"Identifying Heterogeneous Decision Rules From Choices When Menus Are Unobserved"
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Supplementary Appendix: Two examples

The following two examples relate to questions that arise from Barseghyan et al (2021) and Azrieli and Rehbeck (2023) respectively.

Example 1 (singleton menus): If $\mathcal{A}$ includes all singletons and there is complete ignorance about menus, then $C_d = X$ for all d and any λ is rationalized by any Q. Barseghyan et al (2021) assume that the minimum menu size is at least two to avoid this scenario in their setup. However, in our setup we can allow a subset of all singleton menus, in which case they can affect (strictly expand) the identified region.

We illustrate here for the case where every decision rule d is derived from maximization of a preference order, and where complete ignorance is assumed. Then each Q describes a probability distribution over preferences. Let $X = \{a, b, c\}$. The six possible preference orders are:

$$\begin{gathered} a \succ_1 b \succ_1 c, a \succ_2 c \succ_2 b \\ b \succ_3 a \succ_3 c, b \succ_4 c \succ_4 a \\ c \succ_5 a \succ_5 b, c \succ_6 b \succ_6 a. \end{gathered}$$

Finally, let $\mathcal{A} = \{\{a\}, \{a, b\}, \{a, b, c\}\}$.[1] Then, after deleting redundant inequalities, (2.9) reduces to:

$$\begin{aligned} \lambda(a) &\geq Q(\succ_1) + Q(\succ_2) \\ \lambda(\{a, b\}) &\geq Q(\succ_1) + Q(\succ_2) + Q(\succ_3) + Q(\succ_4) \\ \lambda(\{a, c\}) &\geq Q(\succ_1) + Q(\succ_2) + Q(\succ_5) \end{aligned}$$

The first inequality provides an upper bound on the probability of the set of preferences that rank alternative a highest, the second provides an upper bound on the probability of preferences that rank a or b highest, and the third gives an upper bound on the probability of preferences that rank a above b.

Consider now what happens if the singleton $\{a\}$ is deleted from $\mathcal{A}$. Let $\mathcal{A}' = \{\{a, b\}, \{a, b, c\}\}$ be the new set of menus. Then the following *additional* inequalities are implied by (2.9):

$$\lambda(b) \geq Q(\succ_3) + Q(\succ_4)$$

[1] Thus menus are nested. Think, for example, of expanding budget sets.

$$\lambda(\{b,c\}) \geq Q(\succ_3) + Q(\succ_4) + Q(\succ_6)$$

As a result, the sharp identification region shrinks strictly. (The intuition is that when $\{a\}$ is removed, then the sets C_3 and C_4 shrink, leading to the lower bound for $\lambda(b)$, and C_6 shrinks, which leads to the lower bound for $\lambda(\{b,c\})$.)

For a numerical example, take $\lambda(a) = 1/2, \lambda(b) = 1/4$, and $\lambda(c) = 1/4$. The preference distribution given by $Q(\succ_1) = Q(\succ_3) = Q(\succ_4) = Q(\succ_6) = 1/4$ rationalizes λ when $\{a\}$ is included, but not if it is removed. The presence of a singleton menu does not preclude meaningful inference, but it does weaken inference by expanding the sharp identification region.

Example 2 (menu homogeneity): Menu-rationalizability as defined in (2.10) permits heterogeneity in both decision rules *and* in the menu formation processes, the latter because π_d and $\pi_{d'}$ are allowed to differ. Refer to *menu-homogeneity* if $\pi_d = \pi$ for all d. This hypothesis has been adopted in several applied works where one can interpret the different menus as arising from feasibility rather than consideration (Tenn and Yun 2008, Tenn 2009, Conlon and Mortimer 2013, Lu 2022), and in the theoretical contribution by Azrieli and Rehbeck (2023, section 4), while its limitations have been noted by Barsheghyan et al (2021). Where menus are based on consideration one would expect them to depend on preference (or decision rule), as in the applied papers by Goeree (2008), and Abaluck and Adams-Prassl (2021). Here we demonstrate that imposing menu-homogeneity can lead to different conclusions about the sharp identified set.

Let $X = \{a,b,c\}$, and assume preference maximization, with only two possible preference orders

$$a \succ_1 b \succ_1 c, \;\; a \succ_2 c \succ_2 b.$$

Finally, let $\mathcal{A} = \{\{a,b\},\{a,c\},\{b,c\},\{a,b,c\}\}$, and let the empirical measure be given by

$$\lambda(a) = \lambda(b) = \lambda(c) = 1/3.$$

Then λ can be rationalized by Q, where $Q(\succ_1) = 2/3$ and $Q(\succ_2) = 1/3$, because the inequalities (2.9) can be verified. A corresponding version of (2.10) uses the (heterogeneous) distributions over menus given by

$$\pi_1(\{b,c\}) = 1/2 \text{ and } \pi_2(\{b,c\}) = 1.$$

However, Q cannot rationalize λ if one insists on menu-homogeneity: Under the latter condition, (2.10) implies

$$\lambda(b) = Q(\succ_1)[\pi(\{b,c\})] = 1/3$$

$$\lambda(c) = Q(\succ_2)[\pi(\{b,c\})] = 1/3,$$

which would force Q to assign equal probabilities to both preferences, a contradiction.

In general, any restriction on the distributions over menus admitted in (2.10) makes menu-rationalizability more difficult and thus shrinks the sharp identified set. The example confirms that in the case of menu-homogeneity the shrinkage can be strict. (Finally, note that in the example the sharp identification set with menu-homogeneity is not empty. For the two preference orders given, the unique rationalizing measure Q assigns equal probability to the two preference orders.)

References

[1] Conlon C.T. and Mortimer J.H. Demand estimation under incomplete product availability. *AEJ: Micro* (2013) 5, 1-30.

[2] Tenn S. Demand estimation under limited product availability. *Appl. Econ. Letters* (2009) 16, 465-468.

[3] Tenn S. and Yun J. Biases in demand analysis due to variation in retail distribution. *Int. J. Ind. Organ.* (2008) 26, 984-997.