



PUBLIC REPORT, PRICE, AND QUALITY

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A monopolist produces a good with two qualities. All consumers have the same valuation of the first quality, but their valuations of the second vary, and are their private information. A public agency can verify qualities, and make credible reports to consumers. In Full Quality Report, the public agency reports both qualities. In Average Quality Report, it reports a weighted average of qualities. The equilibrium prices and qualities in Full Quality Report can be implemented by Average Quality Report. Equilibrium prices and qualities in Average Quality Report give higher consumer surplus than Full Quality Report. Bertrand competition under Average Quality Report yields first-best prices and qualities.

1. INTRODUCTION

Empowering consumers with product information is often regarded as a plausible way to solve the problems of experience goods, whose qualities are unknown before purchase. This information often comes in the form of ratings and rankings. For example, *US News and World Report (USNWR)* ranks colleges and graduate schools. The National Commission on Quality Assurance (NCQA), which works with the Centers for Medicare & Medicaid Services, compares health care providers, insurance plans, and nursing homes across the United States. *Consumer Reports* rates consumer products and services by indexes. Such information generally can be regarded as a form of quality index.

What is the impact of quality index on market outcomes? It is clear that firms react to quality indexes. For example, when the British hospital rating system assigned zero weight to women and children services, clinicians reported that hospitals reduced those qualities (Mannion et al., 2005). Stake (2006) discusses how the *USNWR* Law School Ranking has affected resource allocation in law schools. Obviously, consumers should be mindful about firms' reactions. Regulatory agencies and watchdog organizations

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should educate consumers on how quality indexes are to be used. A deeper question, however, is how quality indexes should be constructed. In this paper, we study optimal quality indexes.

We study quality index as a mechanism design problem. In our model, a public agency announces how quality indexes are to be constructed. A monopolist produces goods with multiple quality attributes, then chooses prices to offer them to consumers. The qualities of each offered good are observed by the public agency, and the indexes will be constructed according to the decided rules. The quality indexes are then disclosed to consumers. Finally, consumers decide whether to purchase at the set prices. We derive the optimal quality indexes for the maximization of consumer welfare.

Our main result is that the optimal quality indexes generate more consumer and social surpluses than when all quality information is disclosed to consumers. If the public agency releases all quality information, the firm freely chooses its second-degree price-quality discrimination strategies. If the public agency discloses only quality indexes, but not all information, the firm must choose its qualities optimally to achieve any index. Consumers therefore infer that only such qualities are equilibrium choices. This actually restrains the firm's price-quality discrimination strategies. Therefore, carefully chosen indexes can benefit consumers.

We call the above the *Average Quality Report* regime, where indexes are weighted averages of a firm's product qualities. When the public agency discloses all quality information, we call that the *Full Quality Report* regime. Qualities and prices under Average Quality Report generate higher consumer welfare than Full Quality Report. The result remains true if the public agency aims to maximize a weighted sum of consumer surplus and profits (with a lower weight on profits), or if the public agency bans second-degree price-quality discrimination.

The two regimes generate the same consumer surplus when the market is competitive. In this case, the public agency releasing all information will implement the first best, so Full Quality Report is optimal. The interesting observation is that the public agency can implement the first best by Average Quality Report. The public agency can construct the indexes to implement first-best quality strategic responses.

A provocative policy implication of our model is the regulation of quality certification. Average Quality Report imposes a constraint on the monopolist. Therefore, the monopolist would like to hire a third party to verify its product qualities to remove the constraint. Consumers are better off if the government disallows the monopolist to hire its own certification agent. The government then intervenes by observing the qualities and reporting optimal quality indexes. Our model provides a justification for why many reports are indexes and compiled by nonprofit and public organizations.

Many papers study how quality information influences the interaction between firms and consumers. In Matthews and Postlewaite (1985) and Schlee (1996), quality is one dimensional, exogenously given, but may be unknown to either the firm, consumers, or both. These papers compare full quality disclosure and nondisclosure. They show that quality information may harm consumers due to the seller responding with higher prices. By contrast, in this paper, the firm chooses both prices and multiple qualities. Here, nondisclosure will lead to the collapse of the market. Full disclosure allows the firm to perform second-degree price-quality discrimination. Partial disclosure in our model benefits consumers by restraining the firm's price and quality strategies.

Several recent papers analyze disclosure of multiple quality attributes. In Sun (2011) and Hotz and Xiao (2013), firms produce a good with both horizontal and vertical attributes. These papers show that neither a monopolist nor duopolists may disclose an

attribute even if it is costless to do so. In Bar-Isaac et al. (2012), consumers can verify one of two vertical attributes at a cost; when the verification cost of one attribute falls, the monopolist reacts by underinvesting in the other attribute, and welfare decreases. These papers conclude that mandatory full disclosure benefits consumers. By contrast, we compare Full Quality Report and Average Quality Report.

Another strand of the literature focuses on how an intermediary manipulates reports to maximize profit. In Albano and Lizzeri (2001), a monopolist's quality choice is only observed by a profit-maximizing agency. They show that the agency fully reports quality only if reporting fees can be made contingent on quality. When the agency charges a fixed fee, quality information is reported with noise. Lizzeri (1999) focuses on markets with adverse selection. He shows that a monopoly agency maximizes profit by withholding quality information. In our model, the reporting agency aims to maximize consumer surplus, but in equilibrium, withholding some information raises consumer welfare.

Quality reporting has become increasingly popular in health markets. Lu (2012) finds that after the implementation of the Nursing Home Quality Initiative, reported qualities have improved, but unreported ones have deteriorated. Glazer et al. (2008a) propose a method to mitigate the problem of unmeasured attributes in quality reports. Glazer et al. (2008b) show how user-satisfaction ratings can correct a provider's incentive to skim on high-cost consumers.

Glazer and McGuire (2006) first point out the use of a quality index to mitigate adverse selection in a Rothschild–Stiglitz class of models. They show that a single quality index can implement pooling equilibria in a competitive insurance market. In equilibrium, consumers with different preferences pay the same premium, but receive first-best qualities. We consider a monopolist's price and quality decisions. Equilibrium prices and qualities will screen consumer types, and equilibria are never first best.

In Glazer and McGuire (2007), a firm sells a single good with two qualities. These qualities may be unknown to consumers, but a quality index can be constructed. Given a demand function, the firm produces one set of qualities and charges a single price to clear the market. The paper shows situations where average information may or may not improve welfare. By contrast we specify consumer preferences, allow for price-quality screening, and derive optimal quality weight functions for maximizing either consumer surplus or social surplus in monopoly and competition regimes.

In Section 2, we present the model. We characterize equilibria under Full Quality Report in Section 3. Next, in Section 4, we analyze the monopolist's choice of qualities under Average Quality Report. Section 5 derives the equilibrium weights under Average Quality Report, and includes two subsections to examine robustness. In Subsection 5.3, the public agency maximizes social surplus instead of consumer surplus, whereas in Subsection 5.4, the monopolist cannot screen consumers. In Section 6, we let many firms compete in the market. Some final remarks are in Section 7. The Appendix contains all proofs.

2. THE MODEL

2.1 A MONOPOLIST AND CONSUMERS

A monopolist produces a good to sell to consumers. The good consists of two qualities, which we call Quality 1 and Quality 2. The nonnegative vector (q, r) describes the levels of these qualities. We use the term good (q, r) to mean a good with q units of Quality 1

and r units of Quality 2. The monopolist incurs a unit cost $c(q) + d(r)$ for producing good (q, r) . The cost functions c and d are both strictly increasing and strictly convex, with $c(0) = d(0) = c'(0) = d'(0) = 0$. We further assume that both c and d are three-times differentiable, with c''' and d''' nonnegative, and c''/c' and d''/d' nonincreasing; the derivative of c''/c' is $[c'c''' - (c'')^2]/(c')^2$, so the third-order derivative c''' cannot be too big, and similarly for d''' . There are no fixed costs. If the monopolist sells a unit of good (q, r) at a price p , it makes a profit $p - c(q) - d(r)$.

There is a continuum of consumers, with the total mass normalized at 1. Each consumer buys, at most, one unit of the good. A consumer's per-unit valuations of the good's Quality 1 and Quality 2 are given by two parameters, respectively, u and v . The parameter $u > 0$ is the same for all consumers. The parameter v is random, and follows distribution F and density f on the positive support $[\underline{v}, \bar{v}]$; we assume that $f > 0$. Consumers' valuations of Quality 2 are independent. We sometimes call a consumer with valuation parameter v a type- v consumer or consumer v . If a type- v consumer purchases good (q, r) at price p , her utility is $uq + vr - p$. If a consumer does not purchase, she obtains a utility 0.

Although there are two qualities, consumers' preferences are only heterogenous in Quality 2. We use this setup for several reasons. First, our model is tractable. If consumers' preferences were heterogenous in both Qualities 1 and 2, the pricing problem would become much more difficult. This will distract us from our study of effects of quality reports on market outcomes. Second, our assumption is plausible in many applications. For example, in the nursing home market, Quality 1 can be general nursing care, whereas Quality 2 can be specialty nursing care (such as care specific for patients with heart, cognitive, or mobility problems); in the education market, Quality 1 can be the general resources at a college such as library and athletic facility, whereas Quality 2 refers to more specific attributes such as educational and research achievements of faculty.

Third, we can appeal to a more general interpretation. Suppose that, in fact, the good has a single quality, and this is Quality 2, for which consumer preferences are heterogenous. Now consider a second good, which has a single quality, namely Quality 1. Now Quality 1 may be a standardized attribute for which consumers have homogenous preferences. Our model would correspond to this setup when the firm offers these two goods as a bundle. A regulator can mandate that the two goods must be sold together, and implement the quality-report policies studied here. The issue about a quality report is whether the quality information of these two goods should or should not be disclosed together, and, if so, in what form.

2.2 INFORMATION AND QUALITY REPORT

A consumer's valuation of Quality 2 is her private information. Qualities may not be directly observed by consumers. However, a public agency can find out about these qualities. Alternatively, the monopolist may be required to supply quality information to a public agency. The public agency can verify and credibly report information about these qualities. Consumers can obtain quality information only through the public agency.

The public agency commits to a quality-report policy before the monopolist makes its quality and pricing decisions. We consider two report regimes. In Full Quality Report, the public agency commits to reporting both Qualities 1 and 2 of each good the monopolist produces. In Average Quality Report, the public agency commits to reporting a linearly weighted average of qualities of each good that the monopolist produces.

Because the consumer's valuation of Quality 2 varies, the monopolist may want to use a price-quality menu to screen consumers. From the revelation principle, the firm only needs to consider price-quality menus indexed by consumer types: $\{p(v), q(v), r(v)\}$, $v \in [\underline{v}, \bar{v}]$. In fact, we specify that the monopolist must produce such a menu, although it is allowed to set identical values for different items in the menu. In Full Quality Report, this entire menu will be reported by the public agency. In Average Quality Report, the regulator chooses a specific weight to compute the average quality of a good produced by the monopolist. The weights are represented by a function $\theta : [\underline{v}, \bar{v}] \rightarrow [0, 1]$. At v , the weights on Qualities 1 and 2 are $\theta(v)$ and $1 - \theta(v)$, respectively, and $0 < \theta(v) < 1$. For good $(q(v), r(v))$, the weighted-average quality is $A(v) \equiv \theta(v)q(v) + (1 - \theta(v))r(v)$. The menu $\{p(v), A(v)\}$, $v \in [\underline{v}, \bar{v}]$, will be reported to consumers. In each report regime, a consumer picks an item from the menu.

Weights in Average Quality Report are allowed to vary according to different goods produced by the monopolist. Given that the monopolist uses a price-quality menu, it is formally unsatisfactory to limit the quality report to a single weight for all items in the menu. Besides, variable weights in quality reports are common. In the health care industry, NCQA uses different weights to evaluate commercial Preferred Provider Organization (PPO) and Health Maintenance Organization (HMO) plans offered by any insurer in California. PPO and HMO plans target different consumers, and have different qualities in general and specialty cares. *USNWR* also uses different weights to rank national and regional colleges (details at opa.ca.gov and usnews.com/education).

2.3 EXTENSIVE FORMS

The extensive form for Average Quality Report is as follows:

- Stage 0: Nature determines v according to distribution function F , and an independent realization of v is made known to each consumer.
- Stage 1: The public agency chooses a weight function $\theta : [\underline{v}, \bar{v}] \rightarrow [0, 1]$ for reporting the average quality, and the function is public information.
- Stage 2: The monopolist chooses a menu of prices and qualities, $\{p(v), q(v), r(v)\}$, $v \in [\underline{v}, \bar{v}]$.
- Stage 3: After observing the menu, the public agency constructs the average quality $A(v) \equiv \theta(v)q(v) + (1 - \theta(v))r(v)$ of each good in the menu, and then $\{p(v), A(v)\}$, $v \in [\underline{v}, \bar{v}]$ is reported to consumers.
- Stage 4: Each consumer decides to buy an item from the menu $\{p(v), A(v)\}$, $v \in [\underline{v}, \bar{v}]$, or buy nothing.

We do not write down the extensive form for Full Quality Report. It can be obtained straightforwardly by replacing $\{p(v), A(v)\}$, $v \in [\underline{v}, \bar{v}]$ in the above by $\{p(v), q(v), r(v)\}$, $v \in [\underline{v}, \bar{v}]$. We study perfect Bayesian equilibria of both games.

The monopolist maximizes expected profit, and each consumer maximizes her utility. In the Average Quality Report game, the public agency chooses a weight function θ to maximize aggregate consumer utility. Consumers form their beliefs about Qualities 1 and 2 upon observing the reported menus. We will focus on weight functions θ that induce the monopolist to offer incentive-compatible menus. That is, we consider weight functions θ such that, in the continuation equilibrium after Stage 1, the firm offers a menu $\{p(v), q(v), r(v)\}$, $v \in [\underline{v}, \bar{v}]$, the public agency reports $\{p(v), A(v)\}$, $v \in [\underline{v}, \bar{v}]$, where $A(v) \equiv \theta(v)q(v) + (1 - \theta(v))r(v)$, and consumer v picks item $(p(v), A(v))$.

In the Full Quality Report game, consumers have perfect information about qualities. From the revelation principle, we only need to let the monopolist offer menus $\{p(v), q(v), r(v)\}$, $v \in [\underline{v}, \bar{v}]$, that are incentive compatible, so consumer v in equilibrium chooses item $(p(v), q(v), r(v))$ from the offered menu.

2.4 FULL INFORMATION BENCHMARK

We now write down the full information benchmark. Here, consumers' valuations are public information, and the firm's quality choices can be perfectly observed. The monopolist can perfectly price-quality discriminate consumers to extract all surplus. The total surplus of good (q, r) for type- v is $uq + vr - c(q) - d(r)$. The first-best qualities, $(q^{FB}, r^{FB}(v))$, are given by $u = c'(q^{FB})$ and $v = d'(r^{FB}(v))$, respectively. The first-best Quality 1, q^{FB} , depends only on u , which is the same for all consumers. The first-best Quality 2, $r^{FB}(v)$, depends only on the realization of v for a consumer. Under full information, the monopolist offers the first-best qualities to consumer v at a price $p(v) = uq^{FB} + vr^{FB}(v)$. The equilibrium allocation is first best, but each consumer obtains no surplus from the purchase. The monopolist makes a profit $uq^{FB} + vr^{FB}(v) - c(q^{FB}) - d(r^{FB}(v))$ from each consumer.

Finally, we note that if no reporting of quality information is feasible, the market collapses. The monopolist cannot convey any information about qualities, and all consumers refuse to buy at any positive price. At a zero price, the firm does not produce.

3. FULL QUALITY REPORT

In this section, we present the equilibria in the Full Quality Report game. In Stage 3, the public agency reports both Qualities 1 and 2. Consumers' valuations on Quality 2 remain their private information. By the revelation principle, the monopolist offers an incentive-compatible and individually rational menu. A menu $\{p(v), q(v), r(v)\}$, $v \in [\underline{v}, \bar{v}]$, is incentive compatible if $v = \operatorname{argmax}_{v'} uq(v') + vr(v') - p(v')$, $v, v' \in [\underline{v}, \bar{v}]$, and is individually rational if $uq(v) + vr(v) - p(v) \geq 0$, $v \in [\underline{v}, \bar{v}]$. The equilibrium is an incentive-compatible and individually rational menu that maximizes expected profit $\int_{\underline{v}}^{\bar{v}} [p(v) - c(q(v)) - d(r(v))] f(v)dv$.

The following derivation is standard in the screening literature (see, for example, Section 2 of Laffont and Martimont, 2001). Let $W(v) \equiv \max_{v'} uq(v') + vr(v') - p(v')$, $v, v' \in [\underline{v}, \bar{v}]$. Applying the envelope theorem on W , simplifying, applying integration by parts, and substituting for p in the monopolist's profit function, we can obtain the equilibrium by finding \tilde{q} and an increasing function $\tilde{r}(v)$ that maximize

$$\int_{\underline{v}}^{\bar{v}} \left[uq + \left(v - \frac{1 - F(v)}{f(v)} \right) r(v) - c(q) - d(r(v)) \right] f(v)dv.$$

We assume the common *monotone hazard rate* condition,

$$\frac{1 - F(v)}{f(v)} \text{ decreasing in } v.$$

Furthermore, for ease of exposition, we assume that $\frac{1 - F(v)}{f(v)} < v$ for all v , but will note how results are to be adjusted when this inequality is violated for some v . The following is presented with the standard proof omitted:

PROPOSITION 1: *In the Full Quality Report game, under monotone hazard rate, the equilibrium \tilde{q} and $\tilde{r}(v)$ are given by*

$$u = c'(\tilde{q}), \tag{1}$$

$$v - \frac{1 - F(v)}{f(v)} = d'(\tilde{r}(v)). \tag{2}$$

Consumer v 's equilibrium utility is given by $\tilde{W}(v) \equiv \int_v^v \tilde{r}(x)dx$. The equilibrium price of good $(\tilde{q}, \tilde{r}(v))$ is

$$\tilde{p}(v) \equiv u\tilde{q} + v\tilde{r}(v) - \int_v^v \tilde{r}(x)dx. \tag{3}$$

The monopolist's decisions on the two qualities are separable. Consumers' valuation on Quality 1 is fixed at u , so, for each v , the firm chooses the first-best Quality 1, $\tilde{q} = q^{FB}$, and extracts all surplus $u\tilde{q} - c(\tilde{q})$ from Quality 1. For Quality 2, the firm misses information of consumers' valuations v . The Myerson virtual valuation, $\frac{1-F(v)}{f(v)}$, is subtracted from v to adjust for this missing information. Proposition 1 records the marginal conditions.¹ Compared to the full information benchmark, the equilibrium Quality 2 $\tilde{r}(v)$ is lower, but consumer v earns information rent equal to $\int_v^v \tilde{r}(x)dx$. Reporting full quality information prevents the market from collapsing but also lets the monopolist practice standard second-degree price-quality discrimination. In the next two sections, we consider Average Quality Report, and will show that consumers benefit from less information about product qualities.

4. AVERAGE QUALITY REPORT

We begin by characterizing the firm's choices of Qualities 1 and 2 to achieve any average quality.

4.1 COST-MINIMIZING QUALITIES

Consider an average quality $A(v)$ reported by the public agency in Stage 3. Consumers have to infer which Qualities 1 and 2 give rise to that average. Using this inference consumers decide whether they will purchase or not. Many combinations of Qualities 1 and 2 can give rise to an average. Each of these combinations will generate the same inference. Our first result says that Qualities 1 and 2 must be chosen to achieve this average at the minimum cost.

LEMMA 1: *Given a weight function θ chosen by the public agency in Stage 1, in equilibrium the monopolist chooses Qualities 1 and 2, $\hat{q}(v)$ and $\hat{r}(v)$, that minimize $c(q(v)) + d(r(v))$ subject to $\theta(v)q(v) + (1 - \theta(v))r(v) = A(v)$, for any average quality $A(v)$ to be reported in Stage 3. Hence, $(\hat{q}(v), \hat{r}(v))$ satisfy*

1. If $v < \frac{1-F(v)}{f(v)}$ for some values of v , then (2) will fail to hold as an equality at $v < v^*$ where $\frac{1-F(v^*)}{f(v^*)} = v^*$. This implies that $\tilde{r}(v) = 0$ for $v < v^*$.

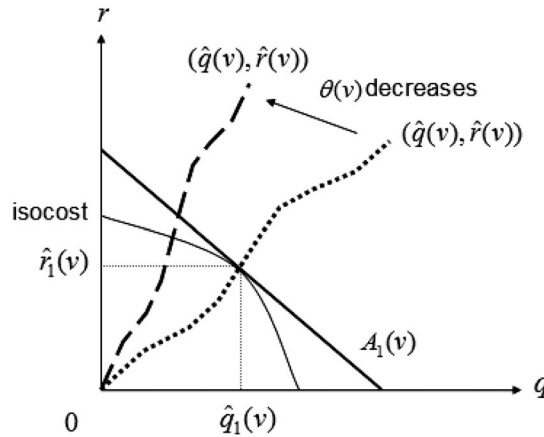


FIGURE 1. COST-MINIMIZING QUALITY PAIRS

$$\frac{c'(\hat{q}(v))}{d'(\hat{r}(v))} = \frac{\theta(v)}{1 - \theta(v)}, \tag{4}$$

$$A(v) = \theta(v)\hat{q}(v) + (1 - \theta(v))\hat{r}(v). \tag{5}$$

When choosing qualities, the monopolist is, in fact, choosing the average which the public agency will report in Stage 3. If an item $(p(v), A(v))$ is to be purchased by consumers, the monopolist raises profits by choosing qualities that minimize the cost of achieving average quality $A(v) = \theta(v)q(v) + (1 - \theta(v))r(v)$. Equation (4) is the familiar optimality condition: the ratio of the marginal costs, c'/d' , should be equal to the ratio of quality weights contributing to the average, $\theta(v)/[1 - \theta(v)]$. For any average quality, in equilibrium, consumers believe that only cost-minimizing qualities will be chosen to achieve that average. We emphasize that Lemma 1 is independent of the distribution of consumer types, so it is valid even when consumers have heterogenous valuations on both Qualities 1 and 2.

For any given $A(v)$ and $\theta(v)$, the monopolist's choice of $(\hat{q}(v), \hat{r}(v))$ is the solution of a standard cost-minimizing problem. Figure 1 illustrates the solution. The isocost line contains $(q(v), r(v))$ pairs that have the same total cost. From the convexity of the c and d functions, the isocost line is concave to the origin. The iso-average quality line contains $(q(v), r(v))$ pairs that achieve the same average quality, and is a negatively sloped straight line. The cost-minimizing qualities for achieving $A_1(v)$ are at the tangency point $(\hat{q}_1(v), \hat{r}_1(v))$. Holding $\theta(v)$ fixed, we can trace out all cost-minimizing quality pairs as $A(v)$ changes. This is the dotted line in Figure 1. For a fixed $A(v)$, the slope of the isoaverage quality line is $-\theta(v)/[1 - \theta(v)]$, which is decreasing in $\theta(v)$. If $\theta(v)$ decreases, the isoaverage quality line becomes flatter, and the entire dotted line will pivot in an anticlockwise direction to become the dashed line.

4.2 EQUILIBRIUM QUALITIES

By Lemma 1, if average quality $A(v)$ is reported in Stage 3, consumers must believe that that average quality comes from good $(\hat{q}(v), \hat{r}(v))$, implicitly defined by equations (4) and

(5). Hence, from the item $(p(v'), A(v'))$, consumer v obtains a utility $u\hat{q}(v') + v\hat{r}(v') - p(v')$. A menu $\{p(v), A(v)\}$ is then equivalent to $\{p(v), \hat{q}(v), \hat{r}(v)\}$, with $(\hat{q}(v), \hat{r}(v))$ given by (4) and (5). A menu is incentive compatible if $v = \operatorname{argmax}_{v'} u\hat{q}(v') + v\hat{r}(v') - p(v')$, $v, v' \in [\underline{v}, \bar{v}]$, and individually rational if $u\hat{q}(v) + v\hat{r}(v) - p(v) \geq 0$.

As in Full Quality Report, we can apply the envelope condition to simplify the incentive-compatibility and individual-rationality constraints. Again, the Myerson adjustment has to be applied to v to account for consumers' private information. After the usual simplification via integration by parts, the monopolist will choose $\hat{q}(v)$ and $\hat{r}(v)$ to maximize

$$\int_{\underline{v}}^{\bar{v}} \left[u\hat{q}(v) + \left(v - \frac{1 - F(v)}{f(v)} \right) \hat{r}(v) - c(\hat{q}(v)) - d(\hat{r}(v)) \right] f(v)dv. \tag{6}$$

Incentive compatibility also requires that $\hat{r}(v)$ be an increasing function. (Details are in the Appendix.) Also, for each v , there is the new constraint (4) on $\hat{q}(v)$ and $\hat{r}(v)$ according to Lemma 1. The Lagrangian of the constrained maximization problem is

$$\int_{\underline{v}}^{\bar{v}} \left[u\hat{q}(v) + \left(v - \frac{1 - F(v)}{f(v)} \right) \hat{r}(v) - c(\hat{q}(v)) - d(\hat{r}(v)) - \lambda(v) \left((1 - \theta(v))c'(\hat{q}(v)) - \theta(v)d'(\hat{r}(v)) \right) \right] f(v)dv, \tag{7}$$

where $\lambda(v)$ is the multiplier at v .

PROPOSITION 2: For a given weight function θ that induces the monopolist to offer incentive-compatible menus, under monotone hazard rate equilibrium Qualities 1 and 2, $\hat{q}(v)$ and $\hat{r}(v)$, in the Average Quality Report game are given by

$$\left[u - c'(\hat{q}(v)) \right] \frac{\theta(v)}{c''(\hat{q}(v))} + \left[v - \frac{1 - F(v)}{f(v)} - d'(\hat{r}(v)) \right] \frac{1 - \theta(v)}{d''(\hat{r}(v))} = 0, \tag{8}$$

$$\frac{c'(\hat{q}(v))}{d'(\hat{r}(v))} - \frac{\theta(v)}{1 - \theta(v)} = 0. \tag{9}$$

Consumer v 's equilibrium utility is $\widehat{W}(v) \equiv \int_{\underline{v}}^v \hat{r}(x)dx$. The equilibrium price of an item with average quality $A(v)$ is

$$p(v) \equiv u\hat{q}(v) + v\hat{r}(v) - \int_{\underline{v}}^v \hat{r}(x)dx. \tag{10}$$

Proposition 2 presents two conditions, (8) and (9), for the equilibrium qualities. The two terms in square brackets in (8) have the forms of the equilibrium quality characterizations, (1) and (2), in Proposition 1 of Full Quality Report. The sum of these two square-bracketed terms multiplied by terms that are dependent on $\theta(v)$ must be equal to 0. For most weight functions θ , the qualities $\hat{q}(v)$ and $\hat{r}(v)$ are different from those equilibrium qualities $\tilde{q}(v)$ and $\tilde{r}(v)$ in Full Quality Report. Furthermore, even

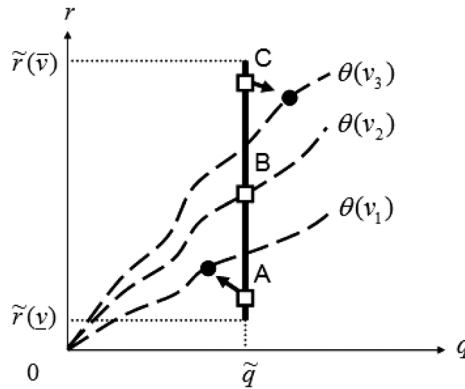


FIGURE 2. EQUILIBRIUM QUALITY PAIRS

though consumers have identical valuations on Quality 1, each consumer v may be offered a different level of that quality.²

The monopolist's choices of Qualities 1 and 2 are no longer separable. Although the marginal profit from increasing Qualities 1 and 2 remains at $u - c'(q)$ and $v - \frac{1-F(v)}{f(v)} - d'(r)$, the monopolist cannot convince consumers that these qualities are provided independently. Consumers only observe the average quality, and believe that Qualities 1 and 2 are jointly chosen to minimize the cost to achieve this average.

How does the cost-minimization belief restrict the monopolist's quality choices? Consider the first-order derivatives of the Lagrangian (7) with respect to q and r :

$$u - c'(\hat{q}(v)) - \lambda(v)(1 - \theta(v))c''(\hat{q}(v)), \tag{11}$$

$$v - \frac{1 - F(v)}{f(v)} - d'(\hat{r}(v)) + \lambda(v)\theta(v)d''(\hat{r}(v)). \tag{12}$$

The last terms in these expressions, $-\lambda(v)(1 - \theta(v))c''(\hat{q}(v))$ and $\lambda(v)\theta(v)d''(\hat{r}(v))$, are due to the constraint (4), and cause the monopolist to deviate from the equilibrium in the Full Quality Report game.

Figure 2 illustrates these distortions. In Figure 2, we have plotted the equilibrium qualities under Full Quality Report by the vertical, solid line. The equilibrium Quality 1 is constant at \tilde{q} . The equilibrium Quality 2 increases from $\tilde{r}(v)$ to $\tilde{r}(\bar{v})$ as v increases from \underline{v} to \bar{v} ; this range is depicted by the thick vertical line. Consider $v = v_2$, the corresponding average quality report weight being $\theta(v_2)$. Point B denotes the equilibrium qualities in the Full Quality Report game at $v = v_2$. The positively sloped graph passing through point B is the set of Qualities 1 and 2 that minimize the monopolist's cost for various average qualities, given weight $\theta(v_2)$ on Quality 1. Point B also happens to be qualities that minimize costs. The monopolist chooses the same combination of qualities at B under Average Quality Report, and the multiplier $\lambda(v)$ at $v = v_2$ becomes 0; equilibrium qualities at B across both games happen to coincide.

2. If $v < \frac{1-F(v)}{f(v)}$ for some v , then for some weight function θ there may be no positive value of r to make the first-order derivative (12) vanish. In this case, the function θ implements a zero Quality 2 for some values of v .

Now consider point A , which denotes the equilibrium qualities in the Full Quality Report game at $v = v_1$. The dashed curve right above A graphs the qualities that minimize costs when the quality weight is $\theta(v_1)$. Nevertheless, qualities at A do not minimize costs. Under Average Quality Report, the monopolist distorts the qualities at A in a northwestern direction, reducing Quality 1 and raising Quality 2. The equilibrium is the solid point to the northwest of A . Here, the multiplier $\lambda(v)$ at $v = v_1$ is positive, so there is underprovision of Quality 1 and overprovision of Quality 2 compared to the Full Quality Report game.

Finally, consider point C . This denotes the equilibrium qualities in the Full Quality Report game at $v = v_3$. The dashed curve right below C graphs the qualities that minimize costs when the quality weight is $\theta(v_3)$. Again, qualities at C do not minimize costs. Under Average Quality Report, the monopolist distorts the qualities at C in a southeastern direction, raising Quality 1 and lowering Quality 2. The equilibrium is the solid point to the southeast of C . Here, the multiplier $\lambda(v)$ at $v = v_3$ is negative, so there is overprovision of Quality 1 and underprovision of Quality 2 compared to the Full Quality Report game.

5. REPORT WEIGHTS AND CONSUMER UTILITIES

We now characterize the public agency's choice of the weight function θ in Stage 1. We begin by showing that the public agency can implement Full Quality Report equilibrium outcomes by an appropriate choice of the weight function.

5.1 IMPLEMENTING FULL QUALITY REPORT OUTCOMES

Under Average Quality Report, consumers receive partial quality information from the public agency, and their beliefs about qualities are restricted to those in Lemma 1. However, the following lemma says that the public agency can choose the weight function to nullify consumers' cost-minimization belief restrictions.

LEMMA 2: *Under Average Quality Report, the public agency can implement the equilibrium qualities in the Full Quality Report game by the weight function*

$$\tilde{\theta}(v) = \frac{c'(\tilde{q})}{c'(\tilde{q}) + d'(\tilde{r}(v))} \tag{13}$$

where \tilde{q} and $\tilde{r}(v)$ are the equilibrium qualities in Full Quality Report, and given by (1) and (2), respectively.

In Average Quality Report, the firm must choose cost-minimizing quality pairs, but the public agency can assign weights on qualities according to the marginal costs of \tilde{q} and $\tilde{r}(v)$, as in (13), effectively neutralizing the constraints. Given $\tilde{\theta}(v)$, the $(\tilde{q}, \tilde{r}(v))$ pair minimizes the cost of achieving average quality $\tilde{A}(v) = \tilde{\theta}(v)\tilde{q} + (1 - \tilde{\theta}(v))\tilde{r}(v)$ at each v . Upon observing $\tilde{A}(v)$, consumers must infer that the average quality comes from good $(\tilde{q}, \tilde{r}(v))$. But then this is the best the monopolist can ever hope to achieve, so choosing $(\tilde{q}, \tilde{r}(v))$ is an equilibrium.

Figure 3 illustrates how the result applies to three items in the equilibrium menu. As in Figure 2, the solid line in Figure 3 plots the equilibrium qualities under Full Quality Report. Points A , B , and C are the Full Quality Report equilibrium qualities for consumers v_1 , v_2 , and v_3 , respectively, where $v_1 < v_2 < v_3$. The three dashed lines in Figure 3 graph the cost-minimizing quality pairs that the monopolist may offer to

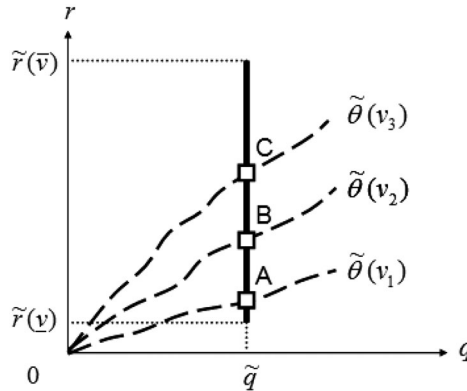


FIGURE 3. IMPLEMENTING FULL QUALITY REPORT QUALITIES

consumers v_1 , v_2 , and v_3 , respectively, under Average Quality Report. For $\tilde{\theta}$ defined by (13), the three dashed lines must pass through the Full Quality Report equilibrium qualities, points A, B, and C. The constraints due to cost minimization do not bind.

5.2 QUALITY WEIGHTS TO MAXIMIZE CONSUMER UTILITY

Lemma 2 implies that consumers cannot become worse off if the public agency switches from Full Quality Report to Average Quality Report. The public agency can always implement the Full Quality Report equilibrium qualities $\{\tilde{p}(v), \tilde{q}, \tilde{r}(v)\}$, $v \in [\underline{v}, \bar{v}]$. In fact, the public agency can do more. Under Full Quality Report, the monopolist leaves consumers only their information rent, $\int_{\underline{v}}^{\bar{v}} (\int_{\underline{v}}^v \tilde{r}(x)dx) f(v)dv$. The public agency can reduce rent extraction by a reduction of the report weights from $\tilde{\theta}(v)$ in (13), which will raise Quality 2 from $\tilde{r}(v)$.

PROPOSITION 3: *In the equilibrium of the Average Quality Report game, $\hat{r}(v)$ is higher than $\tilde{r}(v)$ at every v , whereas $\hat{\theta}(v)$ is smaller than $\tilde{\theta}(v)$ at every v . Each consumer (except type- \underline{v}) obtains a higher equilibrium utility in the Average Quality Report game than in the Full Quality Report game. At every v , the Average Quality Report equilibrium weight and qualities are characterized by (8), (9) and*

$$\hat{\theta}(v) = \frac{\left[d'(\hat{r}(v)) - \left(v - \frac{1-F(v)}{f(v)} \right) \right]}{\left[d'(\hat{r}(v)) - \left(v - \frac{1-F(v)}{f(v)} \right) \right] + d''(\hat{r}(v)) \left[\frac{c''(\hat{q}(v))}{c'(\hat{q}(v))} - \frac{c'''(\hat{q}(v))}{c''(\hat{q}(v))} \right]^{-1}}. \tag{14}$$

In Proposition 3, the equilibrium Quality 2 has been increased from the equilibrium in the Full Quality Report regime. This is achieved by reducing the report weight on Quality 1. Because the firm is missing information on consumers' valuation v , consumers earn information rent, which is increasing in Quality 2. Consumers' valuation of Quality 1 is fixed, and the firm leaves no rent to consumers from Quality 1. From the equilibrium under Full Quality Report, further changes in Quality 1 have no consequence for consumer welfare, but an increase in Quality 2 raises it.

Lemma 1 constrains the way in which Qualities 1 and 2 change together: the ratio of the marginal costs, c'/d' , must be equal to the ratio of quality weights, $\theta(v)/[1 - \theta(v)]$.

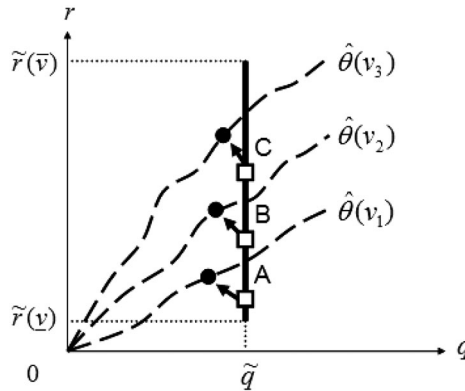


FIGURE 4. EQUILIBRIUM QUALITIES

To see the effect of changing the quality weight function from $\tilde{\theta}$ to $\hat{\theta}$ on equilibrium qualities, we rearrange (14) to get

$$v - \underbrace{\frac{1 - F(v)}{f(v)}}_{\text{Screening distortion}} = d'(\hat{r}(v)) - \underbrace{\left[d''(\hat{r}(v)) \left(\frac{c''(\hat{q}(v))}{c'(\hat{q}(v))} - \frac{c'''(\hat{q}(v))}{c''(\hat{q}(v))} \right)^{-1} \frac{\hat{\theta}(v)}{1 - \hat{\theta}(v)} \right]}_{\text{Cost-minimization distortion}}. \tag{15}$$

Here, on the left-hand side, we have the consumer’s virtual valuation—the Myerson adjustment accounting for the distortion from the firm’s attempt to screen different consumer types. On the right-hand side, we first have the marginal cost of Quality 2, and then must take into account the distortion due to cost minimization. This distortion depends on the slopes and the curvatures of the marginal cost functions, c' and d' . This is because the firm must equate the ratio of the marginal costs and the ratio of quality weights to minimize the cost of producing average quality $\hat{A}(v) = \hat{\theta}(v)\hat{q}(v) + (1 - \hat{\theta}(v))\hat{r}(v)$. Because c''/c' is nonincreasing, the square-bracketed term in (15) is strictly positive. By lowering the weight at v from $\tilde{\theta}(v)$ to $\hat{\theta}(v)$, the public agency lowers the relative cost of using Quality 2 to achieve every level of average quality. This implements a higher level of Quality 2 at every v .

Figure 4 illustrates the equilibrium choices. The diagram builds on the presentation in Figure 3. Points A , B , and C are the quality pairs for consumers v_1 , v_2 , and v_3 under the weight function $\tilde{\theta}$. The three dashed lines represent the cost-minimizing quality pairs under equilibrium weight function $\hat{\theta}$. In the equilibrium of the Average Quality Report game, the monopolist distorts the qualities at each v in a northwestern direction, lowering Quality 1 and raising Quality 2. The three solid points on the dashed lines are the equilibrium qualities obtained by consumers v_1 , v_2 , and v_3 , respectively. Quality 2 has increased, raising consumer welfare, whereas Quality 1 has decreased, but it has no effect on consumer welfare. As the three solid points show, the equilibrium qualities $\hat{q}(v)$ and $\hat{r}(v)$ are both increasing in v . The equilibrium menu $\{\hat{p}(v), \hat{q}(v), \hat{r}(v)\}$, $v \in [\underline{v}, \bar{v}]$, induced by $\hat{\theta}$ is incentive compatible.

Proposition 3 yields a policy implication. The firm earns less profit when the public agency uses Average Quality Report. If the firm could find a credible intermediary to verify its product qualities, it would be able to raise profit by implementing the equilibrium under Full Quality Report. By banning a firm’s own quality certification,

the public agency reinforces the equilibrium in Average Quality Report. Indeed, quality information should be provided by public and nonprofit organizations.

5.3 QUALITY WEIGHTS FOR GENERAL WELFARE

In the previous subsection, we have shown that the public agency can implement a higher consumer surplus in the Average Quality Report game than the Full Quality Report game. Can Average Quality Report also implement a higher general welfare index that is a weighted sum of consumer surplus and profit?

By Proposition 2, consumer surplus under Average Quality Report is $\int_{\underline{v}}^{\bar{v}} (\int_{\underline{v}}^v \hat{r}(x) dx) f(v) dv$. Using integration by parts, we can rewrite consumer surplus as $\int_{\underline{v}}^{\bar{v}} [1 - F(v)] \hat{r}(v) dv$. A general welfare index, $I(\alpha)$, can be written as

$$I(\alpha) \equiv \int_{\underline{v}}^{\bar{v}} \left\{ \alpha \left[\frac{1 - F(v)}{f(v)} \hat{r}(v) \right] + (1 - \alpha) \left[u\hat{q}(v) + \left(v - \frac{1 - F(v)}{f(v)} \right) \hat{r}(v) - c(\hat{q}(v)) - d(\hat{r}(v)) \right] \right\} f(v) dv, \tag{16}$$

where $\hat{q}(v)$ and $\hat{r}(v)$ are given by (8) and (9), and where α , $1/2 \leq \alpha \leq 1$, is the weight on consumer surplus. When $\alpha = 1/2$, (16) becomes social surplus $\int_{\underline{v}}^{\bar{v}} [u\hat{q}(v) + v\hat{r}(v) - c(\hat{q}(v)) - d(\hat{r}(v))] f(v) dv$. When $\alpha = 1$, the second square-bracketed term in (16) vanishes, so (16) becomes consumer surplus. The public agency’s objective now is to maximize the general social welfare in (16). Let θ^I and r^I be the equilibrium weight and Quality 2, respectively, when the public agency maximizes (16).

Recall that the public agency can implement the equilibrium under Full Quality Report using the weights $\tilde{\theta}$ in (13) of Lemma 2, but that in equilibrium, it uses the lower weights $\hat{\theta}$ in Proposition 3. Also, the equilibrium Quality 2 $\hat{r}(v)$ is always strictly greater than the equilibrium Quality 2 $\tilde{r}(v)$ under Full Quality Report. The next proposition says that when the public agency maximizes the general social welfare (16), the equilibrium is in-between equilibria under Full Quality Report and Average Quality Report for consumer utility maximization.

PROPOSITION 4: *When the public agency maximizes the general welfare index (16), the equilibrium weight $\theta^I(v)$ satisfies $\hat{\theta}(v) \leq \theta^I(v) < \tilde{\theta}(v)$, and the equilibrium quality $r^I(v)$ satisfies $\tilde{r}(v) < r^I(v) \leq \hat{r}(v)$ at every v . Moreover, the equilibrium welfare under Average Quality Report is higher than Full Quality Report.*

The intuition behind Proposition 4 is this. By Lemma 2, the public agency can implement the Full Quality Report Qualities 1 and 2, \tilde{q} and $\tilde{r}(v)$, by $\tilde{\theta}$. By lowering the weight at v from $\tilde{\theta}(v)$, say to $\tilde{\theta}(v) - \epsilon$, the public agency raises the cost of using Quality 1 and lowers the cost of using Quality 2 to produce an average quality. The new weight $\tilde{\theta}(v) - \epsilon$ thus reduces Quality 1 but raises Quality 2. This change improves welfare. This is because under Full Quality Report, Quality 1 is first best but Quality 2 is lower than first best. Therefore, the increase in Quality 2 results in a first-order welfare gain whereas the decrease in Quality 1 merely results in a second-order welfare loss. Because the welfare weight α is greater than 0.5, the weight function will never fall below $\hat{\theta}$.

The increase in equilibrium welfare in Proposition 4 also results in a redistribution of surplus from the monopolist to the consumers. Because $r^I(v)$ is higher than $\tilde{r}(v)$ at each v , all consumers (except type- \underline{v}) obtain higher utilities in the equilibrium in Proposition 4

than in the Full Quality Report game. On the other hand, the monopolist's profit in Proposition 4 is lower than in the Full Quality Report game. This is because $\theta^I(v)$ is lower than $\tilde{\theta}(v)$, so the profit-maximizing Qualities 1 and 2 in the Full Quality Report game cannot be equilibrium in Proposition 4.

5.4. QUALITY REPORT AND NONSCREENING PRICE AND QUALITIES

In this subsection, we disallow the firm to offer screening price-quality menus. For example, due to regulatory restrictions or high menu costs the firm can only sell a single good, rather than multiple goods at different qualities and prices to screen consumers. Consider Full Quality Report. Here, the firm's strategy consists of a price p , and Qualities 1 and 2, respectively, q and r . A type- v consumer obtains utility $uq + vr - p$ when he buys the good. The marginal consumer is type v^* where $uq + v^*r - p = 0$. The mass of consumers who buy is $1 - F(v^*)$. Therefore, the firm's profit is

$$[1 - F(v^*)][p - c(q) - d(r)].$$

The monopolist chooses p , q , r , and v^* to maximize profit subject to the definition of v^* .

By standard computation, we obtain the following characterization of the equilibrium:³

$$u = c'(q), \tag{17}$$

$$v^* = d'(r), \tag{18}$$

$$uq + v^*r - c(q) - d(r) = r \frac{1 - F(v^*)}{f(v^*)}. \tag{19}$$

The interpretations of these results are familiar (see, for example, Spence 1975). The determination of the marginal consumer follows the usual trade-off. From (19), the price-cost margin $[p - c(q) - d(r)]/p$ is inversely proportional to the elasticity of demand.⁴ Both qualities are efficient for the marginal consumer v^* because the utilities from qualities are separable from price. The firm simply chooses the best qualities for the marginal consumer v^* , and sets the price to extract the marginal consumer's surplus.

The nonscreening equilibrium in Full Quality Report generally gives rise to a low consumer surplus. All types of the consumer receive a suboptimal Quality 2 (except the marginal v^*). The price is so high that some types of consumer do not purchase.

Average Quality Report can always implement the Full Quality Report equilibrium. The report weight for Quality 1 can be set at

$$\theta^* = \frac{c'(q)}{c'(q) + d'(r)} = \frac{u}{u + v^*},$$

3. We substitute p in the profit function from $p = uq + v^*r$ to simplify the profit-maximization program into the unconstrained maximization of $[1 - F(v^*)][uq + v^*r - c(q) - d(r)]$. The first-order conditions with respect to q , r , and v^* are in (17), (18), and (19).

4. In terms of price p , the demand is $1 - F(p/r - uq/r)$. The demand elasticity with respect to p is the reciprocal of the expression in the right-hand side of (19) multiplied by p .

where $q, r,$ and v^* are those in (17), (18), and (19), respectively. Equilibrium Qualities 1 and 2 in Full Quality Report will indeed minimize the cost $c(q) + d(r)$ for achieving an average quality. Clearly, the public agency can manipulate the report weight to raise consumer surplus. For example, if for a higher consumer surplus Quality 2 should be increased from the equilibrium under Full Quality Report, then the public agency decreases the weight from θ^* .

6. QUALITY REPORT AND COMPETITION

In this section, we let there be many competing firms in the market. Because we have not included horizontal product differentiation in the model, it is natural to continue the analysis with a Bertrand game. We let firms simultaneously make decisions on Qualities 1 and 2, and prices, all as functions of v . When quality information is unavailable, the market collapses, as in the case of monopoly. A public agency can remedy the problem by credibly reporting quality information. Under Full Quality Report, the public agency reports all quality pairs of all firms that engage in production. Under Average Quality Report, the public agency first chooses a quality weight function, computes the weighted averages of qualities of all firms that engage in production, and then reports the averages.

Recall that the first-best Qualities 1 and 2 are, respectively, $q^{FB} \equiv \operatorname{argmax}_q uq - c(q)$, and $r^{FB}(v) \equiv \operatorname{argmax}_r vr - d(r), v \in [\underline{v}, \bar{v}]$. The Bertrand equilibrium in Full Quality Report is simply the first best. Each firm produces quality pairs $(q^{FB}, r^{FB}(v))$, and charges prices $c(q^{FB}) + d(r^{FB}(v)), v \in [\underline{v}, \bar{v}]$. Consumer v picks $(q^{FB}, r^{FB}(v))$ because the equilibrium offers are incentive compatible.⁵ Each firm makes a zero profit.

Consumer welfare is highest under Bertrand competition and Full Quality Report. Actually, Average Quality Report also can implement the first best. Consumers must form their beliefs about qualities when an average is reported by the public agency. Lemma 1 applies to any firm’s reported average quality; each firm must minimize the cost for achieving the average. Now Lemma 2 can be applied. The public agency can choose a quality weight function $\theta^{FB} : [\underline{v}, \bar{v}] \rightarrow [0, 1]$ as follows:

$$\theta^{FB}(v) = \frac{c'(q^{FB})}{c'(q^{FB}) + d'(r^{FB}(v))} = \frac{u}{u + v}. \tag{20}$$

Given this weight function, the first-best qualities are cost-minimizing. It is therefore an equilibrium for firms to produce first-best qualities. In fact, the function θ^{FB} depends only on consumer valuations of the two qualities; information of the cost functions c and d are not needed.

COROLLARY 1: *When firms compete in the Bertrand fashion, under Average Quality Report the public agency implements the first-best Qualities 1 and 2, q^{FB} and $r^{FB}(v)$, by the quality weight function θ^{FB} in (20). Equilibria under Full Quality Report and Average Quality Report are identical.*

In Average Quality Report we can compare equilibrium qualities between monopoly and competition. These are, respectively, those in Proposition 3 and Corollary 1. Figure 5 illustrates these three functions. Equilibrium Quality 2 under competition is first best, and described by the solid line $r^{FB}(v)$. For monopoly, equilibrium

5. By definitions of q^{FB} and $r^{FB}(v)$, we have $uq^{FB} + vr^{FB}(v) - c(q^{FB}) - d(r^{FB}(v)) \geq uq^{FB} + vr^{FB}(v') - c(q^{FB}) - d(r^{FB}(v'))$, all $v, v' \in [\underline{v}, \bar{v}]$.

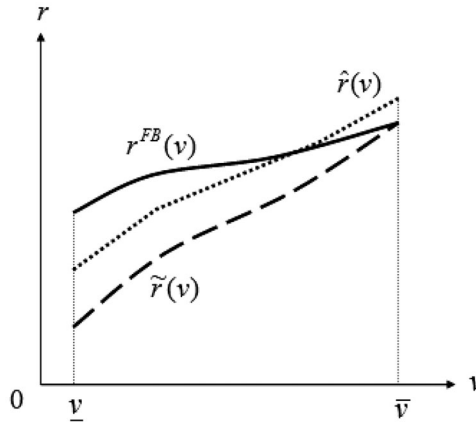


FIGURE 5. EQUILIBRIUM QUALITY 2

Quality 2 under Full Quality Report is the dashed line $\tilde{r}(v)$, while it is the dotted line $\hat{r}(v)$ under Average Quality Report.

For monopoly, at each v , equilibrium Quality 2 under Average Quality Report is higher than Full Quality Report (Proposition 3), so $\hat{r}(v) > \tilde{r}(v)$. For competition, at each v equilibrium Quality 2 is first best (Corollary 1). But then under monopoly, the equilibrium Full Quality Report Quality 2 is lower than first best, so $\tilde{r}(v) < r^{FB}(v)$, but $\tilde{r}(v)$ and $r^{FB}(v)$ have the same limit as v tends to \bar{v} . Figure 5 shows a case where $\hat{r}(v)$ intersects $r^{FB}(v)$ only once (although we have been unable to prove that this is true in general). There, Quality 2 is excessive at the “top” of the distribution of v , deficient at the “bottom,” and efficient at the “middle.”

In the Average Quality Report regime, switching from monopoly to competition must reduce Quality 2 for those consumers with high valuations (those v near \bar{v}), but vice versa for consumers with low valuations (those v near \underline{v}). However, this switch will raise Quality 1 because $\hat{q}(v) < q^{FB}$. Consumer utility must increase when the monopolist is replaced by a set of competitive firms.

7. CONCLUDING REMARKS

We have studied optimal quality indexes for an experience good. In Full Quality Report, a monopolist underprovides the quality that consumers have different and private valuations. In equilibrium, the monopolist extracts surplus by second-degree price-quality discrimination. In Average Quality Report, the public agency can restrain the monopolist’s pricing and quality decisions by the choices of quality weights. The public agency implements a higher consumer surplus through Average Quality Report by assigning higher weights on the quality that has been deficient. Our results also hold when the public agency maximizes a general welfare function, and when the monopolist cannot use menus to screen consumers. Under Bertrand competition, Full Quality Report yields first-best qualities and the maximum consumer surplus, whereas Average Quality Report implements the same outcome by suitable choices of quality weights.

For tractability, we let a single parameter describe consumers’ private preferences of a good with many qualities. Armstrong (1996), and Rochet and Choné (1998) analyze screening models in which qualities and preferences have the same number of

dimension. Multidimensional screening equilibria are complex. The monopolist typically depresses qualities to limit consumers' information rent, but equilibria may exhibit pooling (different consumer types obtain the same good), or exclusion (some consumer types do not purchase). Tractable solutions are uncommon in the literature, but almost all known results involve inefficient qualities. However, the belief-restriction result in Lemma 1 is independent of the consumer's type distribution. We think that Average Quality Report remains an instrument for changing equilibrium qualities, and may remedy deficient qualities in multidimensional screening models.

Our analysis has been based on a linear quality index. This is for tractability and practicality. More complex and nonlinear indexes can be constructed, of course. It would not be very surprising that complex indexes might perform better (since they would include linear indexes as special cases). Analytically, the task of optimizing over the set of all (measurable) functions on qualities would seem rather impossible. Our analytical approach here is sufficient to illustrate the basic economic principle.

APPENDIX

Proof of Lemma 1. Consider an item $(p(v), A(v))$ in a menu $\{p(v), A(v)\}$, $v \in [\underline{v}, \bar{v}]$. The monopolist earns a profit $p(v) - c(q(v)) - d(r(v))$ if it chooses to achieve the average $A(v)$ by good $(q(v), r(v))$. By definition, $p(v) - c(q(v)) - d(r(v)) \leq p(v) - c(\hat{q}(v)) - d(\hat{r}(v))$, with a strict inequality if $(q(v), r(v)) \neq (\hat{q}(v), \hat{r}(v))$ due to the strict convexity of c and d . Regardless of consumers' beliefs, the monopolist earns a higher profit by choosing $(\hat{q}(v), \hat{r}(v))$. Hence, any $(q(v), r(v)) \neq (\hat{q}(v), \hat{r}(v))$ cannot be equilibrium choices for achieving the average $A(v)$. Finally, (4) and (5) are obtained from the first-order conditions of the constrained maximization problem. \square

Proof of Proposition 2. Using pointwise optimization, the first-order conditions of (7) with respect to $\hat{q}(v)$, $\hat{r}(v)$, and the Lagrangian $\lambda(v)$ are, respectively,

$$u - c'(\hat{q}(v)) - \lambda(v)(1 - \theta)c''(\hat{q}(v)) = 0, \quad (\text{A1})$$

$$v - \frac{1 - F(v)}{f(v)} - d'(\hat{r}(v)) + \lambda(v)\theta d''(\hat{r}(v)) = 0, \quad (\text{A2})$$

$$\frac{c'(\hat{q}(v))}{d'(\hat{r}(v))} - \frac{\theta(v)}{1 - \theta(v)} = 0. \quad (\text{A3})$$

We combine (A1), (A2) and eliminate the Lagrangian $\lambda(v)$ to get (8). \square

Proof of Lemma 2. First, use the definition of $\tilde{\theta}$ in (13), and substitute it in (4). We then easily verify that $(\tilde{q}, \tilde{r}(v))$ satisfies the cost-minimization constraint in Lemma 1. Second, by Proposition 1, the quality pair $(\tilde{q}, \tilde{r}(v))$ is incentive compatible and maximizes the monopolist's profit at each v when the monopolist's quality choices are not subject to the cost-minimization constraint. Therefore, we conclude that $(\tilde{q}, \tilde{r}(v))$ is an equilibrium in the continuation game defined by $\tilde{\theta}$ in (13). \square

Proof of Proposition 3. By Proposition 2, total consumer utility under Average Quality Report is $\int_{\underline{v}}^{\bar{v}} (\int_{\underline{v}}^v \hat{r}(x) dx) f(v) dv$, where $\hat{r}(v)$ is given by (8) and (9). Using integration by parts, we rewrite total consumer utility as

$$\int_{\underline{v}}^{\bar{v}} [1 - F(v)] \hat{r}(v) dv, \tag{A4}$$

because $-[[1 - F(v)] \int_{\underline{v}}^v \hat{r}(x) dx]_{\underline{v}}^{\bar{v}}$ is equal to 0.

We begin by showing that in the equilibrium of the Average Quality Report game, $\tilde{r}(v) \leq \hat{r}(v)$ at each v . First, by the Maximum Theorem, $\tilde{r}(v)$ and $\hat{r}(v)$ are continuous. Also, since $\tilde{r}(v)$ and $\hat{r}(v)$ are equilibrium qualities, each must be increasing. So suppose that in equilibrium there exist $v_1 < v_2$ such that for all $v \in (v_1, v_2) \subset [\underline{v}, \bar{v}]$, $\hat{r}(v) < \tilde{r}(v)$. Now replace $\hat{r}(v)$ by $\tilde{r}(v)$ for $v \in (v_1, v_2)$. This new Quality 2 function can be implemented simply by replacing $\hat{\theta}(v)$ by $\tilde{\theta}(v)$ for $v \in (v_1, v_2)$. We have raised the consumer's utility in (A4). This is a contradiction. We conclude that $\tilde{r}(v) \leq \hat{r}(v)$ at each v .

By Proposition 1, $\hat{q}(v)$ and $\tilde{r}(v)$ are given by (1) and (2). If $\hat{q}(v)$ and $\hat{r}(v)$ in (8) are, respectively, substituted by $\hat{q}(v)$ and $\tilde{r}(v)$, each square-bracketed term in (8) becomes 0. Now for (8) to hold, we must have $\hat{q}(v) \leq \tilde{q}(v)$ if and only if $\tilde{r}(v) \leq \hat{r}(v)$. Substituting $(\hat{q}(v), \tilde{r}(v))$ and $(\hat{q}(v), \hat{r}(v))$ into (9), we have

$$\frac{c'(\hat{q}(v))}{\hat{\theta}(v)} - \frac{d'(\hat{r}(v))}{1 - \hat{\theta}(v)} = 0 = \frac{c'(\tilde{q}(v))}{\tilde{\theta}(v)} - \frac{d'(\tilde{r}(v))}{1 - \tilde{\theta}(v)}. \tag{A5}$$

Because $\tilde{r}(v) \leq \hat{r}(v)$, we have $\hat{q}(v) \leq \tilde{q}(v)$, so from (A5) $\hat{\theta}(v) \leq \tilde{\theta}(v)$ at each v .

We now derive the equilibrium weight function $\hat{\theta}$. At an equilibrium, (A4) is maximized. By pointwise optimization, we have $\frac{d\hat{r}(v)}{d\hat{\theta}(v)} = 0$ at every v . By Proposition 2, equations (8) and (9) implicitly define the equilibrium \hat{q} and \hat{r} as two functions in terms of v and $\theta(v)$. After totally differentiating these equations, and simplifying, we obtain

$$\frac{d\hat{r}(v)}{d\hat{\theta}(v)} = \frac{\frac{c'}{\theta(v)(1-\theta(v))d''} \left[\left(v - \frac{1-F(v)}{f(v)} - d' \right) \left(\frac{c'''}{c''} - \frac{c''}{c'} \right) \frac{1-\theta(v)}{\theta(v)} - d'' \right]}{\left[d' \frac{c''}{c'} + c' \frac{d''}{d'} \right] - \left[\left(v - \frac{1-F(v)}{f(v)} - d' \right) \frac{c'''}{c''} + (u - c') \frac{d'''}{d''} \right]}. \tag{A6}$$

We get (14) by setting (A6) equal to 0 and rearranging terms.

We have shown that $\tilde{r}(v) \leq \hat{r}(v)$ and $\hat{\theta}(v) \leq \tilde{\theta}(v)$ at each v . By (14), we can rule out $\tilde{r}(v) = \hat{r}(v)$ and $\tilde{\theta}(v) = \hat{\theta}(v)$. Suppose not; that is, suppose that at some v^* , $\tilde{r}(v^*) = \hat{r}(v^*)$ and $\tilde{\theta}(v^*) = \hat{\theta}(v^*)$. By (2) and (14), then at v^* we have $\hat{\theta}(v^*) = 0$. This contradicts Lemma 2. Finally, because $\frac{c'''}{c''}$ is nonincreasing, we have $\frac{c'''}{c''} < \frac{c''}{c'}$ and the denominator of (14) is strictly positive. We conclude that $0 < \hat{\theta}(v) < \tilde{\theta}(v)$ and $\tilde{r}(v) < \hat{r}(v)$ at each v .

We now verify that the equilibrium menu is incentive compatible. It is sufficient to show that $\hat{r}(v)$ is increasing. We totally differentiate (8) and (9) with respect to $\hat{r}(v)$ and v , then evaluate the derivative at $\theta(v) = \hat{\theta}(v)$. After simplification, we have

$$\left. \frac{d\hat{r}(v)}{dv} \right|_{\theta(v)=\hat{\theta}(v)} = \frac{\frac{1}{d''\hat{\theta}(v)} \left[\frac{d}{dv} \left(v - \frac{1-F(v)}{f(v)} \right) \right]}{\left[d' \frac{c''}{c'} + c' \frac{d''}{d'} \right] - \left[\left(v - \frac{1-F(v)}{f(v)} - d' \right) \frac{c'''}{c''} + (u - c') \frac{d'''}{d''} \right]}. \tag{A7}$$

We show that (A7) is strictly positive. First, using (9) to eliminate $\widehat{\theta}(v)$ in (8) and rearranging terms, we have

$$\left[d' \frac{c''}{c'} + c' \frac{d''}{d'} \right] - \left[\left(v - \frac{1 - F(v)}{f(v)} \right) \frac{c''}{c'} + u \frac{d''}{d'} \right] = 0. \tag{A8}$$

Next, subtracting the left-hand side of (A8) from the denominator of (A7), we obtain

$$\left[\left(v - \frac{1 - F(v)}{f(v)} \right) \left(\frac{c''}{c'} - \frac{c'''}{c''} \right) \right] + \left[u \left(\frac{d''}{d'} - \frac{d'''}{d''} \right) \right] + \left[d' \frac{c'''}{c''} + c' \frac{d'''}{d''} \right]. \tag{A9}$$

Since $\frac{c''}{c'}$ and $\frac{d''}{d'}$ are nonincreasing, we have $\frac{c'''}{c''} < \frac{c''}{c'}$ and $\frac{d'''}{d''} < \frac{d''}{d'}$. Therefore, the first two square-bracketed terms in (A9) are strictly positive. The last square-bracketed term in (A9) is nonnegative because c and d are convex and c''' and d''' are nonnegative. Because expression (A9) is strictly positive, the denominator of (A7) is strictly positive. Finally, by the monotone hazard rate assumption, the square-bracketed term in the numerator of (A7) is strictly positive. We conclude that the equilibrium menu is incentive compatible. \square

Proof of Proposition 4. By pointwise optimization, the first-order condition of (16) with respect to $\theta(v)$ is

$$\alpha \left[\frac{1 - F(v)}{f(v)} \frac{d\widehat{r}(v)}{d\theta(v)} \right] + (1 - \alpha) \left[(u - c'(\widehat{q}(v))) \frac{d\widehat{q}(v)}{d\theta(v)} + \left(v - \frac{1 - F(v)}{f(v)} - d'(\widehat{r}(v)) \right) \frac{d\widehat{r}(v)}{d\theta(v)} \right] = 0, \tag{A10}$$

where (8) and (9) implicitly define $\frac{d\widehat{r}(v)}{d\theta(v)}$ and $\frac{d\widehat{q}(v)}{d\theta(v)}$ as (A6) and

$$\frac{d\widehat{q}(v)}{d\theta(v)} = \frac{\frac{d'}{\theta(v)(1-\theta(v))c''} \left[(u - c') \left(\frac{d''}{d'} - \frac{d'''}{d''} \right) \frac{\theta(v)}{1-\theta(v)} + c'' \right]}{\left[d' \frac{c''}{c'} + c' \frac{d''}{d'} \right] - \left[\left(v - \frac{1-F(v)}{f(v)} - d' \right) \frac{c'''}{c''} + (u - c') \frac{d'''}{d''} \right]}, \tag{A11}$$

respectively. Using (A6) and (A11) to replace $\frac{d\widehat{r}(v)}{d\theta(v)}$ and $\frac{d\widehat{q}(v)}{d\theta(v)}$ in (A10) and rearranging terms, we get

$$\theta^I(v) = \frac{\left[d' - \left(v - \frac{1-F(v)}{f(v)} \right) \right]}{\left[d' - \left(v - \frac{1-F(v)}{f(v)} \right) \right] + [d'' - \Psi(\alpha)] \left[\frac{c''}{c'} - \frac{c'''}{c''} \right]^{-1}} \tag{A12}$$

at every v , where $\Psi(\alpha)$ is given by

$$\Psi(\alpha) \equiv \frac{\alpha (u - c') \left[(u - c') \left(\frac{d''}{d'} - \frac{d'''}{d''} \right) \frac{d''}{c''} + d'' \frac{d'}{c'} \right]}{\left[\alpha \left(v - \frac{1-F(v)}{f(v)} - d' \right) + (1 - \alpha) \left(\frac{1-F(v)}{f(v)} \right) \right]}. \tag{A13}$$

Conditions (8), (9), (A12), and (A13) characterize the equilibrium weight and qualities at each v .

We now show that $\widehat{\theta}(v) \leq \theta^I(v) < \widetilde{\theta}(v)$ at each v . First, by Lemma 2, the monopolist's profit is maximized at $\theta(v) = \widetilde{\theta}(v)$. Hence, the second square-bracketed term in (A10) is strictly positive if $\theta(v) < \widetilde{\theta}(v)$ and nonpositive if $\widetilde{\theta}(v) \leq \theta(v)$. Second, by Proposition 3, the first square-bracketed term in (A10) is strictly positive if $\theta(v) < \widehat{\theta}(v)$ and is strictly

negative if $\widehat{\theta}(v) < \theta(v)$. Finally, Proposition 3 also says $\widehat{\theta}(v) < \widetilde{\theta}(v)$. Therefore, if $\theta^I(v) < \widehat{\theta}(v)$ at some v , the left-hand side of (A10) must be strictly positive for $\frac{1}{2} \leq \alpha \leq 1$, which is a contradiction. We have $\widehat{\theta}(v) \leq \theta^I(v)$. Similarly, if $\widetilde{\theta}(v) \leq \theta^I(v)$ at some v , the left-hand side of (A10) must be strictly negative for $\frac{1}{2} \leq \alpha \leq 1$. This is a contradiction, so we have $\theta^I(v) < \widetilde{\theta}(v)$.

We have shown that $\widehat{\theta}(v) \leq \theta^I(v) < \widetilde{\theta}(v)$ at each v . By condition (A6), $\widehat{r}(v)$ is maximized at $\widehat{\theta}(v)$ and strictly decreasing in $\theta(v)$ if $\widehat{\theta}(v) < \theta(v)$. Therefore, we conclude that $\widetilde{r}(v) < r^I(v) \leq \widehat{r}(v)$ at every v . Moreover, because $\theta^I(v) < \widetilde{\theta}(v)$ and $I(\alpha)$ is maximized at $\theta^I(v)$, the equilibrium welfare under Average Quality Report is higher than Full Quality Report. \square

MONOTONICITY AND INCENTIVE COMPATIBILITY

A menu $\{p(v), A(v)\}$, $v \in [\underline{v}, \bar{v}]$, is incentive compatible if

$$v = \operatorname{argmax}_{v'} u\widehat{q}(v') + v\widehat{r}(v') - p(v'), \quad v, v' \in [\underline{v}, \bar{v}]. \tag{A14}$$

Define $W(v) \equiv \max_{v'} u\widehat{q}(v') + v\widehat{r}(v') - p(v')$. A menu $\{p(v), A(v)\}$, $v \in [\underline{v}, \bar{v}]$, is individually rational if

$$W(v) \geq 0, \quad v \in [\underline{v}, \bar{v}]. \tag{A15}$$

LEMMA 3: *A menu $\{p(v), A(v)\}$, $v \in [\underline{v}, \bar{v}]$, is incentive compatible and individually rational if and only if*

$$\widehat{r}(v) \text{ is increasing in } v, \tag{A16}$$

$$W(v) = W(\underline{v}) + \int_{\underline{v}}^v \widehat{r}(x)dx, \tag{A17}$$

$$W(\underline{v}) \geq 0. \tag{A18}$$

Proof of Lemma 3. We first show that conditions (A14) and (A15) imply conditions (A16)–(A18). Since $W(v)$ defined by (A14) is the maximum of an affine function of v , it is continuous, convex on $[\underline{v}, \bar{v}]$ and almost everywhere differentiable. Thus, $W(v)$ satisfies the envelope condition: $W'(v) = \widehat{r}(v)$. Upon integration, the envelope condition gives (A17). Moreover, the convexity of W says that $W'(v)$ is increasing in v . Hence, $\widehat{r}(v)$ is also increasing in v . Finally, (A18) is implied by (A15).

Next, we show that conditions (A16)–(A18) imply conditions (A14) and (A15). Obviously conditions (A17) and (A18) imply condition (A15). It remains to show that (A16) and (A17) imply (A14). From any increasing function $\widehat{r}(v')$ and $W(\underline{v})$, construct a price function $p(v')$ by

$$p(v') \equiv u\widehat{q}(v') + v'\widehat{r}(v') - W(\underline{v}) - \int_{\underline{v}}^{v'} \widehat{r}(x)dx.$$

Adding $v\widehat{r}(v')$ to both sides, and rearranging terms, we have

$$u\widehat{q}(v') + v\widehat{r}(v') - p(v') = (v - v')\widehat{r}(v') + W(\underline{v}) + \int_{\underline{v}}^{v'} \widehat{r}(x)dx,$$

the left-hand side being the utility for a type- v consumer picking item $(p(v'), A(v'))$.

Subtracting the expression above from condition (A17) gives

$$W(v) - [u\widehat{q}(v') + v\widehat{r}(v') - p(v')] = \int_{\underline{v}}^v \widehat{r}(x)dx - \int_{\underline{v}}^{v'} \widehat{r}(x)dx - (v - v')\widehat{r}(v').$$

If $v > v'$,

$$W(v) - [u\widehat{q}(v') + v\widehat{r}(v') - p(v')] = \int_{v'}^v \widehat{r}(x)dx - (v - v')\widehat{r}(v') > 0,$$

because $\widehat{r}(v)$ is increasing. If $v' > v$,

$$W(v) - [u\widehat{q}(v') + v\widehat{r}(v') - p(v')] = - \int_v^{v'} \widehat{r}(x)dx + (v' - v)\widehat{r}(v') > 0$$

again because $\widehat{r}(v)$ is increasing. □

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