The Family as an Incomplete Annuities Market

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Families can self-insure against uncertain dates of death through implicit or explicit agreements with respect to consumption and interfamily transfers. Interfamily transfers need have nothing to do with altruistic feelings; they may simply reflect risk-sharing behavior of completely selfish family members. Although family annuity markets are incomplete, even small families can substitute by more than 70 percent for perfect market annuities. Given adverse selection and transaction costs, family risk pooling may be preferred to public market annuities. In the absence of public annuities, these risk-sharing arrangements provide powerful incentives for marriage and family formation.

The institution of the family provides individuals with risk-sharing opportunities which may not otherwise be available. Within the family there is a degree of trust and a level of information which alleviates three key problems in the provision of insurance by markets open to the general public, namely, moral hazard, adverse selection, and deception. In addition, provision of insurance within the family may entail smaller transaction costs than arise in the purchase of insurance on the open market. There are a number of important risks for which

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the "public" market problems of moral hazard, adverse selection, and deception are especially severe. The risk of loss of job or earnings because of changes in the pattern of demand or partial disability is one example. Here the ability of the public market to determine the extent to which the individual actually suffered an earnings loss or is simply lying about his backache is highly questionable. Other examples are the risk of bankruptcy and the default risk on personal loans. Many family practices in dealing with these types of risks can be explained as implicit insurance contracts made ex ante by completely selfish family members. Love and affection may be important for the enforcement of some of these implicit contracts, but they need not be their sole or even chief determinant. Healthy brother A's support for disabled brother B may simply be the quid pro quo for brother B's past implicit promise to support A if A became disabled instead of B.

The existing economics literature on marriage and the family (including Schultz [1974] and Becker, Landes, and Michael [1977]) has not, to our knowledge, explicitly considered the family's role in providing insurance to family members.

This paper is concerned with family provision of insurance against the risk of running out of consumption resources because of greater than average longevity. The problem is how fast to consume over time when one does not know how long one will continue to live. Too much consumption when young may mean relative poverty later on if one lives "too long"; alternatively, excessive frugality when young involves the risk of dying without ever having satisfied one's hunger. A complete annuity market permits an individual to hedge this uncertainty of the date of death by exchanging his initial resources for a stream of payments that continue as long as the individual survives. We demonstrate here that implicit risk-sharing arrangements within marriage and the family can substitute to a large extent for the purchase of annuities in public markets. Since the number of family members involved in the risk pooling is generally small, these family risk-sharing arrangements constitute an incomplete annuities market. However, our findings suggest that even small families can substitute by more than 70 percent for a complete annuity market in pooling the risk of death. When the economic structure of society is sufficiently developed to sustain organized public insurance markets, implicit risk pooling within an incomplete family annuity market may well be preferred to public purchase of annuities because of adverse selection and transaction costs.[1] When organized insurance markets do not

[1] The transaction costs we have in mind here include the time costs involved in negotiating individual specific annuity contracts. As we demonstrate in the text, each individual's optimal annuity contract depends on his rate of time preference, his degree

exist, the analysis here indicates that implicit risk-sharing arrangements can provide powerful economic incentives for marriage and family formation.

Throughout the paper individuals are assumed to be completely selfish; that is, they obtain utility only from their own consumption. One implication of this approach is that voluntary transfers from children to parents or bequests and gifts from parents to children need have nothing at all to do with altruistic feelings; rather, they may simply reflect risk-sharing behavior of completely selfish individuals. While altruism per se is not required, some level of mutual trust and honesty *is* required since elements of these arrangements are not legally enforceable.

This paper is divided into four sections, the first of which describes optimal consumption behavior for a single individual in both the presence and absence of a complete annuity market. The welfare gains from access to a complete and fair annuity market are calculated for the case of the iso-elastic utility function. This welfare gain is, in turn, decomposed into income and substitution effects. This decomposition suggests that an important component of the gains from access to complete or incomplete annuity markets is the desirability of substituting future for current consumption.

Section II develops the theoretical argument for Pareto-efficient implicit family annuity contracts and explores potential welfare gains arising from these arrangements.[2] Although the complexity of the calculations precluded analysis of large families, quantitative results for families of two and three persons are presented. The analysis considers cases in which family members both do and do not have identical survival probabilities (i.e., are of similar and dissimilar ages and sexes). This framework permits us to ask whether marriage between individuals with similar survival probabilities is more efficient than marriage between individuals with dissimilar survival probabilities.

Optimal family annuity contracts involve agreements on the consumption path of each family member as well as a commitment on the part of each member to name the other members as sole heirs in his estate. Section III discusses the problems of enforcing both aspects of these agreements. Section IV summarizes the paper and suggests areas for future research.

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of risk aversion, and his survival probabilities. Some individuals may prefer a constant annuity stream, others an increasing or decreasing stream of annuity payments.

2 Kotlikoff and Spivak (1979) present a proof that family annuity contracting converges to a complete annuities market as the number of family members increases.

I. A Single Person's Consumption Plans with and without Fair Annuities[3]

In the absence of an annuity market, a single individual's consumption choice problem is to maximize his expected utility, equation (1), from current and future consumption subject to the budget constraint, equation (2):

$$EU = \sum_{t=0}^{D} P_t U(C_t), \tag{1}$$

$$\sum_{t=0}^{D} C_t R^{-t} = W_0. \tag{2}$$

The P_t's of equation (1) are probabilities of surviving from age zero through age t; P_0 equals one. The term D is the maximum longevity. For simplicity, we assume the utility function is separable in consumption (C_t) over time. In (2) R, the discount factor, is one plus the interest rate. The initial wealth of the individual is W_0; we ignore possible streams of future labor earnings or inheritances.[4]

The budget constraint written in equation (2) is identical to the budget constraint that would arise in a certainty world in which individuals never died before age D. While individuals will, on the average, die prior to age D, equation (2) reflects the nonzero probability that an individual will live through age D; that is, equation (2) is the relevant budget constraint because the individual may actually live through age D, in which case his realized present value of consumption cannot exceed his budget.

Let us now assume that the single person is free to purchase actuarially fair annuities in a complete public annuities market. The budget constraint in this case is

$$\sum_{t=0}^{D} P_t C_t R^{-t} = W_0. \tag{3}$$

[3] Yaari (1965) is the pioneering paper on this subject. Sheshinski and Weiss (1981) provide an illuminating discussion on the interaction of annuities and social insurance. Barro and Friedman (1977) provide an analysis of the risks of the uncertainty of the date of death.

[4] The gains from access to an annuities market are greatest when the individual has all his resources up front. This assumption, then, dramatizes the demand for annuities; but dropping this assumption would not alter the theoretical point that families can substitute for annuity markets. For the sake of completeness one can think of the individuals described in this paper as having received all their resource streams prior to their current age. In the no-annuity, no-family world involuntary bequests can be thought of as being collected by the government and redistributed to individuals at their birth.

In contrast with (2), (3) requires only an equality between the expected present value of consumption and initial wealth. The single individual now chooses his optimal consumption path by maximizing (1) subject to (3); he then exchanges his initial wealth W_0 with the insurance company in return for its promise to pay out the C_t stream as long as the person continues to live.

The P_tR^{-t}'s in (3) may be thought of as prices. Since each of the P_t's in (3), except P_0 which equals unity, is less than one, the consumption choice in the case of a fair annuity market is equivalent to the consumption choice without an annuity market but with lower prices of future consumption. Obviously, access to a fair annuity market increases utility by expanding the budget frontier; it also alters the optimal consumption path because of the income and substitution effects resulting from the lower prices of future consumption.

The iso-elastic utility function (4) is convenient for assessing the potential gains from access to a fair public annuities market as well as the gains from family annuity arrangements:

$$EU = \sum_{t=0}^{D} P_t \frac{C_t^{1-\gamma}}{1-\gamma} \alpha^t. \tag{4}$$

In (4), γ is the constant relative risk-aversion parameter, and α is the time preference parameter. By considering different values of γ we indicate for this family of utility functions how the gains from annuities and family arrangements depend on the specification of tastes.

In the no-annuities case maximization of (4) subject to (2) leads to the consumption plan, (5):

$$C_t = \frac{W_0(R\alpha)^{t/\gamma}P_t^{1/\gamma}}{\sum_{j=0}^{D} R^{j(1-\gamma)/\gamma}\alpha^{j/\gamma}P_j^{1/\gamma}}. \tag{5}$$

In the case of fair annuities, maximizing (4) subject to (3) leads to

$$C_t = \frac{W_0(R\alpha)^{t/\gamma}}{\sum_{j=0}^{D} R^{j(1-\gamma)/\gamma}\alpha^{j/\gamma}P_j}. \tag{6}$$

Figure 1 compares equations (5) and (6) for the case $R = \alpha = 1$. The ability to trade in a fair annuities market may raise or lower initial consumption, depending on whether γ is less than or greater than unity (fig. 1). Intuitively, the higher the degree of risk aversion, γ, the greater the concern for running out of money because of excessive longevity and, hence, the lower the initial consumption. At γ equal to infinity, equation (5) dictates equal consumption in each period.

Plugging (5) or (6) into (4), we arrive at two indirect utility functions for the no-annuity and annuity cases with initial wealth, the interest

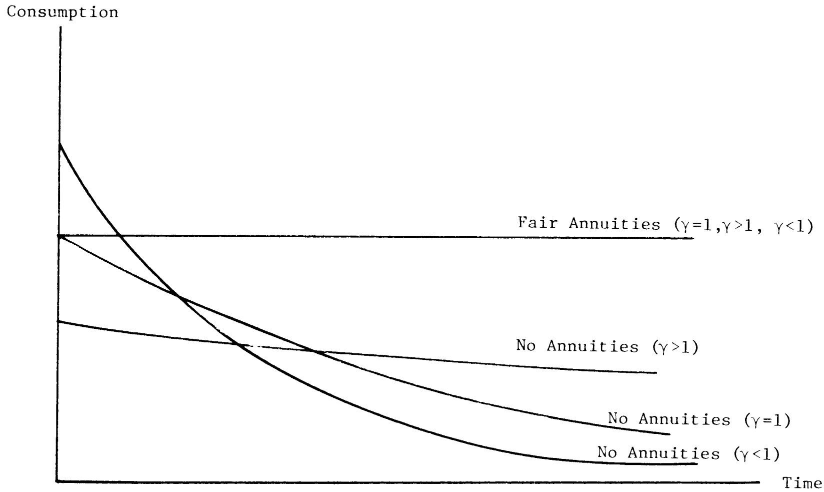

FIG. 1.—Consumption paths with and without fair annuities

rate, and survival probabilities as arguments. These functions are presented in equations (7) and (8), respectively,

$$H_0(W_0) = \frac{1}{1-\gamma} W_0^{1-\gamma} \left[\sum_{j=0}^{D} \alpha^{j/\gamma} R^{j(1-\gamma)/\gamma} P_j^{1/\gamma} \right]^{\gamma}, \tag{7}$$

$$V_0^*(W_0) = \frac{1}{1-\gamma} W_0^{1-\gamma} \left[\sum_{j=0}^{D} \alpha^{j/\gamma} R^{j(1-\gamma)/\gamma} P_j \right]^{\gamma}. \tag{8}$$

The increase in utility resulting from access to fair annuities can be measured in terms of dollars. Equation (9) solves for the value of M, which represents the percentage increment in a single person's initial wealth required, in the absence of an annuity market, to leave him as well off as he would be with no additional wealth but with access to an annuities market:

$$H_0(MW_0) = V_0^*(W_0). \tag{9}$$

For the iso-elastic utility function this calculation is independent of the initial level of wealth. Table 1 reports values of M for different ages and levels of risk aversion using both male and female survival probabilities. Friend and Blume (1975) estimate the degree of relative risk aversion from individual portfolio choices. They conclude that risk aversion, on average, exceeds unity. We present our results for risk-aversion coefficients of 0.75, 1.25, and 1.75, a range that we feel

TABLE 1

Percentage Increase in Initial Wealth Required to Obtain Fair Annuities Utility Level

Age	Relative Risk Aversion (γ)	Males	Females
30	.75	24.5	18.5
55	.75	46.9	34.4
75	.75	71.2	63.0
90	.75	99.8	100.2
30	1.25	30.3	22.7
55	1.25	59.2	43.4
75	1.25	97.0	85.3
90	1.25	152.6	152.9
30	1.75	34.7	26.1
55	1.75	68.9	50.7
75	1.75	119.1	104.6
90	1.75	199.1	199.4

Note.—Throughout table $\alpha = .99$ and $R = 1.01$.

encompasses reality. The survival probabilities used in this and all subsequent calculations are actuarial estimates from the Social Security Administration.[5] Maximum longevity is taken to be 120 throughout the paper.

Table 1 indicates that the utility gain measured in dollars from access to an annuities market can be quite large. For a relative risk-aversion parameter value of 0.75, the gain to a 55-year-old male is equivalent to a 46.90 percent increase in his initial wealth. The utility gain is age dependent; for $\gamma = 0.75$, the 30-year-old male's gain is 24.46 percent, while the 90-year-old male's gain is 99.81 percent. Annuities are less important to young people because a large fraction of their lifetime utility from consumption is fairly certain due to their lower mortality probabilities in the immediate future. Higher levels of risk aversion naturally increase the gains from access to an annuities market. The male-female differences in the table reflect the higher male age-specific mortality rates. The calculation is somewhat sensitive to the choice of α and R. Raising the interest rate to 5 percent while holding α constant increases the age 55 wealth-equivalent factor from 46.90 to 55.57 for the case of $\gamma = 0.75$. The 90-year-old wealth equivalent is increased from 99.81 to 115.34.

[5] We use the low mortality male and female probabilities reported on pp. 17 and 19 of the Social Security Administration Actuarial Study no. 62 (see U.S. Department of Health, Education, and Welfare 1966).

Income, Substitution Effects, and Unintended Bequests

Without access to an annuity market a single, nonaltruistic individual will always die prior to consuming all his wealth and, accordingly, will make involuntary bequests. The level of these unintended bequests can be quite large. From equation (5) we calculated the consumption path as well as the corresponding wealth path for the no-annuity case. By multiplying the probability of dying at each age times the wealth at each age and discounting back to the initial age, the present expected value of these unintended bequests can be computed. For $\gamma = 0.75, R = 1.01$, and $\alpha = .99$, the present expected value of unintended bequests represents 24.47 percent of initial wealth for a single male aged 55. This number means that a 55-year-old male with no annuity market will, on average, fail to consume about one-quarter of his wealth because he is risk averse. Increasing the risk-aversion coefficient to 1.75 raises the ratio of present unintended bequests to initial wealth to 0.3583. These large unintended bequests occur despite a fairly rapid rate of consumption. Current mortality probabilities dictate a fairly rapid rate of consumption even for high levels of risk aversion. For $\gamma = 1.75$, a single male who survives to age 85 consumes at age 85 less than a third of his age 55 consumption level.[6]

The homothetic property of the iso-elastic utility function permits a decomposition of the utility gains from fair annuities into income and substitution effects. Suppose a fair insurance company approached a single, 55-year-old ($\gamma = 0.75$) male and offered to pay him 24.47 percent of his initial wealth in exchange for his naming the insurance company as his heir. The single male would take the 24.47 percent gain and, because of homotheticity, consume it according to his original no-annuity consumption path. This additional wealth would give rise to an additional $.2447 \times .2447$ in present expected bequests. By letting the insurance company also pay for this second round of expected but involuntary bequests as well as further rounds, the insurance company ends up paying $32.40 = .2447/(1 - .2447)$ percent of the single individual's initial wealth. This 32.40 percent figure represents the utility gain from the pure income effect. In this scenario the individual continues to consume at the no-annuity set of prices. Since the total gain from being able to purchase fair annuities and thus face lower prices for future consumption is 46.90 percent, the income effect represents 69.08 percent and the substitution effect 30.92 percent of the total gain. Hence, the ability to alter the age

[6] The ratio of consumption at age 75 to consumption at age 55 is 0.62. When risk aversion equals 0.75, the ratio of consumption at age 85 to consumption at age 55 equals 0.06; it is 0.33 at age 75.

consumption profile is an important part of the total welfare gain from annuities.

II. The Family as an Incomplete Annuities Market

Decisions by family members concerning consumption expenditures and interfamily transfers may reflect implicit though incomplete annuity contracts. In the case of marriage both individuals commonly agree to pool their resources while both marriage partners are alive and to name each other as the major, if not the sole, beneficiary in their wills. For each partner the risk of living too long is somewhat hedged by the other partner's potential death; if one partner lives to be very old, there is a high probability that his (or her) spouse has already died leaving him a bequest to help finance his consumption. While each spouse gains simply from the exchange of wills, the two can further increase their expected utilities by agreeing on a joint consumption path that takes into account each spouse's expected bequest to the other. The importance of joint consumption planning is highlighted in the case of an implicit contract between a parent and a child. Here the parent implicitly promises to name the child in his will in exchange for the child's implicit promise to care for the parent if the parent lives too long. Although the child may have zero probability of dying while the parent is still alive, both can gain because the child agrees to share consumption resources with the parent.

This view of bequest and consumption arrangements within marriage as an incomplete annuity market becomes intuitive when one contemplates increasing the number of members in the family. To simplify the issue, let us assume that all individuals within the family have identical survival probabilities and that they enter this multiperson family with identical resources. In the limit as the family (or "tribe") gets large, the consumption path of an individual within the tribe converges to the path a single individual would choose in a complete and actuarially fair annuities market (Kotlikoff and Spivak 1979).

Quantitative Analysis of Family Risk Pooling

In the case of two family members the frontier of efficient marriage contracts is obtained as the solution to the following recursive dynamic programming problem:

$$V_{t-1}(W_{t-1}) = \max_{W_t, C^H_{t-1}, C^S_{t-1} \geq 0,\, t = T, \ldots, 1} [u^H(C^H_{t-1}) + \theta u^S(C^S_{t-1}) + \alpha P_{t|t-1} Q_{t|t-1} V_t(W_t) + \alpha P_{t|t-1}(1 - Q_{t|t-1}) H_t(W_t) + \theta \alpha Q_{t|t-1}(1 - P_{t|t-1}) S_t(W_t)], \tag{10}$$

subject to

$$W_t/R + C^H_{t-1} + C^S_{t-1} = W_{t-1}, \tag{11}$$

where

$$V_T(W_t) = \max_{C^H_T, C^S_T} u^H(C^H_T) + \theta u^S(C^S_T).$$

In (10) $V_t(W_t)$ is the period t maximum-weighted expected utility of the two family members with joint wealth W_t. In the expression the letters H and S denote the two family members, C^H_t and C^S_t are the consumptions of the two, u^H and u^S are their utility functions, $P_{t/t-1}$ and $Q_{t/t-1}$ are their respective period t survival probabilities conditional upon surviving through period $t - 1$, and $H_t(W_t)$ and $S_t(W_t)$ are the maximum expected utilities for each member if he or she alone survives to period t. These expressions are obtained from equation (7) by replacing W_0 with W_t and applying the appropriate probabilities. The term θ is the differential weight applied to member S's expected utility.

The first two terms on the right-hand side of (10) represent utility from certain period $t - 1$ consumption. The third term is the family's expected period t utility multiplied by the probability that both members survive to period t. The last two terms represent expected utilities when one member dies and the other survives.

The Appendix presents an algorithm to solve (10). The algorithm for solving the three-family-member maximization problem is available from the authors.[7]

The Gains from Family Annuity Contracts

The solution to (10) permits a comparison of consumption paths and utility levels of married people with those of single persons, assuming throughout that there is no public annuities market. Both spouses are assumed to have identical iso-elastic utility functions in the sense of the same degrees of risk aversion and rates of time preference.

The shape of consumption paths for married couples while they are both alive may differ from that of single individuals for two reasons. First, even if the two spouses have identical survival probabilities, the reduction in risk within the marriage rate acts like a reduction in the price of future consumption. If the relative risk-aversion parameter γ exceeds (is less than) one, the identical survival probabilities marriage profile will start above (below) the single person's profile. For γ equal

[7] In the three-member family we maximize a weighted sum of the three members' expected utility taking all survival contingencies into account. If one of the three dies first, the other two jointly inherit the remaining wealth and consume according to the optimal two-person plan.

to unity the profiles are identical. In terms of figure 1, the consumption profiles for married persons lie between the no-annuity and complete annuity profiles. The second reason for different consumption profiles for married people relative to single individuals is possible differences in spousal survival probabilities. Higher survival probabilities act like lower rates of time preferences. When an old man marries a young woman the slope of the optimal marriage consumption profile reflects the survival probabilities of both the old husband and the young wife. The two spouses compromise with respect to the rate at which they eat up their joint wealth while they are both alive. The old husband would prefer to eat up the wealth more rapidly, and the young wife would prefer to consume at a slower rate. The formula for each spouse's consumption when married takes both spouses' survival probabilities into account as well as the relative spousal utility weights. To our knowledge empirical studies of consumption and savings at the household level have not considered this point—that the time preference rate for a household may depend on the age-sex composition of the household.

Table 2 reports the gains from marriage as well as three-person polygamy among individuals who have identical survival probabilities and identical initial endowments and who are weighted equally in the contract. The marriage and three-person polygamy gains are calculated as the percentage increase in a single person's initial wealth needed to make him as well off as he would be in the marriage or polygamous relationship. The table also reports the dollar gain as a fraction of the table 1 total dollar gain from complete and fair annuities. Since utility is concave in wealth, the dollar gain from these family contracts as a fraction of the dollar gain from fair annuities is smaller than the actual utility gain from these contracts as a fraction of the utility gain from fair annuities. Table 2 also reports this latter fraction.[8]

The figures in table 2 indicate that marriage can offer substantial risk-pooling opportunities. For a 55-year-old man using male survival probabilities, pooling risk through marriage is equivalent to about a 20 percent increase in his wealth had he stayed single. The gains from marriage increase as one becomes older since the risks incurred are much greater as one ages. At age 75 marriage is equivalent to increasing one's wealth by 30 percent when risk aversion is 1.25. Death-risk-pooling through marriage can be quite important even at young ages. The table reports gains from 11.7 to 13.6 percent at age 30 using the male probabilities.

[8] This fraction is calculated as $[(1 + m)^{1-\gamma} - 1]/[(1 + a)^{1-\gamma} - 1]$, where m is the fractional wealth equivalent gain from marriage, and a is the fractional wealth equivalent gain from fair annuities.

TABLE 2

THE ANNUITY GAINS FROM MARRIAGE AND THREE-PERSON POLYGAMY

		Marriage			Three-Person Polygamy		
Age	Risk Aversion	Dollar Marriage Gain (%)	Dollar Marriage Gain/Dollar Annuity Gain	Utility Marriage Gain/Utility Annuity Gain	Dollar Polygamy Gain (%)	Dollar Polygamy Gain/Dollar Annuity Gain	Utility Polygamy Gain/Utility Annuity Gain
30	.75	11.7	.478	.499	15.8	.645	.665
55	.75	20.0	.426	.461	28.0	.597	.632
75	.75	25.4	.357	.405	37.2	.522	.571
90	.75	28.2	.283	.339	43.1	.432	.496
30	1.25	13.0	.429	.470	18.0	.594	.635
55	1.25	22.3	.377	.446	32.1	.542	.612
75	1.25	30.1	.310	.419	45.2	.466	.571
90	1.25	37.1	.243	.367	58.2	.381	.524
30	1.75	13.6	.392	.456	19.3	.556	.619
55	1.75	23.5	.341	.451	34.5	.501	.613
75	1.75	33.2	.279	.449	50.7	.426	.596
90	1.75	43.0	.216	.420	68.7	.345	.579

NOTE.—The table uses male mortality probabilities. $R = 1.01$ and $\alpha = .99$.

Marriage can also close much of the utility gap between no annuities and complete annuities. For example, for a 55-year-old with risk aversion of 0.75, marriage substitutes 46.10 percent for complete and fair annuities. Marriage is a better substitute for fair annuities at younger ages because at younger ages the probability that both spouses will die simultaneously is quite small relative to the probability that one spouse will die before the other. In addition, there appears to be an interaction between age and the degree of risk aversion, making marriage a better substitute for fair annuities at young ages when risk aversion is low and at old ages when risk aversion is high.

Over a wide range of ages and parameter values, three people appear to be capable of capturing about 60 percent of the gains from fair annuities. While the complexity of the calculations precluded considering a four-person arrangement, we can conjecture using table 2 how well four people would do together. In the case of a 55-year-old male with risk aversion of 0.75, adding one marriage partner is equivalent to a 20 percent increase in his wealth had he remained single. The marginal dollar gain from adding a third person (table 2) is 8.04 percent. If the marginal dollar gain fell at a constant rate in this range, the fourth person would add $8.04 \times (8.04/20.0) = 3.23$ percent.[9] By adding 3.23 to 28.04, we can roughly calculate the extent to which four people can close the utility gap. The procedure suggests that four people can substitute by 70 percent for a fair annuities market.

Diminishing returns to risk pooling appear, then, to set in at a fairly rapid rate. In this example two people substitute by 46 percent, three people by 63 percent, and four people by over 70 percent for full insurance.

Table 3 considers incomplete annuity arrangements between two parents and one child and between one parent and two children. In both cases we assume equal consumption by all family members but permit the initial wealth of the child or children to vary. All individuals are assigned the male survival probabilities; the children are age 30 and the parents age 55. In the case of two parents with one child, if the child has an initial wealth of $35,000 and the parents have an initial wealth of $20,000, entering into an equal consumption–will-swapping arrangement is equivalent to a 32 percent increase in wealth for each parent and a 10.6 percent increase for each child. For the parent this arrangement captures 71.2 percent of the utility gain from

[9] This is probably a lower bound estimate for the contribution of the fourth person; the marginal dollar gain cannot fall at a constant 40 percent rate forever, because if it did the total dollar gains would, in the limit, not sum up to 46.9 percent, the full annuity gain of table 1. Presumably the marginal dollar gain falls at a decreasing rate, and 3.23 percent probably underestimates the fourth person's marginal contribution.

TABLE 3

GAINS FROM INCOMPLETE ANNUITY ARRANGEMENTS IN THE FAMILY

	TWO PARENTS WITH ONE CHILD		TWO CHILDREN WITH ONE PARENT	
INITIAL WEALTH OF EACH CHILD ($)	Dollar Gain to Parent (%)	Dollar Gain to Child (%)	Dollar Gain to Parent (%)	Dollar Gain to Child (%)
25,000	14.4	34.2	2.3	24.8
30,000	23.2	20.4	16.9	18.9
35,000	32.0	10.6	31.5	14.6
40,000	40.8	3.2	46.1	11.5

NOTE.—The calculations assume equal consumption by all family members. Initial wealth of parent or parents is $20,000. R = 1.01, α = .99, and γ = 0.75.

full annuities; for the child the arrangement substitutes by 45.4 percent for full annuities. The last two columns of table 3 present the case of two children contracting with one parent. When each child contributes $35,000, the gain to the parent is 31.5 percent, while each 30-year-old child enjoys a 14.6 percent gain relative to consuming as a single person. The numerical differences in the table for the two different types of families reflect, on the one hand, different monetary contributions of parents relative to children and, on the other hand, differences in the rate at which resources are consumed when all family members are alive. Resources are consumed at a slower rate in the two-children–one-parent case than in the one-child–two-parent case, since each individual's survival probabilities are given equal weight in determining the optimal rate of consumption.

Is Marrying People of Similar Ages More Efficient?

Suppose one had to decide how to pair up four people, two who are old and two who are young. Is it more efficient to marry the old people together and the young people together than it is to mix ages? Intuitively, marrying two 90-year-olds together and two 20-year-olds together leaves a large chance that both 90-year-olds will die in the immediate future, and resources that they have failed to consume will be lost to the 20-year-olds who, on average, will still be alive.[10] The countervailing argument against mixed-age marriages is that mixing

[10] We assume here that any involuntary bequests that arise from the simultaneous death of both marriage partners or from the death of a surviving spouse are not inherited by any of these four individuals. Again the government can be thought of as collecting these residual bequests and distributing them each year to the newborn. We thank Finis Welch for a helpful discussion on this section.

ages involves greater risk to one of the two partners; the utility cost of this greater risk may exceed the utility gain from the increase in expected resources arising in mixed marriages.[11]

We investigated potential efficiency gains from mixed marriages between two 55-year-olds and two 30-year-olds, where each individual was risk averse at the 0.75 level. The 55-year-olds were assigned male survival probabilities, while the 30-year-olds were assigned female survival probabilities. When risk aversion equals 0.75, weights of 1.7 for the old person yield utility levels for both old and young which exceed those in the old-old, young-young marriages of table 2. The additional dollar gain to the old person from this weighted marriage with the young person is 3.1 percent; the added gain to the young person is 1.6 percent.

These additional gains from mixed-age marriages require, however, a fairly skewed distribution of consumption within the marriage. For this example the young-old weighting scheme that dominates old-old, young-young coupling involves the older spouse's consuming about 86 percent more than the younger spouse while they are both alive. If it is too costly to negotiate such an arrangement within the marriage or if the type of consumption (e.g., housing) within marriage is nonexcludable, then equal consumption marriages of individuals with similar survival probabilities (of similar ages) will be the rule rather than the exception. Of course, we have been discussing here marriages in which each spouse has the same initial dowry. The old-young marriages can dominate old-old, young-young marriages even under an equal consumption arrangement provided the dowry of the young spouse sufficiently exceeds that of the old spouse.

III. Enforcement with and without Altruism

In the absence of altruism would family members voluntarily maintain these implicit contracts as family members age? The answer is

[11] To see this consider an old-young marriage in which the young person promises to consume less than the old person in the state of nature in which both spouses survive. Suppose that this promise to the old person of higher consumption in the "both survive" state is large enough to exactly compensate the old person for the loss in expected utility from the state in which his spouse dies but he survives. The old person's expected utility from this latter "bequest" state is lower when he marries someone young, rather than someone old, because the probability of the young person actually dying is smaller. While the old person is by assumption no worse off in this compensated old-young marriage, the young person could be worse off than if he had married someone young. By entering into the compensated old-young marriage, the young person reduces his payoff in the both survive state while leaving the payoff in the bequest state unchanged. He also increases the probability of the bequest state and lowers the probability of the both survive state. Although expected consumption for the young spouse rises, the spreading of the payoffs may lower expected utility, depending on the young spouse's degree of risk aversion.

that there always are ways of structuring payments to individuals within the family so that each individual at each moment in time has a selfish interest in maintaining the original implicit contract. An equal consumption marriage contract between two individuals with the same survival probabilities and the same initial endowment is a good first example. If each spouse maintains control over his own wealth while both spouses are alive and consumes at the same rate as the other spouse, then each will separately have an incentive to continue the contract at each point in time. A similar type of individual control can be maintained in family arrangements; rather than have the parents use up all their resources before the children begin contributing to their support, the children can contribute each period in return for that period's expected parental bequest. This scenario of parents' maintaining control over their wealth until the very end, as enforcement leverage over their children, may partly explain the limited use of gifts as a tax-saving intergenerational transfer device.

The proof of this proposition is immediate from equation (10). Given their initial endowments at time $t - 1$, W^H_{t-1} and W^S_{t-1}, family members choose a value θ^* such that the contract to consume the contingent plan $[C^H_{t-1}(\theta^*), C^S_{t-1}(\theta^*), C^H_t(\theta^*), C^S_t(\theta^*), \ldots, C^H_D(\theta^*), C^S_D(\theta^*)]$ is in the core at time $t - 1$. The consumption plan at time t $[C^H_t(\theta^*), C^S_t(\theta^*), \ldots, C^H_D(\theta^*), C^S_D(\theta^*)]$ represents the period t Pareto-efficient contract corresponding to the initially chosen utility weight θ^*. This plan is in the core for a set S_t of individual endowments of family members in period t, W^H_t and W^S_t, which satisfy $W^H_t + W^S_t = W_t$. To insure that the initial contract remains a core allocation for each family member, side payments are made at time $t - 1$ when consumption in period $t - 1$ occurs. The side payments leave the period t individual endowments in set S_t. Since the initial contract $[C^H_{t-1}(\theta^*), C^S_{t-1}(\theta^*), C^H_t(\theta^*), C^S_t(\theta^*), \ldots, C^H_D(\theta^*), C^S_D(\theta^*)]$ is in the core, each selfish family member will have a personal incentive to make or accept these side payments.

There are two additional questions of enforcement to consider. One problem is that a spouse may covertly name a third party as beneficiary in his will in exchange for the same commitment by the third party or in exchange for a particular service. A second type of cheating may occur when one or both spouses covertly consume in excess of the consumption levels dictated by an optimal implicit marriage contract; while each spouse may correctly believe that he or she is the beneficiary in the other spouse's will, each may try to take advantage of the other by increasing his own consumption and thus reducing the potential bequest available to the other spouse.

These two types of cheating will be more problematic for implicit incomplete annuity agreements between friends or relatives who are physically separated. The consumption cheating scenario can be modeled as a Nash equilibrium in which each partner chooses his consumption path by taking the other partner's consumption path and potential bequest path as given. Resources are consumed at a faster rate in the Nash equilibrium as each partner fails to consider how his consumption will diminish his expected bequest and thus the expected utility of his partner.

Using male survival probabilities we calculated for two 55-year-olds the dollar equivalent utility gain from engaging in a Nash consumption-cheating partnership. The gains in the Nash equilibrium proved to be almost identical to those in the more efficient marriage contract. For levels of risk aversion of 0.75, 1.25, and 1.75, the percentage dollar increments are, respectively, 19.9, 22.2, and 23.5. While the rate of consumption is faster in the Nash equilibrium, it is not much faster than in the marriage contract. Intuitively cheating by overconsuming is fine provided one's partner actually dies; but if one's partner survives, the early excessive consumption will require relative deprivation later on. Apparently this latter consideration dominates the former, leaving utility in the cheating equilibrium at essentially the same level as under a marriage contract. These examples suggest that consumption cheating does not represent a substantial impediment to consumption–risk-sharing arrangements.

Another means of enforcing these implicit contracts is simply altruism. All of our calculations have involved maximizing a weighted sum of individual family members' utilities. If, however, each family member is altruistic toward each other and each weights each family member's utility from consumption in the same way, then all family members would unanimously agree on the utility maximand. The calculations we have presented can, therefore, be thought of as resulting from the maximization of an agreed-upon altruistic family utility function. Since all family members agree on the maximand, there is no problem of enforcement.

IV. Summary and Conclusion

This paper has demonstrated that consumption and bequest-sharing arrangements within marriage and larger families can substitute to a large extent for complete and fair annuity markets. In the absence of such public markets, individuals have strong economic incentives to establish relationships which provide risk-mitigating opportunities. Within marriages and families there is a degree of trust, information, and love which aids in the enforcement of risk-sharing agreements. Our calculations indicate that pooling the risk of death can be an important economic incentive for family formation; the paper also suggests that the current instability in family arrangements may, to some extent, reflect recent growth in pension and social security public annuities. The methodological approach of this paper can be applied to the study of family insurance against other types of risks. Of chief interest are those types of risks that are handled very poorly by anonymous public markets. Disability insurance and insurance against earnings losses are good examples.

Our approach has been to compare family insurance with perfect insurance. It would seem worthwhile to compare family insurance with public market insurance where the market insurance is subject to adverse selection and moral hazard problems and family insurance is not. Realistic specification of the degree of adverse selection and moral hazard may indicate that family insurance dominates public market insurance even in small families.

Finally, the paper suggests the empirical difficulty of determining whether intergenerational transfers reflect altruism or simply risk-mitigating arrangements of essentially selfish individuals in the absence of perfect insurance markets. Distinguishing between the selfish and altruistic models is fundamental to a number of major economic questions, including the impact of the social security system on national saving and the effectiveness of fiscal policy.[12]

Appendix

Computational Algorithm for the Two-Family-Members Dynamic Risk-pooling Problem

This Appendix indicates the algorithm used to solve the two-family-members dynamic programming problem, copied here as equation (A1). The algorithm for the case of three family members is similar to that for two members and is available from the authors. While we consider the iso-elastic family of utility functions, our algorithm can be applied to any homothetic utility function.

$$V_{t-1}(W_{t-1}) = \max_{W_t, C^H_{t-1}, C^S_{t-1} \geq 0,\, t = T, \ldots, 1} [u^H(C^H_{t-1}) + \theta u^S(C^S_{t-1}) + \alpha P_{t|t-1} Q_{t|t-1} V_t(W_t) + \alpha P_{t|t-1}(1 - Q_{t|t-1}) H_t(W_t) + \theta\alpha Q_{t|t-1}(1 - P_{t|t-1}) S_t(W_t)], \tag{A1}$$

subject to

$$W_t/R + C^H_{t-1} + C^S_{t-1} = W_{t-1}. \tag{A2}$$

Again, the letters H and S correspond to the two family members with respective conditional survival probabilities $P_{t|t-1}$ and $Q_{t|t-1}$. The expression W_t is joint family wealth, θ is the weighting factor, and $H_t(W_t)$ and $S_t(W_t)$ are the expected utility levels for each family member if he alone survives to period t.

[12] See Barro 1974.

Optimal values for C_t^H and C_t^S are found recursively starting at period T and proceeding to period 0. We demonstrate that $V_t(W_t)$ may be written in the form:

$$V_t(W_t) = v_t \frac{W_t^{1-\gamma}}{1-\gamma}, \tag{A3}$$

where v_t is a constant. We also show that total family consumption, C_t, is given by

$$C_{t-1} = W_{t-1} \frac{v_T^{1/\gamma}}{v_T^{1/\gamma} + (\alpha K_t R)^{1/\gamma} R^{-1}}, \tag{A4}$$

where K_t is another constant. Given total family consumption, consumption of the two members is

$$C_{t-1}^H = \frac{C_{t-1}}{1+\theta^{1/\gamma}}, C_{t-1}^S = C_{t-1} \frac{\theta^{1/\gamma}}{1+\theta^{1/\gamma}}. \tag{A5}$$

We demonstrate that K_t is a function of v_t and that v_{t-1} is a function of K_t. Starting then at the initial value for K_t, K_{T+1}, we can compute v_T; v_T in turn gives K_T, which in turn gives v_{T-1}. Proceeding in this fashion to period zero we compute the entire sequence of v_t's and K_t's. These values can then be used in equation (A4) to compute the ratio of consumption to wealth at each period. These ratios together with an initial level of wealth plus (A2) and (A5) generate the optimal consumption path. The homotheticity of the utility function permits us to calculate recursively the shape of the consumption path independently of the initial level of wealth.

Starting with period T the maximization problem for equation (A1) is

$$V_T(W_T) = \max \frac{1}{1-\gamma} (C_T^H)^{1-\gamma} + \theta \frac{1}{1-\gamma} (C_T^S)^{1-\gamma}$$
$$\text{s.t.} \quad C_T^H + C_T^S \leq W_T, \quad C_T^H, C_T^S \geq 0.$$

Solving this maximization and computing the indirect utility function for V_T, we have

$$V_T(W_T) = v_T \frac{1}{1-\gamma} W_T^{1-\gamma}, \text{ where } v_T = (1+\theta^{1/\gamma})^\gamma, \tag{A6}$$

$$C_T^H = W_T \frac{1}{1+\theta^{1/\gamma}}, C_T^S = W_T \frac{\theta^{1/\gamma}}{1+\theta^{1/\gamma}}. \tag{A7}$$

For $t < T$, (A1) for the iso-elastic case is written as

$$\begin{aligned} V_{t-1}(W_{t-1}) &= \max_{C_{t-1}^H, C_{t-1}^S} \frac{1}{1-\gamma} (C_{t-1}^H)^{1-\gamma} + \frac{\theta}{1-\gamma} (C_{t-1}^S)^{1-\gamma} \\ &+ \alpha \frac{P_t}{P_{t-1}} \frac{Q_t}{Q_{t-1}} v_t \frac{1}{1-\gamma} W_t^{1-\gamma} + \alpha \frac{P_t}{P_{t-1}} \left(1 - \frac{Q_t}{Q_{t-1}}\right) h_t \frac{1}{1-\gamma} W_t^{1-\gamma} \\ &+ \theta\alpha \left(1 - \frac{P_t}{P_{t-1}}\right) \frac{Q_t}{Q_{t-1}} s_t \frac{1}{1-\gamma} W_t^{1-\gamma} \end{aligned} \tag{A8}$$
$$\text{s.t.} \quad C_{t-1}^H + C_{t-1}^S + \frac{W_t}{R} = W_{t-1}.$$

In going from (A1) to (A8) we use the fact that $H_t(W_t) = h_t[W_t^{1-\gamma}/(1-\gamma)]$ and $S_t(W_t) = s_t[W_t^{1-\gamma}/(1-\gamma)]$ for the iso-elastic utility function. The values for h_t and s_t are implicitly defined as the bracketed term in equation (7) in the text with $j = 0$ corresponding to time t and with each family member's survival probabilities from time t substituting for P_j.

It is easy to see from (A8) that for given total family consumption, C_t, C_t^H and C_t^S will always satisfy (A5). Hence we may rewrite (A8) as

$$V_{t-1}(W_{t-1}) = \max_{C_{t-1}} v_T \frac{1}{1-\gamma} C_{t-1}^{1-\gamma} \tag{A9}$$

$$+ \alpha \frac{W_t^{1-\gamma}}{1-\gamma}\left[\frac{P_t}{P_{t-1}}\frac{Q_t}{Q_{t-1}} v_t + \frac{P_t}{P_{t-1}}\left(1 - \frac{Q_t}{Q_{t-1}}\right)h_t + \theta\left(1 - \frac{P_t}{P_{t-1}}\right)\frac{Q_t}{Q_{t-1}} s_t\right].$$

Denoting the term in brackets by K_t we now have

$$V_{t-1}(W_{t-1}) = \max_{C_{t-1}} v_T \frac{1}{1-\gamma} C_{t-1}^{1-\gamma} + \alpha \frac{1}{1-\gamma} W_t^{1-\gamma} K_t \tag{A10}$$

$$\text{s.t.} \quad C_{t-1} + \frac{W_t}{R} = W_{t-1}.$$

Maximizing (A10) and computing the indirect utility functions yields

$$v_{t-1} = [v_T^{1/\gamma} + (\alpha R K_t)^{1/\gamma} R^{-1}]^\gamma, \tag{A11}$$

$$C_{t-1} = W_{t-1} \frac{v_T^{1/\gamma}}{v_T^{1/\gamma} + (\alpha K_t R)^{1/\gamma} R^{-1}}. \tag{A12}$$

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