Energy levels of quark atoms

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(Received 31 August 1992)

We assume that free quarks exist in nature. The energy levels of atoms composed of these quarks and various nuclei are calculated. We note some important aspects of these energies.

PACS number(s): 14.80.Dq, 36.10.−k

It is currently believed that all hadrons are composed of quarks which are confined. Whether or not quark confinement can be broken, however, is a purely experimental question. So far, experiments have shown that free, unconfined quarks, if they exist at all, must exist in concentrations of <1 per 10^{20} nucleons [1]. A recent proposal may push this limit by a factor of 10^5 [2]. Theories which accommodate this data can still allow free quarks to exist. For example, the glow model [3] predicts the existence of qq or $\bar{q}\,\bar{q}$ states when the $\mathrm{SU}^{\text{color}}(3)$ symmetry of QCD is spontaneously broken to $\mathrm{SO}^{\text{glow}}(3)$ symmetry. Other models allow for free single quarks [4]. These free quarks or diquarks would most likely be remnants from the big bang [5], with the negatively charged ones existing as quark atoms, such as $Q\,^4\mathrm{He}$ (where Q is a quark or diquark with charge $Z_Q e$, $Z_Q<0$), today.

The existence of free Q, in addition to its purely theoretical implications, would make catalyzed fusion feasible and useful [6]. Because the mass m_Q of a quark is most likely greater than 1 GeV, much larger than the mass of an electron, it would be easy to fuse the deuterons in a ddQ molecule. In particular, a $\bar{u}\,\bar{u}\,Q$ with $Z_Q=-\frac{4}{3}$ would be much more effective than a μ in this catalysis [7]. To be able to isolate the negatively charged Q from bulk material and use the quarks for fusion catalysis, it is important to know the energy levels of quark atoms $Q\,^A Z$, which is the subject of this paper.

To calculate the ground-state energy level E_0 of a quark atom with nucleus $^A Z$ having mass m_A and charge $+Ze$, we numerically solve the nonrelativistic Schrödinger equation with $l=0$ and reduced mass $\mu=m_Q m_A/(m_Q+m_A)$. The potential $V(r)$ that models the quark-nucleus interaction can be separated into two parts, a Coulomb potential $V_C(r)$ and a quark-nucleus potential, $V_Q(r)$, so that $V=V_C+V_Q$. The Coulomb potential between the Q and the nucleus is given simply by $V_C(r)=Z_Q Z e^2/r$ for $r>r_A$, where $r_A=1.1\,A^{1/3}$ fm is the radius of the nucleus. It is the attractive well from $V_C(r)$ which allows the quark atoms to have bound states. $V_Q(r)$, on the other hand, is not based on the electromagnetic force but, instead, on the strong force and thus has a somewhat different contour. Even though the nucleons are colorless, when the free Q is close to a nucleon, there is a color polarization so that the Q sees the quarks inside the nearby nucleon. Thus we approximate the quark-nucleus potential as proportional to the quark-quark potential. Now the quark-quark QCD potential begins with a "Coulomb"-like attraction and then increases linearly, providing a large constant restoring force consistent with QCD theories. For our quark-nucleus potential, we neglect the attractive, short-range forces and let the potential increase linearly starting from $r=r_A$. Then, at some threshold $r=\lambda_G$, the breaking of $\mathrm{SU}^{\text{color}}(3)$ symmetry occurs and there is a strong repulsion force, given by an exponentially decreasing potential. So,

$$V_Q(r)=V_0(r-r_A)\exp[-(r-r_A)/\lambda_G], \quad r>r_A\ . \tag{1}$$

Here, λ_G, the QCD-breaking distance, is related to the mass m_G of the five massive gluons resulting from the lifting of QCD degeneracy [3,8] by

$$\lambda_G=\hbar c/m_G\ . \tag{2}$$

V_0 is a scaling factor determining how steep the QCD attraction will be; as we show, the energies are not sensitive to V_0. The neglect of the attractive forces for $r<r_A$ is justified since the Q never sees this region, the probability that the Q will tunnel through the barrier being extremely small (e.g., for a $Q\,^4\mathrm{He}$ with $m_Q=4$ GeV, $Z_Q=-\frac{4}{3}$, and $V_0=1.5$ GeV/f, the tunneling probability is $\approx 10^{-200}$, corresponding to an atom lifetime of 10^{150} years). Because the potential inside the nucleus is unknown, the wave function is taken to be zero at r_A so

TABLE I. The average radii of $Q\,^4\mathrm{He}$ atoms are listed. ($V_0=1$ GeV/f, $Z_Q=-\frac{4}{3}$, $m_Q=1.5$ GeV.) Without the quark-nucleus repulsion, $\langle r\rangle$ would be greatly reduced (e.g., to 9 f for $m_Q=1.5$ GeV).

m_Q (GeV)	$\langle r\rangle$ (f)
1.5	86
2.0	105
4.0	188
6.0	276
8.0	368
9.5	438

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that the wave function cannot penetrate into the nucleus. The potential in the region $r > r_A$ thus becomes

$$V(r) = V_0(r - r_A)\exp[-(r - r_A)/\lambda_G] + Z_Q Z e^2/r \ . \quad (3)$$

The most prominent feature of V is that the repulsion due to V_Q acts like an impenetrable barrier as seen in Figs. 1(a) and 1(b), so the binding energy of the quark atom will be much less than the corresponding energy if V consisted only of the attractive V_C. Because of this barrier, it is very difficult to combine two quarks which have already been separated.

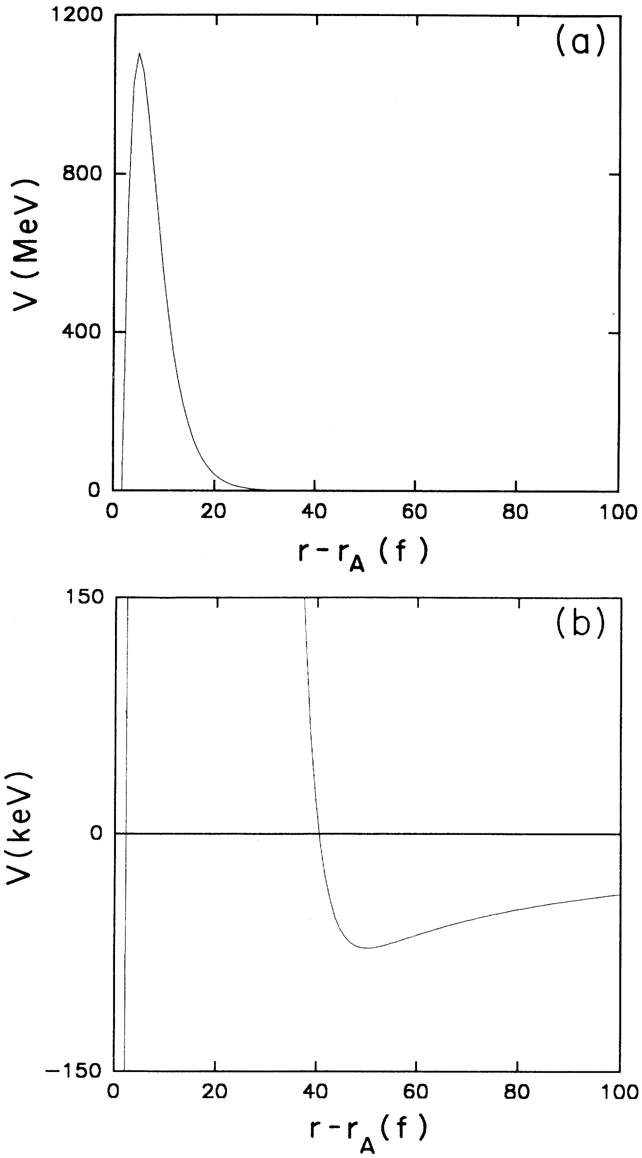

FIG. 1. (a) The potential, Eq. (3), for a $Q\,^4$He atom is graphed with $m_Q = 1.5$ GeV, $V_0 = 1$ GeV/f, and $Z_Q = -\frac{4}{3}$. Note that since the maximum of the potential is on the order of GeV's, the potential acts as a barrier. (b) The same potential as in a with a much larger scale to show the attractive well. Note the near vertical curve at $r \approx 40\ f$. This allows the actual potential to be approximated by an infinite barrier.

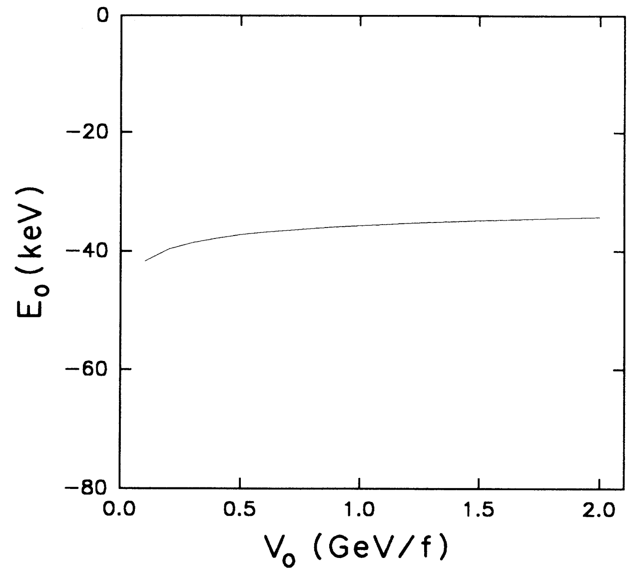

FIG. 2. E_0 is plotted for a $Q\,^4$He atom while V_0 is varied. ($m_Q = 1.5$ GeV, $Z_Q = -\frac{4}{3}$.) We see that the energy levels depend very little on large variations in V_0. (See text.)

The quark atoms we chose to study were $Q\,^1$H, $Q\,^4$He, $Q\,^{12}$C, and $Q\,^{16}$O as an illustration. (In this paper, only the energies for $Q\,^4$He and $Q\,^{12}$C atoms are shown, however, since the other energies vary with m_Q in the same

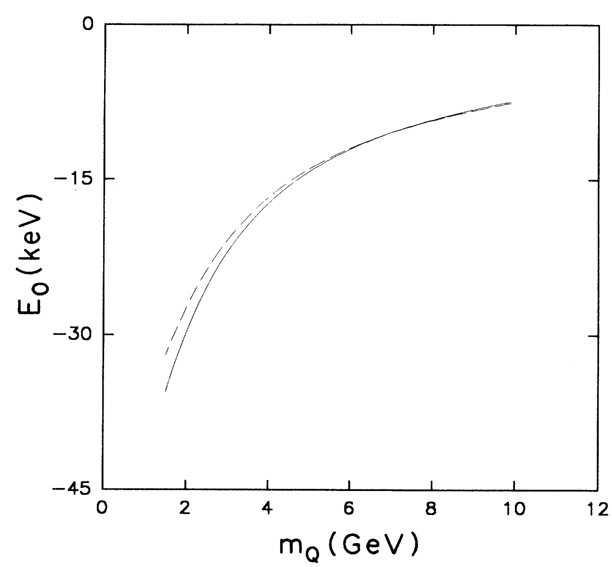

FIG. 3. E_0 for a $Q\,^4$He atom is plotted against the mass of the Q. ($V_0 = 1$ GeV/f, $Z_Q = -\frac{4}{3}$.) The dashed points are the energies if the quark-nucleus repulsion is replaced by an infinite barrier. (See text.) If there was no repulsive quark-nucleus potential V_Q, the binding energy would increase (e.g., to about 200 keV for $m_Q = 1.5$ GeV).

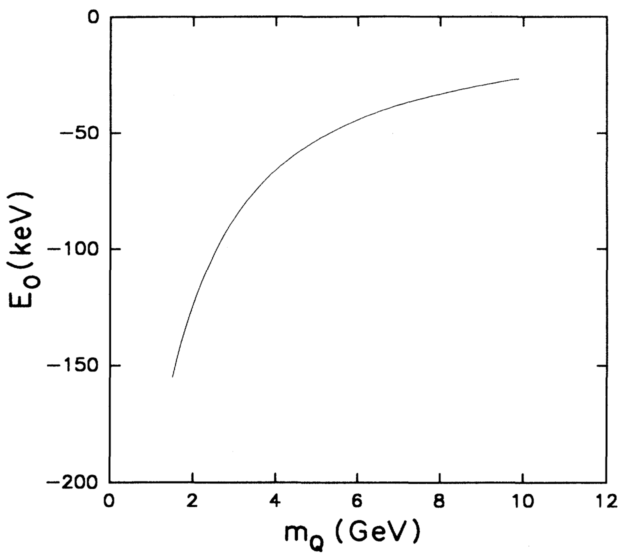

FIG. 4. E_0 for Q ^{12}C atom is plotted against m_Q. ($V_0=1$ GeV/f, $Z_Q=-\frac{4}{3}$.)

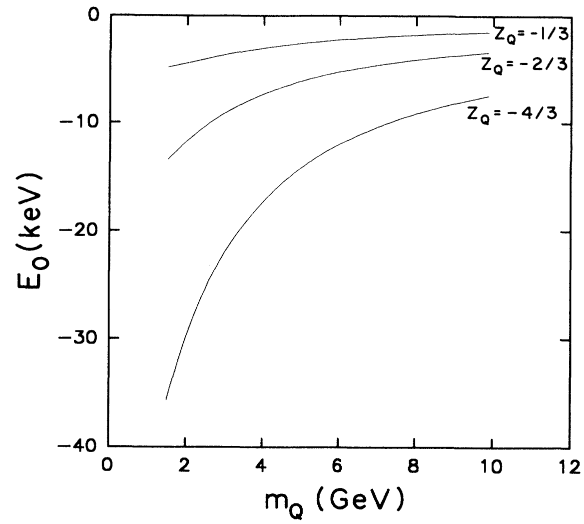

FIG. 5. E_0 is plotted versus m_Q for a Q ^{4}He atom while Z_Q is varied from $-\frac{1}{3}$ to $-\frac{2}{3}$ to $-\frac{4}{3}$. ($V_0=1$ GeV/f, $m_Q=1.5$ GeV.)

manner.) These calculations could be extended similarly for higher Z nuclei. We calculate the lowest energies E_0 of the quark atoms while varying m_Q from 1.5 GeV to 10 GeV. Since we expect m_Q to depend on the size $1/m_G$ of the broken SU$^{\text{color}}$(3) field, we take m_Q proportional to $1/m_G$, with the constant of proportionality chosen as follows: A typical meson has a radius of 1 fm and a mass of 500 MeV. Thus, we want $\lambda_G=1$ fm to correspond to $m_Q=500$ MeV. But by (2), $\lambda_G=1$ fm when $m_G=200$ MeV. So, we set

$$m_Q=(500\text{ MeV})(200\text{ MeV}/m_G)\ . \tag{4}$$

In order not to contradict experimental limits of $\lambda_G>3$ fm [2,8], we must take $m_Q>1.5$ GeV. Given m_Q, λ_G is uniquely determined (for example, $m_Q=1.5$ GeV yields $\lambda_G=3$ fm), and the energy levels are then calculated with a standard search pattern. First, we vary V_0 from 0.1 GeV/f to 2 GeV/f. Then Z_Q was taken to be $-\frac{1}{3}$, $-\frac{2}{3}$, or $-\frac{4}{3}$ (corresponding to Q being a $d/\bar{u}\bar{d}$, $\bar{u}$, or $\bar{u}\,\bar{u}$, respectively).

The results are given in Figs. 2–5. First, notice that the energies E_0 do not depend significantly on V_0 as seen in Fig. 2. Even if V_0 is doubled or tripled, the energy levels change by only a few percent. This shows that although V_0 is not well known, we can still calculate the energies to high accuracy. To understand the reason for this result, we constructed a "hard core" potential $V_{\text{HC}}(r)$ where we replace the quark-nucleus repulsion with an infinite barrier. Energies using this potential were calculated and closely approximate the actual energies as seen in Fig. 3. Now, if we vary V_0 in Eq. (1) by a large amount, then, due to the exponential factor in the quark-nucleus repulsion, the width of the approximating barrier changes very little. But the energies for $V_{\text{HC}}(r)$ depend solely on the width of the barrier, so the energies will only vary slightly, too.

The most important aspect of the energies (using the actual potential) is their magnitude as shown in Figs. 3 and 4. Typical values of E_0 for quark atoms range from a few keV to hundreds of keV. If there were no QCD potential V_Q, though, these energies would be the usual Coulomb result $E_0=-(Z_QZ)^2\alpha^2\mu c^2/2$, which is on the order of MeV. So, we demonstrate that the binding energies of the atoms are reduced by more than an order of magnitude due to the repulsion potential V_Q. Also, the E_0 are inversely proportional to m_Q as expected. If the charge Z_Q of the quark is changed, the energy levels change proportionally as seen in Fig. 5.

In addition to the energy levels, we also calculated $\langle r\rangle$, the expectation value of the radius. The radii are on the order of hundreds of fm as seen in Table I. If the potential consisted of only the Coulomb attraction, $\langle r\rangle$ would shrink by about an order of magnitude.

Our findings show that the quark-nucleus repulsion potential dramatically reduces the quark atom binding energy. This may be significant in fusion catalysis schemes involving free quarks and in the process of isolating the free charge from bulk material.

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