Steve Reich's Signature Rhythm, and an Introduction to Rhythmic Qualities

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There are few composers for whom we could identify anything like a "signature rhythm," but for Steve Reich, the rhythm shown in Figure 1 definitely merits the title. Most musicians will immediately recognize it as the basic material of his *Clapping Music* (1972), which is where it first appears in his works. It is also the main rhythm of his next work *Music for Pieces of Wood* (1973), which can be understood as an exploration of this rhythm's properties. The signature rhythm continues to feature prominently in Reich's works for decades after he wrote these two pieces: it is the basis of movements 1, 2, 3a, 4, 7, and 9 of *Music for 18 Musicians* (1974–6), the framing parts of the third central movement of *Desert Music* (1984), the first and third movement of *Sextet* (1985), the first movement of *Three Movements* (1986), and the third movement of *Electric Counterpoint* (1987). Closely related rhythms are the basis of canons in the first movement of *Vermont Counterpoint* (1982), *New York Counterpoint* (1985), and the outer movements of *Desert Music*. And, as we will also see below, is central to the rhythmic design of *Nagoya Marimbas* (1994).

Figure 1: Reich's signature rhythm

Reich himself identified *Clapping Music* as a turning point in his career, saying it "marks the end of my use of the gradual phase shifting process. [. . .] By late 1972, it was time for something new." (Reich 2002, 68) The "something new" that Reich is talking about, is, from one perspective, not really that much different than the abandoned gradual phase-shifting process. Subsequent works are still based on the same rhythm played in multiple parts in canon, shifting in phase relationships with one another. The difference is that the gradual process is different: rather than shifting phase through a slow accelerando in one part, new canonically related parts are introduced by a gradual process of adding the notes of the new rhythm one or a few at a time. The technique actually appeared first in *Drumming* (1971), but that piece also utilizes the accelerando phase-shifting procedure.

Reich's sense of a turning point may have to do with the abandonment of the radical aesthetic position, stated in his manifesto "Music as Gradual Process,"[1] that Reich had become strongly identified with in the later 1960s. Many commentators have noted that this doctrine is abandoned by the 1980s.[2] Robert Schwartz, for instance, identifies *Drumming* (1971) and *Music for 18 Musicians* (1978) as the turning point, and Dean Paul Suzuki similarly identifies *Drumming* as the beginning of a gradual abandonment of minimalism, while Reich himself suggested on one occasion that he had abandoned minimalist principles at least by 1973 with *Music for Mallet Instruments, Voices, and*

[1] Reich 2002, 34–36

[2] For instance, Bakker (2019)

Percussion.[3] While virtually all music involves processes, the kind of process Reich refers to in this essay is one that is gradual and directly perceivable. By advocating for music in which such immediate processes constitute the entirety of the music, Reich stakes out an extreme aesthetic philosophy, one that prohibits what he calls "hidden structural devices." In other words, he rejects compositional artifice, the many devices a composer might use that have a definite and predictable effect regardless of whether the listener is consciously aware of the methods and mechanics behind them. Like any extreme aesthetic philosophy, this one is cripplingly limited, and it is predictable that it could only maintained for so long before the banned substance, compositional artifice, should seep back into circulation. I will make the case that this occurs in *Music for Pieces of Wood*, a piece that appears on the surface to be a process pieces like *Drumming*, but where the process is carefully engineered, through a bit of compositional artifice, to generate a definite musical form. The vehicle for this is the signature rhythm.

The idea of a "signature rhythm" seems at odds with the principles of the European concert music tradition, perhaps, because something so specific and concrete could hardly seem to provide sufficient mortar for the construction of a compositional style. Something more abstract, a compositional method or philosophy, would seem more fitting to the task. But it is precisely such an aesthetic philosophy, one that was put forth expressly to compete with the then-dominant compositional philosophies of Darmstadt serialism and Cagean indeterminism, that Reich abandoned in 1972. The mystery that remains, then, is how could such a distinctive and manifold universe of music grow from such a humble source as the rhythm of Figure 1. A bit of theory, the concept of *rhythmic qualities* introduced below, will help explain how this rhythm is uniquely suited to such a task, a rhythmic kaleidoscope that can project a seemingly endless number of patterns by tilting it in different directions.

Reich's study of African music in the 1960s, culminating in his trip to Ghana in 1970, is a logical place to look as a source of the signature rhythm. Martin Scherzinger has documented the formative role of African music on Reich's early work, beginning with his discovery of A.M. Jones's *Studies in African Music*[4] in 1963.[5] Indeed, the signature rhythm is very close to the rhythm of Figure 2, a foundational bell pattern in certain dances of the Ewe tribe of Ghana, as well as many other West and Central African musical traditions, which Jones called the *standard pattern*. The standard pattern is in abundant evidence in Jones's transcriptions and figures prominently in a significant discussion of African rhythm in which he argues that African musical traditions widely separated geographically and culturally nonetheless belong to a common musical tradition. The use of the standard pattern as a rhythmic basis for the music of two dances from distant musical cultures is the key to that argument.

Figure 2: The African standard pattern

[3] Schwartz 1981–2, 1990; Suzuki 1991, 492, 505–506.
[4] Jones 1959.
[5] Scherzinger 2018, 2019; Reich 2002, 55–56.

Reich himself makes the connection in a published interview. When Russel Hartenberger asks him where the *Clapping Music* rhythm came from he says "It's just a variation on the African pattern" (referring to the rhythm of Figure 2). He then elaborates, "I didn't want to use the African pattern at that time; I wanted to make my own variation on it. [. . .] Probably it was that I wanted to do something that came out of the African bell pattern but had my own stamp on it."[6] Indeed, Reich's exact rhythm does not appear anywhere in Jones' transcriptions. Reich's concerns about direct appropriation are apparent in much of his commentary on the influence of African music. For example:

> What I don't want to do is go and buy a bunch of exotic-looking drums and set up an Afrikanische Musik in New York City. In fact, what I think is going to happen more and more is that composers will study non-Western music seriously so that it will have a natural and organic influence on their music.[7]
>
> One can create a music with one's own sound that is constructed in the light of one's knowledge of non-Western *structures*. [. . .] One can study the rhythmic structure of non-Western music and let that study lead one where it will, while continuing to use the instruments, scales, and any other sound one has grown up with. This brings about the interesting situation of the non-Western influence being there in the thinking, but not in the sound.[8]

Indeed, pre-empting any possible perception of appropriation seems to have been a central motivating force in his compositional process. In a sketch for the signature-rhythm-based canon of *Music for Pieces of Wood*, Reich pencils in a note to himself: "Too African?".[9]

The standard pattern may be transformed into Reich's signature rhythm by either (1) rotating by five eighth-notes and adding one note, or (2) adding a note and moving the last note of the measure back one place. The added note in either case is the second eighth of the measure. We could also describe the difference by drawing upon a pitch/rhythm analogy proposed by Jeff Pressing and Jay Rahn[10] that has proved effective in previous analyses of Reich's canon-based music.[11] They describe the standard pattern as "diatonic," meaning that it reproduces the same pattern in *beat classes* as a C major diatonic scale does in pitch classes. Milne et al. (2017) suggest the term *time-class* a somewhat preferable replacement for the traditional term "beat-class." Using the time-class/pitch-class analogy, we could say that Reich "transposes" the standard pattern from "C major" to the closely related "F major" and adds a "C-sharp." We will investigate the relationship between these rhythms further after developing some theoretical tools in the next section.

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[6] Hartenberger 2016, 156–8.
[7] Reich 2002, 55.
[8] *ibid.*, 71, original emphasis.
[9] The sketch is reproduced and discussed by Scherzinger (2019).
[10] Pressing 1983; Rahn 1983, 1996
[11] Cohn 1992, Roeder 2003

Rhythmic Qualities and Reich's Signature Rhythm

The most basic and important properties that Reich's music has in common with the music he found transcribed in Jones' book and learned to play in Ghana was that it is built on a steady pulse and is inherently cyclic, based on rhythmic patterns that can be repeated *ad lib*. This allows us to think of rhythms as time-class sets. In Figure 3, Reich's signature rhythm is drawn on a circle.[12] Dots, representing note onsets, are placed at twelve equally spaced point around the circle, representing all of the possible positions on the eighth-note grid of a 12/8 measure. This is therefore a *12-cycle*, twelve points equally spaced around a circle, and I will refer to the space in which it is drawn as the *f_1 space* of the 12-cycle for reasons that will become apparent shortly. In the f_1 space, every pulse has a distinct location that is a distance of 1 from the origin. Hence, all of the pulses fall on the *unit circle*, with their angle from zero (which is on the vertical axis per the musical "clockface" convention) determined by their place in the measure. If we think of each pulse as a *vector* from zero to a point on the unit circle, they all have the same length (1) but point in different directions.

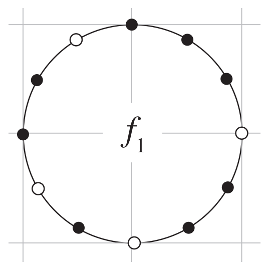

Figure 3: Reich's signature rhythm on the unit circle

A question that we might ask about rhythms in f_1 space is, are they relatively *balanced*? In other words, do the note onsets tend to occur more frequently in one part of the measure or another? If we think of each beat as a force, pulling towards its place on the circle, a balanced rhythm is one where the forces cancel one another out, and in an imbalanced one they tend to reinforce one another. By thinking of each onset as a vector, and each vector as a force, we can find the balance by adding them. Figure 4 does this for Reich's signature rhythm and the standard pattern. Balance is indicated by how close the result is to zero. Both rhythms sum to a point inside the unit circle, making them relatively balanced in f_1, with the standard pattern being more balanced than the signature rhythm. When a rhythm is imbalanced, relatively concentrated to one part of the cycle so that it sums to a point outside the unit circle, we will say it has the *quality* associated with that space. The rhythms in Figure 4 do not have strong f_1 quality. This term, and the metaphors of forces and balance, come from Ian Quinn, who defines quality as a property of pitch-class sets.[13] The extension of the concept I propose here uses the time-class/pitch-class analogy to define a corresponding concept of rhythmic quality. It builds upon many recent developments of the *discrete Fourier transform*

[12] Such circular representations of rhythms are used by many theorists, such as London (2012) and Toussaint (2013).
[13] Quinn 2006

on pitch-class sets[14] and some applications of the method to rhythm.[15] This section will explain the discrete Fourier transform from the perspective of rhythm and meter.

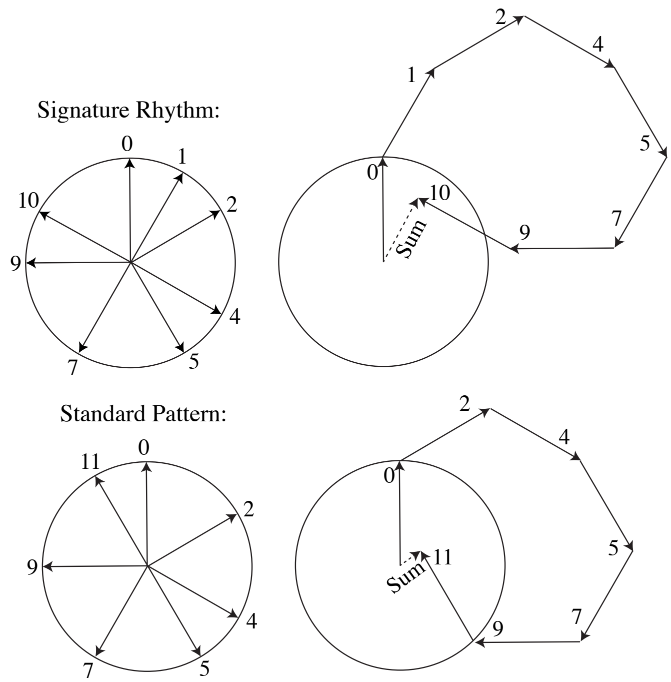

Figure 4: Sums of time-class vectors in f_1 for two rhythms

Imagine hearing Reich's rhythm in a variety of different possible meters, as shown in Figure 5: 6/4, 3/2, 12/8. In each case, the meter imposes one or more subcycles upon the larger cycle of the measure. In 6/4, the dotted half-note beats create a six-element subcycle, dividing the full cycle in half. Dividing the measure by six defines quarter-note subcycles. Other possible subcycles are significant periodicities in other meters: In 3/2, in addition to the quarter-note cycles, the meter divides the measure into three half-note subcycles, while in 12/8 there is a dotted-quarter subcycle dividing the measure by four. We can also assess the balance of each rhythm on any of these subcycles. For instance, consider dividing the standard pattern and the signature rhythm into two dotted half-note subcycles of 6 elements. If we imagine these as beads on necklaces, by wrapping the necklaces in two circles, as shown in Figure 6, we create a six-element space. This is called the f_2 space, meaning it corresponds to a division of the f_1 space into two subcycles. We can assess balance in this space by adding vectors the same way we did for the f_1 space, and it will tell us whether the rhythms are consistently weighted toward a particular place in the half measures. We find that the standard pattern sums to a point directly on the unit circle, meaning it has a moderate f_2 quality, whereas Reich's rhythm is perfectly balanced in f_2, summing to zero, so that f_2 quality is absent. The

[14] Amiot and Sethares 2013; Amiot 2016; Yust 2015a–b, 2016.
[15] Amiot 2016; Milne et al. 2015; Milne, Bulger, and Herff 2017.

angle of vector sums is the *phase* of the component, indicating where in the subcycle it is weighted towards. In the standard pattern, the phase is one eighth-note ahead of the downbeat, or 1/6th cycle counterclockwise, in the half-measure cycle, where two onsets occur a half-note apart (positions 5 and 11). For Reich's signature rhythm, the phase of f_2 is undefined.

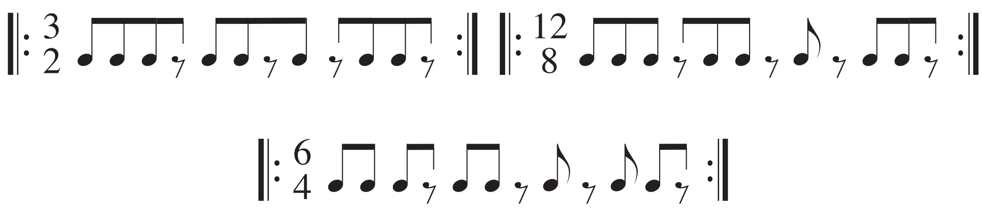

Figure 5: Reich's signature rhythm in different meters

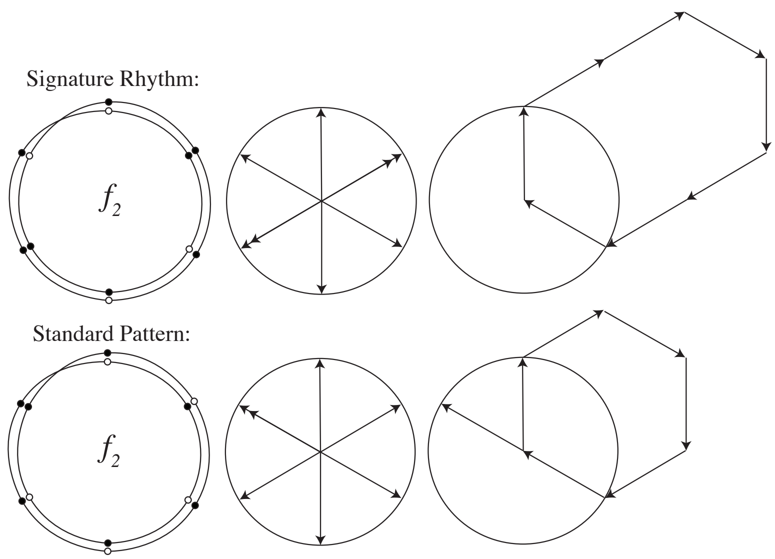

Figure 6: Calculation of f_2 for Reich's signature rhythm and the standard pattern

"Wrapping the necklace" two times around the circle is equivalent to multiplying the *angles* of the time-class positions by 2 (NB, not their *lengths*). We can do the same, multiplying the angles by 3, 4, or 6, to see how each rhythm balances on the half-note, dotted quarter, or quarter-note subcycles, as shown in Figure 7. These vector sums are called f_3, f_4, and f_6, where the subscript again refers to the division of the full cycle. Notice that f_6 has only two possible phases, corresponding to on-beat and off-beat. Reich's rhythm is equal parts on and off the quarter-note beat, so it is balanced on f_6. The f_3 and f_4 sums are length 2, well outside the unit circle, so these are substantial qualities. They are out of phase with the downbeat, however, being rotated 1/4th and 1/3rd cycle clockwise, respectively, which amounts to an eighth note behind the downbeat in both cases (1/4th of a half note or 1/3rd of a dotted quarter). Therefore, although the rhythm has the potential to align with a 3/2 or 12/8 meter, Reich's choice of rotation makes this less apparent.

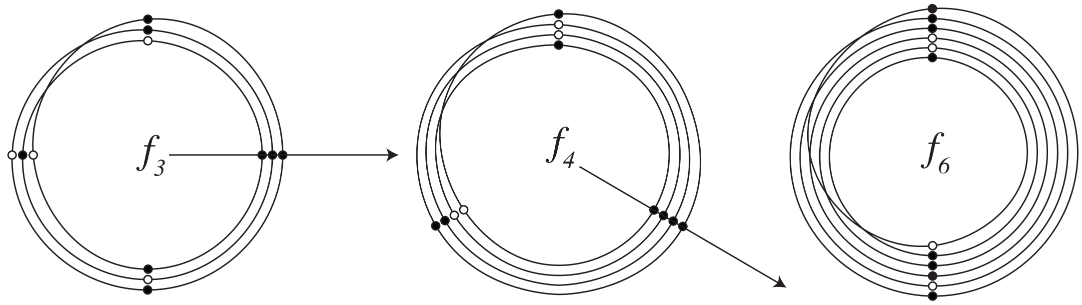

Figure 7: "Wrapped necklaces" for f_3, f_4, and f_6 of Reich's signature rhythm and the resulting vector sums

An important aspect of rhythmic qualities, besides their relationship to possible meters (which will be explored further momentarily), is that if we give a complete list of qualities, we also have a complete description of the rhythm, a property that will be defined more formally below as invertibility. So far, we have considered qualities of a rhythm in a 12-element universe that divide these twelve elements evenly into subcycles: f_1, f_2, f_3, f_4, and f_6. Divisions greater than 6 will not produce new qualities; for instance wrapping the necklace 8 times is equivalent to wrapping it 4 times counter-clockwise. However, there is one other important quality, which is given by dividing the measure into *five* parts, f_5. Even though 12/5 is not an integer, we can still define f_5 with the same necklace-wrapping procedure we used for f_2, f_3, f_4, and f_6. Figure 8a shows what happens when we do this: now *none* of the beads line up with one another. Instead, we reorder them so that timepoints separated by distances of 5 or 7 ♪s are adjacent, the rhythmic analog of a circle of fifths. The significance of the 5♪ interval is that it gives the closest approximation to a multiple of 12/5, $4^4/_5$. Therefore a best approximation to a meter of 5 in a 12-cycle is given by choosing five beads next to one another on the f_5 cycle, as in Figure 8(b). The resulting rhythm is one that maximizes f_5—i.e., this is the maximum length f_5 can have for a 5-note rhythm (3.73). It is also a maximally even rhythm, which is generally true for an n-note rhythm that maximizes f_n.[16] Both the standard pattern, which is maximally even, and Reich's rhythm, which has a maximally even subset, have strong f_5 qualities, as the vector sums in Figure 9 show. The distinctive feel of f_5 quality might be described as an asymmetrical 5-in-12 feel, or as a kind of "funkiness," to Cohn's term for similar kinds of rhythms, though differently defined.[17] In the pitch-class context (the original context for Quinn's definition of quality[18]) f_5 gives the balance of a collection on the circle of fifths, achieving maximum

[16] Amiot 2007.
[17] Cohn 2016.
[18] Quinn 2006.

values for the diatonic hexachord and pentatonic or diatonic scales (where the latter is the pitch-class analogue of the standard pattern).

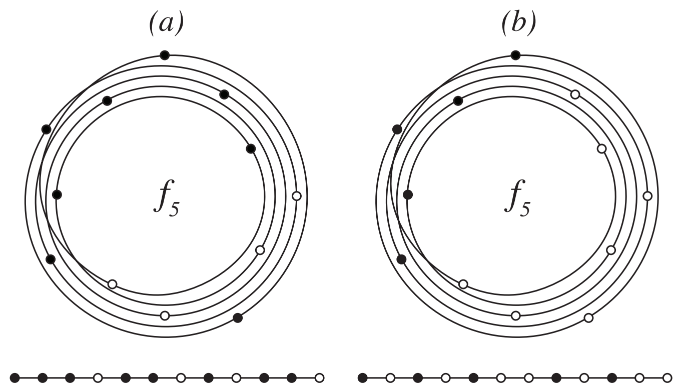

Figure 8: Reich's signature rhythm, (a), and the 5/12 maximally even rhythm, (b), in f_5-space

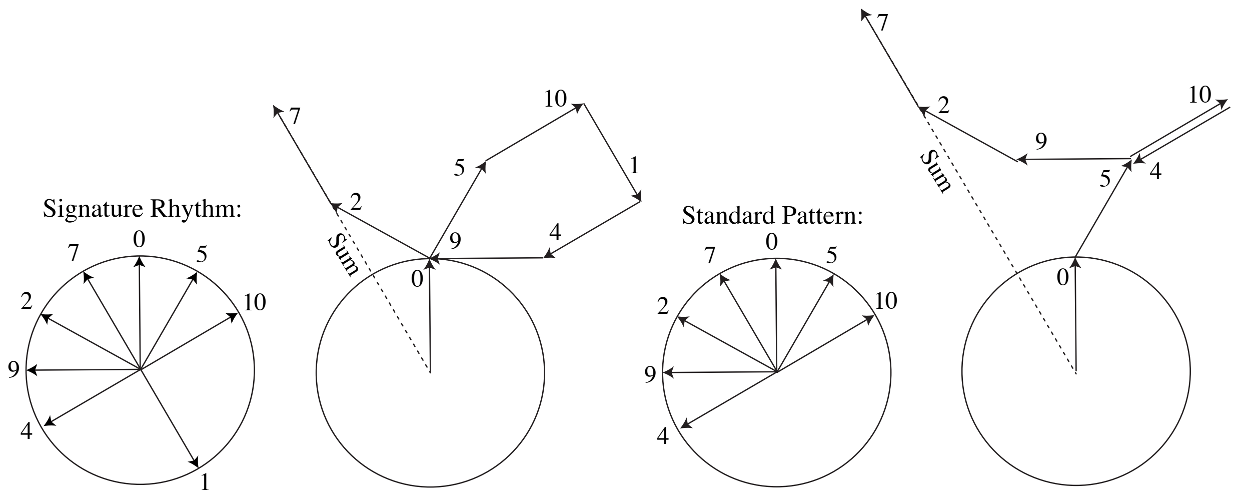

Figure 9: Derivation of f_5 for the signature rhythm and standard pattern

What I have essentially just defined is the *discrete Fourier transform* (or DFT) of a rhythm. Specifically, the DFT of an n-cycle rhythm is the set of n vectors, f_0–f_{n-1}, where f_0 has zero phase and is simply equal to the cardinality of the rhythm, and $f_{n-k} = f_k$, leaving $n/2$ or $(n-1)/2$ non-trivial rhythmic qualities, f_k, for $1 \leq k \leq n/2$. Each f_k is a two-dimensional vector, which I will call the DFT *components*. We are typically interested not in the Cartesian (x and y) values of these vectors, but their *magnitude* (length), denoted $|f_k|$, and *phase* (direction), denoted φ_k.[19] A magnitude indicates the size of

[19] Mathematically, Fourier components are *complex numbers*, and the Cartesian coordinates referred to here are the real and imaginary parts. For a thorough and approachable introduction to the DFT see Briggs and Henson 1995.

a quality, and a phase indicates its orientation with respect to the downbeat. Table 1 summarizes the DFT of the signature rhythm, using squared magnitudes in observance of the conservation of energy property described below.[20] Much of the time the cycle length, n, will be assumed to be 12. Where this is clear from context, Fourier components will be indicated with a simple subscript, f_k. Some analyses below will also compare qualities across different lengths of cycle, in which case n must be specified. In that case, the notation will be $f_{k/n}$.

Table 1: The DFT of the signature rhythm, using squared magnitudes and normalized phases

		f_1	f_2	f_3	f_4	f_5	f_6
Sig. rhythm	Mag.^{2}	0.54	0	4.	4.	7.46	0
	Ph.	1	—	3	4	11	—

The Fourier transform is typically described as a transformation of a signal (a function over time) from the time domain to the frequency domain. Mathematically the domain of the function does not have to be time (for instance, an image can be analyzed with DFT over spatial dimensions, and in Quinn's[21] usage the domain is pitch), but in our application to rhythm it is. We treat a rhythm as a function of time taking a value of 1 at onset points and zeros elsewhere. A *discrete* Fourier transform is one where the signal is discretized. For analysis of rhythms, we use the notated temporal grid to discretize the signal with no loss of information. The discretized signal is then an n-place vector, and the DFT is then the n-place vector $(f_0, \ldots, f_{n-1})$ just defined. The components of the DFT are discretized sinusoidal functions over time. In presenting the theory here, I have replaced sinusoidal functions with vector addition in a two-dimensional space, which is mathematically equivalent, with the length of a vector corresponding to the amplitude of the corresponding sinusoid, and its angle the phase.

Because of the correspondence of DFT components to metrical periodicities, it is tempting to posit a simple one-to-one relationship between the DFT of a rhythm and its meter, with the maximum qualities determining the type of meter, and their phases indicating the position of the downbeat. According to this idea, qualities like f_5 (ones that do not evenly divide their universe) might produce an alternative type of non-isochronous meter, of the type posited by Justin London.[22] Such a theory is tempting, but stated too strongly it has an obvious flaw: the DFT is purely a representation of the rhythm only, and a rhythm alone does not determine its meter, which is

[20] This will be the practice throughout this article. One may use the raw magnitudes instead, as Amiot (2016) does, and arrive at essentially the same conclusions.
[21] Quinn 2006–2007
[22] London (2012) in fact proposes exactly this limiting condition for non-isochronous meters, that they correspond to some maximally even division, even though he did not have anything like a concept of rhythmic qualities in mind.

subjective and can be manipulated through many other factors.[23] For example, the rhythm in Figure 10(a), the beginning of the first canon in *The Desert Music*, is a variant on the signature rhythm. It is, in integer notation, {012479t}, a subset of the signature rhythm, {0124579t}. The DFTs in Figure 10(b) and Table 2 show that removing the onset at 5 shifts the quality profile decisively in favor of f_4. This might suggest that the rhythm implies a 12/8 meter, but it does not induce such a meter, for a number of reasons. First, the downbeat is clearly defined by the context, and f_4 does not align with the downbeat in phase. Furthermore, the articulation and contour reinforce the 6/4 or 3/2 meter implied by the notation. The strong f_4 of the rhythm is not insignificant, but instead of defining a meter, it interacts with one, with the tension between the dotted-quarter periodicity and the quarter-note beat giving the rhythm an anti-metrical quality.

(a)

(b)

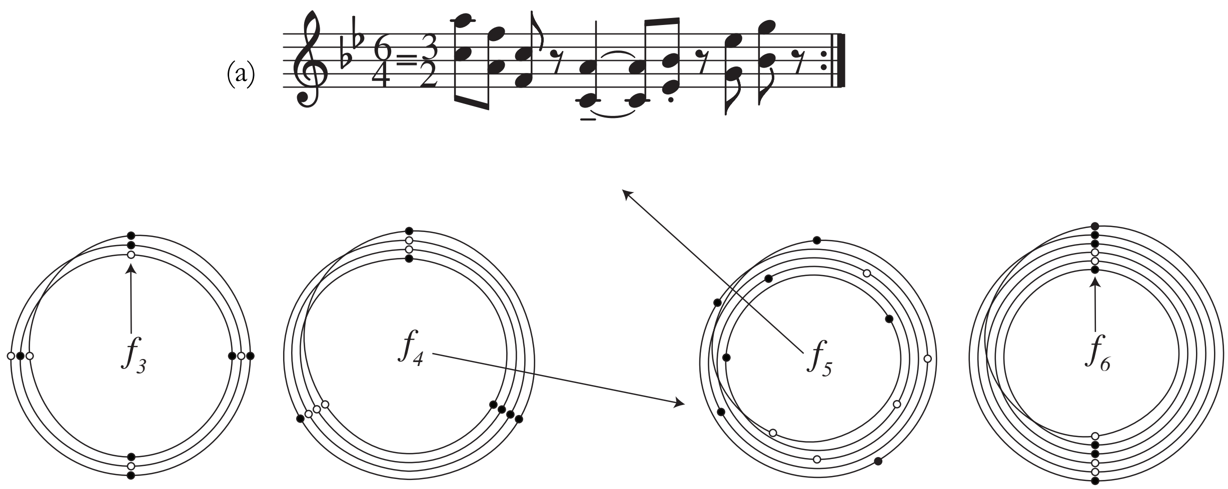

Figure 10: (a) *The Desert Music* reh. 21, Synthesizers and violins,
(b) Qualities f_3–f_6 of the rhythm as vectors

Table 2: The DFT of the variant of the signature rhythm in Figure 10(a), using squared magnitudes and normalized phases

Rhythm		f_1	f_2	f_3	f_4	f_5	f_6
{012479t}	Mag.^{2}	2.27	1.	1.	7.	5.73	1.
	Ph.	11.83	4	3	3.36	10.29	0

It is these kinds of tensions between rhythmic qualities and an established meter that give life and musical interest to many of Reich's rhythms. Since qualities correspond one-to-one to possible metrical divisions if we admit the type of non-isochronous meter defined by maximally

[23] In Reich's canon-based pieces of the 1980s and 1990s, for instance, melodic contour plays a particularly important role in the metrical sense of the canons. Analyses by Horlacher (2000–1) and Roeder (2003) demonstrate this well.

even patterns, these possible tensions between qualities and meter can be classified into two types: the quality may have a non-metrical periodicity, or a metrical periodicity that is out of phase. Instances of the latter type do not necessarily challenge the meter; they may in fact reinforce it, by being regularly aligned with some off-beat part of the meter.[24] A particularly interesting case of the former type (non-metrical periodicity) are the "funky" qualities, the ones like f_5 that divide the cycle by a non-integer periodicity. These are anti-metrical with respect to any conventional meter.

Another example worth comparing to the signature rhythm is the basic rhythm of *Music for Mallet Instruments, Voices, and Organ* of 1973, Figure 11(a), an important stepping stone on the way to Reich's landmark work of the 1970s, *Music for 18 Musicians*. This rhythm was in competition, we may suppose, for the style-defining role that the signature rhythm was to adopt. The DFT in Table 3, reveals some notable similarities to the signature rhythm: f_3, f_4, and f_5 are all relatively strong, with f_5 being slightly larger than the other two. The main differences are that f_6 is particularly high, whereas it is zero for the signature rhythm, and f_3 is closer to being in phase with the downbeat and f_4 further out of phase. As a result, the rhythm is firmly grounded in the 3/2 meter: the particularly strong eighth-note periodicity (f_6) fully agrees with the meter, and the f_3 quality is consistent with it. The f_3 quality is less than it might otherwise be because the eighth-note off-beats cancel out the on-beat onsets. These do not really weaken the 3/2 metrical quality; rather, they make it possible to shift the downbeat by eighth note without a marked sense of syncopation. In other words, the location of the half-note beat is not overly fixed. The dotted-eighth-note periodicity of f_4 cannot coexist with the eighth-note periodicity in any conventional meter, so despite being larger than f_3, it does not compete to establish any semblance of 12/16 meter. Nonetheless, it is not irrelevant: Figure 11(b) shows a melody that appears in the voice part after the first canon is fully established in the marimbas. This part draws partially upon the rhythm of the *dux* and partially upon the *comes*, which is shifted by a dotted quarter, a half cycle. The resulting rhythm differs from the main marimba rhythm by a single "voice leading" from onset 0 to 11. The effect on the rhythmic qualities is momentous: now f_4 is overwhelmingly dominant and the odd-numbered qualities absent. The 12/16 metrical implications of the part are now distinctly perceptible as a cross-meter to the established 3/2 across the rest of the texture.

[24] An instructive example might be the mandolin chop in bluegrass music. This rhythm is wholly off-beat, so if we reckon the meter as 4/4 with bass notes on one and three, the mandolin chop (on two and four), is a perfect representative of a division of the meter in two (f_2), like the bass but opposite in phase. A listener unfamiliar with the style, hearing the mandolin chop in the absence of the bass, would infer the wrong meter (a quarter-note out of phase). But a well enculturated listener can immediately and intuitively infer the correct meter from the mandolin chop alone. For instance, the mandolin can set the tempo before a song begins with a count-off using the off-beat chop, and for expert musicians inferring meter from this is automatic and effortless. Similar points could be made about rhythmic conventions of other genres, such as the rock backbeat, or the salsa *montuno*.

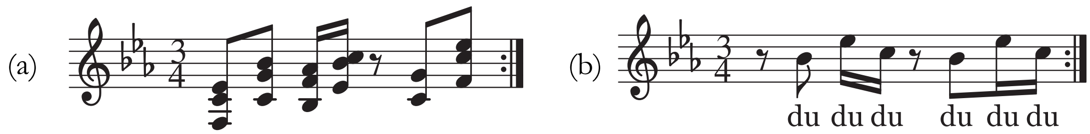

Figure 11: (a) The *dux* of the marimba canons in *Music for Mallet Instruments, Voices, and Organ*, (b) The voice part at Reh. 3C

Table 3: The DFT of the rhythms from Music of Mallet Instruments *in Figure 11, using squared magnitudes and normalized phases*

Rhythm		f_1	f_2	f_3	f_4	f_5	f_6
{02458t}	Mag.^{2}	0.17	1.	2.	3.	3.73	16
	Ph.	2.5	8	1.5	7	0.5	0
{2458te}	Mag.^{2}	0	4.	0	12.	0	4.
	Ph.	—	8	—	7	—	0

These examples illustrate that metrical qualities only enforce a meter when they are overwhelmingly dominant compared to all other qualities. In that case the possible metrical implications of other qualities are occluded, unless they have an index that divides that of the principal quality (as f_3 does to f_6 in the *Music for Mallet Instruments* rhythm). In the absence of a dominating quality on a divisor of the cycle number—for instance in the signature rhythm or the African standard pattern—qualities on divisors of the cycle interact with meters established by other means, such as context, melodic contour, or articulation. These interactions include the “locked-in” or grounded effect of agreement with the meter, and the syncopated effect of agreeing with the meter in periodicity but not in phase. A quality with a non-metrical periodicity will float across the meter, going in and out of phase with the metrical layers in a regular pattern, relatively balanced between different parts of the metrical cycles. This kind of quality can therefore be described as light or lilting, and might be understood as enriching the meter, making a more complex rhythm by complementing the qualities of the meter with independent ones. Assuming a conventional, fully isochronous meter, co-prime qualities like f_5 always have this extra-metrical character, and so are generally mercurial (i.e., they cannot be pinned down by any meter), a point related to the idea of “funkiness” mentioned above. Reich himself refers to the lack of specific metrical affinity as a positive quality of the diatonic rhythm in African music.[25]

Much of the theoretical value of the DFT comes from the fact that it is an information-preserving transformation of a rhythm that separate its purely intervallic properties from those relating to its rotational orientation within the temporal cycle (analogous to transposition-dependent

[25] Reich 2016, 175–6.

properties of a pitch-class set). More specifically, the DFT has the following basic mathematical properties[26]:

(1) ***Orthogonality:*** Components f_1–f_6 (or f_1–$f_{n/2}$) are mathematically independent, meaning that a particular value of one component does not constrain the possible values of any of the others.[27] Thus it is *not* necessarily true, for instance, that a weighting of a half-note cycle, an f_3 quality in the 12-cycle, implies a weighting of the quarter-note cycle, or an f_6 quality, or vice versa.

(2) ***Bijectivity:*** As a direct consequence of orthogonality, the DFT is in *one-to-one* correspondence with the original rhythm, meaning that the rhythm's DFT is unique, and the DFT can be uniquely transformed back into the original rhythm.

(3) ***Relation to translation:*** The magnitude of any f_k is independent of translation (i.e., rotation or time-class transposition) of a rhythm. Therefore, rotating a pattern does not change what qualities are present in the rhythm, only how they are positioned with respect to the meter. Translation only affects the phases of the Fourier components. Hence, the division of Fourier qualities into magnitudes and phases isolates the presence of some quality (magnitude) from its alignment with a downbeat (phase). Note: this is also true of retrograde (time-class inversion); it only affects the phases of the Fourier components, not the magnitudes.

(4) ***Conservation of energy*** (*Parseval–Plancherel)*: An important basic result of Fourier theory says that the Fourier transform preserves the *energy* of a signal, up to a scaling factor, where energy is defined (in analogy with a physical signal) as the sum of squared weights for each timepoint. In our application, the timepoints are weighted either 1 or 0 (an onset is either present in the rhythm or not), so the energy is equivalent to cardinality (the number of onsets). This means, then, that the sum of squared magnitudes of f_0–f_{11} is equal to 12 times the cardinality (12, or u, is the scale factor).

(5) ***Complementation:*** The complement of a rhythm, the rhythm obtained by replacing onsets with rests and rests with onsets, has the same DFT magnitudes as the original rhythm, and opposite phases. (This is the same as pitch-class complementation.)

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[26] Amiot (2016) and Briggs and Henson (1995) include proofs of all of these. See also Yust 2015a.

[27] A caveat, however, is that this independence is only fully realized by allowing each timepoint to be arbitrarily weighted. Thus far we have assumed that a rhythm either contains a time point (which can be understood as a weighting of 1) or not (weighting of 0). In principle, these weights can vary continuously. Component f_0 is left out because if we exclude the possibility of *negative* weightings (which we would typically want to do), then there is a limited dependence of all components on f_0 in the sense that no other component or combination of components can exceed f_0 in magnitude. We may also note that independence does *not* apply to components of index larger than $n/2$, because of the strict dependence between complementary components (f_k and $f_{(n-k)}$) noted above. This dependence comes from the constraint that the weightings of timepoints are real valued. If we relax this constraint and allow for complex-valued weightings, then the independence property applies to all components from f_0 to f_{n-1}. However, it is not clear what a complex weighting would correspond to musically, and no theory involving weightings of pitch-classes or timepoints with non-zero imaginary parts has been proposed as of yet, to my knowledge.

We will take note of one more important property below (the convolution theorem). But let us first consider the significance of these basic properties for the concept of rhythmic qualities and its application in music theory and analysis, starting with the first three.

(1)–(2) The *orthogonality* properties imply that the DFT of a rhythm contains all the information of the rhythm itself, seen from a different perspective, and redistributed into *n* new parameters (magnitudes and phases of f_1–$f_{n/2}$). If the DFTs of two rhythms are different, then the rhythms are different, and if the DFTs are the same, then the rhythms are the same. Every difference between rhythms corresponds to some difference in the DFTs, and every possible difference in the DFTs corresponds to some difference in the rhythms.

(3) The DFT neatly isolates what we may understand as the purely intervallic properties of a rhythm, the properties that involve only the way onsets relate to one another, not their absolute positions with respect to a downbeat. These intervallic qualities are isolated in the DFT magnitudes, with all information related to absolute position with respect to a downbeat channeled into the phase values. In this sense, the DFT accomplishes what Allen Forte set out to achieve with the concept of set class (and does so in the rhythmic as well as the pitch domain).[28]

(4) When we are considering just the DFT magnitudes (the purely intervallic aspect of a rhythm, per (3)), the conservation of energy property allows us to apply the following kind of reasoning: a rhythm that lacks some quality must compensate by increasing the presence of another, given a fixed cardinality. Conversely, an especially strong representation of one quality entails the suppression of other qualities.

Considering these properties, a rhythmic *spectrum*, a graph of squared DFT magnitudes, can be a powerful tool for rhythmic analysis. Figure 12 shows the spectrum of Reich's signature rhythm compared to the standard pattern. This is a summary of all the quality magnitudes we have calculated above: $|f_1|$ in Figure 4, $|f_2|$ in Figure 6, $|f_3|$, $|f_4|$, and $|f_6|$ in Figure 7 (for Reich's rhythm only), and $|f_5|$ in Figure 9. The spectrum shows just the lengths of each of these vector sums, ignoring their phases. Therefore it applies to all rotations of the rhythm, per property (2). Figure 13 shows rotations of the standard pattern and Reich's signature rhythm. Figure 13(a) is another common African timeline.[29] Similarly, Figure 13(b) is a rotation of Reich's signature rhythm. By comparing Figure 13(a) to the original rotation of Reich's rhythm, or comparing 11(b) to the standard pattern, we can see that Reich's pattern, modulo phase differences, contains the standard pattern, extending it by one added note.

[28] In fact, the DFT does somewhat better than the concept of set class, since in addition to transpositional and inversional invariance, the magnitudes are also invariant with respect to the Z-relation. See Amiot 2016, Yust 2016.

[29] See Agawu 2003. Colannino et al. (2009) also compare this rhythm to Reich's signature rhythm.

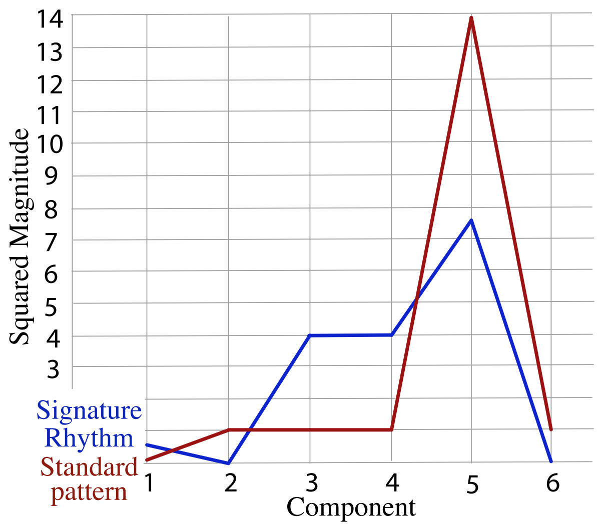
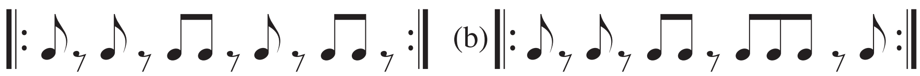


Figure 12: Spectra of the signature rhythm and standard pattern

(a)  (b)

Rotated standard pattern — Signature rhythm rotated

Figure 13: Rotations of the standard pattern, (a), and signature rhythm, (b)

The conservation of energy property, (3), is an important aid in interpreting the spectra in Figure 12. Let us suppose that our goal were to suppress, to the greatest extent possible, all aspects of a rhythm that would align it with any periodicity that would belong to any possible 3/2, 12/8, or 6/4 meter (in any possible orientation within the cycle). The idea here might be to create a mercurial rhythm, one that can be put in all sorts of different meters, and rotated in different ways with respect to the downbeat, without ever sounding appreciably more or less syncopated. This means we want a low f_1 (associated with strength of downbeat in all meters), f_2 (whose subcycle belongs to 12/8 and 6/4), f_3 (3/2), f_4 (12/8), and f_6 (3/2 and 6/4). Conservation of energy says that, assuming cardinality to be roughly fixed, the only way to do this is to compensate for the suppression of f_1, f_2, f_3, f_4, and f_6 by intensifying some other quality, and there is only one more, f_5. In other words, this idea of "metrical mercuriality," as we just formulated it, is *equivalent* (in the n = 12 case) to maximizing f_5. Since 5 is coprime to 12, it can be shown that the rhythm maximizing f_5 is a *generated* rhythm,[30] in this case generated by interval 5 or 7. A generated rhythm is analogous to a generated scale (via the pitch-class/time-class analogy), long recognized as an important property of the diatonic collection.[31] Because the standard pattern, the rhythm analogue of the diatonic scale, is a generated rhythm, its particular musical value may be understood in one of two equivalent ways: it maximizes the f_5 quality of the rhythm, and it is the rhythm that suppresses, to the greatest extent possible, all qualities other than f_5.

[30] Amiot 2007, 2016.
[31] Initially articulated by Gamer (1967).

Reich's signature rhythm, as we have already noted, is similar to the standard pattern, and therefore it also has a relatively strong f_5 quality. However, it is not a generated rhythm, and therefore Reich's choice of this as his signature rhythm cannot be explained purely as a matter of maximizing f_5, which is a sufficient explanation (up to rotation) of the standard pattern. It may be useful then to compare the spectra two other 8-onset supersets of the standard pattern, in Figure 14, one of which, (a), is a generated pattern, maximizing f_5.

The differences between Reich's pattern and the other two possibilities are that Reich's pattern actually better suppresses f_1 and f_2, and at the same time includes prominent f_3 and f_4. Low f_1 and f_2 may be understood as types of evenness properties, since high f_1 indicates as weighting towards one part of the measure (or cycle), and high f_2 a weighting towards one part of the half-cycle. The other special property of Reich's rhythm is that it has *perfect balance* (zero magnitude) on two components, f_2 and f_6.

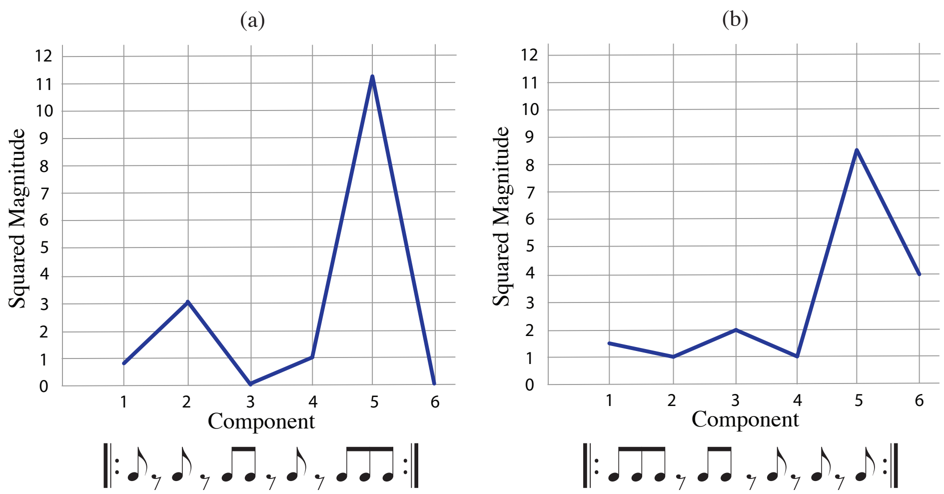


Figure 14: Rhythms that add one additional onset to the standard pattern, but are not rotations of Reich's signature rhythm

This may give us some insight into the question of how Reich's rhythm relates to his study of African music. We already have seen that Reich did not want to directly appropriate material but sought rather to derive it by incorporating internalized features of African music and into an existing aesthetic. In rhythmic qualities, the primary feature of the standard pattern is its high f_5, which is also a trait of Reich's rhythm. This does not, however, distinguish the signature rhythm from those of Figure 14, and does not explain the choice of rotation. These choices may relate to the presence of f_3 and f_4 qualities (and absence of f_1, f_2, and f_6) which may have been influenced by ways that Jones (1959) explains African rhythm.

Jones writes at length about polyrhythm as a basis of African rhythm (to the chagrin of later critics like Agawu[32]). The metrical mercuriality of the standard pattern does indeed facilitate the polyrhythmic interpretation, in that it allows the rhythm to favor no specific meter type. That Reich took a conscious interest in the metrical duplicity of his rhythms is confirmed by some of his writings. His note for the *Sextet* (1985) says "Techniques influenced by African music, where the basic ambiguities in meters of 12 beats between 3 groups of 4 and 4 groups of 3 appear in the third and fifth movements" (Reich 2002, 134).[33] The third movement uses the signature rhythm in alternation with the standard pattern in the piano. In later pieces (like the third movement of *Electric Counterpoint*) he explicitly changes time signatures between 3/2 and 12/8, using an independent layer (the bass lines in *Electric Counterpoint)* to enforce the hearing of the same signature-rhythm canons in the two meters.[34]

By itself, the standard pattern does not strongly align with any possible division of the measure into beats (whether quarter notes, in the manner of a 6/4, dotted quarters as in 12/8, or half notes as in 3/2). In practice, significant layers of the ensemble often play a rhythm in straight dotted quarters, implying a 12/8, and this is the meter expressed in typical dances associated with the pattern (Agawu 2003). As shown in Figure 15(a), only two of the onsets in the dotted quarter layer align with those of the bell pattern. By shifting the pattern ahead one place, as in Figure 15(b), it can be made to align with three onsets.

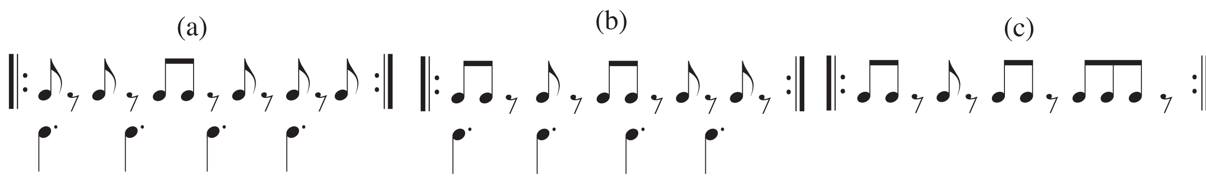


Figure 15: (a) The standard pattern with added dotted-quarter layer, (b) The same, shifted to better align with the dotted-quarter layer, (c) The resultant of (b), which is a rotation of Reich's pattern

The resultant rhythm in Figure 15(c) is a rotation of Reich's signature rhythm.[35] The added note, in addition to giving the rhythm a subset equally spaced by dotted quarters, also gives it a subset equally spaced by half notes. The beats of a possible 12/8 measure and of a possible 3/2 measure are therefore both now present in the rhythm, and it is for this reason that Reich's rhythm, unlike the standard pattern, has strong f_3 and f_4 components. Jones talks explicitly about resultant rhythms

[32] Agawu 2003

[33] It should be noted here, that although Jones's and others' notion of African polyrhythm has been discredited by Agawu (2003), Reich espouses something different and not as objectionable, metrical flexibility. He does not claim that his rhythms, or African rhythms, can be heard in two meters at once, but rather can easily be transferred from one meter to another.

[34] Note, however, that this piece is one example of the signature rhythm appearing in a rotation different than its usual one.

[35] Toussaint (2013, 113–15) gives a similar explanation of the rhythm.

created by combining the standard pattern with layers expressing a 12/8 meter (1959: 53–4, 76, 191–3), so hypothetically, Reich needed only to reposition the layer of dotted quarters with respect to the standard pattern in such a way as to get a rhythm with eight onsets instead of nine. We can see something like this process at work in the early sketch where the rhythm first appears, albeit not in its typical rotation. This sketch, which can be found in Potter 2011, is based on the two-part rhythm in Figure 16. The lower part of the sketch articulates an f_4 prototype rhythm, meaning it maximizes $|f_4|$ (= 4). It is also balanced on all other components. The upper part, which is a rotation of the signature rhythm back by a quarter note, is a variation of the standard pattern designed to fill in the four equally spaced gaps of the lower rhythm. To do so, one onset needs to be added, which here is in the one just ahead of the downbeat.

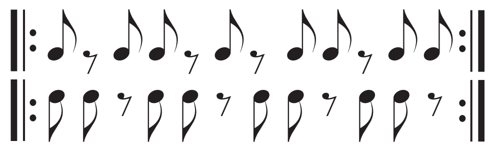


Figure 16: Rhythm for an early sketch of *Clapping Music*

These observations may capture what is special about this rhythm, but only up to rotation. Reich is quite consistent across works in treating the rhythm of Figure 1 as the prime rotation (the one that initiates canons and remains consistent across multiple canons). This may also relate to the importance of the f_3 and f_4 qualities: the rhythm has exactly one perfectly even 3-onset subset and one perfectly even 4-onset subset, and all onsets except for two belong to one or the other of these, or both. In the prime rotation, the downbeat (onset position 0) is assigned to one of the two onsets belonging to neither of these subsets, thereby favoring neither a 3/2-type or 12/8-type meter.

The rotational alignment of a rhythmic quality is expressed in the phase values of the DFT. These are, by usual mathematical convention, given as angles in radians, from 0 to 2π, but it is helpful in the present case to multiply them by $2\pi/n$ so that they all vary instead from 0 to n (with n = 12 here).[36] With that change the phases (denoted Ph_k) of the signature rhythm are:

Ph_1	Ph_2	Ph_3	Ph_4	Ph_5	Ph_6
1	—	3	4	11	—

(These can be seen in the angles of vectors shown in Figures 4, 6, 7, and 9.) Two phases (Ph_2 and Ph_6) are undefined because the components are zero-valued (perfectly balanced). Each Ph_k is

[36] Mathematically, the phase values should go in reverse order, so that, for instance, the first eighth-note after the downbeat should have phase value 11 rather than 1. To be consistent with the figures above, however, I have reversed the Ph_n values here from x to $12 - x$.

measured on a n/k subcycle, with the eighth note equal to k, so the balance of the signature rhythm is on the eighth note after the downbeat in f_1, f_3, and f_4. This is a small proportion (1/12) of the f_1 subcycle, but a relatively large proportion of the f_3, f_4 subcycles (1/4 and 1/3). The rhythm is therefore syncopated with respect to possible 3/2 or 12/8 meters. Thus we can say that the signature rhythm contains the seeds of two possible meters of different types, but they are hidden by the rotation. The other rotation that would syncopate both of these rhythmic qualities is the one from Reich's early sketch in Figure 16. This instead aligns both qualities an eighth note ahead of the beat, $Ph_3 = 9$ and $Ph_4 = 8$, which is a weaker syncopation (since it is normal for simple rhythms to be weighted towards the later part of metrical cycles—e.g., ♩. ♪ vs. ♪♩.).

There is another hypothetical route to Reich's rhythm through Jones's book that invokes property (5) of the DFT above (complementation). Jones (1959, 3, 210–13, 222–4) describes at length the many manifestations, in the music of different tribes, of a rhythm he calls the African "signature tune":

He treats it as a version of the standard pattern and points out that his African informants seem to understand it in some sense as equivalent. This pattern is (up to rotation) a complement of the standard pattern, meaning that it is the rhythm we get by exchanging eighth-notes for eighth-note rests and vice versa. If the standard pattern is the "diatonic rhythm," this is the "pentatonic rhythm." The idea of complementation might seem abstract, but according to property (5) above complements are intimately related: they have exactly the same spectrum (excluding f_0), and opposite phases. If there is a rotation that approximately reverses the phases of the main components of the rhythm, then the complement can also be identified as a subset or superset. The standard pattern has one principal quality, f_5, and the largest intervals in f_5 space are the eighth note (±1, becoming ±5 in f_5 space) and the half-measure (6). Any of these work to translate the complement to a subset. For instance, translating it back by an eighth note gives:

Standard pattern: 

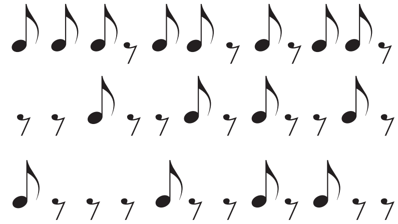
Complementary subset: 

This, and the other two like it, are therefore special quality-preserving subsets.

Reich's signature rhythm has similar easily derived complementary subsets. Taking the first or last of each group of adjacent onsets is equivalent to rotating the complement by eighth note in either direction:

Signature rhythm:

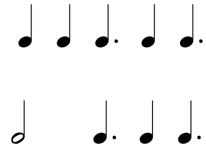
Complementary subsets:



The second of these appears in Jones's discussion of a dance called *Asafo*, both as drumming pattern and a clapping pattern (211):

"Signature tune":

Asafo clap:



Reich's signature rhythm is a complementary superset of the *Asafo* clapping rhythm in the same way that the standard pattern is a complementary superset of Jones' "signature tune."

Whether either of these explanations bears any biographical significance is impossible to know for certain, but the possibility of such derivations lends additional credence to Scherzinger's (2019) arguments about the formative importance of African music on Reich. Disentangling possible sources for the rhythm becomes impossible in part because of its overdetermination given the multiple uses to which it is put; there may indeed be multiple converging reasons that the rhythm took the shape it did and became such an integral element of Reich's style. Whatever their historical significance, the explanations above point to the rhythmic qualities as an effective summary of the properties of interest: a dominant non-metrical, or mercurial, f_5 quality, and two opposed qualities of approximately equal weight, f_3 and f_4, such that, whatever meter is chosen, one will be non-metrical, and one will be syncopated. In *Clapping Music*, each canon will bring different sets of these properties to the fore, so that the entire process is akin to gradually rotating a single multidimensional object to project it onto a complete set of different perspectives. Later works continue to use canon in this capacity, along with other contextualizing factors (such as varying melodic contours and combining with other more clearly metrical rhythms). Constant throughout these examples is the play with the dual metrical affinities made possible by the rhythm's competing f_3 and f_4 qualities, and the use of canons. Rhythmic canon, it turns out, has a special relationship to the DFT, allowing us to generalize further about the role of these in Reich's music, as the next section explains.

Convolution and Reich's Canons

The signature rhythm usually serves as the subject of rhythmic canons when it appears in Reich's music. His first use of the rhythm in *Clapping Music* is the purest example of this, in that the piece presents *all* possible two-part canons on the rhythm, and this process constitutes the entire piece.

The DFT is particularly useful for understanding the properties of such rhythmic canons, because of another mathematical property not listed in the previous section, called the *convolution theorem*. Convolution is essentially the mathematical term for canon, it refers an operation that adds together translated versions of some pattern. For instance, say we have a pattern representing a rhythm, expressed in 1s and 0s,

1 1 1 0 1 1 0 1 0 1 1 0

(this being the signature rhythm), and another pattern representing the configuration of canonic parts. For instance, if one part starts on the downbeat and the other an eighth note after the downbeat, we have the configuration,

1 1 0 0 0 0 0 0 0 0 0 0

The *convolution* of these is given by reproducing the first pattern rotated to start at each place where a 1 occurs in the second pattern and adding them. For this example, the first "1" of the canonic configuration gives an unrotated version of the rhythm, and the second rotates it by one place:

$$
\begin{array}{r}
1\ 1\ 1\ 0\ 1\ 1\ 0\ 1\ 0\ 1\ 1\ 0 \\
+\ 0\ 1\ 1\ 1\ 0\ 1\ 1\ 0\ 1\ 0\ 1\ 1 \\
=\ 1\ 2\ 2\ 1\ 1\ 2\ 1\ 1\ 1\ 1\ 2\ 1
\end{array}
$$

Note that the result is not necessarily made up of only 1s and 0s, so it is a rhythmic multiset rather than a simple rhythm. In the pitch-class domain, this is called a *transpositional combination*, except that in transpositional combination, the multisets are usually reduced to simple pitch-class sets by changing all non-zero integers to 1s.[37] As a multiset, however, the result has a DFT, and it can be determined from the original sets by the simple rule (which we may now add to the list in the previous section):

(6) ***Convolution:*** The DFT of a convolution (transpositional combination) is given by multiplying the magnitudes and adding the phases of the convolved sets.[38]

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[37] As defined by Cohn (1988). The relationship of Reich's canons to transpositional combination was also noted by Cohn (1992). On the DFT and transpositional combination as pitch-class multiplication, see Yust 2015a.

[38] To see roughly why this is the case using the vector spaces developed above, consider that, for a given space, f_k, the convolution of A and B sums a copy of the vector for A rotated by the angle of each pitch class in B. For a more rigorous proof see Briggs and Henson 1995.

This is an especially significant property, as Emmanuel Amiot shows.[39] For instance, Lewin's interval function and common-tone functions and Forte's interval vector are all instances of convolution in the pitch-class domain.[40] For present purposes, it is a convenient result because it allows us to quickly determine the properties of different kinds of rhythmic canons.

Using (6) we can then evaluate all two-part canons by comparing the DFT of the signature rhythm (Table 1) to that of the dyads describing the first six canons of *Clapping Music* (Table 4). Dyads are indicated here and elsewhere with a integer notation for time-class sets. The first canon of *Clapping Music* is the convolution of the signature rhythm with the dyad {01}, the second is the convolution of the signature rhythm with {02}, and so forth. Because f_1 is very small for the signature rhythm and f_2 and f_6 are zero, these will be weak or perfectly balanced in all of the canons. The qualities of the canons are therefore determined primarily by the $|f_3|$, $|f_4|$, and $|f_5|$ values for the dyads. Dyads {01} and {02} are relatively low on all of these, so these canons (the first two of *Clapping Music*) will therefore be relatively flat except for some reinforcement of f_3 on the {01} canon. When the canon flattens a rhythm, the result is an interlocking effect: the two parts tend to coincide as little as possible. The canons at 3♪ {03} and 4♪ {04} strongly reinforce f_4 and f_3 respectively. The {05} canon reinforces f_5. The half-measure canon, {06}, the one that occurs at the midpoint of *Clapping Music*, is particularly noteworthy in that, when used on the signature rhythm, it cancels out all components except f_4, because it has zeros on f_1, f_3, and f_5, while the signature rhythm has zeros on f_2 and f_6. The resultant of the canon is therefore a pure-f_4 rhythm. An onset occurs at every position, and attacks coincide at every third position.

Reich's sketches suggest that he had the interlocking effect in mind when writing the piece[41]: he began with the idea of hocketing parts, the tried to achieve the interlocking effect with a variation on the African standard pattern, the first attempt leading to the sketch in Figure 16. When he settled on the final version, a single rhythm in all possible two-part canons, he wrote out all of the twelve canons along with a "result" (resultant) that shows the transpositional combination of rhythms without doublings. These clearly show a progression from interlocking patterns (canons 1 and 2) whose resultants have no rests, to more reinforcing canons with one or two rests (3–5), to another interlocking canon (6), then back through the same pattern in reverse.

This makes the signature rhythm a particularly interesting choice for *Clapping Music*. In what seems to be a pure process piece, a design is hidden in the one aspect of the piece determined by artifice rather than the simple process, the initial rhythm. The phasing process transfers aspects of this rhythm to the timescale of the form. By building in the 3/2 and 12/8 rhythms, Reich enables the phasing process to narrate a struggle between these meters, with the 3/2 initially dominant, 12/8 emerging strongly as an alternative in the third canon, {03}, and then finally, at the climactic midpoint of the piece, overtaking the entire texture in the {06} canon. Because the continuation

[39] Amiot 2016.
[40] Lewin 2001, 2007; Forte 1973.
[41] See Potter 2011 and Hartenberger 2016.

goes through the inversionally related canon patterns in reverse order, the overall result is what Reich calls an arch form, an idea he would later use to structure most of his multi-movement works.

Table 4: The DFT of six dyads, using squared magnitudes and normalized phases

Set		f_1	f_2	f_3	f_4	f_5	f_6
{01}	Mag.^{2}	3.73	3	2	1	0.27	0
	Ph.	0.5	1	0.5	2	2.5	—
{02}	Mag.^{2}	3	1	0	1	3	4
	Ph.	1	2	—	10	11	0
{03}	Mag.^{2}	2	0	2	4	2	0
	Ph.	1.5	—	10.5	0	1.5	—
{04}	Mag.^{2}	1	1	4	1	1	4
	Ph.	2	10	0	2	10	0
{05}	Mag.^{2}	0.27	3	2	1	3.73	0
	Ph.	2.5	11	1.5	10	0.5	—
{06}	Mag.^{2}	0	4	0	4	0	4
	Ph.	—	0	—	0	—	0

This account only considers magnitudes, however. Reich's choice of rotation for the initial rhythm is also important. The phase values $Ph_3 = 3$ and $Ph_4 = 4$ indicate that both qualities are shifted ahead by an eighth note, putting them in an unstable position with respect to the notated meter. Since convolution *sums* the phases, values close to 9 and 8 would be needed in the dyads to bring these qualities into phase with the notated meter. This only happens, however, in places where the given quality is weak, f_4 in {02} and {05}, and f_3 in {03}. Where the qualities are strongest the phase values are zero, meaning they remain considerably out of phase. This is most notable in the {06} canon. In my own experience of listening to *Clapping Music*, when not performing or looking at a score, is that at some point the original downbeat gets lost, then a meter locks back in on one of the high-f_3 or high-f_4 canons in the middle of the piece, so that by the end my subjective meter has shifted ahead of the notated one by an eighth-note. On this hearing, the piece ends on a different rhythm from the one it began on, even though, from the performers' perspective (and on the page), the beginning and ending are exactly the same.

Reich's absolutist stance towards process essentially ends with *Clapping Music* and *Drumming*, with *Music for Pieces of Wood* as transitional. The latter may be understood largely as a process piece, and is based on similar musical elements as previous pieces—rhythmic canons, built one note at a time as in much of *Drumming*, but using the signature rhythm from *Clapping Music* in the first section. The piece, written for five claves, has three parts in three different meters, with the rhythms and canon configurations listed in vector form in Table 5. Parts sometimes double on the same rhythm where the canon configuration rhythms have 2s. Throughout the entire piece, Clave 1 plays a steady stream of quarter notes.

Table 5: Design of Music for Pieces of Wood

Meter	Rhythm	Canons
6 4	1 1 1 0 1 1 0 1 0 1 1 0	1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 2 0 0 0 0 0 2 0 0 0 0 0 2 0 0 0 0 0
4 4	1 1 0 1 0 1 1 0	1 0 1 0 0 0 0 0 1 0 1 0 1 0 0 0 2 0 1 0 1 0 0 0
3 4	1 1 0 1 1 0	1 1 0 0 0 0 1 1 1 0 0 0 2 1 1 0 0 0

A compositional design is evident in the canonic arrangements listed in Table 5, which are not a simple enumeration of possibilities as in *Clapping Music*. For the first, Reich chooses the central {06} canon of *Clapping Music*, the one that has the special property of creating a pure-f_4 resultant. This is notable, since Clave 1 enforces a quarter-note beat throughout, a pure-f_6 rhythm, making f_4 (a dotted-quarter periodicity) distinctly extra-metrical. Rather than construct a three-part canon, as he does in the other sections, Reich simply doubles both parts of this {06} canon.

Another aspect of compositional design is the changing rhythm and meter from one section to the next. Each subsequent rhythm may be understood as the truncation of the previous one as Figure 17 shows. Still, choices are made as to which part of the rhythm to cut. We may make sense of these choices by considering the spectrum of each rhythm, but first we must address the problem of how to compare spectra for different values of n (the cycle length), here 12, 8, and 6.

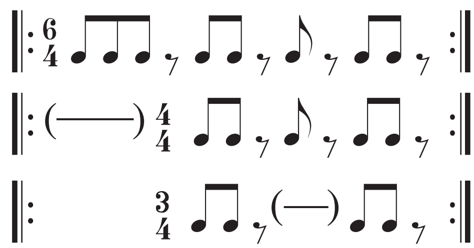

Figure 17: The rhythms of *Music for Pieces of Wood* as truncations

There are two ways to relate qualities across different values of n, one that treats the full cycle as fixed and varies how it is divided, and one for a fixed minimal time-unit where the length of the cycle varies. If the full cycle is fixed, then spectra relate directly, since the component number indicates a division of the full cycle. Reich's changing meters, however, treat the minimal time unit as fixed and vary the length of the cycle. To relate spectra between different cycles, n_1 and n_2, in this scenario, the indexes for the smaller cycles must be stretched by the ratio n_1/n_2. For the meters of *Music for Pieces of Wood*, the 4/4 component numbers can be multiplied by 12/8 = 1.5 and the 3/4 component numbers by 12/6 = 2. The scaling factor for 3/4 is an integer, meaning we are essentially treating a rhythm like (1 1 0 1 1 0) as the same rhythm repeated twice to get a 6/4 rhythm (1 1 0 1 1 0 1 1 0 1 1 0). The repeated rhythm will have perfect balance, i.e. zero magnitude, on all odd components, and non-trivial components will coincide with the even components of a 6/4 rhythm. In the 4/4 rhythm, on the other hand, some components will line up with 6/4 components—for example f_2 in 4/4 becomes f_3 in 6/4—while others will not.

Figure 18 superposes the three spectra using this method. The relationship between the 3/4 and 6/4 rhythms may be described most succinctly by noting that they have exactly the same values on all even components. The 3/4 rhythm, by that account, is simply an elimination of f_1, f_3, and f_5 qualities from the 6/4 rhythm, leaving only f_4. We might say, then, that the change of rhythm over the course of the whole piece has precisely the same effect as the {06} canon in the first section. It cancels out all odd components, leaving only the one expressing the extra-metrical dotted-quarter periodicity, f_4.

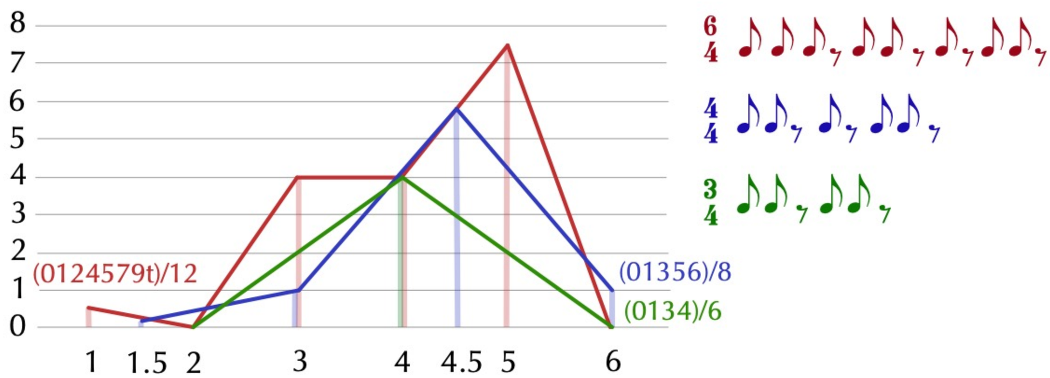

Figure 18: Comparison of spectra for the three rhythms of *Music for Pieces of Wood*

The presence of the 4/4 rhythm as intermediary, however, elaborates upon this picture. This rhythm is known as *cinquillo* in Cuban music, and is the maximally even 5-in-8 rhythm. The generator is a dotted quarter, and it maximizes $f_{3/8}$. In the stretched spectrum of Figure 18 the $f_{3/8}$ value is at 4.5 (/12). (Recall that the $f_{k/n}$ notation specifies the cycle length, n, where that is not clear from the context.) In a sense, then, this rhythm is an attempt to approximate the dotted quarter periodicity of f_4 that is dominant in the outer sections, but does not exist as an even division in 4/4. It also might be compared, though, to the maximum peak of the signature rhythm (the 6/4 rhythm), which is f_5, and similarly coprime to its universe ("funky"). The process of truncation might thus be understood as a gradual directed shift and rationalization of the principal quality from $f_{5/12}$ to $f_{3/8}$ to $f_{2/3}$. Since the end result is the same as what is achieved by the first canon (a pure dotted quarter periodicity), the entire form is an uncanny expansion of one local process (the build-up of the first canon) into a more complex large-scale process of rhythmic and metric transformation over the three sections.

This analysis does not yet address Reich's choice of canons for the second and third sections, however. Figure 19 overlays the spectrum of each rhythm with its first and second canon configuration. Note that we are making comparisons within cycles n = 8 and n = 6 now, so components are numbered accordingly. Each graph plots the spectrum of the basic rhythm against that of the first (two-part) canon and the second (three-part). It is easy to see what the 3/4 canons do: only the f_2 component matters (because the others are zero-valued), and the first canon depresses f_2 while the third cancels it out entirely, resulting in a completely flat rhythm. Only the final addition of Clave 5 reinforcing the *dux* rhythm restores some f_2 quality, re-establishing the basic 2-against-3 feel with Clave 1 that frames the piece.

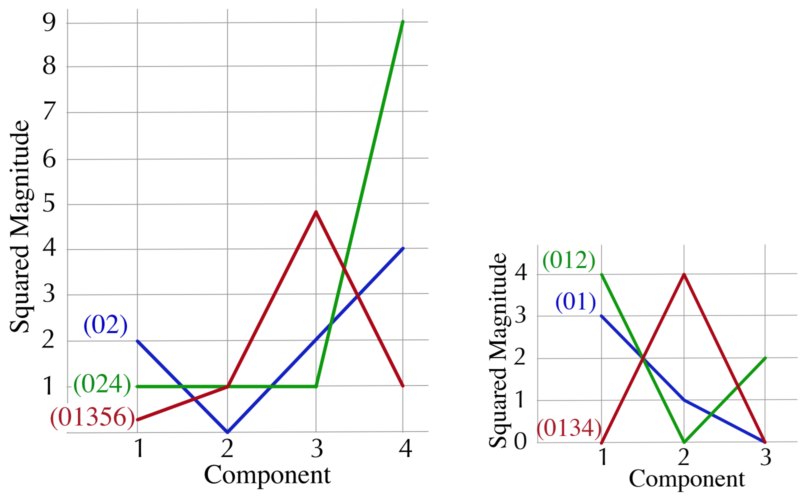

Figure 19: Spectra for the 4/4 and 3/4 rhythms of *Music for Pieces of Wood* and their two-voice and three-voice canon configurations

The process in the middle section is different, though. Here this first canon does not flatten the rhythm, but to the contrary, reinforces its principal component (f_3). This distinct process, one of intensification, makes sense both because this is a middle section, and the overall form gains greater shape through greater contrast, and because the principal component here is not conventionally metrical. The canon accentuates the funkiness of the *cinquillo.* The canon also brings another quality, f_4, to the forefront, which is significant because it is the quality of Clave 1, and resolutely absent from the non-Clave 1 rhythms of the framing sections (in which contexts it is f_6 and f_3 respectively). The second (three-part) canon abandons the task of reinforcing f_3 and doubles down on f_4, so that it becomes the principal quality. This is also a contrast with the framing sections, where the basic rhythms are perfectly balanced on the $n/2$ component, the quarter-note quality of Clave 1. The middle section instead traces a process of coming into coordination with Clave 1. However, this process relates only to the magnitude of the f_4 quality—the basic rhythm has a phase of 4, meaning it is *syncopated* with respect to the Clave 1 beat. Because the canonic configurations have $Ph_4 = 0$, they do not affect the phase of the resultant f_4 according to the convolution theorem. The f_4 quality is therefore consistently opposite in phase ($Ph_4 = 4$) from Clave 1 ($Ph_4 = 0$). The resultant of the three-part canon therefore agrees with the Clave 1 in periodicity but disagrees in phase, making a syncopated rhythm that fills in the gaps of Clave 1's simple metrical rhythm.

Music for Pieces of Wood thus exhibits classical ideals of formal design. It has a three part form in which the outer sections express a similar idea (developing the 2-against-3 polyrhythm) framing a contrasting middle section (that develops a syncopated rhythm). The choice of canons is essential to this arch design. Furthermore, the canon of part one reflects in miniature the process delineated by the changing rhythm of the three sections, creating a resonance between levels. Given the way that this structure is created by identifiable compositional choices (the manner of rhythmic truncation and choice of canons), the role of process in *Music for Pieces of Wood* is no longer that of a self-determining automaton but of a functionary in a stage-managed sequence of actions. In the next analysis, we jump ahead by two decades and find that the rhythmic language and use of canon change little, while the role of compositional intuition only comes more to the forefront.

Nagoya Marimbas

Nagoya Marimbas, a short piece for marimba duet that Reich wrote to commission in 1994, illustrates how the seemingly simple principles that formed the basis of his practice in the 1970s and 80s—the signature rhythm and cyclic canons—could serve as an enduring aesthetic resource. Its varying of the basic rhythmic cycle invites comparison to *Music for Pieces of Wood*, with a much more refined technique evident in the later piece, relying little on mechanical processes, but with a similar concern for large-scale form.

It might not seem at first that the signature rhythm is of fundamental importance to the piece. It only appears sporadically until the end of the piece. Over the first half it appears in three places: measures 28–30, 41, and 49. Not until mm. 58–72, about 3 minutes into the 5 minute piece, does it occur with some consistency.

The piece is designed, as Reich succinctly describes it, as "a series of two-part unison canons."[42] The canonic parts repeat cyclically in one of two meters, 4/4 and 3/4, both with a sixteenth-note pulse.[43] The main meter at the beginning is 4/4 (doubled 2/4), a 16-cycle of sixteenth notes. Over the first half of the piece the meter of the signature rhythm, 3/4, a 12-cycle, only in mm. 28–30 and m. 41, and always with the signature rhythm. The two meters then occur in regular alternation in mm. 58–68, and the piece is entirely in 3/4 from m. 69 to the end (m. 84). Until this last section (in fact, up to measure 72) all 3/4 measures use the signature rhythm with one exception: m. 46. This measure is a special and important moment in the piece, marked by an unusually regular rhythm that occurs nowhere else.

Excluding the isolated 3/4 measures, over the first part of the piece (up to m. 46), Reich uses five different 16-cycle rhythms in Marimba 1. (Marimba 2 is always playing either the same rhythm in canon, or building up to such a canon.) Two of these are special: the rhythm in mm. 31–34 is a 9-in-16 maximally even pattern, generated by an interval of 7♪s. This rhythm therefore maximizes the f_7 quality, which has a "funkiness" similar to the f_5 quality of the signature rhythm, as a periodicity that does not regularly divide the cycle.

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[42] Reich 2002, 184.

[43] Reich actually writes in 2/4 for most of the first part of the piece, but this is best understood as 4/4 in double measures, and I will ignore the distinction in this analysis and refer to all of this music as 4/4.

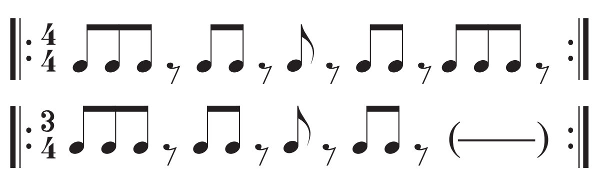
Two other rhythms dominate the music that leads up to the appearance of this maximally even rhythm, the most prominent being the one that occupies mm. 1–20. Most of this passage (mm. 1–15) composes out the construction of a single canon, using a procedure typical of Reich's pieces of the 1970s and 80s. As the *dux* repeats the subject, the *comes* part adds notes one, two, or three at a time until the canon is complete. After he completes the canon he changes the offset without changing the melody (mm. 19–22), then alters the rhythm and melody in m. 23. These first two rhythms, which I will label A_1 and A_2, are closely related to the 9-in-16 maximally even rhythm (ME). They can be related by successively removing one onset:

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A_1: mm. 1–20	●	o	●	o	●	●	o	●	o	●	o	●	●	●	●	●
A_2: mm. 23–6 / mm. 35–38	●	o	●	o	●	●	o	●	o	●	o	●	●	●	●	o
ME, mm. 31–34	●	o	●	o	●	●	o	●	o	●	o	●	●	o	●	o

Note that the time-class numbering here is with respect to the sixteenth-note pulse and black dots indicate onsets. The intermediate A_2 rhythm comes back in mm. 35–38, so that the whole passage can be understood rhythmically as a gradual progression to the maximally even rhythm, followed by a partial retreat.

Figure 20 gives the spectra for these rhythms, which are very similar. The main process is a gradual increase in $|f_7|$, which is the overwhelmingly dominant quality in all of them.

This account excludes only mm. 27–30, which interrupt the process. Measures 28–30 are the first appearance of 3/4 meter and the signature rhythm, and measure 27 is another special 16-cycle rhythm. I will call this the "extended signature rhythm," (XS) because it can be understood as a 16-cycle extension of the signature rhythm, much as the 4/4 and 3/4 rhythms of *Music for Pieces of Wood* are readily understood as truncations of it:



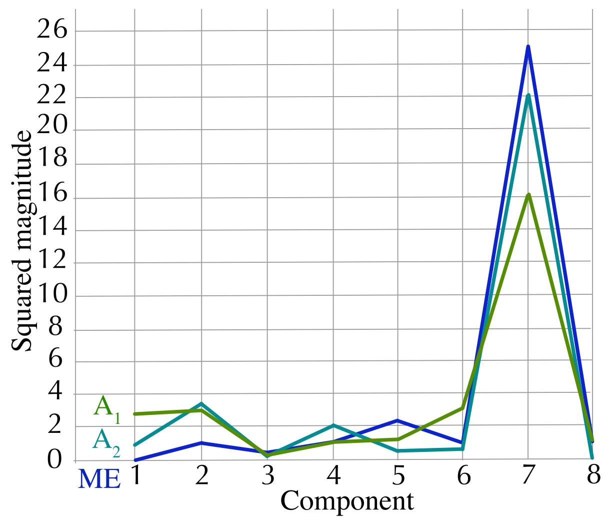


Figure 20: Spectra for three rhythms from the beginning of *Nagoya Marimbas*, mm. 1–20 (A_1), mm, 23–26 (A_2) and mm. 31–34 (“ME”)

This new rhythm evidently appears in m. 27 because Reich is using it to prepare the shift to 3/4 and the signature rhythm. He uses the extended signature rhythm in this way also in mm. 47–48. One other 16-cycle rhythm, which I will label B, appears in the piece, and this also may be understood as part of a gradual process of transformation, in this case from the maximally even rhythm to the extended signature rhythm:

	0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
ME, mm. 31–34	● – o – ● – o – ● – ● – o – ● – o – ● – o – ● – ● – o – ● – o –
A_2: mm. 23–6 / mm. 35–38	● – o – ● – o – ● – ● – o – ● – o – ● – o – ● – ● – ● – ● – o –
B: mm. 39–45	● – ● – ● – o – ● – ● – o – ● – o – ● – o – ● – ● – ● – ● – o –
Extended Sig. Rhythm (**XS**)	● – ● – ● – o – ● – ● – o – ● – o – ● – ● – o – ● – ● – ● – o –

We have already considered A_2 as an intermediary in the first process. Rhythm B adds one more note to A_2, towards the beginning of the measure. The last stage of the process, from B to XS, is a “rhythmic voice leading,” in which one note moves back by a sixteenth.

Again, we can examine this process by comparing spectra, given in Figure 21. Two directed processes are evident over the four rhythms. One is a reversal of the previous process, a gradual diminishing of f_7. The other is a gradual intensification of f_4, so that by the end of the process, the extended signature rhythm, f_4 and f_7 are roughly equal in size.

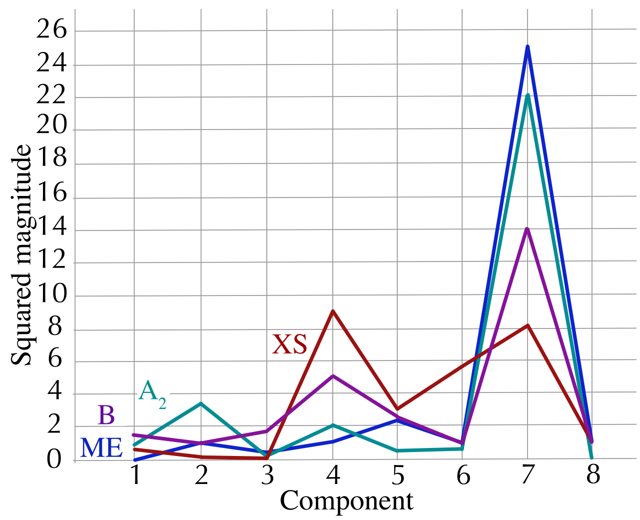


Figure 21: Spectra for 4/4 rhythms in *Nagoya Marimbas*

A large section of the piece, from m. 47 to m. 70, consists of alternations between the extended signature rhythm in 4/4 with the signature rhythm in 3/4. These can be compared directly using the same method as the analysis of *Music for Pieces of Wood* above. Figure 22 adjusts the 16-cycle component numbers by a factor of ¾ to compare the spectra. The similarities between the rhythms are noteworthy. The $f_{5/12}$ peak of the signature rhythm is matched by a nearby $f_{7/16}$ (5.25) peak in the extended rhythm. Both are low on the small-numbered components, and also the eighth-note component ($f_{6/12}$ and $f_{8/16}$). The only difference is that the signature rhythm has energy in two other components, $f_{3/12}$ and $f_{4/12}$, whereas the extended rhythm concentrates most of this energy in one other component, $f_{4/16}$, which corresponds to the same quarter-note periodicity as $f_{3/12}$. Two qualities in each universe therefore account for most of the rhythmic interest of the piece: one metrical, f_3 in the 12-cycle and f_4 in the 16-cycle, and one ametrical, f_5 in the 12-cycle and f_7 in the 16-cycle. The metrical qualities have the same periodicity, while the ametrical qualities are very close in periodicity.

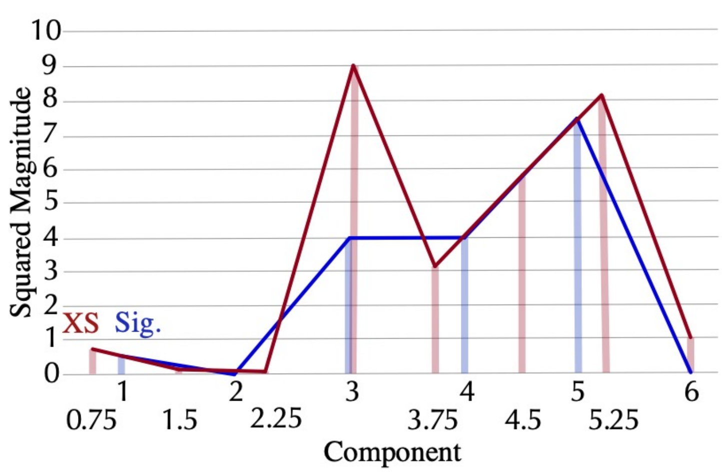


Figure 22: Spectrum for the extended signature rhythm compared to the signature rhythm.

The canons that Reich uses over the course of the piece are listed in Table 6. In addition to the rhythms already discussed (A_1, A_2, B, Sig., XS, ME), four other rhythms of 9 onsets in the 12-cycle are labeled C–F:

	0	1	2	3	4	5	6	7	8	9	10	11
C: m. 46	●	●	●	o	●	●	●	o	●	●	●	o
D: m. 74 (and 72 *comes*)	●	●	●	o	●	●	o	●	o	●	●	●
E_1: mm. 73/78–9	●	o	●	●	●	●	o	●	●	●	●	o
E_2: mm. 80–4	●	●	●	o	●	●	o	●	●	●	●	o
F: mm. 76–77	●	●	●	●	●	o	●	●	o	●	●	o

Rhythm C is a special perfectly even rhythm, a pure representative of f_3. Two (D and E_2) are supersets of the signature rhythm. Rhythm E_1 maximizes f_5 while F maximizes f_4. Rhythm E_2, which ends the piece, is similar to the signature rhythm in that it combines prominent f_3, f_4, and f_5 qualities. Using the time-class/pitch-class analogy, we could think of E_2 as the complement of a major triad, where the signature rhythm is the complement of a minor seventh chord containing it.

Table 6: Canons in Nagoya Marimbas

Meas.	11–16	19–20	23–6	27	28–9	30	31–2	33–4	35–6	37–8	39–40
Meter	4/4	—	—	—	3/4	—	4/4	—	—	—	—
Canon	{06}	{03}	{04}	{04}	{02}	{05}	{0,–4}	{0,–4}	{0,–4}	{0,–5}	{02}
Rhythm	A_1	A_1	A_2	XS	Sig.	Sig.	ME	ME	A_2	A_2	B

41	42–3	44–5	46	47–8	49	50–2	53–4	55	56–7	58	59–61	62–5
3/4	4/4	—	3/4	4/4	3/4	4/4	—	—	—	3/4	4/4–3/4	4/4–3/4
{02}	{08}	{02}	{02}	{02}	{02}	{08}	{02}	{06}	{02}	{02}	{02}	{0,–6}
Sig.	B	B	C	XS	Sig.	XS	XS	XS	XS	Sig.	XS-Sig.	XS-Sig.

66–7	68–9	70	71–2	73	74	75	76–7	78–9	80	81–2	83–4
3/4	4/4–3/4	3/4	—	—	—	—	—	—	—	—	—
{02}	{02}	{02}	{04}	{04}	{0,–4}	{02}	{04}	{04}	{02}	{04}	{06}
Sig.	XS-Sig.	Sig.	Sig./D	E_1	D	Sig.	F	E_1	E_2	E_2	E_2

Let us begin with some general observations from Table 6. The cycle lengths (3/4 versus 4/4 contexts) are indicated with the slash notation "/12" and "/16."

• Odd canons, {03}/16, {05}/12, and {0,–5}/16, occur only in three places toward the beginning of the piece. All others are even.

• The {04}/16 canon, along with its translation {0,–4}/16, dominates the first part of the piece up to m. 36.

• Starting in m. 39, where {02}/16 first appears, this canon and its 3/4 counterpart, {02}/12 predominate, with some other even canons interspersed.

• Although the {04}/16 canon never reappears after m. 36, the last part of the piece, starting from m. 71, is dominated by its 3/4 counterpart, {04}/12, and its translation {0,–4}/12.

• The piece begins on a {06} canon in 4/4, and ends on a {06} canon in 3/4.

We can therefore give a greatly simplified summary of the canonic configurations over the course of the whole piece, showing a kind of arch form:

Table 7: Summary of main rhythmic canons

Meas.	1–16	23–36	39–68	41–70	71–79	83–4
Meter	4/4	—	—	3/4	—	—
Canon	{06}	{04}	{02}	{02}	{04}	{06}
Rhythm	A_1	A_2/ME	B-XS	Sig.	Sig./D/E_1	E_2

The {06} canons are isolated incidents, but gain prominence from their placement at the beginning and end. Note in particular the retracing of the {02}–{04}–{06} process in miniature in mm. 80–84. Also, the transition from {02} in 4/4 to {02} in 3/4 does not occur at a discrete point in time. The substantial middle section of the piece (mm. 39–70) is shown in the chart as two overlapping sections. In reality it is a continual alternation between the two meters, usually with an {02} canon in both.

In discussing *Clapping Music* above, I noted that a relatively flat resultant, lacking any prominent qualities, results when the spectrum of the rhythm and canonic configuration differ substantially. This corresponds to an interlocking between the parts, where the *comes* tends to fill in the gaps of the *dux*. If the canonic interval, on the other hand, has peaks in similar places to the rhythm, then it will reinforce these qualities, so that they are particularly strong in the resulting canon (and the result will then also will be less flat). As we have already seen, the qualities that tend to be prominent, and of particular interest, in Reich's rhythms are f_4 and f_7 in 4/4 and f_3 and f_5 in 3/4.

Figure 23 shows the spectra of the even 16-cycle dyads. All even dyads have maximum values at f_8, the component that distinguishes strong from weak metrical positions at the eighth-note level. We have already seen that this component is weak or non-existent for the rhythms that Reich chooses, so his even canons (the vast majority) will have a certain baseline interlocking quality. The next component of interest is f_7. This is high for {02} and gradually decreases to zero as the canonic interval widens to {04}, {06}, and {08}. Towards the beginning of the piece, then, when the energy of the rhythm is highly focused in f_7, Reich avoids the excessive coincidence of parts that would result from the {02} canon. Nonetheless the {0,–4} canons (with the same spectrum as {04}) on the maximally even rhythm do accentuate its f_7 quality. In the middle of the piece, the {02} canon accentuates the less extreme f_7 of the extended signature rhythm. The resultant $|f_7|$ is therefore roughly constant over the principal canons of each section, shifting from a quality of the rhythm itself to one emerging from the canonic interaction. At the same time, from m. 42–65 Reich regularly intersperses {06} and {08} canons. The low f_7 values of these give flatter results, providing local contrast with the {02} canons. In particular, the {08} canons in the middle section of the piece strikingly eliminate the f_7 quality altogether, similar to the elimination of $f_{5/12}$ quality in the C rhythm of m. 46. The alternation of high-$f_{7/16}$ or high-$f_{5/12}$ rhythms with ones that eliminate these qualities lends to the turbulent character of this middle section.

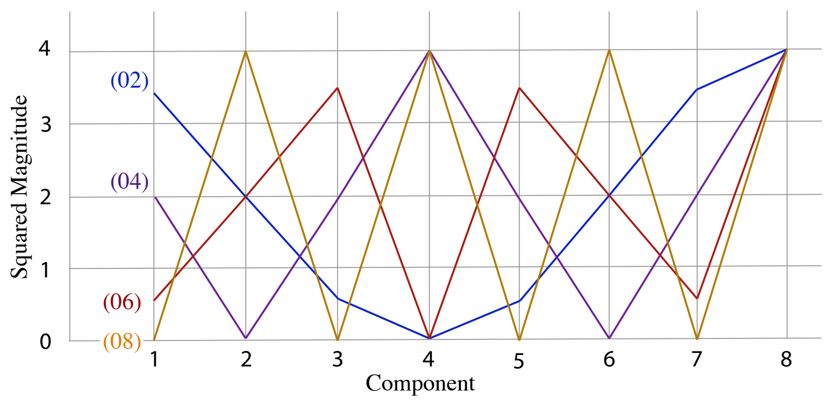


Figure 23: Spectra of even dyads in a 16-cycle, used as canon configurations in *Nagoya Marimbas*

The other noteworthy feature of the even dyads is f_4, which is maximum for {04} and {08} and zero for {02} and {06}. This creates the essential contrast between sections: the A_2 and extended signature rhythms appear with {04} canons towards the beginning of the piece, accentuating the f_4 quality. However, in the long middle section of the piece, the {02} canons cancel out the f_4 quality entirely.

We can make similar observations about the even dyads in the 12-cycle, whose spectra are given in Figure 24. The {02} canon reinforces f_5 of the signature rhythm while canceling out f_3, whereas the {04} canon, neutral with respect to f_5, accentuates f_3. If we compare the spectra across cycle lengths, as in Figure 25, we see that the two {02} canons are quite similar, as are the {04} canons. (This is an instance of *zero padding*.[44]) The structure in Table 7 can therefore be succinctly summarized as a shift away from the metrical qualities of {04} canons and back again overlaid onto a unidirectional process of shifting from 4/4 to 3/4 .

[44] See, e.g., Briggs and Henson 1995.

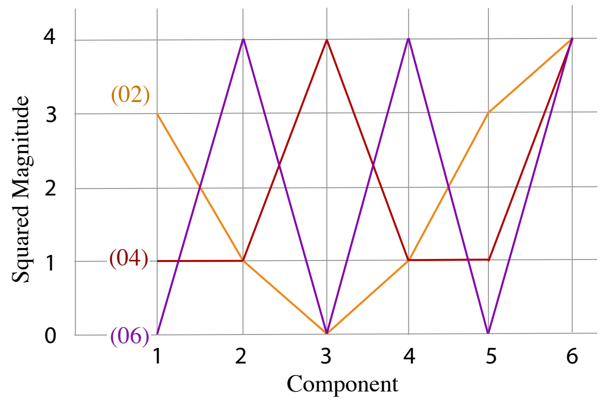


Figure 24: Spectra of even dyads in a 12-cycle

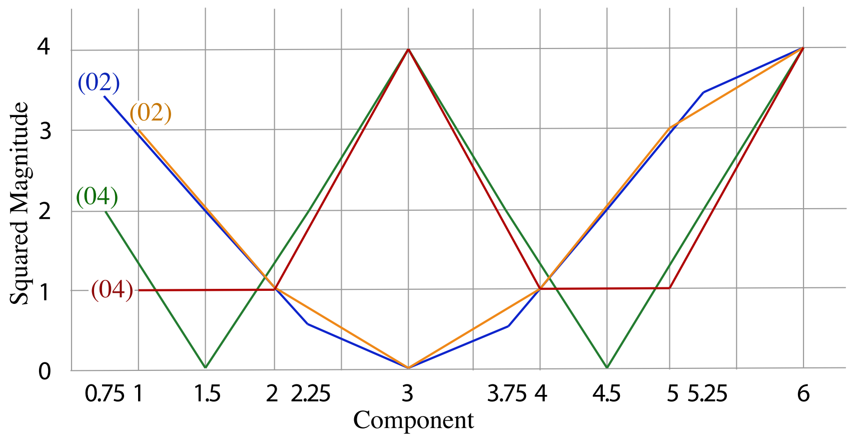


Figure 25: Comparison of {04} and {02} canons in a 12-cycle and a 16-cycle

Harmonic processes of the piece parallel the rhythmic processes. It has an arch from consonance to dissonance and back, with pentatonic collections representing maximum consonance, which parallels the shift away from metrical rhythmic qualities and back. There is also a unidirectional shift up a fifth from G pentatonic to D pentatonic paralleling the change of meter. The addition of dissonance over the middle section involves the introduction of semitones, an interval absent from the pentatonic collections, and greater circle-of-fifths spread. The first part of the piece builds both in dissonance and registral range as the canons and rhythms correspondingly move towards a greater focus on the ametrical $f_{7/16}$ and $f_{5/12}$ qualities. The tonal process of increasing dissonance consists of, first, and addition of F to the collection (in m. 27) and then G♯ (in m. 39). The latter change leads to the only non-diatonic collection, a subset of harmonic minor, to appear in the piece, and the state of greatest circle-of-fifths spread. (Interestingly, the collection we would get by adding G♯ to the white-note diatonic collection is the pitch-class-space analog, up to transposition, of Reich's signature rhythm.) The harmony then gradually reduces the circle-of-fifths spread, dropping the G♯ (in m. 47), then the F (m. 50), and finally adding the F♯ (m. 66) and paring down to the final pentatonic collection (m. 73) as the alternation of meters finally gives way to a consistent 3/4. The return to consonance is matched by the return to the more metrical quality of the {04} canons.

Table 8 shows how these processes coordinate. The major rhythmic events can be divided into those that progress through the arch form of Table 7, and those that interrupt this process with contrasting canons. In the first part of the piece, the progress through the main formal structure coincides with the addition of dissonances F and G♯. In the middle of the piece there are a number of interruptions in the rhythmic structure, one of which, the isolated occurrence of the unusual symmetrical rhythm C, is the last canon with the G♯. At the end of the piece, the return of the {04} canon and the beginning of consistent 3/4 meter coincides with the return of the bass note of the pitch collection to E, and the completion of the 1♯-diatonic (G major/E minor) collection. At the end of the piece, Reich staggers the main tonal and rhythmic processes, first paring the pitch collection down to a D pentatonic, then moving to the final E_2 rhythm and {06} canonic configuration.

Table 8: Coordination of Rhythmic and Tonal Design

Rhythmic			Tonal	
Meas.	Structure	Interruptions	Meas.	
1	{06} canon/A_1		1	E in bass / G pentatonic
19		{03} canon		
23	{04} canon/A_2		24	Add F
27		XS-Sig.	27	Drop G
39	{02} canon / alt. meters		39	Add G♯
42		{08} canon		
46		Rhythm C	47	Drop G♯
50		{08} canon	50–53	Drop F, A in bass
62		{0,–6} canon		
			66	B in bass / add F♯
71	{04} canon / triple meter		70	E in bass / complete 1♯-diat.
			73	Drop C
76		Rhythm F	76	Drop G, D pentatonic
80	Rhythm E_2	{02} canon		
83	{06} canon			

While *Nagoya Marimbas* is not amongst Reich's major works of the 1990s, its close connection to his earlier methods makes it an enlightening study in the aesthetic distance he traveled over the previous two decades. The central role of mechanical process, which we could already see deteriorating in *Music for Pieces of Wood*, is nearly gone in *Nagoya Marimbas*. The importance of canon, however, remains strong, and the signature rhythm that Reich first used in *Clapping Music* continues to be a generative force. Even as the signature rhythm recedes from omnipresence at the surface, it continues to act as a source for all rhythmic material. Two of its principal qualities, f_3 and f_5, and their nearest 16-cycle counterparts therefore, are the main vehicle for Reich's formal design. The arch design we have found in *Nagoya Marimbas* is also evident in *Music for Pieces of Wood*, a telltale sign of formal artifice and the abandonment of pure process as a constructive principle.

References

Amiot, Emmanuel. 2007. "David Lewin and Maximally Even Sets." *Journal of Mathematics and Music* 1 (3): 157–72.

———. 2016. *Music through Fourier Space: Discrete Fourier Transform in Music Theory*. Heidelberg: Springer.

Agawu, Kofi. 2003. *Representing African Music: Postcolonial Notes, Queries, Positions*. New York: Routledge.

Bakker, Twila. 2019. "Steve Reich's *Counterpoints* and Computers: Rethinking the 1980s." In *Rethinking Reich*, ed. Sumanth Gopinath and Pwyll ap Siôn (New York: Oxford University Press): 239–256.

Briggs, William L., and Van Emden Henson. 1995. *The DFT: An Owner's Manual for the Discrete Fourier Transform*. Philadelphia: SIAM.

Clough, John, and Jack Douthett. 1991. "Maximally Even Sets." *Journal of Music Theory* 35: 93–173.

Cohn, Richard. 1988. "Transpositional Combination and Inversional Symmetry in Bartok." *Music Theory Spectrum* 10: 19–42.

———. 1992. "Transpositional Combination of Beat-Class sets in Steve Reich's Phase shifting Music," *Perspectives of New Music* 30(2): 146–77.

———. 2016. "A Platonic Model of Funky Rhythms." *Music Theory Online* 22 (2). At http://mtosmt.org/issues/mto.16.22.2/mto.16.22.2.cohn.html.

Colannino, Justin, Francisco Gómez, and Godfried Toussaint. 2009. "Emergent Beat-Class Sets in Steve Reich's 'Clapping Music' and the Yoruba Bell Timeline." *Perspectives of New Music* 41 (1): 111–34.

Forte, Allen. 1973. *The Structure of Atonal Music*. New Haven: Yale University Press.

Gamer, Carlton. 1967. "Some Combinational Resources of Equal-Tempered Systems," *Journal of Music Theory* 11 (1): 32–59.

Hartenberger, Russell. 2016. *Performance Practice in the Music of Steve Reich*. Cambridge: Cambridge University Press.

Horlacher, Gretchen. 2000–2001. "Multiple Meters and Metrical Processes in the Music of Steve Reich." *Intégral* 14–15: 265–97.

Jones, A.M. 1959. *Studies in African Music*. London: Oxford University Press.

Lewin, David. 2001. "Special Cases of the Interval Function between Pitch-Class Sets X and Y." *Journal of Music Theory* 45/1: 1–29.

——— 2007. *Generalized Musical Intervals and Transformations*, 2d Ed. New Haven: Yale University Press.

London, Justin. 2012. *Hearing in Time: Psychological Aspects of Musical Meter*, second edition. New York: Oxford University Press.

Milne, Andrew J., David Bulger, and Steffen A. Herff. 2017. "Exploring the Space of Perfectly Balanced Rhythms and Scales." *Journal of Mathematics and Music* 11 (2).

Milne, Andrew J., David Bulger, Steffen A. Herff, and William A. Sethares. 2015. "Perfect Balance: A Novel Organizational Principle for Musical Scales and Meters." In *Mathematics and Computation in Music, Fifth International Conference, MCM 2015*, ed. T. Collins, D. Meredith, and A. Volk (Heidelberg: Springer): 97–108.

Potter, Keith. 2011. "Drummed Out?: What Steve Reich did after Drumming." *Mittielungen der Paul Sacher Stiftung* 24: 35–41.

Pressing, Jeff. 1983. "Cognitive Isomorphisms in Pitch and Rhythm in World Musics: West Africa, the Balkans, and Western Tonality." *Studies in Music* 17: 38–61.

Quinn, Ian. 2006. "General Equal-Tempered Harmony" (in two parts). *Perspectives of New Music* 44/2–45/1: 114–159 and 4–63.

Rahn, Jay. 1983. *A Theory for All Music: Problems and Solutions in the Analysis of Non-Western Forms.* Toronto: University of Toronto Press.

———. 1996. "Turning the Analysis Around: Africa-Derived Rhythms and Europe-Derived Music Theory." *Black Music Research Journal* 16 (1): 71–89.

Reich, Steve. 2002. *Writings About Music, 1965–2000*. New York: Oxford University Press.

———. 2016. "Thoughts about Percussion and Rhythm." In *Cambridge Companion to Percussion*, ed. Russel Hartenberger (Cambridge, Eng., Cambridge University Press): 173–86.

Roeder, John. 2003. "Beat-Class Modulation in Steve Reich's Music" *Music Theory Spectrum* 25/2: 275–304

Scherzinger, Martin. 2018. "'Piano Phase' in Global Perspective: Myths and Realities." In *Clash! Generationen – Kulturen – Identitäten in der Gegenwartsmusik*, ed. Jörn Peter Hiekel (Mainz, Schott): 144–163.

———. 2019. "Afro-Electric Counterpoint." In *Rethinking Reich*, ed. Sumanth Gopinath and Pwyll ap Siôn (New York: Oxford University Press): 259–302.

Schwartz, Robert K. 1981–2. "Music as Gradual Process Part II" *Perspectives of New Music* 20 (1–2): 225–286.

———. 1990. "Process versus Intuition in the Recent Works of Steve Reich and John Adams." *American Music* 8 (3): 245–273.

Suzuki, Dean Paul. 1991. "Minimal Music: Its Evolution as Seen in the Works of Phillip Glass, Steve Reich, Terry Reily, and La Monte Young." PhD Diss., Uinversity of Southern California.

Toussaint, Godfried. 2013. *The Geometry of Musical Rhythm: What Makes a Good Rhythm Good?* CRC Press.

Yust, Jason. 2015a. "Applications of DFT to the Theory of Twentieth-Century Harmony." In *Mathematics and Computation in Music, Fifth International Conference, MCM 2015*, ed. T. Collins, D. Meredith, and A. Volk (Heidelberg, Springer): 207–18.

———. 2015b. "Schubert's Harmonic Language and Fourier Phase Space." *Journal of Music Theory* 60 (2): 213–62.

———. 2016. "Special Collections: Renewing Set Theory." *Journal of Music Theory* 60 (2): 213–262.