

# Growth Uncertainty, Generalized Disappointment Aversion and Production-based Asset Pricing\*

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## Abstract

We study a production economy with regime switching in the conditional mean and volatility of productivity growth. The representative agent has generalized disappointment aversion (GDA) preferences. We show that volatility risk in productivity growth carries a positive and sizable risk premium in levered equity. Our model can endogenously generate long-run risks in the volatility of consumption growth observed in the data. We show that introducing leverage with a procyclical dividend process consistent with the data is critical for the GDA preferences to have a large impact on equity returns.

JEL CLASSIFICATION: D81, E32, E44, G12

KEYWORDS: Equity premium, Asset pricing, Business cycles, Disappointment aversion, Volatility risk, DSGE model, Markov switching.

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# 1 Introduction

Financial data have provided many stylized facts that are challenging for standard economic models to explain. For example, the mean and volatility of the equity premium are high, the mean and volatility of the riskfree rate are low, and the conditional equity premium is time varying and moves with business cycles countercyclically (Shiller (1981), Mehra and Prescott (1985), and Campbell (1999)). Using the consumption-based asset pricing approach in the finance literature, one needs to introduce either various sources of risk into the exogenously given aggregate consumption process or nonstandard preferences into the representative agent model. An important source of risk is the long-run risk in the mean and volatility of consumption growth (Bansal and Yaron (2004), Hansen et al. (2008), and Bansal et al. (2013)). Nonstandard preferences are crucial for how risks are perceived and priced in the market.<sup>1</sup>

The importance of volatility risk or time-varying macroeconomic uncertainty has also been noticed in the recent macroeconomics literature.<sup>2</sup> This literature typically studies dynamic stochastic general equilibrium (DSGE) models and focuses on the implications for quantities rather than asset prices. While DSGE models are more coherent in that consumption and dividends are endogenous, they typically fail to explain many asset pricing puzzles (e.g., Rouwenhorst (1995)).

The goal of this paper is to provide a production-based asset pricing model with time-varying macroeconomic uncertainty by linking the preceding two strands of literature. In addition to capital adjustment costs and financial leverage, our model has two key features. First, we assume that aggregate productivity growth follows a Markov-switching process (Hamilton (1989)). In particular, the conditional mean and volatility of aggregate productivity growth follow two independent two-state Markov chains. The persistent changes in the mean and volatility capture long-run risks in the expected productivity growth and the time-varying macroeconomic uncertainty, respectively. We estimate the Markov-switching model for the historical productivity growth data from 1956:Q1 to 2012:Q4. Our empirical estimates strongly support the presence of shifts in mean and volatility regimes, with volatility regimes being more persistent. We show that an increase in the volatility

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<sup>1</sup>Important nonstandard preferences include Epstein-Zin preferences (Epstein and Zin (1989)), habit formation preferences (Campbell and Cochrane (1999)), disappointment-aversion preferences (Gul (1991) and Routledge and Zin (2010)), and ambiguity-sensitive preferences (Hansen and Sargent (2010) and Ju and Miao (2012)).

<sup>2</sup>See, e.g., Justiniano and Primiceri (2008), Bloom (2009), Gilchrist et al. (2013), Born and Pfeifer (2012), Fernandez-Villaverde et al. (2011), Bloom et al. (2013), Basu and Bundick (2012), and Leduc and Liu (2013).

of productivity growth reduces consumption as in the data and raises marginal utility. Thus the model implied market price of productivity volatility risk is negative. Since aggregate stock returns are negatively exposed to this risk both in the data and in the model, the volatility risk carries a positive risk premium.

Second, we assume that the representative agent has generalized disappointment aversion (GDA) preferences that are recently introduced by Routledge and Zin (2010). As in Routledge and Zin (2010), we embed GDA in the Epstein and Zin (1989) recursive utility framework to further disentangle risk aversion from the elasticity of intertemporal substitution (EIS). Compared with an agent with Epstein-Zin preferences, the agent with GDA preferences puts more weight on disappointing outcomes below a threshold set at a proportion of the certainty equivalent of the continuation value. This causes the pricing kernel to be more countercyclical than that for the Epstein-Zin utility, thereby helping raise the mean and volatility of the equity premium. Unlike the Epstein-Zin utility, GDA preferences imply first-order risk aversion, which has a large quantitative impact on the risk premium in levered equity. Moreover, GDA preferences generate stronger precautionary saving motives and hence help lower the riskfree rate.

The two welfare theorems hold true in our model. We thus solve the social planner's problem to derive the equilibrium allocation. Our calibrated model can generate endogenous long-run risks in the mean and volatility of consumption growth observed in the data. We estimate a Markov-switching model of consumption growth using the US data from 1956:Q1 to 2012:Q4. This model is a nonlinear version of the long-run risks model studied in Bansal and Yaron (2004). Our estimates are broadly consistent with Lettau et al. (2008) and Boguth and Kuehn (2013). Using model simulated consumption data, we can closely replicate persistent volatility regimes estimated using the US data. Thus our production-based model provides a foundation for the consumption process adopted in the long-run risk literature.

We find that preference parameters related to risk attitudes have a very small effect on the volatility of the equilibrium allocation, which is determined mainly by the EIS parameter and the capital adjustment cost parameter. This result is related to the finding of Tallarini (2000) for the Epstein-Zin utility. Our analysis shows that, like the risk aversion parameter, the GDA parameters do not matter much for the volatility of the allocation.

We turn to the asset pricing implications by studying a decentralized competitive equilibrium. We find that preference parameters related to risk attitudes have a large effect on asset returns and the first moments of the allocation. A higher degree of disappointment aversion induces a stronger precautionary saving motive and hence a lower riskfree rate and higher average capital. Analyzing equity returns in the model is less straightforward since firm payouts in the model do not correspond to the observed aggregate public equity dividends in the data. In our model the market value of the firm is equal to Tobin's Q times the capital stock and the return to firm payouts (the unlevered equity return) is equal to the investment return. This return does not correspond to the equity return in the data. We show that unlevered equity premium is very small even though we calibrate the risk aversion parameter and the GDA parameter at high values. The reason is that firm payouts are countercyclical in the model: investment rises too much relative to output in response to a positive productivity shock.

We then follow Jermann (1998) and introduce financial leverage to the model. This modeling raises the (levered) equity premium significantly. But it is still too low, about a half of the mean equity premium in the data. This is because dividend growth is still countercyclical even though its volatility is much higher than that in the data. To overcome this problem, we calibrate dividends as levered claims to consumption so that their moments are consistent with the data. In this case our model can match the historical mean riskfree rate and mean equity premium based on much more reasonable values of the risk aversion and GDA parameters.

In our calibration with GDA, capital adjustment costs are small and the EIS is greater than unity. These parameter choices enable our model to produce smooth consumption growth and volatile investment growth observed in the data. Small adjustment costs imply a small real friction in capital accumulation, causing investment to react strongly to productivity fluctuations. A high EIS implies a large substitution effect and a low desire to smooth consumption. This causes consumption to rise less in response to a permanent shock to productivity growth. Thus a high EIS reduces consumption growth volatility in our model.

The long-run risks literature typically assumes that the EIS is greater than unity. But we recognize that the estimate of the EIS is under debate in the literature (e.g., Hall (1988), Campbell and Mankiw (1989), among others). Bonomo et al. (2011) show that a model of endowment

economies with GDA and an EIS less than unity can deliver asset pricing results that are almost fully consistent with the well-known stylized facts. They then conclude that the EIS is not important for asset pricing once GDA is incorporated. However, in a production economy, the value of the EIS is crucial for matching moments of macroeconomic quantities. We show that an EIS lower than unity implies that investment growth is too smooth while consumption growth is excessively volatile. Consequently, this model is less successful in reproducing low-frequency movement in the volatility of consumption growth, generating a low equity premium and a high riskfree rate.

The combination of the volatility risk in productivity growth and GDA preferences is the key ingredient of our model. To see its importance, we consider three comparison models. First, we remove GDA and keep other parameter values fixed. We find that shutting down GDA does not affect significantly the second moments of macroeconomic quantities since preference parameters related to risk attitudes are not important for quantity dynamics. However, this model performs much worse in the asset pricing dimension. In particular, we find that the market price of risk, the mean equity premium, and the equity volatility are too low, while the mean riskfree rate is too high, compared to the data.

Second, we ask whether the Epstein-Zin model with high risk aversion can generate similar results to those in a model with GDA. To address this question, we recalibrate the risk aversion parameter to match the mean equity premium generated from the model with GDA. We find that the recalibrated risk aversion parameter is too high according to the discussion in Mehra and Prescott (1985). The recalibrated Epstein-Zin model generates results similar to the model with GDA at the expense of an extremely high degree of risk aversion.

Third, we shut down the channel of economic uncertainty by fixing the productivity growth volatility at its mean value. We find that this comparison model cannot generate endogenous long-run risks in the volatility of consumption growth. This model also generates lower mean equity premium and equity volatility. This result suggests that the volatility risk in productivity growth carries a positive and economically significant risk premium. We show that, for the model with a calibrated dividend process consistent with the data, the volatility risk in productivity growth raises the annual mean equity premium and the annual equity volatility by about 1.6 and 1.2 percentage points respectively.

This paper is related to two strands of literature cited earlier, namely, the recent macroeconomics literature focusing on the effect of time-varying uncertainty on business cycles and the long-run risks literature exploring the impact of economic uncertainty on asset prices. Our result that macroeconomic volatility risk carries a positive and economically significant risk premium is consistent with the finding of Bansal and Yaron (2004) and Bansal et al. (2013). Our economic mechanism is also similar to theirs: Volatility risk raises marginal utility of consumption and hence the market price of volatility risk is negative. But the stock return has a negative exposure to the volatility risk. The key difference is that consumption growth is endogenously determined in our production-based model rather than exogenously assumed.

We now discuss some closely related recent studies on production-based asset pricing models. Croce (2014) studies a DSGE model with Epstein-Zin preferences and long-run risks in productivity growth. He shows that volatility risk carries a negative risk premium in levered equity using different modeling of leverage. Kaltenbrunner and Lochstoer (2010) show that long-run risks in expected consumption growth can endogenously arise in an otherwise standard DSGE model with Epstein-Zin preferences. However, their model cannot account for long-run risks in the volatility of consumption growth. Our analysis suggests that the time-varying volatility in productivity growth is important to generate this result.

Campanale et al. (2010) study GDA preferences, but do not take into account long-run productivity risks, which are, however, the focus of our paper. They consider both transitory and permanent productivity shocks. But their main focus is on transitory shocks by calibrating a small EIS less than unity and a large capital adjustment cost. Their model implies an excessively high volatility of the riskfree rate as in models with habit formation preferences (Jermann (1998) and Boldrin et al. (2001)). Moreover, their model does not generate significant time variations in equity premium and thus does not produce significant predictability of stock returns.

Jahan-Parvar and Liu (2014) investigate a model with regime switching in the mean of productivity growth, learning, and ambiguity aversion by adapting the model of Ju and Miao (2012) to a production economy. As Hansen and Sargent (2010) and Ju and Miao (2012) show, ambiguity aversion raises the countercyclicality of the pricing kernel because the agent pessimistically attaches more weight to bad states, just like disappointment aversion.

## 2 Regime Shifts in Productivity Growth

In standard real business cycle (RBC) models, the aggregate productivity shock is typically assumed to follow a homoscedastic AR(1) process. To capture time-varying macroeconomic uncertainty, the recent macroeconomics literature introduces stochastic volatility to the productivity process. In the finance literature on long-run risks, economic uncertainty is typically modeled as stochastic volatility in consumption growth in endowment economies. While most papers in these two strands of literature adopt linear processes, we will use a nonlinear Markov-switching process to model productivity growth because this process is easy to estimate. We will estimate this process in Section 2.1 and provide empirical evidence of the impact of volatility risk on macroeconomic quantities and asset returns in Section 2.2.

### 2.1 Estimation of the Markov-Switching Model

Aggregate total factor productivity (TFP)  $Z_t$  is typically constructed by specifying an aggregate production function  $Y_t = Z_t K_t^\alpha N_t^{1-\alpha}$ ,  $\alpha \in (0, 1)$ , where  $Y_t$ ,  $K_t$  and  $N_t$  denote aggregate output, capital, and labor, respectively. Let  $\Delta z_t \equiv \ln(Z_t/Z_{t-1})$  denote the growth rate. The empirical specification for productivity growth takes the form

$$\Delta z_t = \mu(s_{1t}) + \sigma(s_{2t}) \epsilon_t, \quad \epsilon_t \sim N(0, 1), \quad (1)$$

where  $s_{1t}$  and  $s_{2t}$  determine the regimes of the conditional mean and volatility of productivity growth, respectively. The conditional mean has two states,  $\mu(s_{1t}) \in \{\mu_l, \mu_h\}$  with  $\mu_l < \mu_h$ . The conditional volatility also has two states,  $\sigma(s_{2t}) \in \{\sigma_l, \sigma_h\}$  with  $\sigma_h > \sigma_l$ . To keep the model parsimonious, we assume that  $s_{1t}$  and  $s_{2t}$  follow two independent Markov chains. This assumption allows us to characterize the joint transition matrix by only four parameters. The transition probabilities of the two Markov chains are given by

$$\begin{aligned} P(\mu_t = \mu_l | \mu_{t-1} = \mu_l) &= p_{ll}^\mu, & P(\mu_t = \mu_h | \mu_{t-1} = \mu_h) &= p_{hh}^\mu, \\ P(\sigma_t = \sigma_l | \sigma_{t-1} = \sigma_l) &= p_{ll}^\sigma, & P(\sigma_t = \sigma_h | \sigma_{t-1} = \sigma_h) &= p_{hh}^\sigma. \end{aligned}$$

Table 1: **Parameter estimates of productivity growth**

Panel A: Markov switching estimates							
$\mu_l$	$\mu_h$	$\sigma_l$	$\sigma_h$	$p_{ll}^\mu$	$p_{hh}^\mu$	$p_{ll}^\sigma$	$p_{hh}^\sigma$
-0.227	0.479	0.548	0.933	0.781	0.919	0.979	0.962
(0.887)	(3.698)	(8.524)	(7.289)	(7.464)	(11.205)	(27.007)	(17.893)
Panel B: AR(1)-GARCH(1,1) estimates							
$\mu_a$	$\vartheta$	$\alpha_0$	$\theta$	$\alpha_1$			
0.003	0.173	0.000	0.842	0.0767			
(5.246)	(2.338)	(1.011)	(7.213)	(1.447)			

Panel A reports the maximum likelihood estimates of parameters in a four-state Markov-switching model for productivity growth. Panel B reports the estimates of the AR(1)-GARCH(1,1) model.  $t$ -values are shown in parentheses.

Combining together the two Markov chains, we obtain a four-state Markov chain. We use quarterly data on TFP growth from 1956:Q1 to 2012:Q4 constructed by Fernald (2012) to estimate the model. We apply the expectation maximization (EM) algorithm developed by Hamilton (1990) to obtain the maximum likelihood estimates of the parameters. The estimation results are reported in Panel A of Table 1. The average growth rate is 0.48% per quarter in the high expected growth regime and  $-0.23\%$  per quarter in the low expected growth regime. The estimated transition probabilities,  $p_{ll}^\mu = 0.78$  and  $p_{hh}^\mu = 0.92$ , suggest that the expansion state is more persistent than the contraction state. The implied average duration of the expansion state is about 13 quarters and that of the contraction state is about 4 quarters. These results are in line with Hamilton (1989) and Cagetti et al. (2002). The filtered probabilities of the low mean growth states ( $\{\mu_l, \sigma_h\}$  and  $\{\mu_l, \sigma_l\}$ ) are plotted in Figure 1. The plot reveals that the filtered probability spikes during recessions, suggesting that expected growth regime switching has a strong linkage with the business cycle. The contemporaneous correlation between the plotted filtered probability and the dummy variable of the NBER recessions is about 0.63.

The estimated transition probabilities for volatility regimes are  $p_{ll}^\sigma = 0.98$  and  $p_{hh}^\sigma = 0.96$ . These estimates imply that average duration of the low volatility states is about 50 quarters, and that of the high volatility states is about 26 quarters. Compared to expected growth regimes, volatility regimes are far more persistent, suggesting that long-run volatility risk in productivity growth is pronounced in the data. Figure 1 plots the filtered probabilities of the high conditional volatility states. The figure shows that (1) the TFP growth of the U.S. economy features a significant decline

in volatility since the 1980s, and this pattern is persistent in decades, and (2) the variation in the filtered probability of the high volatility states is mostly detached from the business cycle. In fact, the contemporaneous correlation between the plotted filtered probability and the dummy variable of the NBER recessions is only about 0.12. In the next section, we present a production-based general equilibrium model to show that the low-frequency movement in the conditional volatility of productivity growth can induce time-varying risk premium, and this variation does not appear to be strongly connected with the business cycle.

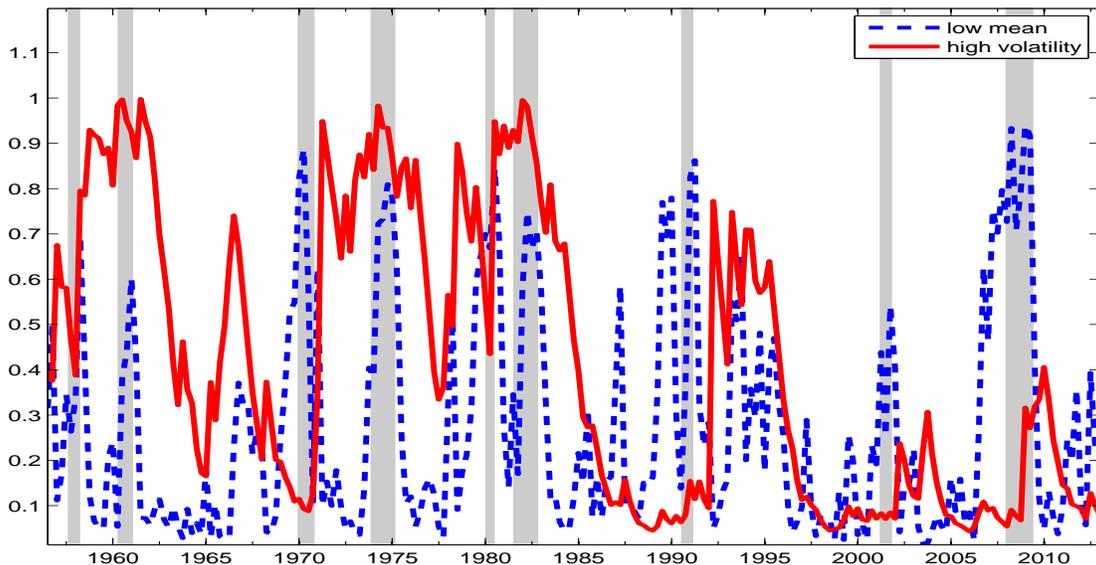


Figure 1: Filtered probabilities. This figure plots the filtered probabilities of the low expected growth states and high volatility states in the next period for the historical TFP growth from 1956:Q1 to 2012:Q4.

## 2.2 Impact of Volatility Risks

What is the impact of volatility risks in productivity growth on the macroeconomic quantities and asset prices in the data? To answer this question, we first estimate an AR(1)-GARCH(1,1) model of productivity growth by the maximum likelihood method,

$$\begin{aligned}\Delta z_t &= \mu_a + \vartheta \Delta z_{t-1} + e_t, \quad e_t \sim N(0, h_t^2), \\ h_t^2 &= \alpha_0 + \alpha_1 e_{t-1}^2 + \theta h_{t-1}^2,\end{aligned}$$

where  $h_t^2$  is the variance of  $e_t$ . Panel B of Table 1 presents the estimates for this model. It shows that the estimates of  $\theta$  and  $\vartheta$  are both significant, indicating strong evidence of time-varying volatility of productivity growth. We then extract data for  $h_t$  from the preceding estimation and use  $\ln(h_t)$  to proxy volatility risk.

We then estimate a VAR model by OLS, where the state vector includes seven variables in the estimation order:  $\ln(h_t)$  and the logs of dividends, investment, consumption, output, price-dividend ratios, and stock returns. Dividends, investment, consumption, and output are detrended. We also use the maximum likelihood method and the BVAR method (Sims and Zha (1998)) to estimate this model and find similar results. We take stock returns data on the NYSE/AMEX/NASDAQ value-weighted index and 3-month T-bill rates (riskfree rates) from the Center for Research in Security Prices (CRSP). We use the CPI data from FRED at St. Louis to deflate all nominal variables. The sample period is from 1956:Q1 to 2012:Q4. We take the data on real fixed investment ( $I$ ), nondurable consumption goods and services ( $C$ ), and aggregate output (defined as  $I + C$ ) from the national income product accounts, Bureau of Economic Analysis. We follow Bansal et al. (2005) and use monthly returns data including and excluding dividends to generate quarterly dividend growth data and price-dividend ratio data for the period 1956:Q1–2012:Q4.

Figure 2 presents the impulse responses to a positive one-standard-deviation shock to the volatility of productivity growth from the VAR estimation. This figure shows that investment, consumption, and output fall, consistent with the finding in the literature.<sup>3</sup> Thus an increase in the volatility risk in productivity growth reduces consumption and raises marginal utility. This implies that the market price of the volatility risk is negative. Figure 2 also shows that dividends, the price-dividend ratio, and stock returns all fall in response to a positive volatility shock. This result is in line with the finding of Bansal and Yaron (2004), Lettau et al. (2008), and Bansal et al. (2013) that high macroeconomic uncertainty is associated with low equity valuation. Our result indicates that stock returns are negatively exposed to the volatility risk in productivity growth. Consequently the volatility risk in productivity growth carries a positive risk premium. In the next section we build a model to generate this result and use the calibrated model to quantify the size of the volatility

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<sup>3</sup>There are several different measures of uncertainty shock in the literature. For example, Bloom (2009) uses the VIX index, Bloom et al. (2013) use firm-specific uncertainty, and Leduc and Liu (2013) use survey data. They all find similar results.

risk premium.

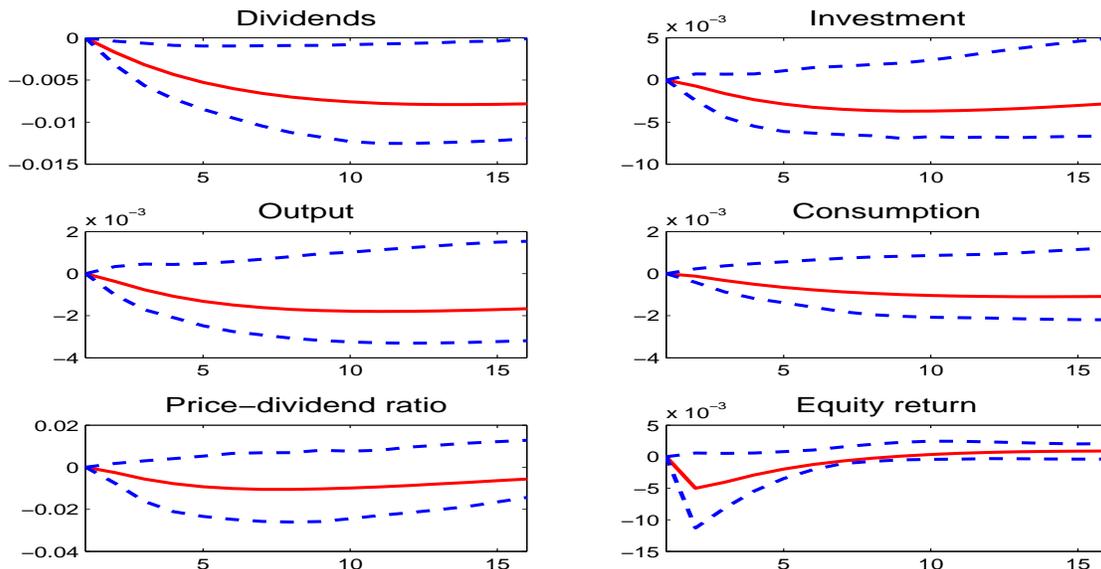


Figure 2: Impulse responses to a positive shock to the conditional volatility of productivity growth. The dashed lines describe 90% confidence bands. The VAR is estimated according to the order:  $\ln(h_t)$  and the logs of dividends, investment, consumption, output, price-dividend ratios, and stock returns. Dividends, investment, consumption, and output are linearly detrended.

### 3 A Benchmark Model

In this section we present a simple production-based asset pricing model. As is standard in the literature (e.g., Jermann (1998), Campanale et al. (2010), Croce (2014), and Kaltenbrunner and Lochstoer (2010), among others), we assume that labor is exogenously given. We first describe the utility function and then discuss the social planner’s problem. Finally, we study decentralization and present asset pricing formulas.

#### 3.1 Generalized Disappointment Aversion

We follow Routledge and Zin (2010) to model the representative agent’s utility function. Routledge and Zin (2010) generalize Gul (1991)’s disappointment aversion utility and embed this static utility in the recursive utility framework of Epstein and Zin (1989). Formally the agent derives utility from a consumption stream  $\{C_t\}$  only. Suppose that the labor supply is exogenously given and

normalized to 1. The agent's continuation utility at date  $t$  is given by

$$V_t = \left[ (1 - \beta)C_t^{1-\frac{1}{\psi}} + \beta\mathcal{R}_t(V_{t+1})^{1-\frac{1}{\psi}} \right]^{\frac{1}{1-\frac{1}{\psi}}}, \quad (2)$$

where  $\psi > 0$  represents the elasticity of intertemporal substitution (EIS), and  $\mathcal{R}_t(V_{t+1})$  represents the (conditional) certainty equivalent of future utility  $V_{t+1}$ . The conditional certainty equivalent is defined as follows

$$\frac{\mathcal{R}_t(V_{t+1})^{1-\gamma}}{1-\gamma} = \mathbb{E}_t \left[ \frac{V_{t+1}^{1-\gamma}}{1-\gamma} \right] - \eta \mathbb{E}_t \left\{ \mathcal{I} \left( \frac{V_{t+1}}{\mathcal{R}_t(V_{t+1})} < \kappa \right) \left[ \frac{(\kappa\mathcal{R}_t(V_{t+1}))^{1-\gamma}}{1-\gamma} - \frac{V_{t+1}^{1-\gamma}}{1-\gamma} \right] \right\}, \quad (3)$$

where  $\mathbb{E}_t$  represents the conditional expectation operator given information at time  $t$ ,  $\gamma > 0$  represents the risk aversion parameter, and the parameters  $\eta > 0$  and  $0 < \kappa \leq 1$  capture disappointment aversion.<sup>4</sup> We use  $\mathcal{I}(A)$  to denote an indicator function that is equal to 1 if the event  $A$  is true, and zero otherwise. When  $\psi = 1$ , the utility function (2) reduces to  $V_t = C_t^{1-\beta} [\mathcal{R}_t(V_{t+1})]^\beta$ .

When  $\eta = 0$ ,  $\mathcal{R}_t(V_{t+1})$  is equal to the certainty equivalent for expected utility and the model in (2) reduces to the familiar Epstein-Zin recursive utility. If  $\eta > 0$ , there is a utility cost for outcomes below the scaled certainty equivalent  $\kappa\mathcal{R}_t(V_{t+1})$ . This represents the agent's aversion to disappointing outcomes. If  $\kappa = 1$ , the model reduces to the pure disappointment aversion model of Gul (1991). Routledge and Zin (2010) generalize Gul's model of disappointment aversion by moving the disappointment reference point with  $\kappa \in (0, 1)$ . The idea is that utility outcomes are disappointing only if they lie sufficiently far below the certainty equivalent. This allows for first-order-risk-aversion effects away from certainty and is useful for asset pricing. More importantly, GDA allows the disappointing outcomes correspond to tail events and hence generate countercyclical risk aversion (or pricing kernel). This is useful to generate time-varying equity premium.

It is straightforward to show that the pricing kernel for the utility model in (2) is given by (see Bonomo et al. (2011))

$$M_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left( \frac{V_{t+1}}{\mathcal{R}_t(V_{t+1})} \right)^{\frac{1}{\psi}-\gamma} \left( \frac{1 + \eta \mathcal{I} \left( \frac{V_{t+1}}{\mathcal{R}_t(V_{t+1})} < \kappa \right)}{1 + \kappa^{1-\gamma} \eta \mathbb{E}_t \left[ \mathcal{I} \left( \frac{V_{t+1}}{\mathcal{R}_t(V_{t+1})} < \kappa \right) \right]} \right). \quad (4)$$

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<sup>4</sup>Routledge and Zin (2010) also consider the case  $\kappa > 1$ , in which states with continuation value greater than the conditional certainty equivalent are also deemed as disappointing.

There are three components in the pricing kernel. The first component is the pricing kernel under standard power utility. The second component reflects the adjustment due to the separation between risk aversion and the EIS as in the Kreps-Porteus and Epstein-Zin utility. When  $\gamma > 1/\psi$ , the agent prefers earlier resolution of uncertainty and the ratio of continuation utility  $V_{t+1}$  to the certainty equivalent of this continuation utility  $\mathcal{R}_t(V_{t+1})$  adds a premium for long-run risk. If consumption growth has a persistent component, a bad shock to productivity growth today will reduce  $V_{t+1}/\mathcal{R}_t(V_{t+1})$ , which will raise the pricing kernel when  $\gamma > 1/\psi$ . The third component reflects the impact of disappointment aversion with  $\eta > 0$ . Whenever the ratio of future utility to its certainty equivalent is less than the threshold  $\kappa$ , the agent attaches a larger weight to the pricing kernel. Thus the last two components make the pricing kernel more countercyclical and help raise the equity premium. We will discuss this point further in Section 4.

What is the impact of the time-varying volatility of productivity growth on the pricing kernel? In Section 4.5 we will show that the time-varying volatility of productivity growth is the key to generating long-run risks in the volatility of consumption growth observed in the data. An increase in the volatility of consumption growth will have persistent effects and make future utility more volatile because disappointing outcomes will be more likely. As a result, the increased consumption growth volatility will raise the volatility of the third component in the pricing kernel. Persistent changes in expected productivity growth can also increase the volatility of future utility and of the GDA pricing kernel, but the effect is indirect. The comparative statics analysis in Section 6 shows that this effect is also important.

### 3.2 Social Planner's Problem

To facilitate detrending, we suppose that aggregate output is produced according to the production function  $Y_t = K_t^\alpha (A_t N_t)^{1-\alpha}$ , where the aggregate productivity shock  $A_t$  and the TFP  $Z_t$  described in Section 2 have the relation  $Z_t = A_t^{1-\alpha}$ .

Capital adjustment is costly and the law of motion of capital is given by

$$K_{t+1} = (1 - \delta) K_t + \phi \left( \frac{I_t}{K_t} \right) K_t, \quad (5)$$

where the adjustment cost function is given by

$$\phi\left(\frac{I_t}{K_t}\right) = a_1 + \frac{a_2}{1 - 1/\xi} \left(\frac{I_t}{K_t}\right)^{1-1/\xi}, \quad a_2 > 0, \quad \xi > 0.$$

Here  $\xi$  represents the elasticity of the investment rate to Tobin's  $Q$  and the parameters  $a_1$  and  $a_2$  are chosen such that there is no adjustment cost in the steady state.<sup>5</sup>

The social planner's problem is to choose  $\{C_t, I_t\}$  so as to maximize (2) subject to (5) and the resource constraint  $Y_t = C_t + I_t$ .

### 3.3 Asset Prices

The decentralization of the planner's solution is standard (e.g., Jermann (1998)). The representative household works for the firm and trades firm shares and riskfree bonds. The representative firm chooses labor and investment demand to maximize its firm value. Firm value is equal to the discounted present value of its future cash flows. The cash flows in period  $t$  are given by

$$F_t = Y_t - w_t N_t - I_t = \alpha Y_t - I_t, \quad (6)$$

where  $w_t$  represents the wage rate and the second equality follows from the fact that  $w_t = \partial Y_t / \partial N_t = (1 - \alpha) A_t^{1-\alpha} K_t^\alpha N_t^{-\alpha}$ . Optimal investment is characterized by the  $Q$  theory (Hayashi (1982)). In particular, Tobin's marginal  $Q$  satisfies

$$Q_t = \frac{1}{\phi'\left(\frac{I_t}{K_t}\right)} = a_2 \left(\frac{I_t}{K_t}\right)^{1/\xi}.$$

This implies that the investment rate increases with Tobin's  $Q$ . As is well known, unlevered firm value  $FV_t$  is given by  $FV_t = Q_t K_{t+1}$ .

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<sup>5</sup>In particular,  $\phi(I/K) = I/K$  and  $\phi'(I/K) = 1$ . This means that

$$a_1 = \frac{\exp(\bar{\mu}) - 1 + \delta}{1 - \xi}, \quad a_2 = (\exp(\bar{\mu}) - 1 + \delta)^{1/\xi},$$

where  $\bar{\mu}$  denotes the unconditional mean of  $\mu(z_t)$ .

In equilibrium the following asset pricing equation must hold for any asset  $j$

$$\mathbb{E}_t \left[ M_{t+1} R_{t+1}^j \right] = 1, \quad (7)$$

where  $R_{t+1}^j$  denotes asset  $j$ 's return between  $t$  and  $t + 1$ . In particular, it holds for the investment return  $R_{t+1}^I$ , defined by

$$R_{t+1}^I = \frac{1}{Q_t} \left\{ Q_{t+1} \left[ 1 - \delta + \phi \left( \frac{I_{t+1}}{K_{t+1}} \right) \right] + \frac{\alpha Y_{t+1} - I_{t+1}}{K_{t+1}} \right\}. \quad (8)$$

This return is also equal to the return on the unlevered firm with cash flows given by (6).

As is well known, unlevered equity premium is typically low compared to the data because cash flows in (6) are relatively smooth (e.g., Jermann (1998)). Financial leverage can raise dividends volatility and hence raise the levered equity premium. In addition, the observed equity prices and returns in the data correspond to leveraged corporations. The price of unlevered cash flows  $FV_t$  and the investment return  $R_{t+1}^I$  are not directly observable. Thus it is important to study levered equity value and returns in order for the model to be confronted with the data.

We now follow Jermann (1998) to introduce financial leverage. Suppose that the Modigliani and Miller Theorem holds so that capital structure does not matter for a firm's investment decision and firm value. There are no corporate taxes or default. Suppose that in each period  $t$ , a fraction  $\omega$  of the firm's capital stock  $K_t$  is financed by long-term discount bonds. We use  $B_{t,n}$  to denote the price of the  $n$ -period discount bonds.<sup>6</sup> We can then write dividends as

$$D_t = Y_t - w_t N_t - I_t + \omega K_t - \omega K_{t-n} / B_{t-n,n}, \quad (9)$$

where  $\omega K_t$  represents the bond proceeds and  $\omega K_{t-n} / B_{t-n,n}$  represents the repayment to the bond holders purchasing the  $n$ -period discount bonds in period  $t - n$  at the price  $B_{t-n,n}$ . This modeling of leverage helps reduce the countercyclicality of dividends because a shock that raises capital will raise debt. This will dampen the decline in dividends caused by higher investment.

The  $n$ -period bond price satisfies the no-arbitrage equation  $B_{t,n} = \mathbb{E}_t [M_{t+1} B_{t+1,n-1}]$ , with the

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<sup>6</sup>The payoff of one unit  $n$ -period discount bond is equal to one at all states  $n$  periods later in the future.

boundary condition  $B_{t,0} = 1$  for any  $t$ . The one-period riskfree rate is given by  $R_{t+1}^f = 1/B_{t,1} = 1/\mathbb{E}_t[M_{t+1}]$ . Let  $P_t^e$  denote the (levered) equity price. The (levered) equity return is given by  $R_{t+1}^e = (D_{t+1} + P_{t+1}^e)/P_t^e$ . By (7), equity value satisfies  $P_t^e = \mathbb{E}_t[M_{t+1}(D_{t+1} + P_{t+1}^e)]$ . By (9), we can compute that equity value is equal to firm value minus debt value,  $P_t^e = FV_t - DV_t$ , where debt value is equal to the total market value of all outstanding bonds from period  $t - n + 1$  to period  $t$ ,

$$DV_t = \sum_{j=1}^n \frac{B_{t,j}\omega K_{t-n+j}}{B_{t-n+j,n}}.$$

Using (7), we can show that the conditional equity premium satisfies

$$\mathbb{E}_t \left[ R_{t+1}^e - R_{t+1}^f \right] = \frac{\sigma_t(M_{t+1})}{\mathbb{E}_t(M_{t+1})} \frac{-Cov_t(M_{t+1}, R_{t+1}^e)}{\sigma_t(M_{t+1})}, \quad (10)$$

where  $\sigma_t(M_{t+1})/\mathbb{E}_t(M_{t+1})$  represents the conditional market price of risk and the last term represents the conditional quantity of risk. Here,  $\sigma_t(\cdot)$  and  $Cov_t(\cdot, \cdot)$  denote the conditional standard deviation and covariance operators given date  $t$  information, respectively. The above equation shows that in order for the conditional equity premium to be positive, the pricing kernel and stock returns must be negatively correlated. The intuition is that the pricing kernel reflects the marginal utility of consumption and is high in bad times. The agent must be compensated for holding stocks when they do poorly in bad times. When the pricing kernel and stock returns are more negatively correlated, the equity premium will be larger. Thus raising the countercyclicality of the pricing kernel is important for explaining asset pricing puzzles.

We can also derive an unconditional version of equation (10). Moreover, we can derive the Hansen-Jagannathan bound (Hansen and Jagannathan (1991)) as follows

$$\frac{\sigma(M_{t+1})}{\mathbb{E}(M_{t+1})} \geq \frac{\mathbb{E}(R_{t+1}^e - R_{t+1}^f)}{\sigma(R_{t+1}^e - R_{t+1}^f)},$$

where  $\sigma(M_{t+1})/\mathbb{E}(M_{t+1})$  is the unconditional market price of risk and the expression on the right-hand side represents the Sharpe ratio. The above inequality shows that the Sharpe ratio provides a lower bound for the market price of risk. Another version of the equity premium puzzle is that the market price of risk implied by many asset pricing models is smaller than the Sharpe ratio observed

in the data, violating the above inequality.

## 4 Quantitative Results

Since the model does not admit a closed-form solution, we use numerical methods to solve the model. We solve the social planner’s problem numerically using the standard value function iteration method with Chebyshev interpolation and then derive asset pricing results using formulas in Section 3.3. An online appendix describes the numerical procedure. We calibrate the model at a quarterly frequency and obtain unconditional moments of macroeconomic quantities and financial variables from 20,000 Monte Carlo simulations, where each simulation contains 228 periods of data.

### 4.1 Calibration

We use the estimates for the TFP growth process  $\ln(Z_t/Z_{t-1})$  presented in Table 1 to assign parameter values in the process  $\ln(A_t/A_{t-1}) = \ln(Z_t/Z_{t-1}) / (1 - \alpha)$ . We fix the capital share at  $\alpha = 0.36$  and the depreciation rate at  $\delta = 0.02$  as in the RBC literature. We then choose other parameter values in the model to match moments in the data described in Section 2.

The quantity dynamics are insensitive to preference parameters related to risk attitudes such as  $\gamma$ ,  $\eta$ , and  $\kappa$ . But the consumption dynamics are sensitive to the EIS parameter  $\psi$ , a finding similar to Croce (2014). When the EIS parameter  $\psi$  increases, the agent is more willing to substitute consumption intertemporally by adjusting the investment level. Thus, investment absorbs most of the (permanent) shock to productivity growth such that consumption growth becomes less volatile.<sup>7</sup> We choose  $\psi = 2.0$ , consistent with the long-run risk literature (Bansal and Yaron (2004) and Bansal et al. (2013)). This value is still under debate in the literature. In Section 6 we will conduct a sensitivity analysis for various values of the EIS parameter. We set  $\xi = 9$  to match the volatility of investment growth relative to output growth in the data. The parameter values for  $a_1$  and  $a_2$  are normalized such that there are no adjustment costs in the deterministic steady state.

We fix the risk aversion parameter  $\gamma$  at 10 in the benchmark calibration as in Bansal and Yaron (2004). Mehra and Prescott (1985) argue that a plausible risk aversion parameter should

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<sup>7</sup>Campanale et al. (2010) and Kaltenbrunner and Lochstoer (2010) show that the case of transitory shocks will generate a different result.

Table 2: **Benchmark parameter values**

Parameter	Description	Value
$\beta$	Subjective discount factor	0.9957
$\gamma$	Risk aversion	10
$\psi$	EIS	2.0
$\kappa$	Disappointment cutoff	0.989
$\eta$	Disappointment aversion	2.45
$\alpha$	Capital share	0.36
$\delta$	Depreciation rate	0.02
$\xi$	Adjustment cost parameter	9
$a_1$	Normalization	-0.003
$a_2$	Normalization	0.662
$\omega$	Leverage parameter	0.01
$n$	Bond maturity	60

be between 1 and 10. Our choice is moderate compared with the existing production-based asset pricing models such as Tallarini (2000), given the enormous difficulty to match the size of equity premium in production-based models. We set the GDA parameter  $\eta$  at 2.45. Epstein and Zin (1991) find that the estimate of  $\eta$  is 1.63 for nondurable consumption and 2.45 for nondurables and services. We then choose  $\kappa = 0.989$  as in Bonomo et al. (2011). Since the riskfree rate is sensitive to the subjective discount factor  $\beta$ , we choose  $\beta$  to match the mean riskfree rate in the data.

For financial leverage, we consider 15-year discount bonds, i.e.,  $n = 60$  quarters. We set  $\omega = 0.01$  in the benchmark calibration to keep the average debt-to-equity ratio to be about 1:1. The leverage ratio is held constant across models in the calibration and sensitivity analysis. Using a comprehensive dataset on debt structure, Rauh and Sufi (2013) document that the leverage ratio is about 50% in their dataset. Jermann and Quadrini (2012) report a similar level of leverage, using the US Flow of Funds data. Table 2 presents the calibrated benchmark parameter values.

## 4.2 Impulse Responses and Price of Risk

Our model features three sources of independent aggregate shocks: an innovation shock to productivity growth and two regime switching shocks to the mean and volatility of productivity growth. We now examine the responses of quantities and asset prices to each of these shocks. We assume that the economy initially stays in the high mean and low volatility state  $(\mu_h, \sigma_l)$  and the capital

stock is at the stochastic steady-state level.<sup>8</sup> We then study the impact of a shock that occurs in the third period and only lasts for that period. An online appendix describes the details of the implementation.

Figure 3 presents the impulse responses of several key variables to a positive innovation shock  $\varepsilon_t$  with the size  $2\sigma_l$ . Because the impact of the innovation shock on the productivity level is permanent, both consumption and investment rise on impact and then grow at the original rate  $\mu_h$ . This effect decreases the representative agent's marginal utility and the pricing kernel. Thus the price of the innovation risk in productivity growth is positive. The immediate increase in investment leads to a temporary decline in dividends. But dividends rise eventually and grow at the rate  $\mu_h$ . Since the equity price grows at the rate of productivity and productivity growth rises on impact, the equity price also rises on impact and then grows at the rate  $\mu_h$ . This leads to an increase in the realized equity return on impact. Thus the pricing kernel and the realized equity return react in opposite directions, generating a positive risk premium. The conditional market price of risk and conditional equity premium are, however, not quite responsive to the innovation shock because the innovation shock is transitory.

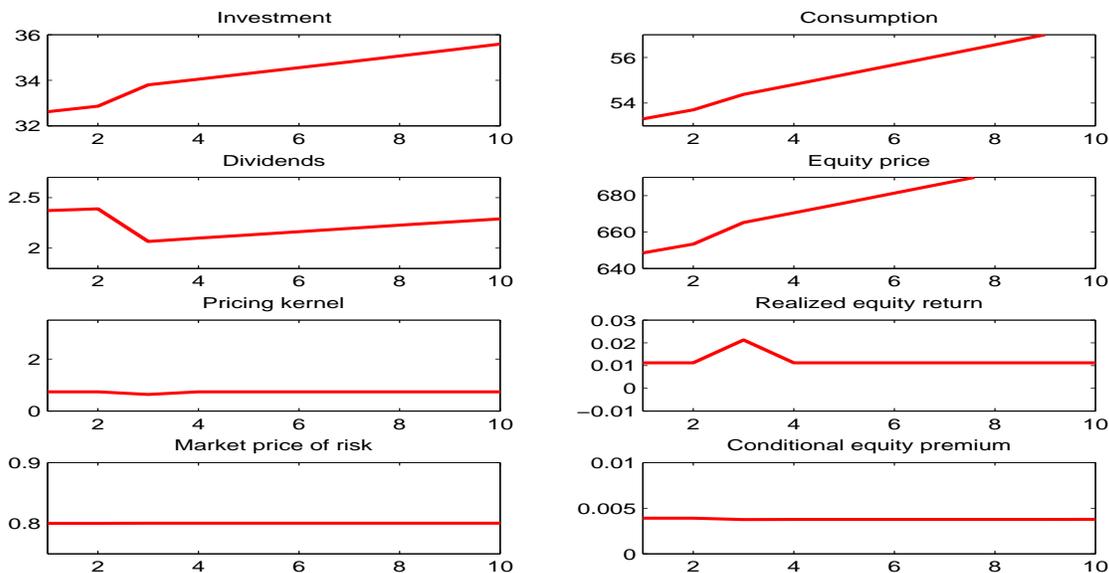


Figure 3: Impulse responses to a positive growth innovation shock. The shock occurs in period 3 and its size is  $2\sigma_l$ . The economy is initially in regime  $(\mu_h, \sigma_l)$ .

<sup>8</sup>Starting at any other state will give a similar result and will not change our insights.

Figure 4 displays the impulse responses to a shock that makes the economy switch from state  $(\mu_h, \sigma_l)$  to state  $(\mu_l, \sigma_l)$ . This shock can be interpreted as a negative long-run risk shock to the expected productivity growth, in the spirit of Bansal and Yaron (2004). As expected, investment and consumption fall and then move to the new long-run trend because aggregate productivity decreases to a permanently lower level. Since the pricing kernel rises when the mean productivity growth decreases, the price of the regime shift in mean is positive. Dividends rise on impact and then grow at the original rate  $\mu_h$ . Equity prices decrease on impact and then rise and grow at the rate  $\mu_h$ . Stock returns also fall on impact. This implies that stock returns are positively exposed to the risk of regime switching in mean. Thus this risk carries a positive risk premium.

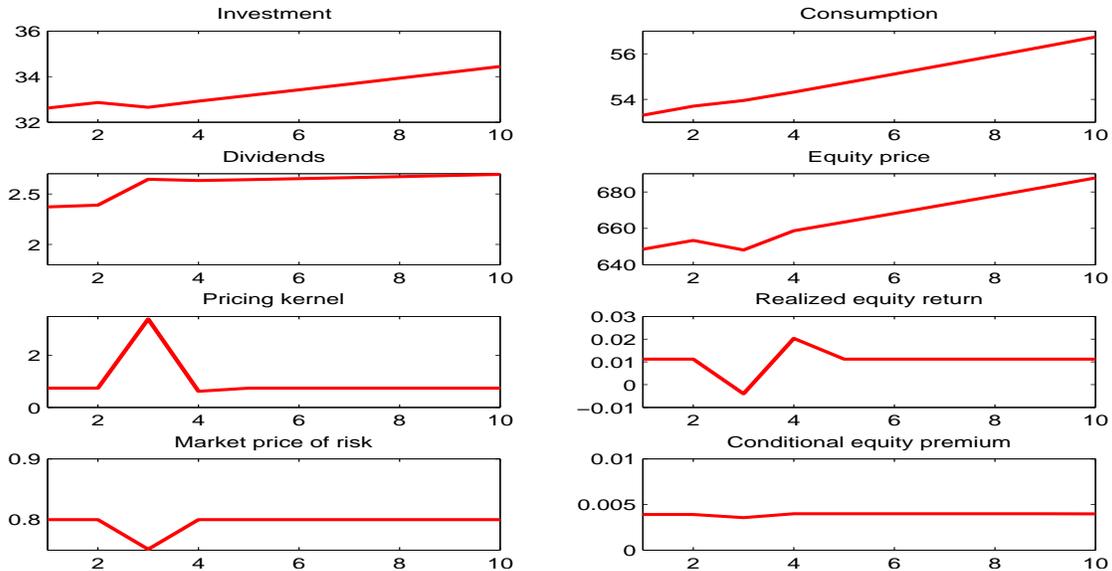


Figure 4: Impulse responses to a mean regime shift from  $\mu_h$  to  $\mu_l$  in period 3. The economy is initially in regime  $(\mu_h, \sigma_l)$ .

We now study the impulse responses to an uncertainty shock that makes the economy switch from state  $(\mu_h, \sigma_l)$  to state  $(\mu_h, \sigma_h)$ . Figure 5 presents the results. Due to increased uncertainty, the precautionary saving motive induces the representative agent to reduce consumption. Note that output  $Y_t = K_t^\alpha A_t^{1-\alpha}$  does not respond to the uncertainty shock on impact because (1) capital  $K_t$  is predetermined, (2) labor is exogenously fixed at 1, and (3) the uncertainty shock does not change the productivity level  $A_t$ . It follows that investment must rise on impact. Thus our benchmark model cannot explain the comovement among consumption, investment, and output in response

to uncertainty shocks observed in the data presented in Section 2.2. However, the consumption response in our model is consistent with the data. The decreased consumption causes the marginal utility and the pricing kernel to rise. Thus the market price of the volatility risk in productivity growth is negative.

Figure 5 also shows that dividends, equity values, and stock returns all fall on impact, a result consistent with the data. This implies that stock returns are negatively exposed to the volatility risk in productivity growth. Thus this risk carries a positive risk premium.

Unlike the first two shocks discussed earlier, the responses of the pricing kernel, the conditional market price of risk, and the conditional equity premium to the uncertainty shock are large, suggesting that volatility risks should play a quantitatively important role in explaining asset pricing phenomena. The intuition is as follows. When the economy switches to the high uncertainty state, it is more likely for disappointing outcomes to occur. Thus the third component of the pricing kernel in (4) rises substantially. In addition, because the agent prefers early resolution of uncertainty ( $\gamma > 1/\psi$ ), the second component of the pricing kernel also rises. These two effects together cause the responses of the pricing kernel and the conditional market price of risk to the uncertainty shock to be significant.

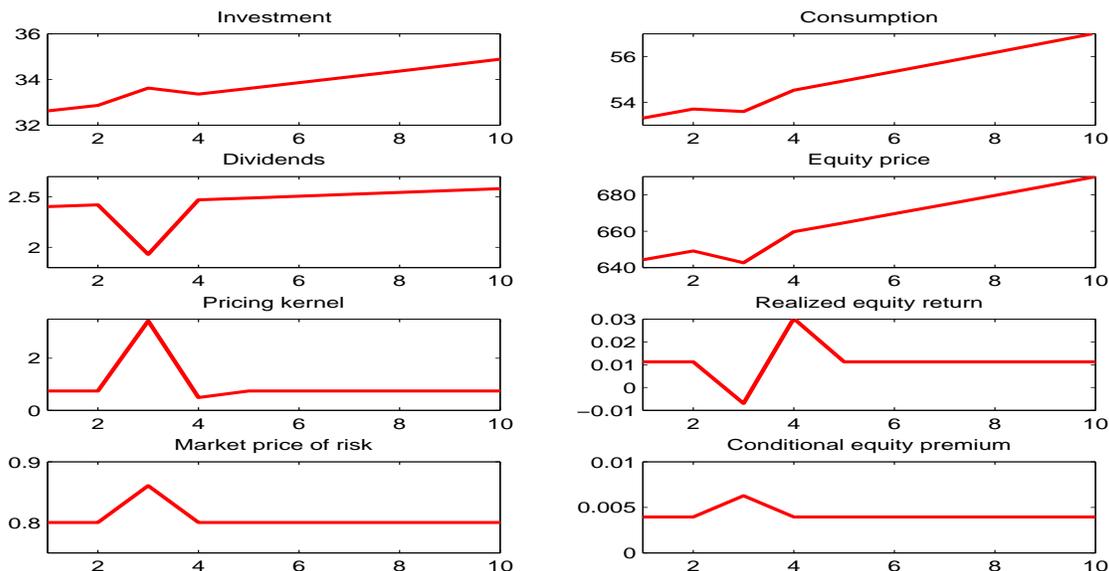


Figure 5: Impulse responses to a volatility regime shift from  $\sigma_l$  to  $\sigma_h$  in period 3. The economy is initially in regime  $(\mu_h, \sigma_l)$ .

### 4.3 Unconditional Moments

Table 3 presents results related to unconditional moments of quantity and financial variables. In terms of quantities, the benchmark model matches the following statistics reasonably well: the volatilities of consumption growth and investment growth relative to output growth, the correlation between consumption growth and output growth, and the correlation between investment growth and output growth. However, the model implied output growth volatility is lower than that in the data. This reflects the fact that capital is a slow-moving state variable and labor is exogenously given. It is challenging for production-based models to simultaneously match investment volatility and equity premium. Previous research such as Jermann (1998) and Campanale et al. (2010) relies on high adjustment costs to increase the equity claim risk and equity premium, but high adjustment costs will counterfactually imply a low investment volatility. In the benchmark model, we assume low adjustment costs to raise investment volatility, while we rely on GDA to raise equity premium.

Recent research in macroeconomics has focused on time variations in the volatility of macroeconomic quantities (e.g., Justiniano and Primiceri (2008), Fernandez-Villaverde et al. (2011), and Bloom et al. (2013)). It is important for DSGE models to match the variability of the volatility in quantities observed in the data. Table 3 shows that the vol-of-vol in investment growth is five times as high as the vol-of-vol in consumption growth in the data. Standard DSGE models with linear homoscedastic productivity shocks cannot explain this salient feature, due to the lack of variations in the second moment of productivity growth. The benchmark model matches well the vol-of-vol in consumption, investment, and output growth in the data. However, the benchmark model cannot match the autocorrelations of consumption, investment, and output growth. This is a common problem in the RBC literature because most RBC models lack a propagation mechanism.

Turning to financial moments, we find that the mean and volatility of the investment return are very small (0.25 and 0.38 percent, respectively) in the benchmark model. But they do not correspond to the observed data for equity. When introducing financial leverage as in Jermann (1998), the mean equity premium and equity volatility become much larger (2.36 and 2.06 percent, respectively).<sup>9</sup> But these numbers are still much lower than the data. Note that explaining the

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<sup>9</sup>The levered equity return satisfies  $R_{t+1}^e = \frac{FV_t}{P_t^e} R_{t+1}^I - \frac{DV_t}{P_t^e} R_{t+1}^D$ , where  $R_{t+1}^D$  denotes the return on debt. Given the calibrated debt-equity ratio  $\mathbb{E}[DV_t/P_t^e] \approx 1$ , the high equity premium is partly due to the result that the mean debt premium is negative and the high equity volatility is partly due to the result that the unlevered equity premium

Table 3: Unconditional moments

Statistic	Data	Benchmark	EZ1	EZ2	GDAC
		$\gamma = 10$ $\kappa = 0.989$ $\eta = 2.45$	$\gamma = 10$	$\gamma = 34$	$\sigma_h = \sigma_l$
Panel A: Macroeconomic moments					
$\sigma_{\Delta c}$ (%)	1.00	0.83	0.83	0.82	0.78
$\sigma_{\Delta i}$ (%)	4.54	3.10	3.11	3.15	2.94
$\sigma_{\Delta y}$ (%)	2.00	1.57	1.57	1.57	1.53
$\rho(\Delta c_t, \Delta c_{t+1})$	0.45	0.23	0.24	0.24	0.23
$\rho(\Delta i, \Delta y)$	0.94	0.98	0.99	0.97	0.99
$\rho(\Delta c, \Delta y)$	0.77	0.89	0.96	0.87	0.96
$\sigma(\sigma_{\Delta c})$ (%)	0.30	0.29	0.27	0.28	0.23
$\sigma(\sigma_{\Delta i})$ (%)	1.55	1.12	1.12	1.14	0.94
$\sigma(\sigma_{\Delta y})$ (%)	0.62	0.55	0.55	0.55	0.48
Panel B: Financial moments					
$\mathbb{E}[R_f] - 1$ (%)	1.44	1.46	2.29	1.37	1.50
$\sigma(R_f)$ (%)	1.07	0.23	0.20	0.20	0.23
$\mathbb{E}(R^I - R_f)$ (%)	n.a.	0.25	0.04	0.29	0.27
$\sigma(R^I - R_f)$ (%)	n.a.	0.38	0.37	0.38	0.35
$\mathbb{E}(R^e - R_f)$ (%)	5.40	2.36	0.34	2.38	2.00
$\sigma(R^e - R_f)$ (%)	15.03	2.06	2.00	2.31	1.76
$\mathbb{E}[\Delta d]$ (%)	1.60	1.80	1.65	1.80	1.79
$\sigma_{\Delta d}$ (%)	4.60	28.84	94.45	33.45	30.25
$\rho(\Delta d, \Delta c)$	0.08	-0.51	-0.28	-0.45	-0.67
$\sigma(M)/\mathbb{E}(M)$	n.a.	1.62	0.31	1.32	1.59
$\mathbb{E}(K/A)$	n.a.	67.18	61.12	67.78	66.61

Empirical moments are computed using US quarterly data from 1956:Q1 to 2012:Q4. Lower case variables denote log values. All means and standard deviations are expressed in annualized terms. All correlations and autocorrelations are in quarterly terms.  $\mathbb{E}(\cdot)$ ,  $\sigma(\cdot)$ , and  $\rho(\cdot, \cdot)$  denote mean, volatility, and correlation, respectively.

equity premium and equity volatility puzzles in the production-based framework is much more challenging than in endowment economies because production-based models generally imply countercyclical dividend growth as discussed in Section 4.2 (see Kaltenbrunner and Lochstoer (2010) and Favilukis and Lin (2013)). However, dividend growth is procyclical in the data. Thus excessively volatile dividend growth is needed to generate a high equity volatility in the model. Financial leverage helps generate high dividend volatility and reduce countercyclicality of dividend growth. In Section 5 we will directly calibrate the firm’s equity payout to the dividends data and assess the model’s ability in reproducing the key moments of asset returns.

The benchmark model with an EIS greater than 1 generates a smooth riskfree rate, with a standard deviation of 0.23 percent per year. This is in contrast to production-based models with habit formation that generate an excessively volatile riskfree rate. For example, in the habit formation model of Jermann (1998) or Boldrin et al. (2001), the agent displays strong aversion to intertemporal substitution of consumption. Given high capital adjustment costs, the riskfree rate must vary a lot in response to productivity shocks.<sup>10</sup> In a closely related paper Campanale et al. (2010) consider Epstein-Zin recursive utility and calibrate the EIS parameter at a small value substantially less than 1. This inevitably generates a high volatility of the riskfree rate due to the implied low intertemporal substitution.

To see the role of the GDA preference, we compare the benchmark model with the Epstein-Zin model. We first set  $\eta = 0$  and fix other parameter values as in Table 2. The resulting model is called EZ1.<sup>11</sup> Table 3 reveals that the benchmark model and model EZ1 generate similar values for unconditional moments of quantities. It turns out that the GDA parameters ( $\eta$  and  $\kappa$ ) do not matter much for quantity dynamics. This result is generally true for parameters related to risk attitudes, a point first made by Tallarini (2000) using the Epstein-Zin model.

By contrast, the GDA parameters matter a lot for financial moments. Specifically, model EZ1 implies a high riskfree rate (2.29 percent) and a low unlevered equity premium close to zero (0.04 percent). GDA preferences induce a large precautionary saving motive, raising the mean detrended

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and the debt premium are highly negatively correlated.

<sup>10</sup>Campbell and Cochrane (1999) overcome this issue in an endowment economy by designing their external habit formation model such that the riskfree rate is held constant.

<sup>11</sup>Simulated dividends are sometimes negative in this model. We delete the simulated data points whenever  $D_{t+1}/D_t < 0$ .

capital stock from 61.12 in model EZ1 to 67.18. This motive in turn reduces the riskfree rate. Since outcomes below the GDA threshold receive relatively more weight, GDA preferences raise effective risk aversion and therefore the price of risk. In particular, the unconditional market price of risk,  $\sigma(M)/\mathbb{E}(M)$ , is 1.62 in the benchmark model but only 0.31 in model EZ1. Model EZ1 also violates the Hansen-Jagannathan bound because the Sharpe ratio is equal to  $5.4/15.03 = 0.36$  in the data. Even though model EZ1 entertains time-varying uncertainty in productivity growth, this uncertainty is insufficiently priced due to low risk aversion.

Can the Epstein-Zin model generate similar results to those for the GDA preferences by raising the relative risk aversion parameter? To answer this question, we consider the case in which we set  $\eta = 0$  and take other parameter values as in Table 2 except for  $\gamma$ . We recalibrate  $\gamma$  to match the mean equity premium in the benchmark GDA model. We call this calibrated model EZ2. We find that  $\gamma = 34$ , which is unreasonably high, given the abundant experimental evidence.

To see the role of the volatility risk in productivity growth, we compare our benchmark model with a third model, called GDAC, in which we fix the conditional volatility of productivity growth at its steady-state level implied by the Markov-switching model. When the volatility risk is shut down, the model implied volatilities of macroeconomic quantities are moderately lower. More pronounced is the impact on financial moments. The volatility risk reduces the risk premium in investment returns from 0.27 percent in model GDAC to 0.25 percent in the benchmark model. The reason is that increased uncertainty raises investment and Tobin's Q. Croce (2014) derives a similar result and argues that volatility risk carries a negative risk premium and reduces equity premium in a production-based model. This conclusion does not apply to the levered equity in our model due to different modeling of leverage.<sup>12</sup> Table 3 shows that volatility risk raises the levered equity premium from 2 percent in model GDAC to 2.36 percent in the benchmark model. The intuition comes from the impulse responses analyzed in Section 4.2. Increased uncertainty raises debt value more than firm value so that equity value falls.

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<sup>12</sup>Unlike our paper, Croce (2014) assumes that the levered equity return is equal to a multiple of the investment return plus an independent noise term.

#### 4.4 Cyclicalities of Asset Prices

Asset prices and asset returns move with business cycles as evidenced in the data. Our production-based asset pricing model is suitable to address this issue. Following Gourio (2012), we use cross-correlograms to illustrate cyclicalities of several financial variables in Figure 6.

We first consider the cyclicalities of equity premium in Panel A of Figure 6. We detrend the log of output using the one-sided version of the Baxter and King (1999) filter and obtain  $\tilde{y}_t$ . Gourio (2012) notes that using the one-sided Baxter-King filter can avoid look-ahead bias and better capture the covariation of output with conditional financial moments. We compute the correlation between  $\tilde{y}_t$  and excess stock returns at various leads and lags, i.e.,  $\rho(\tilde{y}_t, R_{t+j}^e - R_{t+j}^f)$ , for  $j = -5$  to 5 quarters. In the data (lines with diamonds), this correlation is positive for  $j < 0$ , suggesting that excess stock returns positively lead output. But this correlation becomes negative for  $j > 0$ , implying that output negatively leads excess stock returns, i.e., future equity premia are lower when current output is high. Panel A shows that the benchmark model, model EZ2, and model GDAC can all match this pattern reasonably well.

Panel B of Figure 6 presents the cyclicalities of the price-dividend ratio. In the data, the correlation between detrended output and the log price-dividend ratio is positive at all leads and lags, indicating that the price-dividend ratio is procyclical. Again, all three models can match this pattern, but the magnitude of the correlation is larger than in the data.

Panel C of Figure 6 presents the cyclicalities of conditional equity volatility. This panel shows that in the data, equity volatility is countercyclical and negatively related to output at leads and lags up to 5 quarters. All three models can match this pattern. But the negative correlation between output and future equity volatility implied by model GDAC is too large compared to the data.

Panel D of Figure 6 displays the relationship between the stock market and investment. We compute the correlation between the HP filtered log investment  $i_t$  and the log market-to-book ratio, i.e.,  $\rho(i_t, \ln(P_{t+j}/K_{t+j}))$ ,  $j = -5$  to 5. In the data, this covariance is positive for  $j < 2$ , indicating that the stock market positively leads investment. All three models match well the magnitude of the association between stock prices and investment. But the exact timing of the relation between the stock market and investment cannot be replicated.

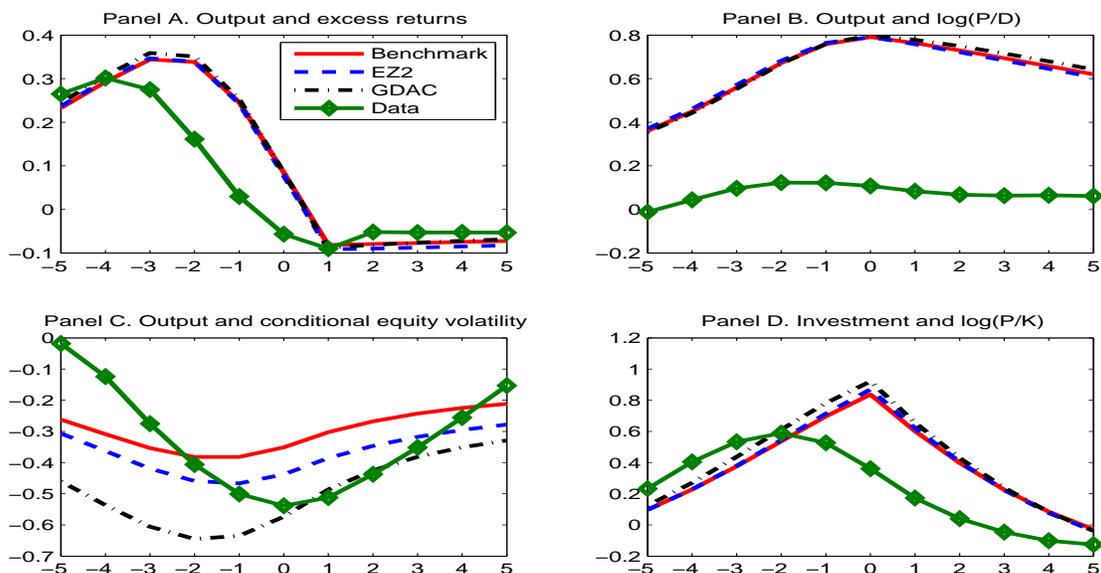


Figure 6: Cross-correlograms with 5 period leads and lags. Data are from 1956:Q1 to 2012:Q4.

In summary, Figure 6 shows that neither the GDA preferences nor the time-varying volatility of productivity growth plays an important role in determining the cyclical patterns of asset prices and returns. This suggests that Epstein-Zin preferences and the regime shifts in the expected growth of aggregate productivity are the two key elements for generating the cyclicity patterns in the data.

## 4.5 Consumption Dynamics

Understanding consumption dynamics is fundamental for consumption-based asset pricing models. For example, models with long-run consumption risks including Bansal and Yaron (2004), Lettau et al. (2008) and Bansal et al. (2012) find considerable evidence in the data for time-varying expected consumption growth and consumption volatility. In particular, Bansal and Yaron (2004) and Lettau et al. (2008) both emphasize the importance of time-varying consumption volatility. In this section, we demonstrate that our production economy model can generate long-run risks in both expected consumption growth and consumption volatility, and thereby lends support to the long-run consumption risks literature.

We begin by documenting empirical evidence that the historical consumption growth data feature time-varying expected growth and volatility. Following Lettau et al. (2008), we estimate a

Table 4: **Estimated consumption dynamics**

Parameter	Data	Benchmark	EZ2
	1956:Q1–2012:Q4		
$\mu_l^c$	0.37 (7.46)	0.25 (20.81)	0.25 (22.38)
$\mu_h^c$	0.87 (27.79)	0.61 (32.91)	0.62 (38.78)
$\sigma_l^c$	0.24 (8.67)	0.25 (39.26)	0.25 (35.99)
$\sigma_h^c$	0.56 (7.29)	0.46 (46.79)	0.45 (41.77)
$p_{ll}^{\mu,c}$	0.92 (21.74)	0.93 (78.11)	0.93 (84.58)
$p_{hh}^{\mu,c}$	0.97 (57.82)	0.95 (138.11)	0.95 (140.07)
$p_{ll}^{\sigma,c}$	0.91 (18.48)	0.96 (139.47)	0.97 (138.02)
$p_{hh}^{\sigma,c}$	0.86 (8.42)	0.96 (119.97)	0.96 (108.71)

This table reports the maximum likelihood estimates of the parameters in the Markov-switching model for consumption growth. The  $t$ -statistics are reported in parentheses.

four-state Markov switching process,

$$\Delta c_t = \mu^c(s_{1t}^c) + \sigma^c(s_{2t}^c)\varepsilon_{c,t}, \quad \varepsilon_{c,t} \sim N(0, 1),$$

where  $s_{1t}^c$  and  $s_{2t}^c$  follow independent two-state Markov chains and  $\varepsilon_{c,t}$  is a white noise and independent of other shocks. The consumption growth data are quarterly growth rates of consumption of nondurable goods and services from 1956:Q1 to 2012:Q4. We use the EM algorithm to compute the maximum likelihood estimates. The estimation results are reported in the second column of Table 4. The estimation clearly identifies two regimes for expected consumption growth and another two distinct regimes for the conditional volatility of consumption growth. Expected consumption growth is positive in both regimes and the growth rate in the high mean regime is about twice as high as that in the low mean regime. Moreover, the estimated conditional volatility in the high volatility regime is about twice as high as that in the low volatility regime.

More importantly, it is apparent that both the mean and volatility regimes are persistent, according to the estimated transition probabilities. The probabilities of staying in the low mean

regime and in the high mean regime are given by 0.92 and 0.97, respectively. The volatility regimes exhibit moderately less persistence. The probabilities of staying in the low volatility regime and in the high volatility regime are given by 0.91 and 0.86, respectively. The estimated transition probabilities imply that the first-order autocorrelation of expected consumption growth is about 0.96 in monthly terms and that of conditional consumption growth volatility is about 0.92, largely in line with the parameter values used by Bansal and Yaron (2004).

Next we estimate the Markov-switching process using simulated consumption growth data from the benchmark model and model EZ2. To avoid the small sample bias, we simulate 5,000 consumption growth data under each model. Further increasing the simulated data points generates very similar estimation results. Table 4 demonstrates that both the benchmark model and model EZ2 can generate long-run risks in the mean and volatility of consumption growth in line with the data. Kaltenbrunner and Lochstoer (2010) show that long-run risks in expected consumption growth can endogenously arise in a DSGE model with Epstein-Zin preferences and IID productivity growth. But their model lacks a mechanism to reproduce time-varying consumption volatility because they assume homoscedastic productivity growth. Our GDAC model shares the same feature when volatility regime switching is shut down.<sup>13</sup> Our analysis suggests that it is important to introduce time-varying volatility in productivity growth in order to generate consumption volatility risk.

## 4.6 Return Predictability

Many researchers have documented empirical evidence of return predictability at an aggregate level by valuation ratios. One of the most widely used predictors is the dividend yield, for example, see Campbell and Shiller (1988), Fama and French (1988a), and Welch and Goyal (2008). Recently, Lettau and Ludvigson (2001) show that the consumption-wealth ratio (*cay*) is another important predictive variable. In a production-based model, Cochrane (1991) shows that macroeconomic quantities such as output growth and the investment rate also have predictive power for stock returns. In particular, empirical evidence suggests that low dividend yields and low consumption-

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<sup>13</sup>When we apply the EM algorithm to the simulated data from model GDAC, the algorithm fails to converge. This suggests that the four-state Markov-switching specification for consumption growth cannot be supported under the GDAC model.

Table 5: **Return Predictability**

Horizon	Data		Benchmark		EZ2		GDAC	
	Slope	$R^2$	Slope	$R^2$	Slope	$R^2$	Slope	$R^2$
Panel A: $i - k$								
1Y	-0.411	0.062	-0.073	0.058	-0.091	0.070	-0.068	0.064
2Y	-0.782	0.118	-0.138	0.110	-0.172	0.132	-0.125	0.119
3Y	-1.370	0.231	-0.195	0.155	-0.241	0.185	-0.175	0.168
4Y	-1.902	0.316	-0.245	0.195	-0.302	0.231	-0.220	0.211
5Y	-2.355	0.388	-0.289	0.229	-0.354	0.270	-0.259	0.249
Panel B: $d - p$								
1Y	0.085	0.036	0.009	0.032	0.011	0.041	0.008	0.041
2Y	0.143	0.061	0.017	0.060	0.021	0.076	0.015	0.075
3Y	0.182	0.079	0.024	0.085	0.029	0.107	0.020	0.104
4Y	0.193	0.082	0.030	0.107	0.036	0.133	0.025	0.129
5Y	0.238	0.101	0.035	0.126	0.043	0.156	0.029	0.150
Panel C: $c - w$								
1Y	2.799	0.079	0.344	0.094	0.365	0.091	0.261	0.068
2Y	5.227	0.156	0.635	0.171	0.681	0.168	0.476	0.124
3Y	7.465	0.242	0.881	0.232	0.951	0.233	0.663	0.172
4Y	9.046	0.317	1.091	0.283	1.181	0.286	0.829	0.215
5Y	9.766	0.297	1.267	0.323	1.375	0.330	0.974	0.253

Univariate OLS regressions of cumulative excess log equity returns onto log investment rates ( $i - k$ ), log dividend yields ( $d - p$ ), and log consumption-wealth ratios ( $c - w$ ), respectively.

wealth ratios forecast low future returns, while low investment rates forecast high future returns.

In our production-based model, the dividend yield and investment rate can be endogenously determined. Moreover, we can follow Epstein and Zin (1989) and Ju and Miao (2012) to show that the wealth-consumption ratio satisfies

$$\frac{W_t}{C_t} = \frac{1}{1 - \beta} \left( \frac{V_t}{C_t} \right)^{1 - \frac{1}{\psi}}, \quad (11)$$

where  $W_t$  is the wealth level in period  $t$ . Thus we can compute the consumption-wealth ratio in the model.

Table 5 presents empirical evidence of return predictability and simulation results for the benchmark model and models EZ2 and GDAC. We regress the 1-, 2-, 3-, 4-, and 5-year cumulative excess log returns onto log investment rates ( $i - k$ ), log dividend yields ( $d - p$ ), and log consumption-wealth ratios ( $c - w$ ), respectively. We construct investment rates and dividend yields in a standard way.

The empirical measure of the consumption-wealth ratio is the *cay* variable constructed by Lettau and Ludvigson (2001). The empirical results in Table 5 are largely in line with the extant literature on return predictability.

We simulate 20,000 samples of data from each of the benchmark model, models EZ2 and GDAC. Each sample contains 228 periods of data. Table 5 reports the estimation results averaging over the simulated samples. For simplicity, we do not report the  $t$ -values adjusted for Hansen and Hodrick (1980) standard errors. All the three models can reproduce the aforementioned predictability patterns in the data, though the magnitude of the simulated slope estimates and  $R^2$ s cannot match the empirical estimates exactly. Specifically, the investment rate negatively forecasts future returns, while the dividend yield and the consumption-wealth ratio positively forecast future returns. In addition, the  $R^2$ s are increasing in the horizon and are large for long-horizon returns.

## 5 Pricing a Claim to Calibrated Dividends

In consumption-based asset pricing models, aggregate dividends are usually defined as a levered claim to aggregate consumption. The stock market dividends, which are defined as the sum of cash dividends and net equity repurchases, only represent a small fraction of the payouts of all productive capital including private equity, small businesses, real estate, etc. Kaltenbrunner and Lochstoer (2010) have a related discussion. Thus, in a production economy, it is reasonable not to deem the firm's payouts as equivalent to the stock market dividends.

In this section we follow Ju and Miao (2012) and directly calibrate dividend growth as containing a component proportional to consumption growth and an idiosyncratic component,

$$\Delta d_{t+1} \equiv \ln \left( \frac{D_{t+1}}{D_t} \right) = \lambda \Delta c_{t+1} + g_d + \sigma_d \varepsilon_{d,t+1},$$

where  $\varepsilon_{d,t+1}$  is an IID standard normal random variable and is independent of all other shocks in the model. The parameter  $\lambda$  can be interpreted as the leverage ratio on expected consumption growth as in Abel (1999). The specification implies that dividend growth is procyclical as opposed to being countercyclical in standard RBC models. This modeling feature allows us to calibrate dividend growth dynamics to the data and simultaneously match equity premium and equity volatility in

the data.

We set the leverage parameter  $\lambda = 2.74$  as in Ju and Miao (2012) and Abel (1999). We follow Bansal et al. (2005) and use monthly returns data including and excluding dividends to generate the dividend series and obtain quarterly dividend growth data for the period 1956:Q1–2012:Q4. In the data, the annualized standard deviation of dividend growth is about 4.6 percent. We choose  $\sigma_d$  to match the volatility of dividend growth, which implies  $\sigma_d = 0.02$ , given that the model implied volatility of consumption growth is about 0.45 percent per quarter. We follow Bansal and Yaron (2004) and choose  $g_d = -0.0078$  such that the average rate of dividend growth is equal to that of consumption growth. Our calibration implies that the correlation between consumption growth and dividend growth is about 0.6, close to the level considered by Bansal and Yaron (2004) and Kaltenbrunner and Lochstoer (2010).

We set the relative risk aversion parameter at  $\gamma = 5$ , which is lower than that in the benchmark model in Section 4.3. With procyclical dividend growth, the market price of risk required to match the mean equity premium in the data becomes lower than that in the benchmark model. The subjective discount factor  $\beta$ , GDA parameters  $\eta$  and  $\kappa$  are chosen to match the mean riskfree rate, the mean equity premium and equity volatility. The calibration leads to the parameter choices  $\beta = 0.9968$ ,  $\eta = 1.23$ , and  $\kappa = 0.983$ . Other parameter values are kept the same as in Table 2. This model is labelled as GDA\*. For comparison, we solve three alternative models: (1) the Epstein-Zin model with a low degree of risk aversion (model EZ1\*), (2) the Epstein-Zin model with a high degree of risk aversion (model EZ2\*), and (3) the GDA model with a constant volatility of productivity growth (model GDAC\*).

Table 6 shows that when the model is calibrated to fit the observed volatility of dividend growth, both the mean equity premium and equity volatility in model GDA\* are high and close to the data. Model EZ2\* assumes a high risk aversion parameter ( $\gamma = 20$ ) and generates similar results. Model EZ1\* implies a very low market price of risk and thus a mean equity premium close to zero. Model GDAC\* lacking time-varying productivity volatility implies a lower mean equity premium of 3.84 percent per year, as opposed to 5.40 percent in model GDA\*, and a low equity volatility of 8.89 percent per year, as opposed to 10.09 percent in model GDA\*. This means that volatility risk carries a sizable risk premium (about 1.6 percent). The reason is that equity value as a levered claim to

Table 6: **Pricing calibrated dividends claim**

	GDA*	EZ1*	EZ2*	GDAC*	
	$\gamma = 5$	$\gamma = 5$	$\gamma = 20$	$\sigma_h = \sigma_l$	
	$\beta = 0.9968$				
	$\kappa = 0.983$				
Statistic	Data	$\eta = 1.23$			
Panel A: Macroeconomic moments					
$\sigma_{\Delta c}$ (%)	1.00	0.89	0.82	0.84	0.77
$\sigma_{\Delta i}$ (%)	4.54	3.03	3.04	3.02	2.95
$\sigma_{\Delta y}$ (%)	2.00	1.57	1.57	1.57	1.53
Panel B: Financial moments					
$\mathbb{E}[R_f] - 1$ (%)	1.44	1.44	2.07	1.47	1.60
$\sigma(R_f)$ (%)	1.07	0.21	0.20	0.20	0.19
$\mathbb{E}(R^e - R_f)$ (%)	5.40	5.40	0.55	5.64	3.84
$\sigma(R^e - R_f)$ (%)	15.03	10.09	11.98	10.22	8.89
$\sigma(M)/\mathbb{E}(M)$	n.a.	0.83	0.15	0.65	0.79
$\mathbb{E}[\Delta d]$ (%)	1.60	1.80	1.80	1.80	1.80
$\sigma_{\Delta d}$ (%)	4.60	4.60	4.60	4.60	4.60
$\rho(\Delta d, \Delta c)$	0.08	0.60	0.60	0.60	0.60

Empirical moments are computed using US quarterly data from 1956:Q1 to 2012:Q4. Lower case variables denote log values. All means and standard deviations are expressed in annualized terms. All correlations are in quarterly terms.  $\mathbb{E}(\cdot)$ ,  $\sigma(\cdot)$ , and  $\rho(\cdot, \cdot)$  denote mean, volatility, and correlation, respectively.

consumption falls in response to increased volatility risk. This intuition is slightly different from that according to Jermann's (1998) modeling of leverage analyzed in Section 4.3.

## 6 Sensitivity Analysis

In this section we conduct an extensive sensitivity analysis to examine the impact of different assumptions or different parameter values on macroeconomic and financial moments. When changing a particular parameter or assumption, we hold everything else constant as in Table 6. In particular, we use model GDA\* studied in the last section as the reference model for comparison.

We first examine the role of the specification of the productivity process by studying two alternative models. In model GDAC1 we assume that expected productivity growth is constant at the steady-state level implied by the estimated Markov switching model of productivity growth. The third row of Table 7 reveals that shutting down regime shifts in expected productivity growth alter results significantly relative to model GDA\*. In particular, the equity return to calibrated

dividends is cut down by a half. We then further shut down the regime shifts in volatility of productivity growth by fixing it at the steady-state level. In this case productivity growth is IID. We call this model GDAC2. The fourth row of Table 7 shows that model GDAC2 performs much worse than model GDA\*, especially in the asset pricing dimension, suggesting that time-varying volatility of productivity growth is an important model element.

Next we study the role of GDA parameters  $\eta$  and  $\kappa$ . Table 7 shows that the magnitude of macroeconomic moments does not change much when we vary the GDA parameter  $\eta$ . However, the impact on financial moments is significant. When  $\eta$  decreases, disappointing outcomes receive less weight. The riskfree rate rises as the agent deems the one-period riskfree bond less valuable. The mean equity premium and the price of risk fall since a less disappointment averse agent demands smaller risk premium. Reducing the relative risk aversion parameter  $\gamma$  has similar effects.

Similarly quantity moments are not very sensitive to changes in the GDA threshold parameter  $\kappa$ , but the mean equity premium is quite sensitive. In particular, when  $\kappa$  increases from 0.989 to 0.995, the mean equity premium rises from 5.71 to 6.18 percent. This result is consistent with that in Bonomo et al (2011). The intuition is that a larger value of  $\kappa$  implies that it is more likely for continuation values to be below the GDA threshold. The agent must be compensated more for holding stocks because there are more disappointing outcomes. Note that the mean riskfree rate is not very sensitive to changes in  $\kappa$ .

Routledge and Zin (2010) emphasize the importance of GDA for generating countercyclical price of risk in an endowment economy in which aggregate consumption randomly switches between two states. If  $\kappa = 1$ , only one state can be disappointing. Thus the conditional equity premium is similar across the two states. But when  $\kappa < 1$ , it is possible that there is only one disappointing continuation value conditional on the bad state and there is no disappointing continuation value conditional on the good state. This can generate a large variation in the conditional equity premium. Thus GDA is important for generating large equity volatility and significant return predictability.

The EIS parameter  $\psi$  is an important parameter for matching macroeconomic and financial moments in the data. The sensitivity analysis with respect to the EIS parameter  $\psi$  suggests that when the agent is less willing to substitute consumption intertemporally, the volatility of consumption growth rises whereas the volatility of investment growth falls. The mean riskfree rate

Table 7: **Sensitivity analysis**

	$\sigma_{\Delta c}$	$\sigma_{\Delta i}$	$\mathbb{E}[R_f] - 1$	$\mathbb{E}(R^I - R_f)$	$\mathbb{E}(R^e - R_f)$	$\sigma(R^e - R_f)$	$\sigma(M)/\mathbb{E}(M)$
GDA*	0.89	3.03	1.44	0.11	5.40	10.09	0.83
GDAC1	0.77	2.82	1.77	0.08	2.14	8.46	0.61
GDAC2	0.66	2.74	1.98	0.07	0.74	7.29	0.42
$\eta = 2.45$	0.91	3.04	0.98	0.22	8.23	8.76	1.38
$\eta = 2.00$	0.92	3.04	1.13	0.16	7.47	9.19	1.20
$\eta = 1.60$	0.90	3.03	1.28	0.13	6.51	9.59	1.02
$\kappa = 0.995$	0.81	3.00	1.56	0.18	6.18	9.83	0.94
$\kappa = 0.992$	0.84	2.98	1.55	0.12	5.90	10.05	0.90
$\kappa = 0.989$	0.82	2.97	1.49	0.13	5.71	9.88	0.89
$\gamma = 10$	0.92	3.04	1.19	0.15	7.24	9.37	1.04
$\gamma = 7$	0.90	3.04	1.34	0.13	6.17	9.76	0.92
$\gamma = 2$	0.88	3.04	1.59	0.08	4.21	10.44	0.72
$\psi = 1.5$	1.05	2.76	1.77	0.08	5.01	9.17	0.83
$\psi = 1.2$	1.19	2.60	2.08	0.06	4.63	8.42	0.82
$\psi = 0.9$	1.37	2.45	2.58	0.04	4.12	7.46	0.81
$\psi = 0.7$	1.51	2.39	3.13	0.03	3.73	6.72	0.80
$\xi = 5$	0.96	2.79	1.44	0.23	4.67	9.22	0.83
$\xi = 3$	1.09	2.52	1.42	0.40	4.38	8.61	0.83
$\xi = 1.5$	1.34	2.10	1.39	0.77	4.21	7.89	0.83

Model GDA\* is described in Section 5. Model GDAC1 assumes a constant expected growth rate of productivity at the steady-state level. Model GDAC2 assumes that both the expected growth rate and the volatility of productivity growth are constant at their steady-state levels. Lower case variables denote log values. When varying a particular parameter or specification, we keep everything else fixed as in Table 6.  $\mathbb{E}(\cdot)$  and  $\sigma(\cdot)$  denote mean and volatility, respectively.

is inversely related to the EIS parameter because a high riskfree rate induces more intertemporal substitution. The mean equity premium and equity volatility both increase with EIS  $\psi$ . For a high EIS, investment is more responsive to fluctuations in productivity growth, leading to a high volatility of returns on investment and a high equity volatility in consequence. Thus, even though the price of risk does not change much in response to a change in the EIS, the mean equity premium rises with EIS. Bonomo et al. (2011) show that the EIS is not important for understanding financial data in a consumption-based model with GDA preferences. Our analysis suggests that their finding is not robust to the extension to a production-based model.

For a higher capital adjustment cost (smaller  $\xi$ ), consumption rises more in response to a positive permanent productivity shock. Thus consumption growth is more volatile and investment growth is less volatile. This implies that the short-run risk is larger for a smaller value of  $\xi$ . But expected consumption growth is lower, implying that the mean riskfree rate is lower and the long-run risk is smaller (Kaltenbrunner and Lochstoer (2010)). Since the impact of the long-run risk dominates in our model, the risk premium of a claim to aggregate consumption decreases with the capital adjustment cost. By our modeling of dividends as a levered claim to consumption in the last section, equity premium also decreases with the adjustment cost. By contrast, the investment return increases with the adjustment cost because Tobin's Q is more volatile for a higher adjustment cost.

## 7 Concluding Remarks

In this paper we use a production-based model to show that the volatility risk in aggregate productivity growth has important asset pricing implications. Combined with GDA preferences, this risk carries a large and positive risk premium and helps explain some asset pricing puzzles. Our model can generate endogenous long-run risks in the mean and volatility of consumption growth and the predictability pattern of equity returns. We show that GDA preferences have a very small effect on macroeconomic quantity dynamics, but have a large effect on the riskfree rate. We also show that introducing leverage with a procyclical dividend process consistent with the data is critical for GDA preferences to have a large impact on equity returns.

The analysis in Section 4.2 indicates that our baseline model cannot generate comovement among investment, consumption, and output in response to uncertainty shocks as in the data. Endogenizing labor choice cannot solve this issue either. If the representative agent can adjust his labor supply and if consumption and leisure are both normal goods, then an increase in uncertainty induces the agent to supply more labor. As current aggregate productivity and the capital stock remain unchanged, labor demand remains unchanged as well. Thus higher uncertainty reduces consumption but raises output, investment, and hours worked. This lack of comovement is a robust prediction of simple RBC models with uncertainty shocks.<sup>14</sup>

Recently Basu and Bundick (2012) propose a solution to this problem using a dynamic new Keynesian model with monopolistic competition and sticky prices. While generalizing our model to the new Keynesian framework would be interesting, we argue that this generalization will not change our key insights. Our key model mechanism relies on two results. First, consumption falls in response to an increase in productivity growth uncertainty. This result is consistent with the data and is robust, while the responses of output and investment in our baseline model are not. This implies that marginal utility of consumption and the pricing kernel rise in response to an increase in productivity growth risk and hence the market price of this risk is negative. Second, stock returns are negatively exposed to uncertainty shocks. This result is also consistent with the data. These two results together imply that productivity growth risk carries a positive risk premium. This model prediction is robust to the extension that can generate comovement.

Finally, it would be interesting to introduce sticky wages as in Favilukis and Lin (2013) because this modeling can make profits more volatility and dividends more procyclical. This extension could raise equity volatility and improve model performance.

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<sup>14</sup>The model of Gourio (2012) also has a similar comovement problem in response to a change in the disaster probability.

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