

# How Much Would You Pay to Resolve Long-Run Risk?

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*Though risk aversion and the elasticity of intertemporal substitution have been the subjects of careful scrutiny when calibrating preferences, the long-run risks literature as well as the broader literature using recursive utility to address asset pricing puzzles have ignored the full implications of their parameter specifications. Recursive utility implies that the temporal resolution of risk matters and a quantitative assessment of how much it matters should be part of the calibration process. This paper gives a sense of the magnitudes of implied timing premia. Its objective is to inject temporal resolution of risk into the discussion of the quantitative properties of long-run risks and related models.*

The long-run risks model of Bansal and Yaron (2004) has delivered a unified explanation of several otherwise puzzling aspects of asset markets.<sup>1</sup> Since Mehra and Prescott (1985) posed the equity premium puzzle, it has been understood that the asset market puzzles are *quantitative* and that an explanation must be consistent with observations in other markets and also with introspection. Imposing such discipline led Mehra and Prescott to exclude rationalization of the observed equity premium by levels of risk aversion exceeding their well known upper bound of 10. This bound on risk aversion has been largely respected since, including in long-run risk models (henceforth LRR). However, we suggest in this paper that quantitative discipline has been lax in another equally important aspect of the long-run risks model.

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<sup>1</sup>In his opening remarks, Bansal (2007) lists the following puzzles: the level of equity premium, asset price volatility, the large cross-sectional differences in average returns across equity portfolios such as value and growth portfolios, and in bond and foreign exchange markets, the violations of the expectations hypothesis and the ensuing return predictability that is quantitatively difficult to explain. He then writes: “What risks and investor concerns can provide a unified explanation for these asset market facts? One potential explanation of all these anomalies is the long-run risks model.” For elaboration and many additional references see Bansal (2007), Piazzesi and Schneider (2007), Hansen, Heaton, and Li (2008), Colacito and Croce (2011) and Chen (2010).

As a representative agent model, LRR has two key components—the endowment process and preferences. The former is modeled as having a persistent predictable component for consumption growth and its volatility; it will be described more precisely below. The representative agent has Epstein-Zin preferences (Epstein and Zin, 1989; Weil, 1990), which permit a partial disentangling of the elasticity of intertemporal substitution (EIS) and the coefficient of relative risk aversion (RRA). Denoting time  $t$  consumption by  $c_t$ , continuation utilities  $U_t$  satisfy the recursion

$$(1) \quad U_t^\rho = (1 - \beta) c_t^\rho + \beta [E_t(U_{t+1}^\alpha)]^{\rho/\alpha},$$

when  $\rho \neq 0$ , and otherwise

$$(2) \quad \log U_t = (1 - \beta) \log c_t + \beta \log [E_t(U_{t+1}^\alpha)]^{1/\alpha}.$$

We assume that  $\rho < 1$ ,  $0 \neq \alpha < 1$  and  $0 < \beta < 1$ . The utility of a deterministic consumption path is in the CES class with the elasticity of intertemporal substitution

$$\text{EIS} = \frac{1}{1 - \rho}.$$

Epstein and Zin (1989) show that  $1 - \alpha$  is the measure of relative risk aversion for timeless wealth gambles and also for suitable gambles in consumption where all risk is resolved at a single instant, justifying thereby the identification

$$\text{RRA} = 1 - \alpha.$$

The noted disentangling is possible because a decrease in  $\alpha$  increases risk aversion without affecting the attitude towards consumption smoothing over time given certainty, unlike in the standard additive power utility model where  $\rho = \alpha$ . With these interpretations of  $\rho$  and  $\alpha$ , parameter values in the LRR literature are specified with due care paid to evidence about the elasticity of substitution and the degree of risk aversion. However, as is clear from the theoretical literature,  $\rho$  and  $\alpha$  affect also another aspect of preference in addition to the EIS and RRA. Clearly, judging the plausibility of parameter values requires that one consider their full quantitative implications for all dimensions of preference.

The above model of utility belongs to the recursive class developed by Kreps and Porteus (1978) in order to model nonindifference to the way in which a given risk resolves over time. For a simple example, suppose that consumption is fixed and certain in periods 0 and 1, and that it will be constant thereafter either at a high level or at a low level, depending on the outcome of the toss of an unbiased coin. Do you care whether the coin is tossed at  $t = 1$  or at  $t = 2$ ? We emphasize that it is risk about consumption, and not income, that is at issue, so that there is no apparent planning advantage to having the coin tossed early. According to the standard additive power utility model ( $\rho = \alpha$ ), the time of resolution of

the given risk is a matter of indifference. But not so more generally for recursive utility. For the specification (1)–(2), it is well known that early resolution of a given risk (here tossing the coin at  $t = 1$ ) is always preferable if and only if

$$(3) \quad 1 - \alpha = \text{RRA} > \frac{1}{\text{EIS}} = 1 - \rho.$$

This condition is satisfied by the parameter values typically used in LRR models where both EIS and RRA are typically taken to exceed 1. Moreover, there is clear intuition that nonindifference to temporal resolution of risk might matter in matching asset market data: because long-run risks are not resolved until much later, they are treated differently, and penalized more heavily, than are current risks thus permitting a large risk premium to emerge even when shocks to current consumption are small. This begs the question whether the differential treatment required to match asset returns data is plausible, which is obviously a quantitative question and calls for evidence about the attitude towards temporal resolution.

We are not aware of any market-based or experimental evidence that might help with a quantitative assessment.<sup>2</sup> In principle, the attitude towards the temporal resolution of risk may underlie behavior in many multi-period settings. However, it is not clear how to disentangle the attitude towards the psychic benefit of early resolution of consumption risk, which is the issue at hand, from either the instrumental benefit of early resolution of income risk, which is plausibly more directly observable at the micro level, or the pricing of consumption risk which is observable from asset market data.<sup>3</sup> Of course, the approach of the long-run risk literature yields information about the former *under the assumption* of Epstein-Zin utility and a suitable endowment process. However, our objective is to judge whether this approach is a good one. To do so, we suggest a simple thought experiment that through introspection may help to judge plausibility of the parameter values used in the LRR literature. Thought experiments and introspection play a role also in assessing risk aversion parameters (see, for example, Kandel and Stambaugh, 1990, 1991, Mankiw and Zeldes, 1991, and Rabin, 2000). In the latter context one considers questions of the form “how much would you pay for

<sup>2</sup>The small experimental literature that we are aware of, see, e.g., Ahlbrecht and Weber (1996), Brown and Kim (forthcoming), and references therein, focuses on whether individuals prefer early or late resolution, not on the strength of this preference. Our paper may provide stimulus for more work along these lines; an important question is how to extrapolate from the experimentally feasible risks and time intervals. There is also some evidence from field experiments that many individuals choose not to learn their test results for various diseases, see, e.g., Thornton (2008) and Oster et. al. (2013); given the clear instrumental value of information this implies that the psychic benefit of early resolution is negative; however, it seems even harder to extrapolate from health outcomes that those studies focus on to consumption outcomes that are relevant here.

<sup>3</sup>There is a literature that seeks to understand risk pricing across maturities (see e.g. Hansen, Heaton and Li, 2008, and Hansen and Scheinkman 2009). In particular, Binsbergen, Brandt and Koijen (2012) use data on dividend strips prices to show that the the long run risks model (as well as other classic models) have counterfactual predictions for the pricing of securities with varying maturities. Since there seem to be no parameterizations of these models that can resolve these puzzles and simultaneously match the moments addressed by Bansal Yaron (2004), the work of Binsbergen et. al. (2012) is complementary to ours in that it provides motivation for search for alternative (endowment and/or preference) models.

the following hypothetical gamble?”. Here we ask instead “what fraction of your consumption stream would you give up in order for all risk to be resolved next month?” We call this fraction the *timing premium* and study its dependence on the parameters of the model.

A picture that seems to emerge is that models that assume high persistence of the consumption process (as in Bansal–Yaron, 2004) tend to imply a timing premium of the order of 25–30%, much higher than in an i.i.d. model where it is of the order of 7–10%. The intuition that persistence inflates the timing premium is corroborated with the rare disaster model: assuming high persistence of the jump process (as in Wachter, 2013) implies a timing premium of the order of 40%, much higher than in the i.i.d. model of Barro (2009), where it is around 20%.

Section I presents our theoretical and numerical results for the LRR model. Section II offers an extended discussion of the results framed as answers to the following series of questions: Why pay a premium for early resolution? Is introspection possible/useful? How is the premium for early resolution related to the welfare cost of risk (Lucas, 1987)? What is the effect of modifying the endowment process to be i.i.d., or to correspond to rare disasters (Barro, 2006,9) or persistent rare disasters (Wachter, 2013)? What if a nonexpected utility model of risk preferences is adopted? Section III concludes and includes a brief comparison with related papers by Ai (2007) and D’Addona and Brevik (2011).

## I. How Much Would You Pay?

### THE LRR CONSUMPTION PROCESS

Consider a consumption process of the following form:

$$(4) \quad \begin{aligned} \log \frac{c_{t+1}}{c_t} &= m + x_t + \sigma_t W_{c,t+1} \\ x_{t+1} &= ax_t + \varphi \sigma_t W_{x,t+1} \\ \sigma_{t+1}^2 &= \sigma^2 + \nu(\sigma_t^2 - \sigma^2) + \sigma_w W_{w,t+1}, \end{aligned}$$

where  $0 < a < 1$  and  $W_{c,t}$ ,  $W_{x,t}$ , and  $W_{w,t}$  are standard Gaussian innovations, mutually independent and i.i.d. over time.

Here  $x_t$  is a persistently varying predictable component of the drift in consumption growth. Though  $\varphi$  should be thought of as much smaller than unity, small innovations to  $x_t$  are important because they affect not only consumption prospects in the short run but also consumption for the indefinite future. The parameter  $a$  determines persistence of the expected growth rate process.

The volatility of consumption growth, represented by  $\sigma_t$ , is time-varying with unconditional variance given by  $\sigma^2$ . The empirical importance of stochastic volatility is emphasized by Bansal et al (2012) and Beeler and Campbell (2012). Setting  $\nu = 0 = \sigma_w$  turns off stochastic volatility and leads to a process with a constant variance of consumption growth; Bansal and Yaron refer to this model as Case I and to the model with stochastic volatility as Case II.

The LRR literature also distinguishes between consumption and dividends and specifies a suitable process for the latter. But it is the consumption process as a whole, and not its components, that is important here in trying to understand the nature of preferences.

In LRR models, a consumption process similar to the above is the endowment of a representative agent in a Lucas-style exchange economy. It is well known that there is limited theoretical justification for the assumption of a representative agent; here it requires that everyone have identical Epstein-Zin (hence homothetic) preferences. Regardless, we treat the representative agent as a real individual when introspecting about her preferences. The infinite horizon can be understood as arising from a bequest motive, or as a rough approximation to a long but finitely-lived individual.

#### DEFINITION OF THE TIMING PREMIUM

Here is the thought experiment. You are facing consumption described by (4) for  $t = 0, 1, \dots$ . In particular, the riskiness of consumption resolves only gradually over time ( $c_t$  and  $x_t$  are realized only at time  $t$ ). How much would you pay at time 0 to have all risk resolved next period? More precisely, you are offered the option of having all risk resolved at time 1. The cost is that you would have to relinquish the fraction  $\pi$  of both current consumption and of the consumption that is subsequently realized for every later period. What is the maximum value  $\pi^*$  for which you would be willing to accept this offer? Call  $\pi^*$  the *timing premium* for the consumption process in (4). Formally, let  $U_0$  be the utility of the consumption process in (4) with risk resolved gradually, and let  $U_0^*$  be the utility of the alternative process where all risk is resolved at time 1. Then<sup>4</sup>

$$\pi^* = 1 - \frac{U_0}{U_0^*}.$$

#### THEORETICAL DERIVATION FOR EIS= 1

The magnitude of EIS, particularly whether it is less than or greater than 1, is a source of debate. Bansal and Yaron argue for an elasticity larger than 1 (in fact,  $EIS > 1$  is important for the empirical performance of their model). Because closed-form solutions are not available for  $EIS \neq 1$ , we compute values of the timing premium numerically below. However, first we derive a closed-form expression for the timing premium under the assumption of a unitary elasticity of substitution and restricting attention to the case of constant volatility for consumption growth.

<sup>4</sup>Utility admits an interpretation in terms of consumption perpetuities. For any consumption process  $c$ , its utility as defined in (1)-(2) equals that level of consumption which if received in every period and state would be indifferent to  $c$ . Thus  $\pi^*$  can be described as the fraction of the consumption perpetuity that if relinquished would just offset the benefit of early resolution of risk.

Continuation utilities of the consumption process in (4), with risk resolved gradually, solve a recursive relation. Guess and verify that utility is given by

$$\log U_0 = \log c_0 + \frac{\beta}{1-\beta a} x_0 + \frac{\beta}{1-\beta} m + \frac{\alpha}{2} \frac{\beta \sigma^2}{1-\beta} \left( 1 + \frac{\varphi^2 \beta^2}{(1-\beta a)^2} \right)$$

Denote by  $U_0^*$  the utility of the alternative process where all risk is resolved at time 1. Then the continuation utility  $U_1^*$  at time 1 is given by

$$\log U_1^* = (1-\beta) [\log c_1 + \beta \log c_2 + \beta^2 \log c_3 + \dots] = \log c_0 + \sum_{t=1}^{\infty} \beta^{t-1} \log (c_t/c_{t-1}).$$

Therefore, from the time 0 perspective  $\log U_1^*$  is normally distributed with mean  $\log c_0 + \frac{m}{1-\beta} + \frac{a}{1-\beta a} x_0$  and variance  $\frac{\sigma^2}{1-\beta^2} \left( 1 + \frac{\varphi^2 \beta^2}{(1-\beta a)^2} \right)$ . Conclude that

$$\begin{aligned} \log U_0^* &= (1-\beta) \log c_0 + \beta \log (E_0 (U_1^*)^\alpha)^{1/\alpha} \\ &= \log c_0 + \frac{\beta}{1-\beta a} x_0 + \frac{\beta}{1-\beta} m + \frac{\alpha}{2} \frac{\beta \sigma^2}{1-\beta^2} \left( 1 + \frac{\varphi^2 \beta^2}{(1-\beta a)^2} \right). \end{aligned}$$

Accordingly, one arrives at the following expression for the timing premium:

$$\pi^* = 1 - \exp \left[ \frac{\alpha}{2} \frac{\beta^2 \sigma^2}{1-\beta^2} \left( 1 + \frac{\varphi^2 \beta^2}{(1-\beta a)^2} \right) \right].$$

The premium is positive, that is, early resolution is preferred, if and only if  $\alpha < 0$ , consistent with (3). In that case, the premium is increasing in RRA,  $\sigma^2$ ,  $\varphi$ ,  $\beta$  and  $a$ , as one would expect. The first column of Table 1 gives a sense of the quantitative meaning of this formula for the parameter values (other than EIS) specified in Bansal and Yaron (2004) for a monthly frequency. (The risk premium described in the last row is defined in Section II.)

#### NUMERICAL RESULTS

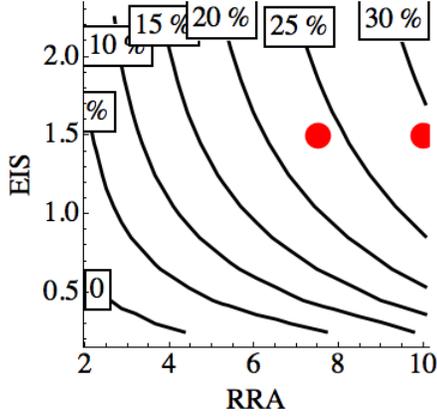
For values of EIS different than 1, we rely on numerical methods. To obtain the value of  $U_0$  we note that the value function  $U$  in (1) can be written as  $U(c, x, \sigma) = cH(x, \sigma)$ , where  $H : \mathbb{R} \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$  is a solution to the functional equation

$$(5) \quad H(x, \sigma) = \left\{ 1 - \beta + \beta e^{\rho \left( m + x + \frac{\alpha \sigma^2}{2} \right)} E_{x, \sigma} \left( H(x', \sigma'^\alpha)^{\frac{\rho}{\alpha}} \right)^{\frac{1}{\rho}} \right\};$$

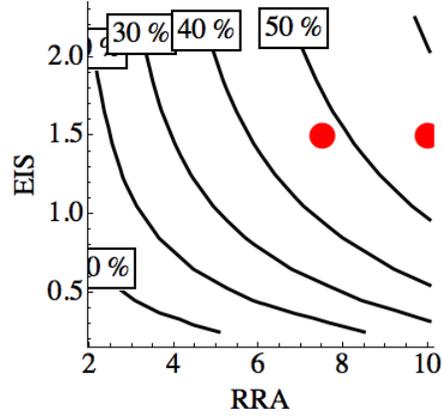
$E_{x, \sigma}$  is the expectation conditional on  $x$  and  $\sigma$ . We discretize  $x$  and  $\sigma$ , approximate  $H$  by a Chebyshev polynomial, and approximate the expectation by a quadrature; thus our approximation to (5) can be written as a system of nonlinear equations. We solve this system using AMPL. To compute the value of early resolution  $U_0^*$  we run Matlab Monte Carlo simulations with a fixed time horizon

$T = 2500$  months (pasting  $U_0$  as the continuation value at  $T$ ).

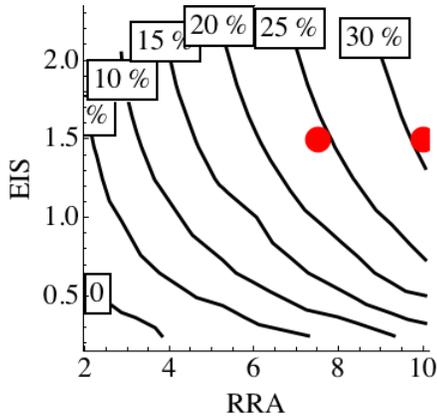
Table 1 reports our numerical results for  $EIS = 1.5$ . Figure 1 plots the isoquants of the timing and risk premia for the two specifications and a range of preference parameters.<sup>5</sup>



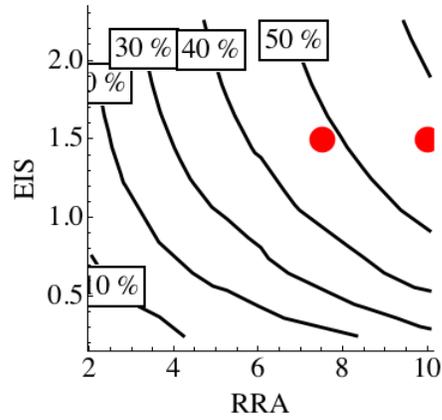
(a) Timing Premium constant volatility



(b) Risk Premium constant volatility



(c) Timing Premium stochastic volatility



(d) Risk Premium stochastic volatility

FIGURE 1. PREMIUM ISOQUANT MAPS FOR THE LRR MODEL. THE RED DOTS DENOTE THE CALIBRATED VALUES OF EIS AND RRA. OTHER PARAMETERS ARE AS IN TABLE 1.

<sup>5</sup>We limit RRA to be no greater than 10, the upper bound considered reasonable by Mehra and Prescott (1985) despite the fact that in the literature many calibrated, as well as estimated (see Chen, Favilukis, and Ludvigson, 2012), parameter values exceed 10. Those parameter values would inflate the timing premium further.

TABLE 1—PREMIA IN THE LRR MODEL

	BY (but EIS=1)	BY (Case I)	BY (Case II)
$\sigma$	.0078	.0078	.0078
$\varphi$	.044	.044	.044
$a$	.9790	.9790	.9790
$\sigma_w^2$	0	0	$.23 \times 10^{-5}$
$\nu$	0	0	.987
$\beta$	.998	.998	.998
RRA	7.5 or 10	7.5 or 10	7.5 or 10
EIS	1	1.5	1.5
Timing Premium $\pi^*$	20% or 27%	23% or 29%	24% or 31%
Risk Premium $\bar{\pi}$	38% or 48%	48% or 56%	48% or 57%

## II. Discussion and Perspective

### WHY PAY A PREMIUM?

Would you give up 25 or 30% of your lifetime consumption in order to have all risk resolved next month? Keep in mind that it is risk about consumption that is at issue rather than risk about income or security returns. Thus early resolution does not have any apparent instrumental value. Kreps and Porteus (1978, 1979) suggest that an instrumental value might arise because of an unmodeled underlying planning problem. Essentially, there are more primitive preferences defined over deeper variables that are the ultimate source of satisfaction; utility defined on consumption is an indirect utility function, and early resolution has value for reasons familiar from Spence and Zeckhauser (1972), for example. This sounds plausible in theory, but one needs a more concrete story in order to believe that it could generate a sizable timing premium.<sup>6</sup>

At a psychic level, early resolution of risk may reduce anxiety. However, anxiety is plausibly more important when risk must be endured for a long time. Therefore, the risk premium required for bearing a lottery is greater the longer is the time that the individual has to live with the anxiety of not knowing how the lottery will be resolved. In other words, the willingness to bear a given risk declines as the date of resolution approaches, a form of dynamic inconsistency. However, such dynamic inconsistency is precluded when utility is recursive and thus anxiety cannot be a rationale for a timing premium given the utility functions considered here. (This argument is due to Grant et al, 2000; Caplin and Leahy, 2001; and Epstein, 2008.) To the extent that introspection is based in part on considerations of anxiety, stated timing premia overstate premia that are consistent with LRR.

<sup>6</sup>Ergin and Sarver (2012) characterize behavior, in terms of choice between ‘lotteries over sets of lotteries’, that indicates (or can be represented via) a hidden planning problem. It remains to see if this work will help in assessing the magnitudes of timing premia.

For perspective, note that modeling nonindifference to the temporal resolution of risk is the *objective* in the Kreps and Porteus papers. Such nonindifference is plausible in theory as a property of ‘rational’ preferences. Further, Epstein and Zin (1989) show that permitting a nonzero timing premium has the modeling benefit of allowing a partial separation between EIS and RRA. Thus even if one is skeptical about the descriptive importance of a nonzero timing premium, one might view its use as a *cost* of separating EIS and RRA.<sup>7</sup> How costly is a quantitative question. Similarly, it is a quantitative issue whether nonindifference to timing makes sense as an important component of an empirical model. But applied papers using Epstein-Zin utility have accepted such nonindifference uncritically.

#### IS INTROSPECTION POSSIBLE? USEFUL?

One might question whether introspection is possible or reliable given the artificial nature of the question posed in the thought experiment: how much would you pay to have your lifetime risk resolved next month, keeping in mind that you cannot use that information. But starkness of the question arguably helps introspection. For example, one might feel strongly, as we do, “*why should I give up 25% ... just to know earlier, when I can’t even use that information?*” In fact, it is arguably easier to introspect than if one is allowed to use the information to reoptimize, in which case self assessment of the timing premium would involve introspection about all of substitutability, risk aversion *and* early resolution, as well as about the available financial instruments and more generally the collection of all consumption processes in an expected budget set.

Introspection is at best a matter of opinion and is inherently subjective. While we are not arguing that a consensus is possible, we are hoping that our exercise may help *some* people understand the LRR model more fully. The alternative is to leave the modeling exercise completely undisciplined, which we find unsatisfactory.

#### HOW IS THE TIMING PREMIUM RELATED TO THE WELFARE COST OF RISK?

Perspective on the timing premium is provided by examining also what the representative agent would be willing to pay to eliminate risk entirely. Lucas (1987) introduced such a calculation into macroeconomics as a way to measure the welfare costs of business cycle fluctuations. His conclusion that consumption risk has very small welfare costs stimulated many others to see how different model specifications might lead to larger costs. Our interest here is less in the total cost of risk per se than in using the latter to provide further perspective on the size of the timing premium. Specifically, are the timing premia reported in Table 1 large *relative* to the total welfare cost of risk?

<sup>7</sup>This has always been Epstein’s view.

Consider the deterministic consumption process  $\bar{c} = (\bar{c}_t)$  where, for every  $t$ ,  $\bar{c}_{t+1}/\bar{c}_t$  equals  $E_0(c_{t+1}/c_t)$ , the unconditional mean of consumption growth that is implied by the LRR process (4). The indicated mean is given by

$$\log E_0(c_{t+1}/c_t) = m + a^t x_0 + \frac{\sigma^2}{2} [1 + \varphi \sum_{i=0}^{t-1} a^{2i}].$$

Therefore let  $\bar{c}$  be defined by  $\bar{c}_0 = c_0$  and

$$\log(\bar{c}_{t+1}/\bar{c}_t) = m + a^t x_0 + \frac{\sigma^2}{2} [1 + \varphi \sum_{i=0}^{t-1} a^{2i}].$$

Its utility at time 0 is  $\bar{U}_0$ . Whenever  $\alpha < 1$ , risk is costly ( $\bar{U}_0 > U_0$ ) and the cost may be measured by the *risk premium*  $\bar{\pi}$ , where<sup>8</sup>

$$\bar{\pi} = 1 - U_0/\bar{U}_0.$$

The last row of Table 1 shows the welfare costs implied by the LRR model. For the parameter values used by Bansal and Yaron, an individual giving up roughly 50% of her deterministic consumption  $\bar{c}_t$  in every period would still be no worse off than with the long-run risk process in (4).

It can be verified further that, as one would expect,

$$\bar{U}_0 > U_0^* > U_0 \text{ if } \alpha < 0.$$

This suggests the following decomposition:

$$\frac{U_0}{\bar{U}_0} = \frac{U_0}{U_0^*} \cdot \frac{U_0^*}{\bar{U}_0},$$

whereby the total cost of risk is decomposed into the cost of bearing risk that is resolved ‘late’ (after time 1), and the cost of bearing risk all of which is resolved ‘early’ (at time 1). The relative importance of the first factor is given by

$$\frac{U_0^*/U_0}{\bar{U}_0/U_0} = \frac{U_0^*}{\bar{U}_0} = \frac{1 - \bar{\pi}}{1 - \pi^*}.$$

For the parameter values in Table 1, the indicated ratio is between .62 and .77. Thus, between 2/3 and 3/4 of the cost (in constant consumption perpetuity) of risk is attributable to the cost of late resolution.

<sup>8</sup>Lucas uses  $\frac{\bar{\pi}}{1-\bar{\pi}}$  to measure the benefit of eliminating risk rather than  $\bar{\pi}$  to measure its cost. The difference between the two measures parallels the difference between the compensating variation (used here) and the equivalent variation (used by Lucas) of a policy change.

## WHAT IS THE ROLE OF THE ENDOWMENT PROCESS?

The numbers presented in Table 1 and Figure 1 depend on the parameters of the endowment process and in particular on the degree of persistence.<sup>9</sup> To examine the importance of persistence and to offer perspective on the improved fit of asset market data provided by the LRR model, we compare its timing premia to those implied by the benchmark i.i.d. model.<sup>10</sup> An i.i.d. growth process for consumption is a workhorse model, fits U.S. data well and is hard to distinguish statistically from the LRR process. It is assumed, for example, in Campbell and Cochrane (1999), Calvet and Fisher (2007), where dividends are separated from consumption, and in Barberis et al (2001).

TABLE 2—TIMING PREMIA FOR IID GROWTH RATE

RRA \ EIS	1.5	1	.2	.1
10	9.5 %	7.8%	1.0%	0.0%
7.5	6.8 %	5.6%	0.4%	-0.5%
5	4.3%	3.5%	0.0 %	-0.8%
2	1.2 %	0.9%	-0.9%	-1.1%
1	0.4%	0.0%	-1.0%	-1.2%

Table 2 assumes  $\beta = .998$ , and that  $\log(c_{t+1}/c_t)$  is i.i.d.  $N(m, \sigma^2)$ , with  $m = .0015$  and  $\sigma^2 = .00007$ . These latter values are roughly consistent with the annual mean (1.8%) and standard deviation (2.9%) for real per-capita consumption growth used by Bansal and Yaron (2004) to calibrate their model. Comparison with Table 1 shows that timing premia here are considerably smaller than for the LRR model.

## RARE DISASTERS

Another specification of the endowment process that is prominent in the asset pricing literature is based on rare disasters. Barro (2009) also uses Epstein–Zin utility but he assumes an i.i.d. consumption process where in every period there is a small probability  $p$  of a negative shock that shrinks consumption by the factor  $b_t$ .<sup>11</sup> Specifically, the consumption process has the following representation:

$$\log \frac{c_{t+1}}{c_t} = m + \sigma W_{c,t+1} + \log(1 - b_t)W_{d,t+1},$$

<sup>9</sup>Bansal, Kiku, and Yaron (2012) use higher volatility persistence,  $\nu = .999$ . Our algorithm failed to find a solution to the value function. Hansen, Heaton, Lee, and Roussanov, (2007, Section 5.3) study a continuous time model where volatility follows a Feller square root process; they find an upper bound on volatility persistence beyond which the value function does not exist.

<sup>10</sup>The next subsection examines yet another endowment process.

<sup>11</sup>He follows Barro (2006) and Rietz (1988).

where  $m$  is mean consumption growth,  $W_{c,t+1} \sim N(0, 1)$ ,  $W_{d,t+1} \sim \text{Bernoulli}(p)$ , and  $b_t$  follows a categorical distribution of disaster sizes obtained from data.<sup>12</sup> All those random variables are mutually independent and i.i.d. over time.

Wachter introduces persistence into this model by assuming that the disaster probability varies over time. Specifically, she assumes that  $W_{d,t+1} \sim \text{Bernoulli}(p_t)$ , where  $p_t = 1 - e^{-\lambda t}$  and  $\lambda$  follows the square root process

$$\lambda_{t+1} = (1 - \kappa)\lambda_t + \kappa\bar{\lambda} + \sigma_\lambda\sqrt{\lambda_t}\epsilon_{t+1},$$

where  $\bar{\lambda}$  is the mean value of  $\lambda$ ,  $\kappa$  measures persistence, and  $\sigma_\lambda$  measures the standard deviation.<sup>13</sup>

The parameter values used by Barro (2009) are for an annual frequency:  $RRA = 4$ ,  $EIS = 2$ ,  $p = .017$ ,  $m = .025$ ,  $\sigma = 0.02$ , and  $\beta = 0.951$ . The distribution of  $b_t$  has mean .29, minimum .15, maximum .73 and was obtained from the author. With these parameter values (and  $T = 200$  for the Monte Carlo simulation) the computed value of the timing premium is 18% and the risk premium is 29%.<sup>14</sup>

We discretize the parameter values used by Wachter (2013):  $RRA = 3$ ,  $EIS = 1$ ,  $\bar{\lambda} = 0.0355$ ,  $\kappa = 0.08$ ,  $\sigma_\lambda = 0.067$ ,  $m = 0.0252$ ,  $\sigma = 0.02$ , and  $\beta = 0.988$ . The categorical distribution of  $b_t$  has mean .22, minimum .1, maximum .68 and was obtained from the author. With these parameter values (and  $T = 200$  for the Monte Carlo simulation) the computed value of the timing premium is 42% and the risk premium is 65%.<sup>15</sup> It is instructive to compare these premium values to those obtained in the model without persistence: setting  $\sigma_\lambda = 0$  and  $\kappa = 1$  yields the timing premium of 22% and the risk premium of 46%.<sup>16</sup> Thus, allowing for long run shocks to the probability of disasters heavily inflates both premia.

#### WHAT ABOUT OTHER PREFERENCE PARAMETER VALUES?

Are there parameter values that allow for sensible values of the timing premium and at the same time provide a good fit of the asset pricing data? Surely, setting EIS in the vicinity of the reciprocal of RRA leads to a small timing premium. With a high RRA needed to accommodate a high equity premium, this would require that EIS be significantly below one. However, Bansal and Yaron (2004) point out that in their model, EIS below 1 would lead to excessive levels and/or volatility for the risk-free rate. This is also true in the variable rare disaster model of Wachter (2013). In addition, Barro (2009) points out that EIS below 1 leads

<sup>12</sup>See also Barro and Jin (2011) who fit power laws to the distribution of disaster sizes and Tsai and Wachter (2013) who allow for rare booms as well as rare disasters.

<sup>13</sup>Wachter's model is in continuous time. We use a discretized version of her process.

<sup>14</sup>For comparison, we also computed the values for the monthly parametrization and they are close: 19% and 29% respectively.

<sup>15</sup>As in the case of the Bansal–Yaron model, we compute the value of early resolution by Monte Carlo simulations. To compute the value of gradual resolution we use value iteration.

<sup>16</sup>The difference between these numbers and those that we obtain for Barro's specification can be accounted for by the different preference parameters and slightly different empirical distributions of disaster sizes.

to the counterfactual prediction that an increase in economic uncertainty would lead to an increase in price dividend ratios. It is possible that there exists a model of the endowment process that fits asset pricing data well with EIS smaller than one; however, we are not aware of such a process.

#### MORE GENERAL RISK PREFERENCES

In (1)-(2) and in the Kreps-Porteus model more generally, risk preferences are in the vNM class. Epstein and Zin (1989) describe a more general class of recursive utility functions in which risk preferences that are consistent with the Allais paradox are also permitted. Some of these specifications have been used to address the equity premium and related puzzles (references are given below). Therefore, we explore briefly the quantitative implications of such generalizations for timing premia.

To preserve simplicity while generalizing preferences, as well as for the convenience of closed forms and for the clarity of intuition delivered thereby, we simplify the endowment process and assume that the (log) growth rate is i.i.d. with  $\log(c_{t+1}/c_t)$  distributed as  $N(m, \sigma^2)$ . Generalize (2) and consider utility defined by:

$$(6) \quad \log U_t = (1 - \beta) \log c_t + \beta \log \mu(U_{t+1}).$$

Here  $\mu(\cdot)$  is the certainty equivalent of random future utility using its conditional distribution at time  $t$ .<sup>17</sup> Assume that  $\mu(x) = x$  for any deterministic random variable  $x$ , that  $\mu$  respects first-order and second-order stochastic dominance, and that  $\mu$  is linearly homogeneous (constant relative risk aversion).

It is convenient to use the renormalized certainty equivalent  $\mu^*$ , where for any positive random variable  $X$  and associated distribution,  $\mu^*(\log X) \equiv \log \mu(X)$ . Then (see the appendix) the timing premium is given in closed-form by  $\pi^* = 1 - \exp(-\beta\Delta)$ , where

$$(7) \quad \Delta \equiv \mu^*(\sum_0^\infty \beta^t \log(c_{t+1}/c_t)) - (1 - \beta)^{-1} \mu^*(\log(c_1/c_0)).$$

For the expected utility-based certainty equivalent,  $\mu^*(\log X) = \frac{1}{\alpha} \log E(X_{t+1}^\alpha)$ , and one obtains the Epstein-Zin implied timing premium; denote the corresponding  $\Delta$  by  $\Delta_{EZ}$ .

As an alternative, consider the following *disappointment aversion* certainty equivalent:<sup>18</sup> Fix  $0 < \gamma \leq 1$ , and for any positive random variable  $X$  (with

<sup>17</sup>In general it depends on the information at  $t$ , but with the i.i.d. assumption such time dependence can be safely suppressed.

<sup>18</sup>The model is due to Gul (1991). For applications to finance, see Epstein and Zin (2001), Ang et al (2005). Routledge and Zin (2010) present and apply a generalization, which is investigated further empirically by Bonomo et al (2011). See also Epstein and Zin (1990) and Bekaert et al (1997).

distribution  $P$ ), define  $\mu_{da}(X)$  implicitly by

$$\log \mu_{da}(X) = E \log(X) - (\gamma^{-1} - 1) \int_{x \leq \mu_{da}(X)} (\log \mu_{da}(X) - \log x) dP(x),$$

or equivalently, (let  $Y = \log X$  and  $Q$  its induced distribution),

$$(8) \quad \mu_{da}^*(Y) = EY - (\gamma^{-1} - 1) \int_{y \leq \mu_{da}^*(Y)} (\mu_{da}^*(Y) - y) dQ(y).$$

The interpretation is that outcomes of  $X$  that are disappointing because they fall below the certainty equivalent are penalized relative to  $E \log(X)$ . If  $\gamma = 1$ , then  $\mu(X) = E \log(X)$  and, when substituted into (6), one obtains the expected utility model where  $RRA = EIS = 1$ . Accordingly, nonindifference to timing arises herein only from the disappointment factor when  $\gamma < 1$ . Because the latter adds to risk aversion, the effective degree of risk aversion is greater than 1. We compare this way of increasing risk aversion to using Epstein-Zin utility with  $\alpha < 0$ .

We show in the appendix that the difference  $\Delta$  in (7), written now  $\Delta_{da}$ , can be expressed in the form

$$(9) \quad \Delta_{da} = \frac{m - \mu_{da}^*(\log(c_1/c_0))}{1 - \beta} \left( 1 - \left[ \frac{1 - \beta}{1 + \beta} \right]^{1/2} \right),$$

which expression involves the certainty equivalent of the single period gamble only. Compare  $\Delta_{EZ}$  and  $\Delta_{da}$  to see the differing implications for timing premia of the expected utility versus disappointment aversion risk preferences. A meaningful comparison requires that the respective parameters  $\alpha$  and  $\gamma$  be suitably related. For example, suppose that the two certainty equivalents assign the same value to the distribution of  $\log(c_1/c_0)$ . Then substitute  $\mu_{da}^*(\log(c_1/c_0)) = m + \frac{1}{2}\alpha \cdot \sigma^2$  into (9) to deduce that

$$(10) \quad \Delta_{da} = \left( 1 - \left[ \frac{1 - \beta}{1 + \beta} \right]^{1/2} \right) \frac{1 + \beta}{\beta} \Delta_{EZ} \simeq 2\Delta_{EZ}.$$

Roughly, disappointment aversion implies timing premia twice as large as those reported in Table 2 when  $\gamma$  is calibrated as described to  $\alpha = -9, -4, -1$ .

An alternative calibration is to assume that the two certainty equivalents assign the same value to the distribution of  $\sum_0^\infty \beta^t \log(c_{t+1}/c_t)$ . Then similar reasoning leads to the relation

$$(11) \quad \Delta_{da} = \frac{1}{\beta} \left( \left[ \frac{1 + \beta}{1 - \beta} \right]^{1/2} - 1 \right) \Delta_{EZ} \simeq 30\Delta_{EZ},$$

and hence to much larger timing premia under disappointment aversion. (For example, the timing premium for  $\gamma$  that corresponds to  $\alpha = -1$  is about 23%.) Thus with either calibration, timing premia are larger than with Epstein-Zin utility.

The appendix shows that (10) and (11) are valid for a broader class of risk preferences.

### III. Concluding Remarks

Though risk aversion and the elasticity of intertemporal substitution have been the subjects of careful scrutiny when calibrating preferences, the long-run risks literature and the broader literature using recursive utility to address asset pricing puzzles have ignored the full implications of their parameter specifications. Recursive utility implies that the temporal resolution of risk matters and a quantitative assessment of how much it matters should be part of the calibration process. This paper is not intended to provide an exhaustive or definitive assessment of parameters used in the literature. Its objective is to give a sense of the magnitudes of implied timing premia and to inject temporal resolution of risk into the discussion of the quantitative properties of LRR and related models.

Timing premia depend on both the parameters of preference and on the nature of the endowment process. In the latter connection, we have demonstrated that, given Epstein-Zin utility, high persistence of the consumption process, as assumed in the LRR literature or in a version of the rare disaster model (Wachter, 2013), inflates timing premia to levels that seem implausible to us based on introspection (20-30% in the former case and 40% in the latter case). Though some may disagree with this admittedly subjective judgement, we believe that we have at least alerted readers to the need to be more cautious when calibrating asset pricing models that rely on nonindifference to temporal resolution as a key component. There are endowment and parameter specifications that imply much smaller timing premia, but while they can account for some asset pricing moments, they yield counterfactual predictions for others. Another alternative is to seek a different model of preference. In Epstein-Zin utility (1) and (2), the two parameters  $\alpha$  and  $\rho$  govern three seemingly distinct aspects of preference, with the result that setting them to match values for EIS and RRA yields timing premia that are beyond direct control of the analyst. This limitation has been recognized from the start in (1989), but this paper may provide renewed impetus to the search for a more flexible model of preference.

For other thought experiments that reflect on parameter values in the LRR model, see D'Addona and Brevik (2011) and Ai (2007). D'Addona and Brevik assert that an agent with Epstein-Zin utility achieves higher utility levels if he can commit to ignoring information about the state variable  $x_t$  appearing in (4). Though they describe their results as concerning information, their analysis does not admit that interpretation: instead of changing the information structure of the agent, they endow the agent with a different consumption process that does

not involve long-run risk (and has the appropriately adjusted unconditional variance). Thus, they de facto study aversion to autocorrelation of consumption instead of the (conceptually distinct) preference for ignoring information or non-indifference to the temporal resolution of risk. In a continuous-time economy with production, Ai (2007) considers the preference for early resolution from a quantitative perspective by asking how much consumption the agent is willing to forgo to learn perfectly the autocorrelated component of the production process instead of having just a noisy signal of it. Our starker thought experiment where early resolution means that all risk is fully resolved, and the discrete-time exchange economy setting, arguably permit a sharper focus and make it easier for introspection to operate. (See Section II for our related comments on whether introspection is useful.)

An important alternative to models based on recursive utility is the external habits model of Campbell and Cochrane (1999). Corresponding scrutiny of that model seems in order. Thus far plausibility of the habits formation process assumed for the representative agent has been judged solely by how it helps to match asset market data. The discipline urged by Mehra and Prescott (1985) suggests that at least one should examine also whether it seems plausible based on introspection about the quantitative effects of past consumption on current preferences. The difficulty of finding market-based evidence concerning external habits, or about timing premia, does not justify leaving them as "free parameters."

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## APPENDIX: DETAILS FOR MORE GENERAL RISK PREFERENCES

To derive (7), use the fact that utilities are given by

$$\begin{aligned}\log U_0 &= \log c_0 + \beta \left[ \frac{1}{1-\beta} \mu^* (\log(c_1/c_0)) \right] \\ \log U_0^* &= \log c_0 + \beta \left[ \mu^* (\sum_0^\infty \beta^t \log(c_{t+1}/c_t)) \right],\end{aligned}$$

Let  $Y = \log(c_1/c_0)$  and  $Y' = \sum_0^\infty \beta^t \log(c_{t+1}/c_t)$ . They are distributed as  $N(m, \sigma^2)$  and  $N\left(\frac{m}{1-\beta}, \frac{\sigma^2}{1-\beta^2}\right)$  respectively. Therefore,

$$Y'' \equiv (1-\beta^2)^{1/2} Y' - m \left( \left[ \frac{1+\beta}{1-\beta} \right]^{1/2} - 1 \right) \text{ is } N(m, \sigma^2).$$

Because  $\mu_{da}(Y'')$  and  $\mu_{da}(Y)$  depend only on the distributions of  $Y''$  and  $Y$ , they must be equal. Note that  $\mu_{da}^*$  satisfies: for all  $\lambda \geq 0$ ,

$$(A1) \quad \mu_{da}^*(Y + \lambda) = \mu_{da}^*(Y) + \lambda \text{ and } \mu_{da}^*(\lambda Y) = \lambda \mu_{da}^*(Y),$$

that is, it exhibits both CARA (constant absolute risk aversion) and CRRA (constant relative risk aversion). Conclude that the two certainty equivalent values appearing in (7) are related by the equation

$$(A2) \quad \begin{aligned} & (1-\beta^2)^{1/2} \mu_{da}^* (\sum_0^\infty \beta^t \log(c_{t+1}/c_t)) - \mu_{da}^* (\log(c_1/c_0)) \\ &= m \left( \left[ \frac{1+\beta}{1-\beta} \right]^{1/2} - 1 \right).\end{aligned}$$

The preceding, and hence also equations (10) and (11), rely only on lognormality and on the fact that  $\mu_{da}^*$  satisfies (A1). Thus the comparative analysis of timing premia applies to any certainty equivalent function satisfying the latter properties. For example, it applies also to the following generalization of (8):

$$\mu_{gda}^*(Y) = EY - (\gamma^{-1} - 1) \int_{y \leq \delta \mu_{gda}^*(Y)} (\delta \mu_{gda}^*(Y) - y) dQ(y),$$

where  $0 < \delta \leq 1$ . Here outcomes are disappointing if they are smaller than the fraction  $\delta$  of the certainty equivalent. This generalization of disappointment aversion (which corresponds to the special case  $\delta = 1$ ) is in the spirit of that provided by Routledge and Zin (2010). (In our setting, their model would take the form  $\mu_{RZ}^*(Y) = EY - (\gamma^{-1} - 1) \int_{y \leq \log \delta + \mu_{RZ}^*(Y)} (\log \delta + \mu_{RZ}^*(Y) - y) dQ(y)$ , which violates the second condition in (A1).)