"Identifying Heterogeneous Decision Rules From Choices When Menus Are Unobserved"
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Supplementary Appendix

This appendix provides an example that relates to a question arising from Barseghyan et al (2021) and proof for an assertion in section 2.2 regarding which data λ can be rationalized under complete ignorance.

Example (singleton menus): If $\mathcal{A}$ includes all singletons and there is complete ignorance about menus, then any alternative can be chosen by any decision rule. Formally, let $C_d = \{d(A) : A \in \mathcal{A}\}$; that is, the set of all alternatives that can be chosen by decision rule d. We have that $C_d = X$ and any λ is rationalized by any Q. Barseghyan et al (2021) assume that the minimum menu size is at least two to avoid this scenario in their setup. However, in our setup we can allow a subset of all singleton menus, in which case they can affect (strictly expand) the identified region.

We illustrate here for the case where every decision rule d is derived from maximization of a preference order, and where complete ignorance is assumed. Then each Q describes a probability distribution over preferences. Let $X = \{a, b, c\}$. The six possible preference orders are:

$$\begin{gathered}
a \succ_1 b \succ_1 c, a \succ_2 c \succ_2 b \\
b \succ_3 a \succ_3 c, b \succ_4 c \succ_4 a \\
c \succ_5 a \succ_5 b, c \succ_6 b \succ_6 a.
\end{gathered}$$

Finally, let $\mathcal{A} = \{\{a\}, \{a, b\}, \{a, b, c\}\}$. Then, after deleting redundant inequalities, (2.10) reduces to:

$$\begin{aligned}
\lambda(a) &\geq Q(\succ_1) + Q(\succ_2) \\
\lambda(\{a, b\}) &\geq Q(\succ_1) + Q(\succ_2) + Q(\succ_3) + Q(\succ_4) \\
\lambda(\{a, c\}) &\geq Q(\succ_1) + Q(\succ_2) + Q(\succ_5)
\end{aligned}$$

The first inequality provides an upper bound on the probability of the set of preferences that rank alternative a highest, the second provides an upper bound on the probability of preferences that rank a or b highest, and the third gives an upper bound on the probability of preferences that rank a above b.

Consider now what happens if the singleton $\{a\}$ is deleted from $\mathcal{A}$. Let $\mathcal{A}' = \{\{a, b\}, \{a, b, c\}\}$ be the new set of menus. Then the following *additional* inequalities are implied by (2.10):

$$\lambda(b) \geq Q(\succ_3) + Q(\succ_4)$$
$$\lambda(\{b,c\}) \geq Q(\succ_3) + Q(\succ_4) + Q(\succ_6)$$

As a result, the sharp identification region shrinks strictly. The intuition is that when $\{a\}$ is removed, then the sets C_3 and C_4 shrink, leading to the lower bound for $\lambda(b)$, and C_6 shrinks, which leads to the lower bound for $\lambda(\{b,c\})$.

For a numerical example, take $\lambda(a) = 1/2, \lambda(b) = 1/4$, and $\lambda(c) = 1/4$. The preference distribution given by $Q(\succ_1) = Q(\succ_3) = Q(\succ_4) = Q(\succ_6) = 1/4$ rationalizes λ when $\{a\}$ is included, but not if it is removed. The presence of a singleton menu does not preclude meaningful inference, but it does weaken inference by expanding the sharp identification region.

Claim: If $\Pi_d = \Delta(\mathcal{A})$ for each d, then every λ with support in $\cup_d\{d(A) : A \in \mathcal{A}\}$ can be rationalized by some Q.

Proof: Let λ be such that

$$\text{supp}(\lambda) \subset \cup_d\{d(A) : A \in \mathcal{A}\}.$$

Partition the support of λ as follows. For each $a \in \text{supp}(\lambda)$, let $D(a)$ denote the (nonempty) set of decision rules d for which there exists some menu $A \in \mathcal{A}$ with $d(A) = a$. Fix a selection function s that assigns to each $a \in \text{supp}(\lambda)$ one decision rule $s(a) \in D(a)$, and define an equivalence relation $\sim$ on $\text{supp}(\lambda)$ by

$$a \sim a' \quad \text{if and only if} \quad s(a) = s(a').$$

For each decision rule $d \in \mathcal{D}$, let

$$\mathcal{P}(d) \equiv \{a \in \text{supp}(\lambda) : s(a) = d\}.$$

The collection $\{\mathcal{P}(d)\}_{d\in\mathcal{D}}$ is the partition of $\text{supp}(\lambda)$ induced by $\sim$.

For every d, let[1]

$$Q(d) = \sum_{a\in\mathcal{P}(d)} \lambda(a).$$

[1]The sum is 0 if $\mathcal{P}(d)$ is empty.

For every $a \in X$, define $\pi_d \in \Delta(\mathcal{A})$ by

$$Q(d) \cdot \pi_d(\{A : d(A) = a\}) = \lambda(a) \quad \text{if } a \in \mathcal{P}(d)\,, \text{ and}$$
$$\pi_d(\{A : d(A) = a\}) = 0 \qquad \text{if } a \notin \mathcal{P}(d)\,. \tag{0.1}$$

Fix a such that $\lambda(a) > 0$. Let d_a be such that a is associated (uniquely) with d_a, (equivalently, $a \in \mathcal{P}(d)$). Then

$$\lambda(a) = Q(d_a) \cdot \pi_{d_a}(\{A : d_a(A) = a\})\,.$$

It follows from (0.1) that

$$\lambda(a) = \sum_d Q(d)\, \pi_d(\{A : d(A) = a\})\,.$$

This equality is valid also when $\lambda(a) = 0$, in which case both sides equal zero. Thus Q rationalizes λ given complete ignorance. ■