of the pool, there are two inflows. First, the land on which houses had depreciated returns to the pool. Second, the government may convert agricultural or industrial land into residential land. Modeling the dynamics of the pool is beyond the scope of this paper.

3.6 Equilibrium

The equilibrium of the economy is defined as follows.

Definition 1 *An equilibrium is a sequence of prices* $\{w_t, R_t, Q_t, p_{Lt}\}_{t=0}^{\infty}$*, savings* $\{b_t, k_t, h_t\}_{t=0}^{\infty}$*, consumption* $\{c_{1,t}^w, c_{2,t+1}^w, c_{1,t}^e, c_{2,t+1}^e\}_{t=0}^{\infty}$*, labor supply/demand* $\{N_{c,t}, N_{h,t}\}_{t=0}^{\infty}$*, and infrastructure* $\{A_t\}_{t=0}^{\infty}$ *such that (i) workers and entrepreneurs maximize their lifetime utilities; (ii) firms maximize profits; (iii) the government budget constraint* (6) *is satisfied; and (iv) the labor, capital, land, and housing markets clear.*

In the rest of this paper, we call an equilibrium *bubbleless* if $Q_t = 0$ for all t, and call an equilibrium *bubbly* if $Q_t > 0$ for all t. While the former equilibrium always exists, the latter depends on parameters. In the bubbleless equilibrium, the housing price is zero and hence the land price p_{Lt} is also zero. Thus the housing and land markets disappear. The government finances infrastructure investment using output taxes only.

There are two steady states in our basic model: one is bubbleless and the other is bubbly. We use a variable without a time subscript to denote its steady-state value and add superscript n or b to denote its bubbleless or bubbly steady-state value, respectively. We first consider the bubbleless steady state and show that

$$A^n = \delta_a^{-1}\tau Y^n, \tag{10}$$

$$K^n = \frac{\beta}{1+\beta}\alpha\psi(1-\tau)Y^n, \tag{11}$$

where (10) follows from (6) with $p_L^n = 0$, and (11) follows from (5) with $\phi^n = 0$. Moreover, the bubbleless steady-state return on capital is equal to

$$R^n = \frac{(1-\psi)(1-\tau)\alpha Y^n}{K^n} = \frac{(1-\psi)(1+\beta)}{\psi\beta}. \tag{12}$$

Next we analyze the bubbly steady state in the following proposition. Its proof and the proofs of other results in the paper are relegated to Appendix B.

Proposition 1 *A unique bubbly steady state exists if*

$$z \equiv \frac{(1-\psi)(1+\beta)}{(1-\delta_h)\psi\beta} < 1. \tag{13}$$

Moreover,

$$\phi^b = 1 - z, \tag{14}$$

$$N_c^b = \frac{(1-\phi^b)(1-\alpha)(1-\delta_h)}{(1-\phi^b)(1-\alpha)(1-\delta_h) + \phi^b\delta_h(1-\alpha_l)\alpha(1-\psi)}, \tag{15}$$

$$A^b = \delta_a^{-1}\alpha_l\delta_h\phi^b\frac{\beta}{1+\beta}\alpha\psi(1-\tau)Y^b + \delta_a^{-1}\tau Y^b, \tag{16}$$

$$K^b = (1-\phi^b)\frac{\beta}{1+\beta}\alpha\psi(1-\tau)Y^b. \tag{17}$$

The variable z defined in (13) is equal to the proportion of capital holdings in the entrepreneur's savings by (14). There are two senses in which condition $z < 1$ is needed for the existence of a bubbly steady state. First, the bubble accounts for a fraction $\phi^b = 1 - z$ of the entrepreneur's saving. We need $z < 1$ to guarantee $\phi^b > 0$. Second, it follows from (12) that condition (13) requires that $R^n < 1 - \delta_h = R^b$. Imposing an upper bound on the bubbleless steady-state capital return is a standard assumption in the literature for a bubble to exist. As pointed out by Tirole (1985), only if the bubbleless steady-state capital return is sufficiently low can a bubbly asset be traded as an alternative channel to save. Our upper bound $1 - \delta_h$ is less than the standard value of 1 from the literature on bubbles, because the bubbly steady-state return on housing (which is also equal to the capital return) is $1 - \delta_h$ due to housing depreciation by (3).

Equations (16) and (17) show that the bubbly steady-state levels of infrastructure and capital are linear in output. Equation (16) shows that the bubbly steady-state infrastructure level is financed by output taxes and the land-sale revenue generated by the housing bubble.

The following proposition characterizes the global equilibrium dynamics.

Proposition 2 *Consider an economy with given initial condition $\{K_0, A_0, H_0\}$. If $z \geq 1$ where z is given in (13), then no bubbly equilibrium exists. Otherwise, there is a unique $\hat{Q}_0 > 0$ such that $0 < Q_0 \leq \hat{Q}_0$ in any bubbly equilibrium. Moreover,*

(i) if $0 < Q_0 < \hat{Q}_0$, then $\lim_{t\to\infty} Q_t = 0$;

(ii) if $Q_0 = \hat{Q}_0$, then $Q^b \equiv \lim_{t\to\infty} Q_t > 0$.

As in Tirole (1985), equilibria around the bubbleless steady state are indeterminate. There is a continuum of equilibria indexed by the initial housing price Q_0 in $(0, \hat{Q}_0)$. For all these equilibria, housing bubbles disappear asymptotically as $\lim_{t\to\infty} Q_t = 0$. By contrast, the bubbly steady state is a saddle. There is a unique initial price $Q_0 = \hat{Q}_0 > 0$ such that the equilibrium starting at this price converges to the bubbly steady state as $\lim_{t\to\infty} Q_t = Q^b > 0$. There also exists another type of equilibria for which bubbles burst stochastically (Weil 1987) as discussed in Section 4. This type of equilibria is most relevant for the interpretation of the Chinese time series. As shown in Proposition 4 of Weil (1987), there is also a continuum of equilibria with stochastic bubbles. We select the equilibrium with the maximum initial bubble size such that the economy converges to the stochastic bubbly steady state before the bubble collapses in Section 4.

To understand the intuition for Proposition 2, we explain how the long-run housing price, $\lim_{t\to\infty} Q_t$, depends on the initial Q_0 for a given initial low level of capital stock K_0. With a higher Q_0, more private capital K_1 is crowded out, and capital return R_1 becomes higher due to the diminishing marginal product of capital. The no-arbitrage condition (3) then implies a higher growth rate Q_1/Q_0. Using this argument for all the future dates, we conclude that higher Q_0 raises the growth rate Q_{t+1}/Q_t for all t. If Q_0 is sufficiently high, then the housing price explodes and cannot be sustained in equilibrium, so bubbly equilibrium does not exist. If Q_0 is sufficiently low, then the housing price eventually declines to zero in the long run. There is a unique value $\hat{Q}_0 > 0$ such that when $Q_0 = \hat{Q}_0$, the housing price converges to a positive limit.

3.7 Inspecting the Mechanism

In Tirole (1985) and Chen and Wen (2017), a bubble crowds out private capital and lowers output in the steady state. This is the traditional *crowding-out* effect of a bubble. In our model, however, a housing bubble also helps the government accumulate more infrastructure, thus raising the productivity and production of the nonhousing sector. This is the *crowding-in* effect introduced in our paper. Moreover, there is a factor *reallocation effect* in our model in that a housing bubble causes labor to flow from the nonhousing sector into the housing sector. We study these three effects in this subsection.

First we compare the steady-state infrastructure, capital, and output with and without a bubble.

Proposition 3 *We have the following relationships in the bubbly and bubbleless steady states:*

$$\frac{K^b}{K^n} = (1-\phi^b)\frac{Y^b}{Y^n},$$

$$\frac{A^b}{A^n} = \left(1 + \frac{\alpha_l \delta_h \phi^b \beta \alpha \psi (1-\tau)}{(1+\beta)\tau}\right)\frac{Y^b}{Y^n},$$

$$\left(\frac{Y^b}{Y^n}\right)^{1-\alpha-(1-\rho)\theta} = \left(1 + \frac{\alpha_l \delta_h \phi^b \beta \alpha \psi (1-\tau)}{(1+\beta)\tau}\right)^{\theta} (1-\phi^b)^{\alpha-\rho\theta} (N_c^b)^{1-\alpha-(1-\rho)\theta}, \tag{18}$$

where N_c^b is given by (15).

From this proposition we can see that

$$\frac{K^b}{K^n} < \frac{Y^b}{Y^n} < \frac{A^b}{A^n}.$$

Thus, it is possible that $Y^b > Y^n$, but $K^b < K^n$. That is, the crowding-out effect on capital is dominated by the crowding-in effect on infrastructure. If the expression on the right-hand side of (18) is greater than 1, then $Y^b > Y^n$. This expression gives the determinants of Y^b/Y^n.

Under Assumption 1, a higher θ strengthens the crowding-in effect because it increases the sensitivity of output to infrastructure. The first term on the right-hand side of (18) captures the impact of infrastructure funded by the land sale, while $-\phi^b$ in $(1-\phi^b)^{\alpha-\rho\theta}$ captures the crowding-out effect on capital. The last term related to $N_c^b < 1$ captures the reallocation effect on labor. If there is no housing bubble in the basic model, we have $N_c^b = 1$ because the housing sector does not exist.

In general the above three effects are time varying along a transition path. The next two numerical examples illustrate these time-varying effects by comparing equilibrium paths with and without a bubble. Both paths start from the same initial condition (A_0, K_0), and the land supply is fixed at L^* in each period.

In the first example, we set $(A_0, K_0) = (A^n, K^n)$. Then the bubbleless equilibrium stays forever in the bubbleless steady state. We choose parameter values such that $Y^b > Y^n$. The top six panels of Figure 2 present the transition dynamics. Panel A shows that bubbly output Y_t^b in the nonhousing sector is slightly lower at $t = 0$ due to the reallocation effect on labor, but eventually higher than bubbleless output Y^n. Panels B and C show residential investment $(Q_t Y_{h,t})$ and GDP, respectively.[12] Panels D, E, and F illustrate the crowding-in effect of the housing bubble on infrastructure, the reallocation effect on labor, and the crowding-out effect on capital, respectively. The crowding-out and reallocation effects dominate during the early stage of the transition. But the crowding-in effect gradually catches up and eventually dominates.

In the second example, we set small initial values for A_0 and K_0, which are all smaller than the bubbleless steady-state values. Choose parameter values such that $Y^b < Y^n$. The bottom six panels of Figure 2 present the transition dynamics. We find that nonhousing output Y_t^b in the bubbly equilibrium is initially $(t = 0)$ lower than Y_t^n in the bubbleless equilibrium due to the reallocation effect on labor. From $t = 1$ until $t = 49$, Y_t^b is higher than Y_t^n because the crowding-in effect on infrastructure dominates the crowding-out effect on capital and the reallocation effect on labor. But Y_t^b is eventually lower than Y_t^n in the long run as the crowding-out effect dominates. Even though the reallocation effect of a housing bubble raises residential investment and maximizes GDP in a *static* sense, GDP is lower in the bubbly steady state than in the bubbleless steady state due to the *dynamic* crowding out of capital in Panel L, i.e., young entrepreneurs accumulate less capital if they must allocate more resource toward bubbly assets.

To summarize, our basic model has illustrated the traditional crowding-out effect of a housing bubble, the reallocation effect on labor, and a new crowding-in effect on infrastructure associated with the land sale by the Chinese local governments. To quantify these effects on the macroeconomy, we will calibrate our model in the next section.

4 Quantitative Analysis

To confront China's data, we extend our basic model in several ways. In particular, we introduce population growth and technology growth. We allow the housing asset to pay rents and thus its

[12]Notice that residential investment is zero in the bubbleless equilibrium of the basic model because $Q_t = 0$ when housing does not pay rents.

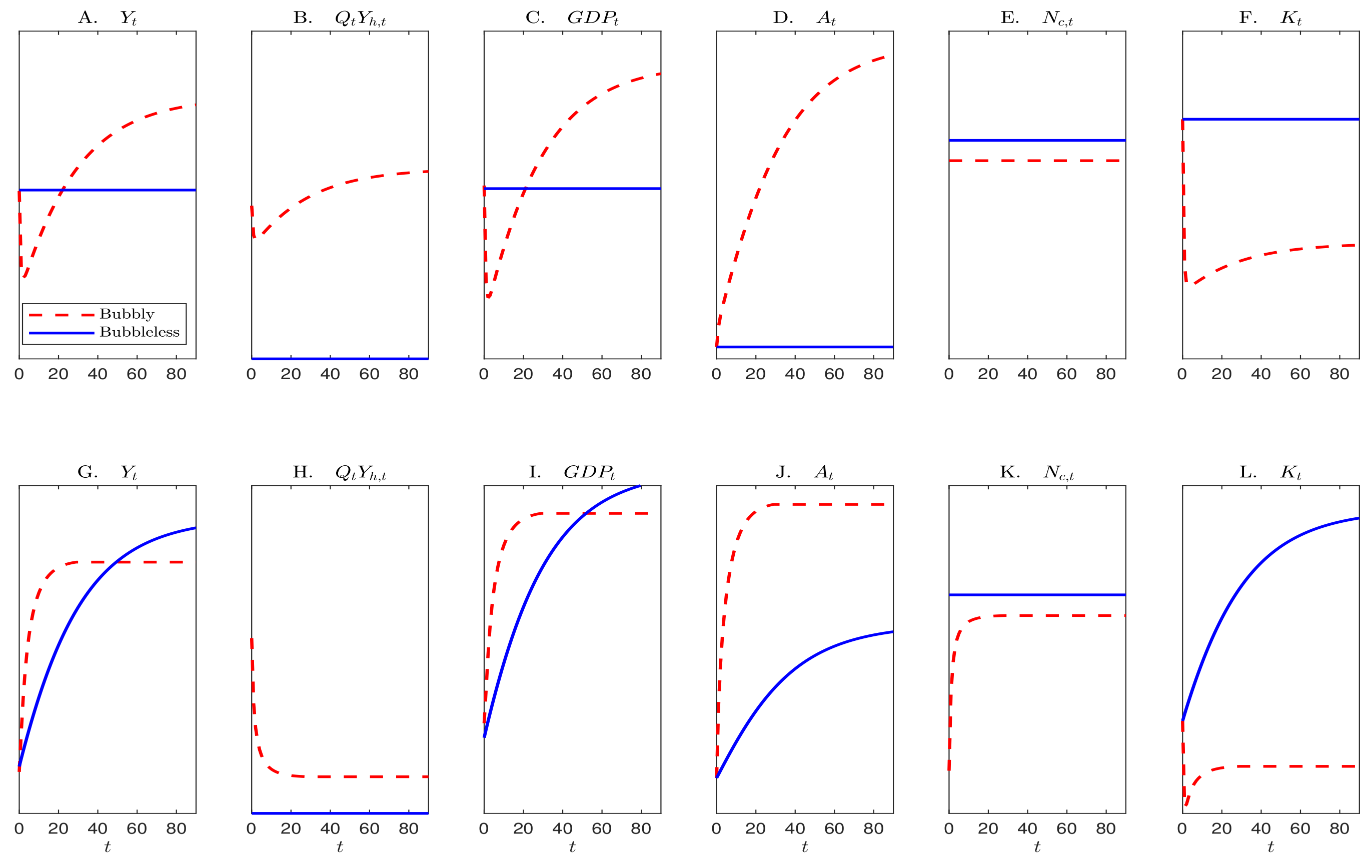

Figure 2: Transitional dynamics. The top six panels present the first numerical example in Section 3.7 where nonhousing output Y^b in the bubbly steady state is above nonhousing output Y^n in the bubbleless steady state, while the bottom six panels present the second example with $Y^b < Y^n$. Here Y_t, $Q_tY_{h,t}$, GDP_t, A_t, $N_{c,t}$, and K_t denote, respectively, nonhousing output, residential investment, total output, infrastructure, nonhousing labor, and capital.

fundamental value is nonzero. The presence of a bubble permits housing prices to grow much faster than rents. We also introduce a stochastic housing bubble to conduct counterfactual experiments (Weil 1987). We then calibrate this extended model and analyze its quantitative predictions. Appendix C presents the technical details for this model.

4.1 Stochastic Bubbles

Assume that all agents have common beliefs that the housing price is random and follows a two-state Markov process. In the bubbly state the housing price contains a bubble component. The bubble collapses with probability $p > 0$ in each period. After the bubble collapses, the economy enters the fundamental state and stays there forever. The housing bubble cannot reappear. Given rational expectations, all equilibrium variables are stochastic and contingent on the state. There is no other shock in the model. When necessary, we use a variable with superscript $+$ $(-)$ to denote its value in the bubbly (fundamental) state.

Assume that both workers and entrepreneurs live for $T > 2$ years, and workers retire at age J. The population of both workers and entrepreneurs grows at a constant rate g_n. A newborn worker of age 1 solves the following utility maximization problem:

$$\begin{aligned}
\max \quad & E\left[\sum_{j=1}^{T} \beta^{j-1} \log(c_j^w)\right] \\
s.t. \quad & c_j^w + b_{j+1}^w = \begin{cases} w + R^f b_j^w, & 1 \leq j \leq J; \\ R^f b_j^w, & J+1 \leq j \leq T, \end{cases} \\
& b_1^w = 0, \quad b_{T+1}^w = 0,
\end{aligned}$$

where c_j^w is age-j worker's consumption and b_j^w denotes bonds held at the beginning of age j. Here the expectation is taken with respect to the probability distribution of the stochastic bubble. A newborn worker does not have any asset so that $b_1^w = 0$. They do not leave any debt/asset when they die. After retirement, the worker has no labor income and accumulates wealth from savings only. For simplicity, we have removed the time subscripts for all variables without risk of confusion.

A newborn entrepreneur in period t has initial endowment m_t and chooses their lifetime consumption and investment in bonds, capital, and housing. The endowment m_t comes from a fraction ψ of the firms' after-tax profits. Housing delivers exogenous rents r_t, which grow at the rate g_r. We do not consider endogenous rents because they would make a bubbly model very hard to analyze.[13] Tirole (1985) and Miao and Wang (2018) have discussed the difficulty. Chen and Wen

[13]Moreover, endogenous rents would complicate our analysis without delivering significant new economic insight into the questions addressed in this paper. Suppose we embed housing service in households' consumption and let rents be endogenously determined by the demand of housing service. This alternative model allows us to study the feedback effect from infrastructure to housing (i.e., local infrastructure may improve housing quality and increase the value of housing service), while our benchmark model only considers how housing price affects infrastructure. Had this feedback effect been strong, we would observe strong growth of housing rents after the boom in infrastructure. Rents growth in the data, however, was low throughout our sample period.

(2017) simply set rents to zero in their model.

The entrepreneur's utility maximization problem is given by

$$\begin{aligned}
&\max && E\left[\sum_{j=1}^{T}\beta^{j-1}\log(c_j^e)\right] \\
&s.t. && Qh_{j+1}+k_{j+1}+c_j^e+b_{j+1}^e \\
&&= & \begin{cases} m, & j=1; \\ Rk_j+\left(Q(1-\delta_h)+r\right)h_j+R^f b_j^e, & 2\le j\le T; \end{cases} \\
&&& b_{j+1}^e \ge -\xi k_{j+1}, \\
&&& k_1=b_1^e=h_1=h_{T+1}=k_{T+1}=b_{T+1}^e=0,
\end{aligned} \tag{19}$$

where c_j^e denotes an age-j entrepreneur's consumption, and h_j and k_j are, respectively, their holdings of housing and capital at beginning of age j. Again we have removed the time subscripts. Notice that we allow entrepreneurs to save or borrow in the international financial market up to some borrowing limit. An entrepreneur can borrow against at most a fraction ξ of their capital assets. We assume that entrepreneurs cannot use residential housing as collateral because such a practice is uncommon in China (see Wu et al. 2015).

We make three changes to the firms' production technologies in the extended model. First, we introduce labor-augmenting technology growth to match the Chinese economic growth. Let the labor efficiency $e_t = (1+g_e)^t$ grow exogenously at the rate g_e. Second, we introduce capital to the housing production function. Assume that both capital and labor are freely mobile across the housing and nonhousing sectors. Third, we allow the land quality to decline over time at the rate g_l. The reason is that newly supplied land generally has a less preferred location (Davis and Heathcote 2007 and Fang et al. 2015). Real estate developers first build houses in cities and then build houses outside cities over time during the Chinese urbanization process. The quality of land in cities is better than that in rural areas as housing prices in cities are more expensive than in rural areas (Fang et al. 2015).

Formally, let the production function in the housing sector be given by

$$y_{h,t} = \left((1-g_l)^t\, l_t\right)^{\alpha_l}(k_{h,t})^{\alpha_k}(e_t n_{h,t})^{1-\alpha_l-\alpha_k},$$

where l_t denotes the land input and $k_{h,t}$ denotes the capital input in the housing sector.[14]

Each entrepreneur after age 1 runs both a nonhousing firm and a housing firm, and maximizes

[14] To simplify our calibration, we assume that infrastructure doesn't enter the housing sector's productivity. This assumption is unlikely to change our model prediction because (1) the housing sector is relatively small, (2) based on the estimation in Wu et al. (2021) and Chinese input-output table, we find that the output elasticity of infrastructure in the housing sector is only half of the nonhousing sector.

total profits:

$$\begin{aligned}
R_t k_t = \max_{n_{c,t},k_{c,t},n_{h,t},k_{h,t},l_t} \quad & \Big\{(1-\psi)\big[(1-\tau)(\hat{A}_t)^{\theta}(k_{c,t})^{\alpha}(e_t n_{c,t})^{1-\alpha} - w_t n_{c,t} \\
& +(1-\tau_h)Q_t((1-g_l)^t\, l_t)^{\alpha_l}(k_{h,t})^{\alpha_k}(e_t n_{h,t})^{1-\alpha_l-\alpha_k} - w_t n_{h,t} - p_{Lt} l_t\big] \\
& +(1-\delta_k)k_{c,t} + (1-\delta_k)k_{h,t}\Big\} \\
s.t. \quad & k_{c,t} + k_{h,t} = k_t,
\end{aligned}$$

where k_t is total demand for capital across the two sectors, δ_k is the depreciation rate of capital, and τ_h is the tax rate in the housing sector. Here productivity satisfies

$$\hat{A}_t = \frac{A_t}{K_{c,t}^{\rho}\,(e_t N_{c,t})^{1-\rho}},$$

where $K_{c,t}$ and $N_{c,t}$ denote the aggregate capital stock and aggregate labor in the nonhousing sector.

We allow the government to borrow at rate R^f in the extended model. Since more than half of the local government debt in China uses the land-sale revenue as collateral, we assume the amount of borrowing, B^g_{t+1}, is proportional to the land-sale revenue, i.e., $B^g_{t+1} = \xi_g p_{Lt} L_t$, where $\xi_g > 0$ is a parameter.[15] The government uses debt, taxes, and land-sale revenue to finance infrastructure investment $A_{t+1} - (1-\delta_a)A_t$ and non-infrastructure expenditure G_t.[16] The government budget constraint is given by

$$\begin{aligned}
& A_{t+1} - (1-\delta_a)A_t + G_t + R^f B^g_t - B^g_{t+1} \\
(20) \qquad = \; & \tau(\hat{A}_t)^{\theta}(K_{c,t})^{\alpha}(e_t N_{c,t})^{1-\alpha} + \tau_h Q_t\left((1-g_l)^t L_t\right)^{\alpha_l}(K_{h,t})^{\alpha_k}(e_t N_{h,t})^{1-\alpha_l-\alpha_k} + p_{Lt} L_t,
\end{aligned}$$

where $K_{h,t}$ and $N_{h,t}$ are the aggregate capital and labor in the housing sector.

GDP in this economy is the sum of aggregate nonhousing output Y_t, residential investment (or the value of aggregate housing output) $Q_t Y_{h,t}$, and aggregate rents $r_t H_t$:

$$(21) \quad Y_t + Q_t Y_{h,t} + r_t H_t = (\hat{A}_t)^{\theta}(K_{c,t})^{\alpha}(e_t N_{c,t})^{1-\alpha} + Q_t((1-g_l)^t L_t)^{\alpha_l}(K_{h,t})^{\alpha_k}(e_t N_{h,t})^{1-\alpha_l-\alpha_k} + r_t H_t.$$

4.2 No-Arbitrage Pricing Equation

To understand the dynamics of housing prices, it is important to derive the pricing equation for the housing asset. When the economy is in the fundamental state in period t, it stays in this state

[15] In accordance with Notice No. 162 [2012] of the Ministry of Land and Resources, local governments can use land-sale revenues from the current and the next 2 years as collateral to issue debt. To simplify the problem, we have removed future land-sale revenues from the collateral constraint, but used the current revenue as an proxy. Adding future revenues will probably deliver a quantitatively similar model with a lower parameter ξ_g.

[16] By the Finance Yearbook of China, 34.2 percent of infrastructure investment was directly financed by land-sale revenue during 2003-2013. However, the actual number should be larger than that because many local governments leverage land-sale revenue to finance infrastructure, which is not directly counted in Finance Yearbook of China. Shi and Huang (2014) find that around 50 percent of infrastructure investment was covered by land-sale revenue during 2003-2007.

forever. Under binding collateral constraints (which happens when $R_{t+1}^{-} > R^f$), we can derive the following no-arbitrage condition:

$$\tilde{R}_{t+1}^{-} = \frac{Q_{t+1}^{-}(1-\delta_h) + r_{t+1}}{Q_t^{-}}, \tag{22}$$

where the variable

$$\tilde{R}_{t+1}^{-} \equiv \frac{R_{t+1}^{-} - \xi R^f}{1-\xi}$$

is the *effective capital return*, which takes into account the impact of the collateral constraint.[17] Equation (22) says that the housing return is equal to the effective capital return. Thus, in the fundamental state, Q_t^{-} is equal to the fundamental value, i.e., the present discounted value of future rents

$$Q_t^{-} = \sum_{s=t+1}^{\infty} \frac{(1-\delta_h)^{s-(t+1)} r_s}{\prod_{i=t+1}^{s} \tilde{R}_i^{-}}.$$

Suppose that the economy is in the bubbly state in period t. Then the housing return is $R_{t+1}^{h+} \equiv \left[(1-\delta_h)Q_{t+1}^{+} + r_{t+1}\right]/Q_t^{+}$ when the economy still stays in this state in period $t+1$. But the housing return is $R_{t+1}^{h-} \equiv \left[(1-\delta_h)Q_{t+1}^{-} + r_{t+1}\right]/Q_t^{+}$ when the economy moves to the fundamental state in period $t+1$. Similarly, we can compute the effective capital returns under binding borrowing constraints: $\tilde{R}_{t+1}^{+} \equiv \left(R_{t+1}^{+} - \xi R^f\right)/(1-\xi)$. The no-arbitrage condition in the bubbly state in period t is given by

$$\begin{aligned} & (1-p)u'(c_{j,t+1}^{e+})R_{t+1}^{h+} + pu'(c_{j,t+1}^{e-})R_{t+1}^{h-} \\ = \; & (1-p)u'(c_{j,t+1}^{e+})\tilde{R}_{t+1}^{+} + pu'(c_{j,t+1}^{e-})\tilde{R}_{t+1}^{-}, \end{aligned} \tag{23}$$

where $c_{j,t+1}^{e-}$ and $c_{j,t+1}^{e+}$ are age-j entrepreneur's consumption at $t+1$ in the fundamental and bubbly states, respectively. Equation (23) says that the expected utility-adjusted housing return is equal to the expected utility-adjusted effective capital return as housing bubbles can collapse randomly. This equation shows that the housing return R_{t+1}^{h+} and R_{t+1}^{h-} may not be equal to the capital return in the corresponding state due to the presence of stochastic bubbles.

4.3 Calibration

To calibrate parameter values, we simulate our extended model based on the equilibrium paths when the economy is always in the bubbly state. Suppose that the model economy starts in 2003 and one period in the model corresponds to one year. We focus on the sample period 2003-2013, during which the national housing-price data are available in China (Fang et al. 2015). Some parameters are set exogenously, while the rest are estimated within the model.

[17] Substituting the binding collateral constraints into the budget constraint yields $Q_t^{-} h_{j+1,t+1}^{-} + (1-\xi)k_{j+1,t+1}^{-} + c_{j,t}^{e-} = (R_t^{-} - \xi R^f)k_{j,t}^{-} + \left(Q_t^{-}(1-\delta_h) + r_t\right) h_{j,t}^{-}$, which gives the expression for the effective capital return $\tilde{R}_{t+1}^{-}$.

Parameter	Description
$R^f = 1.003$	Annual interest rate
$g_n = 0.005$	Growth of population
$g_r = 0.005$	Growth of rents
$\tau = 0.13$	Tax rate in nonhousing sector
$\tau_h = 0.16$	Tax rate in housing sector
$\alpha_l = 0.56$	Land income share in housing sector
$\alpha_k = 0.24$	Capital income share in housing sector
$\alpha = 0.54$	Capital income share in nonhousing sector
$\rho = 0.5$	Capital congestion elasticity
$\zeta_b = 0.46$	Share of government expenditure in debt
$\kappa = 0.53$	Share of infrastructure investment in land-sale revenue
$\delta_h = 0.014$	Housing depreciation rate
$\delta_k = 0.1$	Capital depreciation rate
$\delta_a = 0.095$	Infrastructure depreciation rate

Table 3: Parameters estimated outside the model

4.3.1 Parameters Set Exogenously

We start by discussing parameters chosen exogenously as listed in Table 3. The interest rate R^f is set as 1.003, matching the average one-year real deposit rate (Song et al. 2011). Similar to Song et al. (2011) and Chen and Wen (2017), agents enter the economy at age 22 and live for $T = 50$ years, which is consistent with the average life expectancy of 71.4 years from the 2000 Chinese Population Census. Workers retire after working for 30 years. The population growth rate is set to $g_n = 0.5\%$, the average population growth rate during the period 2003-2013 from the National Bureau of Statistics of China (NBSC) data set. The growth rate of rents is set to $g_r = 0.5\%$, the average growth rate of rents for 2003-2013 according to the NBSC data set. Tax rates in the housing and nonhousing sectors are $\tau_h = 0.16$ and $\tau = 0.13$, according to Bai et al. (2006).

We need to identify the housing sector in the data. In the model, Q_tY_{ht} and Y_t represent the value added of the housing and nonhousing sectors, respectively. In the data, we interpret Q_tY_{ht} as aggregate residential investment, r_tH_t as the sum of imputed and market rents, and Y_t as the remainder in the Chinese GDP. Residential investment consists of land-sale revenue, capital income, and labor income in the real estate sector. We set the share of land-sale revenue $\alpha_l = 0.56$ to match the average ratio of the land-sale revenue to residential investment in the data. Following a similar method in Davis and Heathcote (2005), we set the capital income share in the housing sector $\alpha_k = 0.24$ using China's input-output table.[18]

[18] We use capital income share in the construction sector to approximate α_k in our model, as more than two-thirds of production in China's construction sector is housing construction. Unlike Davis and Heathcote (2005), we do not consider manufacturing and services goods to produce new houses. We compute capital share in the construction sector as follows. Although China's input-output table reports capital share α_i for the value added in each sector i, we cannot use α_{constr} directly because construction also uses intermediate goods such as steel and glass. We define

Parameter	Description	Target
$\beta = 0.999$	Discount factor	Average saving rate
$\psi = 0.42$	Wealth transfer share	Capital return in 2003
$\theta = 0.1001$	Output elasticity of infrastructure	Marginal return of infrastructure investment in 2003
$\xi = 0.17$	Leverage ratio of firm	Average capital investment to GDP ratio
$g_e = 0.038$	Growth of labor efficiency	Average GDP growth rate
$g_l = 0.09$	Diminishing speed of land quality	Average residential investment to GDP ratio
$p = 0.012$	Probability of bubble burst	Average housing price growth during 2003-2013
$\zeta_y = 0.1$	Government expenditure/GDP ratio	Average infrastructure investment to GDP ratio
$\xi_g(t) = 2.34$, if $t < 7$	Leverage ratio of government	Average local government debt to GDP ratio during 2003-2008
$\xi_g(t) = 3.01$, if $t \geq 7$	Leverage ratio of government	Average local government debt to GDP ratio during 2009-2013
$K_0 = 1$	Initial capital stock	Output to capital ratio in 2003
$A_0 = 0.37$	Initial infrastructure stock	Infrastructure to capital ratio in 2003
$H_0 = 0.71$	Initial housing stock	Housing stock to capital ratio in 2003
$r_0 = 0.01$	Initial rent	Residential investment to GDP ratio in 2003

Table 4: Parameters calibrated in the model

Similarly, we calibrate the capital income share in the nonhousing sector as $\alpha = 0.54$. Since our model result is insensitive to ρ, we simply set $\rho = 0.5$ (i.e., we assume capital and labor have the same congestion effect). We set depreciation rates $(\delta_k, \delta_a) = (0.10, 0.095)$ to follow Bai et al. (2006) and Jin (2016), and set $\delta_h = 0.014$ to match residential housing's average lifespan of 70 years in China.

capital share α_k as

$$\alpha_k \equiv \sum_{j=1}^{N} g_{\text{constr},j}\alpha_j,$$

where $g_{\text{constr},j}$ is value added of intermediate goods from sector j divided by the total output of the construction sector. Third, we compute $g_{\text{constr},j}$ as follows. In the input-output table, $X = AX + d$, where $X = [X_i]$ is the vector of outputs for each sector, $d = [d_i]$ is the vector of value added, and A is the direct consumption coefficient matrix. Therefore, $X = Bd$, where $B = (I - A)^{-1}$, and X_{constr} is decomposed as $X_{\text{constr}} = \sum_{j=1}^{N} b_{\text{constr},j} d_j$. Therefore, $g_{\text{constr},j} = \frac{b_{\text{constr},j} d_j}{X_{\text{constr}}}$.

4.3.2 Parameters Set within the Model

Now we choose the remaining parameter values within the model to match certain data moments over the sample period 2003-2013 (see Table 4). We set $\beta = 0.999$ to match the average saving rate of 48 percent in China, $\psi = 0.42$ to match the post-tax capital return of 15 percent in 2003, $\theta = 0.1001$ to match the marginal infrastructure-investment return of 23.7 percent in 2003,[19] $\xi = 0.17$ to match the average investment rate of 42 percent, and the growth rate of labor efficiency $g_e = 0.038$ to match the average GDP growth rate of 10 percent for 2003-2013. While the long-run GDP growth rate is equal to $(1 + g_n)(1 + g_e) - 1 = 4.3\%$, the average growth rate during the transition period can be much higher.

Since we will conduct counterfactual experiments in Section 5, we need to calibrate land supply L_t beyond the period 2003-2013. We choose L_t for $0 \leq t \leq 13$ to match the land supply in 2003-2016 taken from the China Land Statistical Yearbook. We normalize the land supply in 2003 to 1 so that $L_0 = 1$. We do not have the land supply data starting from 2017. We assume that the quantity of land supply since 2017 is a constant equal to the average land supply during the period 2003-2016. As pointed out by Davis and Heathcote (2007), to get constant-quality land supply, the quantity of land supply needs to be adjusted by the quality. We assume that the labor quality declines at the rate g_l. We calibrate $g_l = 0.09$ to match the average ratio of residential investment to GDP for 2003-2013.

To study the impact of bubble bursts, we introduce a stochastic bubble similar to Weil (1987). In each period, we set the probability of a bubble burst to $p = 0.012$ to match the average growth rate of housing prices during the period 2003-2013. We find that in the long run the rent-to-housing price ratio converges to zero, and the housing price and its bubble component grow at the same rate 5.8 percent, which is higher than the long-run GDP growth rate of 4.1 percent, according to our calibration. Our model also implies that the housing price can grow much faster during the transition period due to the growing bubble.

The Chinese local government debt to GDP ratio increased over the period 2003-2013.[20] In particular, the average local government debt to GDP ratio increased from 9 percent before 2009 to 17 percent after 2009. This is because, after the U.S. financial crisis in 2009, the Chinese central government implemented a large economic stimulus package, the so-called Four Trillion Project. Over three-fourths of this project's expenditure was financed by local government debt (Bai et al. 2016). Because more than half of that debt was backed by land-sale revenue, the Four Trillion Project only made land sale more important. Moreover, since 2009 Chinese central government

[19] This marginal return of infrastructure investment is estimated by Wu et al. (2021) using Chinese manufacturing firms data. Also, our calibrated $\theta = 0.1001$ is very close to the value of 0.1 estimated by Bom and Ligthart (2014) in OECD countries.

[20] Our local government debt data are from the Audit Report on National Government Debt 2011 and 2013 issued by the National Audit Office of China (NAOC). The data only include the debt that local governments are guaranteed to pay back, but exclude local governments' contingent liability.

allowed local governments to take on more debt through off-balance-sheet companies known as local financing vehicles. To better model this financial deregulation, we allow a structural break in the leverage ratio ξ_g. In particular, we set $\xi_g = 2.34$ before 2009 and $\xi_g = 3.01$ after 2009 to match the average ratio of local government debt to land-sale revenue before and after 2009. We will quantify the impact of financial deregulation in Section 4.5.

The government non-infrastructure expenditure G_t is financed through three sources: output, government debt, and land-sale revenue. We specify the following rule:

$$G_t = \zeta_y Y_t + \zeta_b (B^g_{t+1} - R^f B^g_t) + (1-\kappa) p_{Lt} L_t.$$

Since 54 percent of local government debt was spent on infrastructure investment and the remainder was spent on other expenditures as reported by the National Audit Office of China, we set $\zeta_b = 0.46$. Assume that only a fraction κ of the land-sale revenue is used to finance infrastructure investment and the remaining fraction $1-\kappa$ is used to finance non-infrastructure expenditures. We set $\kappa = 0.53$ to match the average share of infrastructure investment in land-sale revenue in the data. To calibrate ζ_y, we notice that an increase in ζ_y raises the non-infrastructure expenditure and thus reduces the infrastructure investment expenditure by the government budget constraint (20). Then we choose $\zeta_y = 0.1$ to match the average infrastructure investment to GDP ratio of 7.5 percent.

We set the initial condition in the model as follows. First, we normalize the initial population and labor efficiency to 1. Second, we calibrate the initial (K_0, A_0, H_0) to $(1, 0.37, 0.71)$ to match the capital-output ratio, the housing-capital ratio, and the infrastructure-capital ratio in 2003.[21] Third, the initial housing rent is set to $r_0 = 0.01$ to match the ratio (6.5 percent) of the residential investment to GDP in 2003. We can then construct rents r_t for $t \geq 1$ using the average growth rate g_r of rents in the data. Notice that the official data for the growth rates of rents are available, but the data for the level of rents are not. Finally, the initial wealth distribution of entrepreneurs across various generations is set as urban household wealth distribution in 2002 from NBSC.

4.4 Results

Figure 3 presents the data and results based on our calibrated model. While our model is targeted to match either the average values for 2003-2013 or the initial values in 2003 in the data, our model can match both the first moments and the dynamic patterns in the data fairly well due to our model mechanism. In particular, the rapid rise of housing prices is associated with increases in the infrastructure investment to GDP ratio, the land-sale revenue to GDP ratio, and the residential investment to GDP ratio (see Panels D, E, and F). We leave the discussion of GDP growth to the next subsection. Our model does not match the cyclicalities of the data shown in Figure 3. These

[21] Bai et al. (2006) show the capital-output ratio is 1.66 in 2003 and 13 percent of capital is residential housing. Jin (2016) shows 25 percent of total capital is infrastructure in 2003. Note that capital in these studies refers to commercial capital, infrastructure, and housing, while capital K_t in our paper is only commercial capital.

cyclicalities may be due to various business cycle shocks and uncertainties about China's monetary, fiscal, and housing market policies.[22] Our model does not incorporate these features (except for the risk of the bubble bursting) and hence it cannot match the cyclical pattern in the data.

The increase of the residential investment to GDP ratio over time in the model ($Q_t Y_{h,t}/GDP_t$) is due to two effects: the rapid rise of housing prices Q_t and the reallocation of capital and labor to the housing sector. The reallocation effect causes $K_{h,t}$ and $N_{h,t}$ to rise such that $Y_{h,t}$ increases. Since land-sale revenue $p_{Lt}L_t$ is equal to $\alpha_l(1-\tau_h)\left(Q_t Y_{h,t}\right)$ due to the Cobb-Douglas production function, land-sale revenue rises proportionally with residential investment in the model.

As land-sale revenue increases over time, the government can finance more infrastructure investment such that the infrastructure investment to GDP ratio rises over time as in the data. Notice that infrastructure investment rises dramatically in 2009 both in the model and in the data. This is due to the Four Trillion Project in 2009, when the Chinese local government used government debt backed by land-sale revenue to finance infrastructure investment. Our model captures this event by raising the government leverage ratio ξ_g from 2.34 to 3.01 starting in 2009. We are able to match the average ratio of local government debt to GDP during the period 2003-2013 (see Figure 4).

While infrastructure raises productivity, the capital return is high and declines over time (see Figure 3 Panel B). Recall that the capital return in our model satisfies

$$R_t = \alpha(1-\tau)(1-\psi)\hat{A}_t^{\theta} K_{c,t}^{\alpha-1}(e_t N_{c,t})^{1-\alpha} + (1-\delta_k).$$

The decline of capital return during 2003-2013 is mainly driven by the large increase of capital-labor ratio by more than 250 percent during the transition phase with initially low capital stock. The decline of the capital return is partially offset by the existence of housing bubble. Theoretically, the effect of a bubble is ambiguous because it may increase capital return through higher infrastructure and crowding-out of capital, but decrease capital return through reallocating labor out of the nonhousing sector. Quantitatively, we find its effect to be positive but small, i.e., it adds 0.7 percent to the average capital return during 2003-2013.

Next we discuss housing prices presented in Figure 3 Panel C. Our model matches the growing trend of China's housing prices fairly well. In our simulation, the average growth rate of housing prices is 11.4 percent during 2003-2008 and 8.6 percent during 2009-2013, while in the data it is 11 percent and 9 percent. Although the capital return follows a downward trend, the growth rates of housing prices are relatively stable.

It seems puzzling that the growth rate of housing prices stays high on average, while the capital return follows a fast downward trend. To understand the intuition, consider the no-arbitrage

[22] In addition to cyclicalities, China's growth also exhibited transitional dynamics due to the dual-sector nature of its economy. As shown in Song et al. (2011) and Chen and Wen (2017), the labor reallocation from the public to the private sector can temporarily maintain high capital return, which explains why China's GDP growth is high during 2003-2008.

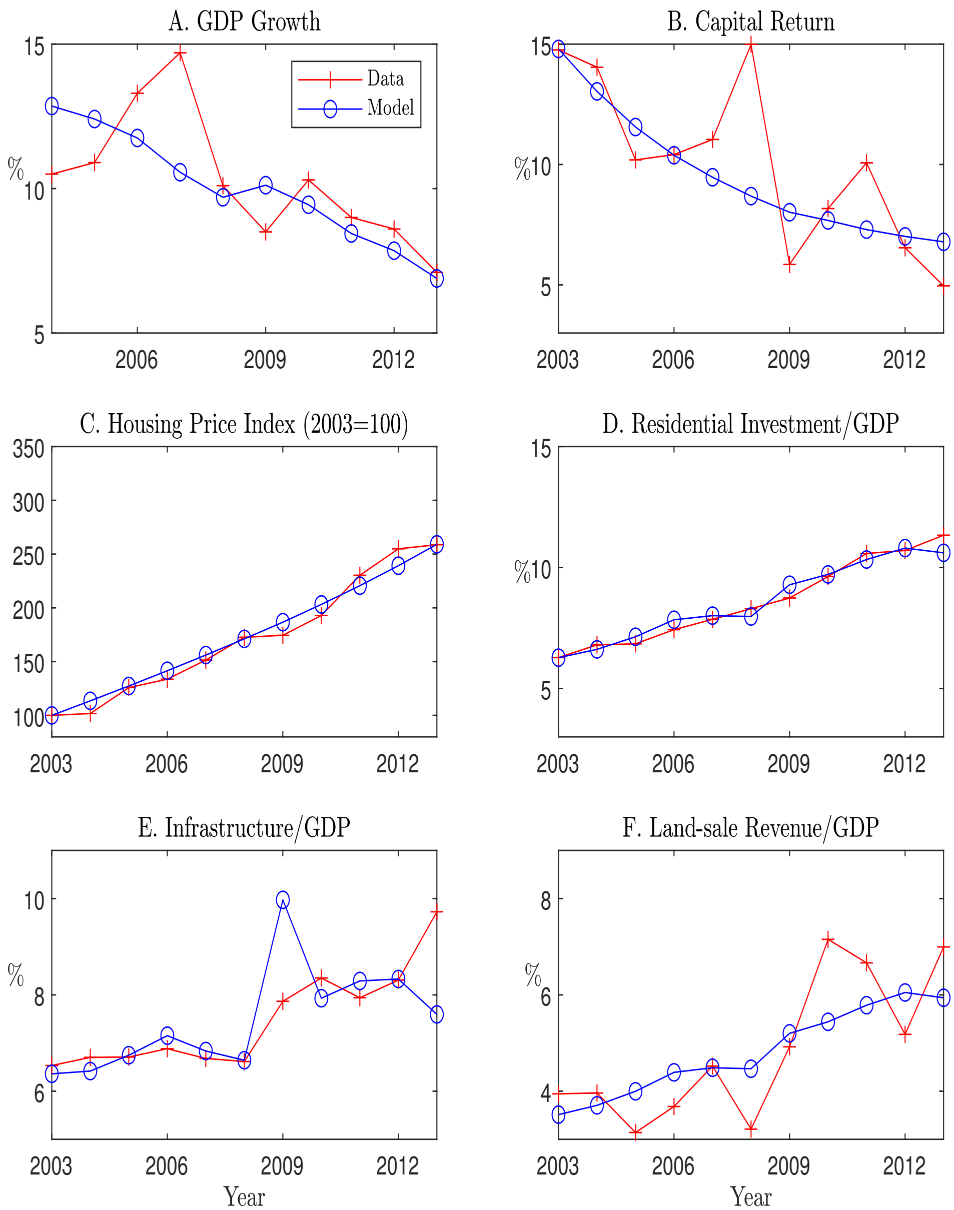

Figure 3: Model results and comparison with the data.

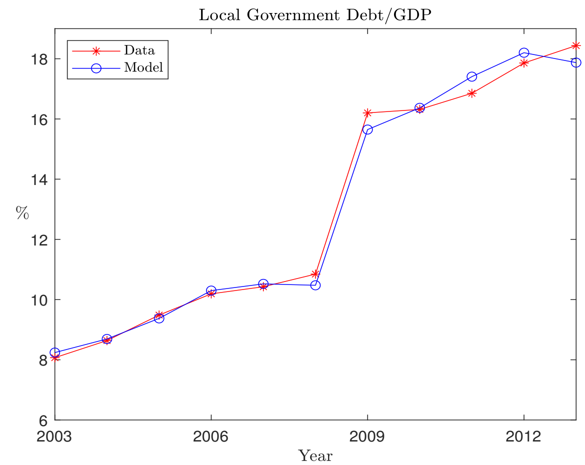

Figure 4: Government debt to GDP Ratio.

equation (23) under risk-neutral utility and zero depreciation of housing ($u'(c) \equiv 1$ and $\delta_h = 0$):

$$\bar{R}_{t+1} = \frac{r_{t+1}}{Q_t^+} + \frac{pQ_{t+1}^- + (1-p)\, Q_{t+1}^+}{Q_t^+}, \tag{24}$$

where $\bar{R}_{t+1}$ denotes the (expected effective) capital return. That is, the expected capital return is equal to the expected housing return, which in turn is equal to the sum of the rent-to-price ratio and expected price appreciation. The rent-to-price ratio is relatively high initially in 2003 and the size of the bubble is also small (i.e., Q_{t+1}^- is close to Q_{t+1}^+). Thus the housing price growth rate Q_{t+1}^+/Q_t^+ is approximately equal to $\bar{R}_{t+1} - r_{t+1}/Q_t^+$, which is less than the capital return $\bar{R}_{t+1}$ in the early years of the 2003-2013 period. As time goes by, both the housing price and the housing bubble grow, but the rents grow at a much lower rate (about 0.5 percent on average in the data). Figure 5 decomposes the housing price into its fundamental and bubble components. The rent-to-price ratio gradually declines to zero in the long run. Thus the fundamental value of housing Q_t^- relative to the bubbly value Q_t^+ approaches zero. In this case, equation (24) implies that the housing price growth rate Q_{t+1}^+/Q_t^+ approaches $\bar{R}_{t+1}/(1-p)$, which is greater than the capital return $\bar{R}_{t+1}$ for $p > 0$. This happens in the later years of the 2003-2013 period in our model. Intuitively, to compensate for the risk of the bubble bursting, the growth rate of housing prices when the bubble never bursts must be higher than the capital return (Weil 1987).

To close this section, we argue that the land-sale revenue channel is essential for our analysis. To see this, let us simply shut down the land-sale revenue channel by assuming that land-sale revenue is transferred to workers without recalibrating the model. We find that the average infrastructure investment to GDP ratio is lowered by 3.5 percentage points, and the average GDP growth rate is lowered by 1 percentage point, compared to our extended model. Moreover we cannot match the increasing pattern of the infrastructure investment to GDP ratio in the data. If we further

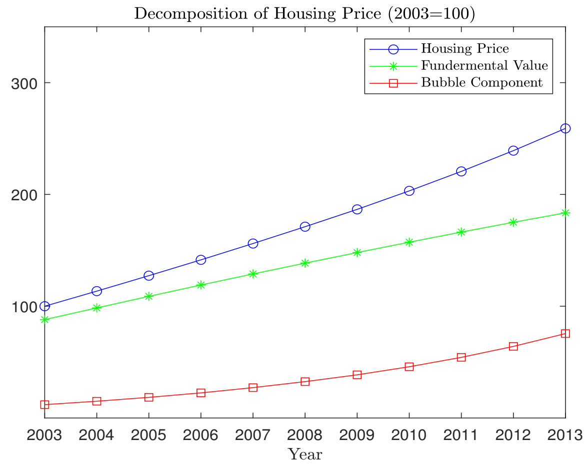

Figure 5: Decomposition of the housing price (2003=100) into its fundamental and bubble components.

recalibrate government spending G_t to match the average infrastructure investment to GDP ratio, we still cannot match the upward trend of this ratio.

4.5 Financial Deregulation

As aforementioned, the Four Trillion Project implemented in 2009 allowed local governments to borrow additional funding for their infrastructure investment. One might think this financial deregulation is so important that it can explain most of the infrastructure boom after 2009. We show below that this is not the case.

To separate the contributions of land-sale revenue and financial deregulation to the infrastructure boom after 2009, we conduct the following experiment. Suppose that there was no financial deregulation in 2009 in the sense that the leverage ratio ξ_g was unchanged after 2009. Then the local government debt to GDP ratio during 2009-2013 would drop from 17.1 percent in the benchmark to 13.4 percent in the counterfactual economy. GDP growth during this period would drop by 0.1 percentage point. The infrastructure boom after 2009 is measured by the 1.7 percentage points increase of the infrastructure investment to GDP ratio in our benchmark model (from 6.7 percent during 2003-2008 to 8.4 percent during 2009-2013).[23] Our counterfactual without financial degregulation and with only land-sale revenue generates an increase of that ratio by 1.2 percentage points, which means that financial deregulation accounts for 0.5 percentage points, or less than a third of the infrastructure boom. We also find that the average infrastructure investment growth during 2009-2013 is 11.4 percent in the benchmark model with financial deregulation, and 11.1 percent in the counterfactual economy without deregulation. We conclude that financial deregulation does not dethrone land-sale revenue as the leading explanation for the Chinese infrastructure

[23] In the data, this ratio is 6.6 percent during 2003-2008 and 8.5 percent during 2009-2013.

Figure 6: Higher land supply after 2009.

boom.

Land-sale revenue accounts for most of the infrastructure boom because local governments dramatically increased their land supply after 2009 and because land prices increased dramatically due to housing bubbles. Figure 6 shows that the supply of residential land nearly doubled after 2009 based on the data from the China Land Statistical Yearbook. The impact on land-sale revenue was even higher because of the ever growing land prices.

4.6 Growth Accounting

In this subsection we discuss GDP growth. Panel A of Figure 3 shows that our model can replicate the average GDP growth rate of 10 percent for 2003-2013, as well as the drop from the highest growth rate of 14 percent to 7 percent during this period. To understand this pattern, we conduct a growth accounting exercise.

Recall that GDP is defined in equation (21). Then we can decompose GDP growth as

$$\frac{\Delta GDP_t}{GDP_t} \approx \frac{Y_t}{GDP_t}\frac{\Delta Y_t}{Y_t} + \frac{Q_t Y_{h,t}}{GDP_t}\frac{\Delta(Q_t Y_{h,t})}{Q_t Y_{h,t}} + \frac{r_t H_t}{GDP_t}\frac{\Delta(r_t H_t)}{r_t H_t}, \tag{25}$$

where $\Delta X_t \equiv X_{t+1} - X_t$ for any variable X_t. Our calibrated model shows that, during the period 2003-2013, the average growth rate of residential investment is 16 percent and the average residential investment to GDP ratio is 8.6 percent, while the average nonhousing output growth rate is only 9.4 percent and the average nonhousing output to GDP ratio is 90 percent. Thus the 10 percent average GDP growth consists of 8.5 percent of nonhousing output growth and 1.4 percent of residential investment growth. Rents contribute only 0.1 percent to GDP growth on average and thus will be ignored in our discussion.[24]

[24]Our model implied average rents to GDP ratio ($r_t H_t / GDP_t$) is about 1.3 percent. This estimate is reasonable

Since aggregate output in the nonhousing sector satisfies

$$Y_t = A_t^{\theta} K_{c,t}^{\alpha-\rho\theta} \left(e_t N_{c,t}\right)^{1-\alpha-(1-\rho)\theta},$$

we can further decompose its growth into

$$\begin{aligned}\frac{\Delta Y_t}{Y_t} &\approx \theta\frac{\Delta A_t}{A_t} + (\alpha-\rho\theta)\frac{\Delta K_{c,t}}{K_{c,t}} + (1-\alpha-(1-\rho)\theta)\frac{\Delta e_t}{e_t} \\ &+ (1-\alpha-(1-\rho)\theta)\frac{\Delta N_{c,t}}{N_{c,t}}.\end{aligned}$$

Based on our calibrated model, we find that

$$\begin{aligned}9.4\% &\approx 0.1 * 10.7\% + 0.49 * 13.7\% + 0.41 * 3.8\% + 0.41 * 0.3\% \\ &\approx 1.1\% + 6.7\% + 1.5\% + 0.1\%.\end{aligned}$$

Thus, to the 9.4 percent average nonhousing sector growth, infrastructure contributes 1.1 percent, capital 6.7 percent, labor efficiency (technology) 1.5 percent, and labor 0.1 percent.

Similarly, we can decompose the residential investment growth into

$$\begin{aligned}\frac{\Delta(Q_t Y_{h,t})}{Q_t Y_{h,t}} &\approx \frac{\Delta Q_t}{Q_t} + \alpha_l\left(\frac{\Delta L_t}{L_t} - g_l\right) + \alpha_k\frac{\Delta K_{h,t}}{K_{h,t}} + (1-\alpha_l-\alpha_k)\frac{\Delta e_t}{e_t} \\ &+ (1-\alpha_l-\alpha_k)\frac{\Delta N_{h,t}}{N_{h,t}}.\end{aligned}$$

Our calibrated model shows that

$$\begin{aligned}16\% &\approx 10.0\% + 0.56 * (-2.1\%) + 0.24 * 21\% + 0.2 * 3.8\% + 0.2 * 6.3\% \\ &\approx 10.0\% + (-1.1\%) + 5.0\% + 0.7\% + 1.3\%.\end{aligned}$$

Thus, to the 16 percent average housing sector growth, housing price contributes 10.0 percent, land -1.1 percent, capital 5 percent, labor efficiency 0.7 percent, and labor 1.3 percent.

To see why GDP growth declined over 2003-2013 in the data, we separate the whole sample into two periods: 2003-2008 and 2009-2013. Average GDP growth is 11.5 percent in the first period, and 8.5 percent in the second period. Using (25), we can decompose these growth rates into

$$\begin{aligned}11.5\% &\approx 0.917 * 10.9\% + 0.073 * 16.9\% + 0.011 * 18.3\% \\ &\approx 10\% + 1.2\% + 0.2\%,\end{aligned}$$

for two reasons: First, total rents are counted as part of the value added in the real estate sector. Since the value added in the real estate sector was 4.4 percent of GDP on average for the period 2003-2013, the rents to GDP ratio should be smaller than 4.4 percent. Second, Bai et al. (2006) estimate that $(K_t + A_t + Q_t H_t)/GDP_t = 1.66$ and $Q_t H_t/(K_t + A_t + Q_t H_t) = 13\%$ for 2003. Thus we have

$$\frac{r_t H_t}{GDP_t} = \frac{r_t}{Q_t}\frac{Q_t H_t}{K_t + A_t + Q_t H_t}\frac{K_t + A_t + Q_t H_t}{GDP_t} = 0.22\frac{r_t}{Q_t}$$

for 2003. Since the rents to price ratio (r_t/Q_t) is around 3 percent to 10 percent, $r_t H_t/GDP_t$ is around 0.66 percent to 2.2 percent for 2003.

Variable (%)	$\Delta A/A$	$\Delta K_c/K_c$	$\Delta N_c/N_c$	$\Delta Q/Q$	$\Delta L/L - g_l$	$\Delta K_h/K_h$	$\Delta N_h/N_h$
2003-2008	8.9	17.4	0.3	11.3	-3.6	23.7	5.7
2009-2013	12.5	10.2	0.2	8.6	-0.5	17.4	6.8
2003-2013	10.7	13.7	0.3	10.0	-2.1	20.5	6.2

Table 5: Growth accounting based on the calibrated model.

and

$$\begin{aligned} 8.5\% &\approx 0.885 * 7.8\% + 0.101 * 14.9\% + 0.014 * 12.2\% \\ &\approx 6.9\% + 1.5\% + 0.2\%. \end{aligned}$$

This decomposition shows that the decline of GDP growth is attributed mainly to the decline of the weighted average nonhousing sector growth from 10 percent to 6.9 percent, while the weighted average housing sector growth increases from 1.2 percent to 1.5 percent.

To see what drives the change of growth pattern from 2003-2008 to 2009-2013, we present the decomposition in terms of factor inputs in Table 5. We find that the decline of the nonhousing sector growth is attributed mainly to the decline of capital growth from 17.4 percent to 10.2 percent, while infrastructure investment growth rises from 9.0 percent to 12.5 percent. Given the rapid rise of housing prices, aggregate capital is crowded out so that capital growth in both housing and nonhousing sectors declines. Capital is also reallocated from the nonhousing sector to the housing sector so that the weight of housing output in GDP increases from 0.073 to 0.101. The increase in the weighted average housing sector growth from 1.2 percent to 1.5 percent partially offsets the decline of GDP growth.

5 Counterfactual Experiments

Due to the dramatic growth trend of housing prices, Chinese policymakers and academic researchers are concerned that housing prices might contain a bubble. Thus they want to understand how much the collapse of a bubble might damage the economy. Chinese leaders are also discussing the potential benefit of implementing a property tax to control housing prices. In this section we use our calibrated model to study the potential impact of the collapse of a housing bubble and the impact of a property tax.

5.1 If the Bubble Bursts

Suppose that the economy stays in the bubbly state until the housing bubble bursts in 2025, then stays in the fundamental state forever. Figure 7 Panel A plots the growth rates of housing prices in the two economies: one with and one without the burst bubble. In the first case, immediately after

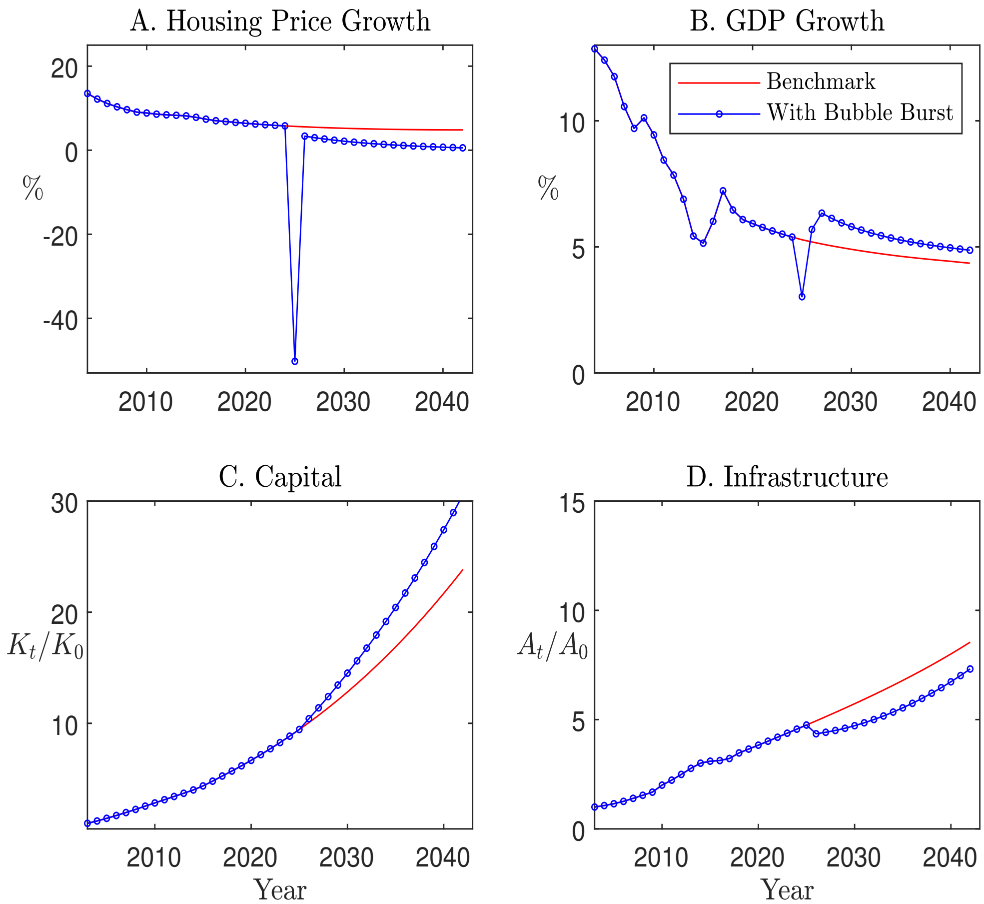

Figure 7: Counterfactual experiment of a future bubble burst.

the burst, the growth rate of housing prices drops from 5.7 percent to −50.2 percent. In the next 30 years on the transition path, it is 1.1 percent on average, much lower than the average growth rate of 5.0 percent without the burst. This can be explained by the low rent growth rate of 0.5 percent. On a balanced growth path in the fundamental state, the growth rate of the bubbleless housing price is equal to the rent growth rate. Thus the average growth rate of the housing price is low during the transition period.

Figure 7 Panel B shows how the bubble burst would affect GDP. After the bubble bursts in 2025, the growth rate of GDP drops from 5.3 percent to 3 percent. This 3 percent GDP growth consists of a 7.1 percent increase of output in the nonhousing sector and a 73.9 percent decrease in the housing sector. Nonhousing output increases because capital and labor flow back from the housing sector into the nonhousing sector, while housing output decreases because newly built houses lose value. Despite the large drop of housing prices, its impact on GDP growth is relatively small because the housing sector accounts for a small share of GDP in 2025. The rise of nonhousing output offsets the large decline of housing output.[25]

One year after the bubble bursts, however, the GDP growth rate is higher than in the case when the bubble never bursts. In the next 30 years the average GDP growth rate after the burst is 0.5 percentage points higher. In the long run on the balanced growth path, GDP growth rates in the two economies with and without a bubble are both equal to the sum of population growth and technology growth. But the bursting of the housing bubble has a level effect. In particular, in 2055, GDP and nonhousing output after the burst are 12.4 percent and 13.6 percent higher than in the case without the burst.

GDP in 2055 is higher after the bubble bursts than it is without the burst due to the following effects (see the second example in Section 3.7 for a similar discussion). After the burst of the bubble, infrastructure investment is reduced because land-sale revenue has declined, but aggregate capital is unleashed: in the 2025-2055 period, the average growth rate of infrastructure decreases from 3.6 percent to 3.4 percent, while the average growth rate of capital increases from 5.2 percent to 6.4 percent (see Panels C and D of Figure 7). This higher growth rate of capital raises GDP.

5.2 Property Tax

The Chinese government has not adopted a comprehensive property tax so far. In this subsection we estimate what would happen if the Chinese government initiated a permanent linear property tax on the entire housing stock in 2025. In our benchmark, the tax rate is 1.5 percent and this tax policy is unexpected by all agents in the model. Since Chinese policymakers have discussed that the property tax revenue can be used to finance the local government spending on infrastructure investment, we simply assume that all property tax revenue is used to finance infrastructure investment. We focus on the equilibrium paths both before and after the tax policy when housing bubbles never burst.[26]

Figure 8 Panel A plots housing prices after the property tax is imposed. The property tax generates a negative wealth effect, which reduces entrepreneurs' housing demand. On impact, the housing price drops by 36.1 percent and its growth rate in 2025 drops from 5.7 percent to -32.6 percent. From 2026 to 2055, the average growth rate of housing prices is higher by 0.3 percentage points than that without the property tax. In the long run, the growth rate of housing prices with the property tax is the same as that without it.

In Figure 8 Panel B, we show how the property tax would change GDP. Based on our simulation, after the property tax is imposed, GDP drops immediately by 1.6 percent compared with the case without the property tax. This 1.6 percent GDP drop consists of a 1.0 percent increase of output in the nonhousing sector and a 55.1 percent decrease in the housing sector. During the 2026-2055

[25] The relatively small impact is also due to the missing collateral channel of housing prices in our model. Unlike the crash in the 1989 Japanese housing market, where reduced land price caused decline in credit and inefficient bankruptcies, this collateral channel is negligible in China because very few Chinese firms use residential land or housing as collateral.

[26] Miao et al. (2015) show that, when the property tax rate is sufficiently high, a housing bubble can never emerge.

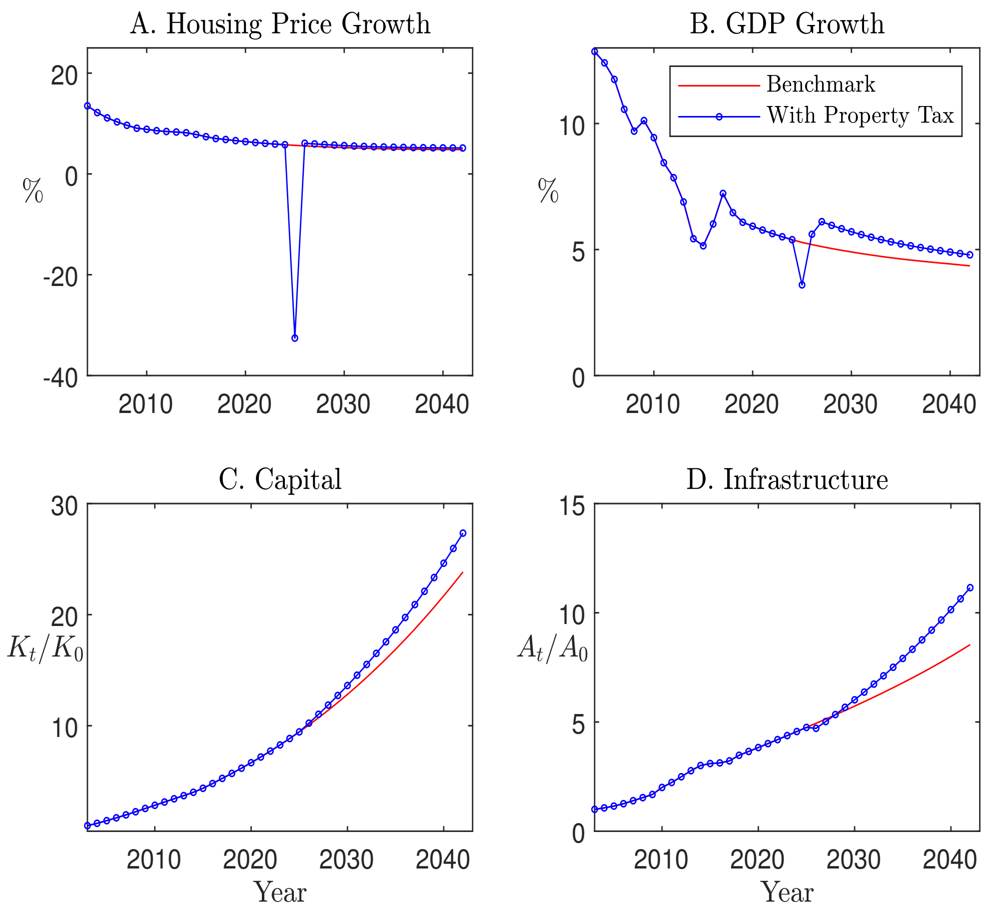

Figure 8: Counterfactual experiment of a future property tax.

period the average growth rate of GDP increases from 4.5 percent to 5.0 percent. In 2055, 30 years after the property tax is started, GDP and nonhousing output are 14.3 percent and 15.0 percent, respectively, higher than they would be without the property tax. The reason that the long-run GDP with the property tax is higher than that without is due to the following three effects. First, the property tax encourages capital accumulation because entrepreneurs invest less in housing assets. The average capital growth increases from 5.2 percent to 5.9 percent (see Panel C of Figure 8). Second, the property tax also increases infrastructure accumulation as we have assumed that the tax revenue is used to finance infrastructure investment. The average infrastructure growth increases from 3.6 percent to 5.0 percent (see Panel D of Figure 8). Third, with the decline of housing prices, more capital and labor are reallocated from the housing sector to the nonhousing sector. The average proportions of capital and labor in the nonhousing sector increase by 0.55 and 0.54 percentage points, respectively.

5.3 Welfare Effects

In this subsection we study the welfare effects of the above two counterfactual experiments on both workers and entrepreneurs alive in 2025. First, consider the impact of the bubble bursting in 2025, presented in Panel A of Figure 9. We measure the welfare change as a percentage deviation in lifetime consumption from the equilibrium in which the housing bubble never bursts.

The oldest cohort living in 2025 enters the economy in 1976 at age 22 in the model. The bubble bursting does not affect wages before 2025, but decreases the wage rate in 2025 because workers flow into the nonhousing sector. The wage rate rises after 2025 because more resources are allocated to capital accumulation such that the marginal product of labor rises. As a result, all workers born before 1996 in our model do not experience welfare changes because these cohorts of workers retire before the bubble bursts. Their lifetime income, which is the present value of wages, is unchanged. Workers born in 1996 suffer a tiny welfare loss as their wages decline in 2025 only. Workers born after 1996 experience welfare gains, because they enjoy an increase in the wage rate during their working periods. The younger the workers are, the larger their welfare gains due to their ability to work for a longer period of time.

By contrast, all entrepreneurs alive in 2025 suffer welfare losses.[27] This is because of the permanent loss of housing values and the decline of capital returns after the bubble bursts. The welfare jump of the latest cohort in 2025 is because the newborn entrepreneurs in our model do not hold any housing assets and start owning houses in subsequent years. Except for this cohort, the younger the entrepreneurs are, the longer they face declining capital returns, and therefore the

[27] Both our paper and Glover et al. (2020) show that a decline of housing price hurts homeowners more than it hurts non-homeowners. The difference is that, in our model, non-homeowners (i.e., workers) benefit from higher labor income, while in Glover et al. (2020) non-homeowners (i.e., renters in the younger cohorts) benefit from the opportunity of buying houses at depressed prices.

Figure 9: Welfare effects.

Variable (%)	Benchmark	$\theta = 0.2$	$\rho = 0.75$	$\rho = 0.25$
GDP growth	10.0	9.0	9.6	10.4
Housing price growth	10.0	8.9	9.8	10.2
Nonhousing sector growth	9.5	8.4	9.0	9.8
Housing sector growth	16	13.8	15.7	16.2
Capital growth	14.0	11.9	13.7	14.3
Infrastructure growth	10.7	9.4	10.6	10.9

Table 6: Sensitivity analysis.

larger their welfare losses.

There are two main differences between the welfare result of Chen and Wen (2017) and ours. First, the wage rate declines in the year when the bubble bursts in our model due to the labor reallocation effect, which is absent in Chen and Wen (2017). Second, the capital return in Chen and Wen (2017) is constant during the transition stage and the bubble burst lowers the capital return only in the post-transition stage. By contrast, the capital return in our model immediately drops so that the welfare losses of entrepreneurs predicted in our model are larger than in Chen and Wen (2017).

Next we consider the welfare effects of the property tax, presented in Panel B of Figure 9. We measure the welfare change as a percentage deviation in lifetime consumption from the equilibrium without a property tax to the equilibrium with the property tax studied earlier. We find that the result is similar to that discussed for the case of the burst bubble, except that the magnitude here is smaller. The intuition is also similar.

6 Sensitivity Analysis

In this section we conduct a sensitivity analysis. Since infrastructure plays an important role in our model, one may wonder whether our results are sensitive to changes in parameters such as the productivity elasticity parameter θ and the congestion effect parameter ρ.

In our benchmark quantitative model we set the elasticity of infrastructure $\theta = 0.1001$ to match the marginal infrastructure-investment return in China. Now we double the value of θ, holding all other parameter values fixed, and report the results in Table 6 column 2. We find that the average growth rates for 2003-2013 of GDP, housing prices, nonhousing output, housing output, aggregate capital, and infrastructure investment all decline with θ. The reason is that a higher value of θ increases not only infrastructure productivity but also the congestion effect of capital and labor. Since capital accumulation is the main driving force of Chinese GDP growth, the congestion effect dominates the increase in infrastructure productivity, causing GDP growth to slow down. This in turn causes all other growth rates reported in Table 6 column 2 to decline.

Next we report the sensitivity analysis of ρ in Table 6 columns 3 and 4, holding all other parameter values fixed. Although we choose $\rho = 0.5$ in our benchmark model, our results are not sensitive to this choice. In particular, GDP growth decreases slightly in Table 6 when ρ increases. Higher ρ strengthens the congestion effect of capital, makes capital less productive, and drives down GDP growth. But higher ρ also weakens the congestion effect of labor, which partially offsets the former negative effect.

In all cases studied above, we have not recalibrated the model to match the same data moments as in our benchmark model. When we recalibrate the model, we find that the new results are almost identical to those in the benchmark.

7 Conclusion

In this paper we study the impact of Chinese housing bubbles on infrastructure investment and economic growth in a two-sector OLG model. Our calibrated model can match the Chinese data reasonably well. Our study makes three contributions. First, we identify a new crowding-in effect of housing bubbles, by introducing a land-sale channel unique to the Chinese economy. With this channel, our model can explain the boom of infrastructure investment in China. Second, we quantify the effects of a bubble bursting and find that, although the crash represents a big negative shock to investors' wealth, the effect on China's real GDP is relatively small due to the reallocation effect on capital and labor. Third, imposing a property tax and using the tax revenue to finance infrastructure investment can lower housing prices and reallocate resources from the housing sector to the nonhousing sector, thereby raising the long-run GDP level.

We have not considered the collateral channel of housing prices in the nonhousing sector because this channel seems weak in the Chinese data (Wu et al. 2015). If nonhousing firms use houses or land as collateral to borrow to finance capital investment, changes in housing prices can have a large impact on nonhousing output and hence GDP (Kiyotaki and Moore 1997). Given that small firms are more likely to use houses as collateral and their investment accounts for a small share of aggregate investment, we expect that incorporating the collateral channel would not change our results significantly. Further study of this issue would be an interesting topic for future research.

Appendix

A The Regression in Section 2.2

Traditional estimation of housing supply elasticity (e.g., Saiz 2010, Mian et al. 2013, and Mian and Sufi 2014) relies on some natural environment variables, such as land supply constraints due to water or steep-slope mountains. As criticized by Davidoff (2016) and Guren et al. (2021), this estimation is correlated with many housing demand factors. For example, good natural environment such as ocean/mountain views may attract highly skilled worker or investment, which bring about higher productivity growth and housing demand. Guren et al. (2021) propose a refinement of the Saiz instrument of housing supply elasticity, by separating all U.S. cities into several geographic regions and regressing the city-level housing price to the regional housing price to obtain a local sensitivity instrument. They argue that this sensitivity instrument could proxy (the inverse of) housing supply elasticity. Their intuition is that if a city's housing price is more sensitive to the regional housing price, then this city's housing supply elasticity is lower.

We separate all Chinese provinces into seven regions (North East, North, Middle, East, South, Southwest and Northwest) by traditional geographic definition. We construct regional housing prices by taking the average of housing prices for provinces in the same region. In the first-stage regression to construct our instrumental variable, we regress local (i.e., province-level) housing price on regional housing price,

$$\Delta Q_{i,t} = \psi_i + \beta_i * \Delta Q_{r,t} + \Gamma * X_{i,t} + \epsilon_{i,t},$$

where ψ_i is the intercept term of province i, $\Delta Q_{i,t}$ and $\Delta Q_{r,t}$ are the log annual change of housing prices in province i and in region r respectively, $X_{i,t}$ is a set of controls, and $\epsilon_{i,t}$ is the error term. Following Guren et al. (2021), we construct the sensitivity instrument $\Delta\hat{Q}_{it} \equiv \hat{\beta}_i \Delta Q_{r,t}$ to proxy (the inverse of) housing supply elasticity, where $\hat{\beta}_i$ is the estimate of β_i. For regressions in Table 1, we use $\Delta\hat{Q}_{it}$ as an instrument for housing prices.

B The Basic Model in Section 3

B.1 Equilibrium Dynamics

The bubbly equilibrium of the basic model can be summarized by the following system of 12 nonlinear difference equations for $t \geq 0$:

$$A_{t+1} = (1-\delta_a)A_t + \tau Y_t + p_{Lt}L_t, \tag{B.1}$$

$$H_{t+1} = (1-\delta_h)H_t + Y_{ht}, \tag{B.2}$$

$$K_{t+1} = \frac{\beta}{1+\beta}M_t - Q_t H_{t+1}, \tag{B.3}$$

$$\text{(B.4)} \qquad R_{t+1} = \frac{Q_{t+1}(1-\delta_h)}{Q_t},$$

$$\text{(B.5)} \qquad R_t = \alpha(1-\tau)(1-\psi)\left[A_t/(K_t^{\rho}N_{c,t}^{1-\rho})\right]^{\theta} K_t^{\alpha-1}N_{c,t}^{1-\alpha},$$

$$\text{(B.6)} \qquad w_t = (1-\alpha)(1-\tau)\left[A_t/(K_t^{\rho}N_{c,t}^{1-\rho})\right]^{\theta} K_t^{\alpha}N_{c,t}^{-\alpha},$$

$$\text{(B.7)} \qquad w_t = (1-\alpha_l)Q_t L_t^{\alpha_l}N_{h,t}^{-\alpha_l},$$

$$\text{(B.8)} \qquad p_{Lt} = \alpha_l Q_t L_t^{\alpha_l-1}N_{h,t}^{1-\alpha_l},$$

$$\text{(B.9)} \qquad 1 = N_{c,t}+N_{h,t},$$

$$\text{(B.10)} \qquad Y_t = \left[A_t/(K_t^{\rho}N_{c,t}^{1-\rho})\right]^{\theta} K_t^{\alpha}N_{c,t}^{1-\alpha},$$

$$\text{(B.11)} \qquad Y_{h,t} = L_t^{\alpha_l}N_{h,t}^{1-\alpha_l},$$

$$\text{(B.12)} \qquad M_t = \psi\alpha(1-\tau)Y_t,$$

for 12 sequences of aggregate variables

$$\{R_t, w_t, p_{Lt}, N_{c,t}, N_{h,t}, Y_t, Y_{h,t}, M_t, K_t, A_t, H_t, Q_t\}_{t=0}^{\infty}.$$

The variables A_t, K_t, and H_t are predetermined and all other variables are nonpredetermined.

Equations (B.1)-(B.2) follow from the definitions of A_{t+1} and H_{t+1}. Equation (B.3) defines the capital holding of young entrepreneurs at t, where M_t given in (B.12) is the total initial endowment of young entrepreneurs derived from (4). Equation (B.4) is the no-arbitrage condition. Equations (B.5)-(B.8) are the firm's first-order conditions with respect to k_t, $n_{c,t}$, $n_{h,t}$, and l_t, respectively. Equation (B.9) is the labor market clearing condition. Equations (B.10) and (B.11) follow from the definitions of Y_t and $Y_{h,t}$.

Our proofs actually rely on a two-variable system, simplified from the system (B.1)-(B.12). We discuss this simplified system next.

B.2 Dynamics of $(H_t, \frac{K_t}{Q_{t-1}})$ in a Simplified System

First, we show that $N_{h,t}$ can be written as a decreasing function of $\frac{K_t}{Q_{t-1}L_t^{\alpha_l}}$, which is useful when we study the equilibrium dynamics below.

Lemma 1 *In any equilibrium,* $N_{h,t} = f\left(\frac{K_t}{Q_{t-1}L_t^{\alpha_l}}\right)$ *for* $t \geq 1$, *where* $f(\cdot)$ *is a fixed strictly decreasing function.*

Proof: Equations (B.6) and (B.7) imply

$$\text{(B.13)} \qquad (1-\alpha)(1-\tau)\left[A_t/(K_t^{\rho}N_{c,t}^{1-\rho})\right]^{\theta} K_t^{\alpha}N_{c,t}^{-\alpha} = (1-\alpha_l)Q_t L_t^{\alpha_l}N_{h,t}^{-\alpha_l}.$$

Substituting (B.5) into (B.13) and simplifying the latter equation, we have

$$\text{(B.14)} \qquad \frac{N_{h,t}^{\alpha_l}}{1-N_{h,t}} = \frac{(1-\alpha_l)\alpha(1-\psi)}{(1-\alpha)\frac{R_t K_t}{Q_t L_t^{\alpha_l}}} = \frac{(1-\alpha_l)\alpha(1-\psi)}{(1-\alpha)(1-\delta_h)\frac{K_t}{Q_{t-1}L_t^{\alpha_l}}},$$

where the second equality uses the no-arbitrage condition (B.4). Because $\frac{N_{h,t}^{\alpha_l}}{1-N_{h,t}}$ is strictly increasing in $N_{h,t} \in (0,1)$, the above equation defines $N_{h,t} \in (0,1)$ as a strictly decreasing function of $\frac{K_t}{Q_{t-1}L_t^{\alpha_l}}$.
■

Second, we show that the equilibrium dynamics of $(H_t, \frac{K_t}{Q_{t-1}})$ satisfy a system of two difference equations for $t \geq 1$:

$$\text{(B.15)} \qquad H_{t+1} = (1-\delta_h)H_t + L_t^{\alpha_l} f\left(\frac{K_t}{Q_{t-1}L_t^{\alpha_l}}\right)^{1-\alpha_l},$$

$$\text{(B.16)} \qquad \frac{K_{t+1}}{Q_t} = \frac{1}{z}\frac{K_t}{Q_{t-1}} - H_{t+1},$$

where z is defined in (13) and $f(\cdot)$ is from Lemma 1.

Since (B.15) follows directly from $H_{t+1} = (1-\delta_h)H_t + Y_{h,t}$ and Lemma 1, we shall focus our discussion on (B.16). The initial wealth m_{t+1} of an entrepreneur born in $t+1$ satisfies

$$m_{t+1} = \alpha(1-\tau)\psi \hat{A}_{t+1}^{\theta} K_{t+1}^{\alpha} N_{h,t+1}^{1-\alpha} = \frac{\psi}{(1-\psi)} R_{t+1}K_{t+1},$$

where the second equality uses (B.5). Therefore,

$$\begin{aligned} \frac{K_{t+1}}{Q_t} + H_{t+1} &= \frac{K_{t+1} + Q_t H_{t+1}}{Q_t} = \frac{\beta}{(1+\beta)}\frac{m_t}{Q_t} \\ &= \frac{\psi\beta}{(1-\psi)(1+\beta)}\frac{R_t K_t}{Q_t} = \frac{(1-\delta_h)\psi\beta}{(1-\psi)(1+\beta)}\frac{K_t}{Q_{t-1}}, \end{aligned}$$

where the second equality uses (2) and the last equality follows from the no-arbitrage condition (B.4).

In period $t = 0$, we have

$$R_0 = \alpha(1-\tau)(1-\psi)\left[A_0/(K_0^{\rho}N_{c,0}^{1-\rho})\right]^{\theta} K_0^{\alpha-1}N_{c,0}^{1-\alpha},$$

and the last equality in (B.14) does not hold because (B.4) does not hold for R_0. Using (B.13) for $t = 0$ and $N_{c,0} + N_{h,0} = 1$, we can show that Q_0 is a function of $N_{h,0}$.

Proof of Proposition 1

First, we show (14). In a bubbly steady state, equations (B.15)-(B.16) become

$$\text{(B.17)} \qquad H^b = (1-\delta_h)H^b + (L^*)^{\alpha_l}(N_h^b)^{1-\alpha_l},$$

$$\text{(B.18)} \qquad \frac{K^b}{Q^b} + H^b = \frac{K^b}{Q^b}/z,$$

where $N_h^b = f\left(\frac{K^b}{Q^b(L^*)^{\alpha_l}}\right)$. It follows from (B.18) that $H^b = (1/z-1)\frac{K^b}{Q^b}$, which implies

$$\phi^b = \frac{H^b Q^b}{H^b Q^b + K^b} = 1 - z.$$

Second, we show (15). It follows from the definition of $f(\cdot)$ in Lemma 1 that

$$\frac{(N_h^b)^{\alpha_l}}{N_c^b} = \frac{(1-\alpha_l)\alpha(1-\psi)}{(1-\alpha)(1-\delta_h)\frac{K^b}{Q^b(L^*)^{\alpha_l}}},$$

which is rewritten as

$$\frac{K^b}{Q^b(L^*)^{\alpha_l}(N_h^b)^{1-\alpha_l}}\frac{1-N_c^b}{N_c^b} = \frac{(1-\alpha_l)\alpha(1-\psi)}{(1-\alpha)(1-\delta_h)}.$$

Substituting

$$\frac{K^b}{Q^b} = \frac{z}{1-z}H^b = \frac{z}{(1-z)\delta_h}(L^*)^{\alpha_l}(N_h^b)^{1-\alpha_l}$$

into the above equation, we have

$$\frac{z}{(1-z)\delta_h}\frac{1-N_c^b}{N_c^b} = \frac{(1-\alpha_l)\alpha(1-\psi)}{(1-\alpha)(1-\delta_h)},$$

which implies (15).

Third, we can derive that

$$p_L^b L^* = \alpha_l Q^b (L^*)^{\alpha_l}(N_h^b)^{1-\alpha_l} = \alpha_l \delta_h Q^b H^b = \alpha_l \delta_h \phi^b \frac{\beta}{1+\beta}\alpha\psi(1-\tau)Y^b,$$

where the first equality follows from (B.8), the second from (B.17), and the last from (5). Then equation (16) follows from the above equation and (6).

Finally, equation (17) follows from (5).

Proof of Proposition 2

We will show that, for any given (K_0, A_0, H_0), there exists a unique $Q_0 > 0$ such that the system (B.1)-(B.12) starting from (K_0, A_0, H_0, Q_0) converges to a bubbly steady state.[28] For simplicity, we first focus on the simplified system (B.15)-(B.16) for $(H_t, \frac{K_t}{Q_{t-1}})$, and extend this system to period $t = 0$ by introducing a variable Q_{-1}. We show that there exists a unique Q_{-1} starting from which the system (B.15)-(B.16) converges.

The proof consists of three steps. Step 1 discusses initial conditions from which the system (B.15)-(B.16) will diverge. Step 2 shows that there exists a unique Q_{-1} from which the system (B.15)-(B.16) converges to a steady state. Step 3 shows that this unique Q_{-1} implies a unique Q_0.

[28] At the beginning of period 0, if (K_0, A_0, H_0, Q_0) are known, then $(R_0, w_0, p_{L0}, N_{10}, N_{20}, Y_0, Y_{h0}, M_0)$ can be solved as functions of (K_0, A_0, H_0, Q_0) from (B.5)-(B.12).

Step 1. To simplify notation, we denote $\frac{K_t}{Q_{t-1}}$ by X_t in the following proof. We introduce two sets of initial conditions from which the system (B.15)-(B.16) will eventually diverge. In particular, we define sets $\mathcal{U}_t$ and $\mathcal{L}_t$ as follows:

$$\begin{aligned}
\mathcal{U}_t &\equiv \{(H,X) : H'(H,X,\overline{L}_t) < H^*(\underline{L}_t) \text{ and } X'(H,X,\overline{L}_t) > X^*(\underline{L}_t)\}, \\
\mathcal{L}_t &\equiv \{(H,X) : H'(H,X,\underline{L}_t) > H^*(\overline{L}_t) \text{ and } X'(H,X,\underline{L}_t) < X^*(\overline{L}_t)\},
\end{aligned}$$

where $(H^*(L), X^*(L))$ denote the steady state when the land supply is always equal to L, and $\underline{L}_t \equiv \inf_{s\geq t} L_s$, $\overline{L}_t \equiv \sup_{s\geq t} L_s$. Here H' and X' represent the right-hand sides of (B.15) and (B.16),

$$\begin{aligned}
H'(H,X,L) \equiv &\quad (1-\delta_h)H + L^{\alpha_l} f\left(\frac{X}{L^{\alpha_l}}\right)^{1-\alpha_l}, \\
X'(H,X,L) \equiv &\quad \frac{1}{z}X - (1-\delta_h)H - L^{\alpha_l} f\left(\frac{X}{L^{\alpha_l}}\right)^{1-\alpha_l}.
\end{aligned}$$

The divergence of $\mathcal{U}_t$ and $\mathcal{L}_t$ is verified in the following lemma.

Lemma 2 *If $(H_t, X_t) \in \mathcal{U}_t$, then $\lim_{s\to\infty} X_s = \infty$. If $(H_t, X_t) \in \mathcal{L}_t$, then $X_s < 0$ for finite s.*

Proof: Suppose $(H_t, X_t) \in \mathcal{U}_t$. Because H' is increasing in L and X' is decreasing in L,

$$\begin{aligned}
H_{t+1} &= H'(H_t, X_t, L_t) \leq H'(H_t, X_t, \overline{L}_t) < H^*(\underline{L}_t), \\
X_{t+1} &= X'(H_t, X_t, L_t) \geq X'(H_t, X_t, \overline{L}_t) > X^*(\underline{L}_t).
\end{aligned}$$

By induction, we can show that $H_s < H^*(\underline{L}_t)$ and $X_s > X^*(\underline{L}_t)$ for all $s \geq t+1$. It follows from (B.16) that for all $s \geq t+1$,

$$X_{s+1} - X^*(\underline{L}_t) = \frac{X_s - X^*(\underline{L}_t)}{z} - H_{s+1} + H^*(\underline{L}_t) > \frac{X_s - X^*(\underline{L}_t)}{z},$$

which implies $\lim_{s\to\infty} X_s = \infty$ since $z < 1$. Similarly, if $(X_t, H_t) \in \mathcal{L}_t$, then by induction we can show $X_s < X^*(\overline{L}_t)$ and $H_s > H^*(\overline{L}_t)$ for all $s \geq t+1$. Moreover,

$$X_{s+1} - X^*(\overline{L}_t) < \frac{X_s - X^*(\overline{L}_t)}{z},$$

which implies $X_s < 0$ for finite s. ■

The following alternative definitions of $\mathcal{U}_t$ and $\mathcal{L}_t$ are used in the proof of Lemma 3 below. The conditions $H'(H,X,\overline{L}_t) < H^*(\underline{L}_t)$ and $X'(H,X,\overline{L}_t) > X^*(\underline{L}_t)$ are, respectively,

$$H < \frac{H^*(\underline{L}_t) - \overline{L}_t^{\alpha_l} f\left(\frac{X}{\overline{L}_t^{\alpha_l}}\right)^{1-\alpha_l}}{1-\delta_h}, \quad H < \frac{X/z - X^*(\underline{L}_t) - \overline{L}_t^{\alpha_l} f\left(\frac{X}{\overline{L}_t^{\alpha_l}}\right)^{1-\alpha_l}}{1-\delta_h}.$$

Therefore,

$$\mathcal{U}_t \equiv \left\{ (H, X) : H < \frac{\min\{H^*(\underline{L}_t), X/z - X^*(\underline{L}_t)\} - \overline{L}_t^{\alpha_l} f\left(\frac{X}{\overline{L}_t^{\alpha_l}}\right)^{1-\alpha_l}}{1-\delta_h} \right\}.$$

Similarly,

$$\mathcal{L}_t \equiv \left\{ (H, X) : H > \frac{\max\{H^*(\overline{L}_t), X/z - X^*(\overline{L}_t)\} - \underline{L}_t^{\alpha_l} f\left(\frac{X}{\underline{L}_t^{\alpha_l}}\right)^{1-\alpha_l}}{1-\delta_h} \right\}.$$

Step 2. We show a unique X_0 from which the system converges. Above X_0, the system enters $\mathcal{U}_t$ eventually. Below X_0, the system enters $\mathcal{L}_t$ eventually.

Lemma 3 *For any $H_0 > 0$, there exists a unique X_0 such that the system starting from (H_0, X_0) converges to $(H^*(L^*), X^*(L^*))$, where L^* is the land supply in the long run. The system starting from $\tilde{X}_0 > X_0$ satisfies $\lim_{t\to\infty} \tilde{X}_t = \infty$, and that from $\tilde{X}_0 < X_0$ satisfies $\tilde{X}_t < 0$ for some $t > 0$.*

Proof: For any $H_0 > 0$, define sets A and B as follows.

$$\begin{aligned} A \equiv \quad & \{X_0 : \quad \text{the system starting from } (H_0, X_0) \text{ satisfies } \lim_{t\to\infty} X_t = \infty\}, \\ B \equiv \quad & \{X_0 : \quad \text{the system starting from } (H_0, X_0) \text{ satisfies } X_t < 0 \text{ for some } t\}. \end{aligned}$$

First, both A and B are nonempty but $A \cap B = \emptyset$. $B \neq \emptyset$ because if X_0 is sufficiently small then $X_1 = X_0/z - H_1 < X_0/z - (1-\delta_h)H_0 < 0$. To prove $A \neq \emptyset$, pick a sufficiently large Y such that $\overline{L}_0^{\alpha_l} f\left(\frac{Y}{\overline{L}_0^{\alpha_l}}\right)^{1-\alpha_l} < \delta H_0$. We show that $\lim_t X_t = \infty$ if $X_0 > \max\{Y, \frac{2H_0}{1/z-1}\}$. To show this, it is sufficient to show $H_t \leq H_0$ and $X_t > \frac{1/z+1}{2} X_{t-1}$ for all $t \geq 1$. If $t = 1$, then

$$\begin{aligned} H_1 &= (1-\delta_h)H_0 + L_0^{\alpha_l} f\left(\frac{X_0}{L_0^{\alpha_l}}\right)^{1-\alpha_l} < (1-\delta_h)H_0 + L_0^{\alpha_l} f\left(\frac{Y}{L_0^{\alpha_l}}\right)^{1-\alpha_l} < H_0, \\ X_1 &= X_0/z - H_1 > X_0/z - H_0 > X_0/z - X_0\frac{1/z-1}{2} = \frac{1/z+1}{2} X_0. \end{aligned}$$

By induction, suppose $H_s \leq H_0$ and $X_s > \frac{1/z+1}{2} X_{s-1}$ for all $s \leq t$, then for $t+1$,

$$\begin{aligned} H_{t+1} &= (1-\delta_h)H_t + L_t^{\alpha_l} f\left(\frac{X_t}{L_t^{\alpha_l}}\right)^{1-\alpha_l} < (1-\delta_h)H_0 + L_t^{\alpha_l} f\left(\frac{Y}{L_t^{\alpha_l}}\right)^{1-\alpha_l} < H_0, \\ X_{t+1} &= X_t/z - H_{t+1} > X_t/z - H_0 > X_t/z - X_t\frac{1/z-1}{2} = \frac{1/z+1}{2} X_t. \end{aligned}$$

$A \cap B = \emptyset$ because the system is terminated after X_t reaches negative values. So X_t cannot converge to ∞ at the same time.

Second, both A and B are open. B is open because if $X_t < 0$ for some finite t, then continuity implies that X_t remains negative if there is a small change to X_0. A is open because if $\lim_{t\to\infty} X_t = \infty$, then equation (B.15) and $f(\infty) = 0$ imply $\lim_{t\to\infty} H_t = 0$. Therefore, $H_t < \frac{\min\{H^*(\underline{L}_t), X_t/z - X^*(\underline{L}_t)\} - \overline{L}_t^{\alpha_l} f\left(\frac{X_t}{\overline{L}_t^{\alpha_l}}\right)^{1-\alpha_l}}{1-\delta_h}$ for large t. It follows from $(H_t, X_t) \in \mathcal{U}_t$ and Lemma 2 that $\lim_{t\to\infty} X_t = \infty$.

Third, $(0,\infty)\setminus(A\cup B)$ is nonempty because $(0,\infty)$ is a connected set. Pick $X_0 \in (0,\infty)\setminus(A\cup B)$ and we show below that the system starting from (H_0, X_0) converges to $(H^*(L^*), X^*(L^*))$, that is, for any $\epsilon > 0$, there exists N such that $(H_t, X_t) \in (H^*(L^*)-\epsilon, H^*(L^*)+\epsilon)\times(X^*(L^*)-\epsilon, X^*(L^*)+\epsilon)$ for all $t \geq N$. We shall repeatedly use the fact that $(H_t, X_t) \notin \mathcal{L}_t \cup \mathcal{U}_t$ for all t.

(i) We show that there exists a small $\epsilon_2 \in (0,\epsilon)$ such that $t > 1/\epsilon_2$ and $H_t \in (H^*(L^*) - \epsilon_2, H^*(L^*) + \epsilon_2)$ imply $(H_{t+1}, X_{t+1}) \in (H^*(L^*) - \epsilon, H^*(L^*) + \epsilon) \times (X^*(L^*) - \epsilon, X^*(L^*) + \epsilon)$. Because $H'(H,X,L)$ and $X'(H,X,L)$ are continuous functions, there exists $\epsilon_3 > 0$ such that $(H', X') \in (H^*(L^*) - \epsilon, H^*(L^*) + \epsilon) \times (X^*(L^*) - \epsilon, X^*(L^*) + \epsilon)$ for all $(H,X,L) \in (H^*(L^*) - \epsilon_3, H^*(L^*) + \epsilon_3) \times (X^*(L^*) - \epsilon_3, X^*(L^*) + \epsilon_3) \times (L^* - \epsilon_3, L^* + \epsilon_3)$. We can choose a sufficiently small $\epsilon_2 < \epsilon_3$ such that $L_t \in (L^* - \epsilon_3, L^* + \epsilon_3)$ for all $t > 1/\epsilon_2$. Because both $\partial\mathcal{L}_t$ and $\partial\mathcal{U}_t$ are upward sloping and continuous, we can also choose a sufficiently small ϵ_2 such that $H_t \in (H^*(L^*) - \epsilon_2, H^*(L^*) + \epsilon_2)$ and $(H_t, X_t) \notin \mathcal{L}_t \cup \mathcal{U}_t$ imply that $(H_t, X_t) \in (H^*(L^*) - \epsilon_3, H^*(L^*) + \epsilon_3) \times (X^*(L^*) - \epsilon_3, X^*(L^*) + \epsilon_3)$ for all $t > 1/\epsilon_2$.

(ii) We show that there exists a small $\epsilon_4 > 0$ such that $t > 1/\epsilon_4$ and $H_t \leq H^*(L^*) - \epsilon_2$ imply $H_{t+1} \in (H_t, H^*(L^*) + \epsilon_2)$. To show $H_{t+1} > H_t$, choose a sufficiently small ϵ_4 such that for $t \geq 1/\epsilon_4$,

(a) $\underline{L}_t^{\alpha_l} f\left(\frac{X^*(L^*)}{\underline{L}_t^{\alpha_l}}\right)^{1-\alpha_l} > \delta(H^*(L^*) - \epsilon_2)$;

(b) $\frac{\min\{H^*(\underline{L}_t), X^*(L^*)/z - X^*(\underline{L}_t)\} - \overline{L}_t^{\alpha_l} f\left(\frac{X^*(L^*)}{\overline{L}_t^{\alpha_l}}\right)^{1-\alpha_l}}{1-\delta_h} > H^*(L^*) - \epsilon_2$.

Then $(H_t, X_t) \notin \mathcal{U}_t$ implies that

$$\frac{\min\left\{H^*(\underline{L}_t), X_t/z - X^*(\underline{L}_t)\right\} - \overline{L}_t^{\alpha_l} f\left(\frac{X_t}{\overline{L}_t^{\alpha_l}}\right)^{1-\alpha_l}}{1-\delta_h} \leq H_t \leq H^*(L^*) - \epsilon_2,$$

which, together with (b), implies $X_t < X^*(L^*)$. It follows from (a) that

$$L_t^{\alpha_l} f\left(\frac{X_t}{L_t^{\alpha_l}}\right)^{1-\alpha_l} \geq \underline{L}_t^{\alpha_l} f\left(\frac{X^*(L^*)}{\underline{L}_t^{\alpha_l}}\right)^{1-\alpha_l} > \delta(H^*(L^*) - \epsilon_2) \geq \delta H_t.$$

Therefore, $H_{t+1} = (1-\delta_h)H_t + L_t^{\alpha_l} f\left(\frac{X_t}{L_t^{\alpha_l}}\right)^{1-\alpha_l} > H_t$. To show $H_{t+1} < H^*(L^*) + \epsilon_2$, choose a sufficiently small ϵ_4 such that for $t \geq 1/\epsilon_4$,

$$\frac{\max\left\{H^*(\overline{L}_t), X^*(L^*)/z - X^*(\overline{L}_t)\right\} - \underline{L}_t^{\alpha_l} f\left(\frac{X^*(L^*)}{\underline{L}_t^{\alpha_l}}\right)^{1-\alpha_l}}{1-\delta_h} < H^*(L^*) + \epsilon_2.$$

By contradiction, suppose $H_{t+1} \geq H^*(L^*) + \epsilon_2$. Then $X_{t+1} = X_t/z - H_{t+1} \leq X^*(L^*)/z - H^*(L^*) - \epsilon_2 < X^*(L^*)$, and

$$
\begin{aligned}
& \frac{\max\left\{H^*(\overline{L}_{t+1}), X_{t+1}/z - X^*(\overline{L}_{t+1})\right\} - \underline{L}_{t+1}^{\alpha_l} f\left(\frac{X_{t+1}}{\underline{L}_{t+1}^{\alpha_l}}\right)^{1-\alpha_l}}{1-\delta_h} \\
\leq\ & \frac{\max\left\{H^*(\overline{L}_{t+1}), X^*(L^*)/z - X^*(\overline{L}_{t+1})\right\} - \underline{L}_{t+1}^{\alpha_l} f\left(\frac{X^*(L^*)}{\underline{L}_{t+1}^{\alpha_l}}\right)^{1-\alpha_l}}{1-\delta_h} \\
<\ & H^*(L^*) + \epsilon_2 \leq H_{t+1},
\end{aligned}
$$

which contradicts the fact that $(H_{t+1}, X_{t+1}) \notin \mathcal{L}_t$.

(iii) Symmetrically, we show that there exists a small $\epsilon_4 > 0$ such that $t > 1/\epsilon_4$ and $H_t \geq H^*(L^*) + \epsilon_2$ imply $H_{t+1} \in (H^*(L^*) - \epsilon_2, H_t)$. To show $H_{t+1} < H_t$, choose a sufficiently small ϵ_4 such that for $t \geq 1/\epsilon_4$,

(a) $\overline{L}_t^{\alpha_l} f\left(\frac{X^*(L^*)}{\overline{L}_t^{\alpha_l}}\right)^{1-\alpha_l} < \delta(H^*(L^*) + \epsilon_2)$;

(b) $\frac{\max\left\{H^*(\overline{L}_t), X^*(L^*)/z - X^*(\overline{L}_t)\right\} - \underline{L}_t^{\alpha_l} f\left(\frac{X^*(L^*)}{\underline{L}_t^{\alpha_l}}\right)^{1-\alpha_l}}{1-\delta_h} < H^*(L^*) + \epsilon_2$.

Then $(H_t, X_t) \notin \mathcal{L}_t$ implies that

$$
\frac{\max\left\{H^*(\overline{L}_t), X_t/z - X^*(\overline{L}_t)\right\} - \underline{L}_t^{\alpha_l} f\left(\frac{X_t}{\underline{L}_t^{\alpha_l}}\right)^{1-\alpha_l}}{1-\delta_h} \geq H_t \geq H^*(L^*) + \epsilon_2,
$$

which, together with (b), implies $X_t > X^*(L^*)$. It follows from (a) that

$$
L_t^{\alpha_l} f\left(\frac{X_t}{L_t^{\alpha_l}}\right)^{1-\alpha_l} \leq \overline{L}_t^{\alpha_l} f\left(\frac{X^*(L^*)}{\overline{L}_t^{\alpha_l}}\right)^{1-\alpha_l} < \delta(H^*(L^*) + \epsilon_2) \leq \delta H_t.
$$

Therefore, $H_{t+1} = (1-\delta_h)H_t + L_t^{\alpha_l} f\left(\frac{X_t}{L_t^{\alpha_l}}\right)^{1-\alpha_l} < H_t$. To show $H_{t+1} > H^*(L^*) - \epsilon_2$, choose a sufficiently small ϵ_4 such that for $t \geq 1/\epsilon_4$,

$$
\frac{\min\left\{H^*(\underline{L}_t), X^*(L^*)/z - X^*(\underline{L}_t)\right\} - \overline{L}_t^{\alpha_l} f\left(\frac{X^*(L^*)}{\overline{L}_t^{\alpha_l}}\right)^{1-\alpha_l}}{1-\delta_h} > H^*(L^*) - \epsilon_2.
$$

By contradiction, suppose $H_{t+1} \leq H^*(L^*) - \epsilon_2$. Then $X_{t+1} = X_t/z - H_{t+1} \geq X^*(L^*)/z - H^*(L^*) + \epsilon_2 > X^*(L^*)$, and

$$
\begin{aligned}
& \frac{\min\left\{H^*(\underline{L}_{t+1}), X_{t+1}/z - X^*(\underline{L}_{t+1})\right\} - \overline{L}_{t+1}^{\alpha_l} f\left(\frac{X_{t+1}}{\overline{L}_{t+1}^{\alpha_l}}\right)^{1-\alpha_l}}{1-\delta_h} \\
\geq\ & \frac{\min\left\{H^*(\underline{L}_{t+1}), X^*(L^*)/z - X^*(\underline{L}_{t+1})\right\} - \overline{L}_{t+1}^{\alpha_l} f\left(\frac{X^*(L^*)}{\overline{L}_{t+1}^{\alpha_l}}\right)^{1-\alpha_l}}{1-\delta_h} \\
>\ & H^*(L^*) - \epsilon_2 \geq H_{t+1},
\end{aligned}
$$

which contradicts the fact that $(H_{t+1}, X_{t+1}) \notin \mathcal{U}_t$.

(iv) We show that set $(H^*(L^*)-\epsilon, H^*(L^*)+\epsilon)$ is absorbing for $t \geq 1/\epsilon_4$. Starting from $(X^*(L^*)-\epsilon, X^*(L^*)-\epsilon_2] \cup [X^*(L^*)+\epsilon_2, X^*(L^*)+\epsilon)$, the path monotonically converges to $(X^*(L^*)-\epsilon_2, X^*(L^*)+\epsilon_2)$; starting from $(X^*(L^*)-\epsilon_2, X^*(L^*)+\epsilon_2)$, the path stays in $(X^*(L^*)-\epsilon, X^*(L^*)+\epsilon)$. Even if $H_0 \notin (H^*(L^*)-\epsilon, H^*(L^*)+\epsilon)$, the path will enter $(H^*(L^*)-\epsilon, H^*(L^*)+\epsilon)$ and stay in it forever.

Fourth, if $\tilde{X}_0 > X_0$, then by induction we can show that $\tilde{H}_t < H_t$ and $\tilde{X}_t > X_t$ for all $t \geq 1$. Therefore,

$$\tilde{X}_{t+1} - X_{t+1} = (\tilde{X}_t - X_t)/z - (\tilde{H}_{t+1} - X_{t+1}) > (\tilde{X}_t - X_t)/z,$$

which implies $\lim_t(\tilde{X}_t - X_t) = \infty$. Therefore, $\lim_t \tilde{X}_t = \infty + X^*(L^*) = \infty$. Similarly, if $\tilde{X}_0 < X_0$, then we can show that $\tilde{H}_t > H_t$ and $\tilde{X}_t < X_t$ for all $t \geq 1$. Therefore, $\lim_t \tilde{X}_t = -\infty$. This step also implies that X_0 is unique. ■

Step 3. We show that for any (A_0, H_0, K_0), there exists a unique Q_0 such that the system (B.1)-(B.12) starting from (K_0, A_0, H_0, Q_0) converges to $(H^*(L^*), X^*(L^*))$. Lemma 3 shows that, given H_0, there exists a unique K_0/Q_{-1} such that the system converges to $(H^*(L^*), X^*(L^*))$. There is an increasing and one-to-one mapping between Q_{-1} and Q_0. Equation (B.13) implies

$$Q_0 = \frac{(1-\alpha)(1-\tau)A_0^\theta K_0^{\alpha-\theta\rho}(1-N_{h,0})^{-\alpha-\theta(1-\rho)} N_{h,0}^{\alpha_l}}{(1-\alpha_l)L_0^{\alpha_l}}.$$

Substituting $N_{h,0} = f\left(\frac{K_0}{Q_{-1}L_0^{\alpha_l}}\right)$ into the above yields

$$Q_0 = \frac{(1-\alpha)(1-\tau)A_0^\theta K_0^{\alpha-\theta\rho}(1-f\left(\frac{K_0}{Q_{-1}L_0^{\alpha_l}}\right))^{-\alpha-\theta(1-\rho)} f\left(\frac{K_0}{Q_{-1}L_0^{\alpha_l}}\right)^{\alpha_l}}{(1-\alpha_l)L_0^{\alpha_l}},$$

which is increasing in Q_{-1}.

Suppose Q_0 corresponds to the unique Q_{-1} in Lemma 3 from which the system (H_t, X_t) converges to a steady state. If $\tilde{Q}_0 < Q_0$, then $\tilde{Q}_{-1} < Q_{-1}$, which implies $\tilde{X}_0 = \frac{K_0}{\tilde{Q}_{-1}} > \frac{K_0}{Q_{-1}} = X_0$ and $\lim_{t\to\infty} \tilde{X}_t = \infty$. This represents an equilibrium in which $\lim_{t\to\infty} \tilde{Q}_t = 0$. However, if $\tilde{Q}_0 > Q_0$, then $\tilde{Q}_{-1} > Q_{-1}$ and $\tilde{X}_t < 0$ for some t. Equilibrium does not exist in the latter case.

Proof of Proposition 3

If A and K are both linear functions of Y,

$$A = \lambda_A Y, \tag{B.19}$$

$$K = \lambda_K Y, \tag{B.20}$$

it follows from (B.19), (B.20) and the production function $Y = A^\theta K^{\alpha-\rho\theta} N_c^{1-\alpha-(1-\rho)\theta}$ that

$$Y = \left(\lambda_A^\theta \lambda_K^{\alpha-\rho\theta} N_c^{1-\alpha-(1-\rho)\theta}\right)^{\frac{1}{1-\alpha-(1-\rho)\theta}}.$$

Therefore, (10), (11) and $N_c^n = 1$ in the bubbleless steady state imply

$$Y^n = \left((\delta_a^{-1}\tau)^\theta \left(\frac{\beta}{1+\beta}\alpha\psi(1-\tau)\right)^{\alpha-\rho\theta}\right)^{\frac{1}{1-\alpha-(1-\rho)\theta}},$$

while (16) and (17) in the bubbly steady state imply

$$\begin{aligned} Y^b &= \Bigg((\delta_a^{-1}\alpha_l\delta_h\phi^b \frac{\beta}{1+\beta}\alpha\psi(1-\tau) + \delta_a^{-1}\tau)^\theta ((1-\phi^b)\frac{\beta}{1+\beta}\alpha\psi(1-\tau))^{\alpha-\rho\theta} \\ &\quad (N_c^b)^{1-\alpha-(1-\rho)\theta}\Bigg)^{\frac{1}{1-\alpha-(1-\rho)\theta}}. \end{aligned}$$

These calculations show that $Y^b > Y^n$ if and only if

$$\left(\frac{Y^b}{Y^n}\right)^{1-\alpha-(1-\rho)\theta} = \left(1 + \frac{\alpha_l\delta_h\phi^b\beta\alpha\psi(1-\tau)}{(1+\beta)\tau}\right)^\theta (1-\phi^b)^{\alpha-\rho\theta}(N_c^b)^{1-\alpha-(1-\rho)\theta} > 1.$$

The left-hand side of the above inequality is increasing in θ because $1 + \frac{\alpha_l\delta_h\phi^b\beta\alpha\psi(1-\tau)}{(1+\beta)\tau} > 1$ and $N_c^b < 1$. The other two equations in the proposition follow from equations (11), (17), (10), and (16).

C The Extended Model in Section 4

C.1 Decision Rules

We show two properties of the optimal consumption-savings allocation in the entrepreneurs' problem. First, an age $j \geq 2$ entrepreneur's consumption $c_{j,t}^e$ in period t satisfies

(C.1) $$c_{j,t}^e = \frac{1-\beta}{1-\beta^{T-(j-1)}}\left(R_t k_{j,t} + (Q_t(1-\delta_h) + r_t)\, h_{j,t} + R^f b_{j,t}^e\right).$$

The newborn age 1 entrepreneur's wealth is inherited from their parents. Then their consumption is given by

(C.2) $$c_{1,t}^e = \frac{1-\beta}{1-\beta^T} m_t.$$

Thus, consumption is a fixed fraction of an entrepreneur's wealth in any period. This fraction depends on the entrepreneur's age, but not on their wealth level or the uncertainty in the rate of return from capital and housing.

Second, all entrepreneurs make the same portfolio choice at time t regardless of age, that is, $h_{j+1,t+1}/k_{j+1,t+1}$ is independent of j. These properties will simplify the calculation of equilibrium allocations in the extended model.

The following two lemmas prove these properties. We need to use an age-j entrepreneur's budget constraints at time t, which are explicitly written as

$$
\begin{aligned}
& Q_t h_{j+1,t+1} + k_{j+1,t+1} + c^e_{j,t} + b^e_{j+1,t+1} \\
= & \begin{cases} m_t, & j = 1, \\ R_t k_{j,t} + (Q_t(1-\delta_h) + r_t)\, h_{j,t} + R^f b^e_{j,t}, & 2 \le j \le T. \end{cases}
\end{aligned}
\tag{C.3}
$$

Lemma 4 *With logarithmic utility and $R_{t+1} > R^f$, we have $b^e_{j+1,t+1} = -\xi k_{j+1,t+1}$ and equations (C.1) and (C.2) hold.*

Proof: We prove (C.1) by backward induction in j. If $j = T$, then $\frac{1-\beta}{1-\beta^{T-(j-1)}} = 1$ and an entrepreneur in their last period of lifespan should obviously consume all the wealth.

Suppose (C.1) holds for $j+1$, i.e.,

$$
\begin{aligned}
c^e_{j+1,t+1} &= \frac{1-\beta}{1-\beta^{T-j}} \Big(R_{t+1} k_{j+1,t+1} + (Q_{t+1}(1-\delta_h) + r_{t+1})\, h_{j+1,t+1} + R^f b^e_{j+1,t+1} \Big) \\
&= \frac{1-\beta}{1-\beta^{T-j}} \Big(R_t k_{j,t} + (Q_t(1-\delta_h) + r_t)\, h_{j,t} + R^f b^e_{j,t} - c^e_{j,t} \Big) \times \\
&\quad (\tilde{R}_{t+1}(1-\phi_{j,t}) + R^h_{t+1}\phi_{j,t}),
\end{aligned}
$$

where we have used (C.3),

$$
\begin{aligned}
\phi_{j,t} &= \frac{Q_t h_{j+1,t+1}}{k_{j+1,t+1} + Q_t h_{j+1,t+1} + b^e_{j+1,t+1}} = \frac{Q_t h_{j+1,t+1}}{(1-\xi)\, k_{j+1,t+1} + Q_t h_{j+1,t+1}}, \\
\tilde{R}_{t+1} &\equiv \frac{R_{t+1} - \xi R^f}{1-\xi}, \quad R^h_{t+1} \equiv \frac{Q_{t+1}(1-\delta_h) + r_{t+1}}{Q_t},
\end{aligned}
$$

and $b^e_{j+1,t+1} = -\xi k_{j+1,t+1}$. Notice that the borrowing constraint always binds when $R_{t+1} > R^f$. Substituting the above consumption equation into the entrepreneur's Euler equation,

$$
u'(c^e_{j,t}) = \beta E_t[u'(c^e_{j+1,t+1})(\tilde{R}_{t+1}(1-\phi_{j,t}) + R^h_{t+1}\phi_{j,t})],
$$

we have

$$
\frac{1}{c^e_{j,t}} = \beta E_t \left[\frac{1}{\frac{1-\beta}{1-\beta^{T-j}} \Big(R_t k_{j,t} + (Q_t(1-\delta_h) + r_t)\, h_{j,t} + R^f b^e_{j,t} - c^e_{j,t} \Big)} \right].
$$

Solving the above equation for $c^e_{j,t}$ yields (C.1).

Finally for $j = 1$, the entrepreneur's wealth is m_t. We then obtain (C.2). ■

Lemma 5 *With logarithmic utility and $R_{t+1} > R^f$, all entrepreneurs make the same portfolio choice at time t regardless of age, i.e., $\phi_{j,t}$ is independent of j.*

Proof: The no-arbitrage condition is

$$(1-p)\frac{u'(c^{e+}_{j,t+1})}{u'(c^{e-}_{j,t+1})}R^{h+}_{t+1}+pR^{h-}_{t+1}=(1-p)\frac{u'(c^{e+}_{j,t+1})}{u'(c^{e-}_{j,t+1})}\tilde{R}^{+}_{t+1}+p\tilde{R}^{-}_{t+1}.$$

It follows from Lemma 4 that $\frac{u'(c^{e+}_{j,t+1})}{u'(c^{e-}_{j,t+1})}=\frac{c^{e-}_{j,t+1}}{c^{e+}_{j,t+1}}$ is equal to the wealth ratio between state $-$ and state $+$. Therefore

$$\begin{aligned}
&(1-p)\frac{(R^{h-}_{t+1}\phi_{j,t}+\tilde{R}^{-}_{t+1}(1-\phi_{j,t}))}{(R^{h+}_{t+1}\phi_{j,t}+\tilde{R}^{+}_{t+1}(1-\phi_{j,t}))}R^{h+}_{t+1}+pR^{h-}_{t+1}\\
\text{(C.4)}\qquad = \;&(1-p)\frac{(R^{h-}_{t+1}\phi_{j,t}+\tilde{R}^{-}_{t+1}(1-\phi_{j,t}))}{(R^{h+}_{t+1}\phi_{j,t}+\tilde{R}^{+}_{t+1}(1-\phi_{j,t}))}\tilde{R}^{+}_{t+1}+p\tilde{R}^{-}_{t+1}.
\end{aligned}$$

From the above equation, we can solve $\phi_{j,t}$ as

$$\phi_{j,t}=\frac{\tilde{R}^{-}_{t+1}\tilde{R}^{+}_{t+1}-(1-p)\tilde{R}^{-}_{t+1}R^{h+}_{t+1}-pR^{h-}_{t+1}\tilde{R}^{+}_{t+1}}{R^{h-}_{t+1}R^{h+}_{t+1}+\tilde{R}^{-}_{t+1}\tilde{R}^{+}_{t+1}-R^{h-}_{t+1}\tilde{R}^{+}_{t+1}-\tilde{R}^{-}_{t+1}R^{h+}_{t+1}}.$$

Because the right-hand side of the above does not depend on j, entrepreneurs of different ages make the same portfolio choice. ■

Let $(K_t(j),H_t(j))_{j=1}^{T}$ denote the aggregate holdings of capital and houses at the beginning of time t for ages $j=1,...,T$. Because Lemma 5 implies that $H_t(j)=\frac{K_t(j)}{K_t}H_t$, we will include K_t, H_t, and $\{K_t(j)\}_{j=1}^{T}$ in our state variables but not $\{H_t(j)\}_{j=1}^{T}$ since $\{H_t(j)\}_{j=1}^{T}$ can be inferred from the others.

C.2 Bubbleless Equilibrium

The dynamic system for the bubbleless equilibrium in the extended model contains $3T+16$ variables $B^g_t,\ A_t,\ H_t,\ K_t(j),\ B^e_t(j),\ K_t,\ Z_t,\ Z_t(j),\ R_t,\ w_t,\ p_{Lt},\ K_{c,t},\ K_{h,t},\ N_{c,t},\ N_{h,t},\ Y_t,\ Y_{h,t},\ M_t,\ Q_t$, for $j=1,...,T$, that satisfy the following system of $3T+16$ difference equations for $t\geq 0$:

$$\begin{aligned}
\text{(C.5)}\qquad B^g_{t+1} &= \xi_g p_{Lt}L_t,\\
\text{(C.6)}\qquad A_{t+1} &= (1-\delta_a)A_t-G_t-R^fB^g_t+\tau Y_t+\tau_hQ_tY_{ht}+p_{Lt}L_t+B^g_{t+1},\\
\text{(C.7)}\qquad H_{t+1} &= (1-\delta_h)H_t+Y_{ht},\\
\text{(C.8)}\qquad K_{t+1} &= \frac{1}{1-\xi}(Z_t-Q_tH_{t+1}),
\end{aligned}$$

$$\begin{aligned}
\text{(C.9)}\qquad K_t(1) &= 0,\\
\text{(C.10)}\qquad K_{t+1}(j+1) &= K_{t+1}\frac{Z_t(j)}{Z_t},\quad \forall j=1,...,T-1,\\
\text{(C.11)}\qquad B^e_{t+1}(j+1) &= -\xi K_{t+1}(j+1),\quad \forall j=0,1,...,T-1,\\
\text{(C.12)}\qquad Z_t(1) &= \left(1-\frac{1-\beta}{1-\beta^T}\right)M_t,
\end{aligned}$$

$$
\begin{aligned}
Z_t(j) &= \left[1 - \frac{1-\beta}{1-\beta^{T-(j-1)}}\right] \times \\
&\quad \left[K_t(1-\xi)\tilde{R}_t + H_t((1-\delta_h)Q_t + r_t)\right] \frac{K_t(j)}{K_t}, \quad \forall j = 2, ..., T, \qquad \text{(C.13)}
\end{aligned}
$$

$$Z_t = \sum_{j=1}^{T} Z_t(j), \qquad \text{(C.14)}$$

$$K_t = K_{c,t} + K_{h,t}, \qquad \text{(C.15)}$$

$$N_t = N_{c,t} + N_{h,t}, \qquad \text{(C.16)}$$

$$R_t = \alpha(1-\tau)(1-\psi)\hat{A}_t^{\theta} K_{c,t}^{\alpha-1}(e_t N_{c,t})^{1-\alpha} + (1-\delta_k), \qquad \text{(C.17)}$$

$$
\begin{aligned}
R_t &= \alpha_k(1-\tau_h)(1-\psi)Q_t \times \\
&\quad ((1-g_l)^t L_t)^{\alpha_l} K_{h,t}^{\alpha_k-1}(e_t N_{h,t})^{1-\alpha_l-\alpha_k} + (1-\delta_k), \qquad \text{(C.18)}
\end{aligned}
$$

$$w_t = (1-\alpha)(1-\tau)\hat{A}_t^{\theta} K_{c,t}^{\alpha} e_t^{1-\alpha} N_{c,t}^{-\alpha}, \qquad \text{(C.19)}$$

$$
\begin{aligned}
w_t &= (1-\alpha_l-\alpha_k)(1-\tau_h)Q_t \times \\
&\quad ((1-g_l)^t L_t)^{\alpha_l} K_{h,t}^{\alpha_k} e_t^{1-\alpha_l-\alpha_k} N_{h,t}^{-\alpha_l-\alpha_k}, \qquad \text{(C.20)}
\end{aligned}
$$

$$p_{Lt} = \alpha_l(1-\tau_h)Q_t(1-g_l)^{t\alpha_l} L_t^{\alpha_l-1} K_{h,t}^{\alpha_k}(e_t N_{h,t})^{1-\alpha_l-\alpha_k}, \qquad \text{(C.21)}$$

$$Y_t = \hat{A}_t^{\theta} K_{c,t}^{\alpha}(e_t N_{c,t})^{1-\alpha}, \qquad \text{(C.22)}$$

$$Y_{h,t} = ((1-g_l)^t L_t)^{\alpha_l} K_{h,t}^{\alpha_k}(e_t N_{h,t})^{1-\alpha_l-\alpha_k}, \qquad \text{(C.23)}$$

$$M_t = \psi((1-\tau)\alpha Y_t + (1-\tau_h)\alpha_k Q_t Y_{h,t}), \qquad \text{(C.24)}$$

$$\tilde{R}_{t+1} = \frac{(1-\delta_h)Q_{t+1} + r_{t+1}}{Q_t}, \qquad \text{(C.25)}$$

where $\hat{A}_t$ and $\tilde{R}_t$ satisfy

$$\hat{A}_t = \frac{A_t}{K_{c,t}^{\rho}(e_t N_{c,t})^{1-\rho}}, \quad \tilde{R}_t = \frac{R_t - \xi R^f}{1-\xi}.$$

The workers' decision problem is much simpler. For our small open economy, their consumption/saving choices do not affect the above equilibrium system. Once obtaining a solution to the above system, we can derive the consumption rules for entrepreneurs and workers. Here we omit the details.

Equations (C.5)-(C.7) follow from the definitions of B_{t+1}^{g}, A_{t+1}, and H_{t+1}. Equation (C.8) computes the aggregate K_{t+1}, using the binding borrowing constraint of entrepreneurs. The variable

Z_t denotes the aggregate wealth net of consumption across all entrepreneurs and $Z_t\left(j\right)$ denotes the total after-consumption wealth of age-j entrepreneurs. Equation (C.9) says that a newborn entrepreneur does not own capital. Equation (C.10) defines an age-$(j+1)$ entrepreneur's capital holding at $t+1$, whose age is j at period t. Here, $\frac{K_{t+1}(j+1)}{Z_t(j)} = \frac{K_{t+1}}{Z_t}$ holds because Lemma 5 shows that

$$\frac{K_{t+1}(j+1)}{Z_t(j)} = \frac{k_{j+1,t+1}}{k_{j+1,t+1} + Q_t h_{j+1,t+1} + b^e_{j+1,t+1}} = \frac{1-\phi_{j,t}}{1-\xi}$$

is independent of j.

Equation (C.11) is the binding borrowing constraint of entrepreneurs. Equation (C.12) defines newly born entrepreneurs' wealth $Z_t\left(1\right)$ after consumption, where M_t given in (C.24) is their total initial endowment. Equation (C.13) defines total age-j entrepreneurs's wealth $Z_t\left(j\right)$ after consumption, for $j = 2, ..., T-1$, where we have used Lemma 5. Here $(K_t(1-\xi)\tilde{R}_t + H_t((1-\delta_h)Q_t + r_t))$ is the total return from holding aggregate capital and houses, while $\frac{K_t(j)}{K_t}$ is the fraction of cohort-j's wealth in the total. Notice that $Z_t\left(T\right) = 0$ because an age-T entrepreneur consumes all their wealth. Equations (C.14), (C.15), and (C.16) define the aggregates. The variable N_t denotes the exogenous worker population.

Equations (C.17)-(C.21) are the firm's first-order conditions with respect to $k_{c,t}$, $k_{h,t}$, $n_{c,t}$, $n_{h,t}$, and l_t, respectively. Equations (C.22)-(C.24) follow from the definitions of Y_t, $Y_{h,t}$, and M_t. Equation (C.25) is the no-arbitrage condition. It shows that the initial fundamental housing value satisfies

$$Q_0 = \sum_{s=1}^{\infty} \frac{(1-\delta_h)^{s-1} r_s}{\prod_{i=1}^{s} \tilde{R}_i}.$$

The predetermined variables for the equilibrium system are A_0, K_0, H_0, $\left\{K_0\left(j\right)\right\}_{j=1}^{T}$, and B_0^g.

C.2.1 Algorithm for Computing the Dynamics Given Q_0

At the beginning of time 0, B_0^g, A_0, H_0, K_0, and $\left\{K_0\left(j\right)\right\}_{j=1}^{T}$ are known. Given Q_0, the dynamics are computed as follows.

(i) Initialize $t = 0$. Given B_0^g, A_0, H_0, K_0, $\left\{K_0\left(j\right)\right\}_{j=1}^{T}$, and Q_0, solve

$$\left(R_0, w_0, p_{L0}, K_{c,0}, K_{h,0}, N_{c,0}, N_{h,0}, Y_0, Y_{h,0}, M_0, Z_0, \left\{Z_0\left(j\right)\right\}_{j=1}^{T}\right),$$

by using equations (C.12)-(C.24) in the previous equilibrium system. The total numbers of equations and unknowns are both $T+11$.

(ii) Given B_t^g, A_t, H_t, K_t, $\left\{K_t\left(j\right)\right\}_{j=1}^{T}$, Q_t, R_t, w_t, p_{Lt}, K_{ct}, K_{ht}, N_{ct}, N_{ht}, Y_t, Y_{ht}, M_t, Z_t, and $\left\{Z_t\left(j\right)\right\}_{j=1}^{T}$,

(a) solve $(B^g_{t+1}, A_{t+1}, H_{t+1}, K_{t+1}, \{K_{t+1}(j)\}_{j=1}^T)$ by using (C.5)-(C.10). The total numbers of equations and unknowns are both $T+4$.

(b) given $(B^g_{t+1}, A_{t+1}, H_{t+1}, K_{t+1}, \{K_{t+1}(j)\}_{j=1}^T)$, solve R_{t+1}, w_{t+1}, p_{Lt+1}, $K_{c,t+1}$, $K_{h,t+1}$, $N_{c,t+1}, N_{h,t+1}, Y_{t+1}, Y_{h,t+1}, M_{t+1}, Z_{t+1}, \{Z_{t+1}(j)\}_{j=1}^T, Q_{t+1}$ by using time-$(t+1)$ versions of (C.12)-(C.25). The total numbers of equations and unknowns are both $T+12$.

(iii) Set $t = t+1$ and go to step (ii).

C.2.2 Algorithm for Computing Equilibrium

We use the shooting method. Set a large time horizon $\bar{T}$ and use the bisection method to compute Q_0 such that the bubbleless equilibrium converges to the balanced growth path.

(i) Choose two initial values of Q_0: (Q_0^h, Q_0^l).

(ii) If $|Q_0^h - Q_0^l| < \epsilon$, then stop. Otherwise, define $Q_0 = \frac{Q_0^h + Q_0^l}{2}$.

(iii) Given B_0^g, A_0, H_0, K_0, $\{K_0(j)\}_{j=1}^T$ and Q_0, solve the system dynamics by using the algorithm in Section C.2.1.

(a) If $\phi_t \equiv Q_t H_{t+1}/Z_t > 0$ for all $t = 0, 1, ..., \bar{T}$, then set $Q_0^h = Q_0$ and go to step (ii).

(b) If $\phi_t < 0$ for finite t (i.e., Q_t becomes negative because the initial guess of Q_0 is too low), then set $Q_0^l = Q_0$ and go to step (ii).

(iv) Increase $\bar{T}$ until the solution for Q_0 does not change much. In this case $\phi_{\bar{T}}$ converges to zero.

C.2.3 Bubbleless Balanced Growth

Notice that g_r, g_e, and g_n are exogenous growth rates of rent, labor-augmented technology, and population, respectively, and that g_l is the exogenous declining rate of land quality. We use g_x to denote the growth rate of a variable x_t. On a balanced growth path, we have

$$g_A = g_K = g_{K_c} = g_{B^e} = g_Z = g_Y = g_M = (1+g_e)(1+g_n) - 1, \tag{C.26}$$

$$g_w = g_e, \quad g_{N_c} = g_n. \tag{C.27}$$

Moreover, the capital return R_t, $\hat{A}_t$, and $\tilde{R}_t$ are constant over time.

It follows from equation (C.25) that the housing price Q_t grows at the growth rate of rents, i.e., $g_Q = g_r$. By equation (C.18), we have

$$1 = (1+g_r)(1-g_l)^{\alpha_l}(1+g_{K_h})^{\alpha_k - 1}\left[(1+g_e)(1+g_{N_h})\right]^{1-\alpha_k-\alpha_l}.$$

By equation (C.20), we have

$$1+g_w=(1+g_r)(1-g_l)^{\alpha_l}(1+g_{K_h})^{\alpha_k}(1+g_e)^{1-\alpha_k-\alpha_l}(1+g_{N_h})^{-\alpha_k-\alpha_l}.$$

From the above two equations, we can solve for g_{K_h} and g_{N_h} :

$$\begin{aligned} g_{N_h} &= \frac{(1+g_r)^{\frac{1}{\alpha_l}}(1-g_l)}{1+g_e}-1, \\ g_{K_h} &= (1+g_r)^{\frac{1}{\alpha_l}}(1-g_l)-1. \end{aligned}$$

Using the housing production function, we can derive

$$g_{Y_h}=(1+g_r)^{\frac{1}{\alpha_l}-1}(1-g_l)-1.$$

Thus the growth rate of residential investment Q_tY_{ht} is given by

$$(1+g_r)^{\frac{1}{\alpha_l}}(1-g_l)-1.$$

By (C.7), $g_H=g_{Y_h}$. It follows from equations (C.5) and (C.21) that

$$g_{p_L}=g_{B^g}=(1+g_r)^{\frac{1}{\alpha_l}}(1-g_l)-1.$$

By the labor market clearing condition,

$$1=\frac{N_{ct}}{N_t}+\frac{N_{ht}}{N_t}.$$

For a bubbleless balanced growth path to exist, we must have $g_n\geq g_{N_h}$ as $g_{N_c}=g_n$, or

$$1+g_n\geq\frac{(1+g_r)^{\frac{1}{\alpha_l}}(1-g_l)}{1+g_e}.$$

Under this condition, we deduce that the growth rate of the housing sector (or residential investment $Q_tY_{h,t}$) is lower than that of the nonhousing sector. Thus ϕ_t converges to zero as $t\to\infty$.

C.3 Equilibrium with Stochastic Bubbles

Once the bubble bursts, it never reappears and the equilibrium system is the same as that in Appendix C.2. Before it bursts, the equilibrium system is also the same as in Appendix C.2, except for two changes. First, we add superscript + to all endogenous variables in equations (C.5) through (C.24) to indicate that these variables are in the bubbly state. Second, the no-arbitrage equation (C.25) is replaced by the following equation:

$$\begin{aligned} &(1-p)\frac{(R_{t+1}^{h-}\phi_t^{+}+\tilde{R}_{t+1}^{-}(1-\phi_t^{+}))}{(R_{t+1}^{h+}\phi_t^{+}+\tilde{R}_{t+1}^{+}(1-\phi_t^{+}))}R_{t+1}^{h+}+pR_{t+1}^{h-} \\ = \; &(1-p)\frac{(R_{t+1}^{h-}\phi_t^{+}+\tilde{R}_{t+1}^{-}(1-\phi_t^{+}))}{(R_{t+1}^{h+}\phi_t^{+}+\tilde{R}_{t+1}^{+}(1-\phi_t^{+}))}\tilde{R}_{t+1}^{+}+p\tilde{R}_{t+1}^{-}, \end{aligned} \tag{C.28}$$

where

$$
\begin{aligned}
R^{h+}_{t+1} &\equiv \frac{(1-\delta_h)Q^+_{t+1}+r_{t+1}}{Q^+_t}, \quad R^{h-}_{t+1} \equiv \frac{(1-\delta_h)Q^-_{t+1}+r_{t+1}}{Q^+_t}, \\
\tilde{R}^+_{t+1} &\equiv \frac{R^+_{t+1}-\xi R^f}{1-\xi}, \quad \tilde{R}^-_{t+1} \equiv \frac{R^-_{t+1}-\xi R^f}{1-\xi}, \quad \phi^+_t \equiv \frac{Q^+_t H^+_{t+1}}{Z^+_t}.
\end{aligned}
$$

The new no-arbitrage equation (C.28) takes into account the stochastic bubble. The variable ϕ^+_t denotes the portfolio share of the housing investment, R^{h+}_{t+1} $\left(R^{h-}_{t+1}\right)$ is the housing return when the bubble persists (bursts), and $\tilde{R}^+_{t+1}$ $\left(\tilde{R}^-_{t+1}\right)$ is the effective capital return when the bubble persists (bursts). When the bubble bursts, it will not reappear and Q^-_{t+1} and R^-_{t+1} represent the housing price and capital return in the bubbleless equilibrium studied in Appendix C.2. The solution algorithm is similar to that described in Appendix C.2 and is omitted here.

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