Paths to the Frontier*

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Abstract

We construct a model of collective search in which players gradually approach the Pareto frontier. The players have imperfect control over which improvements to the status quo will be considered. Inefficiency takes place due to the difficulty in finding improvements acceptable to both parties. The process is path dependent, with early agreements determining long-run outcomes. It may also be cyclical, as players alternate between being more and less accommodating.

KEYWORDS: collective search, bargaining, path dependence, cycling, Raiffa path, delay, inefficiency.

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1 Introduction

When searching for policy improvements, the parties involved often have imperfect control over the scope and direction of the changes that are to be considered. One reason for this is that they must search for ideas on how to improve existing arrangements, and it is hard to anticipate which ideas this search process yields and when. Moreover, it is unlikely that alternatives appear that land them directly on the Pareto frontier. The agents may have to content themselves with making only a series of potentially small improvements over the status quo. This is often the case when the issues that are being dealt with are inherently complex. As Binder and Lee (2013) write when describing the complexity of legislative negotiations:

> "The search for win-win solutions is labor-intensive. Information must be gathered from many sources – for example, interest groups, affected industries, policy experts, activists, and government agencies – before members and their staffs can understand the causes and dimensions of a policy problem and see a pathway to possible solutions."

Besides complex legislation, other real-life examples that feature gradual, step-by-step improvements over existing deals include climate change negotiations, international trade talks, and the effort to reduce the stockpiles of nuclear weapons.

Motivated by these examples, we develop a collective search model in which two players approach the Pareto frontier in a series of interim agreements. Our game has complete information and an infinite time horizon. The set of feasible policies is $X = \{\mathbf{x} \in \mathbb{R}^2_+ : x_1 + x_2 \leq 1\}$. At each period t, player $i = 1, 2$ obtains a flow payoff equal to the coordinate x_i^t of the policy $\mathbf{x}^t = (x_1^t, x_2^t)$ that is in place. The agreement in place at the start of the game is $(0, 0)$. In each period, a new alternative is drawn randomly from the set of feasible policies that are Pareto improvements to the status quo policy, and players decide whether to approve or disapprove the draw. The status quo policy is replaced if and only if both players approve the change; otherwise, it stays in place. Players share a common discount factor $\delta < 1$.

Under a key inter-temporal symmetry assumption, we are able to provide a clean characterization of the set of Markovian equilibria that have a recursive structure.[1] In any period, players only accept alternatives that improve their payoffs by a similar

[1]The assumption, which is made on the distributions from which policies are drawn, implies that the continuation game played from period t onwards starting with a status quo $\mathbf{z} \in X$ is strategically equivalent to the game played from period 0 onwards with status quo $(0, 0)$.

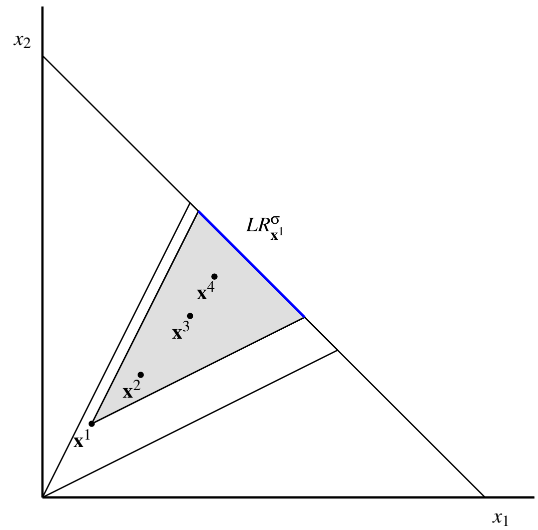

Figure 1: Path of play

amount. In particular, the set of policies that both players find acceptable is a cone defined by two lines with positive slope that pass through the current status quo as its vertex. Figure 1 depicts the first two "acceptance cones" for a possible sequence of policies $\{\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^3, \mathbf{x}^4, ...\}$ that are implemented along the path of play. Policies that lie outside of the cone are rejected even if they are Pareto superior to the status quo. The reason for this is that players cannot commit to approve future policies that disproportionally benefit their opponents. As a result, a player strictly prefers to reject Pareto improvements that favor her opponent significantly more than her, since she (correctly) anticipates that approving these will "close the door" in the future to many policies that she finds attractive. Since players discount the future, the periods of inaction produced by the rejection of Pareto improving alternatives generate inefficiency—an inefficiency that arises simply because of the difficulty in finding moderate policies.

As Figure 1 shows, the distinctive feature of our model is that players will typically reach a sequence of interim agreements, gradually approaching the Pareto frontier. In addition, the randomness of draws and the rigidity of the status quo together imply that the process by which players approach the frontier is necessarily path dependent. In each period, the set of alternatives that players find acceptable depends on the current status quo. As a result, at each point in time, the future path of play depends crucially on the agreements that players reached in the early stages.

This path dependence disappears, however, as players become fully farsighted. In the limit as $\delta \to 1$, the acceptance cone collapses to a line segment connecting the current policy to a point on the frontier. Only policies on this line segment are implemented on the path of play. Intuitively, the cost in terms of forgone future payoff of implementing a policy that is more beneficial to one's opponent increases with δ. In the limit, the only policies that both players accept are those that give a payoff vector on this line segment. When policies are drawn from a smooth distribution, the long run policy converges to an equal split of the surplus. In this case, the unique path to the frontier coincides with the "Raiffa path"— the path of interim agreements proposed by Raiffa (1953) as a plausible outcome in settings in which bargainers engage in step-by-step negotiations.

We also look at the finite horizon version of our game, and give conditions under which equilibria in the infinite horizon limit correspond to a recursive Markovian equilibrium of the infinite horizon game. When these conditions are not met, the equilibria of the finite horizon games may feature cycles under which the players alternate in how accepting they are of the possible Pareto improvements. Given an existing agreement, the acceptance cone may be narrow in some periods but wide in others, following a cyclical pattern. These cycles are not driven by changes in fundamentals, but by self-fulfilling changes in players' expectations about future play.

We also explore a set of extensions in which we characterize the full set of subgame perfect equilibrium payoffs and analyze the case of unequal discounting. In the limit as $\delta \to 1$, we show that the set of subgame perfect equilibrium payoffs coincides with the full set of feasible payoffs. With unequal (but low) discounting, the more patient player has a payoff advantage that is increasing in her discount factor and decreasing in the other player's discount factor.

Finally, we extend the model to allow for "strategic search." This extension is motivated by the fact that our baseline model can be interpreted as a bargaining game in which the players have *no control* over the offers that are generated. This puts the model at the opposite extreme of the standard approach to bargaining theory (e.g. Rubinstein, 1982) in which proposers have *full control* over their offers. A natural extension of our model is, therefore, to the intermediate case in which proposers have *partial control* over the offers that they put on the table. We consider such an extension in which, at each period, a randomly selected proposer chooses the distribution from which the alternative will be drawn. Our main results carry through in this environment.

Related literature. Our paper is primarily related to the literature on collective search. Compte and Jehiel (2010a), Albrecht et al. (2010), Moldovanu and Shi (2013) and Kamada and Muto (2015) study models in which a group of agents sequentially sample alternatives from a distribution and have to choose when to stop. Closer to our model, Roberts (2007) and Penn (2009) also study settings with randomly generated alternatives and with an endogenously evolving status-quo. They consider settings with supermajority rules and focus on how the dynamic nature of the problem affects voting behavior when the set of available alternatives all lie on the Pareto frontier. In contrast, we consider a two-player setting (so necessarily with unanimity) and focus on understanding the process by which policies approach the Pareto frontier.

The interpretation of our model as a bargaining game also connects our paper to prior work on bargaining, especially models featuring delay and inefficiency. However, the inefficiencies that take place in our model are qualitatively different from those that arise in traditional bargaining theory, where players are able to strike agreements that take them directly to the Pareto frontier. Even in models that feature inefficient delay in bargaining (e.g. Cho, 1990, Cramton, 1992, Abreu and Gul, 2000, Fanning, 2018) once an agreement is eventually reached, the outcome typically lies on the frontier.

Because players in our model approach the Pareto frontier in incremental steps, our paper relates to prior work on incremental bargaining and partial agreements. Compte and Jehiel (2004) study a bargaining model in which each player's outside option depends on the history of offers. In this setting players begin negotiations making incompatible offers, and make gradual concessions over time. However, there are no interim agreements in their model: the first agreement that players reach is a point on the Pareto frontier. Acharya and Ortner (2013) analyze a model in which two players bargain over two issues, one of which will only be open for negotiation at a future date. The main result is that players may reach a partial agreement on the first issue, only to complete the agreement when the second issue comes ripe for negotiation.

Our result on commitment and inefficiency relates our paper to the literature on bargaining failures as a result of commitment problems (e.g. Fearon, 1996, Powell, 2004, 2006, Acemoglu and Robinson, 2000, 2001, Ortner, 2017). This work focuses on understanding the conditions under which the players' inability to commit will result in bargaining inefficiencies. Instead, we focus on how the players' inability to commit shapes the way bargainers approach the Pareto frontier.

The rigidity of the agreements in our model relates it to the growing literature on political bargaining with an endogenous status quo (e.g. Kalandrakis, 2004, Duggan and Kalandrakis, 2012, Dziuda and Loeper, 2016, Bowen et al., 2014). We add to this literature by constructing a model in which players bargain over complex issues, and thus have imperfect control over the offers that are generated.

Our connection to the Raiffa path relates our paper to others that also provide foundations for this bargaining solution. Livne (1989), Peters and Van Damme (1991), Diskin et al. (2011) and Samet (2009) provide axiomatizations for the Raiffa path. Myerson (2013), Trockel (2011), Diskin et al. (2011) and Driesen et al. (2017) provide non-cooperative foundations by proposing bargaining models in the tradition of Rubinstein (1982). These models have the property that, in the first round, players reach an agreement at the point at which the Raiffa path intersects the Pareto frontier. In contrast to these studies, our model gives rise to interim agreements, therefore providing foundations for the *path*. Thus, our paper contributes to the "Nash program" of providing non-cooperative foundations to cooperative bargaining solutions.[2]

Finally, our work is related to a set of papers in organizational economics showing how path-dependence can arise in organizations, and arguing that these dynamics may help explain why seemingly identical firms have persistent differences in performance; past work in the literature includes Acharya and Ortner (2017), Callander and Matouschek (2019), Chassang (2010), Halac and Prat (2016), and Li and Matouschek (2013).

2 Model

2.1 Framework

There are two players, $i = 1, 2$. Time is discrete, with an infinite horizon, and indexed by $t = 0, 1, 2, ...$ The set of feasible policies is

$$X := \{(x_1, x_2) \in \mathbb{R}^2_+ : x_1 + x_2 \leq 1\}.$$

At each time t, the players decide whether to change the existing policy from the *status quo* $\mathbf{z}^t = (z^t_1, z^t_2) \in X$ to a new policy $\mathbf{x}$ drawn randomly from a distribution $F_{\mathbf{z}^t}$ with density $f_{\mathbf{z}^t}$ and support over the set

$$X(\mathbf{z}^t) = \{\mathbf{x} \in X : x_i \geq z^t_i \text{ for } i = 1, 2\}$$

[2]Other contributions to the Nash program include Binmore et al. (1986), Gul (1989), Abreu and Pearce (2007, 2015), Compte and Jehiel (2010b) and Fanning (2016).

of alternatives that, given the payoffs we describe below, are Pareto superior to status quo $\mathbf{z}^t$. After policy $\mathbf{x}$ is drawn, the two players simultaneously decide whether or not to accept it. If both accept, then $\mathbf{x}$ becomes the policy in place in period t, so $\mathbf{x}^t = \mathbf{x}$. Otherwise, the status quo is implemented, so $\mathbf{x}^t = \mathbf{z}^t$. The next period's status quo is the previous period policy, so $\mathbf{z}^{t+1} = \mathbf{x}^t$ with $\mathbf{z}^0 = (0,0) =: \mathbf{0}$.

Both players are expected utility maximizers and share a common discount factor $\delta < 1$. If $\mathbf{x}^t = (x_1^t, x_2^t) \in X$ is the policy in place in period t, then player i earns a flow payoff $(1-\delta)x_i^t$ at time t. Player i's payoff from a sequence of policies $\{\mathbf{x}^t\}_{t=0}^{\infty}$ is thus

$$U_i\left(\{\mathbf{x}^t\}\right) = (1-\delta)\sum_{t=0}^{\infty} \delta^t x_i^t.$$

The following assumption will facilitate a tractable analysis of this game, and we will maintain it throughout.

Assumption 1. *For any $\mathbf{x}, \mathbf{z} \in X$ with $\mathbf{x} \in X(\mathbf{z})$, let*

$$P_{\mathbf{z}}(\mathbf{x}) := \left(\frac{x_1 - z_1}{1 - z_1 - z_2}, \frac{x_2 - z_2}{1 - z_1 - z_2}\right)$$

Then, for every $\mathbf{z} \in X$, the density $f_{\mathbf{z}}$ is such that

$$\forall \mathbf{x} \in X(\mathbf{z}), \qquad f_{\mathbf{z}}(\mathbf{x}) = \frac{1}{(1 - z_1 - z_2)^2} f(P_{\mathbf{z}}(\mathbf{x}))$$

where $f := f_{(0,0)}$ is the density from which policies are drawn at the start of the game.

This assumption states that, for any $\mathbf{z} \in X$, the distribution $F_{\mathbf{z}}$ over X from which policies are drawn when the status quo is $\mathbf{z}$ is "identical" to the distribution $F := F_{\mathbf{0}}$ over X from which policies are drawn at the start of the game. The main implication of this assumption will be that a subgame starting with status quo $\mathbf{z} \in X$ is strategically identical to the game starting at status quo $\mathbf{z}^0 = \mathbf{0}$.

Shifting frontier. A special case of our model is one in which the frontier shifts upward in every period, and the players are able to land at points at each period's frontier in every period. This case arises as a special case under the assumption that the support of F is the line segment $\{\mathbf{x} \in \mathbb{R}_+^2 : x_1 + x_2 = c\}$ for some constant $c \in (0,1)$.

Motivation. Our model of sequential search is intended to capture settings in which the parties involved must search for new ideas on how to improve existing policies. Examples include negotiations over complex issues, like climate change, international trade, or healthcare policy. The assumption that the policy draw each period depends on the current status-quo captures the idea that current policies (and their particular shortcomings) may guide agents in deciding what types of improvements to look for. For tractability, our model does not allow agents to recall previous policy draws: if a policy $\mathbf{x}$ was drawn in the past and rejected, players cannot go back and choose it. But allowing players to recall previous policies will not affect our conclusions. The Markovian equilibria that we study below remain equilibria even with recall.[3] Our model also assumes for tractability that the policy drawn each period Pareto dominates the existing policy.

2.2 Solution concepts

The history at the start of time t is $h_t = (\mathbf{x}^\tau, \mathbf{d}^\tau)_{\tau<t}$, where $\mathbf{x}^\tau$ is the policy drawn at period τ and $\mathbf{d}^\tau = (d_1^\tau, d_2^\tau)$ are the voting decisions of players at time τ, with $d_i^\tau = 1$ if player i approved the draw $\mathbf{x}^\tau$, and $d_i^\tau = 0$ otherwise. A pure strategy σ_i is a mapping

$$\sigma_i : (h_t, \mathbf{x}^t) \mapsto d_i^t.$$

For each subgame perfect equilibrium (SPE) $\sigma = (\sigma_1, \sigma_2)$ and each history h_t, we use $V_i^\sigma(h_t)$ to denote player i's continuation payoff under σ at h_t.

A subgame perfect equilibrium (SPE) $\sigma = (\sigma_1, \sigma_2)$ is *Markov Perfect* if, for $i = 1, 2$, all h_t and all $\mathbf{x}^t$, $\sigma_i(h_t, \mathbf{x}^t)$ depends only on the current status quo $\mathbf{z}(h_t)$. For a Markov Perfect equilibrium σ, let $A^\sigma_{i,\mathbf{z}}$ denote the policies that player i accepts under σ when the status quo is $\mathbf{z}$, and $A^\sigma_{\mathbf{z}} := A^\sigma_{1,\mathbf{z}} \cap A^\sigma_{2,\mathbf{z}}$ be the set of mutually acceptable policies.

Consider a Markov Perfect equilibrium $\sigma = (\sigma_1, \sigma_2)$, and let $V_i^\sigma(\mathbf{z})$ be player i's continuation value under σ when the status quo is $\mathbf{z}$.[4] We say that σ is *stage-undominated* if, for $i = 1, 2$, all status quo $\mathbf{z}$, and any draw $\mathbf{x} \in X(\mathbf{z})$, player i approves $\mathbf{x}$ whenever

$$(1-\delta)x_i + \delta V_i^\sigma(\mathbf{x}) \geq (1-\delta)z_i + \delta V_i^\sigma(\mathbf{z}).$$

[3] This is no longer true when we study the finite horizon version of our model in Section 4. There, allowing players to recall previous draws would lead to different equilibrium outcomes.

[4] In a Markov Perfect equilibrium, players' continuation payoffs depend only on the status quo.

Thus, each player approves or disapproves the draw as if their vote is decisive. Throughout the paper, the term *Markovian equilibrium* will refer to stage-undominated Markov Perfect equilibrium.

For any status-quo $\mathbf{z} \in X$ and any subset $A \subset X(\mathbf{z})$, let

$$P_{\mathbf{z}}(A) := \{\mathbf{x} \in X : \mathbf{x} = P_{\mathbf{z}}(\mathbf{y}) \text{ for some } \mathbf{y} \in A\}.$$

In words, set $P_{\mathbf{z}}(A)$ is the projection of $A \subset X(\mathbf{z})$ to the simplex X.

For most of the analysis we focus on Markovian equilibria with the property that $A^{\sigma}_{\mathbf{0}} = P_{\mathbf{z}}(A^{\sigma}_{\mathbf{z}})$ for all $\mathbf{z} \in X$. That is, we focus on Markovian equilibria under which players' voting decisions at each continuation history are "equivalent" to their voting decisions at the start of the game. Formally:

Definition 1. *A* recursive *Markovian equilibrium (RME) σ is a Markovian equilibrium for which $A^{\sigma}_{\mathbf{0}} = P_{\mathbf{z}}(A^{\sigma}_{\mathbf{z}})$ for all $\mathbf{z} \in X$.*[5]

The following lemma holds:

Lemma 1. *Fix an SPE σ, a history h_t, and let $\mathbf{z} = (z_1, z_2)$ be the status quo at this history. Then, there exists an SPE $\hat{\sigma}$ such that, for $i = 1, 2$,*

$$V_i^{\sigma}(h_t) = z_i + (1 - z_1 - z_2)V_i^{\hat{\sigma}}(h_0). \tag{1}$$

Moreover, if σ is an RME, then $\hat{\sigma}$ can be chosen to be equal to σ.

Lemma 1 highlights the recursive nature of our game. When Assumption 1 holds, the continuation game after any history h_t with status-quo $\mathbf{z} = (z_1, z_2)$ is strategically equivalent to the entire game. As a result, player i's continuation payoff under some SPE σ at some history h_t with status quo $\mathbf{z} = (z_1, z_2)$ is equal to the flow payoff z_i that the player is guaranteed to get forever (by the persistence of the status quo) plus the re-scaled payoff $(1 - z_1 - z_2)V_i^{\hat{\sigma}}(h_0)$ that she obtains under some SPE $\hat{\sigma}$ from searching for improvements over the remaining surplus of size $1 - z_1 - z_2$. When σ is an RME, we can take $\hat{\sigma}$ to be equal to σ, so the second term in the right-hand side of (1) is the ex-ante payoff at the start of the game, scaled down by the size of the remaining surplus.

[5] We note that some authors (e.g., Maskin and Tirole, 2001), consider a strategy to be Markov if players play isomorphic strategies in strategically equivalent subgames. Under this definition of Markov strategies, when Assumption 1 holds, all Markovian equilibria of our game are RME.

3 Recursive Markovian Equilibrium

3.1 Characterization

Since in a Markovian equilibrium, players' continuation payoffs at any history depend only on the current status quo, we will abuse notation and write $V_i^\sigma(\mathbf{z})$ to be player i's continuation payoff under RME σ and status quo $\mathbf{z}$. We will also define $W_i^\sigma := V_i^\sigma(\mathbf{0})$ to be player i's ex ante payoffs under RME σ at the start of the game.

We use Lemma 1 to provide a characterization of the path of play in any RME σ. Consider a period t at which the status quo is $\mathbf{z} = (z_1, z_2) \in X$. Player i approves policy $\mathbf{x} = (x_1, x_2) \in X$ only if

$$(1-\delta)x_i + \delta V_i^\sigma(\mathbf{x}) \geq (1-\delta)z_i + \delta V_i^\sigma(\mathbf{z})$$

Using Lemma 1 (noting that in our abuse of notation $V_i^\sigma(h_t) = V_i^\sigma(\mathbf{z}(h_t))$ if σ is an RME) and our definition of W_i^σ, this inequality is equivalent to

$$x_i \geq \ell_{i,\mathbf{z}}(x_{-i}|W_i^\sigma) := z_i + \frac{\delta W_i^\sigma}{1-\delta W_i^\sigma}(x_{-i} - z_{-i})$$

Note that $\ell_{i,\mathbf{z}}(x_{-i}|W_i^\sigma)$ is the line in (x_i, x_{-i})-space with slope $\delta W_i^\sigma/(1-\delta W_i^\sigma)$ that passes through $\mathbf{z}$.

For any $\mathbf{W} = (W_1, W_2) \in X$ and any $\mathbf{z} \in X$, define

$$A_{i,\mathbf{z}}(W_i) := \{\mathbf{x} \in X : x_i \geq \ell_{i,\mathbf{z}}(x_{-i}|W_i)\}$$

to be the set of policies that player i would accept under status-quo $\mathbf{z}$ if her equilibrium payoff was W_i. Then, for any pair of payoffs $\mathbf{W} = (W_1, W_2)$, and any $\mathbf{z} \in X$, the set

$$A_{\mathbf{z}}(\mathbf{W}) := A_{1,\mathbf{z}}(W_1) \cap A_{2,\mathbf{z}}(W_2) \tag{2}$$

is the set of policies that are accepted by both players when the status quo is $\mathbf{z}$ and expected payoffs are $\mathbf{W} = (W_1, W_2)$. Since $1 > \delta(W_1 + W_2)$, the line $\ell_{1,\mathbf{z}}(x_2|W_1)$ has steeper slope than $\ell_{2,\mathbf{z}}(x_1|W_2)$ in (x_1, x_2)-space and $A_{\mathbf{z}}(\mathbf{W})$ is a cone with vertex $\mathbf{z}$. For any pair of values $\mathbf{W}$ we let $A(\mathbf{W}) := A_{\mathbf{0}}(\mathbf{W})$ be the cone with vertex $\mathbf{0}$. Such a cone is depicted in Figure 2.

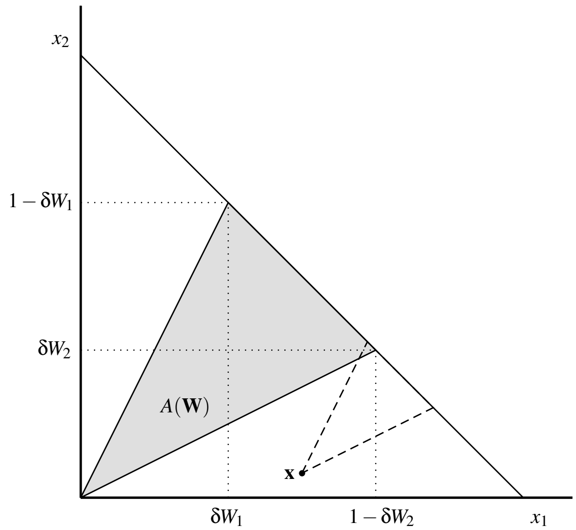

Figure 2: Acceptance region $A(\mathbf{W})$.

From these observations, it follows that player i's payoff at the start of the game under an RME σ satisfies

$$\begin{aligned} W_i^\sigma &= \text{prob}(\mathbf{x} \in A(\mathbf{W}^\sigma))\mathbb{E}[(1-\delta)x_i + \delta V_i^\sigma(\mathbf{x})|\mathbf{x} \in A(\mathbf{W}^\sigma)] + \text{prob}(\mathbf{x} \notin A(\mathbf{W}^\sigma))\delta W_i^\sigma \\ &= \text{prob}(\mathbf{x} \in A(\mathbf{W}^\sigma))\mathbb{E}[x_i - (x_1+x_2)\delta W_i^\sigma|\mathbf{x} \in A(\mathbf{W}^\sigma)] + \delta W_i^\sigma, \end{aligned}$$

where the second line uses $V_i^\sigma(\mathbf{x}) = x_i + (1 - x_1 - x_2)W_i^\sigma$ (by Lemma 1). Therefore, payoffs $\mathbf{W}^\sigma$ under RME σ are a fixed point of operator $\Phi : X \to X$ defined by:

$$\text{for } i = 1,2, \quad \Phi_i(\mathbf{W}) := \text{prob}(\mathbf{x} \in A(\mathbf{W}))\mathbb{E}[x_i - (x_1+x_2)\delta W_i|\mathbf{x} \in A(\mathbf{W})] + \delta W_i. \quad (3)$$

Operator Φ is continuous, and maps points in X into itself, so it has a fixed point. We can also show that for any fixed point $\mathbf{W}$ of Φ, there is an RME σ that has payoffs $\mathbf{W}$.

Proposition 1. *The set of RME is non-empty, and* $\mathbf{W} = (W_1, W_2)$ *is an RME payoff pair if and only if it is a fixed point of the operator* Φ.

Figure 2 plots the acceptance region $A(\mathbf{W})$ at the initial period of the game. As the figure shows, policies that constitute a Pareto improvement over the initial policy $\mathbf{0}$ and lie outside of $A(\mathbf{W})$ are rejected, leading to inefficiency.[6]

[6]As we show in Section 5.1, under an SPE that maximizes the sum of the players' payoffs, players accept all policy draws at every period.

The commitment problem plays an important role in exacerbating the inefficiencies that arise from search frictions. To see how, suppose that in period 0 the alternative $\mathbf{x} > \mathbf{0}$ in Figure 2 is drawn. Policy $\mathbf{x}$ Pareto dominates the initial policy, but if it were to be implemented, then starting in period 1 the set of policies $A_{\mathbf{x}}(\mathbf{W})$ that both players accept would be the area inside the dashed lines in Figure 2. These policies are significantly worse for player 2 than the policies that could be implemented in the future if the status quo $\mathbf{0}$ remains in place. So player 2 strictly prefers to maintain policy $\mathbf{0}$ than to implement $\mathbf{x}$. Player 2 might approve $\mathbf{x}$ if player 1 could commit to accepting policies that are beneficial for player 2 in the future. But player 2 rightly anticipates that player 1 would reject such policies in the future if $\mathbf{x}$ were to be implemented today. This inability to commit implies that only policies that improve both players' payoffs by a similar amount (i.e., moderate policies) will be accepted along the path of play.

Uniqueness. Under certain assumptions, the game has a unique RME. Consider the following two assumptions.

Assumption 2. *There exist $\overline{f} > \underline{f} > 0$ such that $f(\mathbf{x}) \in [\underline{f}, \overline{f}]$ for all $\mathbf{x} \in X$.*

Assumption 3. *In addition to Assumption 2, f is Lipschitz continuous (with respect to the sup norm) with Lipschitz constant $\gamma < \frac{4}{3}\underline{f}$.*

We will occasionally impose Assumption 2 for future results. We use Assumption 3 only as a sufficient condition for uniqueness of RME.

Proposition 2. *Under Assumption 3, there exists $\underline{\delta} < 1$ such that for all $\delta > \underline{\delta}$, the RME is unique.*

Symmetric distributions. Consider the case in which distribution F is *symmetric* about the 45° line, i.e. when its density f satisfies $f(x_1, x_2) = f(x_2, x_1)$ for all $(x_1, x_2) \in X$. In this case, operator Φ always has a symmetric fixed point $\mathbf{W} = (W_1, W_2)$ with $W_1 = W_2$. To see this, for any $W \in [0, 1]$, define

$$\begin{aligned}\Psi(W) &:= \Phi_1(W/2, W/2) + \Phi_2(W/2, W/2) \\ &= \text{prob}(\mathbf{x} \in A(W/2, W/2))\mathbb{E}[x_1 + x_2 | \mathbf{x} \in A(W/2, W/2)](1 - \delta W) + \delta W \qquad (4)\end{aligned}$$

Operator $\Psi(W)$ has a fixed point, corresponding to a symmetric fixed point of Φ.

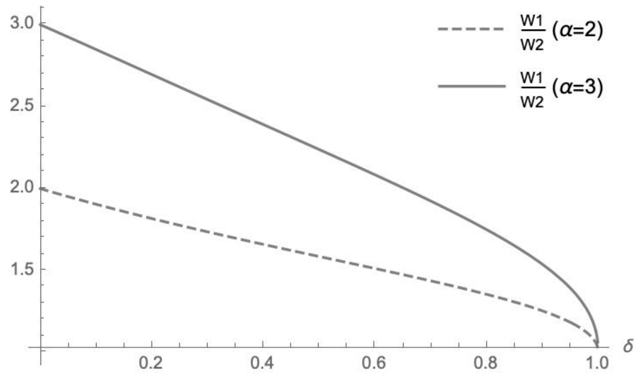

Figure 3: The ratio W_1/W_2 of players' equilibrium payoffs as a function of δ.

Example 1. Assume F is a uniform distribution over X. In this case, for any $W \in [0, 1]$,

$$\Psi(W) = \delta W + \frac{2}{3}(1 - \delta W)^2$$

and thus Ψ has a unique fixed point in $[0, 1]$. The fixed point is

$$W = W(\delta) := \frac{1}{4\delta^2}(3 + \delta - \sqrt{9 + 6\delta - 15\delta^2}).$$

Thus the game has a RME with payoff $W(\delta)/2$ to each player. For $\delta < 1$ we have $W(\delta) < 1$, but $\lim_{\delta \to 1} W(\delta) = 1$, so as $\delta \to 1$ the inefficiency disappears and each player gets a payoff of $1/2$.

Asymmetric distributions. Next, we illustrate with an example how tilting the distribution F away from symmetry affects the players' payoffs.

Example 2. Consider a setting in which policies are drawn from distribution F with support $\{\mathbf{x} \in X : x_1 + x_2 = c\}$ for some $c < 1$; i.e., the shifting frontier model. In particular, suppose y is drawn from a Beta distribution with parameters $\beta = 1$ and $\alpha > 1$. Let $x_1 = cy$ and $x_2 = c - x_1$. Figure 3 plots the ratio W_1/W_2 of players' equilibrium payoffs as a function of discount factor $\delta < 1$, for parameters $c = 1/2$ and $\alpha = 2$ and $\alpha = 3$. For low values of δ, the favored player (player 1) gets larger equilibrium payoffs. However, her advantage disappears in the limit as $\delta \to 1$. In Section 3.2 we show that this is a more general result: when distribution F is sufficiently smooth, players get an equal payoff in the limit as $\delta \to 1$, even if F is asymmetric.

Non-recursive Markovian equilibria. We end this section by noting that, under certain conditions, the game admits Markovian equilibria that are not recursive.

Suppose that operator Φ has multiple fixed points, and let two of these be $\mathbf{W}^*$ and $\mathbf{W}^{**}$. In Appendix A we show that in this case, our game admits a non-recursive Markovian equilibrium with the following structure. For all status quo $\mathbf{z} \in X^- := \{\mathbf{x} \in X \backslash \{\mathbf{0}\} : x_1 \geq x_2\}$, players' continuation strategies are their strategies under the RME that generates payoffs $\mathbf{W}^*$; and for all status quo $\mathbf{z} \in X^+ := \{\mathbf{x} \in X : x_1 < x_2\}$, players' continuation strategies are their strategies under the RME that generates payoffs $\mathbf{W}^{**}$.

3.2 Policy Evolution

In this section, we look at policy evolution in the long run under RME, as well as in the limit as $\delta \to 1$. We start with the following simple observation.

Lemma 2. *(nested acceptance cones) Fix an RME σ, and let $\{\mathbf{x}^t\}_{t=0}^{\infty}$ be a realized sequence of policies under σ. Then, for all $\tau = 0, 1, ...,$*

$$A_{\mathbf{x}^\tau} \supseteq A_{\mathbf{x}^{\tau+1}},$$

with strict inclusion whenever $\mathbf{x}^{\tau+1} \neq \mathbf{x}^\tau$.

This lemma implies that there are policies that are acceptable at some period t, but become no longer acceptable at period $t+1$ (and after) despite also being Pareto improvements relative to the $t+1$ status quo $\mathbf{z}^{t+1}$. The result follows immediately from the fact that, under an RME, the acceptance set $A_{\mathbf{x}}(\mathbf{W})$ under status-quo $\mathbf{x}$ is a cone with vertex $\mathbf{x}$ defined by two lines with slopes $(1-\delta W_1)/\delta W_1$ and $\delta W_2/(1-\delta W_2)$ that pass through the vertex (see Figure 2). Hence, the lines defining all of the acceptance cones are parallel, so the acceptance cones are nested.

Long run policies. Now consider any sequence of policies $\{\mathbf{x}^t\}$ that are implemented along the path of play of an RME. With probability 1, sequence $\{\mathbf{x}^t\}$ has a limit $\mathbf{x}^*$ that lies on the frontier: $x_1^* + x_2^* = 1$. We refer to $\mathbf{x}^*$ as a *long-run policy.* For each RME σ, let LR^σ denote the set of long-run policies that can arise under σ. Similarly, for any status-quo $\mathbf{z} \in X$, let $LR^\sigma_{\mathbf{z}}$ denote the set of long-run policies that can arise under σ, given that current status quo is $\mathbf{z}$.

Proposition 3. *Suppose that F has full support on X, and fix an RME σ. Then,* *(i) (long run policies) for any* $\mathbf{z} \in X$*,* $LR^{\sigma}_{\mathbf{z}} = \{\mathbf{y} \in X : y_1 + y_2 = 1\} \cap A_{\mathbf{z}}(\mathbf{W})$*;*

(ii) (path dependence) $LR^{\sigma}_{\mathbf{z}} \neq LR^{\sigma}_{\mathbf{z}'}$ *for all* $\mathbf{z}' \neq \mathbf{z}$*;*

(iii) (gradual certainty) For every sequence of equilibrium policies $\{\mathbf{x}^{\tau}\}_{\tau=0}^{\infty}$*,* $LR^{\sigma}_{\mathbf{x}^{\tau+1}} \subseteq LR^{\sigma}_{\mathbf{x}^{\tau}}$*, with strict inclusion whenever* $\mathbf{x}^{\tau+1} \neq \mathbf{x}^{\tau}$*.*

At the start of the game, any policy $\mathbf{x}$ on the Pareto frontier with $x_1 \in [\delta W_1, 1-\delta W_2]$ lies in LR^{σ}. As play progresses and the players implement policies that are closer to the frontier, the set of feasible long run policies shrinks. Figure 1 shows $LR^{\sigma}_{\mathbf{x}^1}$ for some interim policy $\mathbf{x}^1$ on the path of play.

Patient players. We now turn to the analysis of the RME path of policies in the limit as $\delta \to 1$. We let $\mathbf{W}^{\delta} = (W_1^{\delta}, W_2^{\delta})$ denote the players' payoffs in an RME of the game with discount factor δ. Abusing our previous notation, let LR^{δ} denote the set of long run policies of the game with discount factor δ.

Proposition 4. *Fix a convergent sequence* $\{\delta_n, \mathbf{W}^{\delta_n}\}$*, with* $\lim_{n\to\infty} \delta_n \to 1$*. Then,*

(i) (determinism) LR^{δ_n} *converges to* $(W_1^*, W_2^*) := \lim_{n\to\infty}(W_1^{\delta_n}, W_2^{\delta_n})$*;*

(ii) (generalized Raiffa path) $\lim_{n\to\infty} A(\mathbf{W}^{\delta_n}) = \{\mathbf{x} \in X : x_1/x_2 = W_1^*/W_2^*\}$*;*

(iii) (efficiency) $\lim_{n\to\infty} W_1^{\delta_n} + W_2^{\delta_n} = 1$*.*

If the distribution F *is twice continuously differentiable, then* $W_1^* = W_2^* = 1/2$*.*

Proposition 4(i) says that as $\delta \to 1$ the path of policies approaches deterministically a particular long run outcome— specifically, the players' RME payoff split. Proposition 4(ii) says that, as $\delta \to 1$, the set of policies that both players find acceptable converges to the line segment connecting $\mathbf{0}$ and the point (W_1^*, W_2^*). Intuitively, the cost in terms of forgone future payoff of implementing a policy that is more beneficial to one's opponent increases with δ. In the limit, the only policies that both players accept are those that give both of them a payoff on this line segment. This implies that, as players become arbitrarily patient, there is no path dependence.

Proposition 4(iii) shows that the inefficiency of delay vanishes as players become fully farsighted. This occurs in spite of the fact that, as $\delta \to 1$, the acceptance region $A(\mathbf{W}^{\delta})$ converges to a straight line, so in any period, there is total gridlock: the probability of changing the existing policy goes to zero. Intuitively, if $\lim_{\delta\to 1} W_1^{\delta} + W_2^{\delta} < 1$, the players would be relatively accommodating in terms of the alternatives that they accept. Hence, the expected delay cost would be zero as the discount factor approaches 1.

Lastly, Proposition 4 shows that, if F is sufficiently smooth, both players obtain the same payoff in the limit as $\delta \to 1$, so $(W_1^*, W_2^*) = (1/2, 1/2)$. That is, even if policies are drawn from a distribution that favors one of the players, as the players become arbitrarily patient they obtain approximately the same payoff in equilibrium. This generalizes the findings in Examples 1 and 2.

Connection to the Raiffa path and other bargaining solutions. The limiting equilibrium outcome in our model relates to the sequential bargaining solution proposed by Raiffa (1953). For any given two-player bargaining set with disagreement point $\mathbf{b}$, let the *utopia* payoff vector $\mathbf{u}$ be the payoff vector that would result if each player obtained her preferred outcome while keeping their opponent at a utility level equal to their disagreement payoff. In our setting, $\mathbf{b} = (0, 0)$ and $\mathbf{u} = (1, 1)$. Fix an integer $n \geq 2$. Under Raiffa's solution, negotiations happen in steps. In the first step of negotiations, players move from the disagreement point $\mathbf{b}$ towards the utopia point by an amount proportional to $\frac{1}{n}$: they move from $\mathbf{x}^0 = \mathbf{b}$ to $\mathbf{x}^1 = \frac{1}{n}\mathbf{u} + \frac{n-1}{n}\mathbf{b}$. In the t-th step of negotiations, players take last period's agreement $\mathbf{x}^{t-1}$ to be the new disagreement point, and move towards the updated utopia point $\mathbf{u}^{t-1}$; i.e., $\mathbf{x}^t = \frac{1}{n}\mathbf{u}^{t-1} + \frac{n-1}{n}\mathbf{x}^{t-1}$.[7] Bargaining continues this way until players reach the Pareto frontier.[8] The path of agreements reached under this sequential solution is sometimes called the *Raiffa path*, and the point at which this path intersects the Pareto frontier is the *Raiffa point*.

When the bargaining set has a linear frontier (as in our model), the Raiffa path is the line-segment connecting the disagreement point to the frontier, which would pass through the utopia payoff vector at the start of negotiations, if extended. In our model, this is the line-segment connecting $(0, 0)$ and $(1/2, 1/2)$, which is exactly the path of play that arises in the limit as $\delta \to 1$ when distribution F is sufficiently smooth.

In our setting with a linear frontier, the Raiffa point coincides with other bargaining solutions like the Nash (1950) solution, and the Kalai and Smorodinsky (1975) solution. Numerous non-cooperative foundations exist for these bargaining solutions. However, prior work does not provide non-cooperative foundations for the *path* to these solutions (see the the literature review above). In Raiffa's account, the players' journey to the

[7]The updated utopia point $\mathbf{u}^{t-1}$ is the utopia point of a bargaining problem with the original bargaining set, and with disagreement payoffs given by $\mathbf{x}^{t-1}$.

[8]Raiffa's *discrete* solution corresponds to the case with $n = 2$. Raiffa's *continuous* solution is obtained by taking the limit as $n \to \infty$.

Pareto frontier is gradual, taking place in a series of steps. Prior work, on the other hand, only provides accounts in which players land directly on the Raiffa point.

4 The Finite Horizon Game

4.1 Framework

In this section we study the finite horizon version of our model. The game is the same as in Section 2, except that players can only draw new policies at times $t = 0, ..., T$; from time $T + 1$ onwards, the policy in place is the policy at time T. In this section, our solution concept is stage-undominated SPE.

It can be shown by backward induction that this game has unique stage-undominated SPE payoffs. We let $V_i(\mathbf{z}, t; T)$ denote player i's equilibrium continuation payoff at time t with status quo $\mathbf{z}$ under deadline T. Let $\mathbf{W}(T) = (V_1(\mathbf{0}, 0; T), V_2(\mathbf{0}, 0; T))$ denote the players' equilibrium payoffs at time $t = 0$. The following result shows that the essence of Lemma 1 carries over to this setting, as does the recursive operator approach that we took to prove Proposition 1.

Lemma 3. *Fix any deadline $T \geq 0$. Then,*

(i) for all $t \leq T$, and all $\mathbf{z} \in X$,

$$V_i(\mathbf{z}, t; T) = z_i + (1 - z_1 - z_2)W_i(T - t) \tag{5}$$

(ii) the players' equilibrium payoffs satisfy $\mathbf{W}(T) = \Phi^{T+1}(\mathbf{0})$, where Φ is the operator defined in (3) and Φ^t denotes its t-th iteration.

In equilibrium, at any time $t \leq T$ with status-quo $\mathbf{z}$, player i accepts policy $\mathbf{x}$ if

$$\begin{aligned}(1-\delta)x_i + \delta V_i(\mathbf{x}, t+1; T) &\geq (1-\delta)z_i + \delta V_i(\mathbf{z}, t+1; T)\\ \iff x_i &\geq z_i + \frac{\delta W_i(T-t-1)}{1-\delta W_i(T-t-1)}(x_{-i} - z_{-i}),\end{aligned}$$

where the second line uses equation (5). Hence, payoffs $\mathbf{W}(t)$ for $t \leq T$ are sufficient to characterize the equilibrium of the game with deadline T. We will therefore study the equilibria of the game in the limit as $T \to \infty$ by studying the sequence $\{\mathbf{W}(0), \mathbf{W}(1), \mathbf{W}(2), ...\}$. We define two types of games.

Definition 2. *We say that the finite horizon games are* convergent *if* $\mathbf{W}(T)$ *converges as* $T \to \infty$. *Otherwise, we say that the finite horizon games are* cycling.

4.2 Convergent Games

It is clear that when the finite horizon games are convergent, equilibrium behavior in the limit as the deadline T grows to ∞ corresponds to equilibrium behavior under an RME of the infinite horizon game studied above.

We now provide two sets of conditions under which the finite horizon games are convergent. Recall that when F is symmetric, we can generate equilibrium payoffs using the operator Ψ defined in (4), and that a fixed point W of this operator corresponds to a symmetric fixed point of Φ.

Proposition 5. *(i) Suppose Assumption 2 holds. Then, there exists* $\underline{\delta} < 1$ *such that, if* $\delta > \underline{\delta}$, *the finite horizon games are convergent.*

(ii) Suppose F *is symmetric. Then, if* $\Psi'(W) > -1$ *for all* $W \in [0, 1]$, *the finite horizon games are convergent.*

Thus, part (i) of the proposition provides a justification for selecting the RME of the infinite horizon game when δ is high, and part (ii) provides conditions under which this selection is justified if F is symmetric. However, this approach to equilibrium selection has its limitations, as the finite horizon games may be cycling.

4.3 Cycling Games

We now turn to cycling games. We start by providing some intuition as to why the finite horizon games may be cycling.

Players in our model trade off implementing a Pareto improving policy today against the benefit of waiting to see if they can change the policy in a more preferred direction tomorrow. At the deadline T, there is no benefit to waiting so the players accept every policy in $X(\mathbf{z}^T)$. In the penultimate period, however, players are less accommodating, since they anticipate that the set of acceptable policies tomorrow will depend on the policy they implement today. Graphically, the acceptance cone becomes smaller (narrower) at period $T-1$, and some extreme policies in $X(\mathbf{z}^{T-1})$ are rejected.

Now consider the third to last period $T-2$. If the probability of changing the policy next period is sufficiently small (i.e., if the distribution F places little mass on the acceptance cone tomorrow), players know that they are unlikely to enact a policy reform in the next period, and, in all likelihood, will have to wait until the final period to change the policy if they don't change it today. Since waiting for two periods is more costly than waiting only one period, players are more accommodating in period $T-2$ than they are in period $T-1$.

This suggests that, for small values of T, equilibrium play may cycle, alternating between periods in which players find it relatively easy to modify existing agreements and periods in which modifying these agreements is harder. Our next result shows that these cycles can also occur in the limit as $T \to \infty$.

To provide simple conditions under which cycling occurs, we focus on the case in which the distribution F is symmetric. Recall that when F is symmetric, players have the same equilibrium payoffs and the sum of these payoffs is the $(T+1)$-th iteration over 0 of the operator Ψ defined in (4).

Proposition 6. *If F is symmetric then Ψ has a unique fixed point $\hat{W}^*$. If, in addition,*

(i) $\Psi(\hat{W}) \neq \hat{W}^*$ for all $\hat{W} \neq \hat{W}^*$, and

(ii) there exists $\varepsilon > 0$ such that $\Psi'(\hat{W}) \leq -1$ for all $\hat{W} \in [\hat{W}^* - \varepsilon, \hat{W}^* + \varepsilon]$,

then the finite horizon games are cycling.

For some intuition as to when the conditions in Proposition 6 hold, note that

$$\Psi(\hat{W}) = H(\hat{W})(1 - \delta\hat{W}) + \delta\hat{W},$$
$$\Psi'(\hat{W}) = \delta(1 - H(\hat{W})) + H'(\hat{W})(1 - \delta\hat{W}),$$

where $H(\hat{W}) := \text{prob}(\mathbf{x} \in A(\hat{W}))\mathbb{E}[x_1 + x_2 | \mathbf{x} \in A(\hat{W})]$. The magnitude of $H'(\hat{W}) < 0$ depends on how much mass the distribution F puts on the boundary of the acceptance set $A(\hat{W})$. Hence, Proposition 6 holds when distribution F places significant mass at the boundary of $A(\hat{W})$ for all $\hat{W}$ close to the fixed point $\hat{W}^*$.

Under the conditions in Proposition 6, the players' equilibrium payoffs $\hat{W}(\tau)/2$ cycle around $\hat{W}^*/2$. Note that, in the symmetric case, the acceptance region $A_{\mathbf{z}}(\hat{W})$ is a cone with vertex $\mathbf{z}$ and lines with slopes $\frac{1-\delta\hat{W}/2}{\delta\hat{W}/2}$ and $\frac{\delta\hat{W}/2}{1-\delta\hat{W}/2}$. Therefore, the fact that payoffs $\hat{W}(\tau)/2$ cycle around $\hat{W}^*/2$ implies that there will be an alternation between periods of high and low probability of agreement; i.e., the game features cycles.

We now present an example to make the cycling result more concrete. The example also shows that the period of the cycle can vary with the model's parameters. For expositional purposes, we consider an example in which distribution F is discrete.[9]

Example 3. Suppose F is such that

$$\text{prob}_F(\mathbf{x} = (1/3, 1/4)) = \text{prob}_F(\mathbf{x} = (1/4, 1/3)) = 1/2.$$

Note that,

$$\Psi(\hat{W}) = \begin{cases} \delta\hat{W} & \text{if } \hat{W} > \frac{6}{7\delta}, \\ \frac{7}{12}(1 - \delta\hat{W}) + \delta\hat{W} & \text{if } \hat{W} \leq \frac{6}{7\delta}. \end{cases}$$

Indeed, when $\hat{W} > \frac{6}{7\delta}$, players' continuation values are too high and the set of acceptable policies has no mass under F. When $\hat{W} \leq \frac{6}{7\delta}$, the set of acceptable policies has probability 1. For $\delta = 0.95$, the sum of players' equilibrium payoffs $\hat{W}(T)$ converges as $T \to \infty$ to a two-period cycle, with payoffs alternating between $\hat{W} \approx 0.93$ and $\hat{W} \approx 0.89$. For $\delta = 0.98$, equilibrium payoffs converge as $T \to \infty$ to a five-period cycle, with payoffs alternating between $\hat{W} \approx 0.94$, $\hat{W} \approx 0.92$, $\hat{W} \approx 0.90$, $\hat{W} \approx 0.88$ and $\hat{W} \approx 0.86$.

The example gives cases in which the cycle has a fixed period of length 2, and one in which it has a fixed period of length 5. Whenever the cycle has a period of fixed length, the equilibria as $T \to \infty$ converge in behavior to some equilibrium of the infinite horizon game (but, of course, not an RME). How can we sustain cycling in the infinite horizon game? The answer is coordination. Consider cycles of period 2. If the players expect that they will both be accommodating tomorrow, they will be less accommodating today. And if they expected to be less accommodating today then they will have been more accommodating yesterday. The cycles are thus driven by self-fulfilling expectations.

5 Extensions

We now return to the infinite horizon game and report some additional results. We first move away from RME by characterizing the full set of SPE payoffs. We then look at two extensions– the case of unequal discounting, and the case in which players can

[9]Two points are worth noting about discrete F. First, Lemma 3 continues to hold when F is discrete. Second, if we endow the space of distributions with the sup norm, operator $\Phi(\mathbf{W})$ is continuous in the distribution F. Hence, Example 3 can be approximated by a sequence of continuous distributions $\{F^n\}$ converging to the discrete distribution.

strategically search for policy improvements by choosing the distributions from which policies are drawn.

5.1 Pareto Frontier of SPE payoffs

In this section, we move away from RME, and look at the set of SPE payoffs. For this analysis we assume that at the start of each period, players have access to a public randomization device.[10] Let Σ denote the set of SPE of the game.

For each Pareto weight $\lambda \in [0, 1]$, we are interested in

$$U_\lambda := \sup_{\sigma \in \Sigma} \lambda V_1^\sigma(h_0) + (1-\lambda) V_2^\sigma(h_0).$$

In words, U_λ is the largest λ-weighted sum of payoffs that can be sustained in a SPE. For each $\lambda \in [0, 1]$, define operator $\Pi^\lambda : [0, 1] \to [0, 1]$ as

$$\Pi^\lambda(U) := \text{prob}(\mathbf{x} \in A_\lambda(U))\mathbb{E}[\lambda x_1 + (1-\lambda)x_2 - (x_1 + x_2)\delta U | \mathbf{x} \in A_\lambda(U)] + \delta U,$$

where $A_\lambda(U) := \{\mathbf{x} \in X : \lambda x_1 + (1-\lambda)x_2 \geq (x_1 + x_2)\delta U\}$. Finally, for any discount factor $\delta < 1$, let $\mathcal{V}^\delta \subset X$ denote the set of SPE payoffs. Then we have:

Proposition 7. *(i) For each $\lambda \in [0, 1]$, U_λ is the largest fixed-point of Π^λ.*

(ii) If f has full support on X, then $\lim_{\delta \to 1} \mathcal{V}^\delta = X$.

In proving part (i) of Proposition 7, we show that U_λ can be attained in an SPE that takes the following form. Along the path of play, if the current period status-quo is $\mathbf{z}$, both players accept draw $\mathbf{x} \in X(\mathbf{z})$ if and only if

$$\lambda x_1 + (1-\lambda)x_2 - \lambda z_1 - (1-\lambda)z_2 \geq (x_1 + x_2 - z_1 - z_2)\delta U_\lambda.$$

If at any period t a player rejects a policy that should have been accepted, the players revert to a continuation equilibrium in which all policies get rejected forever after. So, after such a deviation, player i's continuation payoff is z_i^t.

For part (ii) of the proposition, we show that for $\lambda \in \{0, 1\}$, U_λ converges to 1 as δ goes to 1. That is, as δ approaches 1, the SPE that maximizes player i's payoffs gives player i a payoff converging to 1, and player $j \neq i$ a payoff converging to 0. Since the

[10] Adding public randomization does not affect any of our conclusions thus far: all results up to this point hold as stated.

game always admits a SPE in which both players get payoffs 0 (by having both players reject any draws at each period), and since players have access to a public randomization device, this establishes the result.

We end this section by studying outcomes under Pareto efficient equilibria. Under an equilibrium σ that attains U_λ, at $t = 0$ players accept any policy $\mathbf{x}$ in the agreement region $A_\lambda(U_\lambda)$. Suppose $\lambda \geq 1/2$ (the case of $\lambda < 1/2$ is symmetric and omitted). In this case, the set $A_\lambda(U_\lambda)$ includes all policies $\mathbf{x}$ that satisfy:

$$\lambda x_1 + (1-\lambda)x_2 \geq (x_1 + x_2)\delta U_\lambda \Longleftrightarrow x_1 \geq \frac{x_2(\delta U_\lambda - (1-\lambda))}{\lambda - \delta U_\lambda}, \tag{6}$$

where we used the fact that, for any $\lambda \in [0,1]$, $U_\lambda \leq \max\{\lambda, 1-\lambda\} = \lambda$.[11] When λ is close to 1/2, the right-hand side of (6) is negative, and so $A_\lambda(U_\lambda)$ is equal to the simplex (i.e., when $\lambda \approx 1/2$, players accept any policy). In contrast, when λ is significantly larger than 1/2, $\delta U_\lambda > 1 - \lambda$, so the agreement region $A_\lambda(U_\lambda)$ only includes policies that benefit player 1 significantly more than player 2.

5.2 Asymmetric discounting

Throughout the game, we assumed that players have the same discount factor δ. We now briefly study the case of unequal discounting.

For $i = 1, 2$, let $\delta_i \in (0,1)$ be player i's discount factor. Suppose distribution F is symmetric and that Assumption 3 holds. When $\delta_1 = \delta_2 = \delta > \underline{\delta}$, Proposition 2 says that the game has a unique RME σ, and since F is symmetric, the players have the same payoffs: $W_1^\sigma = W_2^\sigma$.[12] However, if we assume (without loss of generality) that $\delta_1 > \delta_2$, then we can show that the more patient player 1 obtains a higher payoff than player 2.

Proposition 8. *Suppose F is symmetric and Assumption 3 holds. Then, there exists $\hat{\delta} < 1$ such that, if $\delta_1 > \delta_2 > \hat{\delta}$, a unique RME σ exists and the players' RME payoffs $\mathbf{W}^\sigma = (W_1^\sigma, W_2^\sigma)$ satisfy*

$$\frac{W_1^\sigma}{W_2^\sigma} \geq \frac{1 - \delta_2(1 - H(\mathbf{W}^\sigma))}{1 - \delta_1(1 - H(\mathbf{W}^\sigma))}, \tag{7}$$

where $H(\mathbf{W}^\sigma) = prob(\mathbf{x} \in A(\mathbf{W}^\sigma))\mathbb{E}[x_1 + x_2 | \mathbf{x} \in A(\mathbf{W}^\sigma)])$.

The following example illustrates.

[11] Indeed, for any equilibrium σ, $V_1^\sigma(h_0) + V_2^\sigma(h_0) \leq 1$; thus $\lambda V_1^\sigma(h_0) + (1-\lambda)V_2^\sigma(h_0) \leq \max\{\lambda, 1-\lambda\} = \lambda$, where the last equality follows since we are assuming $\lambda \geq 1/2$.

[12] When F is symmetric, the game always has an RME that gives both players the same payoff. By Proposition 2, this is the unique RME whenever Assumption 3 holds and δ is large enough.

Example 4. Consider a setting in which policies are drawn from distribution F with support $\{\mathbf{x} \in X : x_1 + x_2 = c\}$ for some $c < 1$; i.e., the shifting frontier model. In particular, suppose x_1 is drawn from a uniform distribution on $[0, c]$, and $x_2 = c - x_1$. In this case, the acceptance region takes the form $A(\mathbf{W}) = \{\mathbf{x} : x_1 \in [c\delta_1 W_1, c(1-\delta_2 W_2)]\}$, and the probability of agreement is $1 - \delta_2 W_2 - \delta_1 W_1$. Hence, operator Φ is given by:

$$\text{for } i = 1, 2, \qquad \Phi_i(\mathbf{W}) = \frac{(1 - \delta_2 W_2 - \delta_1 W_1)^2}{2} c + \delta_i W_i.$$

Equilibrium payoffs are a fixed point of this operator, and so at the equilibrium payoffs (W_1^σ, W_2^σ) we have $W_1^\sigma / W_2^\sigma = (1 - \delta_2)/(1 - \delta_1)$.

5.3 Strategic Search

Our model with random proposals is intended to capture complexities in the environment that make it difficult for players to find new ideas to improve existing agreements. In this section, we present a natural extension of our framework in which players have some ability to influence the direction in which they will search for new policies.

As we mentioned in the introduction, our model can be interpreted as a bargaining model in which the proposer has no control over the offer that is generated; and, in this sense, our model lies at the opposite extreme of the standard approach to bargaining theory in which proposers have full control over the proposals that are considered. The extension we present in this section bridges the gap between the traditional approach and our baseline model by allowing proposers to have partial control over the payoff consequences of the offers they put on the table. We briefly describe the model here. A formal treatment appears in Appendix C.2.

Two players, $i = 1, 2$, play the following game. Time is discrete and indexed by $t = 0, 1, 2, ...$ The set of policies is X, and players have the same preferences over policies as in our baseline model. At each period t, player $i = 1, 2$ is recognized with probability $1/2$. The recognized player chooses a distribution F from a finite set of distributions $\mathcal{F}_{\mathbf{z}^t}$, where $\mathbf{z}^t$ is the current status quo. We assume that each distribution in $\mathcal{F}_{\mathbf{z}}$ has a density and support in $X(\mathbf{z})$. The alternative $\mathbf{x}$ in period t is then drawn from distribution F.

After the new alternative $\mathbf{x}$ is drawn, the two players simultaneously decide whether or not to accept it. If both players accept it, then the agreement in place in period

t becomes the new policy, so $\mathbf{x}^t = \mathbf{x}$. Otherwise, the status quo is implemented, so $\mathbf{x}^t = \mathbf{z}^t$. The status quo at time $t+1$ is the previous period policy, so $\mathbf{z}^{t+1} = \mathbf{x}^t$.

In Appendix C.2 we show that under a straightforward generalization of Assumptions 1 and 2, this extended model retains all the key features of our baseline model.

6 Conclusion

We constructed a model of collective search in which the players gradually find their way to the Pareto frontier. The difficulty in locating moderate policies that are acceptable to both players results in inefficiency. This inefficiency is driven by the commitment problem. The model also features path dependence as early agreements shape the relative likelihoods of the long run policies. In the limit as players become arbitrarily patient, however, both this path dependence and the inefficiency disappear, and players follow a unique path to a unique policy outcome on the frontier.

We also looked at the finite horizon game, and showed that equilibria of this game may feature cycles as the players alternate between being more and less accommodating. When the equilibrium of this game features a fixed cycle in the limit as the deadline goes to infinity, this equilibrium corresponds to an equilibrium also of the infinite horizon game. Cycling in these equilibria is driven by an alternating pattern of changes in the players' self-fulfilling expectations about the likelihood of making improvements to existing agreements.

Qualitatively similar results hold even if we move the model closer to a traditional bargaining setting by allowing the players to strategically choose the distributions from which new alternatives are drawn. Our model thus provides an answer to an important question (going back at least to Raiffa) of how two bargainers searching for improvements to existing policies approach the Pareto frontier. They do so in steps, while ensuring that these steps fit within the set of trajectories that ensure long-run moderation.

Appendix

A Proofs for Sections 2 and 3

Proof of Lemma 1. Before we begin, as a matter of notation, when we consider the concatenation $h_t \sqcup h_\tau = (\mathbf{x}^s, \mathbf{d}^s)_{s<t+\tau}$ of two histories h_t and h_τ, with $h_\tau = (\tilde{\mathbf{x}}^s, \tilde{\mathbf{d}}^s)_{s<\tau}$, then for all $s = 0, ..., \tau - 1$ we take $(\mathbf{x}^{t+s}, \mathbf{d}^{t+s})$ to be $(P^{-1}_{\mathbf{z}^t}(\tilde{\mathbf{x}}^s), \tilde{\mathbf{d}}^s)$, where $\mathbf{z}^t$ is the status quo at time t under the history h_t.

Fix an SPE σ and a history h_t with status quo $\mathbf{z} = \mathbf{z}(h_t)$. Consider strategy profile $\hat{\sigma}$ such that, for $i = 1, 2$ and for each history h_τ, $\hat{\sigma}_i(h_\tau) = \sigma_i(h_t \sqcup h_\tau)$. Assumption 1 guarantees that $\hat{\sigma}$ is a SPNE of the game. We now show that, for $i = 1, 2$,

$$V_i^\sigma(h_t) = z_i + (1 - z_1 - z_2)V_i^{\hat{\sigma}}(h_0).$$

Suppose for a contradiction that the result is not true. Then there exists $\epsilon > 0$ and $j \in \{1, 2\}$ such that $|V_i^\sigma(h_t) - z_j - (1 - z_1 - z_2)V_j^{\hat{\sigma}}(h_0)| > \epsilon$. Pick T such that $(1 - \delta)\delta^T < \epsilon/4$. Consider strategy profiles σ^T and $\hat{\sigma}^T$ such that: (a) for all histories h_s with $s \leq T$, $\sigma^T(h_t \sqcup h_s) = \sigma(h_t \sqcup h_s)$ and $\hat{\sigma}^T(h_s) = \hat{\sigma}(h_s)$, and (b) for all histories h_s with $s > T$, both players reject all proposals at history $h_t \sqcup h_s$ under σ^T, and both players reject all proposals at history h_s under $\hat{\sigma}^T$.[13]

Since $(1 - \delta)\delta^T < \epsilon/4$, for $i = 1, 2$ we have $|V_i^\sigma(h_t) - V_i^{\sigma^T}(h_t)| < \epsilon/4$ and $|V_i^{\hat{\sigma}}(h_0) - V_i^{\hat{\sigma}^T}(h_0)| < \epsilon/4$. Therefore, since $|V_j^\sigma(h_t) - z_j - (1 - z_1 - z_2)V_j^{\hat{\sigma}}(h_0)| > \epsilon$, we have

$$|V_j^{\sigma^T}(h_t) - z_j - (1 - z_1 - z_2)V_j^{\hat{\sigma}^T}(h_0)| > \epsilon/2.$$

For each history h_T of length T, let $(V_i^{\hat{\sigma}^T}(h_T))_{i=1,2}$ (resp., $(V_i^{\sigma^T}(h_t \sqcup h_T))_{i=1,2}$) denote players' continuation payoffs at history h_T under $\hat{\sigma}^T$ (resp., at history $h_t \sqcup h_T$ under σ^T). Let $\mathbf{z}(h_T)$ denote the status quo under history h_T, and $\mathbf{z}(h_t \sqcup h_T) = \mathbf{z} + (1 - z_1 - z_2)\mathbf{z}(h_T)$ the status quo under history $h_t \sqcup h_T$. Note that:

$$\begin{aligned} V_i^{\hat{\sigma}^T}(h_T) = \ & \text{prob}_{\mathbf{z}(h_T)}(\mathbf{x} \in A^{\hat{\sigma}}(h_T))\mathbb{E}_{\mathbf{z}(h_T)}[x_i | \mathbf{x} \in A^{\hat{\sigma}}(h_T)] \\ & + (1 - \text{prob}_{\mathbf{z}(h_T)}(\mathbf{x} \in A^{\hat{\sigma}}(h_T)))z_i(h_T), \end{aligned}$$

[13] We stress that σ^T and $\hat{\sigma}^T$ need not be equilibria of the game.

where $A^{\hat{\sigma}}(h_T)$ is the set of policies that both players accept under $\hat{\sigma}$, and where the equality follows since policy doesn't change after time T under $\hat{\sigma}^T$. Similarly,

$$\begin{aligned}V_i^{\sigma^T}(h_t \sqcup h_T) &= \text{prob}_{\mathbf{z}(h_t \sqcup h_T)}(\mathbf{x} \in A^{\sigma}(h_t \sqcup h_T))\mathbb{E}_{\mathbf{z}(h_t \sqcup h_T)}[x_i|\mathbf{x} \in A^{\sigma}(h_\tau \sqcup h_T)] \\ &\quad + (1 - \text{prob}_{\mathbf{z}(h_\tau \sqcup h_T)}(\mathbf{x} \in A^{\sigma}(h_\tau \sqcup h_T)))z_i(h_\tau \sqcup h_T) \\ &= \text{prob}_{\mathbf{z}(h_T)}(\mathbf{x} \in A^{\sigma}(h_T))\mathbb{E}_{\mathbf{z}(h_T)}[z_i + (1 - z_1 - z_2)x_i|\mathbf{x} \in A^{\sigma}(h_T)] \\ &\quad + (1 - \text{prob}_{\mathbf{z}(h_T)}(\mathbf{x} \in A^{\sigma}(h_T)))(z_i + (1 - z_1 - z_2)z_i(h_T)) \\ &= z_i + (1 - z_1 - z_2)V_i^{\hat{\sigma}^T}(h_T),\end{aligned}$$

where the second equality uses Assumption 1.

Suppose that $V_i^{\hat{\sigma}^T}(h_t \sqcup h_s) = z_i + (1 - z_1 - z_2)V_i^{\sigma^T}(h_s)$ for histories h_s of length $s = \tau + 1, ..., T$. Consider a history h_τ of length τ. Let $\mathbf{z}(h_\tau)$ be the status quo under h_τ, and $\mathbf{z}(h_t \sqcup h_\tau) = \mathbf{z} + (1 - z_1 - z_2)\mathbf{z}(h_\tau)$ the status quo under $h_t \sqcup h_\tau$. For each $\mathbf{x} \in X$, let $h_{\tau+1}^{\mathbf{x}}$ denote the history of length $\tau + 1$ that follows h_τ if policy $\mathbf{x}$ is implemented at time t. Then

$$\begin{aligned}V_i^{\hat{\sigma}^T}(h_\tau) &= \text{prob}_{\mathbf{z}(h_\tau)}(\mathbf{x} \in A^{\hat{\sigma}}(h_\tau))\mathbb{E}_{\mathbf{z}(h_\tau)}[(1 - \delta)x_i + \delta V_i^{\hat{\sigma}^T}(h_{\tau+1}^{\mathbf{x}})|\mathbf{x} \in A^{\hat{\sigma}}(h_\tau)] \\ &\quad + (1 - \text{prob}_{\mathbf{z}(h_\tau)}(\mathbf{x} \in A^{\hat{\sigma}}(h_\tau)))((1 - \delta)z_i(h_\tau) + \delta V_i^{\hat{\sigma}^T}(h_{\tau+1}^{\mathbf{z}(h_\tau)}))\end{aligned}$$

Similarly,

$$\begin{aligned}V_i^{\sigma^T}(h_t \sqcup h_\tau) &= \text{prob}_{\mathbf{z}(h_t \sqcup h_\tau)}(\mathbf{x} \in A^{\sigma}(h_t \sqcup h_\tau)) \times \\ &\quad \times \mathbb{E}_{\mathbf{z}(h_t \sqcup h_\tau)}[(1 - \delta)x_i + \delta V_i^{\sigma^T}(h_t \sqcup h_{\tau+1}^{\mathbf{x}})|\mathbf{x} \in A^{\sigma}(h_t \sqcup h_\tau)] \\ &\quad + (1 - \text{prob}_{\mathbf{z}(h_t \sqcup h_\tau)}(\mathbf{x} \in A^{\sigma}(h_t \sqcup h_\tau)))((1 - \delta)z_i(h_t \sqcup h_\tau) + \delta V_i^{\sigma^T}(h_t \sqcup h_{\tau+1}^{\mathbf{z}(h_t \sqcup h_\tau)})) \\ &= \text{prob}_{\mathbf{z}(h_\tau)}(\mathbf{x} \in A^{\hat{\sigma}}(h_\tau)) \times \\ &\quad \times \mathbb{E}_{\mathbf{z}(h_t)}[z_i + (1 - z_1 - z_2)((1 - \delta)x_i + \delta V_i^{\hat{\sigma}^T}(h_{t+1}^{\mathbf{x}}))|\mathbf{x} \in A^{\hat{\sigma}}(h_\tau)] \\ &\quad + (1 - \text{prob}_{\mathbf{z}(h_\tau)}(\mathbf{x} \in A^{\hat{\sigma}}(h_\tau)) \times \\ &\quad \times (z_i + (1 - z_1 - z_2)((1 - \delta)z_i(h_\tau) + \delta V_i^{\hat{\sigma}^T}(h_{\tau+1}^{\mathbf{z}(h_\tau)}))) \\ &= z_i + (1 - z_1 - z_2)V_i^{\hat{\sigma}^T}(h_\tau).\end{aligned}$$

Hence, $V_i^{\sigma^T}(h_t) - z_i - (1 - z_1 - z_2)V_i^{\hat{\sigma}^T}(h_0) = 0$, a contradiction.

In the case of RME, the same contradiction follows if we take $\hat{\sigma} = \sigma$. ■

Proof of Proposition 1. By the arguments in the main text, any RME payoffs are a fixed point of operator Φ. Note that Φ takes points in X to points in the same set, and is continuous. So it has a fixed point, $\mathbf{W}$. To show that for any fixed point $\mathbf{W}$ of Φ, there exists an RME σ with payoffs $\mathbf{W}$, let σ_i, $i = 1, 2$, be such that, for every history h_t and all $\mathbf{x}^t$, $\sigma_i(h_t, \mathbf{x}^t) = \mathbf{1}_{\mathbf{x}^t \in A_{i,\mathbf{z}(h_t)}(\mathbf{W})}$, where $\mathbf{z}(h_t)$ is the status-quo at history h_t. The strategy profile $\sigma = (\sigma_1, \sigma_2)$ is an RME that gives payoffs $\mathbf{W}$. ■

Proof of Proposition 2. Let Φ^δ be the operator defined in (3) indexed by $\delta < 1$, and suppose that distribution f satisfies Assumption 3 (and hence Assumption 2). We now show that there exists $\underline{\delta} < 1$ such that, for all $\delta > \underline{\delta}$, Φ^δ has a unique fixed point. Since RME payoffs are a fixed point of Φ, this implies that all RME are payoff equivalent when $\delta > \underline{\delta}$.[14] Let $A^\delta(\mathbf{W})$ denote the acceptance cone at $\mathbf{W}$.

Assumption 3 implies that $\frac{3}{4}\gamma < \underline{f}$ so pick a number $\underline{g} \in (\frac{3}{4}\gamma, \underline{f})$ and define

$$G(V) := \frac{1}{3}\underline{g}\frac{(1-\delta V)^2}{1-\delta} - V.$$

Let $V = \underline{V}^\delta$ denote the smaller of the two solutions to the quadratic equation $G(V) = 0$; specifically,

$$\underline{V}^\delta := \frac{3}{2\underline{g}\delta^2}\left(1 - \delta + \frac{2\underline{g}\delta}{3} - \sqrt{(1-\delta)\left(1 - \delta + \frac{4\underline{g}\delta}{3}\right)}\right) \tag{8}$$

which is clearly a real root. Let $Y^\delta := \{\mathbf{W} \in X : W_1 + W_2 \geq \underline{V}^\delta\}$. To prove the result, Step 1 below will show that Φ^δ cannot have a fixed point in $X \backslash Y^\delta$. The remaining steps will show that when δ is high enough, Φ^δ is a contraction when applied to points in the set Y^δ, so a unique fixed point exists on X, corresponding to the unique RME.

Before we start, it is useful to define a change of variables to calculate integrals of the form $\int_{A^\delta(\mathbf{W})}(x_1 + x_2)f(\mathbf{x})d\mathbf{x}$. For all $\mathbf{W} \in X$, $i = 1, 2$, $j \neq i$, we have

$$\int_{A^\delta(\mathbf{W})}(x_1 + x_2)f(\mathbf{x})d\mathbf{x} = \int_0^1 \int_{a\delta W_i}^{a(1-\delta W_j)} a f(x_i, a - x_i)dx_i da. \tag{9}$$

[14]This payoff-equivalence is sufficient to establish the uniqueness of RME since for all RME σ we have specified that a player accepts a policy change when indifferent, so all of the acceptance sets A_i^σ, $i = 1, 2$ are solely a function of the payoffs.

Step 1. *For all* $\delta < 1$, $\underline{V}^\delta \in (0,1)$, $\lim_{\delta\to 1} \underline{V}^\delta = 1$, *and if* $\mathbf{W}^\delta = (W_1^\delta, W_2^\delta)$ *is a fixed point of* Φ^δ, *then* $\mathbf{W}^\delta \in Y^\delta$.

Proof. We know that $\underline{V}^\delta > 0$, since $G(V) > 0$ for all $V \leq 0$. Moreover, we have that $G(1) = \frac{1}{3}\underline{g}(1-\delta) - 1 < \frac{1}{3}\underline{f}(1-\delta) - 1 \leq \frac{2}{3}(1-\delta) - 1 < 0$, where the second inequality in this chain follows from the fact that $\underline{f} \leq 2$. (If $\underline{f} > 2$ then f cannot be a density: integrating it over X would yield a number larger than 1.) Since $G(0) > 0$ and $G(1) < 0$, the Intermediate Value Theorem implies that $\underline{V}^\delta \in (0,1)$.

By inspecting the right side of (8), we see that $\lim_{\delta\to 1} \underline{V}^\delta = 1$.

Finally, we show that if $\mathbf{W}^\delta$ is a fixed point of Φ^δ, we must have $\underline{V}^\delta \leq W_1^\delta + W_2^\delta$, thus $\mathbf{W}^\delta \in Y^\delta$. To verify this, note that for all $\mathbf{W} \in X$,

$$\begin{aligned}
\Phi_1^\delta(\mathbf{W}) + \Phi_2^\delta(\mathbf{W}) &= \delta(W_1 + W_2) + \text{prob}(\mathbf{x} \in A^\delta(\mathbf{W}))\mathbb{E}[x_1 + x_2 | \mathbf{x} \in A^\delta(\mathbf{W})](1 - \delta(W_1 + W_2)) \\
&= \delta(W_1 + W_2) + \int_{A^\delta(\mathbf{W})} (x_1 + x_2) f(\mathbf{x}) d\mathbf{x}\ (1 - \delta(W_1 + W_2)) \\
&\geq \delta(W_1 + W_2) + \frac{1}{3}\underline{f}(1 - \delta(W_1 + W_2))^2 \\
&> \delta(W_1 + W_2) + \frac{1}{3}\underline{g}(1 - \delta(W_1 + W_2))^2, \qquad (10)
\end{aligned}$$

where the first and second inequalities follow since $f(\mathbf{x}) \geq \underline{f} > \underline{g} > 0$ for all $\mathbf{x}$, and applying the change of variables.[15] Next, note that if $\underline{V}^\delta > W_1 + W_2$ then $\frac{1}{3}\underline{g}(1 - \delta(W_1 + W_2))^2 > (1-\delta)(W_1 + W_2)$, from the equation that defines $\underline{V}^\delta$.[16] Combining this with (10) shows that if $\underline{V}^\delta > W_1 + W_2$ then $\Phi_1^\delta(\mathbf{W}) + \Phi_2^\delta(\mathbf{W}) > W_1 + W_2$, and therefore $\mathbf{W}$ cannot be a fixed point of Φ^δ. Thus, if $\mathbf{W}$ is a fixed point of Φ^δ, it must be that $V^\delta \leq W_1 + W_2$. ■

Step 1 above implies that Φ^δ cannot have a fixed point in $X \backslash Y^\delta$. In the remaining steps we show that when δ is sufficiently large, Φ^δ is a contraction when applied to the points in Y^δ, so it has a unique fixed point in Y^δ.

[15] In particular, for $i = 1, 2$, $j \neq i$,

$$\begin{aligned}
\int_{A^\delta(\mathbf{W})} (x_1 + x_2) f(\mathbf{x}) d\mathbf{x} &= \int_0^1 \int_{a\delta W_i}^{a(1-\delta W_j)} a f(x_i, a - x_i) dx_i da \geq \underline{f} \int_0^1 \int_{a\delta W_i}^{a(1-\delta W_j)} a dx_i da \\
&= \frac{1}{3}\underline{f}(1 - \delta W_i - \delta W_j) > \frac{1}{3}\underline{g}(1 - \delta W_i - \delta W_j).
\end{aligned}$$

[16] Note that $\underline{V}^\delta$ is the smaller real root of the quadratic equation $G(V) = 0$ that defines an upward facing parabola; so for all $W_1 + W_2 < \underline{V}^\delta$ we have $(1-\delta)G(W_1 + W_2) > 0$, as claimed. We later use the fact that since $G(1) < 0$, we must have $(1-\delta)G(W_1 + W_2) \leq 0$ for all $\mathbf{W} \in Y^\delta$.

Step 2. *There exists* $\underline{\delta} < 1$ *such that if* $\delta \in (\underline{\delta}, 1)$ *then* $\Phi^\delta(\mathbf{W}) \in Y^\delta$ *for all* $\mathbf{W} \in Y^\delta$.

Proof. Since $\delta \underline{V}^\delta \to 1$ as $\delta \to 1$ (as implied by Step 1), there exists $\underline{\delta} < 1$ such that for all $\delta > \underline{\delta}$ and all $\mathbf{W} \in Y^\delta$, $\delta - \frac{2\overline{f}}{3}\delta(1-\delta(W_1+W_2)) \geq \delta - \frac{2\overline{f}}{3}\delta(1-\delta\underline{V}^\delta) > 0$.

Now, note that $\delta V + \frac{1}{3}\underline{g}(1-\delta V)^2$ is increasing in V whenever $\delta - \frac{2}{3}\underline{g}\delta(1-\delta V) \geq 0$. Note further that $\underline{g} < \underline{f} < \overline{f}$ implies that $\delta - \frac{2}{3}\delta\underline{g}(1-\delta(W_1+W_2)) > \delta - \frac{2}{3}\overline{f}\delta(1-\delta(W_1+W_2)) \geq 0$. Hence, for all $\delta > \underline{\delta}$ and all $\mathbf{W} \in Y^\delta$, we have $\delta(W_1+W_2) + \frac{1}{3}\underline{g}(1-\delta(W_1+W_2))^2 \geq \delta\underline{V}^\delta + \frac{1}{3}\underline{g}(1-\delta\underline{V}^\delta)^2$. Therefore, for all $\delta > \underline{\delta}$ and all $\mathbf{W} \in Y^\delta$, we have

$$\Phi_1^\delta(\mathbf{W}) + \Phi_2^\delta(\mathbf{W}) \geq \delta(W_1+W_2) + \frac{1}{3}\underline{g}(1-\delta(W_1+W_2))^2 \geq \delta\underline{V}^\delta + \frac{1}{3}\underline{g}(1-\delta\underline{V}^\delta)^2 = \underline{V}^\delta,$$

where the first inequality follows from (10), the second from the argument above, and the equality from the definition of $\underline{V}^\delta$. Thus, $\Phi^\delta(\mathbf{W}) \in Y^\delta$ for all $\mathbf{W} \in Y^\delta$. ■

We now bound the derivatives of Φ_i^δ, $i = 1, 2$, whenever $\delta > \underline{\delta}$ from the step above, and use this bound to show that Φ^δ is a contraction on Y^δ.

Step 3. *Fix* $\delta \in (\underline{\delta}, 1)$ *where* $\underline{\delta} < 1$ *is a threshold satisfying the property claimed in Step 2. Then, for all* $\mathbf{W} \in Y^\delta$ *and* $i = 1, 2$, *we have*

$$\left|\frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_i}\right| + \left|\frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_j}\right| < \delta + \delta(1-\delta) < 1.$$

Proof. Suppose that $\delta > \underline{\delta}$. Note that using our change of variables, we can write Φ_i^δ, $i = 1, 2$, from (3) as

$$\begin{aligned}\Phi_i^\delta(\mathbf{W}) &= \int_{A^\delta(\mathbf{W})} (x_i - (x_1+x_2)\delta W_i) f(\mathbf{x}) d\mathbf{x} + \delta W_i \\ &= \int_0^1 \int_{a\delta W_i}^{a(1-\delta W_j)} (x_i - a\delta W_i) f(x_i, a - x_i) dx_i da + \delta W_i\end{aligned}$$

from which it follows that, for $j \neq i$,

$$\frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_i} = \delta - \delta \int_0^1 \int_{a\delta W_i}^{a(1-\delta W_j)} a f(x_i, a - x_i) dx_i da \tag{11}$$

$$\frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_j} = -\delta(1-\delta(W_i+W_j)) \int_0^1 a^2 f(a(1-\delta W_j), a\delta W_j) da \tag{12}$$

Since we have assumed in Assumption 3 that $f(\mathbf{x}) \leq \overline{f}$ for all $\mathbf{x} \in X$, we know that $\frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_i}$ is contained in the interval $[\delta - \frac{\overline{f}}{3}\delta(1-\delta(W_1+W_2)), \delta]$ while $\frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_j}$ is contained in the interval $[-\frac{\overline{f}}{3}\delta(1-\delta(W_1+W_2)), 0]$.[17] Therefore, for all $\mathbf{W} \in Y^\delta$,

$$\frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_j} \leq 0 \leq \frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_i}. \tag{13}$$

where the second inequality follows from the fact (as claimed in the first line of the proof of Step 2) that for $\delta > \underline{\delta}$ we have $\delta - \frac{2\overline{f}}{3}\delta(1-\delta(W_1+W_2)) > 0$.

Next, note that because f is Lipschitz continuous with respect to the sup norm (Assumption 3), for any $x_i \in [a\delta W_i, a(1-\delta W_j)]$, we have

$$f(x_i, a - x_i) \geq f(a(1-\delta W_j), a\delta W_j)) - \gamma a(1-\delta(W_i+W_j)) \tag{14}$$

From this, it follows that

$$\int_{a\delta W_i}^{a(1-\delta W_j)} f(x_i, a-x_i)dx_i \geq f(a(1-\delta W_j), a\delta W_j))a(1-\delta(W_i+W_j)) \\ - \gamma a^2(1-\delta(W_i+W_j))^2 \tag{15}$$

Then, using inequality (15) in (11), we have

$$\begin{aligned} \frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_i} &\leq \delta - \delta(1-\delta(W_1+W_2)) \int_0^1 a^2(f(a(1-\delta W_j), a\delta W_j)))da \\ &\qquad + \gamma\delta(1-\delta(W_1+W_2))^2 \int_0^1 a^3 da \\ &= \delta + \frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_j} + \frac{1}{4}\gamma\delta(1-\delta(W_i+W_j))^2 \end{aligned} \tag{16}$$

where the final line follows from evaluating the integral and inserting (12).

Thus, combining (13) with the conclusion of (16) we have that for all $\mathbf{W} \in Y^\delta$,

$$\left|\frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_i}\right| + \left|\frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_j}\right| = \frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_i} - \frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_j} \leq \delta + \frac{1}{4}\gamma\delta(1-\delta(W_i+W_j))^2. \tag{17}$$

[17] Indeed, $\int_0^1 \int_{a\delta W_i}^{a(1-\delta W_j)} af(x_i, a-x_i)dx_i da \leq \overline{f}\frac{1}{3}(1-\delta(W_i+W_j)$ and $\delta(1-\delta(W_i+W_j)) \int_0^1 a^2 f(a(1-\delta W_j), a\delta W_j)da \leq \frac{\overline{f}}{3}\delta(1-\delta(W_1+W_2))$.

Note further that for all $\mathbf{W} \in Y^\delta$,

$$\frac{1}{3}\underline{g}\frac{(1-\delta(W_1+W_2))^2}{1-\delta} \leq W_1 + W_2 \tag{18}$$

by definition of $\underline{V}^\delta$ (see the argument in footnote 16). Thus, for all $\mathbf{W} \in Y^\delta$,

$$\left|\frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_i}\right| + \left|\frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_j}\right| \leq \delta + \delta \frac{3}{4}\frac{\gamma}{\underline{g}}(1-\delta) < \delta + \delta(1-\delta) < 1, \tag{19}$$

where the first inequality follows from (18) and the fact that $W_1+W_2 \leq 1$ for all $\mathbf{W} \in Y^\delta$, and the second inequality uses $\gamma < \frac{4}{3}\underline{g}$ from the assumption that $\underline{g} \in (\frac{3}{4}\gamma, \underline{f})$. ■

Step 4. *For all $\delta \in (\underline{\delta}, 1)$, where $\underline{\delta} < 1$ is a threshold satisfying the property claimed in Step 2, Φ^δ is a contraction when applied to points in Y^δ.*

Proof. Let $||\cdot||$ be the sup-norm on $\mathbb{R}^2$. Fix $\mathbf{W}, \mathbf{W}' \in Y^\delta$, $\mathbf{W} \neq \mathbf{W}'$. Fix $i \in \{1, 2\}$ such that $|W_i' - W_i| \geq |W_j' - W_j|$, and suppose wlog that $W_i' > W_i$. Let $W_j(\hat{W}_i) = a + b\hat{W}_i$ be the line passing through $\mathbf{W}$ and $\mathbf{W}'$, with $b = \frac{W_j' - W_j}{W_i' - W_i}$. Note that,

$$\begin{aligned}
|\Phi_i^\delta(\mathbf{W}) - \Phi_i^\delta(\mathbf{W}')| &= \left|\int_{W_i}^{W_i'} \left(\frac{\partial \Phi_i^\delta(\hat{W}_i, W_j(\hat{W}_i))}{\partial W_i} + \frac{\partial \Phi_i^\delta(\hat{W}_i, W_j(\hat{W}_i))}{\partial W_j} b\right) d\hat{W}_i\right| \\
&\leq \int_{W_i}^{W_i'} \left(\left|\frac{\partial \Phi_i^\delta(\hat{W}_i, W_j(\hat{W}_i))}{\partial W_i}\right| + \left|\frac{\partial \Phi_i^\delta(\hat{W}_i, W_j(\hat{W}_i))}{\partial W_j}\right| |b|\right) d\hat{W}_i \\
&\leq \int_{W_i}^{W_i'} \left(\left|\frac{\partial \Phi_i^\delta(\hat{W}_i, W_j(\hat{W}_i))}{\partial W_i}\right| + \left|\frac{\partial \Phi_i^\delta(\hat{W}_i, W_j(\hat{W}_i))}{\partial W_j}\right|\right) d\hat{W}_i \\
&\leq (\delta + \delta(1-\delta))|W_i' - W_i|,
\end{aligned}$$

where the second inequality follows since $|W_i' - W_i| \geq |W_j' - W_j|$ implies $|b| \leq 1$, and the third follows from Step 3. A similar logic implies $|\Phi_j^\delta(\mathbf{W}) - \Phi_j^\delta(\mathbf{W}')| \leq (\delta + \delta(1-\delta))|W_i' - W_i|$. So for all $\delta > \underline{\delta}$ and all $\mathbf{W}, \mathbf{W}' \in Y^\delta$, $||\Phi^\delta(\mathbf{W}) - \Phi^\delta(\mathbf{W}')|| \leq (\delta + \delta(1-\delta))||\mathbf{W} - \mathbf{W}'||$; i.e., Φ^δ is a contraction of modulus $\delta + \delta(1-\delta) < 1$ on Y^δ. ■

Therefore, when $\delta > \underline{\delta}$, there is a unique RME. ■

A non-recursive Markovian equilibrium. We now show that, when Φ has multiple fixed points, the game admits a non-recursive Markovian equilibrium. Suppose that operator Φ has multiple fixed points, and let two of these be $\mathbf{W}^*$ and $\mathbf{W}^{**}$. We now construct a non-recursive Markovian equilibrium σ such that: (i) for all status quo $\mathbf{z} \in X^- := \{\mathbf{x} \in X\backslash\{0, 0\} : x_1 \geq x_2\}$, players' continuation strategies are their strategies under the RME that generates payoffs $\mathbf{W}^*$; and (ii) for all status quo $\mathbf{z} \in X^+ := \{\mathbf{x} \in X : x_1 < x_2\}$, players' continuation strategies are their strategies under the RME that generates payoffs $\mathbf{W}^{**}$.

For any $\mathbf{W}$ and for $i = 1, 2$, let $A_i^+(\mathbf{W}) := \{x \in X^+ : x_i + (1 - x_1 - x_2)\delta W_i^* \geq \delta W_i\}$ and $A^+(\mathbf{W}) := A_1^+(\mathbf{W}) \cap A_2^+(\mathbf{W})$. Similarly, let $A_i^-(\mathbf{W}) := \{x \in X^- : x_i + (1 - x_1 - x_2)\delta W_i^{**} \geq \delta W_i\}$ and $A^-(\mathbf{W}) = A_1^-(\mathbf{W}) \cap A_2^-(\mathbf{W})$. Define operator $\Phi^{NR} : X \to X$ as follows: for $i = 1, 2$,

$$\begin{aligned}\Phi_i^{NR}(\mathbf{W}) := \text{prob}(\mathbf{x} \in A^+(\mathbf{W}))&\mathbb{E}[x_i + (1 - x_1 - x_2)\delta W_i^* | \mathbf{x} \in A^+(\mathbf{W})] \\ &+ \text{prob}(\mathbf{x} \in A^-(\mathbf{W}))\mathbb{E}[x_i + (1 - x_1 - x_2)\delta W_i^{**} | \mathbf{x} \in A^-(\mathbf{W})] \\ &+ \text{prob}(\mathbf{x} \notin A^-(\mathbf{W}) \cup A^+(\mathbf{W}))\delta W_i.\end{aligned}$$

Since Φ^{NR} is continuous, and maps X into itself, it has a fixed point $\mathbf{W}^{NR}$. By construction, these payoffs $\mathbf{W}^{NR}$ can be sustained in a non-recursive Markovian equilibrium under which for all status quo $\mathbf{z} \in X^-$ (resp. $\mathbf{z} \in X^+$), players' continuation strategies are their strategies under the RME that generates payoffs $\mathbf{W}^*$ (resp. the RME that generates payoffs $\mathbf{W}^{**}$). Under this non-recursive Markovian equilibrium, when the status quo is $\mathbf{0}$, the set of policies that both players accept is $A^+(\mathbf{W}^{NR}) \cup A^-(\mathbf{W}^{NR})$.

Proof of Lemma 2. Fix any $\tau \geq t$. Since $\mathbf{x}^{\tau+1} \in A_{\mathbf{x}^\tau}(\mathbf{W})$ we have

$$x_i^{\tau+1} \geq \ell_{i,\mathbf{x}^\tau}(x_{-i}^{\tau+1}|W_i) = x_i^\tau + \frac{\delta W_i}{1 - \delta W_i}(x_{-i}^{\tau+1} - x_{-i}^\tau)$$

for both $i = 1, 2$. For any $\mathbf{x} = (x_1, x_2) \in A_{\mathbf{x}^{\tau+1}}(\mathbf{W})$, add $x_{-i}\delta W_i/(1 - \delta W_i)$ to both sides of the above inequality and rearrange to get

$$x_i^{\tau+1} + \frac{\delta W_i}{1 - \delta W_i}(x_{-i} - x_{-i}^{\tau+1}) \geq x_i^\tau + \frac{\delta W_i}{1 - \delta W_i}(x_{-i} - x_{-i}^\tau)$$

This means that

$$\ell_{i,\mathbf{x}^{\tau+1}}(y_{-i}|W_i) \geq \ell_{i,\mathbf{x}^\tau}(y_{-i}|W_i), \qquad i = 1, 2. \tag{20}$$

If $\mathbf{x} \in A_{\mathbf{x}^{\tau+1}}(\mathbf{W})$ then $x_i \geq \ell_{i,\mathbf{x}^{\tau+1}}(x_{-i}|W_{-i})$ for $i = 1, 2$, and by (20), $x_i \geq \ell_{i,\mathbf{x}^{\tau}}(x_{-i}|W_{-i})$ for $i = 1, 2$. This means that $\mathbf{x} \in A_{\mathbf{x}^{\tau}}(\mathbf{W})$; thus $A_{\mathbf{x}^{\tau+1}}(\mathbf{W}) \subseteq A_{\mathbf{x}^{\tau}}(\mathbf{W})$. ■

Proof of Proposition 3. For each $\mathbf{z} \in X$, define $\widehat{LR}_{\mathbf{z}} := A_{\mathbf{z}}(\mathbf{W}) \cap \{\mathbf{y} \in X : y_1 + y_2 = 1\}$. Since distribution $F_{\mathbf{z}}$ has full support and since $\widehat{LR}_{\mathbf{z}} \subseteq A_{\mathbf{z}}(\mathbf{W})$, any point in $\widehat{LR}_{\mathbf{z}}$ can arise as a long run policy; i.e., $\widehat{LR}_{\mathbf{z}} \subseteq LR_{\mathbf{z}}$.

Consider next a subgame starting at period t with $\mathbf{z}^t = \mathbf{z}$. By Lemma 2, $\mathbf{x}^\tau \in A_{\mathbf{z}}(\mathbf{W})$ for all $\tau \geq t$. Since $\widehat{LR}_{\mathbf{z}} = A_{\mathbf{z}}(\mathbf{W}) \cap \{\mathbf{z} \in X : y_1 + y_2 = 1\}$, any point on the frontier that is not in $\widehat{LR}_{\mathbf{z}}$ cannot arise as a long run policy when $\mathbf{z}^t = \mathbf{z}$. Hence, $LR_{\mathbf{z}} \subseteq \widehat{LR}_{\mathbf{z}}$.

This establishes that $LR_{\mathbf{z}} = \widehat{LR}_{\mathbf{z}}$, and it follows that $LR_{\mathbf{z}} \neq LR_{\mathbf{z}'}$ for $\mathbf{z} \neq \mathbf{z}'$. Lemma 2 then implies that along a realized equilibrium path $\{x_\tau\}_{\tau=t}^{\infty}$, we have $LR_{\mathbf{x}^{\tau+1}} \subseteq LR_{\mathbf{x}^{\tau}}$. The inclusion is strict when $\mathbf{x}^{\tau+1} \neq \mathbf{x}^{\tau}$ since $LR_{\mathbf{x}^{\tau+1}} \neq LR_{\mathbf{x}^{\tau}}$ in this case. ■

Recall that Φ^δ is the operator defined in (3) indexed by $\delta < 1$.

Lemma A.1. *Fix a sequence of discount factors $\{\delta_n\} \to 1$, and let $\mathbf{W}^{\delta_n} = (W_1^{\delta_n}, W_2^{\delta_n}) \in X$ be a sequence such that $\mathbf{W}^{\delta_n} = \Phi^{\delta_n}(\mathbf{W}^{\delta_n})$ for all n. Then, $\lim_{n\to\infty}(W_1^{\delta_n} + W_2^{\delta_n}) = 1$.*

Proof. We begin by deriving an expression for $W_1^\delta + W_2^\delta$ for any fixed $\delta < 1$ so that we can take the limit that is claimed in the lemma. Let $A^\delta(\mathbf{W})$ be the acceptance set defined in equation (2) when the discount factor is δ and status quo is $\mathbf{z} = \mathbf{0}$. If $\mathbf{W}^\delta = (W_1^\delta, W_2^\delta)$ is a fixed point of Φ^δ, then for $i, j = 1, 2, i \neq j$,

$$W_i^\delta = \delta W_i^\delta + \text{prob}(\mathbf{x} \in A^\delta(\mathbf{W}^\delta))\mathbb{E}[x_i - (x_i + x_j)\delta W_i^\delta | \mathbf{x} \in A^\delta(\mathbf{W}^\delta)]$$

and thus

$$W_i^\delta = \frac{\text{prob}(\mathbf{x} \in A^\delta(\mathbf{W}^\delta))\mathbb{E}[x_i | \mathbf{x} \in A(\mathbf{W}^\delta)]}{1 - \delta + \delta \text{prob}(\mathbf{x} \in A^\delta(\mathbf{W}^\delta))\mathbb{E}[x_i + x_j | \mathbf{x} \in A^\delta(\mathbf{W}^\delta)]}.$$

Then, we have

$$W_1^\delta + W_2^\delta = \frac{\text{prob}(\mathbf{x} \in A^\delta(\mathbf{W}^\delta))\mathbb{E}[x_1 + x_2 | \mathbf{x} \in A^\delta(\mathbf{W}^\delta)]}{1 - \delta + \delta \text{prob}(\mathbf{x} \in A^\delta(\mathbf{W}^\delta))\mathbb{E}[x_1 + x_2 | \mathbf{x} \in A^\delta(\mathbf{W}^\delta)]}. \tag{21}$$

Now, to prove the lemma, suppose for the purpose of establishing a contradiction that the result is not true. Hence, there exists a sequence $\{\delta_n\} \to 1$ and a positive number $\eta > 0$ such that $W_1^{\delta_n} + W_2^{\delta_n} < 1 - \eta$ for all n. Note that this implies that there is a set B with nonempty interior such that $B \subseteq A^{\delta_n}(\mathbf{W}^{\delta_n})$ for all n large enough. Therefore, $\text{prob}(\mathbf{x} \in A^{\delta_n}(\mathbf{W}^{\delta_n})) > \text{prob}(\mathbf{x} \in B) > 0$ for all n large enough. It follows from expression (21) that

$$\lim_{n\to\infty} W_1^{\delta_n} + W_2^{\delta_n} = \lim_{n\to\infty} \frac{\text{prob}(\mathbf{x} \in A^{\delta_n}(\mathbf{W}^{\delta_n}))\mathbb{E}[x_1 + x_2 | \mathbf{x} \in A^{\delta_n}(\mathbf{W}^{\delta_n})]}{1 - \delta_n + \delta_n \text{prob}(\mathbf{x} \in A^{\delta_n}(\mathbf{W}^{\delta_n}))\mathbb{E}[x_1 + x_2 | \mathbf{x} \in A^{\delta_n}(\mathbf{W}^{\delta_n})]} = 1,$$

a contradiction. Hence, it must be that $W_1^{\delta_n} + W_2^{\delta_n} \to 1$ as $\delta_n \to 1$. ■

Proof of Proposition 4. Fix a sequence $\{\delta_n\}$ with $\delta_n \to 1$. For each n, let $\mathbf{W}^{\delta_n} = (W_1^{\delta_n}, W_2^{\delta_n})$ be the players' equilibrium payoffs in a game with discount factor δ_n. Since $\mathbf{W}^{\delta_n}$ is a fixed point of Φ^{δ_n}, it follows from Lemma A.1 that $\lim_{n\to\infty} W_1^{\delta_n} + W_2^{\delta_n} = 1$. This establishes part (iii).

Consider next part (i). By Proposition 3, for each n the set LR^{δ_n} is

$$A(\mathbf{W}) \cap \{\mathbf{y} \in X : y_1 + y_2 = 1\} = \{\mathbf{x} \in X : x_1 + x_2 = 1 \text{ and } x_1 \in [\delta W_1^{\delta_n}, 1 - \delta W_2^{\delta_n}]\}.$$

By part (iii), $\delta_n(W_1^{\delta_n} + W_2^{\delta_n})$ converges to 1 as $n \to \infty$. Hence, $[\delta_n W_1^{\delta_n}, 1 - \delta_n W_2^{\delta_n}]$ converges to a point W_1^*, and so LR^{δ_n} converges to (W_1^*, W_2^*).

For part (ii), recall that

$$A^{\delta_n}(\mathbf{W}^{\delta_n}) = \left\{\mathbf{x} \in X : x_i \geq \frac{\delta_n W_i^{\delta_n}}{1 - \delta_n W_i^{\delta_n}} x_{-i} \text{ for } i = 1, 2\right\}.$$

Then using part (iii), $A^{\delta_n}(\mathbf{W}^{\delta_n})$ converges to $\{\mathbf{x} \in X : x_1/x_2 = W_1^*/W_2^*\}$.

Lastly, we show that, when $F \in C^2$, it must be that $W_1^* = W_2^* = 1/2$. Without loss of generality, suppose that $W_1^* \geq W_2^*$. Note that, for each $\delta < 1$ and corresponding equilibrium payoffs $\mathbf{W}$, the agreement region $A(\mathbf{W})$ can be written as

$$A(\mathbf{W}) = \{\mathbf{x} \in X : \exists c \in [0, 1] \ s.t. \ x_1 \in [c\underline{x}, c\overline{x}], x_2 = c - x_1\},$$

where $\underline{x} = \delta W_1$ and $\overline{x} = 1 - \delta W_2$. Since $W_1 + W_2 \to 1$ as $\delta \to 1$, we have that $\epsilon = \overline{x} - \underline{x}$ goes to zero as $\delta \to 1$. Moreover, since $W_1^* \geq W_2^*$, $\lim_{\delta\to 1} \overline{x} = \lim_{\delta\to 1} \underline{x} = W_1^* \geq 1/2$.

Define

$$\lambda := \frac{1}{1 - \delta + \delta \text{prob}(\mathbf{x} \in A(\mathbf{W}))\mathbb{E}[(x_1 + x_2) | \mathbf{x} \in A(\mathbf{W})]}.$$

Hence, for $i = 1, 2$, equilibrium payoffs $\mathbf{W} = (W_1, W_2)$ satisfy

$$W_i = \lambda \text{prob}(\mathbf{x} \in A(\mathbf{W}))\mathbb{E}[x_i | \mathbf{x} \in A(\mathbf{W})].$$

Since $\underline{x} = \delta W_1$ and $\overline{x} = 1 - \delta W_2$, we have

$$\begin{aligned} \underline{x} &= \delta\lambda \text{prob}(\mathbf{x} \in A(\mathbf{W}))\mathbb{E}[x_1 | \mathbf{x} \in A(\mathbf{W})] \\ 1 - \overline{x} &= \delta\lambda \text{prob}(\mathbf{x} \in A(\mathbf{W}))\mathbb{E}[x_2 | \mathbf{x} \in A(\mathbf{W})] \end{aligned}$$

The equations above imply:

$$\frac{\underline{x}}{1 - \overline{x}} = \frac{\text{prob}(\mathbf{x} \in A(\mathbf{W}))\mathbb{E}[x_1 | \mathbf{x} \in A(\mathbf{W})]}{\text{prob}(\mathbf{x} \in A(\mathbf{W}))\mathbb{E}[x_2 | \mathbf{x} \in A(\mathbf{W})]}$$

which is equivalent to

$$\epsilon \text{prob}(\mathbf{x} \in A(\mathbf{W}))\mathbb{E}[x_1 | \mathbf{x} \in A(\mathbf{W})] = \text{prob}(\mathbf{x} \in A(\mathbf{W}))\mathbb{E}[x_1 - \underline{x}(x_1 + x_2) | \mathbf{x} \in A(\mathbf{W})], \tag{22}$$

since $\epsilon = \overline{x} - \underline{x}$. Note next that

$$\begin{aligned} \text{prob}(\mathbf{x} \in A(\mathbf{W}))\mathbb{E}[x_1 | \mathbf{x} \in A(\mathbf{W})] &= \int_{\mathbf{x} \in A(\mathbf{W})} x_1 f(\mathbf{x}) d\mathbf{x} \\ &= \int_0^1 \int_{c\underline{x}}^{c\overline{x}} x_1 f(x_1, c - x_1) dx_1 dc \end{aligned} \tag{23}$$

and

$$\begin{aligned} \text{prob}(\mathbf{x} \in A(\mathbf{W}))\mathbb{E}[x_1 + x_2 | \mathbf{x} \in A(\mathbf{W})] &= \int_{\mathbf{x} \in A(\mathbf{W})} (x_1 + x_2) f(\mathbf{x}) d\mathbf{x} \\ &= \int_0^1 \int_{c\underline{x}}^{c\overline{x}} c f(x_1, c - x_1) dx_1 dc \end{aligned} \tag{24}$$

Since $F \in C^2$, for each $c \in [0, 1]$ and all $\hat{x}_1 \in [c\underline{x}, c\overline{x}]$, we have that

$$\left| f(\hat{x}_1, c - \hat{x}_1) - \frac{\int_{c\underline{x}}^{c\overline{x}} f(x_1, c - x_1) dx_1}{c(\overline{x} - \underline{x})} \right| \leq a\epsilon \tag{25}$$

for some constant $a > 0$ (again, recall that $\epsilon = \overline{x} - \underline{x}$). For each $c \in [0,1]$, define $k(c) := \int_{c\underline{x}}^{c\overline{x}} f(x_1, c - x_1)dx_1$. Using (25) in (23) and (24), we have

$$\text{prob}(\mathbf{x} \in A(\mathbf{W}))\mathbb{E}[x_1|\mathbf{x} \in A(\mathbf{W})] = \int_0^1 k(c)c\left(\underline{x} + \frac{\epsilon}{2}\right)dc + O(\epsilon^2)$$
$$\text{prob}(\mathbf{x} \in A(\mathbf{W}))\mathbb{E}[x_1 + x_2|\mathbf{x} \in A(\mathbf{W})] = \int_0^1 k(c)cdc + O(\epsilon^2)$$

Define $K := \int_0^1 k(c)cdc$. Using these expressions in (22), we get

$$\epsilon K\left(\underline{x} + \frac{\epsilon}{2} + O(\epsilon^2)\right) = K\left(\frac{\epsilon}{2} + O(\epsilon^2)\right)$$

Since $\epsilon \to 0$ as $\delta \to 1$, we have $\lim_{\delta \to 1} \underline{x} = 1/2$. Since $\underline{x} = \delta W_1$, we get that $W_1^* = 1/2$ and $W_2^* = 1 - W_1^* = 1/2$. ■

B Proofs for Section 4

Proof of Lemma 3. We start with part (i). For each $\mathbf{z} \in X$, $\mathbb{E}_{\mathbf{z}}[\cdot]$ is the expectation operator under distribution $F_{\mathbf{z}}$. Let $\mathbb{E}[\cdot]$ be the expectation operator under distribution $F_{\mathbf{0}} = F$. We prove the result by induction.

Consider a subgame starting at period $t = T$ with status quo $\mathbf{z}^T = \mathbf{z} \in X$. Note that

$$V_i(\mathbf{z}, T; T) = \mathbb{E}_{\mathbf{z}}[x_i] = z_i + (1 - z_1 - z_2)\mathbb{E}[x_i],$$

where the first equality follows since, at time T both players accept any policy, and the second equality follows from Assumption 1.

Now, consider the game with deadline $T = 0$. Player i's equilibrium payoffs satisfy $W_i(0) = \mathbb{E}[x_i]$. Hence,

$$V_i(\mathbf{z}, T; T) = z_i + (1 - z_i - z_j)W_i(0)$$

which establishes the basis case.

For the induction step, suppose that (5) holds for all t such that $T - t = 0, 1, ..., n-1$ and for all $\mathbf{z} \in X$. Fix a subgame starting at period $\tilde{t}$ with $T - \tilde{t} = n$ and with status quo $\mathbf{z}^{\tilde{t}} = \mathbf{z} \in X$. We abuse previous notation and in this proof let $A_{\mathbf{z}}(\tilde{t})$ be the set of policies that both players accept at period $\tilde{t}$ when $\mathbf{z}^{\tilde{t}} = \mathbf{z}$; that is,

$$\begin{aligned}A_{\mathbf{z}}(\tilde{t}) &= \left\{\mathbf{x} \in X(\mathbf{z}) : (1-\delta)x_i + \delta V_i(\mathbf{x}, \tilde{t}+1; T) \geq (1-\delta)z_i + \delta V_i(\mathbf{z}, \tilde{t}+1; T) \text{ for } i = 1, 2\right\} \\ &= \left\{\mathbf{x} \in X(\mathbf{z}) : (x_i - z_i) \geq (x_1 + x_2 - z_1 + z_2)\delta W_i(T - \tilde{t} - 1) \text{ for } i = 1, 2\right\},\end{aligned}$$

where the second line follows since, by the induction hypothesis, (5) holds for $t = \tilde{t} + 1$. Note then that

$$\begin{aligned}V_i(\mathbf{z}, \tilde{t}; T) &= \text{prob}(\mathbf{x} \in A_{\mathbf{z}}(\tilde{t}))\mathbb{E}_{\mathbf{z}}\left[(1-\delta)x_i + \delta V_i(\mathbf{x}, \tilde{t}+1; T) \,\middle|\, \mathbf{x} \in A_{\mathbf{z}}(\tilde{t})\right] \\ &\quad + \text{prob}(\mathbf{x} \notin A_{\mathbf{z}}(\tilde{t}))\left((1-\delta)z_i + \delta V_i(\mathbf{z}, \tilde{t}+1; T)\right) \\ &= \text{prob}(\mathbf{x} \in A_{\mathbf{z}}(\tilde{t}))\mathbb{E}_{\mathbf{z}}\left[x_i + (1 - x_1 - x_2)\delta W_i(T - \tilde{t} - 1) \,\middle|\, \mathbf{z} \in A_{\mathbf{z}}(\tilde{t})\right] \\ &\quad + \text{prob}(\mathbf{x} \notin A_{\mathbf{z}}(\tilde{t}))\left(z_i + (1 - z_1 - z_2)\delta W_i(T - \tilde{t} - 1)\right) \\ &= \text{prob}(\mathbf{x} \in A_{\mathbf{z}}(\tilde{t}))\mathbb{E}_{\mathbf{z}}\left[(x_i - z_i) + (z_1 + z_2 - x_1 - x_2)\delta W_i(T - \tilde{t} - 1) \,\middle|\, \mathbf{x} \in A_{\mathbf{z}}(\tilde{t})\right] \\ &\quad + z_i + (1 - z_1 - z_2)\delta W_i(T - \tilde{t} - 1) \qquad (26)\end{aligned}$$

where the second equality follows since, by the induction hypothesis, (5) holds for $t = \tilde{t} + 1$, and the last inequality follows since $\text{prob}(\mathbf{x} \notin A_{\mathbf{z}}(\tilde{t})) = 1 - \text{prob}(\mathbf{x} \in A_{\mathbf{z}}(\tilde{t}))$.

Consider next a game with deadline $T - \tilde{t}$. Let $\tilde{A}$ be the set of policies that both players accept at the first period of the game:

$$\begin{aligned}\tilde{A} &= \left\{\mathbf{x} \in X : (1-\delta)x_i + \delta V_i(\mathbf{x}, 1; T - \tilde{t}) \geq \delta V_i(\mathbf{0}, 1; T - \tilde{t}) \text{ for } i = 1, 2\right\} \\ &= \left\{\mathbf{x} \in X : x_i \geq (x_1 + x_2)\delta W_i(T - \tilde{t} - 1) \text{ for } i = 1, 2\right\},\end{aligned}$$

where the second line follows since, by the induction hypothesis, for all $V_i(\mathbf{x}, 1; T - \tilde{t}) = x_i + (1 - x_i - x_j)W_i(T - \tilde{t})$ for all $\mathbf{x}$. Player i's payoff in this game is equal to

$$\begin{aligned}W_i(T - \tilde{t}) &= \text{prob}(\mathbf{x} \in \tilde{A})\mathbb{E}\left[(1-\delta)x_i + \delta V_i(\mathbf{x}, 1; T - \tilde{t}) \,\middle|\, \mathbf{x} \in \tilde{A}\right] + \text{prob}(\mathbf{x} \notin \tilde{A})\delta V_i(\mathbf{0}, 1; T - \tilde{t}) \\ &= \text{prob}(\mathbf{x} \in \tilde{A})\mathbb{E}\left[x_i - (x_1 + x_2)\delta W_i(T - \tilde{t} - 1) \,\middle|\, \mathbf{x} \in \tilde{A}\right] + \delta W_i(T - \tilde{t} - 1) \qquad (27)\end{aligned}$$

Assumption 1 implies that

$$\begin{aligned}&\text{prob}(\mathbf{x} \in A_{\mathbf{z}}(\tilde{t}))\mathbb{E}_{\mathbf{z}}\left[x_i - z_i + (z_1 + z_2 - x_1 - x_2)\delta W_i(T - \tilde{t} - 1) \,\middle|\, \mathbf{x} \in A_{\mathbf{z}}(\tilde{t})\right] \\ =&(1 - z_1 - z_2)\text{prob}(\mathbf{x} \in \tilde{A})\mathbb{E}\left[x_i - (x_1 + x_2)\delta W_i(T - \tilde{t} - 1) \,\middle|\, \mathbf{x} \in \tilde{A}\right].\end{aligned}$$

Combining this with (26) and (27),

$$V_i(\mathbf{z}, \tilde{t}; T) = z_i + (1 - z_1 - z_2)W_i(T - \tilde{t}).$$

which establishes the result.

Now let us turn to part (ii). The proof is again by induction. Consider the game with deadline $T = 0$. Since it is optimal for both players to accept any alternative $\mathbf{x} \in X$ that is drawn, player i's payoff in this game satisfies $W_i(T) = \mathbb{E}[x_i] = \Phi_i(\mathbf{0})$.

Suppose next that $W_i(\tau) = \Phi_i^{\tau+1}(\mathbf{0})$ for all $\tau = 0, ..., T-1$, and consider game with deadline T. The set of alternatives that both players accept in the initial period are given by

$$\begin{aligned}\tilde{A} &= \{\mathbf{x} \in X : (1-\delta)x_i + \delta V_i(\mathbf{x}, 1; T) \geq \delta V_i(\mathbf{0}, 1; T) \text{ for } i = 1, 2\} \\ &= \{\mathbf{x} \in X : x_i \geq (x_1 + x_2)\delta W_i(T-1) \text{ for } i = 1, 2\},\end{aligned}$$

where the second line follows from part (i). Player i's payoff $W_i(T)$ satisfies

$$\begin{aligned}W_i(T) &= \text{prob}(\mathbf{x} \in \tilde{A})\mathbb{E}\left[(1-\delta)x_i + \delta V_i(\mathbf{x}, 1; T) \,\middle|\, \mathbf{x} \in \tilde{A}\right] + \text{prob}(\mathbf{x} \notin \tilde{A})\delta V_i(\mathbf{0}, 1; T) \\ &= \text{prob}(\mathbf{x} \in \tilde{A})\mathbb{E}\left[x_i - (x_1 + x_2)\delta W_i(T-1) \,\middle|\, \mathbf{x} \in \tilde{A}\right] + \delta W_i(T-1) \qquad (28)\end{aligned}$$

where the equality follows after using part (i). By the induction hypothesis, $\mathbf{W}(T-1) = \Phi^T(\mathbf{0})$, and so $\tilde{A} = A(\Phi^T(\mathbf{0}))$. Using this in (28), $W_i(T) = \Phi(\Phi^T(\mathbf{0})) = \Phi^{T+1}(\mathbf{0})$. ■

Proof of Proposition 5. We start with part (i) and recall various facts from the proof of Proposition 2. First, recall that $\underline{V}^\delta$ is the smaller of the two solutions to the quadratic equation $\frac{1}{3}\underline{g}\frac{(1-\delta\underline{V}^\delta)^2}{1-\delta} = \underline{V}^\delta$, where $\underline{g} \in (0, \underline{f})$.[18] Also from the proof of Proposition 2, for $i, j = 1, 2$, $i \neq j$, $\frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_i}$ is given by (11) and lies in the interval $[\delta - \frac{\overline{f}}{3}\delta(1 - \delta(W_1 + W_2)), \delta]$ while $\frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_j}$ is given by (12) and lies in $[-\frac{\overline{f}}{3}\delta(1-\delta(W_1+W_2)), 0]$. The proof of Proposition 2 also showed that for all $\delta > \underline{\delta}$ and all $\mathbf{W} \in Y^\delta$, $\frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_i} \geq 0 \geq \frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_j}$. Finally, it showed that for all $\delta > \underline{\delta}$ and all $\mathbf{W} \in Y^\delta$, $\Phi^\delta(\mathbf{W}) \in Y^\delta$.

Now fix $\delta > \underline{\delta}$. Towards establishing the result, we first show that if $\mathbf{W} \in Y^\delta$, then $(\Phi^\delta)^T(\mathbf{W})$ converges to a fixed point of Φ as $T \to \infty$. To see why, fix $\mathbf{W}^0 \in Y^\delta$, and let

[18] For this proof, we don't need Assumption 3 to hold, and we also don't need $\underline{g} > \frac{3}{4}\gamma$.

$\{\mathbf{W}^t\}_{t=0}^{\infty}$ be such that, for $t = 1, 2, ...$, $\mathbf{W}^t = (\Phi^\delta)^t(\mathbf{W}) = (\Phi^\delta)(\mathbf{W}^{t-1})$. Note then that $\mathbf{W}^t \in Y^\delta$ for all t.[19]

There are two cases to consider: (a) there exists $s \geq 1$ and $i = 1, 2, i \neq j$ such that $W_i^s \geq W_i^{s-1}$ and $W_j^s \leq W_j^{s-1}$, and (b) for all $s \geq 1$, either $W_1^s \geq W_1^{s-1}$ and $W_2^s \geq W_2^{s-1}$ or $W_1^s \leq W_1^{s-1}$ and $W_2^s \leq W_2^{s-1}$.

Consider first case (a), so there exists $s \geq 1$ and $i = 1, 2, i \neq j$ such that $W_i^s \geq W_i^{s-1}$ and $W_j^s \leq W_j^{s-1}$. Since $\Phi_i^\delta(W_i, W_j)$ is increasing in W_i and decreasing in W_j whenever $\mathbf{W} \in Y^\delta$, it follows that $W_i^{s+1} = \Phi_i(\mathbf{W}^s) \geq \Phi_i(\mathbf{W}^{s-1}) = W_i^s$ and $W_j^{s+1} = \Phi_j(\mathbf{W}^s) \leq \Phi_j(\mathbf{W}^{s-1}) = W_j^s$. Applying the same argument inductively, we get that $\{W_i^t\}$ is an increasing sequence and $\{W_j^t\}$ is a decreasing sequence for all $t \geq s$. Since $\mathbf{W}^t \in X$ for all t, $\mathbf{W}^t$ converges to some $\mathbf{W}^*$ as $t \to \infty$.

Consider next case (b). For $i, j = 1, 2, j \neq i$, define

$$M_{i,i} := \sup_{\mathbf{W} \in Y^\delta} \left| \frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_i} \right| \qquad M_{i,j} := \sup_{\mathbf{W} \in Y^\delta} \left| \frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_j} \right|$$

Note that, for $\delta > \underline{\delta}$, we have that $M_{i,i} \in [0, \delta]$ and $M_{i,j} \in [0, \delta]$.[20] Recall that, in this case, for all $t \geq 1$, either $W_i^t \geq W_i^{t-1}$ for $i = 1, 2$ or $W_i^t \leq W_i^{t-1}$ for $i = 1, 2$. Since $\Phi_i(\mathbf{W})$ is increasing in W_i and decreasing in W_j, for all $t \geq 1$ and for $i = 1, 2$, we have

$$\begin{aligned}
|W_i^{t+1} - W_i^t| &= |\Phi_i^\delta(\mathbf{W}^t) - \Phi_i^\delta(\mathbf{W}^{t-1})| \\
&= |\Phi_i^\delta(\mathbf{W}^t) - \Phi_i^\delta(W_i^{t-1}, W_j^t) + \Phi_i^\delta(W_i^{t-1}, W_j^t) - \Phi_i^\delta(\mathbf{W}^{t-1})| \\
&\leq \max\{|\Phi_i^\delta(\mathbf{W}^t) - \Phi_i^\delta(W_i^{t-1}, W_j^t)|, |\Phi_i^\delta(W_i^{t-1}, W_j^t) - \Phi_i^\delta(\mathbf{W}^{t-1})|\} \\
&\leq \max\{M_{i,i}, M_{i,j}\}||\mathbf{W}^t - \mathbf{W}^{t-1}|| \\
&\leq \delta||\mathbf{W}^t - \mathbf{W}^{t-1}||,
\end{aligned}$$

where the first inequality follows since Φ_i^δ is increasing in W_i and decreasing in W_j. Hence, $\{\mathbf{W}^t\}$ is a Cauchy sequence, and so it is convergent.

We now show that the finite-horizon games are convergent whenever $\delta > \underline{\delta}$. Fix $\delta > \underline{\delta}$. There are two cases to consider: (bi) $\Phi^\delta(\mathbf{0}) \in Y^\delta$, and (bii) $\Phi^\delta(\mathbf{0}) \notin Y^\delta$. Consider case (bi). By our arguments above, $\mathbf{W}(T) = (\Phi^\delta)^T(\Phi^\delta(\mathbf{0}))$ converges as $T \to \infty$.

[19] Indeed, for all $\delta > \underline{\delta}$ and all $\mathbf{W} \in Y^\delta$, $\Phi^\delta(\mathbf{W}) \in Y^\delta$.

[20] For all $\delta > \underline{\delta}$ and all $\mathbf{W} \in Y^\delta$, $\delta - \overline{f}\frac{2}{3}\delta(1 - \delta(W_1 + W_2)) \geq 0$ (see Step 2 in the proof of Proposition 2). Since $\frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_i} \in [\delta - \frac{\overline{f}}{3}\delta(1 - \delta(W_1 + W_2)), \delta]$ and $\frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_j} \in [-\frac{\overline{f}}{3}\delta(1 - \delta(W_1 + W_2)), 0]$, we have that $M_{i,i} \in [0, \delta]$ and $M_{i,j} \in [0, \delta]$ for all $\delta > \underline{\delta}$.

Consider next case (bii), so that $\Phi^\delta(\mathbf{0}) \notin Y^\delta$. By equation (10), for all $\mathbf{W}$ we have

$$\Phi_1^\delta(\mathbf{W}) + \Phi_2^\delta(\mathbf{W}) - (W_1 + W_2) \geq \delta(W_1 + W_2) + \frac{1}{3}\underline{g}(1 - \delta(W_1 + W_2))^2 - (W_1 + W_2)$$
$$+ \frac{1}{3}(\underline{f} - \underline{g})(1 - \delta(W_1 + W_2))^2. \tag{29}$$

For all $\mathbf{W} \in X \backslash Y^\delta$, we have that

$$\delta(W_1 + W_2) + \frac{1}{3}\underline{g}(1 - \delta(W_1 + W_2))^2 > W_1 + W_2.$$

Using (29), for all $\mathbf{W} \in X \backslash Y^\delta$ we have

$$\Phi_1^\delta(\mathbf{W}) + \Phi_2^\delta(\mathbf{W}) - (W_1 + W_2) > \frac{1}{3}(\underline{f} - \underline{g})(1 - \delta(W_1 + W_2))^2 \geq \frac{1}{3}(\underline{f} - \underline{g})(1 - \delta)^2,$$

where the last inequality follows since $W_1 + W_2 \leq 1$. This implies that, when $\Phi(\mathbf{0}) \notin Y^\delta$, there exists $t \geq 1$ such that $\Phi_1^\delta((\Phi^\delta)^t(\mathbf{0})) + \Phi_2^\delta((\Phi^\delta)^t(\mathbf{0})) \geq \underline{V}^\delta$. Hence, by our arguments above, $(\Phi^\delta)^{t+s}(\mathbf{0})$ converges as $s \to \infty$, and so the games are convergent.

Consider next part (ii). Note that when F is symmetric, both players have the same equilibrium payoffs for all deadlines, i.e. $W_1(T) = W_2(T)$ for all $T \geq 0$. Let $\hat{W}(T) = W_1(T) + W_2(T)$, and note that $\hat{W}(T) = \Psi^{T+1}(0)$ (where Ψ is the operator defined in equation (4)).

For any $\hat{W} \in [0, 1]$, define

$$H(\hat{W}) := \text{prob}(\mathbf{x} \in A(\hat{W}))\mathbb{E}[x_1 + x_2 | \mathbf{x} \in A(\hat{W})],$$

so that $\Psi(\hat{W}) = \delta\hat{W} + H(\hat{W})(1 - \delta\hat{W})$. Note that $H'(\hat{W}) \leq 0$. Indeed, $\hat{W}'' > \hat{W}'$ implies that $A(\hat{W}'') \subset A(\hat{W}')$, so for any $\hat{W}'' > \hat{W}'$,

$$\text{prob}(\mathbf{x} \in A(\hat{W}''))\mathbb{E}[x_1 + x_2 | \mathbf{x} \in A(\hat{W}'')] \leq \text{prob}(\mathbf{x} \in A(\hat{W}'))\mathbb{E}[x_1 + x_2 | \mathbf{x} \in A(\hat{W}')].$$

It then follows that $\Psi'(\hat{W}) = \delta(1 - H(\hat{W})) + H'(\hat{W})(1 - \delta\hat{W}) \leq \delta < 1$ for all $\hat{W} \in [0, 1]$. When $\Psi'(\hat{W}) > -1$ for all $\hat{W} \in [0, 1]$, $|\Psi'(\hat{W})| < 1$ for all $\hat{W} \in [0, 1]$. This implies that Ψ is a contraction, and the sequence $\{\hat{W}(T)\}$ converges to its unique fixed point. Hence, the games are convergent. ■

Proof of Proposition 6. First we prove that if F is symmetric, then the fixed point of Ψ is unique. Operator Ψ is continuous and maps $[0,1]$ onto itself, so by Brouwer's fixed point theorem, it has a fixed point.

Let $\hat{W}$ be a fixed point of Ψ. Then, $\hat{W}$ satisfies

$$\hat{W} = \frac{\text{prob}(\mathbf{x} \in A(\hat{W}))\mathbb{E}[x_1 + x_2 | \mathbf{x} \in A(\hat{W})]}{1 - \delta + \delta \text{prob}(\mathbf{x} \in A(\hat{W}))\mathbb{E}[x_1 + x_2 | \mathbf{x} \in A(\hat{W})]}. \tag{30}$$

Note that $A(\hat{W}'') \subset A(\hat{W}')$ for any $\hat{W}'' > \hat{W}'$. Therefore, for any $\hat{W}'' > \hat{W}'$,

$$\text{prob}(\mathbf{x} \in A(\hat{W}''))\mathbb{E}[x_1 + x_2 | \mathbf{x} \in A(\hat{W}'')] \leq \text{prob}(\mathbf{x} \in A(\hat{W}'))\mathbb{E}[x_1 + x_2 | \mathbf{x} \in A(\hat{W}')].$$

Thus, the right side of (30) is decreasing in $\hat{W}$, and so Ψ has a unique fixed point.

Next, the sum of the players' equilibrium payoffs in a game with deadline T is $\hat{W}(T) = \Psi^{T+1}(0)$. By standard results in dynamical systems (e.g., Theorem 4.2 in De la Fuente (2000)), under conditions (i) and (ii) in the statement of the proposition the sequence $\{\hat{W}(T)\}$ does not converge. So the games must be cycling. ■

C Proofs and Details for Section 5

C.1 Proofs for Stated Results

Proof of Proposition 7. We start with part (i).

Fix $\lambda \in [0,1]$, and let W_λ be the largest fixed point of $\Pi^\lambda(\cdot)$. We start by showing that there exists an SPE σ^λ in which the λ-weighted sum of players' payoffs is W_λ. Strategy profile σ^λ is as follows. Along the path of play, at each period t with status-quo $\mathbf{z}_t$, player $i = 1, 2$ accepts policy draw $\mathbf{x} \in X$ if and only if

$$\lambda x_1 + (1-\lambda)x_2 + \delta(1 - x_1 - x_2)W_\lambda \geq \lambda z_1 + (1-\lambda)z_2 + \delta(1 - z_1 - z_2)W_\lambda$$

which is equivalent to

$$\lambda(x_1 - z_1) + (1-\lambda)(x_2 - z_2) \geq \delta(x_1 + x_2 - z_1 - z_2)W_\lambda.$$

If at any period t a player rejects a policy that was supposed to be accepted, then from time $t+1$ onwards both players reject all policies. Note that the payoff player i obtains from rejecting a policy at time t that should have been accepted is z_i^t. Since her continuation payoff at time t from playing according σ^λ is weakly larger than z_i^t, this strategy profile constitutes a SPE. Moreover, players' λ-weighted sum of payoffs under σ^λ is W_λ. Hence, $U_\lambda \geq W_\lambda$.

Next, we show that $U_\lambda \leq W_\lambda$. Fix $\sigma \in \Sigma$, and let $A^\sigma(h_0)$ denote the set of draws that both players accept under σ at history h_0. For each $\mathbf{x} \in X$, let $h_0^{\mathbf{x}}$ denote the history that follows h_0 if $\mathbf{x}$ is drawn and both players accept it. Then,

$$\begin{aligned}&\lambda V_1^\sigma(h_0) + (1-\lambda)V_2^\sigma(h_0)\\ =&\text{prob}(\mathbf{x} \in A^\sigma(h_0))\mathbb{E}^\sigma[(1-\delta)(\lambda x_1 + (1-\lambda)x_2) + \delta(\lambda V_1^\sigma(h_0^{\mathbf{x}}) + (1-\lambda)V_2^\sigma(h_0^{\mathbf{x}})|\mathbf{x} \in A^\sigma(h_0)]\\ &+ \delta\text{prob}(\mathbf{x} \notin A^\sigma(h_0))\mathbb{E}^\sigma[\lambda V_1^\sigma(h_1) + (1-\lambda)V_2^\sigma(h_1)|\mathbf{x} \notin A^\sigma(h_0)] \end{aligned} \tag{31}$$

By Lemma 1, for any $\mathbf{x} \in X$ it must be that $\lambda V_1^\sigma(h_0^{\mathbf{x}}) + (1-\lambda)V_2^\sigma(h_0^{\mathbf{x}}) \leq (\lambda x_1 + (1-\lambda)x_2) + (1 - x_1 - x_2)U_\lambda$. Therefore, by (31),

$$\begin{aligned}&\lambda V_1^\sigma(h_0) + (1-\lambda)V_2^\sigma(h_0)\\ \leq&\text{prob}(\mathbf{x} \in A^\sigma(h_0))\mathbb{E}^\sigma[\lambda x_1 + (1-\lambda)x_2 - (x_1+x_2)\delta U_\lambda|\mathbf{x} \in A^\sigma(h_0)] + \delta U_\lambda\\ \leq&\text{prob}(\mathbf{x} \in A_\lambda(U_\lambda))\mathbb{E}^\sigma[\lambda x_1 + (1-\lambda)x_2 - (x_1+x_2)\delta U_\lambda|\mathbf{x} \in A_\lambda(U_\lambda)] + \delta U_\lambda = \Pi^\lambda(U_\lambda), \end{aligned} \tag{32}$$

where the second inequality follows since $A_\lambda(U_\lambda) = \{\mathbf{x} : \lambda x_1 + (1-\lambda)x_2 \geq (x_1+x_2)\delta U_\lambda\}$. Since (32) holds for any SPE σ, it must be that $U_\lambda \leq \Pi^\lambda(U_\lambda)$.

Finally, we show that $\Pi^\lambda(U) < U$ for all $U > W_\lambda$. To see why, note that

$$\Pi^\lambda(1) = \text{prob}(\mathbf{x} \in A_\lambda(1))\mathbb{E}^\sigma[\lambda x_1 + (1-\lambda)x_2 - (x_1+x_2)\delta|\mathbf{x} \in A_\lambda(1)] + \delta < 1,$$

where the strict inequality follows since, for all $\mathbf{x} \in X$,

$$\lambda x_1 + (1-\lambda)x_2 - (x_1+x_2)\delta = (x_1+x_2)(1-\delta) - (1-\lambda)x_1 - \lambda x_2 < (1-\delta).$$

Towards a contradiction, suppose that there exists $U > W_\lambda$ with $\Pi^\lambda(U) \geq U$. Since W_λ is the largest fixed point of Π^λ, it must be that $\Pi^\lambda(U) > U$. Since $\Pi^\lambda(1) < 1$, and since Π^λ is continuous, there exists $U' \in (U,1)$ such that $\Pi^\lambda(U') = U'$, a contradiction. Hence, $\Pi^\lambda(U) < U$ for all $U > W_\lambda$. Since $U_\lambda \leq \Pi^\lambda(U_\lambda)$, it follows that $U_\lambda \leq W_\lambda$.

Now for part (ii). For $\delta < 1$ and $\lambda \in [0,1]$, let U_λ^δ denote the largest fixed point of Π^λ under discount factor δ. To prove the result, we show that for $\lambda \in \{0,1\}$, $\lim_{\delta\to 1} U_\lambda^\delta = 1$. Note that this implies that payoffs $(1,0)$ and $(0,1)$ both belong in $\lim_{\delta\to 1} \mathcal{V}^\delta$. Since $\mathbf{0} \in \mathcal{V}^\delta$ for all $\delta < 1$ (because the game has a SPE in which both players reject all offers), we have that $\lim_{\delta\to 1} \mathcal{V}^\delta = X$.

Fix $\lambda = 1$ (the proof for $\lambda = 0$ is symmetric and omitted). For each $\delta < 1$, U_1^δ solves:

$$U_1^\delta = \frac{\text{prob}(\mathbf{x} \in A_1^\delta(U_1^\delta))\mathbb{E}[x_1|\mathbf{x} \in A_1^\delta(U_1^\delta)]}{1 - \delta + \text{prob}(\mathbf{x} \in A_1^\delta(U_1^\delta))\mathbb{E}[x_1 + x_2|\mathbf{x} \in A_1^\delta(U_1^\delta)]} \tag{33}$$

Fix a sequence $\delta_n \to 1$, and suppose by contradiction that $\lim_{n\to\infty} U_1^{\delta_n} = k < 1$ (if needed, take a convergent subsequence). Note then that $A_1^{\delta_n}(U_1^{\delta_n}) \to A_1^* := \{\mathbf{x} \in X : x_1 \geq (x_1 + x_2)k\}$. Since $k < 1$, and since f has full support, set A^* has positive measure. Moreover, since f has full support, $\mathbb{E}[x_1 + x_2|\mathbf{x} \in A^*] < \frac{1}{k}\mathbb{E}[x_1|\mathbf{x} \in A^*]$.[21] Using this in (33), we get

$$\begin{aligned} k = \lim_{n\to\infty} U_1^{\delta_n} &= \lim_{n\to\infty} \frac{\text{prob}(\mathbf{x} \in A_1^\delta(U_1^\delta))\mathbb{E}[x_1|\mathbf{x} \in A_1^\delta(U_1^\delta)]}{1 - \delta + \text{prob}(\mathbf{x} \in A_1^\delta(U_1^\delta))\mathbb{E}[x_1 + x_2|\mathbf{x} \in A_1^\delta(U_1^\delta)]} \\ &= \frac{\mathbb{E}[x_1|\mathbf{x} \in A_1^*]}{\mathbb{E}[x_1 + x_2|\mathbf{x} \in A^*]} > \frac{\mathbb{E}[x_1|\mathbf{x} \in A_1^*]}{\frac{1}{k}\mathbb{E}[x_1|\mathbf{x} \in A^*]} = k, \end{aligned}$$

a contradiction. Hence, $\lim_{\delta\to 1} U_1^\delta = 1$. ■

Proof of Proposition 8. Note that in this case RME payoffs are a fixed point of operator $\Phi : X \to X$, with Φ_i now given by

$$\Phi_i(\mathbf{W}) = \text{prob}(\mathbf{x} \in A(\mathbf{W}))\mathbb{E}[x_i - (x_1 + x_2)\delta_i W_i|\mathbf{x} \in A(\mathbf{W})] + \delta_i W_i,$$

where

$$A(\mathbf{W}) = \left\{\mathbf{x} \in X : \text{for } i = 1,2,\ x_i \geq \frac{\delta_i W_i}{1 - \delta_i W_i} x_{-i}\right\}.$$

Proposition 1(ii) extends to this environment. When Assumption 3 holds, there exists $\hat{\delta} < 1$ such that, if $\delta_1 > \delta_2 > \hat{\delta}$, the game has unique RME payoffs. Moreover, as

[21] Indeed, for any $\mathbf{x} \in A^*$, $x_1 \geq (x_1 + x_2)k$, and so $x_1 + x_2 \leq \frac{1}{k}x_1$. Since f has full support, $\mathbb{E}[x_1 + x_2|\mathbf{x} \in A^*] < \frac{1}{k}\mathbb{E}[x_1|\mathbf{x} \in A^*]$.

we showed in the proof of Proposition 5, when players' discount factors are sufficiently high, RME payoffs are given by $\lim_{T\to\infty}\Phi^T(\mathbf{0})$.[22]

Fix $\delta_1 > \delta_2 > \hat{\delta}$, and let $\mathbf{W}^\sigma = (W_1^\sigma, W_2^\sigma)$ denote the players' unique RME payoffs. We first show that $W_1^\sigma > W_2^\sigma$. Define the sequence $\{\mathbf{W}^T\}$ with $\mathbf{W}^T = \Phi^T(\mathbf{0})$ for each $T = 1, 2, ...,$ and note that $\lim_{T\to\infty}\mathbf{W}^T = \mathbf{W}^\sigma$. Note that, for $i = 1, 2$, $W_i^1 = \Phi_i(\mathbf{0}) = \mathbb{E}[x_i]$. Since distribution F is symmetric, $W_1^1 = W_1^2$.

Next, suppose that $W_1^T \geq W_2^T$. We now show that this implies that $W_1^{T+1} > W_2^{T+1}$. Indeed, note that

$$\begin{aligned} W_1^{T+1} - W_2^{T+1} &= \Phi_1(\mathbf{W}^T) - \Phi_2(\mathbf{W}^T) \\ &= \text{prob}(\mathbf{x} \in A(\mathbf{W}^T))\mathbb{E}[x_1 - x_2 | \mathbf{x} \in A(\mathbf{W}^T)] \\ &\quad + (\delta_1 W_1^T - \delta_2 W_2^T)(1 - \text{prob}(\mathbf{x} \in A(\mathbf{W}^T))\mathbb{E}[x_1 + x_2 | \mathbf{x} \in A(\mathbf{W}^T)]). \end{aligned}$$

Since F is symmetric and since $W_1^T \geq W_2^T$, we have $\text{prob}(\mathbf{x} \in A(\mathbf{W}^T))\mathbb{E}[(x_1 - x_2)|\mathbf{x} \in A(\mathbf{W}^T)] \geq 0$. Moreover, using $\text{prob}(\mathbf{x} \in A(\mathbf{W}^T))\mathbb{E}[(x_1 + x_2)|\mathbf{x} \in A(\mathbf{W}^T)] < 1$, $W_1^T \geq W_2^T$ and $\delta_1 > \delta_2$, we have

$$(\delta_1 W_1^T - \delta_2 W_2^T)(1 - \text{prob}(\mathbf{x} \in A(\mathbf{W}^T))\mathbb{E}[(x_1 + x_2)|\mathbf{x} \in A(\mathbf{W}^T)]) > 0.$$

Hence, $W_1^{T+1} > W_2^{T+1}$. Together with $W_1^1 = W_2^1$, this implies that $W_1^\sigma > W_2^\sigma$.

Next, since $\mathbf{W}^\sigma$ is a fixed point of Φ, we have

$$\begin{aligned} W_1^\sigma - W_2^\sigma &= \text{prob}(\mathbf{x} \in A(\mathbf{W}^\sigma))\mathbb{E}[x_1 - x_2 | \mathbf{x} \in A(\mathbf{W}^\sigma)] \\ &\quad + (\delta_1 W_1^\sigma - \delta_2 W_2^\sigma)(1 - \text{prob}(\mathbf{x} \in A(\mathbf{W}^\sigma))\mathbb{E}[x_1 + x_2 | \mathbf{x} \in A(\mathbf{W}^\sigma)]) \\ &= \text{prob}(\mathbf{x} \in A(\mathbf{W}^\sigma))\mathbb{E}[x_1 - x_2 | \mathbf{x} \in A(\mathbf{W}^\sigma)] \\ &\quad + \delta_1(W_1^\sigma - W_2^\sigma)(1 - \text{prob}(\mathbf{x} \in A(\mathbf{W}^\sigma))\mathbb{E}[x_1 + x_2 | \mathbf{x} \in A(\mathbf{W}^\sigma)]) \\ &\quad + (\delta_1 - \delta_2)W_2^\sigma(1 - \text{prob}(\mathbf{x} \in A(\mathbf{W}^\sigma))\mathbb{E}[x_1 + x_2 | \mathbf{x} \in A(\mathbf{W}^\sigma)]) \\ &\geq \delta_1(W_1^\sigma - W_2^\sigma)(1 - \text{prob}(\mathbf{x} \in A(\mathbf{W}^\sigma))\mathbb{E}[x_1 + x_2 | \mathbf{x} \in A(\mathbf{W}^\sigma)]) \\ &\quad + (\delta_1 - \delta_2)W_2^\sigma(1 - \text{prob}(\mathbf{x} \in A(\mathbf{W}^\sigma))\mathbb{E}[x_1 + x_2 | \mathbf{x} \in A(\mathbf{W}^\sigma)]), \end{aligned}$$

[22] While the proof of Proposition 5 is written for the case of equal discounting, the arguments can be readily extended to the case of unequal discounting.

where the last inequality uses $\mathbb{E}[(x_1 - x_2)|\mathbf{x} \in A(\mathbf{W}^\sigma)] > 0$, which holds since F is symmetric and since $W_1^\sigma > W_2^\sigma$. By the inequality above,

$$W_1^\sigma - W_2^\sigma \geq \frac{(\delta_1 - \delta_2)W_2^\sigma(1 - \text{prob}(\mathbf{x} \in A(\mathbf{W}^\sigma))\mathbb{E}[x_1 + x_2|\mathbf{x} \in A(\mathbf{W}^\sigma)])}{1 - \delta_1 + \delta_1 \text{prob}(\mathbf{x} \in A(\mathbf{W}^\sigma))\mathbb{E}[x_1 + x_2|\mathbf{x} \in A(\mathbf{W}^\sigma)]}.$$

Using $H(\mathbf{W}) = \text{prob}(\mathbf{x} \in A(\mathbf{W}))\mathbb{E}[x_1 + x_2|\mathbf{x} \in A(\mathbf{W})]$, this is equivalent to the inequality stated in the proposition. ■

C.2 Details for Strategic Search

In this appendix we flesh out the extension described in Section 5.3. We make the following assumptions on the sets of distributions $\mathcal{F}_\mathbf{x}$. First, for all $\mathbf{x}, \mathbf{y} \in X$, $\text{card}(\mathcal{F}_\mathbf{x}) = \text{card}(\mathcal{F}_\mathbf{y})$; i.e., all the sets $\mathcal{F}_\mathbf{x}$ have the same cardinality. Second, for all $\mathbf{x} \in X$ and all $F_\mathbf{x} \in \mathcal{F}_\mathbf{x}$ with density $f_\mathbf{x}$, there exists $F \in \mathcal{F} = \mathcal{F}_{(0,0)}$ with density f such that $f_\mathbf{x}(\mathbf{y}) = \frac{1}{(1-z_1-z_2)^2} f(P_\mathbf{x}(\mathbf{y}))$ for all $\mathbf{y} \in X(\mathbf{x})$. We further assume that there exists $\overline{f} > \underline{f} > 0$ such that, for all $f \in \mathcal{F}$, $f(\mathbf{x}) \in [\underline{f}, \overline{f}]$ for all $\mathbf{x} \in X$. Note that these assumptions are a generalization of Assumptions 1 and 2 to the new environment.

Fix an RME σ. For each $\mathbf{z} \in X$, let $V_i^\sigma(\mathbf{z})$ be player i's continuation payoff under σ when the status quo is $\mathbf{z}$ and let W_i^σ be player i's payoff at the start of the game under σ. The following result extends Lemma 1 to this environment. The proof is identical to the proof of Lemma 1, and hence omitted.

Lemma C.1. *Fix an RME σ. For all $\mathbf{z} = (z_1, z_2) \in X$,*

$$V_i^\sigma(\mathbf{z}) = z_i + (1 - z_1 - z_2)W_i^\sigma. \tag{34}$$

Lemma C.1 can be used to obtain a recursive characterization of RME payoffs. Fix an RME σ. As in our baseline model, under σ player i approves a policy $\mathbf{x} = (x_1, x_2) \in X(\mathbf{z})$ when the status quo is $\mathbf{z}$ only if

$$(1 - \delta)x_i + \delta V_i^\sigma(\mathbf{x}) \geq (1 - \delta)z_i + \delta V_i^\sigma(\mathbf{z})$$

which, using Lemma C.1, becomes

$$x_i + (1 - x_1 - x_2)\delta W_i^\sigma \geq z_i - (1 - x_1 - x_2)\delta W_i^\sigma,$$

Thus, player i accepts policy $\mathbf{x}$ when the status quo is $\mathbf{z}$ only if $\mathbf{x} \in A_{i,\mathbf{z}}(W_i^\sigma) = \{\mathbf{x} \in X(\mathbf{z}) : x_i \geq \ell_{i,\mathbf{z}}(x_{-i}|W_i^\sigma)\}$, where $\ell_{i,\mathbf{z}}(x_{-i}|W_i^\sigma)$ is defined as in the main text. For any pair of payoffs $\mathbf{W} = (W_1, W_2)$ and for any $\mathbf{z} \in X$, the set $A_{\mathbf{z}}(\mathbf{W})$ defined in the main text is the set of policies that are accepted by both players when the status quo is $\mathbf{z}$, and $A(\mathbf{W})$ is the acceptance set at the start of the game.

Now suppose player $i = 1, 2$ is recognized to choose the distribution from which the policy will be drawn at the initial period. If player i chooses distribution $F \in \mathcal{F}$, she obtains payoffs equal to

$$\text{prob}_F(x \in A(\mathbf{W}))\mathbb{E}_F[x_i - (x_1 + x_2)\delta W_i | \mathbf{x} \in A(\mathbf{W})] + \delta W_i.$$

For any $\mathbf{W} \in X$ and for $i = 1, 2$, let

$$F^*_{\mathbf{W},i} \in \arg\max_{F \in \mathcal{F}} \text{prob}_F(x \in A(\mathbf{W}))\mathbb{E}_F[x_i - (x_1 + x_2)W_i | \mathbf{x} \in A(\mathbf{W})],$$

and let $F^*_{\mathbf{W}} := \frac{1}{2}F^*_{\mathbf{W},1} + \frac{1}{2}F^*_{\mathbf{W},2}$. Note that the initial period policy is drawn from distribution $F^*_{\mathbf{W}}$.

Define the operator $\Phi^S : X \to X$ as follows: for $i = 1, 2$ and for all $\mathbf{W} \in X$,

$$\Phi^S_i(\mathbf{W}) = \text{prob}_{F^*_{\mathbf{W}}}(x \in A(\mathbf{W}))\mathbb{E}_{F^*_{\mathbf{W}}}[x_i - (x_1 + x_2)\delta W_i | \mathbf{x} \in A(\mathbf{W})] + \delta W_i.$$

Let $\mathbf{W}^*$ denote the players' RME payoffs at the start of the game. The following result extends Proposition 1 to the current environment – the proof uses the same arguments as the proof of Proposition 1, and hence we omit it.

Proposition C.1. *An RME exists, and the players' equilibrium payoffs under an RME are a fixed point of* Φ^S.

This characterization of equilibrium payoffs can be used to generalize the main results in the main text to the current environment. First, any RME features inefficient delays. Second, the acceptance regions are nested, and the distribution over long-run outcomes that an RME induces at a subgame starting with status quo payoff $\mathbf{z}$ has support equal to $\{\mathbf{x} \in X : x_1 + x_2 = 1\} \cap A_{\mathbf{z}}(\mathbf{W})$. Therefore, RME also display path-dependence. It can also be shown that Proposition 4 continues to hold in this setting, so the RME outcome also becomes deterministic in the limit as $\delta \to 1$.[23]

[23]The proofs of all of these results are available upon request.

References

Abreu, D. and F. Gul (2000): "Bargaining and Reputation," *Econometrica*, 68, 85–117.

Abreu, D. and D. Pearce (2007): "Bargaining, reputation, and equilibrium selection in repeated games with contracts," *Econometrica*, 75, 653–710.

——— (2015): "A dynamic reinterpretation of Nash bargaining with endogenous threats," *Econometrica*, 83, 1641–1655.

Acemoglu, D. and J. A. Robinson (2000): "Why Did the West Extend the Franchise? Democracy, Inequality, and Growth in Historical Perspective," *The Quarterly Journal of Economics*, 115, 1167–1199.

——— (2001): "A theory of political transitions," *American Economic Review*, 938–963.

Acharya, A. and J. Ortner (2013): "Delays and partial agreements in multi-issue bargaining," *Journal of Economic Theory*, 148, 2150–2163.

——— (2017): "Progressive learning," *Econometrica*, 85, 1965–1990.

Albrecht, J., A. Anderson, and S. Vroman (2010): "Search by committee," *Journal of Economic Theory*, 145, 1386–1407.

Binder, S. A. and F. E. Lee (2013): "Making Deals in Congress," in *Negotiating Agreement in Politics*, ed. by J. Mansbridge and C. J. Martin, APSA, 54–72.

Binmore, K., A. Rubinstein, and A. Wolinsky (1986): "The Nash bargaining solution in economic modelling," *The RAND Journal of Economics*, 176–188.

Bowen, T. R., Y. Chen, and H. Eraslan (2014): "Mandatory versus discretionary spending: The status quo effect," *American Economic Review*, 104, 2941–74.

Callander, S. and N. Matouschek (2019): "The risk of failure: Trial and error learning and long-run performance," *American Economic Journal: Microeconomics*, 11, 44–78.

Chassang, S. (2010): "Building routines: Learning, cooperation, and the dynamics of incomplete relational contracts," *American Economic Review*, 100, 448–65.

Cho, I.-K. (1990): "Uncertainty and delay in bargaining," *The Review of Economic Studies*, 57, 575–595.

Compte, O. and P. Jehiel (2004): "Gradualism in Bargaining and Contribution Games," *Review of Economic Studies*, 71, 975–1000.

——— (2010a): "Bargaining and majority rules: A collective search perspective," *Journal of Political Economy*, 118, 189–221.

——— (2010b): "The coalitional Nash bargaining solution," *Econometrica*, 78, 1593–1623.

Cramton, P. C. (1992): "Strategic delay in bargaining with two-sided uncertainty," *The Review of Economic Studies*, 59, 205–225.

De la Fuente, A. (2000): *Mathematical methods and models for economists*, Cambridge University Press.

Diskin, A., M. Koppel, and D. Samet (2011): "Generalized Raiffa solutions," *Games and Economic Behavior*, 73, 452–458.

Driesen, B., P. Eccles, and N. Wegner (2017): "A non-cooperative foundation for the continuous Raiffa solution," *International Journal of Game Theory*, 46, 1115–1135.

Duggan, J. and T. Kalandrakis (2012): "Dynamic Legislative Policy Making," *Journal of Economic Theory*, 147, 1653–1688.

Dziuda, W. and A. Loeper (2016): "Dynamic collective choice with endogenous status quo," *Journal of Political Economy*, 124, 1148–1186.

Fanning, J. (2016): "Reputational bargaining and deadlines," *Econometrica*, 84, 1131–1179.

——— (2018): "No compromise: Uncertain costs in reputational bargaining," *Journal of Economic Theory*, 175, 518–555.

Fearon, J. D. (1996): "Bargaining over objects that influence future bargaining power," *Unpublished manuscript.*

Gul, F. (1989): "Bargaining foundations of Shapley value," *Econometrica: Journal of the Econometric Society*, 81–95.

Halac, M. and A. Prat (2016): "Managerial attention and worker performance," *American Economic Review*, 106, 3104–32.

Kalai, E. and M. Smorodinsky (1975): "Other solutions to Nash's bargaining problem," *Econometrica*, 513–518.

Kalandrakis, T. (2004): "A Three-Player Dynamic Majoritarian Bargaining Game," *Journal of Economic Theory*, 116, 294–322.

Kamada, Y. and N. Muto (2015): "Multi-agent search with deadline," Tech. rep., University of California Berkeley.

Li, J. and N. Matouschek (2013): "Managing conflicts in relational contracts," *American Economic Review*, 103, 2328–51.

Livne, Z. A. (1989): "On the status quo sets induced by the Raiffa solution to the two-person bargaining problem," *Mathematics of Operations Research*, 14, 688–692.

Maskin, E. and J. Tirole (2001): "Markov perfect equilibrium: I. Observable actions," *Journal of Economic Theory*, 100, 191–219.

Moldovanu, B. and X. Shi (2013): "Specialization and partisanship in committee search," *Theoretical Economics*, 8, 751–774.

Myerson, R. B. (2013): *Game theory*, Harvard university press.

Nash, J. F. (1950): "The bargaining problem," *Econometrica*, 155–162.

Ortner, J. (2017): "A theory of political gridlock," *Theoretical Economics*, 12, 555–586.

Penn, E. M. (2009): "A model of farsighted voting," *American Journal of Political Science*, 53, 36–54.

Peters, H. and E. Van Damme (1991): "Characterizing the Nash and Raiffa bargaining solutions by disagreement point axioms," *Mathematics of Operations Research*, 16, 447–461.

Powell, R. (2004): "The inefficient use of power: Costly conflict with complete information," *American Political Science Review*, 98, 231–241.

——— (2006): "War as a commitment problem," *International organization*, 60, 169.

Raiffa, H. (1953): "Arbitration schemes for generalized two-person games," *Annals of Mathematics Studies*, 28, 361–387.

Roberts, K. (2007): "Condorcet cycles? A model of intertemporal voting," *Social Choice and Welfare*, 29, 383–404.

Rubinstein, A. (1982): "Perfect equilibrium in a bargaining model," *Econometrica: Journal of the Econometric Society*, 97–109.

Samet, D. (2009): "What if Achilles and the tortoise were to bargain? An argument against interim agreements," .

Trockel, W. (2011): "An exact non-cooperative support for the sequential Raiffa solution," *Journal of Mathematical Economics*, 47, 77–83.