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DECISION MAKING AND THE TEMPORAL RESOLUTION OF UNCERTAINTY*

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1. INTRODUCTION

There exist in the literature many two-period analyses of behavior under uncertainty. (See, for example, Sandmo [1970] and [1971], Rothschild and Stiglitz [1971], Turnovsky [1973] and Epstein [1975, 1978].) These works typically assume that a decision must be made in period 1 subject to uncertainty about the environment that will prevail in period 2. At the start of period 2 the true state of the environment becomes known and perhaps some further decisions are made. The effects on the period 1 decision of the prior uncertainty in expectations are closely examined.

Clearly the above framework is inadequate for modelling the more general situation where $n>1$ decisions are made sequentially and subject to improving information about the eventual state of the world. In this general framework the influence on decisions of the way in which uncertainty is resolved through time represents an interesting area of investigation. This paper undertakes such an investigation in the context of several specific decision problems.

For simplicity (see footnote 4), we adopt a minimal extension of the two-period model that makes possible the analysis of the temporal resolution of uncertainty, namely, a three-period model. Thus the decision-making framework considered in this paper may be described as follows: an expected utility maximizing agent faces a three-period planning horizon. He makes decisions in each period. Period 3 decisions are made after all uncertainty has been resolved. In period 1 the decision is made subject to prior expectations about the state of the world that will prevail in period 3. The uncertain future environment is represented by a random variable Z. Before the start of period 2 new information about the ultimate value of Z becomes available. The information is forthcoming through the observation of another random variable Y which in general is correlated with Z. The agent is a Bayesian decision maker and revises his prior probability distribution about Z after observing Y. The amount of additional information about Z provided by Y is a parameter in the model.

The objective of the paper is to compare the decisions in period 1 in two choice problems that differ only with respect to the amount of information provided by Y about Z. Note that in both problems the agent faces the identical prior uncertainty about Z. The difference in the two problems is only in the amount of

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information he can expect to gain about Z before his period 2 decision is made. (Two extreme cases are (a) no additional information, and (b) perfect information. In the latter case the period 2 decision is made under certainty and we have a two-period framework.) Thus it seems justifiable to describe the analysis below as an investigation of the effects on decision making of the temporal resolution of uncertainty. We will adopt the following terminology: if Y' contains more (less) information about Z than does Y, uncertainty is considered to be resolved earlier (later) in the decision problem characterized by Y' than in that characterized by Y.[2]

The notion of greater information adopted in this paper is that due to Blackwell [1951, 1953] and Marschak and Miyasawa [1968], and employed by Kihlstrom [1974] and Grossman, Kihlstrom and Mirman [1977]. Section 2 describes the notion briefly, relying heavily on Marschak and Miyasawa (MM). Of particular importance is Theorem 1 which describes the way in which the behavioral implications of greater future information may be determined. (It plays an analogous role here to that played by the Rothschild and Stiglitz result [1971, p. 67] in the comparative statics analysis of greater uncertainty.)

Note the difference between our study and the two studies above which employ the same definition of greater information. In the latter, the amount of information is endogenous and determined by the agent subject to the costs of acquisition. Here, the amount of information is exogenous to the agent and is forthcoming automatically with the passage of time.[3] Partly as a result of our simpler framework we are able, unlike the other studies, to derive comparative statics propositions which are perfectly general in the sense that they do not depend on a particular specification of the joint probability distribution of Y and Z.

On intuitive grounds one would expect the prospect of greater future information to increase the incentive to maintain some flexibility in period 2 to take advantage of the information. This incentive would discourage the adoption in period 1 of decisions that would limit the agent's options in period 2. Such a behavioral response to greater information is readily established in a model where a totally irreversible decision, i.e., one that fixes the decision in period 2 as well, is in the agent's period 1 choice set. It is shown in Section 3 that the prospect of greater future information discourages the adoption of an irreversible decision. The result is trivial to derive and the model is not of much interest in itself. It is included in order to provide some perspective regarding the other, nontrivial results in the paper that concern less extreme irreversibilities. For example, in

[2] We emphasize that by uncertainty resolving earlier given Y rather than Y' we do not mean that the decision maker learns Y earlier in the sequence of decision making.

[3] More generally we might assume an exogenous cost of information acquisition and investigate the consequences of variations in that cost. Our results, suitably modified, will remain valid if: (i) information costs enter the individual's objective function in an additively separable fashion, and (ii) the demand for information is a downward sloping function of the acquisition cost. This is the case for a reasonable extension of the models in Sections 5 and 7. (See also Kihlstrom [1976].)

Sections 4 and 5 the intuitively expected results are derived in more interesting models but only for a more restrictive class of (intertemporally additive) utility functions than considered in Section 3. A simple consumption-savings model is considered in Section 6. Consumption of period 1 wealth is an irreversible decision in that it limits future consumption-savings options. However, optimal first period consumption does not unambiguously fall in response to an earlier resolution of uncertainty. The corresponding response in the model of the firm analyzed in Section 7 is similarly ambiguous. In both cases we are able to determine the parameters upon which the qualitative responses depend.

Also undertaken in the context of the last two models is a comparison between the qualitative effects of (i) an earlier resolution of a given prior uncertainty, and (ii) a reduction in the prior variability of expectations for a given structure of resolution over time. Clearly the expected utility of all risk averters is unambiguously raised by either change in expectations. One might consequently be led to guess that (i) and (ii) would have similar qualitative impacts on behavior. In fact this is false. We show that there does not exist a general relationship between the qualitative effects of (i) and (ii) on period 1 decisions. Related is the observation that the behavioral consequences of reduced prior variability depend on the way in which the uncertainty is resolved over time.

A few words are in order concerning the literature on the structure of preferences in problems of dynamic choice. Mossin [1969], Drèze and Modigliani [1972] and Spence and Zeckhauser [1972] discuss the preference for random income streams induced from primitive preference for consumption streams defined by a von Neumann-Morgenstern utility index. They observe that such induced preference is not consistent with the von Neumann-Morgenstern axioms and in particular that the timing of the resolution of uncertainty concerning income can be important. Kreps and Porteus [1978a] generalize the von Neumann-Morgenstern axioms to permit the temporal resolution of uncertainty to matter. In [1978b] the authors compare the generalized preference structure with induced preference for random income streams. Kreps [1978] characterizes a "preference for flexibility" which is closely related to the preference structure adopted in this paper. Finally, Selden [1978] provides an alternative axiomatization of preference for dynamic choice problems. This paper investigates the behavioral consequences of the manner in which uncertainty resolves given that preference for random variables is induced from a primitive von Neumann-Morgenstern utility index. A comparable analysis in the context of one of the alternative preference structures mentioned above would be an interesting subject for future research.

2. THE GENERAL DECISION PROBLEM

This paper considers some special instances of the following general decision problem:[4]

$$\max_{x_1 \in C_1} E_Y \max_{x_2} \{E_{Z/Y} U(x_1, x_2, Z) \mid x_2 \in C_2(x_1)\}. \tag{1}$$

x_1 and x_2 are real scalars that represent period 1 and period 2 decision variables respectively. C_1 and $C_2(x_1)$ are convex subsets of the non-negative real line with nonempty interiors.[5] U is a von Neumann-Morgenstern utility index that is concave and twice continuously differentiable in (x_1, x_2). Z is a random variable (r.v.) reflecting the agent's subjective uncertainty about his future economic environment. The true value of Z becomes known at the end of period 2. Before x_2 is chosen, however, the agent gains some information about Z by observing a r.v. Y which in general is correlated with Z. The individual is a Bayesian decision maker. His prior probability distribution for Z, held at the start of period 1, is revised according to Bayes' Rule after observing Y. x_1 is chosen before observing Y. However, the prospect of future information about Z is, of course, taken into account by the agent when choosing x_1. $C_2(x_1)$ represents a constraint set, in general depending on x_1, which faces the agent in choosing x_2.

The objective of the paper is to investigate how the choice of x_1 is influenced by varying amounts of information provided by Y about Z. We adopt the notion of "greater information" due to Blackwell [1951] and [1953] and discussed by MM [1968]. We follow the MM discussion. Adopting their notation we may briefly describe the results of their analysis which are important below. (For further details the reader should refer directly to their work.)

Y and Z are assumed to be discrete r.v.'s with possible realizations $(y_1,\dots, y_n)$ and $(z_1,\dots, z_m)$ respectively. The set $\{y_1,\dots, y_n\}$ is called the set of possible messages corresponding to Y. The corresponding probability vectors are $q^T = (q_1,\dots, q_n)$, $q_i = \Pr(Y=y_i)$ and $r^T = (r_1,\dots, r_m)$, $r_i = \Pr(Z=z_i)$.[6] The likelihood matrix is denoted $\Lambda = (\lambda_{ij})$, where $\lambda_{ij} = \Pr(Y=y_j|Z=z_i)$ and the posterior probability distribution is denoted $\Pi = (\pi_{ij})$, $\pi_{ij} = \Pr(Z=z_i|Y=y_j)$. Y is often called an experiment. If Y' is another experiment, $(y'_1,\dots, y'_{n'})$, q', Λ' an Π' are defined

[4] The general multi-period problem involves a sequence of decisions x_t and experiments represented by Y_t, $t=1, 2,\dots, T$, with x_t chosen after the realization of Y_{t-1} but before the realization of Y_t. Y_t generally provides information about each Y_{t+k}, $k>0$. The effects of a change in the information structure may be determined by extensions of the arguments in this paper. As an example consider the consequences of varying the informativeness of Y_1 with respect to Y_2 alone keeping unchanged all other conditional distributions $Y_t/(Y_{t-1},\dots, Y_1)$, $t>2$. The effects of such a change may be analyzed in a three period framework where U is a utility function derived from the underlying multi-period utility function by standard dynamic programming arguments.

[5] The single exception is in Section 3 where x_1 and x_2 may assume discrete values only.

[6] All vectors are column vectors unless transposed by a superscript T. Note also that partial derivatives are denoted throughout by appropriate subscripted variables.

in an obvious fashion. Z and its prior probability distribution r are fixed and the effects of a change in experiment from Y to Y' are considered. (Clearly, therefore, $q^T = r^T \Lambda$, $q'^T = r^T \Lambda'$, and $\Pi q = \Pi' q' = r$.)

Y is said to be more informative than Y' if every user of information about Z is at least as well off observing Y before making a decision as he would be if he based his decision on an observation of Y'. This definition may be made more precise in the context of decision problems of the type (1). Let $S^{m-1} = \{\xi = (\xi_1, \ldots, \xi_m)/\xi_i \geq 0, \sum \xi_i = 1\}$,

$$J(x_1, \xi) \equiv \max_{x_2} \{\sum_i \xi_i U(x_1, x_2, z_i) \mid x_2 \in C_2(x_1)\}. \tag{2}$$

Denote by π_j and π'_j the j-th columns of Π and Π' respectively. Then Y is more informative then Y' if and only if:

(D.1) $\sum_j q'_j J(x_1, \pi'_j) \leq \sum_j q_j J(x_1, \pi_j)$, for all x_1, U and C_2 for which the maximum in (2) exists.[7] (Two extreme experiments are of interest. Y provides no information about Z if Y and Z are stochastically independent. Y provides perfect information if Z conditional on Y is certain.)

Several alternative characterizations of D.1 are described by MM.[8] One (due initially to Blackwell) which is important for the purpose of comparative statics analyses is the following (Theorem 12.1): Y is more informative than Y' if and only if

$$\text{(D.2)} \quad \sum_{j=1}^{n} q_j \rho(\pi_j) \geq \sum_{j=1}^{n'} q'_j \rho(\pi'_j), \qquad \text{for any convex function } \rho \text{ on } S^{m-1}.$$

The application of this characterization in this paper stems from the following theorem. It is assumed that the maxima in (1) exist and are unique and for simplicity, that the optimal x_1 lies in the interior of C_1. The function $J(x_1, \xi)$ defined by (2) is assumed to be concave and differentiable with respect to x_1. (That will be the case if U and C_2 are "well-behaved" as we assume in all the examples considered below. For example, $J(x_1, \xi)$ is (strictly) concave in x_1 if $U(x_1, x_2, z)$ is (strictly) concave in x_1, x_2 and if $C_2(x_1)$ satisfies the following condition:

$$x_2 \in C_2(x_1),\ t_2 \in C_2(t_1) => \lambda x_2 + (1-\lambda) t_2 \in C_2(\lambda x_1 + (1-\lambda) t_1), \quad 0 \leq \lambda \leq 1.$$

[7] The relationship of this notion of information to entropy is clarified by Marschak [1974] and Marschak and Miyasawa [1968]. Specific parameterizations of the amount of information, consistent with the general definition, are adopted by Kihlstrom [1974], Grossman, Kihlstrom and Mirman [1977], Bradford and Kelejian [1977] and Murota [1976]. Bradford and Kelejian assume essentially that $Y = Z + \delta$ where δ, the observation or prediction error, is normally and independently distributed with zero mean. The amount of information provided by Y varies inversely with the variance of δ. Murota assumes $n = m = 3$, $(z_1, z_2, z_3) = (y_1, y_2, y_3)$ and $\Pr(Y = y_i \mid Z = z_j) = 1 - 2\varepsilon$ for $i = j$ and $= \varepsilon$ for $i \neq j$. The amount of information varies inversely with ε.

[8] Green and Stokey [1978] relate these characterizations of "more informative" to another involving the set S of partitions of the set of possible outcomes of a given experiment. S is naturally ordered by the criterion of refinement.

This condition is satisfied if $C_2(x_1)=\{x_2|f(x_1, x_2)\geq 0\}$, f concave.)

THEOREM 1. *Let x_1^* and x_1^{**} be the solutions of* (1) *given Y and Y' respectively, where Y is more informative than Y'. Let $J(x_1, \xi)$, $\xi \in S^{m-1}$, be defined by* (2). *If $J_{x_1}(x_1^*, \xi)$ is concave (convex) in ξ, then $x_1^* \leq (\geq) x_1^{**}$. If $J_{x_1}(x_1^*, \xi)$ is neither convex nor concave, then the sign of $x_1^* - x_1^{**}$ is ambiguous in the following sense: there exist r.v.'s Z, Y, Y' and Y'' such that Y' and Y'' are each less informative than Y and such that the optimal x_1 for Y' exceeds x_1^* and that for Y'' is less than x_1^*.*

PROOF. By assumption x_1^* and x_1^{**} are the unique solutions to

$$\sum q_j J_{x_1}(x_1^*, \pi_j) = 0 \quad \text{and} \quad \sum q_j' J_{x_1}(x_1^{**}, \pi_j') = 0$$

respectively. Suppose $J_{x_1}(x_1^*, \xi)$ is convex in ξ. Since Y is more informative than Y', $0=\sum q_j J_{x_1}(x_1^*, \pi_j)\geq \sum q_j' J_{x_1}(x_1^*, \pi_j')$. Therefore $x_1^{**}\leq x_1^*$. Similarly, $J_{x_1}(x_1^*, \xi)$ concave in $\xi => -J_{x_1}(x_1^*, \xi)$ convex in ξ and it follows that $x_1^{**}\geq x_1^*$. The final assertion in the theorem is analogous to a similar statement in Rothschild and Stiglitz [1971, p. 67] concerning the comparative static effects of increased uncertainty. The proof here is also analogous to the proof of the Rothschild-Stiglitz assertion which is based on the analysis in Rothschild and Stiglitz [1970, p. 240], and would require a straightforward extension of Theorem 12.1 of MM.

Several comments are in order concerning the framework described in (1) and concerning Theorem 1 before proceeding to applications of the latter. First note that the strict inequalities $x_1^* < (>) x_1^{**}$ may be established if $\sum q_j J_{x_1}(x_1^*, \pi_j) < (>) \sum q_j' J_{x_1}(x_1^*, \pi_j')$. An extension of the MM analysis shows that the latter will be the case if the posterior probability distributions π and π' are not equal and if $J_{x_1}(x_1^*, \xi)$ is strictly concave (convex) in ξ, or indeed (see Sections 4 and 5) only suitably nonlinear in ξ.

Second, if the constraint defined by C_1 is binding given either Y or Y', the qualitative relationship described in the theorem between x_1^* and x_1^{**} remains valid. (For example, suppose that the constraint $x_1 \geq 0$ is binding given Y. Then $\sum q_j J_{x_1}(0, \pi_j)\leq 0$. Therefore $J_{x_1}(0, \xi)$ concave (convex)$\Rightarrow \sum q_j' J_{x_1}(0, \pi_j') \geq (\leq) 0 \Rightarrow x_1^{**} \geq (\leq) x_1^* = 0$.) For simplicity we shall assume interior solutions for x_1 in the problems below.

The decision problem (1) is sufficiently general to include models where a third decision x_3 must be made after Z is observed and all uncertainty is removed. The function U in (1) need only be interpreted as a derived utility function, derived from a more basic underlying function after optimizing with respect to x_3. Thus (1) does represent the three period decision problem referred to in the introduction.

Finally, some of the results discussed by MM have been proven for continuous random variables by Blackwell [1951] and [1953]. However, it does not appear that such an extension of the important Theorem 12.1 has been established. Therefore, all r.v.'s in the sequel are assumed to be discrete.

3. AN IRREVERSIBLE DECISION[9]

Consider problem (1) modified so that x_1 and x_2 can each take on the values 0 and 1 only. Choosing $x_1=1$ is an irreversible decision in the sense that $C_2(1)=\{1\}$. The alternative decision $x_1=0$ is such that $C_2(0)=\{0, 1\}$.

Because future information can be of value only if $x_1=0$, the prospect of more information in the future would tend to discourage the adoption of an irreversible descision in period 1. More formally, adopt the notation of Section 2 with Y being more informative than Y' and x_1^* and x_1^{**} the corresponding optimal decisions. We show that $x_1^{**}=0 => x_1^*=0$.

Suppose that $x_1^{**}=0$ and hence that $\sum q_j' J(0, \pi_j') > \sum q_j' J(1, \pi_j')$. By D.1, $\sum q_j J(0, \pi_j) \geq \sum q_j' J(0, \pi_j')$. Also, $\sum q_j' J(1, \pi_j') = \sum_j \sum_i q_j' \pi_{ij}' U(1, 1, z_i) = \sum_i r_i U(1, 1, z_i) = \sum q_j J(1, \pi_j)$. It follows that $\sum q_j J(0, \pi_j) > \sum q_j J(1, \pi_j)$.

4. HIGHWAYS AND FARMS

Consider a planner who solves the following problem:

$$\max_{1 \geq x_1 \geq 0} u(x_1, 1 - x_1) + \sum_j q_j \max_{x_2 \leq x_1} \sum_i \pi_{ij} v(x_2, 1 - x_2, z_i). \tag{3}$$

There is one unit of farm land available initially. The planner has the option of paving over as much of that land, in the form of highways, as he wishes in period 1. x_1 represents the amount of farm land left intact and $1-x_1$ the amount turned into highway. $u(f, h)$ is a cardinal index of social utility derived in period 1 from farms and highways of sizes f and h respectively. The planner has a two period horizon. At the start of period 2 he may convert more farms into highways. x_2 represents the amount of farm land left intact for period 2. The constraint $x_2 \leq x_1$ corresponds to the fact that the transformation of farms into highway is irreversible. This irreversibility poses a problem to the planner in period 1 because there is uncertainty about the utility that will be derived in period 2 from any f and h. Thus the second period utility index $v(f, h, z_i)$ depends on the realization z_i of a random variable Z. The planner entertains a prior probability distribution for Z when x_1 is chosen. Additional information about Z is forthcoming, however, through an observation of a r.v. Y before x_2 is chosen. q_j and π_{ij} are as defined in Section 2. u and v are assumed to be increasing and strictly concave in (f, h).

Intuitively one would expect that the more information that will be available about Z before the period 2 decision must be made, the less farm land will the planner decide to pave in period 1. This is in fact the case as we now show.

Define

[9] This section applies D.1 rather than D.2 or Theorem 1. See footnote 5.

$$J(x_1, \xi) \equiv \max_{x_2 \le x_1} \sum \xi_i v(x_2, 1 - x_2, z_i) = \sum \xi_i v(x_2^*, 1 - x_2^*, z_i), \tag{4}$$

where x_2^* solves the problem in (4). Then $x_2^* < x_1$ if and only if $\sum \xi_i (dv(x_2, 1-x_2, z_i)/dx_2)_{x_2 = x_1} < 0$. By the envelope theorem

$$J_{x_1}(x_1, \xi) = \begin{cases} 0, & x_2^* < x_1, \\ \sum \xi_i (dv/dx_2)_{x_2 = x_1}, & x_2^* = x_1 \end{cases} \tag{5}$$

or evaluating at $x_1 = x_1^*$,

$$J_{x_1}(x_1^*, \xi) = \max(0, \sum \xi_i (dv/dx_2)_{x_2 = x_1^*}). \tag{6}$$

The second function inside the maximum is linear and hence convex in ξ. Since a maximum of two convex functions is itself convex, it follows that J_{x_1} is convex in ξ. Therefore, by Theorem 1, x_1^* is larger the more informative is Y.

The piecewise linearity of J_{x_1} is also consistent with intuition. For example, suppose that $\pi_1 \ne \pi_2$ (columns of Π) are such that $x_2^* = x_1^*$ given either $\xi = \pi_1$ or $\xi = \pi_2$ in (4). Define Π' by $\pi'_1 = \pi'_2 = (1/2)\pi_1 + (1/2)\pi_2$ and $\pi'_i = \pi_i$, $i = 3, \ldots, n$ and choose q and q' so that $\Pi q = \Pi' q' (= r)$. Y is more informative than Y' since, roughly speaking, two messages of Y are "garbled" into a single message in Y'. (See also MM (p. 154).) But from (6) and Theorem 1, x_1^* is the same for Y and for Y'. The "garbling" of messages both of which, upon receipt, would result in a binding constraint in period 2, has no effect on the period 1 decision, and similarly for two messages both of which would result in an ineffective constraint in period 2. Only when the process of "garbling" involves at least one message y_i that corresponds to $x_2^* < x_1^*$ and another y_j such that $x_2^* = x_1^*$, is the loss of information consequential for behavior. Such "garbling" strictly increases the amount of first period highway construction undertaken.

5. THE TIMING OF ORDERS FOR CAPITAL

There is another interpretation, which we now describe briefly, that may be provided for the mathematical structure of the problem in Section 4.

A producer may order capital equipment in period 1 or in period 2 at prices c_1 and c_2 respectively. The equipment is for production in period 3. Labor, the other factor of production, may be adjusted instanteously to the level desired in period 3. In periods 1 and 2 there is uncertainty about the ouptut and labor prices that will prevail in period 3. $c_2 > c_1$, i.e., the earlier the order is placed, the lower the price that must be paid for the capital. The producer will still, in general, order some capital in period 2 because by that time he will have revised his expectations about period 3 prices.

Formally, the producer solves

$$\max_{x_1 \ge 0} - c_1 x_1 + \sum_j q_j \max_{x_2 \ge 0} \{ \sum_i \pi_{ij} g(z_i; x_1 + x_2) - c_2 x_2 \}. \tag{7}$$

x_i denotes the amount ordered in period i, $i = 1, 2$. The period 1 order is irreversible ($x_2 \ge 0$). $g(p, w; x)$ is the variable profit function corresponding to the producer's technology; it gives the maximum variable profits attainable in period 3 given capital stock x and output and labor prices p and w respectively.[10] Z is a vector random variable, with possible vector realizations $(z_1, \ldots, z_m)$, representing uncertainty about the period 3 values of these prices. q_j and π_{ij} correspond, as before, to a r.v. Y whose realization becomes known before the period 2 order must be placed.

With the change of decision variables $\alpha_1 = x_1$, $\alpha_2 = x_1 + x_2$, problem (7) has a structure similar to that of (3) with the exception that $\alpha_2 \geq \alpha_1$ whereas $x_2 \leq x_1$ in (3). Define $J(\alpha_1, \xi)$ in the obvious fashion. Then $J_{\alpha_1}(\alpha_1^*, \xi)$ is a minimum of two linear functions of ξ and hence is concave in ξ. Therefore, the more informative is Y the smaller is the optimal first period order, i.e., the greater is the flexibility left for the second period.

6. A CONSUMPTION-SAVINGS PROBLEM

An individual has a given wealth w_1 which he wishes to allocate between consumption in three periods. What he does not consume in any period he invests in a single asset. Investment in period 1 yields a sure gross return r and investment in period 2 yields a random gross return Z.[11] The allocation is made so as to maximize expected utility, i.e., the consumer solves

$$\max_{0 \leq x_1 \leq w_1} u(w_1 - x_1) + \frac{1}{\beta} \sum_j q_j \max_{0 \leq x_2 \leq r x_1} \left\{ u(r x_1 - x_2) + \frac{1}{\beta} \sum_i \pi_{ij} u(x_2 z_i) \right\}. \tag{8}$$

x_1 and x_2 denote savings in periods 1 and 2 respectively, r is the sure gross return to first period savings, z_i, $i = 1, \ldots, m$ are the possible gross returns to second period savings and β is a utility discount factor. The utility index has constant relative risk aversion $\text{RRA} = -wu''(w)/u'(w)$, i.e.,

$$u(w) = \begin{cases} w^{1-\alpha}/(1-\alpha), & \alpha \neq 1, \\ \log w, & \alpha = 1 \end{cases}$$

and $\text{RRA} = \alpha > 0$. Again, some information about Z is gained, through observing Y before x_2 is chosen.

One might expect x_1^* to increase as Y becomes more informative since there is an incentive to carry over more wealth to period 2 to take advantage of the increased information. On the other hand, the increase in information and the resulting rise in the expected value of the sum of second and third period utility

[10] See Diewert [1974]. If $F(x, L)$ is the production function, $g(p, w; x) = \max \{pF(x, L) - wL \mid L \geq 0\}$. g uniquely defines the technology and is characterized by the following properties: linear homogeneous and convex in prices, concave in x, increasing in p, x and decreasing in w. g is strictly concave in x if F is strictly concave.

[11] The assumption that r is certain is made for simplifying purposes only. If the rate of return were a r.v. R it would in general provide some information about Z. All our results, suitably modified, remain valid if we compare experiments Y and Y' such that Y is more informative than Y' conditional on R.

for any given x_1 imply that the consumer can worry less about the future and indulge in more consumption in period 1. Thus there is also an incentive to reduce x_1^*.[12]

A referee has provided the following further intuition: the above effects of better information may be interpreted as substitution and income effects respectively. This suggests that the elasticity of substitution will help to determine the final result. The three-period utility index defined above is ordinally equivalent to a CES utility function with elasticity of substitution equal to $(1|\alpha)$. Intuition suggests, therefore, that the substitution (income) effect will dominate if α is small (large). This we now proceed to demonstrate.[13]

Define

$$J(x_1, \xi) \equiv \max_{0 \le x_2 \le r x_1} \left[u(r x_1 - x_2) + \frac{1}{\beta} \sum_i \xi_i u(x_2 z_i) \right], \tag{9}$$

and denote by x_2^* the solution in (9). Straightforward computations yield the solutions $x_2^* = r x_1/(1+\beta)$ if $\alpha = 1$ and $x_2^* = r x_1[1 + (\sum \xi_i z_i^{1-\alpha}/\beta)^{-1/\alpha}]^{-1}$ if $\alpha \neq 1$. By the envelope theorem,

$$J_{x_1}(x_1, \xi) = r u'(r x_1 - x_2^*) = r^{1-\alpha} x_1^{-\alpha} \left[1 + \left(\frac{\sum \xi_i z_i^{1-\alpha}}{\beta} \right)^{1/\alpha} \right]^{\alpha}, \qquad \text{for all } \alpha.$$

$J_{x_1}(x_1, \xi)$ is thus convex (concave) in ξ if $\alpha < (>) 1$, and is independent of ξ if $\alpha = 1$. By Theorem 1, x_1^* is not influenced by future information if $\alpha = 1$. (Note that x_2^* above is independent of ξ if $\alpha = 1$.) Also the prospect of more information in the future increases (reduces) savings if $\alpha < (>) 1$.

It is interesting to compare the behavioral response to an earlier resolution of uncertainty with the response to increased prior uncertainty about Z. We would like to vary the riskiness of Z alone, keeping the informativeness of Y constant in some sense. This partial variation in the probability distribution would seem to be accomplished by keeping the likelihood matrix Λ fixed as r changes in the sense of Rothschild and Stiglitz [1970]. However, we have not been able to determine the behavioral impact of such a change. Therefore, we consider only the polar cases where (a) Y provides no information about Z, and (b) where Y provides perfect information about Z. In (a) Y may be ignored completely since it doesn't enter the utility function and in (b) Z is essentially observed before period 2.

In both cases (a) and (b) the decision problem (8) reduces essentially to a two-period framework. Therefore the now familiar techniques of Rothschild and Stiglitz [1971] (especially section 2.A) may be applied to determine the effects of increased first period uncertainty about Z. In fact, as indicated briefly in the

[12] The resultant ambiguity is akin to that surrounding the effects on savings of greater uncertainty in a two-period model. See Sandmo [1970].

[13] The referee has conjectured that the results below hold more generally and depend only on the elasticity of substitution. It is not obvious, however, that the results depend *only* on ordinal properties of the utility function.

Appendix, the optimal savings rules can be determined explicitly in both (a) and (b).

We find that reduced (increased) prior uncertainty in case (a) has the same qualitative impact on behavior as does an earlier (later) resolution of uncertainty. The same is true in case (b) if $\alpha \geq 1/2$. However, if $0<\alpha<1/2$, the qualitative impact of a change in the prior uncertainty surrounding Z is ambiguous.

Two general conclusions may be drawn: First, the qualitative effects of increased prior uncertainty depend on the temporal resolution of that uncertainty.[14] Second, and more directly related to the theme of this paper, the relationship between the qualitative effects of a change in prior uncertainty on the one hand and of the timing of the resolution of uncertainty on the other hand, is ambiguous.

7. A FIRM'S DEMAND FOR CAPITAL

Consider a modification of the model of Section 5. A firm produces a single output by means of the strictly concave production function $F(K, L)$. Output is produced and sold in period 3 at a price P which becomes known to the firm only in period 3. Both factors of production must be hired or purchased in advance of the actual production date but the lag is larger for capital. Therefore, capital K must be ordered in period 1 and labor L in period 2. By the time the labor decision must be made the producer will have revised his prior expectations about the price as a result of having observed a correlated r.v. Y. The firm maximizes expected profits.

The formal problem solved by the firm is the following:

$$\max_{0 \leq K} -cK + \sum_j q_j \max_{0 \leq L} \left[\sum_i \pi_{ij} p_i F(K, L) - wL\right], \tag{10}$$

where w denotes the wage rate and c the rental or purchase price of capital, which, along with the possible output prices p_i, are assumed to be appropriately discounted.

Define

$$J(K, w, \xi) \equiv \max_{L \geq 0} \sum_i \xi_i p_i F(K, L) - wL. \tag{11}$$

From the definition of the variable profit function $g(p, w; K)$ (see footnote 10) it is clear that $J(K, w, \xi) = g(\sum \xi_i p_i, w; K)$. Therefore, J_K is convex (concave) in ξ if $g_K(p, w; K)$ is convex (concave) in p. In period (1), the firm solves

$$\max_{K \geq 0} \sum q_j J(K, w, \pi_j) - cK.$$

We conclude therefore from Theorem 1 that as Y becomes more informative,

[14] For the two polar cases of temporal resolution, this fact has been recognized implicitly, if not explicitly, in many analyses of firm behavior under uncertainty. See, for example, Turnovsky [1973], Hartman [1976] and Epstein [1978].

K^* increases (decreases) if $g_K(p, w; K)$ is convex (concave) in the output price p.

The noted third order property of the variable profit function has been found to be important also in the two-period models of the firm analyzed by Hartman [1976] and Epstein [1978]. Insight into our result and into its relationship with these two analyses is most easily provided by addressing the corresponding question to that considered at the end of the last section; namely, the relationship between the effects of increased prior uncertainty and an earlier resolution of uncertainty.

Define cases (a) and (b) as in the previous section. In (b), P is observed at the start of period 2 and problem (1) becomes

$$\max_{K \geq 0} \sum r_i g(p_i, w; K) - cK, \tag{12}$$

precisely the two-period problem considered by Hartman and Epstein. From their results we see that increased prior uncertainty increases (reduces) the demand for capital if $g_K(p, w; K)$ is convex (concave) in p. Combining this and our earlier finding, we may conclude that when Y provides perfect information about P, *later* resolution and a *reduction* in prior uncertainty have similar qualitative impacts on K^*.[15]

This result has a simple explanation: Since all uncertainty is removed before the labor decision is made, a reduction in prior uncertainty means that the capital decision alone is made subject to less uncertainty. A later resolution of uncertainty means that in an average sense, less uncertainty will be resolved before L^* is chosen. Therefore, again the environment at the time of the capital decision is made relatively less uncertain than at the time of the labor decision. In both cases there is an incentive, to the extent that capital and labor are highly substitutable, to employ relatively more capital. In fact if F has constant elasticity of substitution σ and degree of homogeneity $\mu > 1$, then it follows from Hartman [1976] that K^* rises in response to a later resolution if $\sigma > 1/(1-\mu)$ and that K^* falls if $\sigma < 1/(1-\mu)$.

(The appeal of the explanation is diminished by the fact that it does not apply to the model of Section 6. The particular mathematical feature of the production model seems to be that $E[P/Y]$ is a sufficient statistic for the period 2 decision. One can show that if Y is more informative than Y', then $E[P/Y]$ is more variable than $E[P/Y']$ in the sense of Rothschild and Stiglitz [1970].)

Turn now to case (a) where Y provides no information about P. Then K and L are essentially chosen simultaneously to solve

$$\max_{\substack{L \geq 0 \\ K \geq 0}} (EP)F(K, L) - wL - cK, \tag{13}$$

where $EP = \sum r_i p_i$ is the expected value of the prior price expectation. Prior

[15] Note that we are comparing the effects of a change in prior variability given perfect information in period 2, with the effects of a change in the structure of information given *any* initial resolution of uncertainty.

uncertainty has no effect on behavior as the expected value of the output price is a sufficient statistic for input decisions.

These findings confirm our earlier conclusions stated at the end of Section 6. In addition they demonstrate the following: the relationship between the qualitative effects of earlier resolution and reduced prior uncertainty *for a given information structure over time* (e.g., perfect information before period 2) is model specific.

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APPENDIX

Suppose that $\alpha \neq 1$. Consider the decision problem (8) in case (a); i.e., when Y provides no information about Z. Define $V(x_2) \equiv \max_{x_1} u(w_1 - x_1) + u(rx_1 - x_2)/\beta$. Then one can show that $V(x_2) = b(w_1 r - x_2)^{1-\alpha}/(1-\alpha)$, where b is a constant (independent of x_2), and that the optimal x_1 is given by $x_1^*(x_2) = [(r/\beta)^{1/\alpha} w_1 + x_2]/[(r/\beta)^{1/\alpha} + r]$. The consumer's choice problem may be expressed in the form

$$\max_{x_2} \frac{b(w_1 r - x_2)^{1-\alpha}}{1-\alpha} + \frac{1}{\beta^2} \frac{E(x_2 Z)^{1-\alpha}}{1-\alpha},$$

a two-period consumption-savings with constant relative risk aversion α utility function. From Rothschild and Stiglitz [1971, pp. 68–72], x_1^* increases (falls) as Z becomes more uncertain if $\alpha > (<) 1$. Since $x_1^*(x_2)$ is increasing in x_2, optimal first period saving responds similarly to changes in prior uncertainty about Z.

In case (b), the consumer solves

$$\max_{x_1} u(w_1 - x_1) + \frac{1}{\beta} EV(rx_1, Z), \tag{14}$$

where $V(w, z) \equiv \max_{x_2} u(w - x_2) + (1/\beta) u(zx_2)$. One can show that $V(w, z) = A(z) w^{1-\alpha}/(1-\alpha)$, $A(z) = [z^{(1-\alpha)/\alpha} + \beta^{1/\alpha}]^{\alpha}$. Thus (14) becomes

$$\max_{x_1} \frac{(w_1 - x_1)^{1-\alpha}}{1-\alpha} + \frac{r^{1-\alpha} x_1^{1-\alpha}}{\beta(1-\alpha)} EA(Z).$$

It follows that x_1^* varies positively with $EA(Z)$. $A(z)$ is convex if $\alpha > 1$, concave if $1/2 \leq \alpha \leq 1$, and neither if $0 < \alpha < 1/2$. The stated (Section 6) influence on x_1^* of uncertainty about Z follows.

The analyses are similar for $\alpha = 1$.

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