Distorted Continuity: Chromatic Harmony, Uniform Sequences, and Quantized Voice Leadings

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Abstract:

Uniform patterns are voice-leading patterns that are purely regular when represented with *generic intervals* which are related to real intervals through *quantization*. Various kinds of chromatic and diatonic sequences are uniform patterns. The generic intervals of uniform patterns provide a way of precisely and quantitatively comparing different kinds of chromatic patterns to diatonic ones by drawing upon the kind of robust, continuous metrics associated with voice-leading spaces. The possibility of deriving chromatic and diatonic logics from common principles suggests a new perspective on the "integration" problem of nineteenth-century harmony—the question of whether chromaticism represents a radical break or evolution from conventional tonal harmony. The first movement of Schubert's String Quartet no. 15 and other passages from Schubert and Liszt illustrate the use of the theory of uniform patterns and generic intervals in analysis.

Introduction:

Musicians often understand chromatic sequences by thinking of them as distortions of familiar diatonic models. But a chromatic sequence can vary in how similar it is to a given diatonic model and many can be referred to more than one such model. This paper defines a particular kind of voice-leading pattern, the *uniform progression*, that characterizes most sequential patterns found in music, and illustrates the concept with a number of chromatic sequences from works of Schubert and Liszt and related diatonic sequences. Each uniform progression is based on *generic intervals*, intervals that vary systematically between two sizes a semitone apart. We can compare various chromatic and diatonic sequence types in a precise and quantitative way through differences between generic interval sizes.

The concept of uniform progression draws upon two parallel developments in recent mathematical music theory that have helped to loosen the strictures of simple algebraic models of pitch. Most recently, the geometrical theory of voice leading (Callender, Quinn, and Tymoczko, 2008) has enriched pitch-class sets and set classes with robust concepts of nearness and distance (see Tymoczko 2009). A previous watershed, Clough and Douthett's

(1991) maximally even scales, perhaps surprisingly, relies on the same continuous and metric conception of pitch class. Though Clough and Douthett do not make explicit reference to the metric properties of continuous pitch class space, it is implicit in their method of defining maximal evenness. Uniform progressions are essentially a generalization of maximal evenness to sequential voice-leading patterns, the music-theoretic value of which is possibility of relating patterns using this implicit notion of distance.

The possibility of reconciling diatonic and chromatic systems suggests a new perspective on a persistent music theory problem: the clear continuities and discontinuities that nineteenth century chromaticism presents with earlier tonal practice. An integrationist theoretical approach, applying tools designed for eighteenth century harmony to chromatic music, easily becomes anachronistic, while analytical methods designed specifically for chromatic harmony may obscure the dialectic with established tonal conventions so important to much of this music. Richard Cohn's (2012) strong advocacy for the anti-integrationist position, which first appeared in Cohn 1996, is intertwined with his use of transformational systems, especially in earlier work, to explain chromatic harmony. Transformational theory originated in the context of atonal music analysis and retains the essential mathematical underpinnings of the Babbitt/Forte language of set classes, transposition, and inversion (Lewin 1982, 1987). This group-theoretic model of pitch relationships is compelling in its simplicity but also rigid, leaving an unbridgeable algebraic gulf between the mod-12 pitch-class universe and the mod-7 diatonic universe of tonal harmony. The theory developed in this paper demonstrates that an approach grounded in a continuous model of pitch can bridge this divide by showing how chromatic sequential patterns may emerge as modifications of familiar diatonic models. At the same time it sharpens the sense of a fundamental independence of chromaticism by providing a clear and definite distinction between chromatic and diatonic logics. While the scope of the paper is primarily limited to making such distinctions and comparisons between sequential voice-leading patterns, in §4.3 an extension of the central analytical example of the paper, Schubert's String Quartet no. 15, suggests the possibility of broader application to chromatic harmony.

The paper begins, in section one, with a motivational example from this String Quartet. The second section then lays the theoretical groundwork of uniformity, quantization, and generic interval and the third section extends this to complete voice-leading patterns. Section four applies these concepts in a more thorough analysis of Schubert's use of sequence in the first movement of the String Quartet. Section five expands upon the idea of quantifiable diatonic interpretations introduced in section two, as a means of evaluating different ways that chromatic patterns can be understood as distortions of familiar diatonic ones, analyzing sequences from various works of Schubert and Liszt.

(1) A Sequence from Schubert's String Quartet no. 15

The main theme of the first movement of Schubert's G major String Quartet (Ex. 1(a)) is based on a chromatic sequence on major triads, G–D–F–C–E♭, alternating $^{5-6}_{3-3}$ so that the bass line descends chromatically from $\hat{1}$ to $\hat{5}$. Example 1(b) summarizes the voice-leading pattern of this sequence. There are two ways that one might imagine this sequence to be modeled on familiar diatonic patterns:

Example 1: (a) Schubert, String Quartet no. 15, mm. 15–22, and (b) its sequential voice-leading pattern, compared to (c) the retrograde of the diatonic ascending 5–6, and (d) the diatonic descending 5–6.

(1) It is a retrograde of the familiar ascending 5–6 sequence (Ex. 1(c)), with chromatic variants used to transform the root position vii° and vi triads into major triads (♭VII and ♭VI).

(2) It is a "descending 5–6" (AKA "Pachelbel sequence"—Ex. 1(d)) using a chromatic scale rather than a diatonic. That is, the familiar diatonic sequence alternates root position and first inversion chords over a bass descending stepwise

in a diatonic scale. Schubert's sequence does the same except that the bass descends stepwise in a chromatic scale.[1]

What makes each of these comparisons plausible? All three sequence patterns share some important characteristics: First, very generally:

(1) In all three patterns the chords are all the same general type of object. In the diatonic sequences, it is always a diatonic triad. In Schubert's sequence, it is always a major triad.

(2) All three patterns alternate root position and first inversion. This is more properly described as a feature of the voice-leading pattern specifically: The first voice leading is root→third, third→fifth, and fifth→root, while the second is root→fifth, third→root, fifth→third. (Adding a bass that doubles roots, for instance, does not change this feature.)

Comparing the voice leadings in more detail, one finds that all of the sequences tend to move voices by similar distances in the same places, but the retrograde ascending 5–6 is generally closer to Schubert's sequence than the descending 5–6. For instance, the retrograde ascending 5–6 always has diatonic steps where Schubert's sequence has whole steps and generally never moves a voice by an interval more than a semitone larger or smaller than Schubert's sequence.

Another important similarity between the retrograde ascending 5–6 and Schubert's sequence is that the overall progressions descend at about the same rate: a diatonic step for each two chords in the retrograde ascending 5–6, and a whole step for each two chords in Schubert's sequence. The descending 5–6 descends a diatonic *third* for each two chords. All the sequences diverge therefore gradually, with the descending 5–6 moving more rapidly and the retrograde ascending 5–6 not quite as rapidly as Schubert's sequence.

The language of similarity and distance invoked in this comparison of sequential patterns cannot be captured within a group-theoretic framework—or, to use Lewin's (1987) term, a transformational model—because in two of the sequences, distances are always measured in a diatonic scale, which are mod-7 (step-class) transformations. There are no

[1] This explanation of the sequence was first proposed to me by Dmitri Tymoczko (personal correspondence, June 2012) and relates to Hook's (2007) "cross-type transformations."

non-trivial algebraic relationships between this step-class group and the chromatic mod-12 group where Schubert's sequence lives.[2]

However, it would be a mistake to think therefore that we are comparing these sequences in imprecise or non-mathematical terms. Rather, it is a different kind of mathematics that underlies the comparison. The concepts of generic intervals, quantization, and uniformity developed in the next sections of this paper can make all of these statements precise.

(2) Quantization, Diatonic Interpretations, and Uniform Sequences

In this section I will define two concepts that will help us compare chromatic and diatonic sequences and explain the ambiguity of diatonic reference for Schubert's sequence described above. The first concept is that of an *elided* voice-leading pattern, which will help us derive sequences like the ascending 5–6 and descending 5–6 from a single underlying pattern. The second is *generic interval size*, a way of quantifying intervals that come in two sizes, like diatonic thirds. Generic intervals are converted into real ones by means of a process of *quantization*, and sequences derived in this way have a property that I call *uniformity*.

2.1 Elided patterns. Typical sequential patterns involve the alternation of two different voice leadings. The descending 5–6 pattern (Ex. 2(a)), for example, has a first voice leading where the root and third move down by step, and a second where the root moves down by third while the third and fifth move down by step. However, it is convenient to think of such patterns as *elisions* of patterns on a single voice-leading type. Example 2(b), for instance, repeats a single voice leading, the first voice leading of the descending 5–6 pattern (root and third down by step). The result is the descending 5–6 with an extra chord for each sequential repetition. Thinking of a sequence as *elided* in this way does not necessarily imply that the elided chord is somehow present as an "imaginary" step in realizations of the progression. Rather, an elided sequence is simply a two-chord sequence in which one voice leading is the other "times two."

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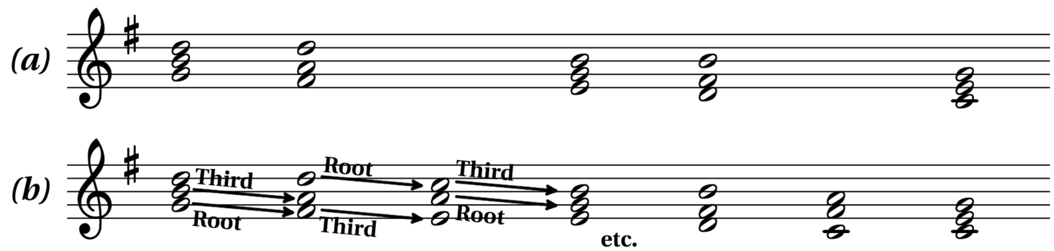
[2] See Tymoczko 2008b (a response to Hook 2007), and Hall 2008 and 2009 for interesting related discussions.



Example 2: (a) The descending 5–6 sequence, and (b) its underlying uniform pattern.

The voice leadings of Example 2(b) are only equivalent in terms of diatonic scale-steps; their real interval sizes vary. Such patterns are *uniform* in the sense that the voice leadings are the same up to a variation of intervals between two sizes a semitone apart. (I will define this more precisely below). The descending 5–6 pattern is then an *elided uniform* pattern, a uniform pattern that skips over one of the chord positions (here the $^{6}_{4}$ position).

2.2 Diatonic interpretations. I have described uniformity as a feature of voice leading, but it will help to think of it first as a feature of root motion and return to matters of chord type and voice leading in the next section. In Example 2(b), the root motion is always by ascending fifth, and the chord is always a triad. The intervals of root motion and of the triads always vary between no more than two adjacent sizes. This is a general fact for all generic interval sizes in a diatonic scale, and was shown by Clough and Douthett (1991) to result from the maximal evenness of the scale.[3] In the remainder of this section, I will temporarily discard the added complication of chord type and chord position to focus on the concept of generic interval size and uniformity in the context of root progression alone.

Example 3 shows the derivation of a diatonic scale as a maximally even set using Clough and Douthett's definition, but replacing their floor function with a rounding function,[4] which I will call *quantization*.[5] Starting from an even division of the octave into

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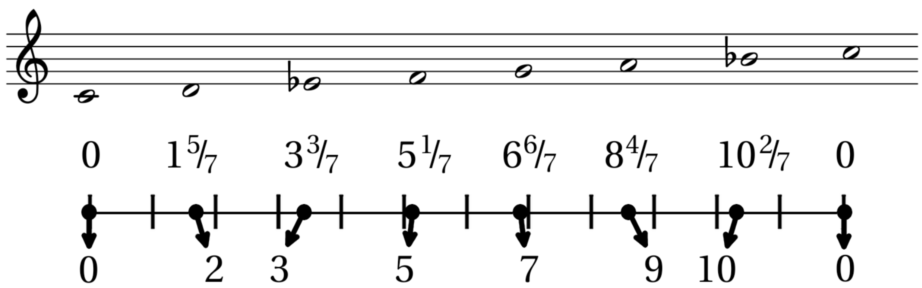
[3] See also Clough and Myerson (1985). They refer to scales with up to two interval sizes per span as having "Myhill's property." If one specifies further that the two interval sizes must be adjacent, the property is equivalent to maximal evenness.

[4] There is no essential mathematical difference between these. A rounding function is the same as adding a constant of 0.5 before applying the floor function. (I assume the rounding function uses a "half-up" tie-breaker.)

seven (a "7-equal scale"[6]), each value is rounded to the nearest integer value. Starting from C = 0, the result is a scale with two flats, but transposing the scale by $^{1}/_{7}$ before rounding changes one accidental, moving it one tic on the circle of fifths (Ex. 4). I will generally refer to such transpositions by fractions of a semitone as *microtranspositions*.[7] There are twelve 7-equal scales in increments of $^{1}/_{7}$-semitone, meaning that each diatonic scale has a unique set of seven pre-quantized values associated with it. The 7-equal scales are thus, in a sense, the perfectly even "preimage" of each diatonic scale. The diatonic need not be arranged in scalar order, though. Example 5 derives it from 7-equal thirds ($3^{3}/_{7}$), which quantize to three major thirds and four minor thirds.



Example 3: The quantization of a 7-equal scale gives a diatonic scale.

Arranging the notes in circle-of-fifths order shows that the fractional part of the unquantized value corresponds to the scale-degree position of the note. (Ex. 6) I will call these 7-equal values *diatonic interpretations*, because there is a single value associated with each scale degree position in each of twelve diatonic scales. Considered as a group of 84 trans-positions (in multiples of $^{1}/_{7}$), they are equivalent to Hook's (2006) signature transformation group.[8] Diatonic interpretations, however, also involve a specific concept of intervallic size

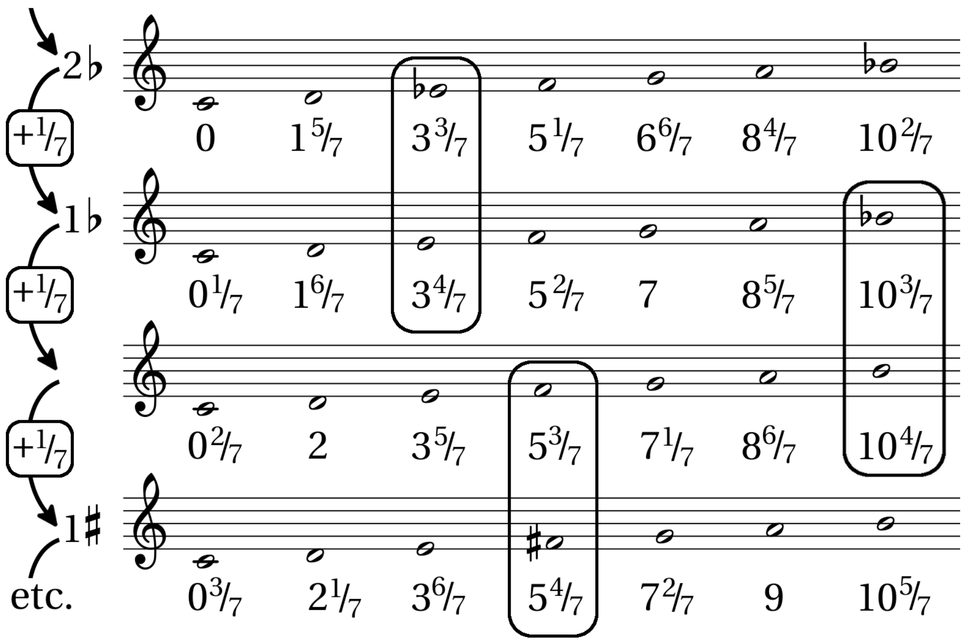
[5] Much of my terminology evolved from earlier versions of this paper (first presented in Yust 2010) through discussions with Dmitri Tymoczko in the preparation for the 2012 John Clough Memorial Conference, New Haven, 2012.

[6] I use "*n*-equal" to refer to equal divisions of the octave by *n* throughout the paper.

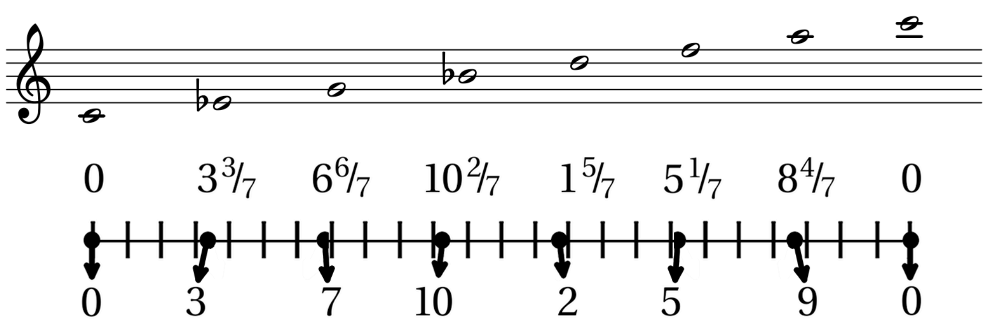
[7] This term was suggested by a reviewer of an earlier draft of this paper.

[8] The table in Example 6 also resembles Rings' scale-degree/pitch-class matrix (2011, 44–54), but Rings' group actually does not map consistently onto the diatonic interpretations,

or distance, which gives them additional mathematical structure beyond the abstract group-theoretic one. This added structure is essential to one of our primary purposes here, to make direct comparisons between diatonic and chromatic uniform patterns. Group theory does not allow for such comparisons, but the metric of generic intervals does.



Example 4: Diatonic scales derived from quantization of 7-equal scales. Microtransposition of a collection by $^1/_7$ causes exactly one note to change its quantized value (these are boxed for emphasis).



Example 5: Quantization of a series of 7-equal thirds

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because he treats scale-degree qualia (changes of scale-degree sense on a given pitch-class) as an order-seven subgroup, rather than scale step (changes of scale-degree within a given scale.) Nevertheless, many of Rings' analytical insights about, e.g., pivot intervals can be translated into a system of transformations based on diatonic interpretations.

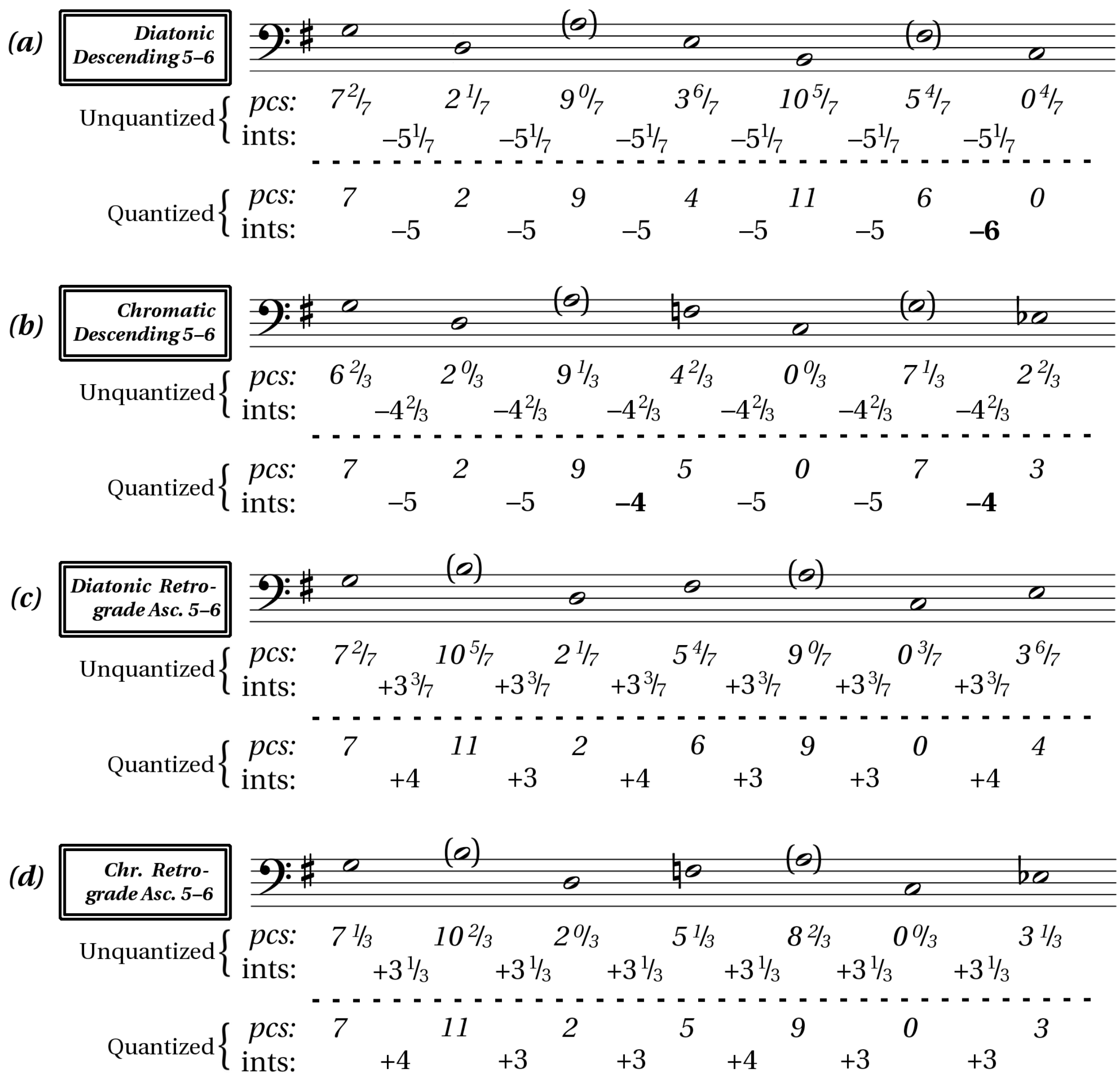
Scale degree	A♭	E♭	B♭	F	C	G	D	A	E	B	F♯	C♯
	Diatonic collection											
$\hat{4}$	$1\tfrac{3}{7}$	$8\tfrac{3}{7}$	$3\tfrac{3}{7}$	$10\tfrac{3}{7}$	$5\tfrac{3}{7}$	$0\tfrac{3}{7}$	$7\tfrac{3}{7}$	$2\tfrac{3}{7}$	$9\tfrac{3}{7}$	$4\tfrac{3}{7}$	$11\tfrac{3}{7}$	$6\tfrac{3}{7}$
$\hat{1}$	$8\tfrac{2}{7}$	$3\tfrac{2}{7}$	$10\tfrac{2}{7}$	$5\tfrac{2}{7}$	$0\tfrac{2}{7}$	$7\tfrac{2}{7}$	$2\tfrac{2}{7}$	$9\tfrac{2}{7}$	$4\tfrac{2}{7}$	$11\tfrac{2}{7}$	$6\tfrac{2}{7}$	$1\tfrac{2}{7}$
$\hat{5}$	$3\tfrac{1}{7}$	$10\tfrac{1}{7}$	$5\tfrac{1}{7}$	$0\tfrac{1}{7}$	$7\tfrac{1}{7}$	$2\tfrac{1}{7}$	$9\tfrac{1}{7}$	$4\tfrac{1}{7}$	$11\tfrac{1}{7}$	$6\tfrac{1}{7}$	$1\tfrac{1}{7}$	$8\tfrac{1}{7}$
$\hat{2}$	10	5	0	7	2	9	4	11	6	1	8	3
$\hat{6}$	$4\tfrac{6}{7}$	$11\tfrac{6}{7}$	$6\tfrac{6}{7}$	$1\tfrac{6}{7}$	$8\tfrac{6}{7}$	$3\tfrac{6}{7}$	$10\tfrac{6}{7}$	$5\tfrac{6}{7}$	$0\tfrac{6}{7}$	$7\tfrac{6}{7}$	$2\tfrac{6}{7}$	$9\tfrac{6}{7}$
$\hat{3}$	$11\tfrac{5}{7}$	$6\tfrac{5}{7}$	$1\tfrac{5}{7}$	$8\tfrac{5}{7}$	$3\tfrac{5}{7}$	$10\tfrac{5}{7}$	$5\tfrac{5}{7}$	$0\tfrac{5}{7}$	$7\tfrac{5}{7}$	$2\tfrac{5}{7}$	$9\tfrac{5}{7}$	$4\tfrac{5}{7}$
$\hat{7}$	$6\tfrac{4}{7}$	$1\tfrac{4}{7}$	$8\tfrac{4}{7}$	$3\tfrac{4}{7}$	$10\tfrac{4}{7}$	$5\tfrac{4}{7}$	$0\tfrac{4}{7}$	$7\tfrac{4}{7}$	$2\tfrac{4}{7}$	$9\tfrac{4}{7}$	$4\tfrac{4}{7}$	$11\tfrac{4}{7}$

Example 6: A table of diatonic interpretations. Each column quantizes to a different diatonic scale. The numbers in each row share the same fractional part and represent the same scale degree in a major scale.

What is constant about all 7-equal scales are the *intervals*. The preimage of the diatonic scale step is the 7-equal value of $1\tfrac{5}{7}$, which sometimes becomes a half step, sometimes a whole step. The fractional part represents the likelihood of the interval rounding larger—so five of the seven scale steps are whole steps. A single scale step is not completely forthcoming about its origins: for instance, the interval C–D could come from five different diatonic scales, represented by values from $11\tfrac{6}{7}$–$1\tfrac{4}{7}$ ($\hat{6}$–$\hat{7}$ in E♭ major) to $0\tfrac{3}{7}$–$2\tfrac{1}{7}$ ($\hat{4}$–$\hat{5}$ in G major). However, the pre-quantized values become definite in the context of a consistent pattern, such as a sequence.

2.3 Diatonic and chromatic sequences. Example 7(a) shows the effect of quantization on the root progression of the descending 5–6 sequence, with elided values in parentheses. The 7-equal interval of the root progression is $-5\tfrac{1}{7}$. This interval is usually realized as a descending perfect fourth, but becomes an augmented fourth when taken from a value whose fractional part is just above $\tfrac{1}{2}$ (such as $\tfrac{4}{7}$, the fractional part that quantizes to $\hat{7}$ of its associated major scale). It returns to its starting point after seven steps ($5\tfrac{1}{7} \times 7 \cong 0 \bmod 12$).

The concept of *uniformity* generalizes the principle of maximal evenness from a property of scalar collections to a property of patterns that exhibit a similar kind of regularity. This generalization permits precise comparisons between conventional sequences whose uniformity derives from the maximal evenness of the diatonic scale, and chromatic sequences whose uniformity does not have to do with the use of scales. Example 7(b) shows the sequence from the main theme of Schubert's string quartet as chromatic version of the descending 5–6. The root progression for this sequence is uniform in the sense that its intervals vary in the range of a semitone (–5 to –4) just like the intervals of (a) do (–5 to –6), and the same is true of the intervals between every second note (2 to 3), every third note (–2), and so on. This kind of uniformity relates to maximal evenness in the sense that registrally unfurling the root progression of (b) results in a maximally even division of the 14-semitone ninths into three intervals (4–5–5).



Example 7: Root progressions for diatonic sequences (a, c) and Schubert's sequence from the main theme of Sting Quartet no. 15*, derived as elisions of different uniform patterns via quantization (b, d).*

The property of uniformity can be derived from quantization of consistent, non-integer, interval sizes (such as $-5^{1}/_{7}$ in 6(a) and $-4^{2}/_{3}$ in 6(b)), which I will refer to as *generic intervals*. The generic interval represents an average of all the real, post-quantization, intervals (imagining the sequence continued infinitely). The generic interval $-5^{1}/_{7}$ restricts the quantized root progression to a single diatonic scale, sometimes being realized as a perfect fourth, sometimes an augmented fourth. The generic interval $-4^{2}/_{3}$ is realized as a perfect fourth $^{2}/_{3}$ of the time and a major third $^{1}/_{3}$ of the time, so that when repeated three times in succession it always adds to exactly $-4^{2}/_{3} \times 3 = -14 = -2 \pmod{12}$. The periodicity of this pattern matches the periodicity of the sequence, so that the resulting sequential pattern is chromatically perfectly regular, unlike diatonic sequences, whose periodicity of intervallic regularity, seven, is too large to match the periodicity of an ordinary sequence.

The chromatic pattern in Example 7(b) also has a *phase*, analogous to the "key" of diatonic patterns, determined by the choice to begin the sequence on $6^{2}/_{3}$. Beginning on 7 would micro-transpose the entire sequence by $^{1}/_{3}$, raising every third chord root by a semitone (which, in this case, would be inconsequential, affecting only the elided chords). This is analogous to the microtransposition of a diatonic pattern by $^{1}/_{7}$, which changes one accidental out of seven. The choice of phase in 7(b) makes the first three chords the same as in 7(a).

The diatonic retrograde ascending 5–6 is based on a root progression by third rather than by fourth or fifth. In Example 7(c) the chord in the second position is elided so that when chromaticized in 7(d), the result is a different derivation of Schubert's sequence. The ambiguity of this sequence, then, has to do with where the elided chord goes in the pattern. This is a remarkable property, since it means that either voice leading in the pattern can be considered a twice-iteration of the other.

The sequences of 7(c) and 7(d) diverge much more gradually than 7(a) and 7(b), because the difference in the generic interval of the root progression ($3^{3}/_{7} - 3^{1}/_{3} = {}^{2}/_{21}$) is much smaller. This means that Schubert's sequence is more similar to a diatonic pattern as a kind of retrograde ascending 5–6. However, the descending 5–6 is much more familiar as a diatonic pattern, making it perhaps more likely that we hear the sequence on this model. In that case, we are committed to thinking of the sequence as highly chromatic: in reference to the diatonic model, the minor third root progression in the sequence is a chromatically enlarged step—an augmented second, in other words. Using Roman numerals to reflect the diatonic reference point, we could say that sequence 7(d) is I–V–♭VII–IV–♭VI while sequence 7(b) is I–V–♯VI–♯III–♯×IV. That is to say, 7(d) is within the bounds of ordinary mixture, while the divergence in 7(b) is beyond what could be achieved through any ordinary diatonic process. Part 5 of the paper explores these kinds of observations further, developing a more systematic quantitative approach to diatonic interpretations.

(3) Voice-leading patterns and generic chord types.

The discussion of uniformity above has been simplified by considering only the root motion of sequences. A complete specification of a sequence also requires a *generic chord type* and a *rotation*. The result is an entire voice-leading pattern in generic intervals.

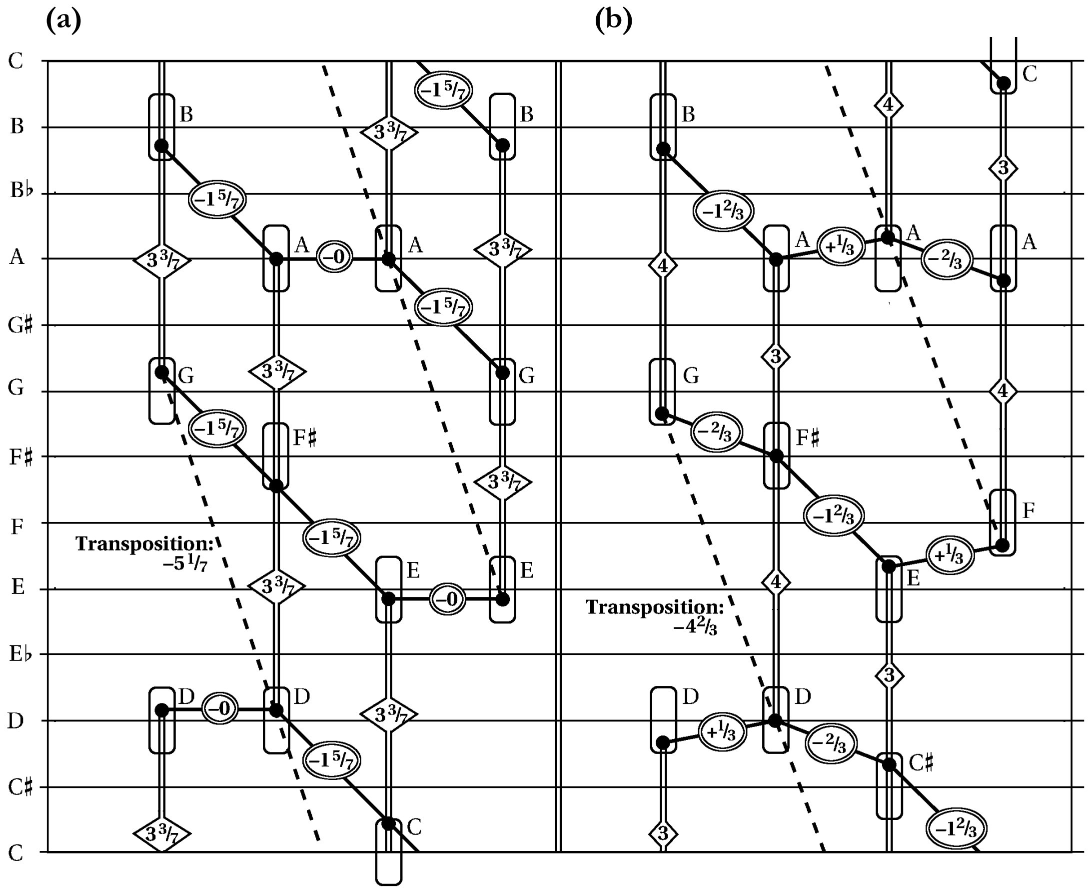
3.1 Generic chord types and idealized voice leadings. Example 8 graphs the uniform patterns for (a) a diatonic descending 5–6 sequence and (b) Schubert's sequence from String Quartet no. 15. In addition to the transpositions (dotted lines)—i.e., the root motions—, the chord types (doubled lines) and voice leadings (solid lines) are defined with generic intervals. For the diatonic sequence, the chord type is a *7-equal triad*, a triad built in 7-equal thirds ($3\frac{3}{7}$), which means that the chords are restricted to the same diatonic scale as the root progression in 7-equal fourths. As the fractional part of the chord roots drifts downward ($\frac{2}{7}$–$\frac{1}{7}$–$\frac{0}{7}$–$\frac{6}{7}$) the quality of the quantized chord changes from major to minor. Schubert's sequence (b) consists of only major triads, so the chord type is a pure major triad, (0 4 7), which quantizes to a major triad under any fractional transposition.

Example 8 also shows voice-leading intervals for each sequence. These are *idealized voice leadings*, meaning that a given triadic progression is represented by its most efficient one-to-one voice leading.[9] Each voice-leading interval is the difference between the transposition and a chordal interval, where the chordal intervals are determined by the *rotation*. For a transposition downward in the range of a third or fourth (as in Ex. 8), the most efficient voice leading uses an upward rotation to roughly cancel out the transposition, leaving relatively small voice-leading intervals. We can call this a "root→third" rotation, because the root of one chord goes to the third of the next. For a transposition upward in the range of a third or fourth, as in the retrograde ascending 5–6 –type patterns, the opposite "root→fifth" rotation gives the idealized voice leading.[10] Note, however, that the elided chords are

[9] See Cohn 2012, p. 6, and §3.3 below.

[10] See Tymoczko 2008a. Tymoczko calls rotations "scalar transpositions."

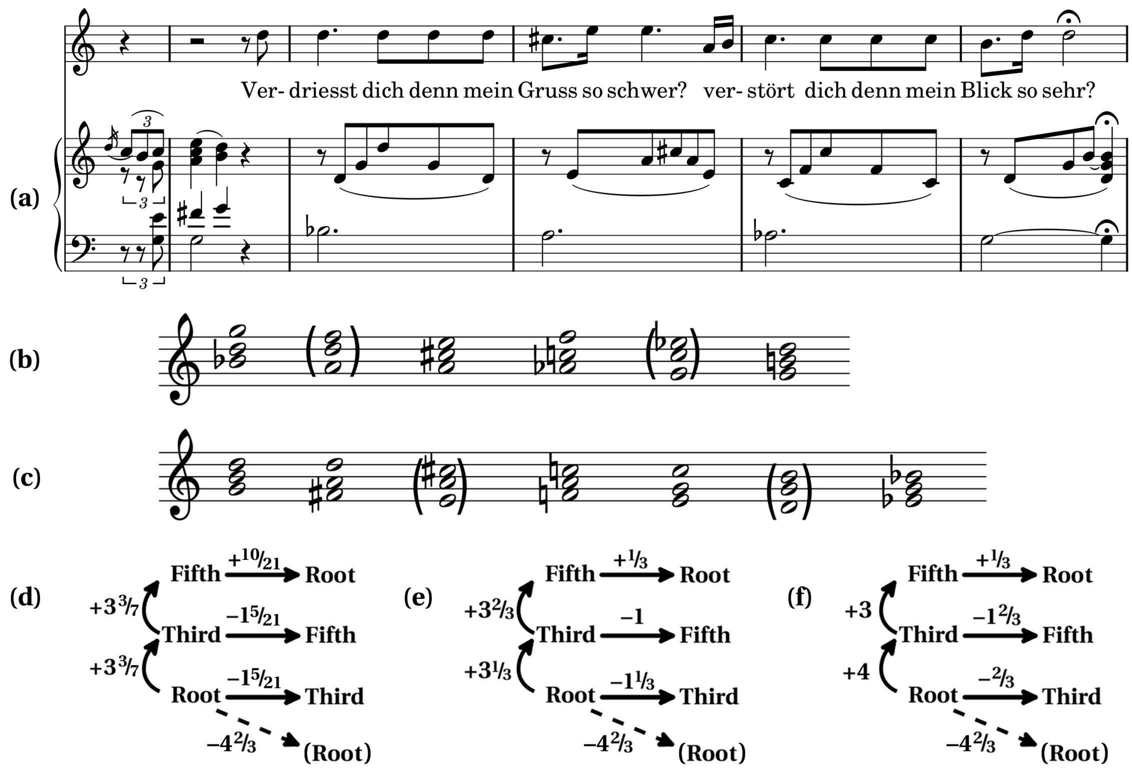
included in the pattern. Without them, the second voice leading in a descending 5–6 pattern would *not* be maximally efficient.



Example 8: (a) Diatonic descending 5–6 and (b) chromatic descending 5–6 sequences as uniform patterns. Doubled lines show the chord type by giving the size of the thirds. Dashed lines show the transpositions, and solid lines show the voice-leading intervals.

Idealized voice leadings are central to many theories of harmony such as that of Cohn 2012. They are equivalent to shortest paths in the chord spaces of Callender, Quinn, and Tymoczko 2008 and Tymoczko 2011. Generic chords correspond to points in these spaces, and quantization takes a generic chord to the nearest point in the lattice of 12-equal chords. Distances between generic chord *types*, which are distances in Callender, Quinn, and Tymoczko's set-class spaces, are also important for evaluating the relationships between different sequential patterns.

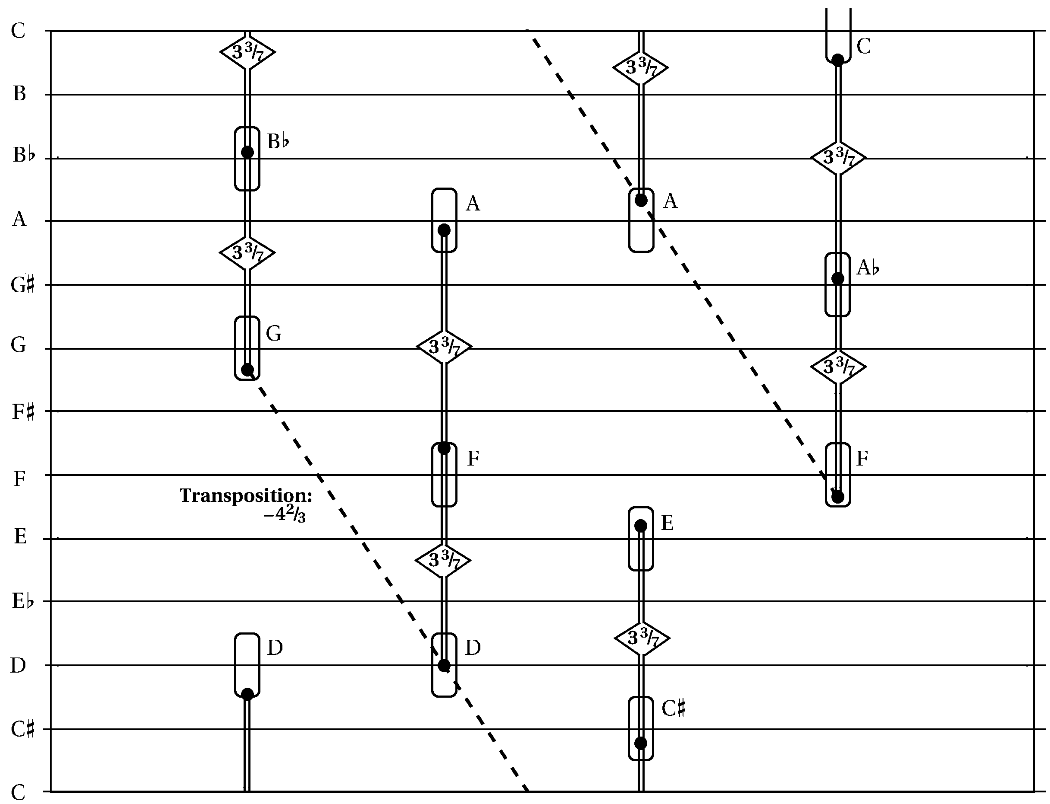
Schubert uses a similar sequence in the middle phrase of the short strophic song, "Morgengruss," from the cycle *Die Schöne Müllerin* (Ex. 9). In the song, the young man courts the miller's daughter in vain. The optimistic melody of the first phrase ends with a questioning half cadence as the girl averts her eyes, and the chromatic turn of the second phrase reflects the doubt creeping over the young man: "Do you dislike my greeting so deeply? / Does my glance upset you so much?"



Example 9: (a) A sequential passage from Schubert's "Morgengruss" (Schöne Müllerin *no. 8), mm. 12–15, and the preceding HC. (b) The underlying uniform progression for the sequence. (c) The uniform progression for the main theme of Schubert's 15th String Quartet, for comparison. (d)–(e) Two sets of generic intervals that produce the sequence in (b), showing voice-leading intervals (straight arrows), chordal intervals (curved arrows) and the transposition (dashed arrow). (f) The generic intervals for (c).*

The sequence in Example 9(b) has the same generic interval of transposition as (c), $-4\tfrac{2}{3}$, so that the root progressions of (b) and (c) are the same; only the location of the elided chord is different. This difference is significant in that it prevents, in (b), the kind of ambiguity we have already observed in (c).

The “Morgengruss” sequence also mixes major and minor triads, unlike (c), which reflects a difference in generic chord type. The sequence can be generated using the 7-equal triad as a chord type, as in (d). These quantize to minor triads when the root is on or below the nearest integer, and to major when it is above (see Ex. 10). If we instead take a simple average of the chords in the pattern, the generic chord type is (0 $3\frac{1}{3}$ 7). Because the sequential transposition of $-4\frac{2}{3}$ produces a limited set of fractional levels of transposition (three), these two chord types are indistinguishable for the given sequence.



Example 10: A graph of the sequence from “Morgengruss” as a uniform pattern of chromatic transpositions on diatonic triads.

The effects of changing the chord type are the direct result of voice-leading distances in set class space. The 7-equal triad, (0 $3\frac{3}{7}$ $6\frac{6}{7}$), is very close to (0 $3\frac{1}{3}$ 7)—a “city block” distance of $\frac{5}{21}$—which makes them indistinguishable for the given transposition. The distance between (0 $3\frac{1}{3}$ 7) and (0 4 7) is more substantial ($\frac{2}{3}$), turning some, but not all, or the chords in the “Morgengruss” pattern into minor triads.

3.2 The complete definition of uniformity. In section 2.3 above uniformity was defined in the context of a root progression alone. The concept applies more generally to entire voice-leading patterns. In a uniform voice-leading pattern each interval of the chord type, each voice-leading interval from the same chord member (root, third, fifth, etc.), and any sum of such intervals vary by no more than one semitone.

For instance, in Example 9(b), the root–third and third–fifth intervals both vary in the range of 3–4 and the root–fifth interval is always 7. The voice-leading interval from, e.g., the chordal third varies in the range 0–1. More generally, we could take any sum of chord intervals and voice-leading intervals and the range would be at most 1. This includes intervals of transposition and voice-leading intervals between non-adjacent chords (for instance, two voice-leading intervals from a root always sum to 2–3 in 9(b).)

3.3 Idealized voice-leadings in other analytical methods. Idealized voice leading is central to recent theoretical approaches to Schubert's harmony. This is true of Cohn 2012 and Damschroder 2006 and 2010, despite their diametrically opposed positions on the integrationist spectrum discussed in the introduction. Damschroder (2006) claims that chromaticism in Schubert's sequences poses no threat to "tonal coherence" and "enhance a broader diatonic initiative" (272–3), because they are framed by conventional functional harmony.[11] However, the essential function of much of Damschroder's newly coined analytical terminology ("5-phases" and "6-phases," "unfurled" $^{6}_{3}$s) is to refer harmonic progressions back to an abstract voice-leading norm. These norms are idealized voice leadings.

Damschroder's Schenker-influenced method explains sequences as linear processes that fill the space between functional harmonies. For the music to substantiate such a description, the sequence need only feature (a) consistently smooth stepwise voice leading, and (b) an identifiably functional harmony at the beginning and at the end. For instance the sequence in Example 9(a) is framed by dominant chords. The relatively smooth voice leading then guarantees the existence of chromatic lines that outline this dominant harmony: B–B♭–A–A♭–G and D–C♯–C–B. The idealization of voice leading, the reference of the real musical pattern to a maximally smooth voice-leading model, allows for flexibility in criterion (a).

[11] This does not mean that a Schenkerian outlook automatically leads to integrationism: one of the earliest proponents of an anti-integrationist position, Proctor (1978), makes his case squarely from a Schenkerian standpoint.

The concept of uniform patterns stakes out something of a middle ground between Damschroder and Cohn on integrationism. It provides a direct comparison between chromatic sequences and diatonic models, suggesting that they are a variation on diatonic habits, rather than a radically different harmonic syntax (*pace* Cohn). However, they also show how chromatic patterns dispense with the mediation of the diatonic scale through its characteristic generic intervals, so that an essential feature of conventional tonality is at least momentarily suspended, a fact that is certainly of deep significance to understanding Schubert's musical language.

Explanations in terms of uniformity also provide more specific and digestible information about the internal mechanics of sequences. The similarity of the sequences in Example 9 and their common reference to the descending 5–6 is not immediately evident from the transformational labels or their identification as linear processes. Generic intervals and chord types go a step further in specifically quantifying essential differences between sequential patterns.

(4) Ramifications of the Main Theme Sequence in Schubert's String Quartet no. 15

The sequence in the main theme of the G major quartet participates in a network of sequential and other chromatic processes throughout the piece. This includes sequences in the transition and the development, and unusual chromaticism in the secondary theme.

4.1 The transition. Schubert uses another chromatic sequence in the transition, an ascending fifths sequence on major triads (Ex. 11(a)). The idealized triadic voice leading for this sequence, (b), is a descending three-voice pattern where the chords go through three positions for each rotation of the sequence. Schubert's sequence repeats after two chords and ascends, but reflects the descending triadic pattern in the third→fifth voice-leading descents, made especially prominent by the striking chromatic passing tones.[12] The three-chord descending pattern is turned into a two-chord ascending pattern by a "reaching-over"

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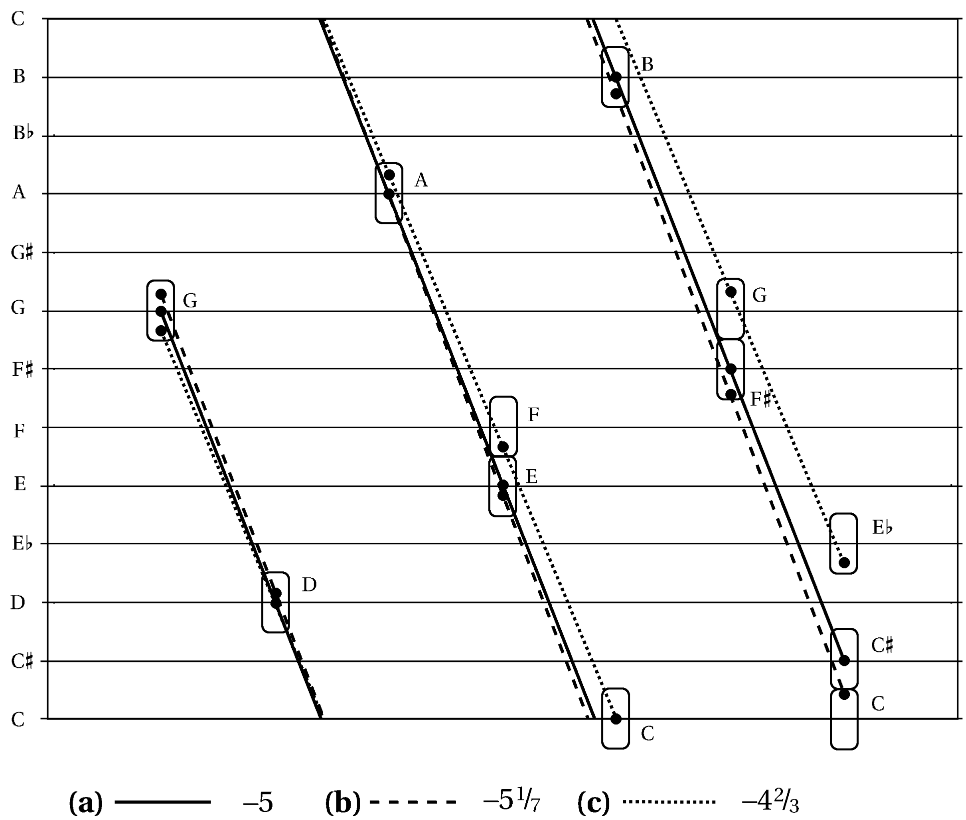
[12] Ascending- and descending-fifths sequences usually have four-voice patterns on triads with doubled roots or seventh chords. These naturally create two-chord sequences by rotating the chord by two positions in each voice leading (dividing the cardinality of the chord in half). The four-voice pattern does not fit this passage, however, because it does not include the third→fifth descents so prominent in Schubert's sequence.

(c). This contrasts with sequences described in the previous sections, which create two-chord patterns on triads through elision.

Example 11: (a) Schubert, String Quartet no. 15, mm. 54–60. (b) The three-voice uniform ascending-fifths progression on triads is a descending three-chord pattern. (c) Schubert's sequence reflects the voice leading of the uniform triadic progression using "reaching over" to create a two-chord ascending sequence.

The transition sequence is *purely uniform*, meaning the chords are all of the exact same type (major triads) and the voice leading is exactly the same at each step: root, third, fifth move by –1, –2, 0 respectively. The sequential voice leading is reproduced in Example 12(a), for comparison to the descending 5–6 and Schubert's sequence with added chords (b)–(c). This reveals a close relationship between Schubert's two sequences: the transition sequence is *another chromatic version of the descending 5–6 progression* (without the elision). Schubert, in fact, explicitly directs us to hear this relationship, because the melodic material for the transition sequence emerges from a liquidation of a varied repeat of the theme. The graph in Example 13 shows how the generic interval of the transition sequence (–5) produces a more gradual departure from the diatonic model ($-5\frac{1}{7}$) than in the main theme sequence ($-4\frac{2}{3}$).

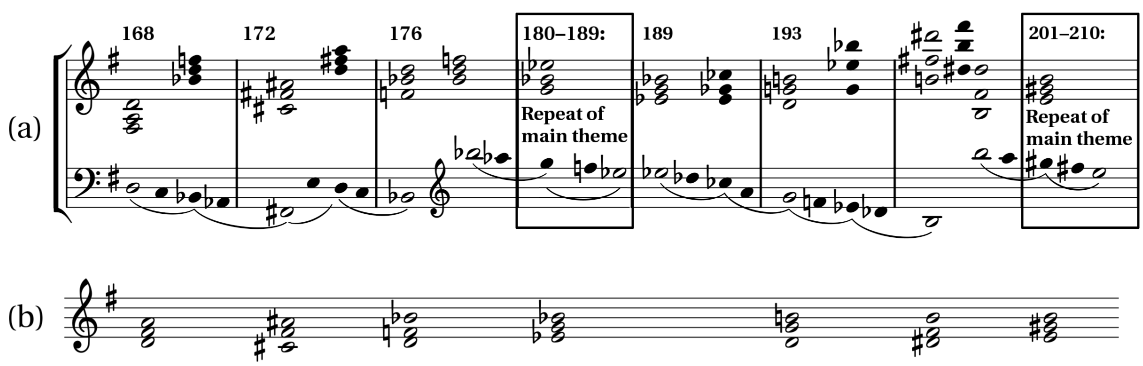
Example 12: (a) The chromatic ascending-fifths progression as a three-chord sequence on triads. (b) The diatonic and (c) Schubert's chromatic descending 5–6 as elided uniform progressions.



Example 13: A graph of root progressions of the sequences shown in Example 12. The slope of each line is proportional to the size of the generic interval.

4.2 Sequence in the development. In the development of the movement, Schubert creates a new thematic idea by distilling the main theme down to the descending whole tone interval that marks the distance between each of its sequential repetitions. He extends this descending whole tone obsessively into a registrally sprawling mechanical melodic descent by whole tone, articulated at each major third by subito tremolo I–V–I progression on the key of the melody note. The new thematic idea is punctuated by reprises of the main theme in E♭ major and E major, giving the effect of floating the theme in a strange and distant cosmos. Example 14(a) summarizes the passage starting from m. 168 where the exposition ends, with a single chord on the upper staff for each I–V–I progression and the outline of the whole-tone melody on the lower staff. The descent is strictly by whole tone up to m. 180, where a single half step (A♭–G) initiates a repeat of the theme in E♭ major. (The full ten-measure theme repeats here, represented by a single chord in the summary). The whole process then repeats a half step higher.

Much of voice leading in the harmonic progression is extravagantly disjunct on the surface. In mm. 168–176 the disjunct pattern helps to articulate every other of the major-third related chords—the more weighted chords in the pattern are shown in open noteheads in Example 14(a). In mm. 189–197 an underlying efficient voice leading is more apparent. Example 14(b) reduces the summary to the weighted chords of the strange new theme and the tonics of the main theme reprises and normalizes the register to produce idealized voice leadings. This is a uniform three-chord progression corresponding to the large-scale sequence (up by half step) that forms the basis of the passage. The root relationships are +4, +4, and +5 semitones, which averages to a generic interval of $+4\frac{1}{3}$.



Example 14: (a) A summary of the beginning of the development of Schubert's String Quartet no. 15, first movement. The melodic line on the lower staff begins in the cello but passes between instruments starting just before the first repeat of the main theme. (b) A uniform three-chord progression underlying this large-scale sequence, in idealized voice leading.

Comparing the sequence of Example 14 to those of Example 12 we find that the uniform progressions of the piece distort the basic interval of the diatonic scale, the generic fifth or fourth, in three ways, ranging from mild to extreme. The transition simply eliminates the augmented fourth / diminished fifth from the diatonic circle so that it spills out into the full chromatic. This represents a small shift in generic interval, $5\frac{1}{7} \rightarrow 5$, making the transition sequence locally similar to a diatonic sequence despite its inherently chromatic logic. The greater distortion of $5\frac{1}{7} \rightarrow 4\frac{2}{3}$ which produces the pattern of the main theme represents the "original sin" of the piece, where the fourth changes from something with perfect and augmented varieties to perfect and diminished varieties. The development exaggerates the effect by taking the interval of the main theme sequence and stretching it further away from diatonicity, to the breaking point where the fourth is *more likely* to be diminished than perfect, $4\frac{2}{3} \rightarrow 4\frac{1}{3}$.[13]

The next logical step would be the point at which the diatonic system, after having expanded into the chromatic like a supermassive star, collapses into itself creating the black hole of the hexatonic system—or, in other words, $4\frac{1}{3} \rightarrow 4$ (see Ex. 15). The uniform progression would then become one of Cohn's (2000) "toggling cycles," a voice leading on major triads whose elements, (–1, 0, 1), sum to zero, making overall progress upward or downward in pitch space impossible. This last step is intimated by Schubert's strange development theme: focusing on just the theme itself (e.g., the first five chords of Ex. 14(a)) the progression is a continual transposition down by 4 semitones, which will, of course, endlessly repeat the same three chords if it is not interrupted. Schubert's development thus

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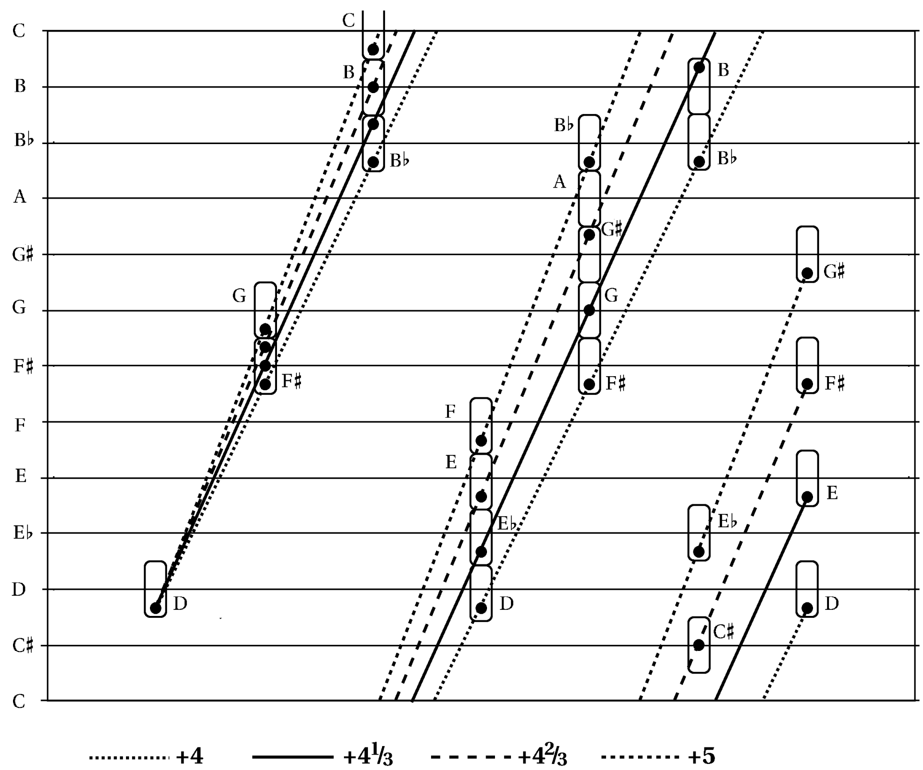
[13] This extension of conventional intervallic nomenclature relies upon the fact that it is systematic in a way demonstrated by Douthett and Hook's (2009) mathematical derivation of it. Their use of quantization is related to its use to define generic intervals here, and can be reframed in the present context as follows: "major" and "minor" intervals are those that result from rounding 7-equal intervals up or down, respectively, by a distance between $\frac{1}{4}$ and $\frac{3}{4}$, while perfect intervals are rounded $< \frac{1}{4}$, and diminished or augmented intervals $> \frac{3}{4}$. The same criteria can be applied to the sum of some alteration of a 7-equal value (my "distortions") plus the rounding error. A sum larger than $1\frac{3}{4}$, is then doubly augmented or doubly diminished, and so forth. In the present case, we have an analytical reason for treating the $4\frac{1}{3}$ generic interval as a distorted fourth, even though it is not much further from the 7-equal third ($3\frac{3}{7}$) than from the 7-equal fourth, and might in other circumstances be considered as an interval that varies between major and augmented thirds.

brings tonality to the brink of annihilation, the tonal framework of the main theme interjections playing the role of *deus ex machina*.



Example 15: A graph of the root progression of Example 13(b) as produced by the generic interval $+4^{1}/_{3}$ and three other hypothetical chromatic root progressions based on alternate values of the generic interval (+5, $+4^{2}/_{3}$, and +4).

4.3 Generic intervals beyond sequential contexts: the secondary theme. While sequences are the main topic of this paper, secondary theme group of Schubert's string quartet affords an opportunity to explore how one may apply the concept of generic interval in a broader context while expanding upon some of the analytical observations already made above.

In sequences, we can reverse-engineer a quantization to obtain generic intervals because the regularity of the sequence enables us to clearly determine what intervals should be the same in the underlying pattern. In non-sequential contexts, other kinds of grounds are required to make this kind of inference. Yet we regularly, almost reflexively, make such inferences of generic interval sizes in standard Roman-numeral–type harmonic analysis. We do so by implicitly referring back to a template of uniform diatonic patterns that may be realized only in a very fragmentary way on the musical surface. Inferring non-diatonic generic intervals in non-sequential contexts, then, is a matter of generalizing some of these procedures to chromatic templates appropriate to the harmonic language of the work at hand. In Schubert's string quartet, the sequences of the main theme and transition provide a guide for interpreting the unusual harmonic features of the secondary theme group.

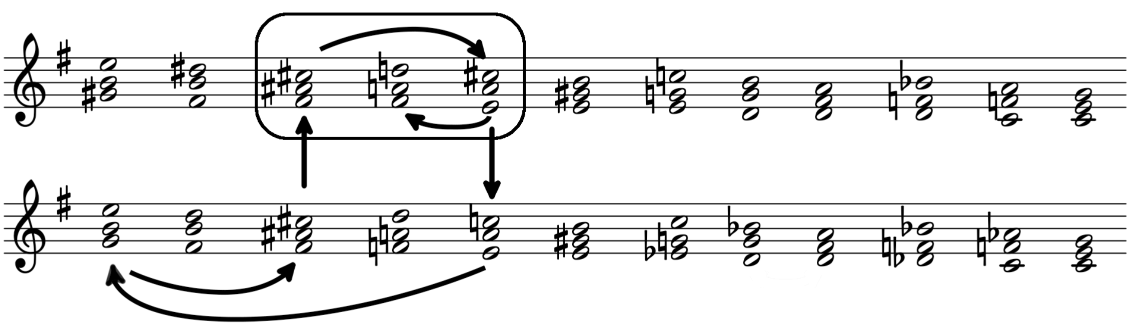
Schubert's modulation to the dominant in the exposition of the movement is strikingly indirect. A clear rhetorical medial caesura ends the transition in m. 63, but rather than arriving on the dominant of D major here Schubert emphatically arrives on an F♯ major triad, which has no obvious relation to either of the primary keys of the exposition. He then begins his secondary theme (Ex. 16) with the F♯ major triad moving directly to V$^4_3$ of D major, as if it had an obvious function in the key. He repeats this progression throughout the theme, never intimating that F♯ major might have any kind of conventional secondary function.

Example 16: The secondary theme of Schubert's String Quartet no. 15, mm. 65–75.

A closer look at the voice leading reveals that Schubert uses the same triadic voice leading in the F♯ major – A major progression (disregarding the voice that goes to the seventh of the V$^4_3$) as in the characteristic progression of the main theme sequence from D major to F major and C major to E♭ major. If the main theme sequence is the distortion of a diatonic descending 5–6, produced by contracting the generic interval of $-5\tfrac{1}{7}$ to $-4\tfrac{2}{3}$, the secondary theme distorts an up-by-step diatonic progression (like IV–V) by replacing the generic interval $1\tfrac{5}{7}$ ($= -5\tfrac{1}{7} - 5\tfrac{1}{7}$) with $2\tfrac{2}{3}$ ($= -4\tfrac{2}{3} - 4\tfrac{2}{3}$).

Ordinary diatonic functions could be thought of as positions in a kind of abstract "pre-compositional" set of uniform progressions based on root motion by $-5\tfrac{1}{7}$. Example 17 shows two such progressions on a pure major triad chord-type (representative of secondary functions) and a 7-equal chord type (giving diatonic functions). Typical tonal progressions that are not necessarily uniform will nonetheless trace relatively simple trajectories within these idealized voice-leading patterns. The "chromatic functionality" of Schubert's secondary theme can be understood analogously as trajectories in abstract uniform progressions based on generic interval of the main theme sequence, $-4\frac{2}{3}$, instead of $-5\frac{1}{7}$. Example 18 plots the characteristic progressions of the secondary theme in two uniform progressions analogous to those in Example 17, on pure major triad and ($0\ 3\frac{1}{3}\ 7$) chord types (the latter being a 36-equal approximation to a 7-equal triad).

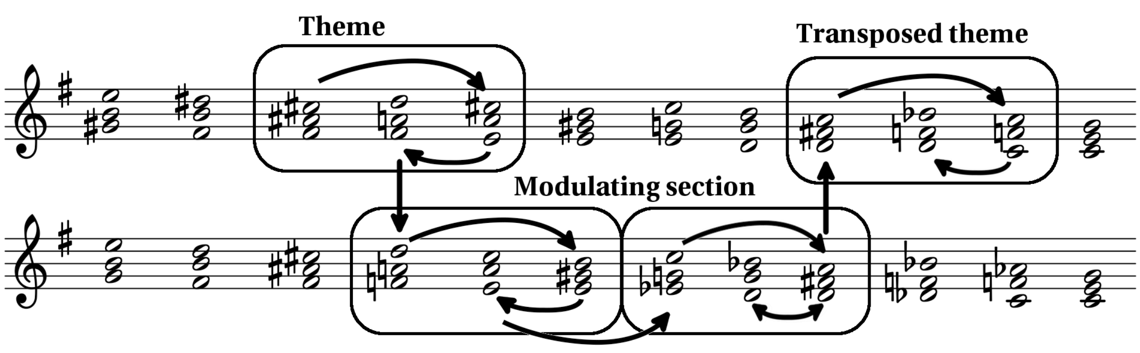
Example 17: Diatonic functions arranged as uniform progressions by $-5\frac{1}{7}$. The top staff reflects the pure major triad chord type (0 4 7), and the bottom staff reflects the 7-equal chord type ($0\ 3\frac{3}{7}\ 6\frac{6}{7}$).



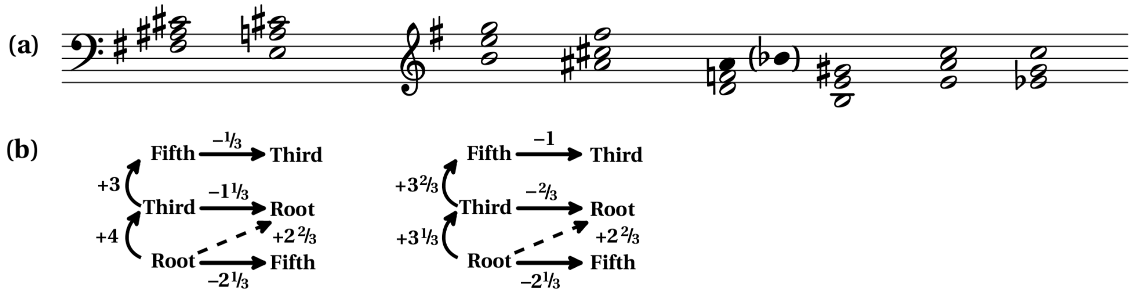
Example 18: Uniform voice-leading patterns based on $-4\frac{2}{3}$ transpositions between (0 4 7) and ($0\ 3\frac{1}{3}\ 7$) chord types, and the harmonic progression of Schubert's secondary theme as a pathway within these.

Such reasoning can inform a harmonic analysis of the entire secondary theme group. Example 19 shows the modulating part of the group, which leads to a transposed repetition of the secondary theme in B♭ major. A summary of this harmonic process charts the trajectory shown in Example 20. The crucial voice leadings throughout the process, four of which are shown in Example 21, are those that move left to right in the $-4\frac{2}{3}$ uniform progressions skipping over one position, like the F♯ major – A major progression that initiates the theme. These all share the root motion by $-4\frac{2}{3} - 4\frac{2}{3} = 2\frac{2}{3}$ (mod 12) and consist of two possible generic voice-leading patterns (b), depending on the chord type.

Example 19: The modulating part of Schubert's secondary theme group



Example 20: Uniform voice-leading patterns based on $-4\frac{2}{3}$ transpositions on (0 4 7) and (0 $3\frac{1}{3}$ 7) chord types, and the essential harmony of Schubert's secondary theme group as a pathway within these.



Example 21: (a) Four significant voice leadings from Schubert's secondary theme group, and (b) the generic intervals of their underlying uniform patterns, according to the interpretation in Example 20

(5) Between Diatonic Interpretations and Chromatic Uniformity

The applicability of conventional Roman numeral analysis to chromatic sequences constitutes a baseline for integrationism, a minimal requirement for claiming that principles of Classical tonal harmony are operative. Translating Roman numerals into diatonic interpretations (as described in section two above) quantifies the central feature of the technique, the assertion of diatonic contexts underlying harmonic patterns. By doing so, we can better evaluate the applicability of Roman numerals to chromatic sequences. The quantitative aspect of generic intervals also enables the direct comparison of diatonic-based accounts with explanations from chromatic uniformity.

Because Roman numeral analysis explains chromatic progressions by referring them to diatonic models, it typically "factors" progressions into two components: (1) a diatonic root progression, which can be represented by a 7-equal generic interval and (2) a change of key, which corresponds to a microtransposition. The latter are "residuals" in the sense that they are the "error" of the diatonic interpretation, the part that is left over after the diatonic part is factored out. Two kinds of residual microtransposition are especially characteristic of Roman-numeral interpretations:

(1) Microtransposition by $\pm{}^{2}/_{7}$ is the maximal distance for a simple triadic *pivot*: major triads only occur when the root has a fractional part of ${}^{1}/_{7}$, ${}^{2}/_{7}$, or ${}^{3}/_{7}$. For instance, an F major triad as V (in B♭ major) has a fractional part of ${}^{1}/_{7}$, but as VI (in A minor) it is ${}^{3}/_{7}$. Therefore, this reinterpretation corresponds to a shift of the triad

(and the implied diatonic framework) by a "pivot interval"[14] of $+\frac{2}{7}$. A larger microtransposition would change at least one note of the triad.

(2) Transposition by $\pm\frac{3}{7}$ represents *mode change*, because it is the interval between major and natural minor scales on the same tonic, and is also the smallest interval that maps all major triads onto minor triads, or vice versa. In this sense it is the directed version of the neo-Riemannian *Parallel* transformation—i.e., $+\frac{3}{7}$ changes any minor triad to major, and $-\frac{3}{7}$ any major to minor, on the same root.

Roman numeral analysis is native to eighteenth-century practice, where pivots and mode changes may generally be applied sparingly and used to relate keys of formal significance. In chromatic sequences, these microtranspositions must be applied simply to satisfy the requirements of the theoretical apparatus, implying keys whose reality is not evident musical fact. Nevertheless, Roman-numeral accounts of chromatic progressions provide a sense of continuity with earlier tonal practice by referring these progressions back to diatonic models, which can be revealed by removing the "residuals," the microtranspositions, leaving behind a purely diatonic uniform progression.

Chromatic uniform progressions explain chromatic sequences differently, without necessarily invoking diatonic models and without imputing imaginary transitory keys. They do not, however, preclude reference to a diatonic model, because they can be compared to "nearby" diatonic models by means of distances between generic intervals. In other words, the two methods relate chromatic sequences to diatonic models differently: where Roman numeral analysis modifies the diatonic model by adding in discrete key changes, chromatic uniform progressions stretch or compress intervals of the diatonic model uniformly. Regardless of which kind of analysis seems preferable in a given instance, distances between generic intervals allow for precise comparisons between them and clarify the mechanics of each.

The excerpt in Example 22, from one of Liszt's piano etudes, uses a familiar enharmonic trick, reinterpreting dominant sevenths as augmented sixths, to create a chromatic sequence that descends by semitone. This explanation implies a Roman numeral

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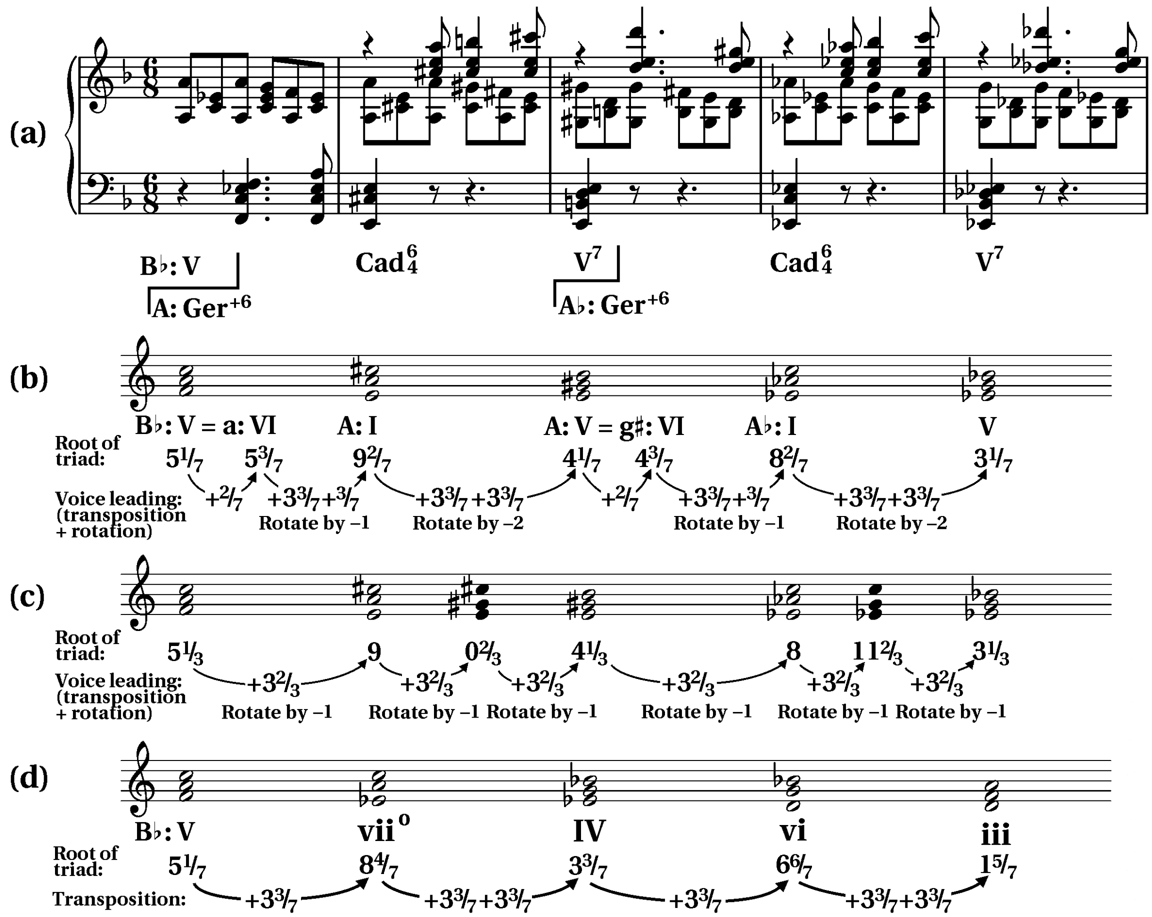
[14] I borrow this term from Rings (2011), although microtransposition works differently than Rings' tonal pitch-class group, as explained in footnote 8 (§2.2) above.

analysis on the triadic skeleton of the progression, shown in (b).[15] Each Roman numeral corresponds to a value representing the diatonic interpretation of its root (this determines the entire triad, which is assumed to be 7-equal). Chords belonging to minor keys (other than dominants) adopt the same value they would have in the relative major. Voice leadings are indicated as a transposition + rotation. (Rotation by –1 is root→fifth, while rotation by –2 is root→third.[16]) The resolution of each augmented sixth involves three components: first, "V" (which refers to a root whose fractional part is $\frac{1}{7}$) is reinterpreted as "VI" (a root whose fractional part is $\frac{3}{7}$—equivalent to "IV") by means of a $+\frac{2}{7}$ pivot. The root motion is then by diatonic third (produced by $+3\frac{3}{7}$), but also simultaneously shifts to the parallel major (because the cadential $^{6}_{4}$ is major quality) requiring the $+\frac{3}{7}$ microtransposition. The resolution of the cadential $^{6}_{4}$ is a diatonic fifth progression, which is the sum of two thirds: $+3\frac{3}{7} + 3\frac{3}{7} = +6\frac{6}{7}$. Note that each third-progression (transposition by $+3\frac{3}{7}$) is accompanied by a –1 rotation, while microtranspositions occur independently of rotations.

The triadic voice leading in Example 22 is also an elided uniform progression with chord type ($0\ 3\frac{2}{3}\ 7$) and transposition $+3\frac{2}{3}$, as shown in (c). Both of these can be compared to the diatonic model, (d), which is derived from (b) by removing the "residual" microtranspositions, and related to (c) as the nearest diatonic approximation. As a chromaticization of the diatonic model, (d), Liszt's sequence reduces the sequential interval of a descending diatonic step, of generic size $-1\frac{5}{7}$, to a pure semitone, –1, a difference of $\frac{5}{7}$. In (b), this difference is divvied up into two tonally interpretable pieces, the pivot interval of $\frac{2}{7}$ and the mode shift of $\frac{3}{7}$, which are then distributed through the progression. In (c), the $\frac{5}{7}$ discrepancy is evenly distributed through the three stages of the underlying uniform progression ($3\frac{3}{7} + (\frac{5}{7})/3 = 3\frac{2}{3}$). In other words, (c) presents the chromaticism as a small and continuously accumulating difference from pure diatonicity, while (b) presents it as discrete shifts of key.

[15] The usual scruples about calling the cadential $^{6}_{4}$ a "I" do not pertain here, since we are labeling voice-leading patterns and making no specific claims about function.

[16] These are expressed as negative rotations to reflect the fact that they cancel out the positive transpositions. See §3.1 above.



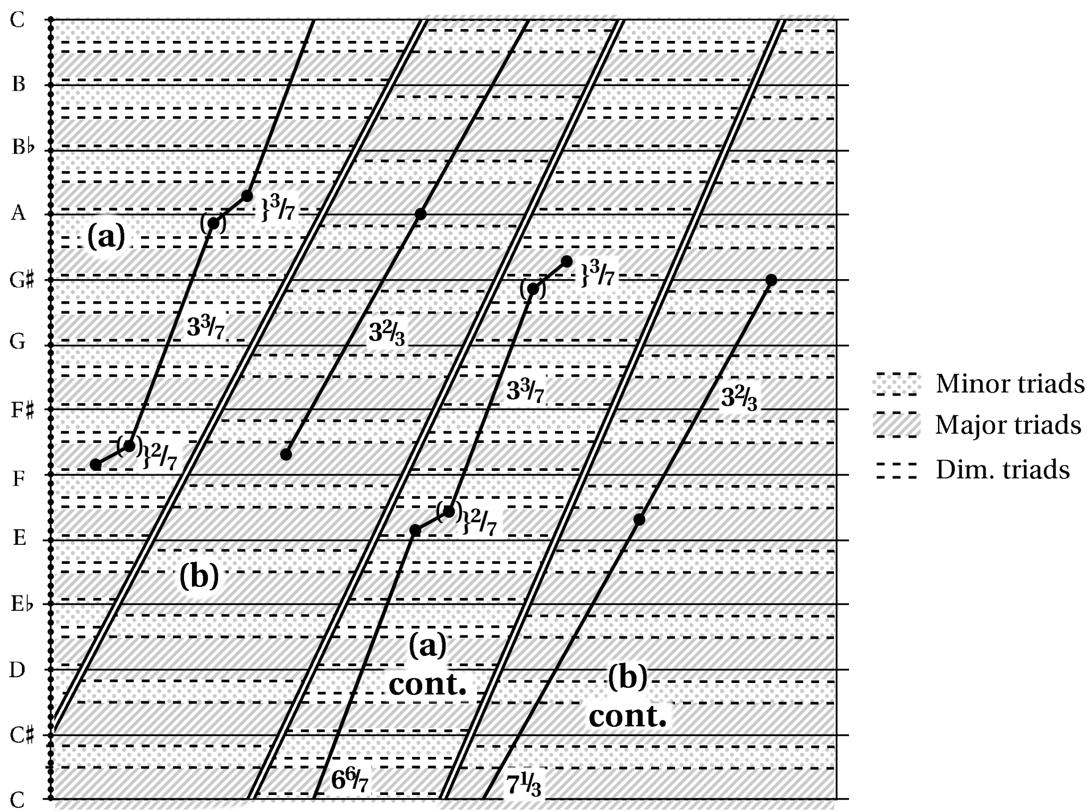
Example 22: (a) A sequence from no. 3 of Liszt's Transcendental Etudes*, "Paysage," mm. 67–71, with Roman numeral analysis. (b) The underlying triadic voice-leading pattern with diatonic interpretation values implied by the Roman numerals, (c) the triadic progression as an elided uniform chromatic sequence on (0 $3^2/_3$ 7) triads, and (d) the diatonic sequence that results from removing residuals from (b).*

It would be accurate to say that, according to the analysis in (c), the key drifts around the circle of fifths at a consistent rate of –$1^2/_3$ tics per chord in the underlying uniform pattern, adding up to –5 tics for each two measures.[17] Quantization thus realizes a suggestion

[17] This metaphor calls to mind the filtered symmetry approach of Douthett 2008, which is closely related to the approach to sequence advocated here in its grounding in the mathematical principles of maximal evenness. The most important difference in Douthett's approach is his use of higher order maximal evenness, to derive, e.g., 7-equal triads. In the present approach I instead allow for free definition of generic chord types.

made by Daniel Harrison (2002) with respect to enharmonicism, that chromatic shifts can be thought of as accumulating over the course of a progression in order to avoid discrete localization. Harrison's example, a trisection of the octave into major thirds in Liszt's first Piano Concerto, is in fact an instance of a uniform sequence. In the context of Harrison's article, it appears that the passage is exceptional in its explanatory demands because it includes an even division of the octave. However, other chromatic intervals can be evenly divided in essentially the same way to create a uniform progression: in this case, 11 semitones divided by three (= $3^2/_3$).

Interpretations (b) and (c) are graphed in Example 23(a) and (b) for comparison, showing how the chromatic interpretation averages the individual pieces of the diatonic interpretation into a single regular root progression.



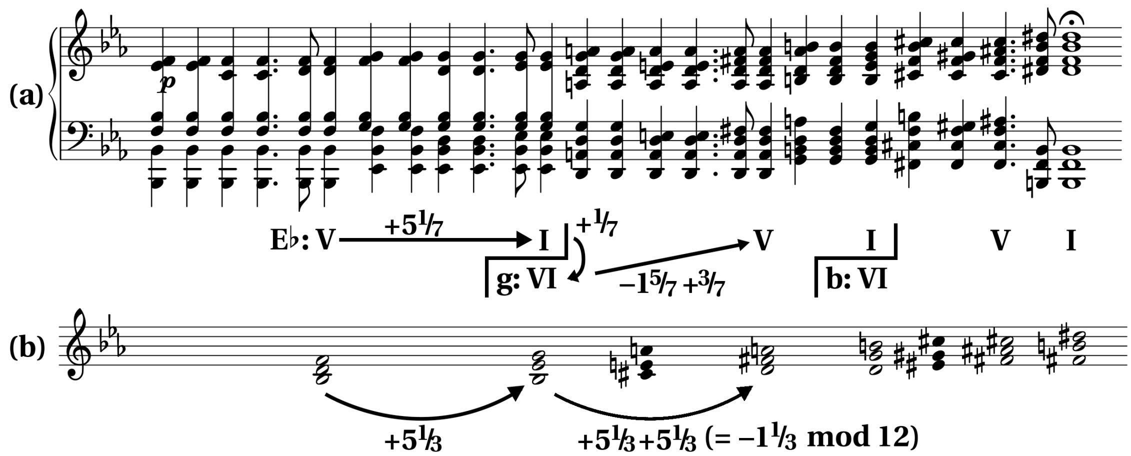
Example 23: The root progressions for the (a) diatonic and (b) chromatic interpretations of the sequence in Example 22. Shading of regions indicates the chord type that results from the given position of the root. These regions reflect the chord types (0 $3^3/_7$ $6^6/_7$) for (a) and (0 $3^2/_3$ 7) for (b). Points in parentheses in (a) refer to intermediate states after the pivot interval and before the mode change interval.

Example 24(a) gives another sequence from one of Liszt's Etudes for comparison to the previous example, with diatonic interpretation (b), chromatic uniform progression (c), and diatonic model (d), as in the previous example. On the surface, the similarity of the progressions is not apparent. Using Cohn's (2012) approach, for instance, the sequence in Example 22 consists of compound transformations LP–LR, while the description of the sequence in Example 24 is more succinct, N–L, and making it a type Cohn discusses explicitly (94–95).[18] However, since both are chromatic sequences by semitone, the difference from the diatonic model is the same: $^5/_7$. As a consequence, the Roman numeral analysis uses the same division of $^5/_7$ into a mode change of $^3/_7$ and a pivot interval of $^2/_7$. These go in the opposite direction in 24(b) because it is an ascending sequence, so that the $-^3/_7$ shift can be accomplished by the ordinary mode mixture on the dominant of the minor key. The pivot is the same as in Example 22 in the opposite direction. The diatonic model in Example 24(d) (ascending 5–6) is the retrograde of the type in Example 22(d). The difference between the two as uniform patterns (22(c) and 24(c)), other than the retrograde, is the position of the elided chord. (The same kind of difference illustrated in Example 9(b)–(c).) The generic interval of $\pm 3^2/_3$ is the same.

An example from Liszt's *Harmonies Poétiques et Religieuses* no. 4 (Ex. 25) shows how changing the V→VI pivot to I→VI reduces the pivot interval from $^2/_7$ to $^1/_7$, creating an overall shift of $^4/_7$ per sequential unit, the difference between major third–related keys. The underlying diatonic model for this sequence is the retrograde of the descending 5–6, chromaticized to have *larger* average voice leadings than the diatonic model ($4 > 3^3/_7$) unlike the chromatic sequences from Schubert's String Quartet no. 15 discussed above (e.g., Example 12(a) and (c) which move 2 semitones and 3 semitones per sequential unit). The uniform progression (b) is based on the generic interval of $+5^1/_3$, which distributes the $^4/_7$ residual between the three $+5^1/_7$s of the diatonic retrograde descending 5–6 ($5^1/_7 + (^4/_7)/3 = 5^1/_3$).

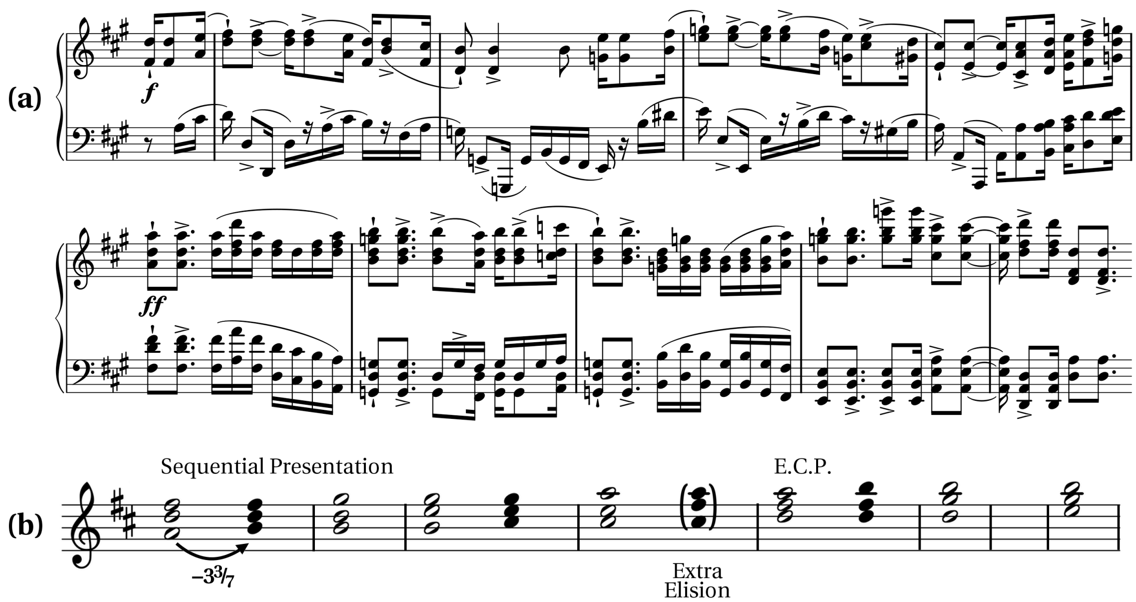
[18] "N" stands for *"Nebenverwandt,"* the relationship of a minor triad and its dominant. This was not treated as a basic transformation in earlier neo-Riemannian theory (e.g., Hyer 1995; Cohn 1997; Gollin 2000, 251–281), but is a fundamental transformation in Cohn 2000 and 2012 because of its status as a "Weitzman-group" operation.

Example 24: (a) A sequential passage from no. 11 of Liszt's Transcendental Etudes*, "Harmonies du Soir," mm. 88–94. (b) The underlying triadic voice-leading pattern with Roman numeral analysis and implied diatonic interpretation values, (c) the triadic progression as an elided uniform chromatic sequence, and (d) the diatonic sequence that results from removing pivot and mode-change intervals from (b).*



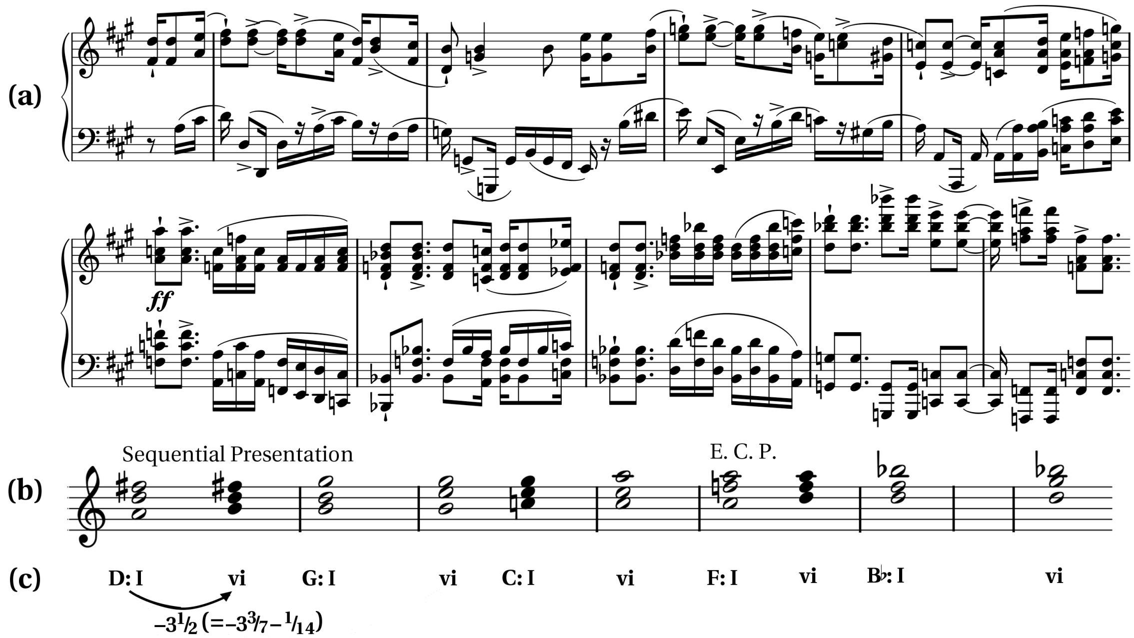
Example 25: (a) A sequential passage from no. 4 of Liszt's Harmonies Poétiques et Religieuses, *S.173, "Pensée du morts," m. 66, with Roman numerals and diatonic interpretation values. (b) The triadic voice-leading pattern as a uniform progression.*

The modeling of a uniform chromatic progression on a familiar diatonic pattern can sometimes be explicitly realized by the composer, as in the following example from the second movement of Schubert's "Gasteiner" Sonata (D. 850). The movement is a five-part double variations. The A part of the second theme (mm. 42–50, Ex. 26(a)) uses an ascending sequence for its presentation (mm. 42–45). The continuation (mm. 46–50) is an expanded cadential progression in which an insistent extra measure of subdominant extends the phrase to five measures. The sequential presentation and expanded cadential progression, though separate in function, can be combined in a single uniform voice-leading pattern, as shown in Example 26(b). Schubert approaches the inverted tonic that begins the cadential progression (m. 46) as if the two-measure sequence will repeat a third time, but replaces the anticipated F♯ minor (iii) chord with a first-inversion tonic. This is an extra elision that shifts the cadential chords one place to the right in the uniform voice-leading pattern.



Example 26: (a) The second theme (A part) from Schubert Piano Sonata D.850, "Gasteiner," second movement, mm. 42–50. (b) The sequential presentation (mm. 42–45) is based on a diatonic uniform voice-leading pattern, and expanded cadential progression (mm. 46–50) continues this pattern with an extra elision.

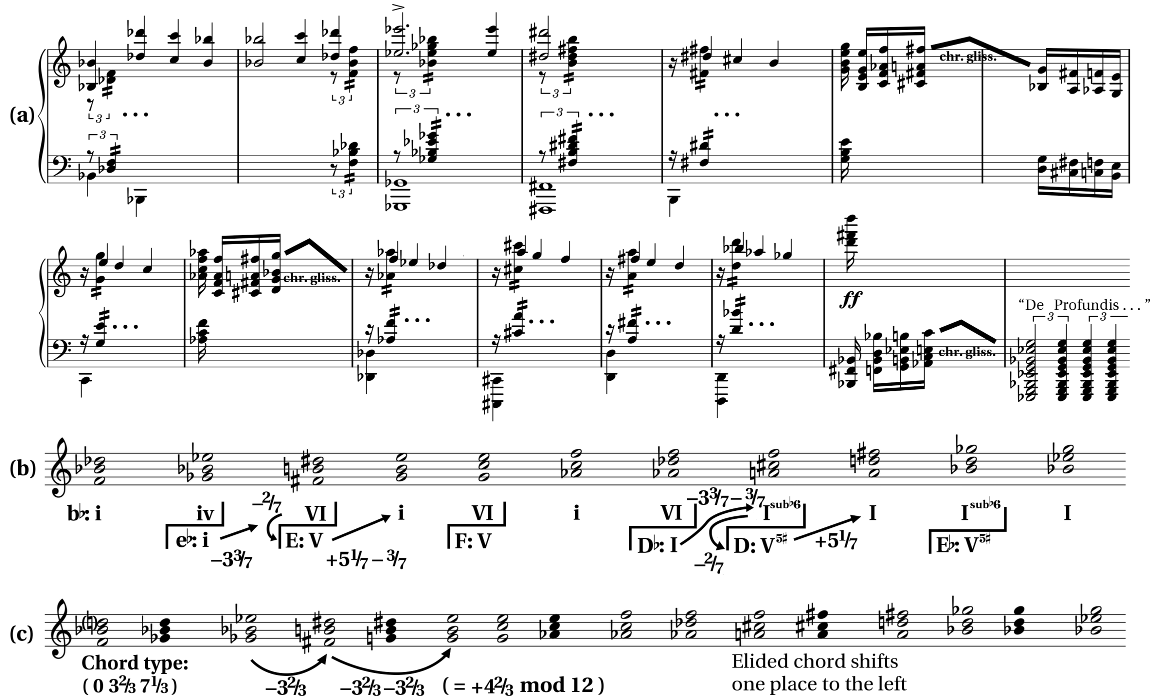
The entire voice-leading pattern in Example 26(b) is diatonic in D major, a representative of the diatonic ascending 5–6 pattern. It can therefore be generated via quantization of the 7-equal triad, $(0\ 3\frac{3}{7}\ 6\frac{6}{7})$, under transposition by $3\frac{3}{7}$. Schubert makes an ingenious modification of the pattern in the dissolving recapitulation ("A''", mm. 68–85) in order to destabilize the key for retransition to the first theme (Ex. 27(a)). In the last measure of the presentation (m. 71) he changes the mode of the dominant chord, using C♮ in place of C♯. The continuation then begins on F major instead of D major $^{6}_{3}$. This reflects two changes to the voice-leading pattern (b): first, Schubert shifts the diatonic set two accidentals flatward over the course of the sequential presentation; second, he continues the pattern of elisions faithfully into the expanded cadential progression (so that there is no extra elision). The cadential progression then proceeds as before, but starting from an F major tonic, so that the entire progression is a third higher and three accidentals flatward.



Example 27: (a) The dissolving recapitulation of the second theme from Schubert's Piano Sonata in D major "Gasteiner," second movement, mm. 68–80. (b) The passage is based on a chromaticized version of the uniform voice-leading pattern from the A part. (c) The chromaticization is a gradual flatward drift in the diatonic collection. A value of $^1/_{14}$ taken from the 7-equal transposition value results in a gradual shift around the circle of fifths that accumulates to a one-accidental change ($^1/_7$) per two chords.

What Schubert has actually done in the dissolving recapitulation is to chromatically regularize the voice-leading pattern of the theme. The purely diatonic pattern is generated from descending-thirds root motions of $3\frac{3}{7}$ semitones. A root motion of $-3\frac{1}{2}$ generates a perfectly regular chromatic pattern that alternates minor and major thirds, the equivalent of a Med sequence (using Lewin's (1987) label—see also Kopp 2002). Since $3\frac{1}{2}$ is just $\frac{1}{14}$ larger than the 7-equal value $3\frac{3}{7}$, we can think of the transpositional operator as "down a third minus half a tic on the circle of fifths," moving the key down a fifth for every descending fifth in the pattern of roots (c).

A final example from Liszt's "Pensée du morts" (Ex. 28(a)) shows even more strongly how an explanation from chromatic uniformity can be much more economical than Roman numerals, without sacrificing the ability to compare the sequence to diatonic models. In the piece, Liszt expands the sequence to great lengths with grandiose flair in virtuosic piano tremolos and glissandos, setting up a melodramatic instrumental recitation of the *De Profundis* with a great climactic crescendo. (The excerpt in Example 25 above follows this recitation.)



Example 28: (a) A summary of a passage from no. 4 of Liszt's Harmonies Poétiques et Religieuses, *S.173, "Pensée du morts," mm. 44–58. (b) The underlying triadic voice-leading pattern with values implied by a diatonic interpretation of the triadic Roman numerals, (c) the values for the triadic progression as an elided uniform chromatic sequence.*

The passage appears to comprise two different sequences when viewed in Roman-numeral terms (b), but actually involves a single uniform progression (c), in which the change in the sequence is simply a shift in the pattern of elisions. The chord type for the pattern quantizes to major, minor, and augmented triads, but in the first part of the progression the augmented triads are elided, producing the same sequence as in Example 24 (Cohn's (2012) N–L pattern). The shift occurs where Liszt realizes the missing augmented triad in m. 54 in place of the minor triad from the established pattern, a moment that also coincides with a fragmentation in the textural pattern. The diatonic interpretations have to allow for a change of chord type in order to account for the augmented triads, which mix elements from different diatonic collections—I and ♭VI, or V and V/iii. Nonetheless we can still trace the same kind of diatonic rationale, albeit somewhat more delicately, within the enharmonic reinterpretation of the augmented triad: a mode change ($-\tfrac{3}{7}$) relates I to the substitute $\flat\hat{6}$, the pivot from $\flat\hat{6}$ to $\hat{5}$ creates a $-\tfrac{2}{7}$ residual. With the residuals (and variations in chord type) removed, the progression reduces to an ascending 5–6 ($-3\tfrac{3}{7}$, $+5\tfrac{1}{7}$).

The chromatic description of the passage, (c), avoids the complication of multiple chord types by defining a single type that varies between major, minor, and augmented under quantization, showing that the progression obeys a chromatic logic not amenable to diatonic interpretation. Nonetheless, the model of the ascending 5–6 can be seen behind interpretation (c), in that the generic interval of $+3\frac{2}{3}$ approximates the 7-equal generic interval of $+3\frac{3}{7}$, just as in Example 24(c).

Conclusion

The traditional intuition of music analysts towards the harmonic styles of the nineteenth century has been to enter the surgical room armed with tools sharpened on the music of Haydn and Mozart, in recognition of continuity evident in the course of the century. The efforts of transformational theory to problematize the universal applicability of these tools is laudable, but has left behind a paradox: two incommensurable systems of harmonic analysis coexisting in the same music. A goal of the present study has been to preserve the insights of recent theory on the inherently chromatic organization of many passages in the nineteenth century repertoire, while describing this inherently chromatic organization in a way that places chromatic and diatonic explanations in the same arena, giving them an opportunity to interact. The essential theoretic resources for making this interaction possible is the robust concept of distance built into the concept of generic interval. The comparison of chromatic uniform patterns to diatonic models, especially in §5, shows how generic intervals greatly expand the concept of voice-leading distances and its potential analytical implications.

Uniform sequential patterns have served as an experimental population for the claims of quantization as an analytic tool. While the focus on sequence certainly furnishes a playing field uniquely hospitable to the present approach, the fact that a theory of generic intervals can assimilate the underlying principles of Roman numeral analysis in the form of diatonic interpretations implicitly sketches a conceptual route to a more general application of uniformity and generic interval. The analysis of the second theme of Schubert's String Quartet no. 15 (§4.3) has laid some groundwork in this direction.

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