Equilibrium Information in Credence Goods

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September 2021

Abstract

In a general credence-good model, a consumer’s potential loss is a continuous random variable. Observing the loss value, an expert can provide a repair. If the expert’s price offer is accepted, the loss will be avoided. Not knowing the true loss, the consumer never learns if the repair would have been worthwhile. We characterize perfect-Bayesian equilibria, all of which are inefficient. In closed form, we derive separating and pooling equilibria; in the former, the consumer can infer losses from prices, in the latter, the consumer can infer that losses reside in an interval. We also endogenize the expert’s information acquisition. The first best is achieved if the expert can commit to only knowing if the loss is below or above a certain threshold. The general framework can be extended to a two-dimensional model in which cost and loss are random and correlated, and to consider a market with multiple experts.

Keywords: credence goods, experts, separating equilibrium, pooling equilibrium, information acquisition

JEL: D80, D82, D83

Acknowledgment: We thank Jacobo Bizzotto, Lester Chan, Winand Emons, Sanghoon Kim, Henry Mak, Chiara Margaria, Juan Ortner, and seminar participants at Boston University, Stony Brook University, and Webinar Series on Credence Goods and Expert Markets for their suggestions and comments.
1 Introduction

Credence goods and services have qualities and values which may not be observed by consumers even after purchase or consumption. A canonical credence-good model is as follows. A consumer may suffer a loss, which is randomly distributed. An expert can find out about the potential loss, but this becomes the expert’s private information. The expert can offer a treatment to prevent the loss. If the consumer accepts the offer, she pays the price, and the loss is avoided. Nevertheless, the consumer never learns the actual repair, so will not know if she has overpaid. The leading examples are automobile repairs and medical services.¹

In this paper, we study a general model of credence goods. We let the consumer’s potential losses follow a continuous distribution. This contrasts with the existing literature (see Pitchik and Schotter (1987), Wolinsky (1993), Emons (1997), Fong (2005), Dulleck and Kerschbamer (2006), and Balafoutas and Kerschbamer (2020)); there, the loss is binary, often labeled as minor or major. Our contributions are fourfold. First, we provide a general and tractable framework to study credence goods. Second, our model exhibits rich equilibrium strategies and outcomes, many of which are absent in the binary model. In particular, we present a novel and intuitive pure strategy equilibrium in which the seller recommends the same price for losses if and only if they are lower than a cutoff. Third, we endogenize the expert’s information acquisition decision, and show that the expert will optimally acquire imperfect diagnosis information. Fourth, we demonstrate that the general model is robust against such issues as a two-dimensional description of correlated loss and cost, and the consumer searching among multiple experts.

First, the continuous loss setup is natural. There is no compelling reason why problems and repairs must fall into only two levels. Our reading of the literature is that the binary assumption is made for tractability. We now provide the necessary tools for the general analysis, and the techniques turn out to be familiar ones in contract theory. The continuous loss model also dispenses with an expert having to make price commitments before he learns about losses, a common assumption in the literature.

¹Here is a stylised example. The instrument panel in a driver’s car flashes a warning, and only a mechanic can be sure of the needed repair. Upon service (and payment to the mechanic), the car runs as before, and the warning has disappeared. The driver can neither ascertain what could eventually have gone wrong nor what service has been done.
Second, we derive necessary conditions for all perfect-Bayesian equilibria: (i) the equilibrium price weakly increases in loss, (ii) the likelihood that the loss is avoided weakly decreases in loss, and (iii) the expert’s equilibrium profit strictly decreases in loss. Then, we derive equilibria in closed forms under the assumption that losses below a threshold are efficient to repair. Among all perfect-Bayesian equilibria, the most compelling ones are a separating equilibrium, in which the expert’s price is the loss, and a pooling equilibrium, in which the expert makes a single price offer if and only if the loss is below a threshold. We do characterize other equilibria, but also adopt a natural refinement to reject most.

The separating equilibrium is reminiscent of the “no-cheating” equilibrium in the literature. The consumer perfectly infers her loss from the equilibrium price offer, but randomizes between accepting and rejecting it. The principle of probabilistic rejection deterring the expert from cheating has figured prominently in the earlier literature. In our general model we need to describe a continuum of acceptance probabilities for all losses (rather than a single probability in the binary version) that deter cheating. Indeed, we solve a differential equation, in closed form, for the acceptance probabilities to construct the separating equilibrium.

In the pooling equilibrium, both the expert and the consumer use pure strategies. The expert recommends a single price for losses lower than a threshold and makes no recommendations for higher losses. The consumer accepts any price up to the single price and rejects any higher prices. The equilibrium outcome has a single price offered, and it is always accepted. The expert’s price offer decision is based on the marginal repair cost; the consumer’s acceptance decision is based on average valuations. The equilibrium property is reminiscent of the lemons problem à la Akerlof (1970): the equilibrium price equates the expert’s marginal cost at the loss threshold to the consumer’s average value for losses below the threshold.

The welfare properties of the pooling and separating equilibria are as follows. The expert’s pooling equilibrium repair loss threshold is lower than the first best, so some large losses are left unrepaired. In the separating equilibrium each loss is left unrepaired with a positive probability, so the allocation is also inefficient. Depending on model primitives, the welfare properties of the pooling equilibrium may dominate the separating equilibrium, or vice versa. The welfare comparison draws a sharp contrast with the existing literature (Fong (2005)), which finds that the separating equilibrium maximizes profit and efficiency.
In the pooling equilibrium, the consumer earns a zero ex ante surplus; in the separating equilibrium, the consumer earns a zero ex post surplus. These are important properties. In the separating equilibrium, no consumer should complain about being cheated. In the pooling equilibrium, consumers with low losses will realize, if they eventually learn about their losses, that they have been overcharged—the equilibrium price being above the loss value. But consumers with high losses will discover that the price they have paid is below the loss value. The same expert, ex post, may be praised by some consumers but complained against by others.\footnote{The credence good model is identical to one in which losses become observed if repairs are not sought, but repair is no longer feasible once losses have occurred.}

In the continuous-loss model, we can also analyze the expert’s information acquisition decision, instead of making the usual assumption that the expert has perfect diagnostic information. Here, the expert chooses an information structure which generates a private signal about the consumer’s loss. After the signal has been observed, the expert makes the price offer. We show that the expert can achieve the first-best profit with a binary-signal information structure, which shows whether the loss is below or above a certain cutoff.

The intuition is this. The expert’s price is based on the signal, so can be contingent on whether the loss is above or below some cutoff. For profit maximization, the expert compares that price with the average cost of providing repair. Likewise, when the consumer receives a price offer, she must realize that it is based on a signal. The consumer compares the price with the average loss contingent on that signal. Both players now make decisions based on averages, and the Akerlof lemons problem is avoided.

We show that our results are robust against (i) two dimensions of uncertainty, (ii) multiple experts, and (iii) a more general cost function. In the two-dimensional uncertainty case, the consumer’s loss and the repair cost are both random, but positively correlated. The previous single dimensional model assumes that the cost is a deterministic and increasing function of loss; in that case, information about loss is equivalent to information about cost. For the two-dimensional, loss-cost uncertainty model, the expert’s equilibrium price offer can only be a function of the expert’s privately observed cost. In equilibrium, the expert’s information about the consumer’s loss cannot be used to set prices. The intuition is that the expert only cares about his own cost and whether his price offer will be accepted, never the consumer’s loss. The two dimensional
model thus reduces to the single dimensional model.

We also consider the case of multiple experts with consumer search. We show that any equilibrium outcome in the single-expert market remains an equilibrium outcome in the multiple-expert market. The idea is that any expert will follow the same equilibrium strategy as in the single-expert model. The consumer recognizes that experts will make the same price offer, so visiting more experts does not pay. In sum, equilibria in the single-expert model are robust. Then we discuss the case in which it is inefficient to fix the problem when the loss is either very high or very low. We show that the pooling equilibrium is robust but the separating equilibrium fails to exist. Moreover, the pooling equilibrium features over provision of treatment for low losses and under provision of treatment for high losses.

The rest of the paper is organized as follows. The next subsection is a literature review. Section 2 presents the model. Section 3 derives properties of all equilibria. We focus on separating and pooling equilibria in Section 4. There we actually present multiple pooling equilibria, but introduce a natural refinement to reject all but the one with the highest equilibrium price. Section 5 is about expert acquiring information. Section 6 has a first subsection that characterizes what we call “hybrid” equilibria; it is then followed by three subsections on robustness: a two-dimensional model in loss and cost, multiple experts, and an alternative cost function. We offer some concluding remarks in Section 7.

1.1 Related literature

Darby and Karni (1973) first introduced the concept “credence goods” and discussed firms’ incentives to mislead consumers to raise demand. The subsequent literature analyzes sellers’ strategic behavior in game-theoretic models with binary losses. Dulleck and Kerschbamer (2006) and Balafoutas and Kerschbamer (2020) are comprehensive literature surveys.

The extant literature has taken two directions. The first line of the literature assumes that the expert is liable for repairing consumers’ problems once consumers agree to offered prices; in other words, the expert must work to eliminate the loss. Papers along this line of research include Pitchik and Schotter (1987),

\[ \text{For example, an automobile’s very minor problems may not be worth the repair cost, and the same may be true for very serious problems. Repairs are cost effective for intermediate conditions.} \]
Wolinsky (1993, 1995), Taylor (1995), Fong (2005), Liu (2011), Fong et al. (2018, 2021). The second line of the literature assumes that only the expert’s repair or treatment expense is verifiable; in other words, the expert must carry out the promised work, but is not responsible for the loss elimination. This line of research includes Emons (1997), Alger and Salanie (2006), Dulleck and Kerschbamer (2009), Fong et al. (2014), Bester and Dahm (2018), and Chen et al. (2021).

The importance of the distinction between these two lines of work is this. Where cost is nonverifiable, cheating or hiding loss information can be regarded as costless because the expert only performs the minimal repair but is free to inflate the loss. However, when cost is verifiable, the expert will have to carry out wasteful work to get a higher price, so cheating or hiding loss information can be regarded as costly. We assume that the expert is liable for the repair, and that treatment cost is not verifiable, so it belongs to the first line of the literature. We do extend the model to one with uncertain cost and loss; there neither loss nor cost is verifiable.

The binary model in the literature has never exhibited any nontrivial pure-strategy equilibria. However, we show that pooling equilibria always exist, and they can be more profitable and efficient than a separating equilibrium. This finding draws a sharp contrast to the idea that the expert makes the most profit by telling the truth (for example, Fong (2005)).

Most of the existing literature assumes that experts can perfectly diagnose the consumer’s problem at zero cost. There is a small but growing literature on the expert’s incentives to acquire diagnostic information by costly hidden actions; these papers include Pesendorfer and Wolinsky (2003), Dulleck and Kerschbamer (2009), Bester and Dahm (2018), and Chen et al. (2021). They study an environment in which treatment is verifiable but the expert is not liable for treatment outcome. Our paper here complements them because we assume that the expert is liable for treatment outcome and assume that information acquisition is costless. We are unaware of a result for the first-best outcome when the expert may choose to acquire (partial or full) information. Our paper also shows that acquiring too much information may harm profits.

A more recent literature has implemented tests of credence-good models with data from the laboratory and from the real world. Most of the laboratory experiments are based on the binary model (Dulleck et
al. (2011), Kerschbamer et al. (2015), Mimra et al. (2016), Balafoutas et al. (2021)). Hence, the subject chooses between whether to make an honest recommendation or not. Our framework permits a general strategy space for the expert; in such an experiment, subjects would have a rich set of ways to mislead consumers. In field experiments, subjects of course are quite unrestricted in their strategies; see, for example, Schneider (2012), Balafoutas et al. (2013), Liu et al. (2019), and Gottschalk et al. (2020), and our general model does allow such rich strategy sets. Our work therefore provides a foundation for future experiments and empirical analyses.

Our work is broadly related to the literature on trading under asymmetric information. Bagwell and Riordan (1991) and Wolinsky (1983) study how firms can signal their goods’ qualities by prices. In both papers, there exists a separating equilibrium in which a high price signals a high quality. A common driving force for the separating equilibrium is that some consumers are informed about the quality prior to consumption. In our model we also identify a separating equilibrium in which a high price signals a high loss. Here, however, consumers are never informed about their losses before accepting the expert’s offer or after the repair. Kim (2012) studies endogenous market segmentation as a way to alleviate information asymmetry. In Kim (2012), sellers send a cheap-talk message to buyers before trade takes place. Kim constructs an equilibrium in which some low-quality sellers reveal their quality, whereas some other low-quality sellers pool with high-quality sellers. In our paper, the seller’s recommendation is the price, which directly affects the consumer’s payoff. We do have hybrid equilibria in which some seller types reveal information through prices, but other seller types pool together by offering the same price.

2 A General Credence-Good Model

A risk-neutral consumer has been enjoying a utility $B$ from a good or a service. However, it has come to light, say from a fault indicator, that the baseline utility $B$ may be reduced. The potential utility or monetary loss is denoted by $\ell$ and is randomly distributed on the strictly positive support $[\ell, \overline{\ell}]$, with an absolutely continuous distribution $F$, and we occasionally denote its density by $f \equiv F'$. Any potential loss can be avoided by a risk-neutral expert’s repair. For now, we assume that the consumer interacts with one expert only, but in Subsection 6.3, we let the consumer interact with many experts. If the loss turns out
to be \( \ell \), the expert can incur a cost \( C(\ell) \) to eliminate it, where \( C : [\underline{\ell}, \bar{\ell}] \to \mathbb{R}_+ \) is a strictly increasing and differentiable function. The distribution of \( \ell \) and the cost function of \( \ell \), respectively \( F \) and \( C \), as well as the baseline utility \( B \), are assumed to be common knowledge. We assume \( \ell > C(\ell) \) if and only if \( \ell < \hat{\ell} \), with \( \underline{\ell} < \hat{\ell} \leq \bar{\ell} \), which says that it is efficient to treat only problems with losses lower than \( \hat{\ell} \). In Subsection 6.4, we discuss the case in which it is inefficient to treat problems with very low or very high losses.

The credence-good nature of the model is this. The expert gets to observe the potential loss \( \ell \) but the consumer does not observe it once her problem is repaired.\(^4\) The function \( C \) exhibits a monotone relationship between the expert’s cost and loss, so we assume that the consumer never learns the expert’s repair cost either; in other words, repair cost is not verifiable. In Subsection 6.2, we will relax the monotonicity assumption; there, costs and losses are random, but positively correlated. The only verifiable event here is that the consumer gets to enjoy the utility \( B \) upon a repair. The expert’s offer to repair the good at some price is a contract that lets the consumer continue to enjoy \( B \) upon paying some repair price.\(^5\)

If the expert repairs the loss \( \ell \) at a price \( p \), his profit is \( p - C(\ell) \); otherwise, profit is zero. If the consumer accepts the repair offer at price \( p \), her utility is \( B - p \); otherwise utility will become \( B - \ell \). Because loss \( \ell \) belongs to \( [\underline{\ell}, \bar{\ell}] \), we can let prices belong to \( [\underline{\ell}, \bar{\ell}] \) without any loss of generality. We denote the expert’s refusal to repair by the notation of a price set at \( +\infty \). The extensive form between the consumer and the expert is as follows:

**Stage 1:** A consumer visits the expert, who then observes the consumer’s potential loss \( \ell \in [\underline{\ell}, \bar{\ell}] \), drawn from distribution \( F \). The expert chooses between refusing to repair and offering to repair at a price \( p \in [\underline{\ell}, \bar{\ell}] \).

**Stage 2:** If the consumer agrees to the repair, she pays the expert \( p \), and the expert incurs cost \( C(\ell) \) for the repair. If the consumer rejects the expert’s offer, she suffers the loss.

The expert’s strategy is a price function that maps the consumer’s losses to prices: \( P : [\underline{\ell}, \bar{\ell}] \to [\underline{\ell}, \bar{\ell}] \cup \{+\infty\} \).

\(^4\)If the repair is not done, the consumer’s utility eventually becomes \( B - \ell \). At that point, the consumer can infer the loss value, but repair at that time has become infeasible.

\(^5\)The expert does not have the option of repairing a fraction of the loss \( \ell \) by incurring some fraction of the cost \( C(\ell) \). This assumption is consistent with the postulate that only the baseline utility \( B \) is verifiable.
\{+\infty\}, with \(P(\ell) \in [\ell, \overline{\ell}]\) denoting the expert’s offer to eliminate loss \(\ell\) and \(+\infty\) denoting the expert’s refusal to treat the consumer. The consumer’s strategy is a probability acceptance function: \(\alpha : [\ell, \overline{\ell}] \rightarrow [0, 1]\), with \(\alpha(p)\) denoting the consumer’s probability of accepting a repair offer \(p \in [\ell, \overline{\ell}]\). Given the expert’s strategy, the consumer updates her belief about her loss according to Bayes rule when it is possible. We characterize perfect-Bayesian equilibria.

Each price constitutes an information set. There can be many unreached information sets in an equilibrium. For example, suppose that an equilibrium price function specifies that \(P([\ell, \overline{\ell}]) = p_1\) and \(P([\ell, \overline{\ell}]) = p_2\), so the consumer responds to prices \(p_1\) and \(p_2\) on the equilibrium path. Prices other than \(p_1\) and \(p_2\) never get offered in equilibrium. Yet, the consumer’s equilibrium strategy must still specify her acceptance probability at these prices. Clearly, the consumer’s responses at off-equilibrium price offers may support many equilibria, but we will impose belief restrictions later to rule out many.

Our key model construction uses a continuum of losses, whereas all earlier papers that we are aware of adopt a binary-loss assumption. There is no compelling reason that the potential loss must be one of only two possible values. Binary-loss models actually rule out many classes of equilibria, as we will see. A second difference is that we do not adopt a price-posting stage in the extensive form, which again has been commonly used in earlier papers. Here, the expert cannot commit to offering a subset of prices before he learns the loss.

3 Perfect-Bayesian equilibrium strategies

We derive equilibria by construction, but begin by deriving necessary conditions on players’ equilibrium strategies and the expert’s equilibrium profit. Consider any perfect-Bayesian equilibrium \((P, \alpha)\), consisting of the expert’s pricing function \(P : [\ell, \overline{\ell}] \rightarrow [\ell, \overline{\ell}] \cup \{+\infty\}\) and the consumer’s acceptance probability function \(\alpha : [\ell, \overline{\ell}] \rightarrow [0, 1]\). (We omit the consumer’s response—the empty set—when the expert refuses to offer treatment.) Let \(Range(P) \subset [\ell, \overline{\ell}]\) denote the set of expert’s equilibrium prices for treatment. Those prices in \([\ell, \overline{\ell}] \setminus Range(P)\) are off-path prices or unreached information sets. The acceptance function \(\alpha\) is defined over all elements of \([\ell, \overline{\ell}]\) which may be a strict super set of \(Range(P)\).
Next, for any given equilibrium strategy profile \((P, \alpha)\), define a function \(\tilde{\alpha}_P : [\ell, \bar{\ell}] \to [0, 1]\) by \(\tilde{\alpha}_P(\ell) \equiv \alpha(P(\ell))\). The function \(\tilde{\alpha}_P\) tracks the acceptance probability on the equilibrium path according to strategy profile \((P, \alpha)\), over all loss values; it does so by compounding the equilibrium acceptance probability and the equilibrium price function. Let \(\Pi(\ell) \equiv \tilde{\alpha}_P(\ell) \times \{P(\ell) - C(\ell)\}\) denote the equilibrium profit from repairing \(\ell\). The expert’s expected equilibrium profit is \(\int \Pi(\ell) dF(\ell)\). We now state properties valid for all equilibria; all proofs of results (unless omitted) are in the Appendix.

**Proposition 1** Any perfect-Bayesian equilibrium strategy profile \((P, \alpha)\) has the following properties:

i) The equilibrium acceptance probability \(\tilde{\alpha}_P(\ell)\) is weakly decreasing.

ii) Suppose the expert offers \(P(\ell)\) at \(\ell \in [\ell, \bar{\ell}]\) in equilibrium. Then price \(P(\ell)\) is weakly increasing if \(\tilde{\alpha}_P(\ell) > 0\), \(\ell \in [\ell, \bar{\ell}]\).

iii) Equilibrium profit \(\Pi(\ell)\) is weakly decreasing in \([\ell, \bar{\ell}]\) and is strictly decreasing in \([\ell_1, \ell_2]\) if \(\tilde{\alpha}_P(\ell) > 0\) for \(\ell \in [\ell_1, \ell_2] \subset [\ell, \bar{\ell}]\).

Any perfect-Bayesian equilibrium \((P, \alpha)\) must satisfy the following no-deviation conditions: at \(\ell'\) and \(\ell\)

\[
[P(\ell') - C(\ell')]\tilde{\alpha}_P(\ell') \geq [P(\ell) - C(\ell')]\tilde{\alpha}_P(\ell) \tag{1}
\]

and

\[
[P(\ell) - C(\ell)]\tilde{\alpha}_P(\ell) \geq [P(\ell') - C(\ell')]\tilde{\alpha}_P(\ell') \tag{2}
\]

Part i) of Proposition 1 is obtained by adding these two inequalities and collecting terms, a common step in asymmetric-information games. Part ii) says that the expert must not decrease his price when the repair cost goes up if the offer may be accepted. Together, these are intuitive results. Reducing prices when costs increase would just be against profit maximization. Parts i) and ii) together say that the consumer must not play an equilibrium strategy that results in higher acceptance probabilities when prices increase. Otherwise, the expert would deviate to recommending the prices meant for high losses when the consumer’s loss is low. Part iii) follows because the expert’s cost increases in loss, and acceptance is only based on offered prices. As loss increases, given the consumer’s acceptance strategy, profit must decrease.

According to Proposition 1, any equilibrium must consist of i) the expert offering prices constant or
strictly increasing in $\ell$, and ii) the consumer accepting offers with constant or decreasing probabilities. Still many candidate equilibria can satisfy Proposition 1. We next turn to different classes of equilibria.

4 Separating and pooling equilibria

We begin by defining separating and pooling equilibria in terms of equilibrium allocations with positive-probability repairs.

**Definition 1 (Separating equilibrium)** An equilibrium $(P, \alpha)$ is said to be a separating equilibrium if for any $\ell$ and $\ell'$ in $[\ell, \overline{\ell}]$ for which $\alpha(P(\ell)) > 0$ and $\alpha(P(\ell')) > 0$, $P(\ell) \neq P(\ell')$.

**Definition 2 (Pooling equilibrium)** An equilibrium $(P, \alpha)$ is said to be a pooling equilibrium if for any $\ell$ and $\ell'$ in $[\ell, \overline{\ell}]$ for which $\alpha(P(\ell)) > 0$ and $\alpha(P(\ell')) > 0$, $P(\ell) = P(\ell')$.

In both definitions, we ignore those prices that will never be accepted in an equilibrium. Separating prices are those that will be accepted with some positive probability, but vary according to losses. A pooling price is one that will be accepted with some positive probability, but does not vary with losses. Notice that the requirement that prices vary in a separating equilibrium and that prices remain constant in a pooling equilibrium is for all losses in $[\ell, \overline{\ell}]$ for which equilibrium acceptance may happen. Hence, these two definitions are not exhaustive. Continuum of equilibria that are neither separating nor pooling do exist. In anticipation, we call these *hybrid equilibria*, which will be defined and analyzed in Subsection 6.1.

4.1 A separating equilibrium

To begin constructing a separating equilibrium, we present the following key lemma.

**Lemma 1** Suppose that the equilibrium price function $P$ is strictly increasing over an interval of losses on the equilibrium path, say $(\ell_1, \ell_2)$, then it must be $P(\ell) = \ell$ if the consumer accepts $P(\ell)$ with some positive probability.

Lemma 1 says that in a separating equilibrium, the price must equal the consumer’s loss, making her just indifferent between accepting and rejecting the offer. Clearly, if the consumer can infer her loss from
the price offer, she will pay at most her loss. If the expert offers to repair a problem at an equilibrium price strictly less than the loss, the consumer will accept it with probability one. However, this will create an incentive problem. The expert would offer this price for all problems with lower losses because he could surely earn this higher price. The argument implies that the consumer must reject the expert’s price with a positive probability, so \( P(\ell) = \ell \). Using this property, can we find the acceptance probability function to support a separating equilibrium?

**Proposition 2** There is a unique separating equilibrium (which involves the threshold \( \hat{\ell} \), defined by \( C(\hat{\ell}) = \hat{\ell} \)). The expert’s strategy is

\[
P(\ell) = \begin{cases} 
\ell & \text{if } \ell \leq \ell \leq \hat{\ell} \\
\infty & \text{if } \hat{\ell} < \ell \leq \ell 
\end{cases}
\]

and the consumer’s strategy is

\[
\alpha(\ell) = \begin{cases} 
\exp \left\{ -\int_{\ell}^{\hat{\ell}} dx \frac{d}{c(x)} \right\} & \text{if } \ell \leq \ell \leq \hat{\ell} \\
0 & \text{if } \hat{\ell} < \ell \leq \ell 
\end{cases}
\]

Uniqueness follows from Lemma 1 and the consumer’s acceptance function. Given the expert’s strategy, the consumer is indifferent between accepting and rejecting the offer. Hence, the acceptance strategy \( \alpha(\ell) \) is a best response. Given the acceptance strategy \( \alpha(\ell) \), the expert strictly prefers to set \( P(\ell) = \ell \) at loss \( \ell \). The rest of the proof of the Proposition consists of deriving the necessary and sufficient condition on the consumer acceptance probability function to support the expert’s separating price function. The construction of the equilibrium acceptance probability function is by means of the envelope condition to obtain the equilibrium profit’s derivative, a familiar method in contract design. Because choosing \( P(\ell) = \ell \) is optimal, the derivative of equilibrium profit \( \Pi(\ell) = (\ell - C(\ell))\alpha(\ell) \) only depends on how it changes with respect to cost:

\[
\Pi'(\ell) = -C'(\ell)\alpha(\ell) < 0. \tag{3}
\]

The expert’s equilibrium profit must be strictly decreasing in the separating equilibrium, consistent with Proposition 1. To prevent the expert from gaining by offering a higher price when the loss is small, the consumer must decrease acceptance probabilities as losses (and prices) increase.
The equilibrium in Proposition 2 does not have any unreached information set in the relevant price range \([\ell, \tilde{\ell}]\). The separating equilibrium, therefore, is robust against any refinement that restricts beliefs off the equilibrium path. The equilibrium does require the consumer to fine tune the acceptance probability according to the solution of a differential equation. This is demanding, perhaps unrealistic, and suffers from the usual criticism of a player randomizing between multiple optimal choices to support a rival’s optimal action. Nevertheless, Proposition 2 does confirm some insights from the binary-loss models in the literature: i) credence goods do not necessarily yield equilibrium lies (Fong (2005)), and ii) the consumer rejecting offers does discipline the expert’s information advantage ((Wolinsky (1993), Fong (2005), Dulleck and Kerschbamer (2006)).

4.2 A pooling equilibrium

Now we set up some preliminary concepts for a pooling equilibrium. First, we denote the expected loss by

\[ \mu = \int \ell dF(\ell). \]

Next, we define conditional average loss values for a loss interval by

\[ AL : [\ell, \tilde{\ell}] \times [\ell, \tilde{\ell}] \to [\ell, \tilde{\ell}] \]

where

\[ AL(\ell_1, \ell_2) = \int_{\ell_1}^{\ell_2} \frac{x dF(x)}{F(\ell_2) - F(\ell_1)}. \]

Of particular relevance here is the expected loss conditional on the loss below \(\ell\), \(AL(\ell, \ell)\). Clearly, \(AL(\ell, \ell) < \ell\), and \(\lim_{\ell \to \tilde{\ell}} AL(\ell, \ell) = \mu\).

We define a set of losses \(LP\) by

\[ LP = \{\ell : AL(\ell, \ell) \geq C(\ell)\}, \]

where \(LP\) can be taken as an acronym of losses in pooling equilibria. The set \(LP\) denotes all losses each of which has a conditional expected repair value \(AL(\ell, \ell)\) larger than cost \(C(\ell)\). Now, both \(AL(\ell, \ell)\) and \(C(\ell)\) are increasing, so may intersect multiple times as \(\ell\) varies between \(\ell\) and \(\tilde{\ell}\). Hence, the set \(LP\) may consist of disjoint loss intervals. However, \(C(\tilde{\ell}) = \tilde{\ell} > AL(\ell, \tilde{\ell})\), so for losses above \(\tilde{\ell}\), \(C(\ell)\) must over take \(AL(\ell, \ell)\).

Finally, we define a threshold \(\ell^*\) by

\[ \ell^* = \max\{\ell : AL(\ell, \ell) \geq C(\ell)\}. \]

Obviously \(AL(\ell, \ell) < C(\ell)\) for \(\ell > \ell^*\); moreover, \(\ell^* < \tilde{\ell}\) because \(C(\ell^*) = AL(\ell, \ell^*) < \ell^*\) and \(C(\ell) < \ell\) if and
only if $\ell < \hat{\ell}$. Figure 1 graphs an example of $AL(\xi, \ell)$ and $C(\ell)$ in which they cross three times. There the set $LP$ consists of those losses between $\xi$ and $\ell_1$ and between $\ell_2$ and $\ell^*$, the maximum loss in $LP$.

![Figure 1: Conditional expected loss value $AL(\xi, \ell)$ and cost $C(\ell)$](image)

In a pooling equilibrium, the expert must set the same price for all repairs that happen with a positive probability. Given that treatment cost $C(\ell) > \ell$ for $\ell > \hat{\ell}$, there does not exist a pooling equilibrium in which all losses are repaired. (In such a (candidate) pooling equilibrium, the expert’s price is at most $\mu$, the consumer’s expected loss. Because $\mu < \ell < C(\ell)$, the expert will refuse to repair those losses that cost more than $\mu$.) In the next proposition, we characterize an equilibrium in which the expert offers a pooling price for losses lower than the threshold $\ell^*$. Many prices in the range $[\xi, \hat{\ell}]$ will be off path, but we only need to impose a minimum restriction on the consumer’s belief upon a price deviation.

**Proposition 3** There is a pooling equilibrium indexed by $\ell^*$ (where $AL(\xi, \ell^*) = C(\ell^*)$). The expert’s strategy is

$$P(\ell) = \begin{cases} AL(\xi, \ell^*) & \text{if } \xi \leq \ell \leq \ell^* \\ +\infty & \ell^* < \ell \leq \hat{\ell} \end{cases}.$$
The consumer’s strategy is
\[
\alpha(p) = \begin{cases} 
1 & \text{if } p \leq AL(\ell, \ell^*) \\
0 & \text{if } p > AL(\ell, \ell^*) 
\end{cases}
\]

When the consumer receives a price offer different from \(AL(\ell, \ell^*)\), she believes that her loss is drawn from \([\ell, \ell^*]\) according to \(F(\ell)\)

There is a single equilibrium price \(AL(\ell, \ell^*)\) offered by the expert that results in equilibrium transactions, so this is a pooling equilibrium. Essentially, the consumer assumes (correctly) that the expert offers \(AL(\ell, \ell^*)\) for \(\ell \in [\ell, \ell^*]\) and that no transaction can occur at higher losses. The consumer accepts all prices below \(AL(\ell, \ell^*)\) and rejects higher prices. Against the consumer’s best response, the expert’s best response, indeed, is to offer price \(AL(\ell, \ell^*)\) for \(\ell \in [\ell, \ell^*]\). At \(\ell = \ell^*\), the expert makes zero profit because \(AL(\ell, \ell^*) = C(\ell^*)\). Part iii) of Proposition 1 says that the expert cannot make any profit for higher losses. Therefore, no transaction will happen for losses greater than \(\ell^*\). The pooling equilibrium is illustrated in Figure 2.

Figure 2: A pooling equilibrium

The pooling equilibrium is simple and intuitive, but such a construction cannot appear in the literature with binary losses. The contrasts between the separating equilibrium and the pooling equilibrium are quite
striking. First, the consumer infers her loss from price in the separating equilibrium but only knows that her loss is below or above a threshold in the pooling equilibrium. Second, the consumer plays a mixed strategy in the separating equilibrium but plays a pure strategy in the pooling equilibrium. Third, in the separating equilibrium, the expert repairs each loss with a positive probability (declining as loss increases), and makes a strictly positive profit (also declining as loss increases) for losses efficient to repair; in the pooling equilibrium, the expert only repairs losses between \( \ell \) and \( \ell^* \), and profit goes to 0 as the loss value goes to \( \ell^* < \hat{\ell} \). Fourth, the consumer has a zero ex-post payoff in the separating equilibrium; by contrast, in the pooling equilibrium, the consumer has a zero ex-ante payoff, but ex post, if the loss eventually became known, her payoff might turn out to be positive or negative. These results also compare squarely with those in the literature. A separating equilibrium is synonymous with truth-telling, or lack of lying. so the expert does not lie in an equilibrium. By contrast, a pooling equilibrium is synonymous with lying. Both truth-telling and lying are equilibrium behavior in the model with a continuum loss. We believe that the pooling equilibrium is more plausible than the separating equilibrium because it is a simple, pure strategy equilibrium. Furthermore, as we show in Subsection 6.4, a separating does not exist when cost is higher than loss at low loss values, so may not be robust. The pooling equilibrium does not suffer from this problem.

It is of interest to compare the expert’s equilibrium profits between pooling and separating equilibria. This comparison is meaningful. Because consumer surplus is zero in both equilibria, comparing profits across the two equilibria is the same as comparing efficiencies. Depending on model primitives, pooling equilibrium profit may be higher or lower than separating equilibrium profit. The following Figure 3 illustrates these possibilities. In these panels the consumer’s loss distribution \( F \), and hence the function \( AL(\ell, \ell) \), stay the same, but the cost function \( C \) differs. In the left panel, the value of \( \ell^* \) is high, near \( \hat{\ell} \), so the pooling equilibrium profit is quite close to the first best, higher than the separating equilibrium profit. In the right panel, the value of \( \ell^* \) is low, near \( \hat{\ell} \), so the profit in the pooling equilibrium is quite low, and the separating equilibrium must yield higher profits. In general, for a given cost function \( C \) and a given distribution \( F \), we can compare equilibrium profits as follows (the proof omitted).

**Corollary 1** The pooling equilibrium in Proposition 3 is more profitable and efficient than the separating
equilibrium in Proposition 2 if and only if
\[
\int_{\ell}^{\ell^*} [\ell - c(\ell)](1 - \alpha(\ell))dF(\ell) > \int_{\ell}^{\hat{\ell}} [\ell - c(\ell)]\alpha(\ell)dF(\ell),
\]
where \(\alpha(\ell) = \exp \left\{ - \int_{\ell}^{\ell^*} \frac{dx}{x - c(x)} \right\} \).

In the pooling equilibrium, the expert repairs any problem with a loss less than \(\ell^*\) and expropriates the ex ante surplus.\(^6\) Hence, we can write the expert’s profit in the pooling equilibrium as \(\int_{\ell}^{\ell^*} [\ell - c(\ell)]dF(\ell)\).

In the separating equilibrium, the expert captures the surplus for \(\ell < \hat{\ell}\) with probability \(\alpha(\ell)\). The left-hand-side of inequality (4) is the expert’s profit gain in the pooling equilibrium relative to the separating equilibrium for losses below \(\ell^*\). However, there is no repair in the pooling equilibrium for \(\ell > \ell^*\) but repairs happen with probabilities \(\alpha(\ell)\) for losses in this range in the separating equilibrium. The right-hand-side of (4) is the expert’s loss in profit in the pooling equilibrium relative to the separating equilibrium. Figure 4 is a heuristic comparison between the separating and pooling equilibria. The consumer’s strategies are the blue and red lines. The “+” region indicates the incremental repair probabilities of pooling over separating equilibria, whereas the “−” region indicates the opposite.

We work out two examples that verify Corollary (4):

\(^6\)By definition, \(AL(\ell, \ell) = \int_{\ell}^{\ell^*} xdF(x)\). Hence \(\int_{\ell}^{\ell^*} [AL(\ell, \ell^*) - C(\ell)]dF(\ell) = \int_{\ell}^{\ell^*} [\ell - C(\ell)]dF(\ell)\).
Example 1: Loss $\ell$ is uniformly distributed on $[1, 3]$ and $C(\ell) = \frac{1}{2} \ell^2$. It follows that $\hat{\ell} = 2$, $\ell^* = \frac{1 + \sqrt{5}}{2}$, $\alpha(\ell) = \left(\frac{2}{\ell} - 1\right)$ for $\ell \leq \hat{\ell}$. From straightforward computation, the left-hand-side of (4) is 2.517 and the right-hand-side is 0.009.

Example 2: Loss $\ell$ is uniformly distributed on $[\underline{\ell}, \overline{\ell}]$, $\ell > 0$, and $C(\ell) = b\ell - k$, with $b > 1$ and $k \in ((b - 1)\underline{\ell}, (b - 1)\overline{\ell})$. In this case, $\hat{\ell} = \frac{k}{b - 1}$, $\ell^* = \frac{\ell + 2k}{2b - 1}$ and $\alpha(\ell) = \left[\frac{k - (b - 1)\ell}{k - (b - 1)\overline{\ell}}\right]^{\frac{1}{(b - 1)^2}}$. The difference between the left-hand side and the right-hand side of (4) is $\frac{k - (b - 1)\ell^2}{(\ell - \ell^*)(2b - 1)^2} > 0$. Note that $k - (b - 1)\ell$ is the surplus from repairing the lowest loss $\underline{\ell}$. Hence, the profit difference between pooling and separating equilibria increases in this surplus.

Corollary 1 and the above two examples show results that would be unavailable in a binary model. Fong (2005) shows that the expert’s profit is maximized by the separating equilibrium. If there are only two losses, say minor $\underline{\ell}$ and major $\overline{\ell}$, and the average loss is smaller than the major repair cost $C(\overline{\ell})$, the only pooling equilibrium has the expert offering price $\underline{\ell}$ if and only if the loss is minor. The binary setting has lumpy equilibrium inefficiency characteristics. However, for continuous losses, all losses up to $\ell^*$ are repaired, and this often yields more profits than the separating equilibrium.
4.2.1 Other pooling equilibria and a belief refinement

The equilibrium in Proposition 3 may not be the only pooling equilibrium. We can apply the same construction and argument for any $\tilde{\ell} \in LP$ where $AL(\ell, \tilde{\ell}) = C(\tilde{\ell})$ (such as points $\ell_1$ and $\ell_2$ in Figure 2). We simply replace $\ell^*$ in Proposition 3 (and its proof) by $\tilde{\ell}$ verbatim. We record this as a corollary (proof omitted).

**Corollary 2** There may be other pooling equilibria indexed by $\tilde{\ell} \in LP$ at which $AL(\ell, \tilde{\ell}) = C(\tilde{\ell})$, and, in each of these equilibria, the strategies and beliefs are the same as those in Proposition 3 with $\ell^*$ there replaced by $\tilde{\ell}$.

A pooling equilibrium indexed by $\tilde{\ell} < \ell^*$ is supported by the off-equilibrium belief that the consumer’s loss belongs to the interval $[\ell, \tilde{\ell})$, irrespective of the price. This off-equilibrium passive belief is not very sophisticated or convincing. We now present a belief refinement, and then shows that it eliminates all pooling equilibria except for the one indexed by $\ell^*$.

**Definition 3 (Nonnegative Profit Principle)** Upon receiving an off-equilibrium price offer $p$, the consumer believes that the loss is drawn from $\{\ell : p \geq C(\ell)\}$ according to $F$. A pooling equilibrium is said to satisfy the Nonnegative Profit Principle if the consumer’s strategy specifies a best response against off-equilibrium price $p$ based on the belief that loss has been drawn from $\{\ell : p \geq C(\ell)\}$ according to $F$.

According to the Nonnegative Profit Principle, the consumer believes that the expert’s price at least covers cost. Offering a price that would not cover cost is a dominated strategy, and the consumer would not believe that the expert would do that. Apart from this requirement, the consumer cannot pin down other motives, so we assume that she believes losses are drawn from a truncated distribution of losses.

**Proposition 4** The pooling equilibrium in Proposition 3 satisfies the Nonnegative Profit Principle. A pooling equilibrium indexed by $\tilde{\ell} < \ell^*$ in Corollary 2 does not satisfy the Nonnegative Profit Principle.

Figure 5 illustrates how the Nonnegative Profit Principle eliminates all pooling equilibria but the one indexed by $\ell^*$. Consider the pooling equilibrium indexed by $\ell_1$ in the figure. The equilibrium price is
Suppose that the expert offers the off-equilibrium price $p'$. Based on the Nonnegative Profit Principle, the consumer believes that her loss belongs to $[\ell, \ell']$, where $p' = C(\ell')$, so her conditional expected loss $AL(\ell, \ell')$ is higher than the price $p'$. Hence, the consumer will accept $p'$ with probability one. Given the consumer’s response to the off-equilibrium price, the expert will deviate to offering $p'$ for losses lower than $\ell'$. The Nonnegative Profit Principle effectively allows an expert to convince the consumer to accept a higher price. Proposition 4 says that among all pooling equilibria, the one in Proposition 3 is most compelling.

Figure 5: The Nonnegative Profit Principle

Propositions 1 to 3 together show that all equilibria are inefficient. Efficiency requires all losses $\ell \in [\hat{\ell}, \ell]$ be repaired with probability one. Proposition 1 says that the equilibrium acceptance rate is weakly decreasing in loss. Hence, the only possible candidate for an efficient equilibrium is a pooling equilibrium. Nevertheless, because the highest loss threshold for repair in any pooling equilibrium, namely $\ell^*$, is strictly less than $\hat{\ell}$, there does not exist such an efficient pooling equilibrium. We now turn to an alternative information structure for
a fully efficient equilibrium.

5 Expert information acquisition

We have taken as given that the expert perfectly observes the consumer’s loss before he makes a price offer. In this section, we study the expert’s information acquisition decision. We show that the expert is better off being imperfectly informed about the consumer’s loss even when information acquisition is costless. We modify the game by adding a “Stage 0,” in which the expert picks an information structure or a diagnostic test. This is defined by a set of signals $S \subset \mathbb{R}$ and a joint distribution, $J : S \times [\ell, \bar{\ell}] \to [0, 1]$, so $J(s, \ell)$ measures the probability that the signal is less than $s$ and the loss is less than $\ell$. To be a valid test, we require: i) $\int_S dJ(s, \ell) = J(\sup(S), \ell) = F(\ell)$ and ii) $J(s, \bar{\ell}) \equiv G(s)$ yields a probability distribution over $S$. The requirement i) implies that $\int_0^{\bar{\ell}} \int_S dJ(s, \ell) = \int_0^{\bar{\ell}} dF(\ell) = \mu$. We do rule out the uninteresting case where $J$ is the product of its two marginal distributions, the case of uninformative signals. The model in Section 2 has a perfectly informative test. 

We let the information structure or diagnostic test be public information once it has been chosen. A cost can be associated with an information structure, and including this is straightforward. For now we just assume that an information structure is costless.

We augment the game in Section 2 as follows. In Stage 0, the expert chooses an information structure or a diagnostic test. In Stage 1, the consumer visits the expert who then privately receives a signal from the test. The expert makes a price offer based on the private signal, or refuses to provide a repair. In Stage 2, upon receiving a price offer, the consumer chooses between accepting or rejecting the offer.

We assume that the consumer’s actual loss $\ell$ will be revealed to the expert during the repair even if the test is imperfect. When the expert provides treatment for loss $\ell$, he still has to incur the cost $C(\ell)$ eventually because he is liable for repair outcomes. More information arrives during the repair process (but after the price has been agreed upon); for example, car mechanics usually learn more about the cost after they disassemble parts during the repair.

That is, there is an increasing, bijective function $\phi : [\ell, \bar{\ell}] \to [s, \bar{s}]$ such that $F(\ell) = G(s)$ where $s = \phi(\ell)$ and $G(s) \equiv \int_\ell^{\bar{s}} dJ(s, \ell)$. The simplest case is where $\phi$ is the identity map.
Recall that the first-best allocation is one with repair for all losses up to \( \hat{\ell} \) (at which point, \( \hat{\ell} = C(\hat{\ell}) \)). If the diagnosis test is perfect, the analysis is the same as in Section 4. Neither the separating equilibrium in Proposition 2 nor the pooling equilibrium in Proposition 3 can yield the first best. The follow diagnosis test, however, can implement the first best. Let the test has binary signals: \( S = \{s_L, s_H\} \), where \( s_L < s_H \). The signal takes the value \( s_L \) when loss is below \( \hat{\ell} \), and \( s_H \) otherwise. Thus, the signal is \( s_L \) if and only if the loss is above cost; the signal \( s_L \) indicates that repair is efficient. We write down its definition as follows.

It is somewhat more familiar to use conditional densities. So let \( f = F' \), and \( j(s_L, \ell) = \frac{\partial J(s_L, \ell)}{\partial \ell} \) and \( j(s_H, \ell) = \frac{\partial J(s_H, \ell)}{\partial \ell} \). Then in terms of joint densities, we have

<table>
<thead>
<tr>
<th>joint density</th>
<th>( j(s_L, \ell) )</th>
<th>( j(s_H, \ell) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \ell \leq \ell \leq \hat{\ell} )</td>
<td>( f(\ell) )</td>
<td>0</td>
</tr>
<tr>
<td>( \hat{\ell} \leq \ell \leq \hat{\ell} )</td>
<td>0</td>
<td>( f(\ell) )</td>
</tr>
<tr>
<td>signal probabilities</td>
<td>( F(\hat{\ell}) = \Pr(S = s_L) )</td>
<td>( 1 - F(\hat{\ell}) = \Pr(S = s_H) )</td>
</tr>
</tbody>
</table>

and the corresponding conditional densities, \( k(\ell | S) \), are

\[
k(\ell | S = s_L) = \begin{cases} 
\frac{f(\ell)}{F(\hat{\ell})} & \text{for } \ell \leq \hat{\ell} \\
0 & \text{for } \ell > \hat{\ell}
\end{cases}
\]

and

\[
k(\ell | S = s_H) = \begin{cases} 
0 & \text{for } \ell \leq \hat{\ell} \\
\frac{f(\ell)}{1 - F(\hat{\ell})} & \text{for } \ell > \hat{\ell}
\end{cases}
\]

Upon observing signal \( s_L \), the expert’s expected repair cost is \( \int_{\ell}^{\hat{\ell}} C(\ell)dF(\ell) \) \( F(\hat{\ell}) \) \( \equiv AC(\ell, \hat{\ell}) \), the average cost of repairing losses below \( \hat{\ell} \). By the definition of \( \hat{\ell}, \ell > C(\ell) \) for all \( \ell < \hat{\ell} \). Hence, \( AL(\ell, \hat{\ell}) > AC(\ell, \hat{\ell}) \).

Given that the information structure is public information, if the consumer knew that the expert received the signal \( s_L \), the maximum acceptable price would be \( AL(\ell, \hat{\ell}) \).

**Proposition 5** The expert can achieve the first best by the information structure in (5) or (6). The expert’s strategy is

\[
P(s) = \begin{cases} 
AL(\ell, \hat{\ell}) & \text{if } s = s_L \\
+\infty & \text{if } s = s_H
\end{cases}
\]

and the consumer’s strategy is

\[
\alpha(p) = \begin{cases} 
1 & \text{if } p \leq AL(\ell, \hat{\ell}) \\
0 & \text{if } p > AL(\ell, \hat{\ell})
\end{cases}
\]
The consumer always believes that the expert has received the signal $s_L$ upon being made a price offer for repair.

The proof of this Proposition is obvious, and omitted. The expert’s equilibrium expected profit is

$$\left[ AL(\hat{\ell}, \hat{\ell}) - AC(\hat{\ell}, \hat{\ell}) \right] F(\hat{\ell}) = \int_{\hat{\ell}}^{\hat{\ell}} (\ell - C(\ell)) dF(\ell),$$

which is the first-best surplus. However, ex post, the repair cost for some losses may be above the equilibrium price. Indeed, $C(\hat{\ell}) = \hat{\ell} > AL(\hat{\ell}, \hat{\ell})$, so for losses close to $\hat{\ell}$, the expert will make a loss ex post, but of course the expert will make a profit for low losses. A similar observation holds for the consumer: at low losses, the consumer will over pay, but at high losses, the consumer will earn a surplus. On average, the consumer earns a zero expected payoff, all ex-ante surplus being expropriated by the expert.

Clearly, the information structure in (6) acts like it could only implement a pooling equilibrium, but this equilibrium is first best, so more efficient than the one in Proposition 3. In Proposition 5, the offer is based on the average cost; in Proposition 3, it is based on the actual or marginal cost. The strategy profile in Proposition 5 would not constitute an equilibrium if the expert had perfect information; the expert would refuse to repair losses with (marginal) costs above the pooling price $AL(\hat{\ell}, \hat{\ell})$. Anticipating the expert’s cream skimming, the consumer would not accept that price offer. By contrast, under the information structure in (6), the consumer and the expert are on a symmetric footing. All the expert manages to learn is whether the average cost is below the average loss. Using a binary signal, the expert commits to using average-cost information to make price offers, and achieves the first best.

Proposition 5 is not meant to report an optimism that the simple diagnostic test entirely solves credence-good problems. The analytical result notwithstanding, the extensive form does require the expert’s choice of a diagnostic test be common knowledge. In practice, consumers may not understand how a diagnostic test actual works, or how a signal is to be interpreted. Proposition 5 succinctly shows that more information may impede economic transaction because the extra information will become private; this is in addition to more information may risk a cognitive overload.
6 Other equilibria and extensions

6.1 Hybrid equilibria

Our analysis has focussed on the separating and the pooling equilibria. There are many other equilibria, indeed, a continuum of them. First we give a definition for equilibria that are neither pooling nor separating.

**Definition 4 (Hybrid equilibrium)** An equilibrium \((P, \alpha)\) is said to be a hybrid equilibrium if, there are disjoint intervals \((\ell_1, \ell_2)\) and \((\ell_3, \ell_4)\), both in \([\underline{\ell}, \overline{\ell}]\), for any \(\ell\) and \(\ell'\) in \((\ell_1, \ell_2)\) for which \(\alpha(P(\ell)) > 0\) and \(\alpha(P(\ell')) > 0\), \(P(\ell) \neq P(\ell')\), and for any \(\ell\) and \(\ell'\) in \((\ell_3, \ell_4)\) for which \(\alpha(P(\ell)) > 0\) and \(\alpha(P(\ell')) > 0\), \(P(\ell) = P(\ell')\). (There may well be many intervals of the sort like \((\ell_1, \ell_2)\) and \((\ell_3, \ell_4)\).)

In a hybrid equilibrium, over some loss interval, prices that may be accepted vary with losses, but over some other loss interval, a price that may be accepted does not vary with losses. There may also be many of these separating and pooling intervals. Now we construct a continuum of hybrid equilibria. First, we present a corollary of Proposition 2, which characterizes a separating equilibrium. However, loss values may be revealed over a subset of \([\underline{\ell}, \overline{\ell}]\), not necessarily for all losses. In fact, the proof of Proposition 2 shows that explicitly. The following presents an acceptance function that supports separation over a loss interval (proof omitted).

**Corollary 3** In an equilibrium, the expert offers \(P(\ell) = \ell\) for each \(\ell \in [\ell_1, \ell_2] \subset [\underline{\ell}, \overline{\ell}]\) if and only if the consumer accepts offer \(\ell\) with probability \(\alpha(\ell) = \alpha(\ell_1) \exp\left(-\int_{\ell_1}^{\ell} \frac{dx}{x-c(x)}\right)\), where \(\alpha(\ell_1)\) is the acceptance probability of offer \(P(\ell_1) = \ell_1\).

The next result presents a family of hybrid equilibria. In each, the expert offers a pooling price for low losses, and then switches to a separating price for high losses; in between there is jump in the offer. The consumer accepts the pooling price with probability one, and then follows the acceptance probabilities prescribed by Corollary 3.

**Proposition 6** For each \(\overline{\ell} \in \{\ell : AL(\underline{\ell}, \ell) > C(\ell)\}\) (which is the interior of \(LP\)), there is a hybrid
equilibrium. The expert’s strategy is

\[ P(\ell) = \begin{cases} 
  AL(\ell, \tilde{\ell}) & \text{if } \ell \in [\tilde{\ell}, \ell] \\
  \ell & \text{if } \ell \in (\tilde{\ell}, \ell] \\
  +\infty & \text{if } \ell \in (\ell, \tilde{\ell}] 
\end{cases} \]

The consumer’s strategy is

\[ \alpha(p) = \begin{cases} 
  1 & \text{if } p \in [\ell, AL(\ell, \tilde{\ell})] \\
  0 & \text{if } p \in (AL(\ell, \tilde{\ell}), \tilde{\ell}) \\
  \alpha(\tilde{\ell}) \exp \left\{ - \int_{\ell}^{p} \frac{dx}{x - C(x)} \right\}, \text{ with } \alpha(\tilde{\ell}) = \frac{AL(\ell, \tilde{\ell}) - C(\tilde{\ell})}{\ell - C(\tilde{\ell})} & \text{if } p \in [\ell, \tilde{\ell}] \\
  0 & \text{if } p \in [\tilde{\ell}, \ell] 
\end{cases} \]

If the consumer is offered a price at or below \(\tilde{\ell}\), she believes that the loss is drawn from \([\ell, \tilde{\ell}]\) according \(F\); if she is offered any price at or above \(\ell\), she believes that the loss is \(\ell\).

This proposition presents a continuum of “hybrid” equilibria; they amalgamate equilibria in Propositions 2 and 3. Each hybrid equilibrium has a pooling interval, for those losses between \(\ell\) and \(\tilde{\ell}\), with the pooling price \(AL(\ell, \tilde{\ell})\) that is always accepted by the consumer, and then a separating interval, for losses above \(\tilde{\ell}\). The expert makes a strictly positive profit at loss \(\tilde{\ell}\) because \(AL(\ell, \tilde{\ell}) > C(\tilde{\ell})\) by the definition of \(\tilde{\ell}\). Therefore, there is scope for more transactions, which happen to be separating prices. At the cutoff loss \(\tilde{\ell}\), the expert is indifferent between recommending the pooling price \(AL(\ell, \tilde{\ell})\) and the higher separating price \(\ell\), by the choice of the acceptance probability \(\alpha(\tilde{\ell})\). But there is a discontinuous jump from the pooling price \(AL(\ell, \tilde{\ell})\) to the lowest separating price \(\tilde{\ell}\). As \(\tilde{\ell}\) goes to \(\ell\), the hybrid equilibrium converges to the separating equilibrium in Proposition 2, and as \(\tilde{\ell}\) goes to \(\ell^*\), the hybrid equilibrium converges to the pooling equilibrium in Proposition 3.

Figure 6 illustrates Proposition 6. There, the expert recommends the pooling price, \(AL(\ell, \tilde{\ell})\), for losses up to \(\tilde{\ell}\). At \(\tilde{\ell}\), the price offers jump up to the 45-degree line. Off-equilibrium prices are those below \(AL(\ell, \tilde{\ell})\), between \(AL(\ell, \tilde{\ell})\) and \(\ell\), and above \(\tilde{\ell}\). At off-equilibrium prices between \(AL(\ell, \tilde{\ell})\) and \(\tilde{\ell}\), the consumer continues to believe that the loss is drawn from the pooling set of losses \((\ell, \tilde{\ell})\), so rejects them.
As we have shown in Proposition 4, the “passive” off-equilibrium belief at prices between $AL(\ell, \tilde{e})$ and $\tilde{e}$ is implausible. The consumer should not ignore the expert’s information advantage and pricing strategy. In fact, we can reject these hybrid equilibria.\footnote{We are aware that hybrid equilibria might resemble two-part tariffs: a base fee for diagnostic and minor repairs, and then a higher fee for more complicated repairs. We are agnostic about whether such two-part tariffs can be convincingly rationalized by our basic model. A full analysis is beyond the scope here.}

**Proposition 7** Hybrid equilibria in Proposition 6 do not satisfy the Nonnegative Profit Principle.

The proof of this Proposition follows closely that of Proposition 4, so we will only give an outline here. Consider a hybrid equilibrium indexed by $\tilde{e}$ where $AL(\ell, \tilde{e}) > C(\ell)$, as in Figure 6. The pooling price of the hybrid equilibrium is $AL(\ell, \tilde{e})$; the profit from repairing a problem with loss $\tilde{e}$ is $AL(\ell, \tilde{e}) - C(\tilde{e}) > 0$. Suppose that the expert offer price $AL(\ell, \tilde{e}) + \varepsilon$. For a sufficiently small $\varepsilon > 0$, the set $\{\ell : AL(\ell, \tilde{e}) + \varepsilon \geq C(\ell)\}$ has an upper bound $\tilde{e} + \delta$, where $AL(\ell, \tilde{e}) + \varepsilon = C(\ell + \delta)$; see Figure 7. At $\tilde{e} + \delta$, we have $AL(\ell, \tilde{e} + \delta) > AL(\ell, \tilde{e}) + \varepsilon$,
which says that the consumer’s willingness to pay is higher than the price if she believes that the loss is drawn from $[\ell, \tilde{\ell} + \delta]$ according to $F$. This belief is what the Nonnegative Profit Principle prescribes, so the consumer should accept the price $AL(\ell, \tilde{\ell}) + \varepsilon$. The same argument can be applied to rule out any other hybrid equilibrium.\(^9\).

More complicated hybrid equilibria can be constructed in a similar fashion. An equilibrium can start with separating prices, turns to pooling prices, and then reverts to separating prices—as long as monotonicity in Proposition 1 and no-deviation conditions are satisfied.\(^10\) We can certainly write down conditions on the

\(^{9}\)A formal argument is the following. For a given $\tilde{\ell} \in \{\ell | AL(\ell, \tilde{\ell}) > C(\ell)\}$, define $\ell_m \equiv \min \{\ell | AL(\ell, \tilde{\ell}) = C(\ell)\}$ and $\ell > \tilde{\ell}$. Because $\tilde{\ell} < \ell_m$, $AL(\ell, \tilde{\ell}) < AL(\ell, \ell_m)$. There exists a small $\varepsilon$ such that $AL(\ell, \tilde{\ell}) + \varepsilon < AL(\ell, \ell_m)$. Following the definition of $\delta$ and $\ell_m$, $\ell_m + \delta < C(\ell_m)$ and hence $\tilde{\ell} < \ell + \delta < \ell_m$. Again, by the definition of $\ell_m$ and $\tilde{\ell}$, $AL(\ell, \tilde{\ell} + \delta) > C(\ell + \delta) = AL(\ell, \ell_m) + \varepsilon$. This argument shows that upon recommended the off-the-equilibrium-path price $AL(\ell, \tilde{\ell}) + \varepsilon$, the consumer’s expected loss is $AL(\ell, \tilde{\ell} + \delta) > AL(\ell, \tilde{\ell}) + \varepsilon$. Hence, the consumer will accept the price.

\(^{10}\)We can also construct equilibria that are made up of only many pooling regions.
primitives for such constructions, but they will be little more than applying Proposition 1 and modifying strategies in Proposition 6.

We use the model primitives in Example 1 above to construct a complicated hybrid equilibrium. Again, suppose that \( \ell \) is uniformly distribution on \([1, 3]\) and \( C(\ell) = \frac{1}{2} \ell^2 \). Then \( AL(\ell_1, \ell_2) = \frac{1}{2}[\ell_1 + \ell_2] \). We construct an equilibrium in which the expert recommends the separating price in two disjoint intervals of losses and the pooling price in between. Consider the following expert strategy:

\[
P(\ell) = \begin{cases} 
\ell & \text{if } \ell \in [1, \ell_1] \\
\frac{1}{2}[\ell_1 + \ell_2] & \text{if } \ell \in (\ell_1, \ell_2] \\
\ell & \text{if } \ell \in (\ell_2, \hat{\ell}] \\
+\infty & \text{if } \ell \in (\hat{\ell}, 3] 
\end{cases}
\]

The expert recommends the separating price for \( \ell \in [1, \ell_1] \cup (\ell_2, \hat{\ell}] \) and the pooling price \( \frac{1}{2}[\ell_1 + \ell_2] \) for \( \ell \in (\ell_1, \ell_2] \). Let \( \alpha(\ell) = \alpha(\ell_0) \exp \left\{ -\int_{\ell_1}^{\ell} \frac{dx}{x - C(x)} \right\} = \alpha(\ell_0) \left[ \frac{2}{\ell} - 1 \right] \), where \( \ell_0 \) is the beginning loss value at which separation occurs. The consumer’s acceptance probability is \( \alpha(\ell) = \frac{2}{\ell} - 1 \) for \( \ell \in [1, \ell_1] \). It follows that the expert’s profit at the threshold \( \ell_1 \) is \( \pi(\ell_1) = \frac{2}{\ell_1} - 1 \left[ \ell_1 - \frac{1}{2} \ell_1^2 \right] \). For \( \ell_1 \), the profit at the pooling price is \( A \times \left[ \frac{1}{2}[\ell_1 + \ell_2] - \frac{1}{2} \ell_1^2 \right] \), where \( A \) is the probability that the consumer accepts the pooling price. Probability \( A \) is chosen to make the expert just indifferent between recommending the separating price \( \ell_1 \) and the pooling price at \( \ell_1 \). So,

\[
\left[ \frac{2}{\ell_1} - 1 \right] \left[ \ell_1 - \frac{1}{2} \ell_1^2 \right] = A \times \left[ \frac{1}{2}[\ell_1 + \ell_2] - \frac{1}{2} \ell_1^2 \right]. \tag{7}
\]

The expert’s profit from pooling at threshold \( \ell_2 \) is \( A \times \left[ \frac{1}{2}[\ell_1 + \ell_2] - \frac{1}{2} \ell_2^2 \right] \), whereas profit from separating at price \( \ell_2 \) is \( \alpha(\ell_2) \left[ \ell_2 - \frac{1}{2} \ell_2^2 \right] \). For the expert to be indifferent between recommending the pooling price and the separating price at \( \ell_2 \), we have

\[
A \times \left[ \frac{1}{2}[\ell_1 + \ell_2] - \frac{1}{2} \ell_2^2 \right] = \alpha(\ell_2) \left[ \ell_2 - \frac{1}{2} \ell_2^2 \right]. \tag{8}
\]
Given \( \ell_1 \) and \( \ell_2 \), equations (7) and (8) define a two-unknown-two-variable linear equation system. We can solve for \( A \) and \( \alpha(\ell_2) \) given \( \ell_1 \) and \( \ell_2 \). In fact, we have

\[
A = \left( \frac{2}{\ell_1} - 1 \right) \left( \ell_1 - \frac{1}{2} \ell_2^2 \right) \left( \frac{1}{2} (\ell_1 + \ell_2) - \frac{1}{2} \ell_1^2 \right),
\]

\[
\alpha(\ell_2) = A \left( \frac{1}{2} (\ell_1 + \ell_2) - \frac{1}{2} \ell_2^2 \right) = \frac{2}{\ell_1} \left( \ell_1 - \frac{1}{2} \ell_2^2 \right) \left( \frac{1}{2} (\ell_1 + \ell_2) - \frac{1}{2} \ell_1^2 \right) \times \frac{1}{2} (\ell_1 + \ell_2) - \frac{1}{2} \ell_2^2.
\]

It can be verified that both \( A \) and \( \alpha(\ell_2) \) are less than one. We also require the boundary condition for pooling to make nonnegative profits: \( \frac{1}{2} (\ell_1 + \ell_2) - \frac{1}{2} \ell_2^2 \geq 0 \). For example, if \( \ell_1 = 1 \), then the largest pooling range, given by \( \ell_2 \), is \( \frac{1}{2} (1 + \ell_2) = \frac{1}{2} \ell_2^2 \), which gives the maximum \( \ell_2 \) to be \( \frac{1 + \sqrt{5}}{2} = 1.618 \) as in Example 1. As \( \ell_1 \) increases from 1, the maximum \( \ell_2 \) to sustain pooling increases.

The two-parameter example includes the separating equilibrium in Proposition 2, and the pooling equilibrium in Proposition 3, as special cases, respectively at \((\ell_1 = 1, \ell_2 = 2)\) and \((\ell_1 = 1, \ell_2 = \frac{1}{2} [1 + \sqrt{5}])\). We can construct other hybrid equilibria following similar constructions.

### 6.2 Random losses and costs

Now we let a consumer’s loss and cost be both random. Denote the cost’ support by \([c; \bar{c}]\), a positive interval. Then the random variable \((\ell, c) \in [\underline{\ell}; \bar{\ell}] \times [\underline{c}; \bar{c}]\) has a joint distribution \( F : [\underline{\ell}; \bar{\ell}] \times [\underline{c}; \bar{c}] \to [0, 1] \) and a density function \( f(\ell, c) > 0, (\ell, c) \in [\underline{\ell}; \bar{\ell}] \times [\underline{c}; \bar{c}] \). (Do note that we abuse notation by using the same \( F \) to denote a random vector of losses and costs.) In other words, now a consumer’s type has two dimensions described by \((\ell, c)\). A single dimensional model is one where \( \ell \) and \( c \) are perfectly correlated, so we would be able to write the repair cost as (a deterministic function) \( C(\ell) \), as in Section 2. The increasing \( C \) assumption in Section 2 now is replaced by the assumption that \( \ell \) and \( c \) are positively correlated. The extensive form is rewritten here:

**Stage 1:** A consumer visits the expert, who then observes the consumer’s potential loss \( \ell \in [\underline{\ell}; \bar{\ell}] \) and repair cost \( c \in [\underline{c}; \bar{c}] \), drawn with the joint distribution \( F \). The expert chooses between offering to repair at a price \( p \in [\underline{\ell}; \bar{\ell}] \) and refusing to repair.
Stage 2: If the consumer agrees to the repair, she pays the expert \( p \), and the expert incurs cost \( c \) for the repair.

The expert's strategy is a map: \( P : [\ell, \overline{\ell}] \times [c, \overline{c}] \to [\ell, \overline{\ell}] \), which prescribes a price offer when the provider observes consumer \((\ell, c)\). A consumer's strategy is the same as before: an acceptance probability of the offered price. The consumer never gets to observe loss \( \ell \) or cost \( c \).

In a perfect-Bayesian equilibrium, the consumer updates belief about the \((\ell, c)\) distribution, but her utility depends only \( \ell \) and the price. The expert does not directly care about the consumer's loss because profit only depends on the repair cost and the price. Due to these preferences, in fact, the two-dimensional model is isomorphic to the one-dimensional model. The precise relationship is given by the following.

**Proposition 8** In any perfect-Bayesian equilibrium, the expert's strategy \( P : [\ell, \overline{\ell}] \times [c, \overline{c}] \to [\ell, \overline{\ell}] \) is (almost everywhere) independent of \( \ell \) whenever the price \( P(\ell, c) \) is accepted with a positive probability.

The expert’s equilibrium price is only dependent on his cost, never on the consumer’s loss. It is the price-cost margin that generates profit; any price function varying with losses cannot be sustained as an equilibrium strategy. The main thrust of Proposition 8 is this. In Section 2, we have assumed a deterministic relationship between loss and cost, and that the expert observes the loss. Obviously, it makes no difference if the expert observes cost (instead of loss). In that case, the pricing strategy is a function of cost. From the (separating or pooling) equilibrium strategy, the consumer will infer losses, and all results remain the same.\(^\text{11}\)

Now Proposition 8 says that for a two-dimensional model, any equilibrium price function depends on cost only. The consumer’s equilibrium inference therefore concerns the expected losses conditional on cost. Let \( L(c) \) denote the expected losses conditional on \( c \):

\[
L(c) = \frac{\int_{\ell}^{\overline{\ell}} \ell dF(\ell, c)}{\int_{\ell}^{\overline{\ell}} dF(\ell, c)}.
\]

\(^{11}\)Suppose that \( \ell = D(c) \) where \( D \equiv C^{-1} \) is the inverse of the cost function in Section 2. The expert’s strategy now is \( P : [c, \overline{c}] \to [\ell, \overline{\ell}] \), with \( \ell = C(\ell) \) and \( \overline{\ell} = C(\overline{\ell}) \). Given a price function \( P \), the consumer will try to infer the loss. For example, if \( P(c) = c + k \), then \( c = P(c) - k \) and the inferred loss is \( D(P(c) - k) \). More generally, if \( P \) is strictly increasing over a range of \( c \), then it is invertible, so the inverted value of \( c \) can be used to infer loss \( \ell \). If \( P \) is a constant function over a range of \( c \), then at that constant, the consumer will infer that losses belong ot an interval.
A positive correlation between loss and cost implies that $L(c)$ is increasing. Hence the function $L$ acts like the inverse of the cost function $C$ in Section 2. In other words, the deterministic, one-dimensional model in Section 2 is the reduced form of the stochastic, two-dimensional model here: they yield the same set of equilibria. Our use of a deterministic cost function is without loss of generality.

### 6.3 Many experts

Now we consider a market with many consumers and many experts. There are $N \geq 2$ identical experts and a consumer with a loss drawn from $F$. The consumer randomly chooses an expert. The expert perfectly observes the consumer’s loss and makes a price offer. The consumer then decides whether to accept the price. If the consumer accepts the price, the expert must perform the repair. If the consumer rejects, she can exit the market or repeat the search process until she visits all experts. The consumer must pay a small search cost to visit an expert, except for the first visit. The search cost captures the consumer’s disutility from waiting. Following the literature (Wolinsky (1993)), we assume that an expert does not know a consumer’s search history. Hence, the expert cannot make a price offer contingent on the consumer’s past searches or price offers, if any.

We claim that the separating equilibrium in Proposition 2 and the pooling equilibrium in Proposition 3 remain equilibria in the market with multiple experts. Precisely, it is an equilibrium for each expert and each consumer to play their respective strategies in Proposition 2; likewise, this is also true for Proposition 3. Recall that the monopolist expert plays pure strategies in each of the separating and pooling equilibria. If all experts play the same pure strategy, a consumer expects to have the same payoff from visiting a different expert, but must incur a search cost. In equilibrium, therefore, a consumer does not search, which reinforces an expert’s ability to maintain monopoly power. Hence, the strategy profiles in the monopoly market will continue to constitute an equilibrium in the market with multiple experts.

To elaborate the argument, suppose that all the experts play the same strategy as in the separating equilibrium in Proposition 2. Given that experts will recommend the same price $P(\ell) = \ell$ and that search involves a small cost, a consumer will not solicit a second opinion. Since the consumer is indifferent between accepting the first expert’s recommendation and exiting the market, it is a best response for the consumer to
accept price $\ell$ with probability $\alpha(\ell)$ as in the separating equilibrium in Proposition 2. If the consumer does not search for more opinions, each expert is dealing with an inexperienced consumer. Given the consumer’s acceptance function $\alpha(\ell)$, it is optimal for an expert to adopt the pricing strategy $P(\ell) = \ell$ for the same argument in Proposition 2.

We have argued that an equilibrium outcome in the single-expert model remains an equilibrium outcome in the many-expert model. This argument requires symmetry: all experts follow the same pooling or separating equilibrium strategy in the single-expert model. There might be asymmetric equilibria in which different experts use different pricing strategies and consumers search for different prices. It also has not escaped our attention that consumers engage in search because experts may possess different information, or experts behave differently. It is beyond the scope of this paper to fully characterize the set of equilibria in markets with multiple homogenous or heterogenous experts.

6.4 Alternative cost function: efficient repair only at medium losses

We have made the assumption that $C(\ell) < \ell$ for all $\ell < \hat{\ell}$. A different scenario may have costs higher than losses at low and high loss values; that is $C(\ell) < \ell$ if and only if $\ell \in [\ell_0, \hat{\ell}]$ where $\underline{\ell} < \ell_0 < \hat{\ell} < \overline{\ell}$, but we maintain the assumption that $C$ is an increasing function.\textsuperscript{12} How do results change under this cost function specification? First, the separating equilibrium will fail to exist. Clearly, if the expert’s price offers at $\ell$ between $\underline{\ell}$ and $\ell_0$ reveal the losses, then all these prices must be below cost $C(\ell)$. The expert must reject them; otherwise the expert would have been better off refusing to offer treatment. Given this, Part i) of Proposition 1 says that there cannot be any repair for any higher $\ell$. But this means that there is no acceptance for any losses, so a separating equilibrium fails to exist.

Second, the pooling equilibrium in Proposition 3 remains robust, and the equilibrium price is still $AL(\underline{\ell}, \ell^*)$. Here, of course, losses between $\underline{\ell}$ and $\ell_0$ will be repaired inefficiently. It follows that the pooling equilibrium involves service overprovision at low losses and underprovision at high losses.

Third, we can continue to construct a continuum of hybrid equilibria, as in Proposition 6, and they still

\textsuperscript{12}For example, an automobile’s very minor problems may not be worth the repair cost, and the same may be true for very serious problems. Repairs are cost effective for intermediate conditions.
will be eliminated by the nonnegative profit principle by the same argument in Proposition 7. However, any hybrid equilibrium cannot have $P(\ell) = \ell$ for low values of $\ell$, so the more complicated example hybrid equilibrium in the previous subsection would not exist under the modified cost function.

7 Conclusion

We study credence goods with two functions as primitives: the consumer’s loss distribution, and the expert’s cost. We characterize perfect-Bayesian equilibria in the canonical extensive form, in which the consumer decides on accepting the expert’s repair price offer after the expert has learned of the loss. Both separating and pooling equilibria exist, and support a rich set of equilibrium outcomes. All equilibria are inefficient, but full efficiency can be restored if the expert gets to choose the diagnostic test.

We have left open a host of other issues in the market of credence goods. First, we have assumed risk neutrality. Obviously consumers may be risk averse, so the expert’s repair may be a way to dampen utility fluctuations. Will the consumer’s demand for insurance give the expert a higher leverage to expropriate surplus? On the other hand, the expert may also have to face risks if the diagnostic test cannot fully reveal the loss level.

Second, we have assumed limited contracting options. Only the only verifiable event is the restoration of the baseline utility, so partial repair is ruled out. In practice, partial repairs are quite common. An analysis of partial repair will have to be based on a larger set of verifiable outcomes.

Third, we have used a static approach to model information about losses. Information about potential losses, such as in health and house repair settings, may be coming along during treatment or repair. Renegotiation about price may have to happen because the ultimate repair costs may change.

Fourth, we consider the market environment in which the treatment cost is not verifiable. Our framework can be used to analyze the case when treatment cost is verifiable but the expert is not liable for treatment outcome.
Appendix

Proof of Proposition 1: We begin with i). Consider two losses \( \ell \) and \( \ell' \). Adding the no-deviation constraints in (1) and (2) we have

\[
-\alpha P(\ell') - C(\ell') \geq -\alpha P(\ell) - C(\ell)
\]

\[
[C(\ell) - C(\ell')] \alpha P(\ell') \leq 0.
\]

Clearly if \( \ell > \ell' \), we have \( C(\ell) > C(\ell') \). so \( \alpha P(\ell) \leq \alpha P(\ell') \).

Next, to prove ii), suppose that it is false. That is, suppose that \( P(\ell) < P(\ell') \) when \( \ell > \ell' \). Then we have

\[
[P(\ell) - C(\ell')] \alpha P(\ell') < [P(\ell') - C(\ell')] \alpha P(\ell) \leq [P(\ell') - C(\ell)] \alpha P(\ell'),
\]

where the first inequality follows from \( \alpha P(\ell) > 0 \) and the second inequality follows from i). But this contradicts (2). We conclude that \( P(\ell) \geq P(\ell') \), and \( P(\ell) \) is weakly increasing.

Finally, we prove iii). Suppose \( \ell > \ell' \). Then,

\[
\Pi(\ell') \equiv [P(\ell') - C(\ell')] \alpha P(\ell') \geq [P(\ell) - C(\ell')] \alpha P(\ell) \geq [P(\ell) - C(\ell)] \alpha P(\ell) \equiv \Pi(\ell).
\]

The first inequality follows from (1) and the second inequality follows from \( C(\ell') < C(\ell) \). If \( \alpha P(\ell) > 0 \), the last weak inequality becomes a strict inequality. \( \blacksquare \)

Proof of Lemma 1: If equilibrium \( P \) is strictly increasing on \((\ell_1, \ell_2)\), then it is invertible, so from \( P(\ell) \), the consumer infers that the loss is \( \ell \), which is her highest willingness to pay for repair. Hence, \( P(\ell) \leq \ell \) if the offer is accepted with a positive probability. Suppose that \( P(\ell) < \ell \) for some \( \ell \); this implies that the offer \( P(\ell) \) will be accepted with probability 1. Consider \( \ell' < \ell \). At loss \( \ell' \), the equilibrium profit is \([P(\ell') - C(\ell')]\alpha(P(\ell'))\). Because \( P \) is strictly increasing, we have \( P(\ell') < P(\ell) \). Let the expert deviate to price \( P(\ell) \) at loss \( \ell' \), the profit now becomes \( P(\ell) - C(\ell') > [P(\ell') - C(\ell')]\alpha(P(\ell')) \), which says that \( P(\ell') \) fails to be an equilibrium price, a contradiction. We conclude that \( P(\ell) = \ell \). \( \blacksquare \)

Proof of Proposition 2: We prove a more general result. Suppose that \( P(\ell) = \ell \) for \( \ell \in (\ell_1, \ell_2) \subseteq [\ell, \hat{\ell}] \). Then the consumer will be indifferent between accepting and rejecting the price offer. We construct an acceptance probability function against which the expert’s best response is indeed \( P(\ell) = \ell \).
Let \( \alpha(P(\ell)) = \alpha(\ell) \) be the equilibrium acceptance probability for \( \ell \in (\ell_1, \ell_2) \). (We have dispensed with the \( \tilde{\alpha}_P \) notation used in Proposition 1 because \( P \) is an identity map.) Define \( \pi(\ell'; \ell) = [\ell' - C(\ell)] \alpha(\ell') \), any \( \ell', \ell \in (\ell_1, \ell_2) \). For \( P(\ell) = \ell \) to be an equilibrium price, we must have

\[
[\ell' - C(\ell')]|\alpha(\ell')| \geq [\ell - C(\ell)]|\alpha(\ell)| \quad \text{and} \quad [\ell - C(\ell)]|\alpha(\ell)| \geq [\ell' - C(\ell)]|\alpha(\ell')|.
\]

In other words, in equilibrium, we have \( \pi(\ell; \ell) \geq \pi(\ell'; \ell) \) and \( \pi(\ell; \ell') \geq \pi(\ell; \ell') \).

Let \( \Pi(\ell) \equiv \pi(\ell; \ell) \). Then, we have

\[
\Pi(\ell) \geq \pi(\ell'; \ell) = [\ell' - C(\ell)] |\alpha(\ell')| = [\ell' - C(\ell')] |\alpha(\ell')| - [C(\ell) - C(\ell')] |\alpha(\ell')| = \Pi(\ell') - [C(\ell) - C(\ell')] |\alpha(\ell')|.
\]

By symmetry, we have

\[
\Pi(\ell') \geq \Pi(\ell) - [C(\ell') - C(\ell)] |\alpha(\ell)|.
\]

Combining and then dividing by \( \ell - \ell' \), we have

\[
\frac{-[C(\ell) - C(\ell')] |\alpha(\ell)|}{\ell - \ell'} \geq \frac{\Pi(\ell) - \Pi(\ell')}{\ell - \ell'} \geq \frac{-[C(\ell) - C(\ell')] |\alpha(\ell')|}{\ell - \ell'}.
\]

By taking limits, we have \( \Pi'(\ell) = -C'(\ell) |\alpha(\ell)| \).

Next, we use the definition of \( \Pi(\ell) = [\ell - C(\ell)] |\alpha(\ell)| \) to obtain

\[
\Pi'(\ell) = [\ell - C(\ell)] |\alpha'(\ell)| + [1 - C'(\ell)] |\alpha(\ell)|.
\]

Substituting by \( \Pi'(\ell) = -C'(\ell) |\alpha(\ell)| \), we have

\[
[\ell - C(\ell)] |\alpha'(\ell)| + [1 - C'(\ell)] |\alpha(\ell)| = -C'(\ell) |\alpha(\ell)|.
\]

Hence, we get a differential equation for \( |\alpha(\ell)| \)

\[
(\ell - C(\ell)) |\alpha'(\ell)| + |\alpha(\ell)| = 0 \quad \text{or} \quad \frac{d \ln |\alpha(\ell)|}{d \ell} = -\frac{1}{\ell - C(\ell)}.
\]

We solve the differential equation for \( |\alpha(\ell)| \):

\[
\ln |\alpha(\ell)| = - \int_{\ell_1}^{\ell} \frac{dx}{x - C(x)} + K \quad \text{some constant } K
\]

\[
|\alpha(\ell)| = K' \exp \left\{ - \int_{\ell_1}^{\ell} \frac{dx}{x - C(x)} \right\} \quad \text{some constant } K'
\]

\[
|\alpha(\ell)| = |\alpha(\ell_1)| \exp \left\{ - \int_{\ell_1}^{\ell} \frac{dx}{x - C(x)} \right\},
\]

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where we have relabelled $K'$ as $\alpha(\ell_1)$, the acceptance probability at $\ell_1$.

The separating equilibrium is the case when $\ell_1 = \tilde{\ell}$, and $\ell_2 = \hat{\ell}$. The consumer must accept the lowest price with probability 1 because $\ell_1 > C(\ell_1)$, so $\alpha(\tilde{\ell}) = 1$. We have obtained the equilibrium acceptance probability function in the Proposition.

For $\ell > \hat{\ell}$, the consumer’s maximum willingness to pay $\ell$ is less than the cost $C(\ell)$, but $P(\ell) \geq C(\ell)$ in an equilibrium. Hence, the consumer’s strategy is optimal, and the expert’s not offering a repair is also optimal. ■

**Proof of Proposition 3**: Given the expert’s strategy, the consumer infers that her loss is drawn from $[\tilde{\ell}, \ell^*]$ when the expert offers $P(\ell) = AL(\tilde{\ell}, \ell^*)$, so her expected loss is $AL(\tilde{\ell}, \ell^*)$. The consumer is indifferent between accepting and rejecting it, so it is a best response for her to accept the offer. If the consumer is recommended an off-the-equilibrium price, she continues to believe that her loss is drawn from $[\tilde{\ell}, \ell^*]$ according to $F(\ell)$. Hence, the consumer will accept any price lower than $AL(\tilde{\ell}, \ell^*)$ and reject any price higher than $AL(\tilde{\ell}, \ell^*)$.

Next, given that the consumer accepts prices below $AL(\tilde{\ell}, \ell^*)$ and and rejects all higher prices, the expert offers price $AL(\tilde{\ell}, \ell^*)$ if and only if $AL(\tilde{\ell}, \ell^*) \geq C(\ell)$ for consumer with loss $\ell$. From the definition of $\ell^*$, we have $AL(\tilde{\ell}, \ell^*) = C(\ell^*) < C(\ell)$ for any $\ell > \ell^*$. Hence offering $P(\ell) = AL(\tilde{\ell}, \ell^*)$ is the expert’s best response if and only if $\ell \leq \ell^*$. Given that the consumer rejects all prices higher than $AL(\tilde{\ell}, \ell^*)$, it is the expert’s best response to recommend no treatment for $\ell > \ell^*$. ■

**Proof of Proposition 4**: We begin with the pooling equilibrium in Proposition 3. Consider a price deviation $p' > AL(\tilde{\ell}, \ell^*)$. Define $\ell'$ by $p' = C(\ell')$. According to the Nonnegative Profit Principle, upon being offered price $p'$, the consumer believes that her loss is drawn from $[\tilde{\ell}, \ell']$ according to $F(\ell')$. Because $C(\ell^*) = AL(\tilde{\ell}, \ell^*) < p' = C(\ell')$, $\ell^* < \ell'$, and $AL(\tilde{\ell}, \ell^*) < C(\ell^*) = p'$. Therefore, the consumer will reject $p'$. Now, consider a price deviation $p' < AL(\tilde{\ell}, \ell^*)$. Such a price deviation is not profitable even if the consumer accepts $p'$ with probability one because the transaction happens at a lower price. We conclude that the pooling equilibrium in Proposition 3 satisfies the Nonnegative Profit Principle.

Consider a pooling equilibrium in Corollary 2 indexed by $\tilde{\ell} < \ell^*$. Suppose the expert offers an off-
equilibrium price \( p' = AL(\ell, \ell^*) - \varepsilon, \varepsilon > 0 \). For a small enough \( \varepsilon \), we apply the same argument in the first paragraph of this proof, so the consumer believes that her loss is drawn from \([\ell, \ell']\), where \( p' = C(\ell') \). Because \( C(\ell') = p' = AL(\ell, \ell^*) - \varepsilon < AL(\ell, \ell^*) = C(\ell^*), \ell' < \ell^* \), and \( \ell' \) is arbitrarily close to \( \ell^* \) when \( \varepsilon \) is sufficiently small. By the definition of \( \ell^* \), \( AL(\ell, \ell) \) crosses \( C(\ell) \) from above at \( \ell^* \). Therefore, \( AL(\ell, \ell^*) > C(\ell^*) = p' \). So, the consumer must accept \( p' \). The expert now has a profitable deviation from the lower price \( AL(\ell, \ell) \) to the higher price \( p' = AL(\ell, \ell^*) - \varepsilon \) for all \( \ell \in [\ell, \ell^*] \). We conclude that pooling equilibria in Corollary 2 do not satisfy the Nonnegative Profit Principle.

**Proof of Proposition 6**: Given the expert’s strategy, the consumer is always indifferent between accepting and rejecting the on-equilibrium price, so her strategy against the price offer at \( AL(\ell, \ell) \) and in \([\ell, \ell^*] \) is optimal. At any off-equilibrium price between \( AL(\ell, \ell) \) and \( \ell^* \), the consumer continues to believe that the loss is drawn from \([\ell, \ell^*] \), so strictly prefers to reject the (off-equilibrium) price. Upon a price offer above \( \ell^* \), the consumer believes that her loss is \( \ell^* \) and hence will reject the offer.

We next consider the expert’s optimal choices at each \( \ell \), given the consumer’s strategy in the Proposition. First, suppose that the expert has a consumer with loss \( \ell < \ell^* \). According to the strategy in the Proposition, his payoff is \( AL(\ell, \ell^*) - C(\ell) > 0 \). He earns less profit by lowering the price, and makes zero profit by offering a price between \( AL(\ell, \ell^*) \) and \( \ell^* \), according to the consumer’s strategy in the Proposition. Suppose that the expert offers a price \( \ell' > \ell^* \). In this case, his profit becomes \( [\ell' - C(\ell)] \alpha(\ell') = \pi(\ell'; \ell) \). Take the derivative

\[
\frac{\partial \pi(\ell'; \ell)}{\partial \ell'} = \alpha(\ell') + [\ell' - C(\ell)] \frac{\alpha'(\ell')}{\ell'}
\]

\[
= \alpha(\ell') \left[ 1 + \frac{\ell' - C(\ell)}{\ell'} \frac{\alpha'(\ell')}{\alpha(\ell')} \right]
\]

\[
< 0,
\]

where the third equality follows from (10) and the inequality follows from \( C(\ell) < C(\ell') \). It follows that the expert’s best deviation is at \( \ell^* \). If the expert does deviate to price \( \ell^* \), his expected profit is \([\ell - C(\ell)]\alpha(\ell^*) \). We
compare this candidate deviation profit with $AL(\tilde{\ell}, \tilde{\ell}) - C(\ell)$ as follows:

$$AL(\tilde{\ell}, \tilde{\ell}) - C(\ell) = \left[ \tilde{\ell} - C(\ell) \right] \alpha(\tilde{\ell})$$

$$= AL(\tilde{\ell}, \tilde{\ell}) - \tilde{\ell} \alpha(\tilde{\ell}) - C(\ell)[1 - \alpha(\tilde{\ell})]$$

$$= C(\tilde{\ell})[1 - \alpha(\tilde{\ell})] - C(\ell)[1 - \alpha(\tilde{\ell})]$$

$$= [C(\tilde{\ell}) - C(\ell)][1 - \alpha(\tilde{\ell})] \geq 0,$$

where the second equality follows from the definition of $\alpha(\tilde{\ell})$, and the last inequality follows from $C(\tilde{\ell}) > C(\ell)$. Hence, the expert’s maximum profit is from offering price $AL(\tilde{\ell}, \tilde{\ell})$ when $\ell$ is between $\underline{\ell}$ and $\tilde{\ell}$.

Second, suppose that the expert has a consumer with loss $\ell \geq \tilde{\ell}$. According to the Proposition, he should obtain an expected profit $[\ell - C(\ell)] \alpha(\ell) > 0$. The proof of Proposition 2 shows that he cannot earn more profit by deviating to any price above $\tilde{\ell}$. Clearly, the expert cannot profit from deviating to a price between $AL(\tilde{\ell}, \tilde{\ell})$ and $\tilde{\ell}$ (because it will be rejected). Because the consumer always accepts the price $AL(\tilde{\ell}, \tilde{\ell})$, it remains to show that he cannot earn more by deviating to price $AL(\tilde{\ell}, \tilde{\ell})$. The profit from this deviation is $AL(\ell, \tilde{\ell}) - C(\ell)$. From the definition of $\alpha(\tilde{\ell})$, we have $AL(\ell, \tilde{\ell}) - C(\tilde{\ell}) = \alpha(\tilde{\ell})[\ell - C(\tilde{\ell})]$, hence

$$AL(\ell, \tilde{\ell}) - C(\ell) < \alpha(\tilde{\ell})[\ell - C(\ell)]$$

$$\leq \alpha(\ell)[\ell - C(\ell)]$$,

where the first inequality is due to $C(\ell) > C(\tilde{\ell})$ and $\alpha(\tilde{\ell}) < 1$, and the second inequality is from the proof of Proposition 2. We have verified that the strategies form an equilibrium.

**Proof of Proposition 8:** Consider an equilibrium, let $P(\ell, c)$ be the equilibrium price when the expert has a type $(\ell, c)$ consumer. Let $\alpha(P(\ell, c)) > 0$ be the corresponding acceptance probability when the consumer is offered the price $P(\ell, c)$. In an equilibrium an expert with a type $(\ell, c)$ consumer chooses a price to obtain equilibrium profit $\max_{\ell', c'}\{\alpha(P(\ell', c'))[P(\ell', c') - c]\} \equiv \Pi(c)$, which is independent of loss $\ell$.

Consider equilibrium prices $P(\ell_1, c_1)$ and $P(\ell_2, c_2)$ for any $(\ell_1, c_1)$ and $(\ell_2, c_2) \in [\underline{\ell}, \tilde{\ell}] \times [c, \overline{c}]$. Hence, $\Pi(c_1) = \alpha(P(\ell_1, c_1))[P(\ell_1, c_1) - c_1]$ and $\Pi(c_2) = \alpha(P(\ell_2, c_2))[P(\ell_2, c_2) - c_2]$. Let $c_2 > c_1$

$$\Pi(c_2) \geq \alpha(P(\ell_1, c_1))[P(\ell_1, c_1) - c_2]$$

$$= \alpha(P(\ell_1, c_1))[P(\ell_1, c_1) - c_1 + c_1 - c_2] = \Pi(c_1) - (c_2 - c_1)\alpha(P(\ell_1, c_1))$$.

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Hence we have
\[
\frac{\Pi(c_2) - \Pi(c_1)}{(c_2 - c_1)} \geq -\alpha(P(\ell_1, c_1)).
\]

Analogously,
\[
\Pi(c_1) \geq \alpha(P(\ell_2, c_2))[P(\ell_2, c_2) - c_1]
\]
\[
= \alpha(P(\ell_2, c_2))[P(\ell_2, c_2) - c_2 + c_2 - c_1] = \Pi(c_2) + (c_2 - c_1)\alpha(P(\ell_2, c_2)).
\]

Hence we have
\[
-\alpha(P(\ell_2, c_2)) \geq \frac{\Pi(c_2) - \Pi(c_1)}{(c_2 - c_1)}.
\]

Combining we have
\[
-\alpha(P(\ell_2, c_2)) \geq \frac{\Pi(c_2) - \Pi(c_1)}{(c_2 - c_1)} \geq -\alpha(P(\ell_1, c_1)), \quad \text{for } c_1 < c_2, \text{ and for any } \ell_1 \text{ and } \ell_2. \quad (12)
\]

Now consider \( c_1 < c < c_2 \). We now have
\[
-\alpha(P(\ell_2, c_2)) \geq \frac{\Pi(c_2) - \Pi(c)}{(c_2 - c)} \geq -\alpha(P(\ell_2, c)), \quad \text{for } c < c_2, \text{ and for any } \ell \text{ and } \ell_2
\]
where we have let \( c \) take the role of \( c_1 \) in (12). Let \((\ell_2, c_2)\) converge to \((\ell, c)\), and we have the right-hand derivative satisfying
\[
\left(\frac{d\Pi(c)}{dc}\right)^+ \geq -\alpha(P(\ell, c)).
\]

Next, we have
\[
-\alpha(P(\ell, c)) \geq \frac{\Pi(c) - \Pi(c_1)}{(c - c_1)} \geq -\alpha(P(\ell_1, c_1)),
\]
\[
-\alpha(P(\ell, c)) \geq \frac{\Pi(c_1) - \Pi(c)}{(c_1 - c)} \geq -\alpha(P(\ell_1, c_1)), \quad \text{for } c_1 < c, \text{ and for any } \ell \text{ and } \ell_1,
\]
where we have let \( c \) take the role of \( c_2 \) in (12). Let \((\ell_1, c_1)\) converge to \((\ell, c)\), and we have the left-hand derivative satisfying
\[
-\alpha(P(\ell, c)) \geq \left(\frac{d\Pi(c)}{dc}\right)^-.
\]

Now \( \Pi(c) \) is the maximum of an affine linear function in \( c \), so it is convex, and almost everywhere differentiable. The left-hand and right-hand derivatives are equal, so we have
\[
\left(\frac{d\Pi(c)}{dc}\right)^- = \left(\frac{d\Pi(c)}{dc}\right)^+ \geq -\alpha(P(\ell, c)) \geq \left(\frac{d\Pi(c)}{dc}\right)^- \quad \text{for any } \ell.
\]

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The above inequalities show that $\alpha(P(\ell, c))$ is independent of $\ell$. Suppose the equilibrium price $P(\ell, c)$ varies in $\ell$ for a fixed $c$. Let $\ell$ denote the loss that leads to the highest equilibrium price $P(\ell, c)$. Because $\alpha(P(\ell, c))$ is constant in $\ell$, the expert will deviate to recommending $P(\ell, c)$ for all $\ell \neq \ell$. We conclude that $P$ must be independent of $\ell$. ■
References


