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Harmonic Qualities in Debussy's "Les sons et les parfums tournent dans l'air du soir"

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This analysis of the fourth of Debussy's *Préludes* book I illustrates his use of harmonic qualities in the sense of Ian Quinn: coefficients of the discrete Fourier transform (DFT) on pitch-class sets. The principal activity of the piece occurs in the fourth and fifth coefficients, the octatonic and diatonic qualities, respectively. The development of harmonic ideas can therefore be mapped out in a two-dimensional octatonic/diatonic phase space. Whole-tone material, representative of the sixth coefficient of the DFT, also plays an important role. I discuss Debussy's motivic work, how features of tonality—diatonicity and harmonic function—relate to his musical language, and the significance of perfectly balanced set classes, a special case of nil DFT coefficients.

Keywords: Discrete Fourier Transform; Debussy; harmonic qualities; diatonicity; octatonicity; whole-tone collection; perfect balance; harmonic function

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A growing body of recent research is showing an increasingly wide range of applications for the DFT on pitch-class sets, pitch-class distributions, and beat-class sets in music theory. Many of these developments are catalogued in Emmanuel Amiot's recent book (Amiot 2016), which also provides an excellent mathematical introduction to the topic that gives a sense of why the DFT provides such powerful tools for understanding harmony, rhythm, and other aspects of music. They also include the work of Andrew Milne and others on perfect balance, some of which is represented in the present issue (Milne, Bulger, and Herff 2017). I myself have found the DFT especially valuable as a framework for harmonic analysis (Yust 2015a,b, 2016). The method can not only synthesize theoretical approaches to tonal harmony, but to harmonic techniques of diverse non-tonal composers as well. As the following analysis of a piano prelude by Debussy illustrates, this kind of breadth is necessary for any analytical method that hopes to apply in a holistic way to the music of Debussy and other composers like him: the harmonic universes of these early non-tonal composers are not apart from tonality but rather encompass it.

1. "Harmonie du soir"

The title of the fourth piece from Debussy's *Préludes* book I, "Les sons et les parfums tournent dans l'air du soir," is a line from Baudelaire's "Harmonie du soir" from *Fleurs du*

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Mal. Debussy would have been intimately familiar with the poem, having set it decades earlier in his *Cinq Poëmes de Charles Baudelaire*. The "sounds" and "perfumes" of the quote are depicted as dancing a "mournful waltz" with "languid vertigo": hence the 3/4 time (waltz time) punctuated by the occasional languid 5/4 measure. The sounds are the music of a violin and the perfumes are the effluence of flowers, which Baudelaire likens to incense for a nightly quasi-religious ritual of remembrance of a deceased lover. The poem culminates with an image of the assembled recollections of the lover glowing on a monstrance, the often elaborately gilded holder used by Catholic churches to display the Eucharist or holy relics.

The line that Debussy chooses for the piece's title highlights the potential metaphorical associations charateristic of what is often referred to as his "impressionist" style. The floral effusions might be likened to different kinds of resonances of a low pedal tone, A. I will liken different sorts of harmonic qualities associated with this pedal tone as resonances of this kind, seeming to emanate from it. As they turn freely in the air, we imagine seeing them from different angles, against different backgrounds. The dancing partners, the sounds and perfumes, invite comparison to Debussy's motivic work, in which distinct motives twirl around one another in different formations. Finally, the reverant tone that ends the prelude, which unites the harmonic elements of Debussy's motives in a single harmonious sound pointing towards eternity, may derive from the similarly worshipful conclusion of Baudelaire's poem.

2. Diatonicity

The status of tonality is a critical issue in Debussy's music of this period. (He wrote the *Préludes* between 1910 and 1913.) Theorists typically regard Debussy's later music as non-tonal, but, unlike contemporaries such as Schoenberg and Webern, there does not seem to be a discrete break with or a self-conscious rejection of tonality. Tonal elements—key signatures, triads, seventh chords, scales—remain in play, but the music seems to be purified of any vestige of the teleological drive associated with tonality, which had been identifiable as a post-Wagnerian strain in Debussy's earlier music. The metaphor of tonal function is often invoked to describe that teleology lacking in this music. As Mark DeVoto puts it: "When we say that Debussy's characteristic harmony is often independent of its tonal function, [. . .] we mean that he chooses a harmony first and foremost for its value as sound and sonority. [. . .] It is the non-functional dominant that is an immediately recognizable signal of Debussy's harmony" (DeVoto 2003, 188–89). The appeal to sound, sonority, or color, however, so common in Debussy criticism, is frustratingly vague. They tend to act as blank placeholders for the withdrawn concept of function, with its rich network of associations, onto which any fancy of the listener may presumably be projected.

At the same time, DeVoto's singling out of the dominant-seventh sonority may be prescient. Certainly sonorities built from dominant sevenths are ubiquitous at the surface of Debussy's music, but this harmony also may also have a deeper significance, as revealed when we consider its *qualities* as defined by Ian Quinn 2006. These are obtained by taking the DFT of the characteristic function (indicator vector) for the pitch-class set of a dominant seventh, $(1, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 0)$ or any of its rotations. If the characteristic function is given by a_n $(0 \leq n \leq 11)$, then the DFT coefficients ("components") are given by:

Table 1. Qualities of the dominant seventh or half-diminished seventh

$\lvert f_1^2\rvert$	$\lvert f_2^2\rvert$	$\lvert f_3^2\rvert$	$\lvert f_4^2\rvert$	$\lvert f_5^2\rvert$	$\lvert f_6^2\rvert$
0.27	1	2	7	3.73	4

$$f_k = \sum_{j=0}^{11} a_j e^{-i2\pi kj/12} \tag{1}$$

More generally, for any universe (i.e., length of indicator vector), u,

$$f_k = \sum_{j=0}^{u-1} a_j e^{-i2\pi kj/u} \tag{2}$$

But only the $u = 12$ case will be relevant to this paper.

The DFT yields twelve complex numbers, f_0–f_{11}, of which we consider just f_1–f_6 because of inherent symmetries that determine the others (see Amiot 2016, Yust 2015a). Table 1 gives the *squared magnitudes* for the coefficients 1–6 of this DFT.[1] By ignoring the phases, we consider just the properties of the dominant seventh as a set class, since the magnitudes are invariant under transposition and inversion. The principal qualities of the dominant seventh, those with the highest magnitudes, are f_4, f_5, and f_6. The prototypes for these qualities (set classes that maximize them) are octatonic scales (or fully-diminished sevenths), diatonic scales or diatonic subsets, and whole-tone scales, respectively, so they have been dubbed "octatonicity," "diatonicity," and "whole-tone quality" (Quinn 2006; Amiot 2016; Yust 2015a, 2016). Amiot (2017b) argues, in particular, that the size of f_5 best reflects the intuitive meaning of "diatonicity," because it measures the similarity of the collection to the most characteristic diatonic subsets (those that are most tightly packed on the circle of fifths). A similar point can be made with regards to octatonicity (f_4) and whole-tone quality (f_6). For the dominant seventh, the presence of these qualities relates to the facts that it is a subset of octatonic and diatonic scales and is more heavily weighted towards one whole-tone collection over the other. These three qualities are also the primary dimensions of activity in "Les sons et les parfums . . . ," as we will see. By contrast, the topography of tonal harmony appears to be largely defined by f_3 and f_5, with f_2 also playing a role (Yust 2015b, 2017a). Diatonicity (f_5), in particular, is carefully controlled in tonal music, and therefore the listener experienced in tracking the modulations and tonicizations of tonal music should be especially sensitive to the position of Ph_5, the diatonic position, which is a circle-of-fifths balance and is closely associated with key in tonal music. Debussy's music shares with tonal music a concern for diatonicity as a primary means of harmonic organization, but privileges other harmonic dimensions—f_4 and f_6—over the coefficient most closely associated with tonal harmonic function, f_3.

One attempt to account more precisely for the sense of sonority in Debussy's music, using the tools of Fortean pitch-class set theory, is made by Richard Parks (1989), who bases his approach on pitch-class set "genera," specifically diatonic, octatonic, chromatic,

[1] The notation for the DFT I use here comes from Yust 2015a: f_n indicates coefficient n, $|f_n|^2$ its squared magnitude, and Ph_n its phase converted to a 0–12 scale (i.e. divided by $\pi/6$). Amiot 2016's equivalent notation for f_n is a_n, whereas Milne, Bulger, and Herff 2017 use $(\mathcal{F}u)_n$.

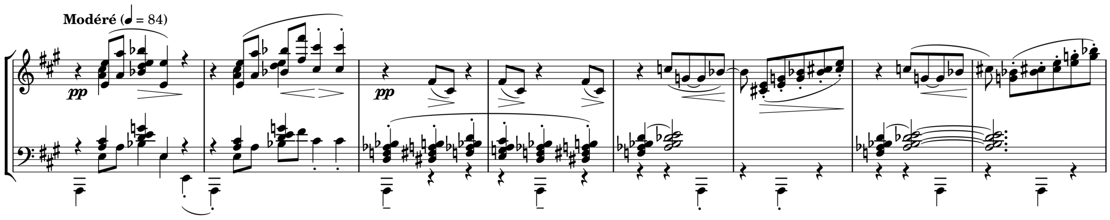

Figure 1. "Les sons et les parfums . . . ," mm. 1–8

and whole-tone genera.[2] As Quinn 2006 shows, DFT qualities accomplish the goals for which classification systems like Park's and Forte's genera were devised, yet with more nuance and a more mathematically robust framework. Parks' genera map neatly onto the DFT qualities of diatonicity (f_5), octatonicity (f_4), whole-tone quality (f_6), and—somewhat more problematically—"chromaticity" (f_1). Parks omits those qualities most identified with harmonic functionality in tonal music: f_3, triadicity, and f_2, "dyadicity" or "quartal quality."[3] Where f_2 and f_3 fluctuate on the basis of the sounding harmony, f_5 tends to be highly stable in tonal music, a feature of key rather than chord. Debussy's retention of diatonicity as a significant harmonic parameter in the absence of the other characteristic qualities of tonal music thus preserves a thread to tonal style and draws upon tonal hearing while shedding specifically those elements of tonal music that involve moment-to-moment goal-directed harmony.

The opening measure of "Les sons et les parfums ..." (Fig. 1) certainly alludes to tonality, even harmonic function, with a progression that might be described as V–ii$^{\varnothing 7}$ in D minor, although the intended tonal center is clearly A, not D. The second measure, stating the full melodic idea, adds an F♯, making a complete "D harmonic major" collection. To avoid erroneous implications of tonal center I will refer to this as the "1♯ harmonic major," following the recommendation of Hook (2011), meaning that its accidentals (2♯s – 1♭) add up to 1♯. Table 2 gives the DFT of both collections, showing that the prominent qualities are those of f_4 and f_5. The introduction of F♯ at the high point of the melodic line in m. 2 is a striking moment, and we may see why by considering the diatonic position of that note in the collection. Fig. 2 illustrates this by plotting the phase values of f_5 of all of the pitch classes and the collection as a whole. The overall collection has a high diatonicity because its pitch classes are tightly grouped around the overall Ph_5 value (i.e. they are closely packed on the circle of fifths). But three of the elements of the collection are relatively remote: B♭, C♯, and F♯. These are precisely the elements juxtaposed in the melody of m. 2. In fact, the melody outlines an F♯ major triad, but it is not spelled—and is unlikely to be heard—as such, at least in the tonal sense, because the context orients the F♯–B♭ interval diatonically the long way around the Ph_5 cycle as a diminished fourth, rather than as a major third. The following measures repeat the B♭–F♯–C♯ melody motivically, with a complete dominant seventh chord built from the B♭.

The A–E fifth in the bass, in contrast to the B♭–F♯–C♯ of the melody, is diatonically central to the opening collection, with the same Ph_5 value (3.5). Debussy thus establishes

[2]Park's genera are not the same as those of Forte (1991) in that they can in principle be much more freely defined.

[3]An additional, 8–17/18/19 genus proposed by Parks is associated with triadic quality, f_3. However, he tends to use this genus to capture high-f_4 pitch-class sets that happen to not be strict octatonic subsets. It is telling, also, that he declines to define a genus on the basis of the more prototypical representative of f_3, the hexatonic collection. This would better serve his purpose of defining genera as distinctly as possible, but would not be of much use in analyzing Debussy's music.

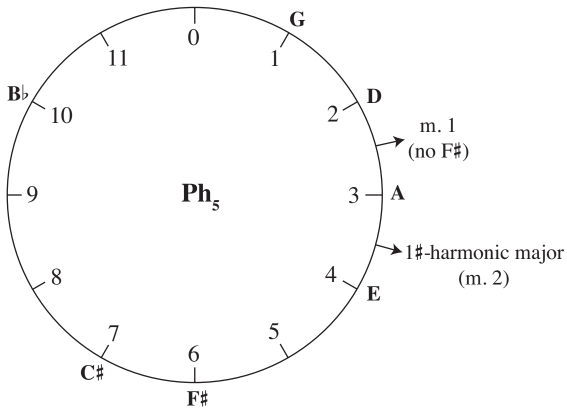

Figure 2. Ph_5 values for the 1♯ harmonic major collection and its elements. (0 is placed by convention at the top of the circle and values ascend clockwise.)

Table 2. DFTs for pitch-class content of measures 1–2

Meas.	pcs	$\lvert f_1^2\rvert$	Ph_1	$\lvert f_2^2\rvert$	Ph_2	$\lvert f_3^2\rvert$	Ph_3	$\lvert f_4^2\rvert$	Ph_4	$\lvert f_5^2\rvert$	Ph_5	$\lvert f_6^2\rvert$	Ph_6
1	AB♭C♯DEG	0.27	0.5	3	7	2	7.5	9	8	3.73	2.5	0	—
2	AB♭C♯DEF♯G	0.27	5.5	1	8	5	6.89	7	8.64	3.73	3.5	1	0

an opposition between what might be understood as the ground (A-E) and the air (B♭-C♯-F♯). Contrasts of diatonicity are central to the effect.

An advantage of using the DFT to define harmonic quality is that the basic theorems associated with it point to attributes of significant theoretical value. One of these is orthogonality, which tells us that the qualities are independent of one another and independent of cardinality. Another is the Parseval-Plancherel theorem, which implies that the total power—the sum of squared DFT magnitudes—is constant for a given cardinality of pitch-class set. This means that a reduction in one quality must be compensated by an increase in one or more others. The diatonically remote pitch-classes in Debussy's melody therefore must contribute some other quality to the total sound of mm. 1–2, which in this case is an octatonic quality. One way to understand the importance of other prominent qualities—octatonic and whole-tone—in Debussy's music, then, is as an alternative to diatonicism, the particular means Debussy chooses to weaken and promote ambiguity in the diatonicity of his harmonic materials.

3. Perfect balance

Diatonicity serves two kinds of functions in Debussy's music. High diatonicity gives a grounded sense of tonal place and association between harmonic elements. Low diatonicity and diatonic oppositions provide ambiguity and instability. The prototypes of diatonic ambiguity are those with zero diatonicity, the *perfectly balanced* collections, to use the terminology proposed by Andrew Milne and others (Milne, Bulger, and Herff 2017; Milne et al. 2015). Perfect balance is a special case of what Amiot (2016) calls *nil coefficients*, meaning collections with zero-valued coefficients in its DFT. It refers to a zero-valued *first* coefficient in the DFT, but as Amiot (2016) shows, a zero in any coefficient whose index is coprime to the universe implies zeros in all such coefficients. (59) Hence, in the 12-tET case, perfect balance could be equivalently defined as a zero-valued

Table 3. DFTs for some perfectly balanced sets

Set	$\lvert f_1^2\rvert$	Ph_1	$\lvert f_2^2\rvert$	Ph_2	$\lvert f_3^2\rvert$	Ph_3	$\lvert f_4^2\rvert$	Ph_4	$\lvert f_5^2\rvert$	Ph_5	$\lvert f_6^2\rvert$	Ph_6
CF♯	0	—	4	0	0	—	4	0	0	—	4	0
CC♯F♯G	0	—	12	11	0	—	4	10	0	—	0	—
CEG♯	0	—	0	—	9	0	0	—	0	—	9	0
CE♭F♯A	0	—	0	—	0	—	16	0	0	—	0	—
CDEF♯G♯B♭	0	—	0	—	0	—	0	—	0	—	36	0
BFCEA♭	0	—	4	2	9	0	4	4	0	—	1	0

f_5, a more musically significant feature as far as Debussy is concerned.[4] Amiot (2016, 2017c) shows further that nil coefficients more broadly speaking—not just in the coprime coefficients—are highly significant in many applications of the DFT to music, making them a natural generalization of perfect balance. Recalling the Parseval-Plancherel theorem, nil first and fifth coefficients imply the one or more other coefficients must have a relatively large value. Hence, most perfectly balanced collections are prototypes of some other qualit(ies). Table 3 gives some examples: the tritone (f_2, f_4, and f_6), the (0167) tetrachord (f_2 and f_4), the augmented triad (f_3 and f_6), the diminished seventh chord or octatonic scale (f_4), and the whole-tone scale (f_6). Not all perfectly balanced sets are transpositionally symmetrical like these; consider the last entry in Table 3 (interestingly, a set of this type may be found as a subset in m. 2 of "Les sons et les parfums . . . ," by removing pitch-classes A and E, the pedal and its fifth). However, the transpositionally symmetrical sets are most representative of specific qualities, as a rule.

Debussy uses perfectly balanced collections in two ways:

(1) By partitioning such a collection into two parts, each of which may itself have a strong diatonicity, he can create an opposition of two diatonic universes on opposite sides of the circle of fifths. The two subsets will have equal magnitude of f_5 and opposite phases. The phase of f_5 may be understood as the location of the likely key—or, more accurately, an implicit key signature—on the circle of fifths. Therefore, partitions of perfectly balanced collections create perfect diatonic oppositions: sets of equal diatonic strength in opposite diatonic positions.

(2) Low diatonicity collections may be understood as highly sensitive in diatonic space. The addition or omission of a single pitch class may push the phase of f_5 a great distance one way or the other. A perfectly balanced collection epitomizes this property: any one or more pitches added to a perfectly balanced collection completely defines its diatonic position. The position of the whole collection is the same as that of the added pitch(es). Similarly, when one or more pitch-classes is *omitted* from a perfectly balanced collection, its diatonic position will be exactly opposite the omitted pitch(es). When subsets and/or supersets of single perfectly balanced collection span different sections of a piece, the diatonic meaning of its individual pitch-classes can be manipulated in this way by minimal changes in the harmonic profile of the music.

We can find examples of both of these strategies on the first page of "Les sons et les parfums . . . ." After he isolates the B♭–F♯-C♯ motive in mm. 3–4, the next melodic element Debussy introduces is the interval C–G in m. 5, which develops the F♯–C♯ motive

[4]The same cannot, however, be said for very small but non-zero f_1, almost perfectly balanced collections. For example, the 5-note and 7-note collections of 12-tET with the smallest non-zero f_1s are the pentatonic and diatonic scales, which have *maximum* f_5. Therefore, maximizing balance under certain constraints, an idea suggested by Milne, Bulger, and Herff 2017, would give different results than maximizing circle-of-fifths balance.

Table 4. DFTs for pitch-class content of measures 3–8. "−(C)" indicates all pitch-classes except C (etc.).

Meas.	pcs	$\lvert f_1^2\rvert$	Ph_1	$\lvert f_2^2\rvert$	Ph_2	$\lvert f_3^2\rvert$	Ph_3	$\lvert f_4^2\rvert$	Ph_4	$\lvert f_5^2\rvert$	Ph_5	$\lvert f_6^2\rvert$	Ph_6
3–4, RH	F♯C♯	0.27	8.5	3	11	2	7.5	1	10	3.73	6.5	0	—
3–4	−(C)	1	6	1	6	1	6	1	6	1	6	1	6
5^2, RH	CG	0.27	2.5	3	11	2	1.5	1	10	3.73	0.5	0	—
5	−(BD♯F♯)	0.27	2.5	1	8	5	9.89	3	7	3.73	0.5	1	0
8	AC♯EGB♭	1	3	1	6	1	9	13	8.46	1	3	1	6

while at the same time harmonically opposing it, at least in the diatonic dimension. The interval of transposition used here, the tritone, is perfectly balanced, which means that the combination of two sets related by tritone (a transpositional combination or multiplication by tritone) is also perfectly balanced. Taken separately, these two elements define diatonically opposing areas. The same may be said of the entire pitch-class content of the measures they occur in, despite an overall complexity of texture and the presence of 9–11 pitch classes at a time (Table 4). Taken as a whole, the set CC♯F♯G is diatonically ambiguous, so it yields to other elements of the texture for the overall diatonic orientation of the passage—here, the persistent A pedal in the bass ($\mathrm{Ph}_5 = 3$) holds the diatonic center and defines F♯-C♯ ($\mathrm{Ph}_5 = 6.5$) as a sharp-side element and C-G ($\mathrm{Ph}_5 = 0.5$) as a flat-side one, as they are spelled. The C-G version of the motive diatonically reinforces the sense of B♭ as a flat-side element, not sharpward as would be implied by identifying it as an element of F♯-major. The triadic status of the B♭–F♯-C♯ remains, as it were, a secret.

The final gesture of the first section of the piece, in mm. 6 and 8, consolidates the diatonically balanced materials presented thus far, while maintaining the centrality of the A pedal. The fully diminished seventh chord in the right-hand is nil in f_5 so that the pedal A takes free rein in defining the diatonic position, $\mathrm{Ph}_5 = 3$. It incorporates pitches from the two motivic fourths, C♯ from F♯-C♯ and G from C-G, the B♭, and the fifth over the pedal, E. The diminished seventh is a prototype of octatonicity. In fact, it is nil in *all* coefficients save for f_4 (and, trivially, f_0 and f_8). The presence of this diminished seventh is a thread through the first part of the piece, as reflected in the stability around $\mathrm{Ph}_4 = 8$, the value for C♯EGB♭ for the harmonic materials in mm. 1–8. We may understand the octatonic context, then, as the foundation from which Debussy establishes the diatonic oppositions of mm. 3–4 and m. 5 on either side of the central diatonic position of the pedal tone.

4. Octatonic complementation

As is often the case in such music, there is an evident octatonic presence in "Les sons et les parfums . . . " despite the absence of any octatonic scales *per se*. The special status of the octatonic collection, which gives us reason to invoke it even when it is not literally present—as, e.g., Forte (1991) does—is its nil coefficients. The octatonic is nil in all coefficients except the fourth (and eighth and zeroth). In this sense, it is a bit like the aggregate, which is nil in all coefficients (except the zeroth). As Amiot (2017a) points out, this implies that complementary collections have equal and opposite DFTs in all but the zeroth coefficients. (prop. 15) That is, the coefficients have the same size and opposite phases. This notion of complementation can be extended to other nil-valued collections, particularly the octatonic. Octatonic complements—collections that share no common notes and combine to complete an octatonic collection—are equal and opposite in all but their fourth (eighth) and zeroth coefficients.

The idea of octatonic complementation may be deployed to simplify the otherwise somewhat complex succession of octatonic sets that Debussy presents in the first part of "Les sons et les parfums . . . ." Fig. 3 shows four of these collections, the melody of m. 2, the harmony on the downbeat of m. 4, the melody of mm. 5 and 7, and the harmony of mm. 6 and 8, all of which are five-note subsets of Oct_{01}. The first important observation here is that all of these are subsets of the same octatonic. Therefore, regardless of the exact content of each, Ph_4 will be highly stable over the section. It provides a consistent context in which the contrasts of diatonicity may be presented. The nil value of f_5 for the octatonic guarantees that it has subsets with a full range of Ph_5 values. Fig. 3 shows the complements of all the octatonic collections Debussy uses. As trichords, these are somewhat easier to digest, and are the inverse images of the five-note collections in all but their octatonicity. The first collection is both relatively diatonic and triadic. The second concentrates the diatonic quality and is similar to the first. The last two are complements of diminished triads and therefore are weak in all but their fourth coefficient. We can also see the abrupt shift of diatonicity from the harmony in m. 4 to the melody of m. 5. Furthermore, all of the complements contain E♭/D♯, the diatonic antipode to A, the pedal tone.

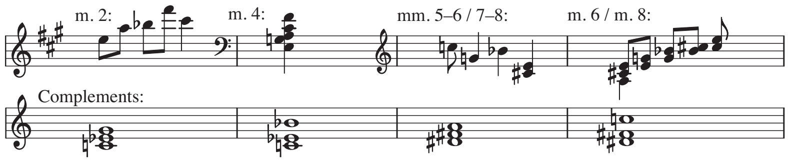

Figure 3. Collections from mm. 1–8 and their octatonic complements.

5. Whole-tone collections and resonance

The next section of music (Fig. 4) introduces two other perfectly balanced collections. The initial chord of m. 9 is an augmented triad, which is nil on all but f_3 and f_6 (and f_0 and f_9). The melodic D♯ therefore initially determines both the diatonicity and octatonicity of the measure by itself, due to the indeterminacy of the augmented triad in both dimensions. D♯ is precisely the pitch-class absent from all of the collections in Fig. 3, and the diatonic antipode to the pedal tone A. The diatonicity therefore swings between opposite poles from m. 8 to m. 9. At the same time, the octatonic position of D♯ is roughly the same as those of the Oct_{01} subsets of mm. 1–8. Thus, we have another instance of diatonic contrasts softened by a stable octatonic context, with an added whole-tone connection (the harmony of m. 8 is weakly affiliated with WT_1), as evident from comparing Tables 4 and 5. The result is a relatively high number of common tones over the bar line despite the abrupt reversal of diatonicity. Musically, Debussy invites us to hear the same pitches from opposite diatonic perspectives: the formerly central A becomes the ambiguous outlier, and C♯, formerly an extreme sharp-side element, at $\mathrm{Ph}_5 = 7$, is now slightly flat of the $\mathrm{Ph}_5 = 9$ context. The mechanism relating common-tone retention to the DFT lies in the convolution theorem, which Amiot (2017a) identifies as a deep property of the DFT. See Yust 2016.

Taking mm. 9–13 as a whole, we find a complete whole-tone collection (the odd collection: WT_1), plus the F♯ that occurs in the chromatic counter-melody played in both hands. Again, the principle of indeterminacy of perfectly balanced collections applies: the

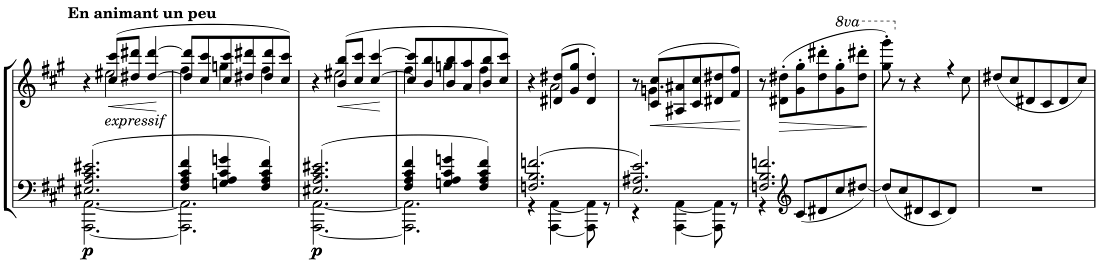

Figure 4. "Les sons et les parfums . . . ," mm. 9–17

Table 5. DFTs for pitch-class content of measures 9–19.

Meas.	pcs	$\lvert f_1^2\rvert$	Ph_1	$\lvert f_2^2\rvert$	Ph_2	$\lvert f_3^2\rvert$	Ph_3	$\lvert f_4^2\rvert$	Ph_4	$\lvert f_5^2\rvert$	Ph_5	$\lvert f_6^2\rvert$	Ph_6
9	AC♯E♯D♯	1	9	1	6	4	9	1	0	1	9	16	6
9–12	ABC♯D♯FF♯G	1	6	1	0	1	6	1	0	1	6	25	6
13	ABD♯FG♯	1	4	3	5	1	0	7	2.64	1	8	9	6
18	C♯FG♯BD♯	0.54	10	0	—	1	0	4	4	7.46	8	9	6
19	CEGB♭C♯D♯	2	10.5	0	—	2	1.5	12	9	2	10.5	0	—

whole-tone collection is nil in all but the sixth coefficient, so that all other coefficients are determined only by the F♯, which has the same Ph_4 value as the D♯, but a distinct Ph_5, closer to that of the preceding material. Besides the shifting diatonic positions, the contrast that Debussy establishes in this section is primarily one of quality, from the previous predominance of f_4 and f_5 to that of f_6. In all cases, pitch-class A remains central within the controlling dimension, as if each section is constructed out of contrasting resonances of that ever-present pedal tone. The method may be compared to the more unsubtle "Voiles," except in the latter case the common center of the whole-tone and diatonic material, A♭, is distinct from the pedal tone, B♭.

Measure 9–12 are the first place in "Les sons et les parfums . . . " where octatonicity as a whole is weakened. The two perfectly balanced collections that Debussy uses here, the augmented triad and whole-tone scale, are among the few pitch-class sets nil in both f_4 and f_5. Debussy often weakens a harmonic quality like this as a preparation for a shift in that dimension. In "Le vent dans la plaine," for instance, as shown in Yust 2016, Debussy uses whole-tone material to smooth over a form-defining large shift of diatonic position that follows very stable diatonic material. In "Les sons et les parfums . . . ," the previously stable component is instead f_4. The collection in m. 13 (see Table 5) is a new kind of octatonic subset, a complement of (026), whose main quality other than octatonicity is whole tone. Indeed, this quality helps it connect smoothly to the preceding whole-tone material. Its only non-WT_1 pitch class is G♯. Octatonically, however, at $Ph_4 = 2.64$ it is opposite the initial harmonic major collection ($Ph_4 = 8.64$) that served as the stable context for the first section.

The following section of the piece, mm. 18–23, is the first place where the A pedal tone—in fact, pitch-class A altogether—is absent (Fig. 5). The C♯-D♯ from the melody of m. 9 instead acts as a kind of upper voice pedal, while two dominant seventh chords a half-step apart (C♯^{7}, spelled C♯-F-G♯-B, and C^7) alternate in the left hand. This kind of semitonal "planing" of dominant seventh chords is now defined as a motivic element of the piece, originating in mm. 3–4. The C♯-D♯, when combined with the C♯^{7}, makes a dominant major ninth chord, an object whose principal qualities are diatonic and whole tone. When the C♯-D♯ is combined with the C^7, on the other hand, it makes the octatonic complement of a minor third, whose principal quality is octatonic.

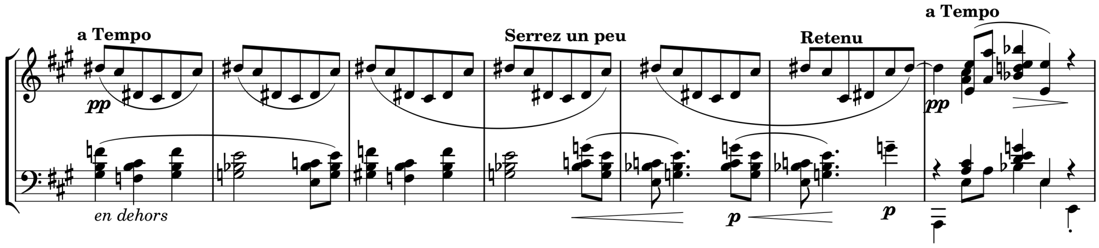

Figure 5. "Les sons et les parfums . . . ," mm. 18–24

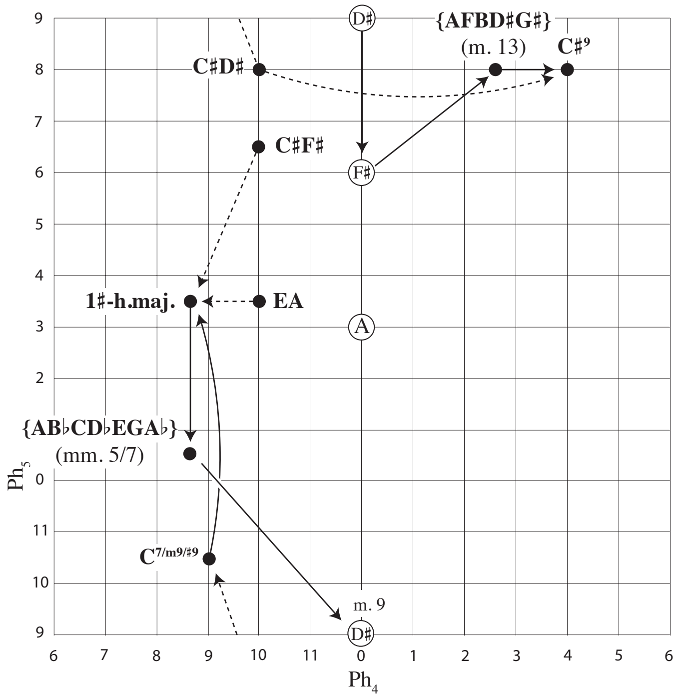

Figure 6. Phase space plots of elements from mm. 1–23. Solid arrows show musical progressions while dashed arrows show subset relationships.

6. Phase spaces

We have seen thus far that significant harmonic activity in the prelude occurs largely in the diatonic and octatonic dimensions. The whole-tone quality, which also plays a role, is a less sensitive instrument because it has no imaginary component (a consequence of the fact that the characteristic functions of pitch-class sets are real-valued only), and therefore Ph_6 takes only values 0 (for WT_0) and 6 (for WT_1). We have also found that phase shifts in f_4 and f_5 have important musical meaning. Phase space plots, showing the values of Ph_4 and Ph_5 in a two-dimensional toroidal space, are a useful method for tracking such relationships in this kind of situation.

The plot in Fig. 6, for instance, illustrates points made above: the A-E fifth is diatonically central to the collection of mm. 1–2, while C♯-F♯ is diatonically peripheral. All are similar in Ph_4. The harmony in the latter part of m. 5 (and m. 7) has exactly the same Ph_4 value, but a large diatonic shift from the initial collection. It is also diatonically opposite the motivic C♯-F♯ fourth.

The phase space plot then sheds further light on what happens in the subsequent sections, where the C♯-D♯ motive emerges from the whole-tone material (mm. 9–12), and then becomes a new pedal over the repeating C^7–$C\sharp^7$ progression in mm. 18–23. The whole-tone material, as previously observed, weakens f_4 for the first major octatonic shift, represented by the chord in mm. 13 and 15 (an $F^{\o 7}$ over the pedal A). This move breaks what had previously been an octatonic barrier at the value of the pedal tone A ($\mathrm{Ph}_4 = 0$). The progression in mm. 18–23 consists of two chords, a C♯ dominant major ninth, and a C dominant minor ninth + ♯9 (C^7 + C♯-D♯). The first of these has strong diatonic and whole-tone qualities and the second a strongly octatonic quality (see Table 2). In phase space (Fig. 6) the two are far apart in Ph_4 and surround D♯, the antipode to the tonic A, in Ph_5. Furthermore, C♯-D♯, the pedal, is a special subset. It represents the approximate Ph_4 value of the octatonic chord, $C^{7/\mathrm{m}9/\sharp 9}$, and the Ph_5 value of the diatonic $C\sharp^9$.

The main theme returns in mm. 24–25, rounding out the expository section. The transition that follows bridges the main formal break of the piece, shown in Fig. 7. Debussy's method here is highly representative of his style. Assuming, as the preceding analysis has endeavored to show, that the main harmonic activity of the piece is in f_4, f_5, and f_6, Debussy chooses a maximally oppositional transposition of the main theme: down by semitone. This oppositional use of semitone transposition has already been foreshadowed, in the semitonal progressions of dominant sevenths A^7–$B\flat^7$–B^7 in mm. 3–4 and $C\sharp^7$–C^7 in mm. 18–23. What is particularly brilliant, though, is how he uses the established motivic material to transition between the distant harmonic areas represented by these two transpositions of the theme. Though measure 26 echoes the F♯–C♯ motive like mm. 3–4, it stops short on the left hand's A^7, thereby isolating a subset of the previous harmonic major collection. This subset is slightly sharpward and, octatonically, a little further towards the A axis, as the phase space plot in Fig. 8 shows. Debussy then further isolates the F♯–C♯ motive, respelling it as G♭–D♭ in m. 27. This dyad is a subset of both the 1♯ and 6♭ (= 6♯) harmonic major collections, and is almost directly between them in $\mathrm{Ph}_4/\mathrm{Ph}_5$-space. The isolation of these subsets thus makes for an impressively smooth harmonic transition between these distant areas. The shift of C♯-F♯ from a contrasting diatonic area, floating in the upper register, to a diatonic center in the bass, enacts a motivic dance like the waltzing sounds and perfumes of Baudelaire's poem.

There is one other common tone between the collections, more subtle but no less important, and that is the pedal tone A. Debussy's selection of the 6♭ collection as the contrasting area preserves this important pitch class but reverses its status. This can be seen in the phase-space plot: A is representative of the *diatonic* position of the tonic 1♯ harmonic major collection, where it is octatonically remote, yet it is is *octatonically* central to the contrasting 6♭ collection, and diatonically remote (as $\flat\hat{6}$). Its status with respect to these collections is analogous to the status of the C♯-D♯ motive with respect to the contrasting $C\sharp^9$ and $C^{7/\mathrm{m}9/\sharp 9}$ collections.

7. Harmonic function

At the beginning of this analysis, I described Debussy's stance with respect to tonal music by dividing tonality into two distinct attributes, diatonicity and harmonic function: diatonicity continues to be a crucial dimension of activity for Debussy, whereas the logic of harmonic function seems to be absent. An important passage in the prelude, however, seems to throw the latter claim into question. The progression in m. 30, as indicated in Fig. 9, admits of a fairly sound functional analysis as an approach to the dominant of

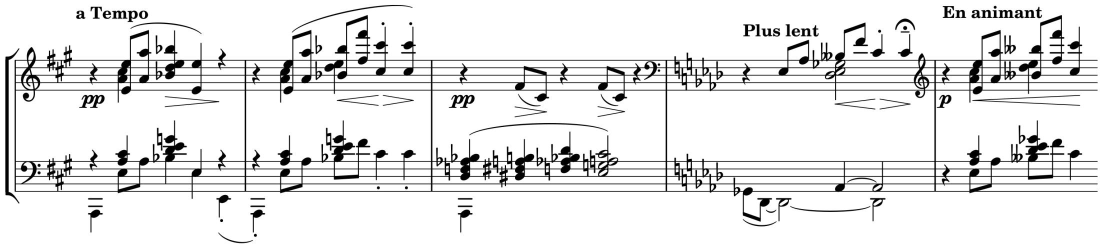

Figure 7. “Les sons et les parfums . . . ,” mm. 24–28

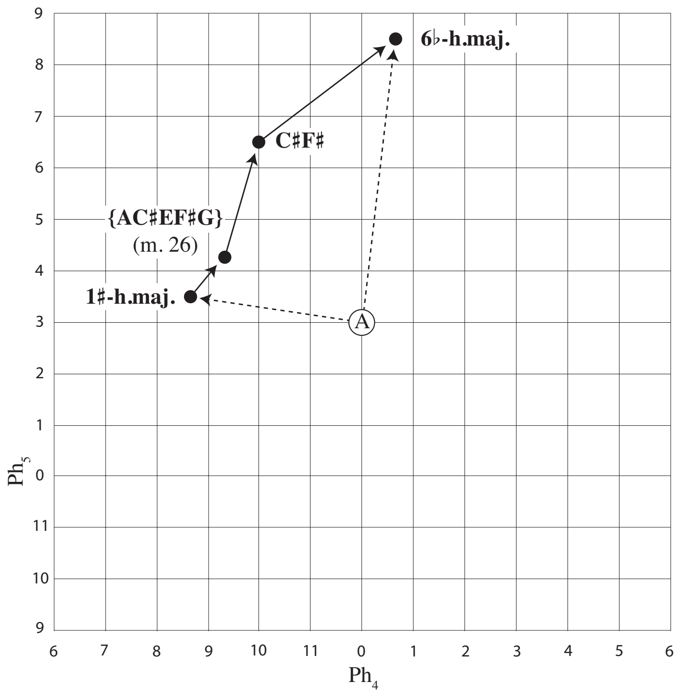

Figure 8. Phase space for the transition to the transposed theme in m. 28. Solid arrows show musical progressions while dashed arrows show subset relationships.

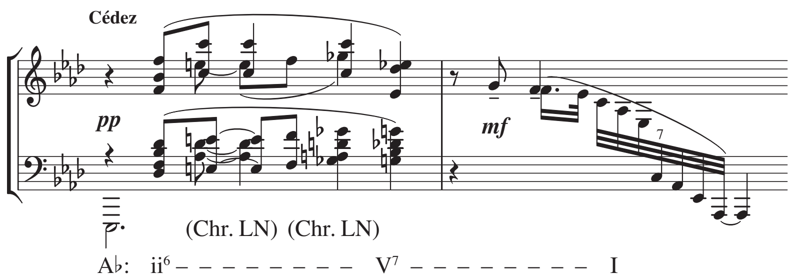

Figure 9. “Les sons et les parfums . . . ,” mm. 30, with functional analysis

A♭ major. The progression is of particular formal significance: Debussy also uses it to end the piece, transposed to A major, in m. 50, creating a large-scale formal rhyme and underlining the invocation of harmonic function all the more, since it provides a sense of tonal resolution on a concluding A major harmony.

For some theoretical approaches, this passage might pose an existential threat: after all, if we propose a non-tonal syntax for explaining the music, the infiltration of a clear tonal syntax implies a deep rupture in the harmonic language, as if a line in old Langue d'Oc were suddenly inserted into a poem in modern French. While such hybridity is certainly a logical possibility, it would not appear to be an appropriate description of the progression in m. 30, which fits seamlessly into the overriding harmonic palette of the piece.

To avoid such an undesirable explanatory disjunction, the theoretical basis for our analysis must be able to subsume the concept of harmonic function. The DFT presents two potential ways to formulate simple models of harmonic function. Triads and seventh chords have two strong coefficients besides f_5: f_3 and f_4. Chords within a given key are close together in Ph_5, but spread out in Ph_3 and Ph_4. These dimensions therefore effectively sort chords within a key by function. Fig. 10 shows the diatonic triads of A♭ major in a $\mathrm{Ph}_3/\mathrm{Ph}_4$-space, connected in their usual "authentic" functional order by solid arrows. We may equate functional prograde motion, then, with *either* descending Ph_3 or ascending Ph_4. However, ultimately both may be necessary to distinguish functional progressions from other, "functionally oblique" kinds of progressions. The latter include, for instance, motion between parallel triads and other kinds of purely semitonal motion (shown with dashed arrows in Fig. 10).

With this sketch of harmonic function, we can now refine our account of Debussy's stance with respect to it. The music of the first major section of the piece is non-functional in two senses. First, it is static: the value of Ph_4 tends to be stable and Ph_5 less so, in contrast to tonal music where the opposite is true. Second, although we have not given much consideration to f_3 up to this point, it is evident that where strong motions in Ph_3 do occur, they tend to be of the functionally oblique sort—purely semitonal motion.

The music in m. 30 unites the functional and functionally oblique kinds of motion in a beautiful way. As the plot in Fig. 11 shows, if we consider the motion from beginning to end of m. 30 broadly, followed by the resolution to A♭ major, we can trace an authentic

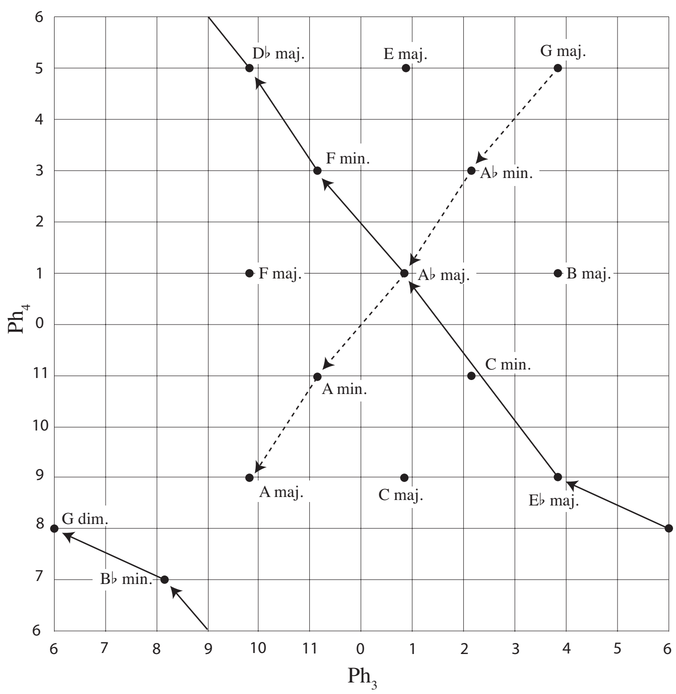

Figure 10. Functionally ordered triads of A♭ major in $\mathrm{Ph}_3/\mathrm{Ph}_4$ space (solid arrows) and functionally oblique relationships (dashed arrows).

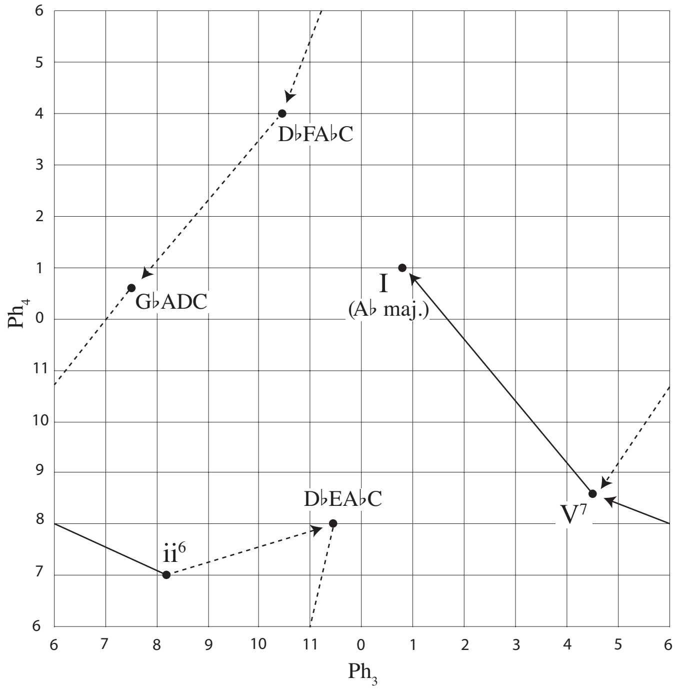

Figure 11. Phase space plot of m. 30–31, with the overriding functional progression (solid arrows) and more local semitonal motion (dashed arrows).

functional path through Ph_3/Ph_4-space. However, if we go chord by chord, without trying to distinguish structural from non-structural tones, we find the kind of semitonal motion on the local level that resembles the harmony elsewhere in the piece. In fact, the approach to the dominant (E♭^{7}) is by pure semitonal planing like that of mm. 3–4 or mm. 18–21 (a fact somewhat obscured by Debussy's orthography).

8. "Every vestige of the luminous past"

The end of the prelude brings together the threads of the preceding music culminating in a celebratory yet detached consummation. Principal among these are harmonic and motivic elements of the first section discussed above: diatonic contrasts within a stable octatonic context, whole tone as a contrasting harmonic quality, the pedal tone A, and its whole-tone effusions, C♯-D♯.

Starting from m. 41, at the "Tranquille et flottant" marking (Fig. 12), Debussy interjects an evasive and seemingly unprecedented musical idea. We may understand the logic of this idea by first observing that its harmonic quality is predominantly whole tone. Here again the idea of the whole tone as a perfectly balanced collection, and one that nullifies octatonicity as well as diatonicity, is useful. The whole-tone scale used here, WT_0 is opposite the one Debussy uses at the beginning of the piece. However, Debussy does not use the entire WT_0 collection, or exclusively that collection. He excludes C♮ from the collection, and includes a C♯ from outside of the collection. Since the whole-tone collection is nil on all coefficients except for Ph_6, when considering f_1–f_5 we may understand the collection as C♯ – C♮, a kind of "anti-semitone." For f_5 the negation of C♮ is equivalent to F♯, so, in its diatonic capacity, this collection recalls the important C♯–F♯ motive from the opening music. For f_4, however, the negation of C♮ has the phase value

Figure 12. “Les sons et les parfums . . . ,” mm. 41–52.

of Oct_{12}. The position of “C♯ – C♮” in Ph_4/Ph_5-space, as shown in Fig. 13, is therefore diatonically aligned with the 4♯ harmonic major and the C♯–F♯ dyad, but to the left in Ph_4, in a zone unexplored in the opening, but to be exploited in the music that follows. Functionally, this region of Ph_4 may be understood as the dominant area with respect to A major.

The “Tranquille et flottant” music thus sets the stage harmonically for what follows, a transposition of the main theme up a major sixth. The transposed theme (which would be in a 4♯ harmonic major collection except that the note B is never sounded) may be understood as the full realization of the contrasting suspended diatonicity of the false F♯ major triad, B♭–C♯–F♯, of the opening melody. This triad, previously absent, but

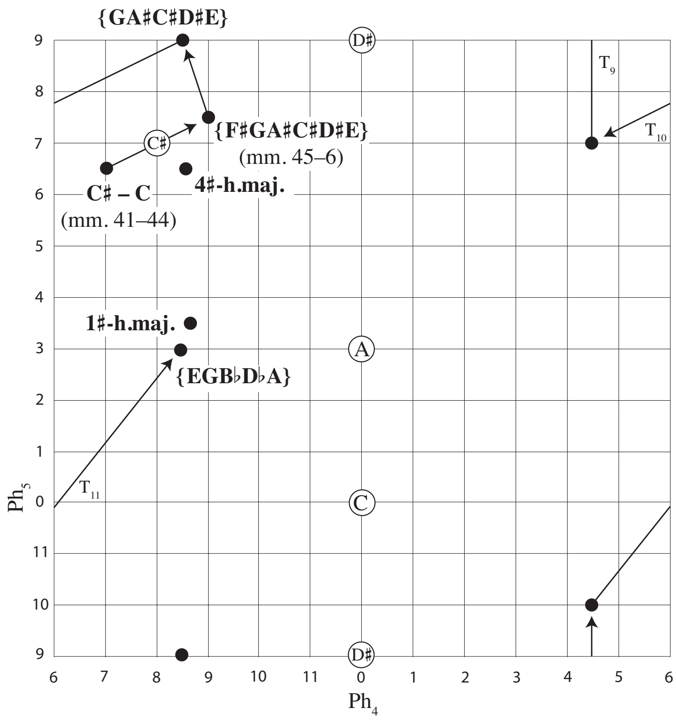

Figure 13. Phase space plots of elements from mm. 41–48.

secretly present, like the departed lover, is now brought fully into view. It initiates an irregular sequence, down by whole step, minor third, then semitone, which completes a tritone descent in the motivic fourth from D$\sharp$–A$\sharp$ to the tonic A–E. As the phase-space trajectory in Fig. 13 shows, this returns to the home-key diatonicity through a sharpward cycle, and by moving within two octatonic regions, the home region and the dominant region to its left.

The ending (m. 46–48) condenses the basic thematic idea, E–A–B$\flat$–F$\sharp$–C$\sharp$ into a quick furtive gesture (with C$\sharp$ moved to the beginning), and alternates C$\sharp$ (D$\flat$) with C$\natural$ in a series of such gestures separated by octatonic chords. The semitonal alternation recalls the opposition built into the "Tranquille et flottant" (C$\sharp$ – C$\natural$) harmony. It also concentrates the previous semitonal fluxuations—between dominant seventh chords, transposed versions of the main theme, etc.—into a single voice and single gesture. As with the opening section, looking at octatonic complements, given in Fig. 14, helps us understand the progression. The sequential descent in complements of diminished triads is primarily octatonic in quality, weakening diatonicity to lubricate the rapid Ph_5 journey. The *léger* gesture with C$\sharp$ then restores the triadic quality of the main theme idea (complement of C minor). The following chord (m. 48) intensifies diatonicity, but also at the same time omits precisely the sharpward elements of motivic importance in the opening, F$\sharp$, C$\sharp$, and D$\sharp$. These sharpward elements contrast with the groundedness of the A–E pedal at the beginning of the piece. Here, the $\mathrm{C}^{7\mathrm{add}6}$ chord is their diatonic opposite, within the prevailing octatonic context. In the second *léger* gesture, which takes the C$\natural$ from the preceding chord but replaces G with the F$\sharp$ of the first *léger* gesture, the quality is instead whole tone—specifically WT_0—opposing the WT_1 collection from which the C$\sharp$-D$\sharp$ motive emerged.

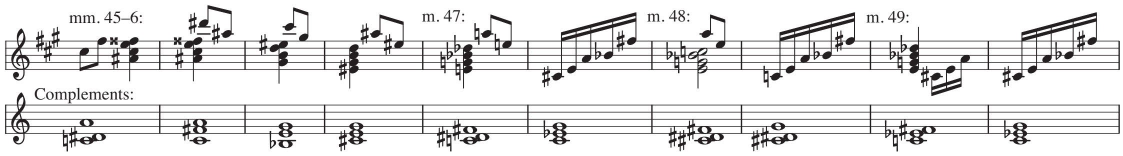

Figure 14. Collections from mm. 45–48 and their octatonic complements.

These are the challenges to unity, then, that are put to rest after the transposed cadence in m. 50. Debussy colors the final A major with a B major upper-neighbor chord, which allows him to include both the motivic F$\sharp$, as well as the D$\sharp$–C$\sharp$ motive. Diatonically, the Lydian tonality of the conclusion ($\mathrm{Ph}_5 = 6$) hovers sharpward from the opening 1$\sharp$ harmonic major collection and its grounding A–E pedal ($\mathrm{Ph}_5 = 3.5$), the tonic harmony, as it were, now drifted aloft from its grounded diatonic position in the region originally occupied by the hovering B$\flat$–F$\sharp$–C$\sharp$ motive.

9. Conclusion

Behind what seems to be an unrestrained diversity of harmonic materials in Debussy's music, with a little bit of digging, we find remarkable consistency in technique. When musicians consider Debussy's style, they usually think of surface features that often characterize short passages: planing harmonies, the modal use of diatonic and pentatonic scales, whole-tone collections, etc. These kind of superficial features give us frustratingly little access to any sense of a musical language, or the possibility of sustained analysis of Debussy's music.

In the interest of putting a more holistic picture of Debussy on solid footing, and as a distillation the above analysis, I would offer the following stylistic traits:

- Contrasts of harmonic quality, especially octatonic (f_4), diatonic (f_5), and whole tone (f_6), particularly as a way of recontextualizing an important pitch, interval, or motive,
- Control of diatonicity as a primary form-defining element,
- Use of non-tonal harmonic qualities, particularly octatonic (f_4) and whole tone (f_6) as a means of smoothing over major shifts of diatonicity,
- Sustained motivic work based on minimal motives (such as individual intervals) and focused specifically on their harmonic properties.

We have seen in "Les sons et les parfums ..." how Debussy uses a stable octatonic context to establish contrasting diatonic areas, and uses a whole-tone context to stage shifts in both diatonic and octatonic position, and this analysis can be compared to analyses of other pieces illustrating similar techniques (Yust 2016). The use of simple pitch-motives, F♯-C♯ and C♯-D♯ and the association of them with more abstract harmonic properties, particularly their diatonic locations, is particularly impressive. All of this points to a remarkable sense, on Debussy's part, for harmonic qualities, and their potential for meaningful musical deployment.

These observations invite the question of how a seemingly erudite mathematical theory, one based on a procedure, the application of the DFT to pitch-class sets, that Debussy himself certainly could have had no knowledge of, could accurately uncover musical techniques implicit in his harmonic language. It may appear that in this analysis I have waved a magic DFT wand over Debussy's music and pulled out a living, breathing hermeneutic rabbit, and that there must be some sleight of hand involved. This is where theory, and mathematical results, become crucial, such as the basic features of the DFT (orthogonality, conservation of power, conversion of convolution into multiplication) referenced in passing above. I have made the case elsewhere (Yust 2015b, 2016, 2017b) that mathematical discoveries about the DFT made by Lewin (2001), Amiot (2016), and others reveal aspects of harmony that are implicit in the materials themselves. It is therefore plausible that sensitive musicians like Debussy should have been able to discover through experimentation and intuition the special properties of perfectly balanced sets and learned to navigate the landscape of harmonic qualities to the particular ends of each composition. He could have discovered, for instance, the special role of perfectly balanced sets in effecting large shifts of diatonicity while retaining a relatively large number of common tones, something which is demonstrable mathematically through the DFT. The task of mathematical music theory, is to explain how and why this is the case, and it is in this sense that mathematics is often an essential prelude to good music analysis.

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