Online Appendix

Collusion in Auctions with Constrained Bids:

Theory and Evidence from Public Procurement

Sylvain Chassang — New York University

Juan Ortner* — Boston University

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Abstract

This Online Appendix to "Collusion in Auctions with Constrained Bids: Theory and Evidence from Procurement Auctions" provides extensions, robustness checks and proofs. We provide additional empirical results and robustness checks in Section OA. Section OB collects all proofs. We analyze variants of our baseline model allowing for non-performing bidders (Section OC), as well as endogenous participation by cartel members (Section OD). Section OF presents a back-of-the-envelope calibration of our model, and lets us get a sense of potential treatments effects as the level of the minimum price varies.

KEYWORDS: collusion, cartel enforcement, minimum prices, entry deterrence, procurement.

*Chassang: chassang@nyu.edu, Ortner: jortner@bu.edu.

OA Further Empirical Exploration

OA.1 Greater entry, and worse collusion

We are interested in the relative importance of greater entry and worse within-cartel enforcement in explaining the impact of minimum prices. Data from Tsuchiura includes bids from all participants (i.e. includes non-winners) and lets us make progress on these questions. We proceed by assessing the impact of minimum prices on entry, and then, by assessing the impact of minimum prices on winning bids, controlling for entry. Since these are, by force, single-city before-after regressions, we first check that before-after regressions yield estimates of the impact of minimum prices that are consistent with estimates obtained from a more reliable difference-in-differences framework.

Policy impact in a single city regression. We perform both OLS and quantile regressions of the linear model

$$norm\_winning\_bid_a = \beta_0 + \beta_1 policy\_change + \beta controls + \varepsilon_a \tag{O1}$$

where *controls* (used throughout the analysis) include Japanese $\log GDP$ as well as a time trend and month fixed effects. We report effects for the subsample of auctions such that the normalized winning bid is above .8, as well as the mean effect for the unconditional sample. Table OA.1 reports the outcome of regression (O1).

	unconditional sample	sample s.t. $norm\_winning\_bid > .8$				
norm_winning_bid	mean effect	mean effect	$q = .2$	$q = .4$	$q = .6$	$q = .8$
policy_change	-0.016***	-0.026***	-0.061***	-0.015***	-0.006***	-0.005***
	(0.006)	(0.004)	(0.014)	(0.003)	(0.002)	(0.002)
ln_gdp	0.519***	0.226***	0.531***	0.129***	0.092***	0.052**
	(0.079)	(0.059)	(0.196)	(0.050)	(0.031)	(0.022)
year	0.005***	0.005***	0.010***	0.005***	0.003***	0.002***
	(0.001)	(0.001)	(0.003)	(0.001)	(0.000)	(0.000)
R-squared	0.083	0.059				
N	1748	1660	1660	1660	1660	1660

Table OA.1: The effect of minimum prices on winning bids. OLS estimates for unconditional sample and quantile regression estimates for conditional sample; regressions include month fixed-effects.

While the results are not precisely identical, these magnitudes match those of our difference-in-differences design (Table 4), which gives us some confidence that our controls are sufficient to make a single-city analysis not-implausible.

Entry and participation. We now study the impact of minimum prices on entry and participation by cartel members.

As expected, minimum prices increase both entry and participation. Table OA.2 reports the results from OLS estimation of the following auction-level linear models:

$$\begin{aligned} num\_entrants_a &= \beta_0 + \beta_1 policy\_change + \beta controls + \varepsilon_a \\ num\_bidders_a &= \beta_0 + \beta_1 policy\_change + \beta controls + \varepsilon_a \\ &= \beta_0 + \beta_1 policy\_change + \beta_2 num\_entrants_a + \beta controls + \varepsilon_a \end{aligned}$$

	num_entrants	num_bidders	num_bidders	num_bidders
policy_change	0.243**	0.516***	0.364**	0.410***
	(0.117)	(0.161)	(0.144)	(0.139)
ln_gdp	1.714	2.535	1.462	2.351
	(1.632)	(2.258)	(2.015)	(1.943)
year	-0.024	-0.350***	-0.335***	-0.378***
	(0.022)	(0.030)	(0.027)	(0.026)
num_entrants			0.626***	0.644***
			(0.030)	(0.029)
ln_reserve				0.382***
				(0.033)
R-squared	0.028	0.126	0.305	0.355
N	1748	1748	1748	1748

Table OA.2: The effect of minimum prices on entry and participation; regressions include month fixed-effects.

The introduction of minimum prices has a significant effect on both entry and participation by long-run bidders, adding on average .24 entrants and .52 bidders to auctions. These numbers are large given that the mean and median number of participants per auction are respectively 3.8 and 3. Note that participation increases even controlling for new entrants, suggesting that participation by cartel members is an endogenous decision. The results are broadly unchanged when controlling for the auction's reserve price. The data suggests that cartel participation itself is affected by minimum prices, which is consistent with the extension of our model discussed in Section 6 and fully exposed in the Appendix OD.

Next, we examine the effect of minimum prices on winning bids controlling for participation, using the linear model

$$norm\_winning\_bid_a = \beta_0 + \beta_1 policy\_change + \beta_2 num\_bidders_a + \beta controls + \varepsilon_a. \quad \text{(O2)}$$

To deal with potential endogeneity problems, we also run regression (O2) using the number of bidders in lagged auctions with similar characteristics as an instrument for the current number of bidders.[1] Table OA.3 reports the estimates.

	unconditional sample		conditional sample	
	OLS	IV	OLS	IV
policy_change	-0.010*	-0.011**	-0.023***	-0.023***
	(0.005)	(0.006)	(0.004)	(0.004)
num_bidders	-0.012***	-0.010***	-0.011***	-0.010***
	(0.001)	(0.003)	(0.001)	(0.002)
ln_gdp	0.550***	0.557***	0.258***	0.257***
	(0.074)	(0.075)	(0.054)	(0.055)
year	0.001	0.002	0.001	0.001
	(0.001)	(0.002)	(0.001)	(0.001)
R-squared	0.194	0.189	0.191	0.191
N	1748	1739	1660	1653
Underid. LM statistic		100.62		113.70
Weak Id. F-Test		105.82		120.89
	First-stage results unconditional sample		First-stage results conditional sample	
lagged_num_bidders		0.310***		0.297***
		(0.030)		(0.027)
R-squared		0.172		0.216

Table OA.3: The effect of minimum prices on winning bids, controlling for participation. OLS and IV estimates for unconditional and conditional samples; regressions include month fixed-effects.

As Table OA.3 shows, the policy change has a negative effect on bids even when controlling for participation.[2] We emphasize that the findings of Table OA.3 do not arise naturally

[1] More precisely, we use the average number of bidders among auctions in the previous date whose reserve price lies in the same quantile of the reserve price distribution. See also Online Appendix OE for a discussion of the likely sign of a potential bias.

[2] Regression (O2) assigns a smaller share of the drop in mean normalized winning bids (-1.6%, Table OA.1) to the "greater-entry" channel (-0.6%) than to the "worse within-cartel collusion" channel (-1.0%).

from a model of competitive bidding: controlling for the number of bidders, minimum prices should not cause a first-order stochastic dominance drop in the right tail of winning bids under competition (Proposition 5).

OA.2 Individual policy group regressions

Aggregate regressions (2Agg) and (3Agg) aggregate results from individual policy group regressions. Tables OA.4 and OA.5 provide a sense of potential heterogeneity in treatment effects by reporting estimates for (2g) and (3g) for individual policy groups. With the exception of Tsukubamirai, individual policy group findings are broadly consistent with the aggregate estimates.

We emphasize that setting a threshold of 0.8 is not necessarily appropriate for all treatment cities.[3] In the case of Hitachiomiya, for instance, we find that the policy has a negative effect on the unconditional mean, but no effect on the conditional one. In the case of Tsukuba, we find that the policy has a negative effect on the upper quantiles of the winning bid distribution. This is consistent with Hitachiomiya having set the minimum prices at lower levels than Tsuchiura, and Tsukuba having set minimum prices at higher levels.[4]

Lastly, Figure OA.1 plots the time-series charts of the normalized winning bids on the conditional sample for each of the treatment cities, before and after the policy change. The figure is in line with our main findings: winning bids of long-run firms are more negatively affected by the introduction of minimum prices than the winning bids of entrants.

OA.3 Robustness

Smooth equilibrium transition. A potential concern with the analysis in Section 5 is that it implicitly assumes that firms' bidding behavior prior to the introduction of the minimum price was not affected by expectations of change, and that their behavior after the introduction of minimum prices adjusted immediately to the new environment. We have argued that this should bias results against our findings.

We further address these concerns by running regressions (2Agg) and (3Agg), excluding the data on auctions that were conducted within six months before or after the policy change.

[3] The threshold of 0.8 is the mid-point of minimum prices we observe in Tsuchiura. We do not observe minimum prices in other cities.

[4] Table OA.10 shows that our results are robust to specifying different thresholds.

norm_winning_bid	unconditional mean effect	sample s.t. *norm_winning_bid* > .8 mean effect	$q = .2$	$q = .4$	$q = .6$	$q = .8$
policy_change	-0.008	-0.026***	-0.084***	-0.021***	-0.006*	0.003
Tsuchiura	(0.007)	(0.005)	(0.010)	(0.005)	(0.003)	(0.002)
N	3705	3459				
policy_change	-0.021**	-0.008	-0.004	0.006	0.009**	0.009***
Hitachiomiya	(0.008)	(0.006)	(0.005)	(0.011)	(0.005)	(0.004)
N	2457	2379				
policy_change	-0.040***	-0.032***	-0.112***	-0.023*	0.011*	0.014***
Inashiki	(0.009)	(0.007)	(0.005)	(0.013)	(0.006)	(0.004)
N	1990	1913				
policy_change	-0.029**	-0.021**	-0.026	-0.003	-0.001	-0.002
Toride	(0.012)	(0.008)	(0.022)	(0.007)	(0.004)	(0.003)
N	2348	2272				
policy_change	0.046***	0.014**	0.021*	-0.014**	-0.019***	-0.006
Tsukuba	(0.010)	(0.007)	(0.011)	(0.007)	(0.006)	(0.008)
N	2650	2276				
policy_change	0.001	0.006	0.035***	0.007	0.006	0.004
Tsukubamirai	(0.017)	(0.009)	(0.009)	(0.007)	(0.009)	(0.011)
N	1070	930				

***, ** and * respectively denote effects significant at the .01, .05 and .1 level.

Table OA.4: Difference-in-differences analysis of the effect of minimum prices on normalized winning bids. OLS estimates for unconditional and conditional samples and quantile regression estimates for conditional sample; regressions include city fixed-effects, year fixed-effects, month fixed-effects and city specific time-trends.

Table OA.6 reports the results. Findings are unchanged.

Separate markets. We now provide support for the assumption that markets are separate. The argument is geographical and uses the fact that bidder names are publicly available for Tsuchiura. This allows us to geolocate all long-run bidders, and compute their (straight line) distance to treatment and control cities. We then compute two measures of proximity indicating that the three markets are not integrated.

The first metric is the proportion of long-run bidders whose closest city is Tsuchiura (treatment) rather than Tsukuba or Ushiku (controls). If the three markets were integrated, given that the population of Tsuchiura is bracketed by that of the control cities, we should

norm_winning_bid	unconditional mean effect	sample s.t. $norm\_winning\_bid > .8$ mean effect	$q = .2$	$q = .4$	$q = .6$	$q = .8$
policy_change	0.024**	-0.007	-0.025	-0.012	0.000	0.006
Tsuchiura	(0.010)	(0.007)	(0.018)	(0.008)	(0.005)	(0.004)
long_run X policy_change	-0.036***	-0.021***	-0.054***	-0.007	-0.007	-0.004
Tsuchiura	(0.009)	(0.007)	(0.016)	(0.007)	(0.004)	(0.004)
N	3705	3449				
policy_change	-0.015	-0.012*	-0.007	-0.001	0.006	0.012**
Hitachiomiya	(0.010)	(0.008)	(0.008)	(0.008)	(0.008)	(0.005)
long_run X policy_change	-0.008	0.003	0.001	0.007	0.002	-0.002
Hitachiomiya	(0.007)	(0.005)	(0.005)	(0.006)	(0.005)	(0.003)
N	2457	2379				
policy_change	-0.017*	-0.016**	-0.091***	-0.006	0.013	0.014***
Inashiki	(0.010)	(0.008)	(0.008)	(0.008)	(0.009)	(0.005)
long_run X policy_change	-0.032***	-0.025***	-0.021***	-0.076***	-0.006	0.004
Inashiki	(0.008)	(0.006)	(0.006)	(0.006)	(0.006)	(0.004)
N	1990	1913				
policy_change	-0.040***	-0.028**	-0.050*	-0.009	0.002	-0.001
Toride	(0.015)	(0.011)	(0.030)	(0.010)	(0.006)	(0.004)
long_run X policy_change	0.017	0.009	0.023	0.004	-0.004	-0.002
Toride	(0.013)	(0.010)	(0.027)	(0.009)	(0.005)	(0.004)
N	2348	2272				
policy_change	0.087***	0.041***	0.034**	0.017*	0.015**	0.030***
Tsukuba	(0.012)	(0.008)	(0.014)	(0.009)	(0.007)	(0.008)
long_run X policy_change	-0.053***	-0.035***	-0.021**	-0.034***	-0.052***	-0.057***
Tsukuba	(0.009)	(0.006)	(0.010)	(0.006)	(0.005)	(0.006)
N	2650	2276				
policy_change	0.011	-0.011	0.005	-0.078***	-0.028*	0.012
Tsukubamirai	(0.030)	(0.016)	(0.018)	(0.016)	(0.016)	(0.018)
long_run X policy_change	-0.007	0.023	0.036**	0.085***	0.033**	-0.004
Tsukubamirai	(0.028)	(0.015)	(0.017)	(0.015)	(0.015)	(0.017)
N	1070	930				

***, ** and * respectively denote effects significant at the .01, .05 and .1 level.

Table OA.5: Difference-in-differences analysis of the effect of minimum prices on normalized winning bids. OLS estimates for unconditional and conditional samples and quantile regression estimates for conditional sample; regressions include city fixed-effects, year fixed effects month fixed-effects and city specific time-trends.

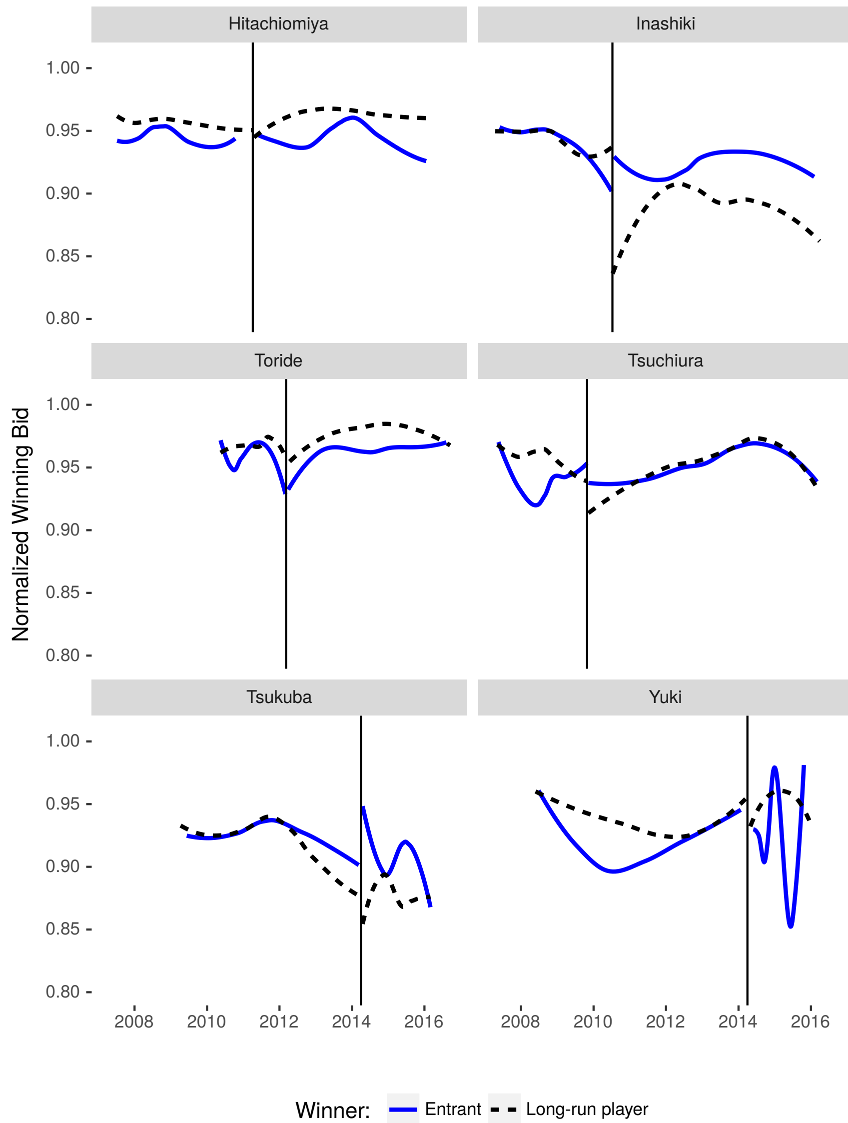

Figure OA.1: Average normalized winning bids of long-run bidders and entrants, conditional sample, before and after treatment.

expect roughly 1/3 of long-run bidders to have Tsuchiura as their closest location. Instead the number in our data is 87%.

Our second metric compares the share of bidders within a fixed radius from each city. Given a quantile Q, we compute the Q^{th} quantile radius for Tsuchiura, i.e. the distance d_Q such that a proportion Q of long-run bidders are within distance d_Q of Tsuchiura. We then compute the proportion of long-run bidders within distance d of either control cities. Since the distance between control cities is roughly equal to the distance between Tsuchiura and

norm_winning_bid	unconditional mean effect		conditional mean effect	
policy_change	-0.022**	-0.000	-0.021***	-0.006
p-value	0.042	0.971	0.002	0.402
policy_change x long_run		-0.021**		-0.015**
p-value		0.042		0.018
R-squared	0.293	0.315	0.311	0.330
N	8234	8234	7611	7611

***, ** and * respectively denote effects significant at the .01, .05 and .1 level.

Table OA.6: Difference-in-differences analysis of the effect of minimum prices on normalized winning bids, excluding auctions occurring around the policy change. OLS estimates for unconditional and conditional samples; regressions include city fixed-effects, year fixed effects month fixed-effects and city specific time-trends. Standard errors are clustered at the (city, year) level and p-values are calculated by wild bootstrap.

each control city, if the markets were integrated, we would expect that a proportion Q of long-run bidders would be within distance d_Q of each control city. This is not the case: for $Q = .5$, the proportion of long-run bidders within distance d_Q of control cities is exactly equal to 0; for $Q = .75$, it is 13%. This suggests that markets are largely separate.

Controlling for reserve prices. We now show that our empirical results continue to hold if we control for the level of the reserve price. We run the following versions of regressions (2Agg) and (3Agg):

$$\begin{aligned}\log\_winning\_bid_a &= \beta_0 + \beta_1 log\_reserve\_price + \beta_2 policy\_change \\ &\quad + \frac{1}{N_a}\sum_{g,\ s.t.\ a\in g}(\beta_g controls + fixed\_effects_g) + \varepsilon_a \\ \log\_winning\_bid_a &= \beta_0 + \beta_1 log\_reserve\_price + \beta_2 policy\_change + \beta_3 \widetilde{long\_run} \\ &\quad + \beta_4 \widetilde{long\_run}\_policy\_change + \frac{1}{N_a}\sum_{g,\ s.t.\ a\in g}(\beta_g controls + fixed\_effects_g) + \varepsilon_a.\end{aligned}$$

The results are presented in Table OA.7.

As a further check, we run the aggregate regressions in Section 5.3 for four subsamples of the data, corresponding to the four quartiles of the reserve price distribution. Results are

log_winning_bid	unconditional mean effect		conditional mean effect	
policy_change	-0.018	0.015	-0.025**	-0.009
p-value	0.462	0.641	0.016	0.214
policy_change x long_run		-0.036***		-0.017***
p-value		0.004		0.004
R-squared	0.995	0.995	0.999	0.999
N	8958	8958	8236	8236

***, ** and * respectively denote effects significant at the .01, .05 and .1 level.

Table OA.7: Difference-in-differences analysis of the effect of minimum prices on log winning bids. OLS estimates for unconditional and conditional samples; regressions include city fixed-effects, year fixed effects month fixed-effects and city specific time-trends. Standard errors are clustered at the (city, year) level and p-values are calculated by wild bootstrap.

presented in Table OA.8

Observability of participation. To assess whether the assumption of observable participants is plausible, we compute OLS estimates of linear models

$$\begin{aligned} norm\_bid_a =& \beta_0 + \beta_1 policy\_change + \beta_2 num\_entrants \\ &+ \beta_3 num\_long\_run\_participants + \beta controls + \varepsilon_a \\ ln\_bid_a =& \beta_0 + \beta_1 policy\_change + \beta_2 num\_entrants \\ &+ \beta_3 num\_long\_run\_participants + \beta_4 ln\_reserve + \beta controls + \varepsilon_a \end{aligned}$$

for all (bidder, auction) pairs using data from Tsuchiura. The results are presented in Table OA.9. For concision we do not reports coefficients for control variables (year and log GDP).

The data supports the assumption that participation is observable. Indeed, even conditional on auction size (proxied here by the reserve price), both the realized number of entrants and the realized number of participating long-run bidders have a significant effect on bids.

Different thresholds for normalized bids. Throughout the paper, we analyzed the effect that the policy change had on the distribution of normalized winning bids truncated at 0.8. Our results are robust to changes in this threshold.

norm_winning_bid	unconditional mean effect		conditional mean effect	
Auctions with reserve price in first quartile				
policy_change	0.017**	0.031**	-0.003	0.003
p-value	0.164	0.010	1.000	0.796
long_run X policy_change		-0.018		-0.009
p-value		0.302		0.511
N	2063	2063	1923	1923
Auctions with reserve price in second quartile				
policy_change	0.006	0.010	-0.023*	-0.018
p-value	0.694	0.276	0.064	0.218
long_run X policy_change		0.001		-0.007
p-value		0.937		0.392
N	2285	2285	2150	2150
Auctions with reserve price in third quartile				
policy_change	-0.023*	0.011	-0.018***	-0.002
p-value	0.070	0.444	0.002	0.847
long_run X policy_change		-0.028*		-0.016
p-value		0.054		0.198
N	2312	2312	2099	2099
Auctions with reserve price in fourth quartile				
policy_change	-0.011	0.023	-0.023	-0.022
p-value	0.669	0.844	0.328	0.228
long_run X policy_change		-0.041		-0.003
p-value		0.134		0.704
N	2298	2298	2064	2064

***, ** and * respectively denote effects significant at the .01, .05 and .1 level.

Table OA.8: Difference-in-differences analysis of the effect of minimum prices on normalized winning bids. OLS estimates for unconditional and conditional samples; regressions include city fixed-effects, year fixed effects month fixed-effects and city specific time-trends. Standard errors are clustered at the (city, year) level and p-values are calculated by wild bootstrap.

To illustrate this, we estimate equations (2Agg) and (3Agg) using thresholds of 0.78 and 0.82. The results are presented in Table OA.10.

	norm_bid	ln_bid
policy_change	-0.025***	-0.024***
	(0.005)	(0.006)
num_entrants	-0.012***	-0.014***
	(0.002)	(0.002)
num_long_run_participants	-0.011***	-0.013***
	(0.001)	(0.001)
ln_reserve		1.008***
		(0.003)
R-squared	0.253	0.996
N	6560	6560

Table OA.9: Bid (winning or not) as a function of realized participation; clustered by auction id.

	$norm\_winning\_bid > 0.78$		$norm\_winning\_bid > 0.82$	
norm_winning_bid	mean effect		mean effect	
policy_change	-0.026**	-0.012	-0.008	0.003
p-value	0.040	0.206	0.122	0.674
long_run X policy_change		-0.016**		-0.012***
p-value		0.018		0.004
R-squared	0.316	0.336	0.313	0.332
N	8418	8418	8057	8057

***, ** and * respectively denote effects significant at the .01, .05 and .1 level.

Table OA.10: Difference-in-differences analysis of the effect of minimum prices on normalized winning bids. OLS estimates for unconditional and conditional samples; regressions include city fixed-effects, year fixed effects month fixed-effects and city specific time-trends. Standard errors are clustered at the (city, year) level and p-values are calculated by wild bootstrap.

OB Proofs

OB.1 Proofs for Section 2

This appendix contains the proofs of Section 2. We start with a few preliminary observations. First, since the game we are studying is a complete information game with perfect monitoring, the set of SPE payoffs is compact (Proposition 2.5.2 in Mailath and Samuelson (2006)). Hence, $\overline{V}_p$ and $\underline{V}_{i,p}$ are attained. Fix an SPE σ and a history h_t. Let $\beta(\mathbf{c})$, $\gamma(\mathbf{c})$ and $T(\mathbf{c}, \mathbf{b}, \gamma, \mathbf{x})$ be the bidding and transfer profile that firms play in this equilibrium after history h_t. Let $\beta^W(\mathbf{c})$ and $\mathbf{x}(\mathbf{c})$ be, respectively, the winning bid and the allocation induced by bidding profile $(\beta(\mathbf{c}), \gamma(\mathbf{c}))$. Let $h_{t+1} = h_t \sqcup (\mathbf{c}, \mathbf{b}, \gamma, \mathbf{x}, \mathbf{T})$ be the concatenated history composed of h_t followed by $(\mathbf{c}, \mathbf{b}, \gamma, \mathbf{x}, \mathbf{T})$, and let $\{V(h_{t+1})\}_{i \in N}$ be the vector of continuation payoffs after history h_{t+1}. We let $h_{t+1}(\mathbf{c}) = h_t \sqcup (\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}), \mathbf{T}(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})))$ denote the on-path history that follows h_t when current costs are $\mathbf{c}$. Note that the following inequalities must hold:

(i) for all $i \in \widehat{N}$ such that $c_i \leq \beta^W(\mathbf{c})$,

$$x_i(\mathbf{c})(\beta^W(\mathbf{c}) - c_i) + T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) + \delta V_i(h_{t+1}(\mathbf{c})) \geq \rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c})(\beta^W(\mathbf{c}) - c_i) + \delta \underline{V}_{i,p}. \tag{O3}$$

(ii) for all $i \in \widehat{N}$ such that $c_i > \beta^W(\mathbf{c})$,

$$x_i(\mathbf{c})(\beta^W(\mathbf{c}) - c_i) + T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) + \delta V_i(h_{t+1}(\mathbf{c})) \geq \delta \underline{V}_{i,p}. \tag{O4}$$

(iii) for all $i \in N$,

$$T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) + \delta V_i(h_{t+1}(\mathbf{c})) \geq \delta \underline{V}_{i,p}. \tag{O5}$$

The inequality in (O3) must hold since a firm with cost below $\beta^W(\mathbf{c})$ can obtain a payoff at least as large as the right-hand side by undercutting the winning bid when $\beta^W(\mathbf{c}) > p$, or, by bidding p and choosing $\gamma_i = 1$ when $\beta^W(\mathbf{c}) = p$. Similarly, the inequality in (O4) must hold since firms with cost larger than $\beta^W(\mathbf{c})$ can obtain a payoff at least as large as the right-hand side by bidding more than $\beta^W(\mathbf{c})$. Finally, the inequality in (O5) must hold since otherwise firm i would not be willing to make the required transfer.

Conversely, suppose there exists a winning bid profile $\beta^W(\mathbf{c})$, an allocation $\mathbf{x}(\mathbf{c})$, a transfer profile $\mathbf{T}$ and equilibrium continuation payoffs $\{V_i(h_{t+1}(\mathbf{c}))\}_{i \in N}$ that satisfy inequalities (O3)-(O5) for some $\gamma(\mathbf{c})$ that is consistent with $\mathbf{x}(\mathbf{c})$ (i.e., $\gamma(\mathbf{c})$ is such that $x_i(\mathbf{c}) = \gamma_i(\mathbf{c}) / \sum_{j : x_j(\mathbf{c}) > 0} \gamma_j(\mathbf{c})$ for all i with $x_i(\mathbf{c}) > 0$). Then, $(\beta^W, \mathbf{x}, \mathbf{T})$ can be supported in an SPE as follows. For all $\mathbf{c}$, all firms $i \in \widehat{N}$ bid $\beta^W(\mathbf{c})$. Firms $i \in \widehat{N}$ with $x_i(\mathbf{c}) = 0$ choose $\tilde{\gamma}_i(\mathbf{c}) = 0$, and firms $i \in \widehat{N}$ with $x_i(\mathbf{c}) > 0$ choose $\tilde{\gamma}_i(\mathbf{c}) = \gamma_i(\mathbf{c})$. Note that, for all $i \in \widehat{N}$, $x_i(\mathbf{c}) = \tilde{\gamma}_i(\mathbf{c}) / \sum_j \tilde{\gamma}_j(\mathbf{c})$ and $\rho_i(\beta^W, \tilde{\gamma}, \mathbf{x})(\mathbf{c}) = \rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c})$. If no firm deviates at the bidding stage, firms make transfers $T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}))$. If no firm deviates at the transfer stage, in the next period firms play an SPE that gives payoff vector $\{V(h_{t+1}(\mathbf{c}))\}_{i \in N}$. If firm i deviates at the bidding stage, there are no transfers and the cartel reverts to an equilibrium that gives firm i a payoff of $\underline{V}_{i,p}$; if firm i deviates at the transfer stage, the cartel reverts to an equilibrium that gives firm i a payoff of $\underline{V}_{i,p}$ (deviations by more than one firm go unpunished). Since (O3) holds, under this strategy profile no firm has an incentive to undercut the winning bid $\beta^W(\mathbf{c})$. Since (O4) holds, no firm with $c_i > \beta^W(\mathbf{c})$ and $x_i(\mathbf{c}) > 0$ has an incentive to bid above $\beta^W(\mathbf{c})$ and lose. Upward deviations by a firm i with $c_i < \beta^W(\mathbf{c})$ who wins the auction are not profitable since the firm would lose the auction by bidding $b > \beta^W(\mathbf{c})$. Finally, since (O5) holds, all firms have an incentive to make their required transfers.

Proof of Lemma 1. Let σ be an SPE that attains $\overline{V}_p$. Towards a contradiction, suppose there exists an on-path history $h_t = h_{t-1} \sqcup (\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}), \mathbf{T}(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})))$ such that $\sum_i V_i(\sigma, h_t) = V(\sigma, h_t) < \overline{V}_p$. Let $\{V_i\}_{i\in N}$ be an equilibrium payoff vector with $\sum_i V_i = \overline{V}_p$.

Consider changing the continuation equilibrium at history h_t by an equilibrium that delivers payoff vector $\{V_i\}_{i\in N}$, and changing the transfers after history $h_{t-1} \sqcup (\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}))$ as follows. First, for each $i \in N$, let $\hat{T}_i$ be such that $\hat{T}_i + \delta V_i = T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) + \delta V_i(\sigma, h_t)$. Note that

$$\sum_i \hat{T}_i = \sum_i \{T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) + \delta(V_i(\sigma, h_t) - V_i)\} < 0,$$

where we used $\sum_i V_i = \overline{V}_p > \sum_i V_i(\sigma, h_t)$ and $\sum_i T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) = 0$. For each $i \in N$, let $\tilde{T}_i = \hat{T}_i + \frac{\epsilon}{n}$, where $\epsilon > 0$ is such that $\sum_i \tilde{T}_i = \sum_i \hat{T}_i + \epsilon = 0$. Replacing transfers $T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}))$ and continuation values $V_i(\sigma, h_t)$ by transfers $\tilde{T}_i$ and values V_i relaxes constraints (O3)-(O5) and increases the total expected discounted surplus that the equilibrium generates. Therefore, if σ attains $\overline{V}_p$, it must be that $V(\sigma, h_t) = \overline{V}_p$ for all on-path histories.

We now prove the second statement in the Lemma. Fix an optimal equilibrium σ, and let $\{V_i\}_{i\in N}$ be the payoff vector that this equilibrium delivers, with $\sum_i V_i = \overline{V}_p$. For each $\mathbf{c}$, let $(\beta(\mathbf{c}), \gamma(\mathbf{c}))$ be the bidding profile that firms use in the first period under σ, and let $\mathbf{x}(\mathbf{c})$ denote the allocation induced by bidding profile $(\beta(\mathbf{c}), \gamma(\mathbf{c}))$. It follows that

$$\overline{V}_p = \mathbb{E}\left[\sum_{i\in\widehat{N}} x_i(\mathbf{c})(\beta_i(\mathbf{c}) - c_i)\right] + \delta\overline{V}_p \Longleftrightarrow \overline{V}_p = \frac{1}{1-\delta}\mathbb{E}\left[\sum_{i\in\widehat{N}} x_i(\mathbf{c})(\beta_i(\mathbf{c}) - \mathbf{x}(\mathbf{c}))\right].$$

We show that there exists an optimal equilibrium in which firms use bidding profile $(\beta(\cdot), \gamma(\cdot))$ after all on-path histories. For any $(\mathbf{c}, \mathbf{b}, \gamma, \mathbf{x})$, let $T_i(\mathbf{c}, \mathbf{b}, \gamma, \mathbf{x})$ denote the transfer that firm i makes at the end of the first period under equilibrium σ when first period costs, bids and allocation are given by $\mathbf{c}$, $\mathbf{b}$, γ and $\mathbf{x}$. Let $V_i(h_1(\mathbf{c}))$ denote firm i's continuation payoff under equilibrium σ after first period history $h_1(\mathbf{c}) = (\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}), \mathbf{T}(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})))$. By our arguments above, $\sum_i V_i(h_1(\mathbf{c})) = \overline{V}_p$ for all $\mathbf{c}$. Since σ is an equilibrium, it must be that $\beta(\mathbf{c})$, $\gamma(\mathbf{c})$, $\mathbf{x}(\mathbf{c})$, $T_i(\mathbf{c}, \mathbf{b}, \gamma, \mathbf{x})$ and $V_i(h_1(\mathbf{c}))$ satisfy (O3)-(O5).

Consider the following strategy profile. Along the equilibrium path, at each period t firms bid according to $(\beta(\cdot), \gamma(\cdot))$. For any $(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}))$, firm i makes transfer $\hat{T}_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}))$ such that $\hat{T}_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) + \delta V_i = T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) + \delta V_i(h_1(\mathbf{c}))$. Note that

$$\sum_i \hat{T}_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) = \sum_i \{T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) + \delta(V_i(h_1(\mathbf{c})) - V_i)\} = 0,$$

where we used $\sum_i T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) = 0$ and $\sum_i V_i(h_1(\mathbf{c})) = \overline{V}_p = \sum_i V_i$. If firm i deviates at the bidding stage or transfer stage, then firms revert to an equilibrium that gives firm i a payoff of $\underline{V}_{i,p}$. Clearly, this strategy profile delivers total payoff $\overline{V}_p$. Moreover, firms have the same incentives to bid according to (β, γ) and make their required transfers than under the original equilibrium σ. Hence, no firm has an incentive to deviate at any stage and this strategy profile can be supported as an equilibrium. ■

Proof of Lemma 2. Suppose there exists an SPE σ and a history h_t at which firms bid according to a bidding profile (β, γ) that induces winning bid $\beta^W(\mathbf{c})$ and allocation $\mathbf{x}(\mathbf{c})$. Let $T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}))$ be firm i's transfers at history h_t when costs are $\mathbf{c}$ and all firms play according to the SPE σ. Let $h_{t+1}(\mathbf{c}) = h_t \sqcup (\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}), \mathbf{T}(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})))$ be the on-path history that follows h_t when costs are $\mathbf{c}$, and let $V_i(h_{t+1}(\mathbf{c}))$ be firm i's equilibrium payoff at history $h_{t+1}(\mathbf{c})$. Since the equilibrium must satisfy (O3)-(O5) for all $\mathbf{c}$,

$$\begin{aligned}
&\sum_{i \in \widehat{N}} \left\{ (\rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c}) - x_i(\mathbf{c})) \left[\beta^W(\mathbf{c}) - c_i\right]^+ + x_i(\mathbf{c}) \left[\beta^W(\mathbf{c}) - c_i\right]^- \right\} \\
\leq &\sum_{i \in N} T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) + \delta \sum_{i \in N} (V_i(h_{t+1}(\mathbf{c})) - \underline{V}_{i,p}) \leq \delta(\overline{V}_p - \sum_{i \in N} \underline{V}_{i,p}),
\end{aligned}$$

where we used $\sum_i T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) = 0$ and $\sum_i V_i(h_{t+1}(\mathbf{c})) \leq \overline{V}_p$.

Next, consider a winning bid profile $\beta^W(\mathbf{c})$ and an allocation $\mathbf{x}(\mathbf{c})$ that satisfy (1) for all $\mathbf{c}$ for some $\gamma(\mathbf{c})$ consistent with $\mathbf{x}(\mathbf{c})$ (i.e., such that $x_i(\mathbf{c}) = \gamma_i(\mathbf{c}) / \sum_{j: x_j(\mathbf{c}) > 0} \gamma_j(\mathbf{c})$ for all $i \in \widehat{N}$ with $x_i(\mathbf{c}) > 0$). We now construct an SPE that supports $\beta^W(\cdot)$ and $\mathbf{x}(\cdot)$ in the first period. Let $\{V_i\}_{i \in N}$ be an equilibrium payoff vector with $\sum_i V_i = \overline{V}_p$. For each $i \in N$ and each $\mathbf{c}$, we construct transfers $T_i(\mathbf{c})$ as follows:

$$T_i(\mathbf{c}) = \begin{cases} -\delta(V_i - \underline{V}_{i,p}) + (\rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c}) - x_i(\mathbf{c}))(\beta^W(\mathbf{c}) - c_i) + \epsilon(\mathbf{c}) & \text{if } i \in \widehat{N}, c_i \le \beta^W(\mathbf{c}), \\ -\delta(V_i - \underline{V}_{i,p}) - x_i(\mathbf{c})(\beta^W(\mathbf{c}) - c_i) + \epsilon(\mathbf{c}) & \text{if } i \in \widehat{N}, c_i > \beta^W(\mathbf{c}), \\ -\delta(V_i - \underline{V}_{i,p}) + \epsilon(\mathbf{c}) & \text{if } i \notin \widehat{N}, \end{cases}$$

where $\epsilon(\mathbf{c}) \ge 0$ is a constant to be determined below. Note that, for all $\mathbf{c}$,

$$\begin{aligned} &\sum_{i \in N} T_i(\mathbf{c}) - n\epsilon(\mathbf{c}) \\ = &- \delta(\overline{V}_p - \sum_{i \in N} \underline{V}_{i,p}) + \sum_{i \in \widehat{N}} \left\{ (\rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c}) - x_i(\mathbf{c})) \left[\beta^W(\mathbf{c}) - c_i\right]^+ + x_i(\mathbf{c}) \left[\beta^W(\mathbf{c}) - c_i\right]^- \right\} \le 0, \end{aligned}$$

where the inequality follows since β^W and $\mathbf{x}$ satisfy (1). We set $\epsilon(\mathbf{c}) \ge 0$ such that transfers are budget balance; i.e., such that $\sum_{i \in N} T_i(\mathbf{c}) = 0$.

The SPE we construct is as follows. At $t = 0$, for each $\mathbf{c}$ all participating firms bid $\beta^W(\mathbf{c})$. Firms $i \in \widehat{N}$ with $x_i(\mathbf{c}) = 0$ choose $\tilde{\gamma}_i(\mathbf{c}) = 0$, and firms $i \in \widehat{N}$ with $x_i(\mathbf{c}) > 0$ choose $\tilde{\gamma}_i(\mathbf{c}) = \gamma_i(\mathbf{c})$. Note that, for all $i \in \widehat{N}$, $x_i(\mathbf{c}) = \tilde{\gamma}_i(\mathbf{c}) / \sum_j \tilde{\gamma}_j(\mathbf{c})$ and $\rho_i(\beta^W, \tilde{\gamma}, \mathbf{x})(\mathbf{c}) = \rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c})$. If no firm deviates at the bidding stage, firms exchange transfers $T_i(\mathbf{c})$. If no firm deviates at the transfer stage, from $t = 1$ onwards they play an SPE that supports payoff vector $\{V_i\}$. If firm $i \in N$ deviates either at the bidding stage or at the transfer stage, from $t = 1$ onwards firms play an SPE that gives firm i a payoff $\underline{V}_{i,p}$ (if more than one firm deviates, firms punish the lowest indexed firm that deviated). This strategy profile satisfies (O3)-(O5), and so β^W and $\mathbf{x}$ are implementable. ■

Proof of Proposition 1. By Lemma 1, there exists an optimal equilibrium in which firms use the same bidding profile (β, γ) at every on-path history. For each cost vector $\mathbf{c}$, let $\beta^W(\mathbf{c})$ and $\mathbf{x}(\mathbf{c})$ denote the winning bid and the allocation induced by this bidding profile under cost vector $\mathbf{c}$.

We first show that $\beta^W(\mathbf{c}) = b_p^*(\mathbf{c})$ for all $\mathbf{c}$ such that $b_p^*(\mathbf{c}) > p$. Towards a contradiction, suppose there exists $\mathbf{c}$ with $\beta^W(\mathbf{c}) \neq b_p^*(\mathbf{c}) > p$. Since $\mathbf{x}^*(\mathbf{c})$ is the efficient allocation, the procurement cost under allocation $\mathbf{x}(\mathbf{c})$ is at least as large as the procurement cost under allocation $\mathbf{x}^*(\mathbf{c})$. Since bidding profile (β, γ) is optimal, it must be that $\beta^W(\mathbf{c}) > b_p^*(\mathbf{c}) > p$. Indeed, if $\beta^W(\mathbf{c}) < b_p^*(\mathbf{c})$, then the cartel would strictly prefer to use a bidding profile that allocates the contract efficiently and has winning bid $b_p^*(\mathbf{c})$ under cost vector $\mathbf{c}$ than to use bidding profile $(\beta(\mathbf{c}), \gamma(\mathbf{c}))$. By Lemma 2, $\beta^W(\mathbf{c})$ and $\mathbf{x}(\mathbf{c})$ must satisfy

$$\begin{aligned}\delta(\overline{V}_p - \sum_{i\in N} \underline{V}_{i,p}) &\geq \sum_{i\in\widehat{N}} \left\{(1 - x_i(\mathbf{c}))\left[\beta^W(\mathbf{c}) - c_i\right]^+ + x_i(\mathbf{c})\left[\beta^W(\mathbf{c}) - c_i\right]^-\right\} \\ &\geq \sum_{i\in\widehat{N}} (1 - x_i^*(\mathbf{c}))\left[\beta^W(\mathbf{c}) - c_i\right]^+,\end{aligned}$$

which contradicts $\beta^W(\mathbf{c}) > b_p^*(\mathbf{c}) > p$. Therefore, $\beta^W(\mathbf{c}) = b_p^*(\mathbf{c})$ for all $\mathbf{c}$ such that $b_p^*(\mathbf{c}) > p$.

Next, we show that $\beta^W(\mathbf{c}) = p$ for all $\mathbf{c}$ such that $b_p^*(\mathbf{c}) \leq p$. Towards a contradiction, suppose there exists $\mathbf{c}$ with $b_p^*(\mathbf{c}) \leq p$ and $\beta^W(\mathbf{c}) > p$. By Lemma 2, $\beta^W(\mathbf{c})$ and $\mathbf{x}(\mathbf{c})$ satisfy

$$\begin{aligned}\delta(\overline{V}_p - \sum_{i\in N} \underline{V}_{i,p}) &\geq \sum_{i\in\widehat{N}} \left\{(1 - x_i(\mathbf{c}))\left[\beta^W(\mathbf{c}) - c_i\right]^+ + x_i(\mathbf{c})\left[\beta^W(\mathbf{c}) - c_i\right]^-\right\} \\ &\geq \sum_{i\in\widehat{N}} (1 - x_i^*(\mathbf{c}))\left[\beta^W(\mathbf{c}) - c_i\right]^+,\end{aligned}$$

which contradicts $\beta^W(\mathbf{c}) > p \geq b_p^*(\mathbf{c})$. Therefore, $\beta^W(\mathbf{c}) = p$ for all $\mathbf{c}$ such that $b_p^*(\mathbf{c}) \leq p$. Combining this with the arguments above, $\beta^W(\mathbf{c}) = \beta_p^*(\mathbf{c}) = \max\{p, b_p^*(\mathbf{c})\}$.

Finally, we characterize the allocation in an optimal equilibrium. Note first that under an optimal bidding profile the cartel must allocate the procurement contract efficiently whenever $\beta_p^*(\mathbf{c}) > p$. Indeed, by construction, the efficient allocation is sustainable whenever the winning bid is $\beta_p^*(\mathbf{c}) > p$. Therefore, if the allocation was not efficient for some $\mathbf{c}$ with $\beta_p^*(\mathbf{c}) > p$, the cartel could strictly improve its profits by using a bidding profile with winning bid $\beta_p^*(\mathbf{c})$ that allocates the good efficiently.

Consider next a cost vector $\mathbf{c}$ such that $\beta_p^*(\mathbf{c}) = p$. In this case, the cartel's bidding profile in an optimal equilibrium induces the most efficient allocation (i.e., the allocation that minimizes expected procurement costs) consistent with (1) when the winning bid is p.
■

Proof of Corollary 1. We begin with part (i). Fix a set of participants $\widehat{N} \subset N$ and a cost realization $\mathbf{c} = (c_i)_{i\in\widehat{N}}$. Note that for any bid b, an increase in the cost c_j of any participating firm $j \in \widehat{N}$ weakly increases the term $\sum_{i\in\widehat{N}}(1 - x_i^*(\mathbf{c}))[b - c_i]^+$. Therefore, any increase in the cost of any participating firm weakly decreases $\beta_p^*(\mathbf{c})$.

Consider next part (ii). Fix $\widehat{N}_0 \subset N$ and $j \in N\backslash\widehat{N}_0$. Fix also a cost realization $\mathbf{c} = (c_i)_{i\in\widehat{N}_0}$ of firms in $\widehat{N}_0$ and cost realization c_j of bidder j. When the set of participants is $\widehat{N}_0$, under cost realization $\mathbf{c}$ the winning bid is $\beta_p^*(\mathbf{c}) = \max\{p, b_p^*(\mathbf{c})\}$. When the set of participants is $\widehat{N}_0 \cup \{j\}$, under cost realization $\hat{\mathbf{c}} = (\mathbf{c}, c_j)$, the winning bid is $\max\{p, b_p^*(\hat{\mathbf{c}})\}$. Note that

$$b_p^*(\mathbf{c}) = \sup\left\{b \le r : \sum_{i\in\widehat{N}_0}(1 - x_i^*(\mathbf{c}))[b - c_i]^+ \le \delta(\overline{V}_p - \sum_i \underline{V}_{i,p})\right\}$$
$$\ge \sup\left\{b \le r : \sum_{i\in\widehat{N}_0\cup\{j\}}(1 - x_i^*(\hat{\mathbf{c}}))[b - c_i]^+ \le \delta(\overline{V}_p - \sum_i \underline{V}_{i,p})\right\} = b_p^*(\hat{\mathbf{c}}),$$

and so $\beta_p^*(\mathbf{c}) \ge \beta_p^*(\hat{\mathbf{c}})$. Since this holds for any cost realization $\mathbf{c}$ of firms in $\widehat{N}_0$ and all cost realizations c_j of bidder j, it follows that $\mathbb{E}[\beta_p^*(\mathbf{c})|\widehat{N} = \widehat{N}_0] \ge \mathbb{E}[\beta_p^*(\mathbf{c})|\widehat{N} = \widehat{N}_0 \cup \{j\}]$. ■

Proof of Corollary 2. Note that, for $\delta = 0$, $b_p^*(\mathbf{c}) = c_{(2)}$ for all $\mathbf{c}$. By Proposition 1, when $\delta = 0$ the winning bid under the best equilibrium for the cartel is equal to $\beta^{\mathsf{comp}}(\mathbf{c}) = \max\{c_{(2)}, p\}$, which is the winning bid under competition. ■

Fix a minimum price p. For every value $V \ge \sum_{i\in N} \underline{V}_{i,p}$ and every $\mathbf{c}$, let

$$b_p(\mathbf{c}; V) \equiv \sup\left\{b \le r : \sum_{i\in\widehat{N}}(1 - x_i^*(\mathbf{c}))[b - c_i]^+ \le \delta(V - \sum_i \underline{V}_{i,p})\right\},$$

and let $\beta_p(\mathbf{c}; V) = \max\{b_p(\mathbf{c}; V), p\}$. Note that $\beta_p(\mathbf{c}; V)$ would be the winning bid in an optimal equilibrium if $V = \overline{V}_p$. Let $\mathbf{x}^p(\mathbf{c}; V)$ be the allocation under an optimal equilibrium when the cartel's total surplus is V. For every $V \ge \sum_{i\in N} \underline{V}_{i,p}$, define

$$U_p(V) \equiv \frac{1}{1-\delta}\mathbb{E}\left[\sum_{i\in\widehat{N}} x_i^p(\mathbf{c}; V)(\beta_p(\mathbf{c}, V) - c_i)\right],$$

to be the total surplus generated under a bidding profile that induces winning bid $\beta_p(\mathbf{c}; V)$ and allocation $\mathbf{x}^p(\mathbf{c}; V)$. The winning bid and allocation in an optimal equilibrium are $\beta_p^*(\mathbf{c}) = \beta_p(\mathbf{c}; \overline{V}_p)$ and $\mathbf{x}^p(\mathbf{c}; \overline{V}_p)$, and so $\overline{V}_p = U_p(\overline{V}_p)$. Define

$$\overline{U}_p \equiv \sup\left\{V \ge \sum_{i\in N} \underline{V}_{i,p} : V \le U_p(V)\right\}.$$

Lemma OB.1. $\overline{V}_p = \overline{U}_p$.

Proof. Since $\overline{V}_p = U_p(\overline{V}_p)$, it follows that $\overline{U}_p \geq \overline{V}_p$. We now show that $\overline{U}_p \leq \overline{V}_p$. Towards a contradiction, suppose that $\overline{U}_p > \overline{V}_p$. Hence, there exists $\tilde{V} \geq \sum_{i\in N} \underline{V}_{i,p}$ such that $U_p(\tilde{V}) \geq \tilde{V} > \overline{V}_p$. Let $\{V_i\}_{i\in N}$ be such that $\sum_i V_i = U_p(\tilde{V})$ and $V_i \geq \underline{V}_{i,p}$ for all i, and consider the following strategy profile. For all on-path histories, cartel members use a bidding profile (β, γ) inducing winning bid $\beta_p(\mathbf{c}; \tilde{V})$ and allocation $\mathbf{x}^p(\mathbf{c}; \tilde{V})$. If firm i deviates at the bidding stage, there are no transfers and in the next period firms play an equilibrium that gives firm i a payoff of $\underline{V}_{i,p}$ (if more than one firm deviates, firms play an equilibrium that gives $\underline{V}_{i,p}$ to the lowest indexed firm that deviated). If no firm deviates at the bidding stage, firms make transfers $T_i(\mathbf{c})$ given by

$$T_i(\mathbf{c}) = \begin{cases} -\delta(V_i - \underline{V}_{i,p}) + (\rho_i(\beta_p, \gamma, \mathbf{x}^p)(\mathbf{c}) - x_i^p(\mathbf{c}; \tilde{V}))(\beta_p(\mathbf{c}; \tilde{V}) - c_i) + \epsilon(\mathbf{c}) & \text{if } i \in \widehat{N}, c_i \leq \beta_p(\mathbf{c}; \tilde{V}), \\ -\delta(V_i - \underline{V}_{i,p}) + \epsilon(\mathbf{c}) & \text{otherwise,} \end{cases}$$

where $\epsilon(\mathbf{c}) \geq 0$ is a constant to be determined.[5] Note that

$$\sum_{i\in N} T_i(\mathbf{c}) - n\epsilon(\mathbf{c}) = -\delta(U_p(\tilde{V}) - \sum_{i\in N} \underline{V}_{i,p}) + \sum_{i\in \widehat{N}} ((\rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c}) - x_i^p(\mathbf{c}; \tilde{V}))[\beta_p(\mathbf{c}; \tilde{V}) - c_i]^+ \leq 0,$$

where the inequality follows since $\beta_p(\mathbf{c}; \tilde{V})$ and $x_i^p(\mathbf{c}; \tilde{V})$ are the winning bid and the allocation under an optimal equilibrium when the cartel's total surplus is $\tilde{V} \leq U_p(\tilde{V})$. We set $\epsilon(\mathbf{c}) \geq 0$ such that $\sum_i T_i(\mathbf{c}) = 0$. If firm i deviates at the transfer stage, in the next period firms play an equilibrium that gives firm i a payoff of $\underline{V}_{i,p}$ (if more than one firm deviates, firms play an equilibrium that gives $\underline{V}_{i,p}$ to the lowest indexed firm that deviated). Otherwise, in the next period firms continue playing the same strategy as above. This strategy profile generates total surplus $U_p(\tilde{V}) \geq \tilde{V} > \overline{V}_p$ to the cartel. One can check that no firm has an incentive to deviate at any stage, and so this strategy profile constitutes an equilibrium. This contradicts $U_p(\tilde{V}) > \overline{V}_p$, so it must be that $\overline{U}_p \leq \overline{V}_p$. ∎

Proof of Proposition 2. We first establish part (i). Suppose that $p \leq \underline{c}$ and fix equilibrium payoffs $\{V_i\}_{i\in N}$. Fix $j \in N$ and consider the following strategy profile. At $t = 0$, firms' behavior depends on whether $j \in \widehat{N}$ or $j \notin \widehat{N}$. If $j \in \widehat{N}$, all firms $i \in \widehat{N}$ bid $\min\{c_j, c_{(2)}\}$ (where $c_{(2)}$ is the second lowest procurement cost). Firm $i \in \widehat{N}$ chooses $\gamma_i = 1$

[5] Recall that $\mathbf{x}^p(\mathbf{c}; \tilde{V})$ is the allocation under an optimal equilibrium when continuation payoff is $\tilde{V}$. Therefore, $\mathbf{x}^p(\mathbf{c}; \tilde{V})$ is such that $x_i^p(\mathbf{c}; \tilde{V}) = 0$ for all i with $c_i > \beta_p(\mathbf{c}; \tilde{V})$.

if $c_i = \min_{k\in\widehat{N}} c_k$ and chooses $\gamma_i = 0$ otherwise. Note that this bidding profile constitutes a Nash equilibrium of the stage game. If $j \notin \widehat{N}$, at $t = 0$ participating firms play according to some equilibrium of the stage game. If all firms bid according to this profile, firm j's transfer is $T_j = -\delta V_j$ at the end of the period regardless of whether $j \in \widehat{N}$ or $j \notin \widehat{N}$. The transfer of firm $i \neq j$ is $T_i = \frac{1}{n-1}\delta V_j$ at the end of the period, so $\sum_i T_i = 0$. If no firm deviates at the bidding or transfer stage, at $t = 1$ firms play according to an equilibrium that delivers payoffs $\{V_i\}$. If firm i deviates at the bidding stage, there are no transfers and at $t = 1$ firms play the strategy just described with i in place of j. If no firm deviates at the bidding stage and firm i deviates at the transfer stage, at $t = 1$ firms play the strategy just described with i in place of j (if more than one firm deviates at the bidding or transfer stage, from $t = 1$ firms play according to an equilibrium that delivers payoffs $\{V_i\}_{i\in N}$). Note that this strategy profile gives player j a payoff of 0. Moreover, no firm has an incentive to deviate at $t = 0$, and so $\underline{V}_{j,p} = 0$ for all $p \leq \underline{c}$.

Suppose next that $p > \underline{c}$, and note that for all $i \in N$,

$$\underline{V}_{i,p} \geq \underline{v}_{i,p} \equiv \frac{1}{1-\delta}\text{prob}(i \in \widehat{N})\mathbb{E}_{F_i}\left[\frac{1}{\widehat{N}}\mathbf{1}_{c_i\leq p}(p - c_i)|i \in \widehat{N}\right] > 0,$$

where the inequality follows since $\underline{v}_{i,p}$ is the minmax payoff for a firm in an auction with minimum price p. This establishes part (i).

We now turn to part (ii). Note that $\beta_0^*(\underline{c}) = \inf_{\mathbf{c}} \beta_0^*(\mathbf{c}) = \underline{c} + \frac{\delta\overline{V}_0}{n-1} > \underline{c}$.[6] We now show that there exists $\eta > 0$ such that $\overline{V}_p - \sum_{i\in N}\underline{V}_{i,p} < \overline{V}_0$ for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$. Fix $\eta > 0$ and $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$. For every $V \geq \sum_{i\in N}\underline{V}_{i,p}$ and every $\mathbf{c}$, let $\tilde{\beta}_p(\mathbf{c}; V) \equiv \max\{b_0(\mathbf{c}; V), p\}$. Since $\underline{V}_{i,p} > 0$ for all $p > \beta_0^*(\underline{c})$, it follows that $b_0(\mathbf{c}; V) \geq b_p(\mathbf{c}; V)$ for all $\mathbf{c}$ and all $V \geq \sum_i \underline{V}_{i,p}$, and so $\tilde{\beta}_p(\mathbf{c}; V) \geq \beta_p(\mathbf{c}; V) = \max\{b_p(\mathbf{c}; V), p\}$ for all $\mathbf{c}$ and all $V \geq \sum_i \underline{V}_{i,p}$. Define

$$\tilde{U}_p(V) \equiv \frac{1}{1-\delta}\mathbb{E}\left[\sum_{i\in\widehat{N}} x_i^*(\mathbf{c})(\tilde{\beta}_p(\mathbf{c}; V) - c_i)\right],$$

and note that $\tilde{U}_p(V) \geq U_p(V)$ for all $V \geq \sum_i \underline{V}_{i,p}$. Define

$$\tilde{V}_p \equiv \sup\left\{V \geq \sum_i \underline{V}_{i,p} : \tilde{U}_p(V) \geq V\right\},$$

[6]Term $\beta_0^*(\mathbf{c})$ attains its lowest value when all cartel members participate in the auction and costs are $\mathbf{c} = (\underline{c})_{i\in N}$ (i.e., all firms have cost $\underline{c}$). For this cost vector, $\beta_0^*(\mathbf{c}) = \underline{c} + \frac{\delta\overline{V}_0}{n-1}$.

and note that $\tilde{V}_p \geq \overline{V}_p$. Recall that, for all V, $U_0(V) = \frac{1}{1-\delta}\mathbb{E}\left[\sum_{i\in\widehat{N}} x_i^*(\mathbf{c})(b_0(\mathbf{c};V)-c_i)\right]$.[7] Therefore, for all V,

$$\tilde{U}_p(V) - U_0(V) = \frac{1}{1-\delta}\mathbb{E}\left[(p - b_0(\mathbf{c};V))\mathbf{1}_{\{\mathbf{c}:b_0(\mathbf{c};V)<p\}}\right] > 0.$$

Note that for all V and all $\mathbf{c}$, $b_0(\mathbf{c};V) \geq \underline{c} + \frac{\delta V}{n-1}$. Let $\hat{V} > 0$ be such that $\underline{c} + \frac{\delta\hat{V}}{n-1} = \beta_0^*(\underline{c}) + \eta = \underline{c} + \frac{\delta\overline{V}_0}{n-1} + \eta$; that is, $\hat{V} = \overline{V}_0 + \frac{(n-1)\eta}{\delta} > \overline{V}_0$. Then, for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c})+\eta]$ and all $V \geq \hat{V}$, $b_0(\mathbf{c};V) \geq p$ for all $\mathbf{c}$, and so $\tilde{U}_p(V) = U_0(V)$. Since $\hat{V} > \overline{V}_0$ and since $\overline{V}_0 = \sup\{V \geq 0 : U_0(V) \geq V\}$, it follows that $V > U_0(V) = \tilde{U}_p(V)$ for all $V \geq \hat{V}$ and all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c})+\eta]$, and so $\hat{V} = \overline{V}_0 + \frac{(n-1)\eta}{\delta} > \tilde{V}_p \geq \overline{V}_p$ for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c})+\eta]$.

Finally, let $\eta > 0$ be such that[8]

$$\frac{(n-1)\eta}{\delta} = \sum_{i\in N} \underline{v}_{i,\beta_0^*(\underline{c})} = \sum_{i\in N} \frac{\text{prob}(i\in\widehat{N})}{1-\delta}\mathbb{E}_{F_i}\left[\frac{1}{\widehat{N}}\mathbf{1}_{c_i\leq\beta_0^*(\underline{c})}(\beta_0^*(\underline{c})-c_i)|i\in\widehat{N}\right].$$

Since $\underline{V}_{i,p} \geq \underline{v}_{i,p} \geq \underline{v}_{i,\beta_0^*(\underline{c})}$ for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c})+\eta]$,

$$\hat{V} = \overline{V}_0 + \frac{(n-1)\eta}{\delta} > \overline{V}_p \Rightarrow \overline{V}_0 > \overline{V}_p - \sum_{i\in N}\underline{V}_{i,p},$$

which completes the proof. ■

OB.2 Proofs of Section 3

Proof of Proposition 3. Consider first a collusive environment. By Propositions 1 and 2, there exists $\eta > 0$ such that $\beta_p^*(\mathbf{c}) \leq \beta_0^*(\mathbf{c})$ for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c})+\eta]$ and all $\mathbf{c}$ such that $\beta_0^*(\mathbf{c}) \geq p$, with strict inequality if $\beta_0^*(\mathbf{c}) < r$. Therefore, for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c})+\eta]$, $\text{prob}(\beta_p^* \geq q|\beta_p^* \geq p) \leq \text{prob}(\beta_0^* \geq q|\beta_0^* \geq p)$, and the inequality is strict for some $q > p$ whenever $\text{prob}(\beta_0^* < r) > 0$. This proves part (i).

Under competition, for all p and all $q > p$, $\text{prob}(\beta_p^{\mathsf{comp}} \geq q|\beta_p^{\mathsf{comp}} > p) = \text{prob}(c_{(2)} \geq q|c_{(2)} > p) = \text{prob}(\beta_0^{\mathsf{comp}} \geq q|\beta_0^{\mathsf{comp}} > p)$. This proves part (ii). ■

Proof of Proposition 4. We first show that there exists a symmetric equilibrium as

[7] Indeed, by Proposition 1, $\mathbf{x}^{p=0}(\mathbf{c};V) = \mathbf{x}^*(\mathbf{c})$ for all V.

[8] Recall that for all p, $\underline{V}_{i,p} \geq \underline{v}_{i,p} = \frac{\text{prob}(i\in\widehat{N})}{1-\delta}\mathbb{E}_{F_i}\left[\frac{1}{\widehat{N}}\mathbf{1}_{c_i\leq p}(p-c_i)|i\in\widehat{N}\right]$.

described in the statement of the proposition, and then we show uniqueness.

Consider first a minimum price $p \le b_0^{AI}(\underline{c})$. Clearly, in this case all firms using the bidding function $b_0^{AI}(\cdot)$ is a symmetric equilibrium of the auction with minimum price p.

Consider next the case in which $b_0^{AI}(\underline{c}) < p$. For any $c \in [\underline{c}, \overline{c}]$, define

$$P(c) \equiv \sum_{j=0}^{\widehat{N}-1} \binom{\widehat{N}-1}{j} \frac{1}{j+1} F(c)^j (1-F(c))^{\widehat{N}-j-1}.$$

$P(c)$ is the probability with which a firm with cost $c' \le c$ wins the auction if all firms use a bidding function $\beta(\cdot)$ with $\beta(c') = b \ge p$ for all $c' \le c$ and $\beta(c') > b$ for all $c' > c$.

Let $\hat{c} \in (\underline{c}, \overline{c})$ be the unique solution to $P(\hat{c})(p-\hat{c}) = (1-F(\hat{c}))^{\widehat{N}-1}(b_0^{AI}(\hat{c}) - \hat{c})$.[9] Let $b_p^{AI}(\cdot)$ be given by

$$b_p^{AI}(c) = \begin{cases} b_0^{AI}(c) & \text{if } c \ge \hat{c}, \\ p & \text{if } c < \hat{c}. \end{cases}$$

Note that if all firms bid according to bidding function $b_p^{AI}(\cdot)$, the probability with which a firm with cost $c < \hat{c}$ wins the auction is $P(\hat{c})$. We now show that all firms bidding according to $b_p^{AI}(\cdot)$ is an equilibrium.

Suppose that all firms $j \neq i$ bid according to $b_p^{AI}(\cdot)$. Note first that it is never optimal for firm i to bid $b \in (p, b_p^{AI}(\hat{c}))$. Indeed, if $c_i < b_p^{AI}(\hat{c})$, bidding $b \in (p, b_p^{AI}(\hat{c}))$ gives firm i a strictly lower payoff than bidding $b_p^{AI}(\hat{c})$: in both cases firm i wins with probability $(1-F(\hat{c}))^{\widehat{N}-1}$, but by bidding $b_p^{AI}(\hat{c})$ the firm gets a strictly larger payoff in case of winning. If $c_i > b_p^{AI}(\hat{c})$, bidding $b \in (p, b_p^{AI}(\hat{c}))$ gives firm i a strictly lower payoff than bidding $b_p^{AI}(c_i)$.

Suppose that $c_i \ge \hat{c}$. Since $b_p^{AI}(x) = b_0^{AI}(x)$ for all $x \ge \hat{c}$, firm i with cost c_i gets a larger payoff bidding $b_p^{AI}(c_i)$ than bidding $b_p^{AI}(x)$ with $x \in [\hat{c}, \overline{c}]$. If $c_i = \hat{c}$, firm i is by construction indifferent between bidding p and bidding $b_p^{AI}(\hat{c})$. Moreover, for all $c_i > \hat{c}$,

$$\begin{aligned} (1-F(c_i))^{\widehat{N}-1}(b_p^{AI}(c_i) - c_i) &\ge (1-F(\hat{c}))^{\widehat{N}-1}(b_p^{AI}(\hat{c}) - \hat{c}) + (1-F(\hat{c}))^{\widehat{N}-1}(\hat{c} - c_i) \\ &= P(\hat{c})(p-\hat{c}) + (1-F(\hat{c}))^{\widehat{N}-1}(\hat{c} - c_i) \\ &> P(\hat{c})(p-\hat{c}) + P(\hat{c})(\hat{c} - c_i), \end{aligned}$$

where the strict inequality follows since $P(\hat{c}) > (1-F(\hat{c}))^{\widehat{N}-1}$ and $c_i > \hat{c}$. Hence, firm i

[9] Note first that such a $\hat{c}$ always exists whenever $b^{AI}(\underline{c}) < p$. Indeed, in this case $P(\underline{c})(p-\underline{c}) = p - \underline{c} > b_0^{AI}(\underline{c}) - \underline{c}$, while $P(p)(p-p) = 0 < (1-F(p))^{\widehat{N}-1}(b_0^{AI}(p) - \overline{c})$. By the Intermediate value Theorem, there exists $\hat{c} \in [\underline{c}, p]$ such that $P(\hat{c})(p-\hat{c}) = (1-F(\hat{c}))^{\widehat{N}-1}(b_0^{AI}(\hat{c}) - \hat{c})$. Moreover, for all $c \le p$, $\frac{\partial}{\partial c} P(c)(p-c) = -P(c) + P'(c)(p-c) \le -P(c) < -(1-F(c))^{\widehat{N}-1} = \frac{\partial}{\partial c}(1-F(c))^{\widehat{N}-1}(b_0^{AI}(c) - c)$, so $\hat{c}$ is unique.

strictly prefers to bid $b_p^{AI}(c_i)$ when her cost is $c_i > \hat{c}$ than to bid p. Combining all these arguments, a firm with cost $c_i \geq \hat{c}$ finds it optimal to bid $b_p^{AI}(c_i)$ when her cost is $c_i \geq \hat{c}$.

Finally, suppose that $c_i < \hat{c}$. Firm i's payoff from bidding $b_p^{AI}(c_i) = p$ is $P(\hat{c})(p - c_i)$. Note that, for all $c \geq \hat{c}$,

$$\begin{aligned}
P(\hat{c})(p - c_i) &= P(\hat{c})(p - \hat{c}) + P(\hat{c})(\hat{c} - c_i) \\
&\geq (1 - F(c))^{\widehat{N}-1}(b_p^{AI}(c) - \hat{c}) + P(\hat{c})(\hat{c} - c_i) \\
&> (1 - F(c))^{\widehat{N}-1}(b_p^{AI}(c) - c_i),
\end{aligned}$$

where the first inequality follows since $P(\hat{c})(p - \hat{c}) = (1 - F(\hat{c}))^{\widehat{N}-1}(b_p^{AI}(\hat{c}) - \hat{c}) \geq (1 - F(c))^{\widehat{N}-1}(b_p^{AI}(c) - \hat{c})$ for all $c \geq \hat{c}$, and the second inequality follows since $P(\hat{c}) > (1 - F(c))^{\widehat{N}-1}$ for all $c \geq \hat{c}$ and since $c_i < \hat{c}$. Therefore, firm i finds it optimal to bid $b_p^{AI}(c_i) = p$ when her cost is $c_i < \hat{c}$.

Next we establish uniqueness. We start with a few preliminary observations. Fix an auction with minimum price $p > 0$ and let b_p be the bidding function in a symmetric equilibrium. By standard arguments (see, for instance, Maskin and Riley (1984)), b_p must be weakly increasing; and it must be strictly increasing and differentiable at all points c such that $b_p(c) > p$. Lastly, b_p must be such that $b_p(\bar{c}) = \bar{c}$.[10]

Consider a bidder with cost c such that $b_p(c) > p$, and suppose all of her opponents bid according to b_p. The expected payoff that this bidder gets from bidding $b_p(\tilde{c}) > p$ is $(1 - F(\tilde{c}))^{\widehat{N}-1}(b_p(\tilde{c}) - c)$. Since bidding $b_p(c) > p$ is optimal, the first-order conditions imply that b_p solves

$$b_p'(c) = \frac{f(c)}{1 - F(c)}(\widehat{N} - 1)(b_p(c) - c),$$

with boundary condition $b_p(\bar{c}) = \bar{c}$. Note that bidding function b_0^{AI} solves the same differential equation with the same boundary condition, and so $b_p(c) = b_0^{AI}(c)$ for all c such that $b_p(c) > p$.

Consider the case in which $p < b_0^{AI}(\underline{c})$, and suppose that there exists a symmetric equilibrium $b_p \neq b_0^{AI}$. By the previous paragraph, $b_p(c) = b_0^{AI}(c)$ for all c such that $b_p(c) > p$. Therefore, if $b_p \neq b_0^{AI}$ is an equilibrium, there must exist $\tilde{c} > \underline{c}$ such that $b_p(c) = p$ for all $c < \tilde{c}$, and $b_p(c) = b_0^{AI}(c)$ for all $c \geq \tilde{c}$. For this to be an equilibrium, a bidder with cost $\tilde{c}$ must be indifferent between bidding $b_0^{AI}(\tilde{c}) = b_p(\tilde{c})$ or bidding p: $P(\tilde{c})(p - \tilde{c}) = (1 - F(\tilde{c}))^{\widehat{N}-1}(b_0^{AI}(\tilde{c}) - \tilde{c})$. But this can never happen when $p < b_0^{AI}(\underline{c})$ since $P(\underline{c})(p - \underline{c}) = p - \underline{c} < b_0^{AI}(\underline{c}) - \underline{c}$, and

[10]This condition holds for the case in which $r \geq \bar{c}$. If $r < \bar{c}$, then b_p must be such that $b_p(r) = r$.

for all $c \in [\underline{c}, p]$, $\frac{\partial}{\partial c}P(c)(p-c) = -P(c) + P'(c)(p-c) \leq -P(c) < -(1-F(c))^{\widehat{N}-1} = \frac{\partial}{\partial c}(1-F(c))^{\widehat{N}-1}(b_0^{AI}(c)-c)$. Therefore, in this case the unique symmetric equilibrium is b_0^{AI}.

Consider next the case with $p > b_0^{AI}(\underline{c})$. By the arguments above, any symmetric equilibrium b_p must be such that $b_p(c) = b_0^{AI}(c)$ for all c with $b_p(c) > p$. Therefore, in any symmetric equilibrium, there exists $\tilde{c} > \underline{c}$ such that $b_p(c) = p$ for all $c < \tilde{c}$, and $b_p(c) = b_0^{AI}(c)$ for all $c \geq \tilde{c}$. Moreover, $\tilde{c}$ satisfies $P(\tilde{c})(p-\tilde{c}) = (1-F(\tilde{c}))^{\widehat{N}-1}(b_p^{AI}(\tilde{c})-\tilde{c})$. When $p > b_0^{AI}(\underline{c})$, there exists a unique such $\tilde{c}$ (see footnote 9). Therefore, in this case the unique symmetric equilibrium is b_p^{AI}. ■

Proof of Corollary 4. Suppose first that $p \leq b_0^{AI}(\underline{c})$. Then, $\text{prob}(\beta_p^{AI} \geq q | \beta_p^{AI} > p) = \text{prob}(\beta_0^{AI} \geq q | \beta_0^{AI} > p)$ for all $q > p$.

Consider next the case in which $p > b_0^{AI}(\underline{c})$. For all $b \in [b_0^{AI}(\underline{c}), b_0^{AI}(\overline{c})]$, let $c(b)$ be such that $b_0^{AI}(c(b)) = b$. Since $\hat{c}$ is such that $b_0^{AI}(\hat{c}) > p$, it follows that $\hat{c} > c(p)$. Note then that, for all $q \geq b_0^{AI}(\hat{c})$, $\text{prob}(\beta_p^{AI} \geq q | \beta_p^{AI} > p) = \frac{(1-F(c(q)))^{\widehat{N}}}{(1-F(\hat{c}))^{\widehat{N}}} > \frac{(1-F(c(q)))^{\widehat{N}}}{(1-F(c(p)))^{\widehat{N}}} = \text{prob}(\beta_0^{AI} \geq q | \beta_0^{AI} > p)$. For $q \in (p, b_0^{AI}(\hat{c}))$, $\text{prob}(\beta_p^{AI} \geq q | \beta_p^{AI} > p) = 1 > \frac{(1-F(c(q)))^{\widehat{N}}}{(1-F(c(p)))^{\widehat{N}}} = \text{prob}(\beta_0^{AI} \geq q | \beta_0^{AI} > p)$. ■

OB.3 Additional results and Proofs for Section 4

This appendix analyzes the model with entry in Section 4. We let $\widehat{N}_e$ denote the set of all participants in the auction; i.e., $\widehat{N}_e = \widehat{N}$ when $E = 0$, and $\widehat{N}_e = \widehat{N} \cup \{e\}$ when $E = 1$. Given a history h_t and an equilibrium σ, we let $\beta(\mathbf{c}|h_t, \sigma)$ be the bidding profile of cartel members and short-lived firm induced by σ at history h_t as a function of procurement costs $\mathbf{c} = (c_i)_{i \in \widehat{N}_e}$.[11] Our first result generalizes Lemma 1 to the current setting.

Lemma OB.2 (stationarity – entry)**.** *Consider a subgame perfect equilibrium σ that attains $\overline{V}_p$. Equilibrium σ delivers surplus $V(\sigma, h_t) = \overline{V}_p$ after all on-path histories h_t.*

There exists a fixed bidding profile β^* such that, in a Pareto efficient equilibrium, firms bid $\beta(\mathbf{c}_t|h_t, \sigma) = \beta^*(\mathbf{c}_t)$ after all on-path histories h_t.

Proof. The proof is identical to the proof of Lemma 1, and hence omitted. ■

Given a bidding profile (β, γ), we let $\beta^W(\mathbf{c})$ be the winning bid and $\mathbf{x}(\mathbf{c}) = (x_i(\mathbf{c}))_{i \in \widehat{N}_e}$ be the induced allocation when realized costs are $\mathbf{c} = (c_i)_{i \in \widehat{N}_e}$. As in Section 2, for all $i \in \widehat{N}_e$

[11] Since the vector of costs $\mathbf{c}$ includes the cost of the short-lived firm in case of entry, the cartel's bidding profile can be different depending on whether the short-lived firm enters the auction or not.

we let

$$\rho_i(\beta^W,\gamma,\mathbf{x})(\mathbf{c}) \equiv \mathbf{1}_{\beta^W(\mathbf{c})>p} + \frac{\mathbf{1}_{\beta^W(\mathbf{c})=p}}{\sum_{j\in\widehat{N}_e\setminus\{i\}:x_j(\mathbf{c})>0}\gamma_j(\mathbf{c})+1}.$$

Lemma OB.3 (enforceable bidding – entry)**.** *A winning bid profile $\beta^W(\mathbf{c})$ and an allocation $\mathbf{x}(\mathbf{c})$ are sustainable in SPE if and only if, for $E\in\{0,1\}$ and for all $\mathbf{c}$,*

$$\sum_{i\in\widehat{N}}\{(\rho_i(\beta^W,\gamma,\mathbf{x})(\mathbf{c})-x_i(\mathbf{c}))[\beta^W(\mathbf{c})-c_i]^+ + x_i(\mathbf{c})[\beta^W(\mathbf{c})-c_i]^-\} \leq \delta(\overline{V}_p - \sum_{i\in N}\underline{V}_{i,p}). \tag{O6}$$

$$E\times\{(\rho_e(\beta^W,\gamma,\mathbf{x})(\mathbf{c})-x_e(\mathbf{c}))[\beta^W(\mathbf{c})-c_e]^+ + x_e(\mathbf{c})[\beta^W(\mathbf{c})-c_e]^-\}\leq 0. \tag{O7}$$

Proof. We start with a few preliminary observations. Fix an SPE σ and a history h_t, and suppose that the entry decision of the short-lived firm at time t is E. For each $\mathbf{c}$, let $\beta(\mathbf{c})$, $\gamma(\mathbf{c})$ and $T(\mathbf{c},\mathbf{b},\gamma,\mathbf{x})$ be the bidding profile of cartel members and short-lived firm and the transfer profile of cartel members in this equilibrium after history $h_t \sqcup (E,\mathbf{c})$. For each $\mathbf{c}$, let $\beta^W(\mathbf{c})$ and $\mathbf{x}(\mathbf{c})$ be winning bid and the allocation induced by bidding profile $(\beta(\mathbf{c}),\gamma(\mathbf{c}))$. For each $h_{t+1} = h_t \sqcup (E,\mathbf{c},\mathbf{b},\gamma,\mathbf{x},\mathbf{T})$, let $\{V(h_{t+1})\}_{i\in N}$ be the vector of continuation payoffs of cartel members after history h_{t+1}. We let $h_{t+1}(\mathbf{c}) = h_t \sqcup (E,\mathbf{c},\beta(\mathbf{c}),\gamma(\mathbf{c}),\mathbf{x}(\mathbf{c}),\mathbf{T}(\mathbf{c},\beta(\mathbf{c}),\gamma(\mathbf{c}),\mathbf{x}(\mathbf{c})))$ denote the on-path history that follows $h_t\sqcup(E,\mathbf{c})$. With this notation, the inequalities (O3)-(O5) in Appendix OB must also hold in this setting. Moreover, if $E=1$, it must also be that

$$\mathbf{x}_e(\mathbf{c})[\beta^W(\mathbf{c})-c_e]^+ \geq \rho_e(\beta^W,\gamma,\mathbf{x})(\mathbf{c})[\beta^W(\mathbf{c})-c_e]^+ \text{ and } x_e(\mathbf{c})[\beta^W(\mathbf{c})-c_e]^-\leq 0. \tag{O8}$$

Conversely, suppose there exists a winning bid profile $\beta^W(\mathbf{c})$, an allocation $\mathbf{x}(\mathbf{c})$, a transfer profile $\mathbf{T}$ and equilibrium continuation payoffs $\{V_i(h_{t+1}(\mathbf{c})\}_{i\in N}$ that satisfy inequalities (O3)-(O5) in Appendix OB for some $\gamma(\mathbf{c})$ that is consistent with $\mathbf{x}(\mathbf{c})$ (i.e., $x_i(\mathbf{c}) = \gamma_i(\mathbf{c})/\sum_{j:x_j(\mathbf{c})>0}\gamma_j(\mathbf{c})$ for all $i\in\widehat{N}_e$ with $x_i(\mathbf{c})>0$) and satisfy (O8) if $E=1$. Then, $(\beta^W,\mathbf{x},\mathbf{T})$ can be supported in an SPE as follows. For all $\mathbf{c}$, all firms $i\in\widehat{N}_e$ bid $\beta^W(\mathbf{c})$. Firms $i\in\widehat{N}_e$ with $x_i(\mathbf{c})=0$ choose $\tilde{\gamma}_i(\mathbf{c})=0$ and firms $i\in\widehat{N}_e$ with $x_i(\mathbf{c})>0$ choose $\tilde{\gamma}_i(\mathbf{c})=\gamma_i(\mathbf{c})$. Note that, for all $i\in\widehat{N}_e$, $x_i(\mathbf{c})=\tilde{\gamma}_i(\mathbf{c})/\sum_j\tilde{\gamma}_j(\mathbf{c})$ and $\rho_i(\beta^W,\tilde{\gamma},\mathbf{x})(\mathbf{c}) = \rho_i(\beta^W,\gamma,\mathbf{x})(\mathbf{c})$. If no firm $i\in\widehat{N}$ deviates at the bidding stage, cartel members make transfers $T_i(\mathbf{c},\beta(\mathbf{c}),\gamma(\mathbf{c}),\mathbf{x}(\mathbf{c}))$. If no firm $i\in N$ deviates at the transfer stage, in the next period cartel members play an SPE that gives payoff vector $\{V(h_{t+1}(\mathbf{c}))\}_{i\in N}$. If firm $i\in\widehat{N}$ deviates at the bidding stage, there are no transfers and the cartel reverts to an equilibrium that gives firm i a payoff of $\underline{V}_{i,p}$; if firm $i \in N$ deviates at the transfer stage, the cartel reverts to an equilibrium that gives firm i a payoff of $\underline{V}_{i,p}$ (deviations by more than one firm go unpunished). Since (O3) holds, under this strategy profile no firm $i \in \widehat{N}$ has an incentive to undercut the winning bid $\beta^W(\mathbf{c})$. Since (O4) holds, no firm $i \in \widehat{N}$ with $c_i > \beta^W(\mathbf{c})$ and $x_i(\mathbf{c}) > 0$ has an incentive to bid above $\beta^W(\mathbf{c})$ and lose. Upward deviations by a firm $i \in \widehat{N}_e$ with $c_i < \beta^W(\mathbf{c})$ who bids $\beta^W(\mathbf{c})$ are not profitable since the firm would lose the auction by bidding $b > \beta^W(\mathbf{c})$. Since (O8) holds, the short-lived firm does not have an incentive to deviate when $E = 1$. Finally, since (O5) holds, all firms $i \in N$ have an incentive to make their required transfers.

We now turn to the proof of the Lemma. The proof that (O6) must hold in any equilibrium uses the same arguments used in the proof of Lemma 2, and hence we omit it. Since (O8) must hold for $E = 1$, it follows that

$$E \times \{(\rho_e(\beta^W, \gamma, \mathbf{x})(\mathbf{c}) - x_e(\mathbf{c}))[\beta^W(\mathbf{c}) - c_e]^+ + x_e(\mathbf{c})[\beta^W(\mathbf{c}) - c_e]^-\} \leq 0.$$

Next, consider a winning bid profile $\beta^W(\mathbf{c})$ and an allocation $\mathbf{x}(\mathbf{c})$ that satisfy (O6) and (O7) for all $\mathbf{c}$ for some $\gamma(\mathbf{c})$ consistent with $\mathbf{x}(\mathbf{c})$ (i.e., such that $x_i(\mathbf{c}) = \gamma_i(\mathbf{c}) / \sum_{j: x_j(\mathbf{c})>0} \gamma_j(\mathbf{c})$ for all i with $x_i(\mathbf{c}) > 0$). We construct an SPE that supports $\beta^W(\mathbf{c})$ and $\mathbf{x}(\mathbf{c})$ in the first period. Let $\{V_i\}_{i \in N}$ be an equilibrium payoff vector with $\sum_i V_i = \overline{V}_p$. For each $\mathbf{c} = (c_i)_{i \in \widehat{N}_e}$ and $i \in N$, we construct transfers $T_i(\mathbf{c})$ as follows:

$$T_i(\mathbf{c}) = \begin{cases} -\delta(V_i - \underline{V}_{i,p}) + (\rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c}) - x_i(\mathbf{c}))(\beta^W(\mathbf{c}) - c_i) + \epsilon(\mathbf{c}) & \text{if } i \in \widehat{N}, c_i \leq \beta^W(\mathbf{c}), \\ -\delta(V_i - \underline{V}_{i,p}) - x_i(\mathbf{c})(\beta^W(\mathbf{c}) - c_i) + \epsilon(\mathbf{c}) & \text{if } i \in \widehat{N}, c_i > \beta^W(\mathbf{c}), \\ -\delta(V_i - \underline{V}_{i,p}) + \epsilon(\mathbf{c}) & \text{if } i \notin \widehat{N}, \end{cases}$$

where $\epsilon(\mathbf{c}) \geq 0$ is a constant to be determined below. Since $\beta^W(\mathbf{c})$ and $\mathbf{x}(\mathbf{c})$ satisfy (O6), it follows that for all $\mathbf{c}$,

$$\begin{aligned} &\sum_{i \in N} T_i(\mathbf{c}) - n\epsilon(\mathbf{c}) \\ =& -\delta(\overline{V}_p - \sum_{i \in N} \underline{V}_{i,p}) + \sum_{i \in \widehat{N}} \left\{ (\rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c}) - x_i) \left[\beta^W(\mathbf{c}) - c_i\right]^+ + x_i \left[\beta^W(\mathbf{c}) - c_i\right]^- \right\} \leq 0. \end{aligned}$$

We set $\epsilon(\mathbf{c}) \geq 0$ such that transfers are budget balance; i.e., such that $\sum_{i \in N} T_i(\mathbf{c}) = 0$.

The SPE we construct is as follows. At $t = 0$, for each $\mathbf{c} = (c_i)_{i \in \widehat{N}_e}$ all firms $i \in \widehat{N}_e$ bid $\beta^W(\mathbf{c})$. Firms $i \in \widehat{N}_e$ with $x_i(\mathbf{c}) = 0$ choose $\tilde{\gamma}_i(\mathbf{c}) = 0$, and firms $i \in \widehat{N}_e$ with

$x_i(\mathbf{c}) > 0$ choose $\tilde{\gamma}_i(\mathbf{c}) = \gamma_i(\mathbf{c})$. Note that, for all $i \in \widehat{N}_e$, $x_i(\mathbf{c}) = \tilde{\gamma}_i(\mathbf{c}) / \sum_j \tilde{\gamma}_j(\mathbf{c})$ and $\rho_i(\beta^W, \tilde{\gamma}, \mathbf{x})(\mathbf{c}) = \rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c})$. If no firm $i \in \widehat{N}$ deviates at the bidding stage, cartel members exchange transfers $T_i(\mathbf{c})$. If no firm $i \in N$ deviates at the transfer stage, from $t = 1$ onwards firms play an SPE that supports payoff vector $\{V_i\}$. If firm $i \in N$ deviates either at the bidding stage or at the transfer stage, from $t = 1$ onwards firms play an SPE that gives firm i a payoff $\underline{V}_{i,p}$ (if more than one firm deviates, then firms punish the lowest indexed firm that deviated). One can check that this strategy profile satisfies (O3)-(O5) in Appendix OB and (O8). Hence, winning bid profile β^W and allocation $\mathbf{x}$ are implementable.

■

Recall that

$$b_p^*(\mathbf{c}) = \sup \left\{ b \leq r : \sum_{i \in \widehat{N}} (1 - x_i^*(\mathbf{c})) \left[b - c_i\right]^+ \leq \delta(\overline{V}_p - \sum_{i \in N} \underline{V}_{i,p}) \right\}.$$

Proposition OB.1. *In an optimal equilibrium, the on-path bidding profile is such that:*

(i) if $E = 0$, the cartel sets winning bid $\beta_p^*(\mathbf{c}) = \max\{b_p^*(\mathbf{c}), p\}$;

(ii) if $E = 1$, the winning bid is $\beta_p^*(\mathbf{c}) = \max\{p, \min\{c_e, b_p^*(\mathbf{c})\}\}$ when a cartel wins the auction, and is $\beta_p^*(\mathbf{c}) = \max\{c_e, p\}$ when the entrant wins the auction.

Proof. The proof of part (i) is identical to the proof of Proposition 1, and hence omitted.

We now turn to part (ii). Note first that, by Lemma OB.3, entry by the short-lived firm reduces the set of sustainable bidding profiles and thus the profits that the cartel can obtain in an auction. Therefore, in an optimal equilibrium the cartel seeks to maximize its payoff and minimize the short-lived firm's payoff from entry.

Suppose $E = 1$. For any $\mathbf{c}$, let $\beta^W(\mathbf{c})$ and $\mathbf{x}(\mathbf{c})$ be, respectively, the winning bid and allocation in an optimal equilibrium. We let $c_{(1)} = \min_{i \in \widehat{N}} c_i$ be the lowest cost among participating cartel members. Consider first cost realizations $\mathbf{c}$ such that $c_{(1)} > c_e \geq p$. In this case, $x_e(\mathbf{c}) = 1$ in an optimal bidding profile. Indeed, by equation (O7), $\beta^W(\mathbf{c}) \leq c_e$ if $x_e(\mathbf{c}) < 1$. Hence, the cartel is better-off letting the short-lived firm win whenever $c_{(1)} > c_e \geq p$. Moreover, by setting $\beta^W(\mathbf{c}) = c_e$, the cartel guarantees that the short-lived firm earns zero payoff.[12]

[12]This is achieved by having all participating cartel members bidding $\beta^W(\mathbf{c}) = c_e$ and $\gamma_i(\mathbf{c}) = 0$, and having the entrant bidding $\beta^W(\mathbf{c}) = c_e$ and $\gamma_e(\mathbf{c}) = 1$.

Consider next $\mathbf{c}$ such that $c_{(1)} > p > c_e$. By (O7), it must be that $x_e(\mathbf{c}) > 0$. In this case, in an optimal equilibrium the cartel sets winning bid equal to $\beta^W(\mathbf{c}) = p$, as this minimizes the short-lived firm's payoff from winning.

Consider next $\mathbf{c}$ such that $c_{(1)} < c_e$ and $c_e \geq p$. Clearly, an optimal bidding profile for the cartel must be such that $x_e(\mathbf{c}) = 0$. Equation (O7) then implies that $\beta^W(\mathbf{c}) \leq c_e$. We now show that, in this case, $\beta^W(\mathbf{c}) = \max\{p, \min\{c_e, b_p^*(\mathbf{c})\}\}$. There are two cases to consider: (a) $b_p^*(\mathbf{c}) > c_e$, and (b) $b_p^*(\mathbf{c}) \leq c_e$. Consider case (a), so $b_p^*(\mathbf{c}) > c_e \geq p$. It follows that

$$\sum_{i \in \widehat{N}} (1 - x_i^*(\mathbf{c}))[c_e - c_i]^+ < \sum_{i \in \widehat{N}} (1 - x_i^*(\mathbf{c}))[b_p^*(\mathbf{c}) - c_i]^+ \leq \delta(\overline{V}_p - \sum_{i \in N} \underline{V}_{i,p}).$$

Therefore, a bidding profile that induces winning bid c_e and allocation $\mathbf{x}^*(\mathbf{c})$ satisfies (O6) and (O7). Since such a bidding profile is optimal for the cartel among all bidding profiles with winning bid lower than c_e, it must be that $\beta^W(\mathbf{c}) = c_e$.

Consider next case (b). Note that for all $b > \max\{b_p^*(\mathbf{c}), p\}$ and any allocation $\mathbf{x}(\mathbf{c})$,

$$\sum_{i \in \widehat{N}} \left\{(1 - x_i(\mathbf{c}))[b - c_i]^+ + x_i(\mathbf{c})[b - c_i]^-\right\} \geq \sum_{i \in \widehat{N}} (1 - x_i^*(\mathbf{c}))[b - c_i]^+ > \delta(\overline{V}_p - \sum_{i \in N} \underline{V}_{i,p}),$$

so $\max\{b_p^*(\mathbf{c}), p\}$ is the largest winning bid that can be supported in an equilibrium. Therefore, in an optimal equilibrium cartel members must use a bidding profile inducing winning bid $\max\{b_p^*(\mathbf{c}), p\}$.

Finally, consider $\mathbf{c}$ such that $c_{(1)} < p$ and $c_e < p$. We now show that, in an optimal equilibrium, $\beta^W(\mathbf{c}) = p$. Indeed, by (O7), a winning bid $\beta^W(\mathbf{c}) > p > c_e$ can only be implemented if $x_e(\mathbf{c}) = 1$. But this is clearly suboptimal for the cartel. Indeed, the cartel could make strictly positive profits by having a firm with cost $c_{(1)}$ bidding p; and doing this would also strictly reduce the short-lived firm's expected payoff from entering. Therefore, in an optimal equilibrium it must be that $\beta^W(\mathbf{c}) = p$. ■

Proposition OB.1 characterizes bidding behavior under an optimal equilibrium. In periods in which the short-lived firm does not participate, the cartel's bidding behavior is the same as in Section 2. Entry by a short-lived firm reduces the cartels profits in two ways: (i) the cartel losses the auction whenever the entrant's procurement cost is low enough, and (ii) entry leads to weakly lower winning bids when the cartel wins the auction.

By Proposition OB.1, the winning bid when the entrant wins the auction is $\beta_p^*(\mathbf{c}) = \max\{c_{(e)}, p\}$. For $p \leq \underline{c}$, the entrant earns zero payoff from participating in the auction.

Therefore, for $p \leq \underline{c}$ the entrant participates in the auction if and only if its entry cost is equal to zero.[13] For $p > \underline{c}$, the entrant's payoff from participating in the auction is strictly positive. From now on we assume that the distribution of entry costs F_k has a mass point at zero, so that there is positive probability of entry for all minimum prices p.

Our last result in this section extends Proposition 2 to the current setting. Recall that $\beta_0^*(\underline{c})$ is the lowest bid under minimum price $p = 0$.

Proposition OB.2 (worse case punishment – entry)**.** *(i)* $\underline{V}_{i,0} = 0$*, and* $\underline{V}_{i,p} > 0$ *whenever* $p > \underline{c}$*;*

(ii) there exists $\eta > 0$ *such that, for all* $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$*,* $\overline{V}_p - \sum_{i \in N} \underline{V}_{i,p} \leq \overline{V}_0 - \sum_{i \in N} \underline{V}_{i,0}$*. The inequality is strict if* $p \in (\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$*.*

Proof. We first establish part (i). Suppose that $p \leq \underline{c}$ and fix equilibrium payoffs $\{V_i\}_{i \in N}$. Fix $j \in N$ and consider the following strategy profile. At $t = 0$, firms' behavior depends on whether $j \in \widehat{N}$ or $j \notin \widehat{N}$. If $j \in \widehat{N}$, all firms $i \in \widehat{N}_e$ bid $\min\{c_j, \hat{c}_{(2)}\}$ (where $\hat{c}_{(2)}$ is the second lowest procurement cost among firms in $\widehat{N}_e$). Firm $i \in \widehat{N}_e$ chooses $\gamma_i = 1$ if $c_i = \min_{k \in \widehat{N}_e} c_k$, and chooses $\gamma_i = 0$ otherwise. Note that this bidding profile constitutes an equilibrium of the stage game. If $j \notin \widehat{N}$, at $t = 0$ participating firms play according to some equilibrium of the stage game. If all firms bid according to this profile, firm j's transfer is $T_j = -\delta V_j$ at the end of the period regardless of whether $j \in \widehat{N}$ or $j \notin \widehat{N}$. The transfer of firm $i \in N \backslash \{j\}$ is $T_i = \frac{1}{n-1} \delta V_j$ at the end of the period, so $\sum_i T_i = 0$. If no firm deviates at the bidding or transfer stage, at $t = 1$ firms play according to an equilibrium that delivers payoffs $\{V_i\}$. If firm i deviates at the bidding stage, there are no transfers and at $t = 1$ firms play the strategy just described with i in place of j. If no firm deviates at the bidding stage and firm i deviates at the transfer stage, at $t = 1$ firms play the strategy just described with i in place of j (if more than one firm deviates at the bidding or transfer stage, from $t = 1$ firms play according to an equilibrium that delivers payoffs $\{V_i\}_{i \in N}$). Note that this strategy profile gives player j a payoff of 0. Moreover, no firm has an incentive to deviate at $t = 0$, and so $\underline{V}_{i,p} = 0$ for all $p \leq \underline{c}$.

Suppose next that $p > \underline{c}$, and note that

$$\underline{V}_{i,p} \geq \underline{u}_{i,p} \equiv \frac{1}{1-\delta} \text{prob}(i \in \widehat{N}) \mathbb{E}_{F_i} \left[\frac{1}{\widehat{N}+1} \mathbf{1}_{c_i \leq p} (p - c_i) | i \in \widehat{N} \right] > 0,$$

[13] We assume that the short-lived firm participates in the auction whenever its indifferent.

where the inequality follows since firm i can always guarantee a payoff at least as large as $\underline{u}_{i,p}$ by bidding p whenever $c_i \leq p$ and bidding $b \geq c_i$ otherwise. This establishes part (i).

We now turn to part (ii). Note that $\beta_0^*(\underline{c}) = \underline{c}$.[14] Fix $\eta > 0$ and $p \in [\underline{c}, \underline{c}+\eta]$. For $E = 0, 1$, let (β^E, γ^E) be the bidding profile that firms use on the equilibrium path at periods in which the short-lived firm's entry decision is E under an equilibrium that attains $\overline{V}_p$ when the minimum price is p. Let $\beta_p^*(\mathbf{c})$ and $\mathbf{x}^p(\mathbf{c})$ denote, respectively, the winning bid and the allocation under this optimal equilibrium. The cartel's expected payoff under this optimal equilibrium satisfies

$$\begin{aligned}(1-\delta)\overline{V}_p =& \text{prob}(E=0|p)\mathbb{E}\left[\sum_{i\in\widehat{N}} x_i^p(\mathbf{c})(\beta_p^*(\mathbf{c})-c_i)\,|E=0\right] \\ &+ \text{prob}(E=1|p)\mathbb{E}\left[\sum_{i\in\widehat{N}} x_i^p(\mathbf{c})(\beta_p^*(\mathbf{c})-c_i)\,|E=1\right].\end{aligned}$$

Suppose there is no minimum price and consider the following bidding profile for cartel members. For $E = 0, 1$ and all $\mathbf{c}$ such that $\beta_p^*(\mathbf{c}) > p$, participating firms bid according to (β^E, γ^E). For $E = 0$ and all $\mathbf{c}$ such that $\beta_p^*(\mathbf{c}) = p$, all participating cartel members bid $c_{(2)}$; firm $i \in \widehat{N}$ with $c_i = c_{(1)} = \min_{j\in\widehat{N}} c_j$ sets $\gamma_i = 1$, and firm $i \in \widehat{N}$ with $c_i > c_{(1)}$ sets $\gamma_i = 0$. For $E = 1$ and all $\mathbf{c}$ such that $\beta_p^*(\mathbf{c}) = p$, all participating firms bid $\min\{c_{(2)}, c_e\}$; firm $i \in \widehat{N}_e$ sets $\gamma_i = 1$ if $c_i = \min_{k\in\widehat{N}_e} c_k$ and sets $\gamma_i = 0$ otherwise. Note that, for $\mathbf{c}$ such that $\beta_p^*(\mathbf{c}) = p$, the bidding profile that firms use constitutes an equilibrium of the stage game when there is no minimum price. Note further that the entrant earns a lower expected payoff under this bidding profile than under the optimal equilibrium for minimum price $p \in [\underline{c}, \underline{c} + \eta]$; indeed, under this bidding profile, the entrant earns the same payoff than under the optimal equilibrium whenever $\beta_p^*(\mathbf{c}) > p$, and earns a payoff of zero whenever $\beta_p^*(\mathbf{c}) = p$. Therefore, the probability of entry under this strategy profile is lower than under the optimal equilibrium when minimum price is p. Let $\beta(\mathbf{c})$ and $\mathbf{x}(\mathbf{c})$ denote the winning bid and the allocation that this bidding profile induces. Let $\hat{V}_p$ be the cartel's total surplus

[14] Indeed, by Proposition OB.1, $\beta_0^*(\mathbf{c}) = \underline{c}$ whenever $E = 1$ and $c_e = \underline{c}$.

under this strategy profile, and note that

$$
\begin{aligned}
(1-\delta)\hat{V}_p =&\text{prob}(E=0|\text{no min price})\mathbb{E}\left[\sum_{i\in\widehat{N}} x_i(\mathbf{c})(\beta(\mathbf{c})-c_i)\,|E=0\right] \\
&+\text{prob}(E=1|\text{no min price})\mathbb{E}\left[\sum_{i\in\widehat{N}} x_i(\mathbf{c})(\beta(\mathbf{c})-c_i)\,|E=1\right] \\
\geq&\text{prob}(E=0|p)\mathbb{E}\left[\sum_{i\in\widehat{N}} x_i^p(\mathbf{c})(\beta_p^*(\mathbf{c})-c_i)\mathbf{1}_{\beta_p^*(\mathbf{c})>p}\,|E=0\right] \\
&+\text{prob}(E=1|p)\mathbb{E}\left[\sum_{i\in\widehat{N}} x_i^p(\mathbf{c})(\beta_p^*(\mathbf{c})-c_i)\mathbf{1}_{\beta_p^*(\mathbf{c})>p}\,|E=1\right],
\end{aligned}
$$

where we used the fact that the prob$(E=0|p)\leq$ prob$(E=0|\text{no min price})$ and that the cartel's payoff conditional on $E=0$ is weakly larger than its payoff conditional on $E=1$.

Note that $b_p^*(\mathbf{c})\geq \underline{c}+\frac{\delta(\overline{V}_p-\sum_{i\in N}\underline{V}_{i,p})}{n-1}>\underline{c}$.[15] By Proposition OB.1, $\beta_p^*(\mathbf{c})=\max\{p,b_p^*(\mathbf{c})\}$ whenever $E=0$. Therefore, for $\eta>0$ small enough and for $E=0$, $\beta_p^*(\mathbf{c})>p$ for all $\mathbf{c}$ and all $p\in[\underline{c},\underline{c}+\eta]$. For all such $\eta>0$ and for all $p\in[\underline{c},\underline{c}+\eta]$, prob$(\beta_p^*(\mathbf{c})=p|E=0)=0$. Moreover, Proposition OB.1 also implies that prob$(\beta_p^*(\mathbf{c})=p|E=1)=F_e(p)$ for all $p\in[\underline{c},\underline{c}+\eta]$.[16] Therefore, for $\eta>0$ small enough and for $p\in[\underline{c},\underline{c}+\eta]$,

$$
\begin{aligned}
(1-\delta)(\overline{V}_p-\hat{V}_p)\leq&\text{prob}(E=1|p)\mathbb{E}\left[\sum_{i\in\widehat{N}} x_i^p(\mathbf{c})(\beta_p^*(\mathbf{c})-c_i)\mathbf{1}_{\beta_p^*(\mathbf{c})=p}\,|E=1\right] \\
\leq&\text{prob}(E=1|p)\frac{n}{n+1}F_e(p)\mathbb{E}[(p-c_{(1)})\mathbf{1}_{c_{(1)}\leq p}].
\end{aligned}
$$

where the second inequality follows since the probability with which the cartel wins the auction when the entrant's cost is below p is bounded above by $\frac{n}{n+1}$, and since the cartel's payoff from winning the auction at price p is bounded above by $(p-c_{(1)})\mathbf{1}_{c_{(1)}\leq p}$. Let $\underline{F}$ be a distribution with support $[\underline{c},\overline{c}]$ such that $\mathbb{E}[(p-c_{(1)})\mathbf{1}_{c_{(1)}\leq p}]\leq\int_{\underline{c}}^{p}(p-c)n(1-\underline{F}(c))^{n-1}\underline{f}(c)dc$.[17]

[15]Indeed, $\inf_{\mathbf{c}} b_p^*(\mathbf{c})$ is attained when all cartel members participate and they all have a cost equal to $\underline{c}$. In this case, $b_p^*(\mathbf{c})=\underline{c}+\frac{\delta(\overline{V}_p-\sum_{i\in N}\underline{V}_{i,p})}{n-1}$.

[16]Indeed, $b_p^*(\mathbf{c})>p$ for all $\mathbf{c}$ and all $p\in[\underline{c},\underline{c}+\eta]$. Therefore, by Proposition OB.1, for all $p\in[\underline{c},\underline{c}+\eta]$ and for $E=1$, the winning bid $\beta_p^*(\mathbf{c})$ is equal to p only when the entrant's cost is below p.

[17]For instance, choose $\underline{F}$ such that for all $i\in N$ and all $c\in[\underline{c},\overline{c}]$, $\underline{F}(c)\geq F_i(c)$.

Note then that

$$(1-\delta)(\overline{V}_p - \hat{V}_p) \leq \text{prob}(E=1|p)\frac{n}{n+1}F_e(p)\int_{\underline{c}}^{p}(p-c)n(1-\underline{F}(c))^{n-1}\underline{f}(c)dc.$$

On the other hand, for each $i \in N$,

$$(1-\delta)\underline{V}_{i,p} \geq \underline{u}_{i,p} \geq \frac{n}{n+1}\text{prob}(i \in \widehat{N})\mathbb{E}_{F_i}[(p-c_i)\mathbf{1}_{c_i \leq p}] = \frac{n}{n+1}\text{prob}(i \in \widehat{N})\int_{\underline{c}}^{p}(p-c)f_i(c)dc.$$

Note that, for $p = \underline{c}$, $\hat{V}_p \geq \overline{V}_p - \sum_{i \in N}\underline{V}_{i,p} = \overline{V}_p$. Note further that

$$\begin{aligned}
\frac{\partial}{\partial p}\Big|_{p=\underline{c}} F_e(p)\int_{\underline{c}}^{p}(p-c)n(1-\underline{F}(c))^{n-1}\underline{f}(c)dc &= 0\\
\frac{\partial^2}{\partial p^2}\Big|_{p=\underline{c}} F_e(p)\int_{\underline{c}}^{p}(p-c)n(1-\underline{F}(c))^{n-1}\underline{f}(c)dc &= 0\\
\frac{\partial}{\partial p}\Big|_{p=\underline{c}} \int_{\underline{c}}^{p}(p-c)f_i(c)dc &= 0\\
\frac{\partial^2}{\partial p^2}\Big|_{p=\underline{c}} \int_{\underline{c}}^{p}(p-c)f_i(c)dc &= f_i(\underline{c}) > 0.
\end{aligned}$$

Therefore, there exists $\eta > 0$ small enough such that $\hat{V}_p \geq \overline{V}_p - \sum_{i \in N}\underline{V}_{i,p}$ for all $p \in [\underline{c}, \underline{c}+\eta]$, with strict inequality if $p > \underline{c}$. To establish part (ii) of the Lemma, we show that $\overline{V}_0 \geq \hat{V}_p$ for all $p \in [\underline{c}, \underline{c}+\eta]$.

Suppose there is no minimum price, and consider the following strategy profile. Along the equilibrium path, bidders bid according to the bidding profile described above, which generates surplus $\hat{V}_p$ for the cartel. If firm $i \in \widehat{N}$ deviates at the bidding stage, there are no transfers and in the next period cartel members play an equilibrium that gives firm i a payoff of $\underline{V}_{i,0} = 0$ (if more than one firm deviates, cartel members punish the lowest indexed firm that deviated). If no firm deviates at the bidding stage, each firm $i \in N$ makes transfer $T_i(\mathbf{c})$ to be determined below. If a firm $i \in N$ deviates at the transfer stage, in the next period firms play an equilibrium that gives firm i a payoff of $\underline{V}_{i,0} = 0$ (if more than one firm deviates, cartel members again punish the lowest indexed firm that deviated). Otherwise, if no firm deviates at the bidding and transfer stages, in the next period firms continue playing the same strategies as above.

Let $\{V_i\}_{i \in N}$ be a payoff profile with $\sum_i V_i = \hat{V}_p$ and $V_i \geq \underline{V}_{i,0} = 0$ for all i. The transfers

$T_i(\mathbf{c})$ are determined as follows. For all $\mathbf{c}$ such that $\beta_p^*(\mathbf{c}) = p$, $T_i(\mathbf{c}) = 0$ for all $i \in N$. Otherwise,

$$T_i(\mathbf{c}) = \begin{cases} -\delta V_i + (1 - x_i^p(\mathbf{c}))(\beta_p^*(\mathbf{c}) - c_i) + \epsilon(\mathbf{c}) & \text{if } i \in \widehat{N}, c_i \leq \beta_p^*(\mathbf{c}), \\ -\delta V_i + \epsilon(\mathbf{c}) & \text{otherwise,} \end{cases}$$

where $\epsilon(\mathbf{c}) \geq 0$ is a constant to be determined.[18] Note that

$$\sum_i T_i(\mathbf{c}) - n\epsilon(\mathbf{c}) = -\delta \hat{V}_p + \sum_i (1 - x_i^p(\mathbf{c}))[\beta_p^*(\mathbf{c}) - c_i]^+ \leq 0,$$

where the inequality follows since $\beta_p^*(\mathbf{c})$ is implementable with minimum price p, and since $\hat{V}_p \geq \overline{V}_p - \sum_{i \in N} \underline{V}_{i,p}$. We set $\epsilon(\mathbf{c}) \geq 0$ such that $\sum_i T_i(\mathbf{c}) = 0$. This strategy profile generates total surplus $\hat{V}_p$ for the cartel. One can check that no firm has an incentive to deviate at any stage, and so this strategy profile constitutes an equilibrium. Hence, it must be that $\overline{V}_0 \geq \hat{V}_p \geq \overline{V}_p - \sum_{i \in N} \underline{V}_{i,p}$ for all $p \in [\underline{c}, \underline{c} + \eta]$, and the second inequality is strict if $p > \underline{c}$. ∎

Proof of Proposition 5. Consider first a collusive environment and suppose that $E \in \{0, 1\}$. By Propositions OB.1 and OB.2, for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$, $\beta_p^*(\mathbf{c}) \leq \beta_0^*(\mathbf{c})$ for all $\mathbf{c}$ such that $\beta_0^*(\mathbf{c}) \geq p$. Therefore, for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$ and all $q > p$, $\text{prob}(\beta_p^* \geq q | \beta_p^* \geq p, E) \leq \text{prob}(\beta_0^* \geq q | \beta_0^* \geq p, E)$. This completes the proof of part (i).

Consider next a competitive environment. Let $\hat{c}_{(2)}$ be the second lowest cost among all participating firms (including the entrant if $E = 1$). Then, for all $p > 0$ and all $q > p$, $\text{prob}(\beta_p^{\mathsf{comp}} \geq q | \beta_p^{\mathsf{comp}} > p, E) = \text{prob}(\hat{c}_{(2)} \geq q | \hat{c}_{(2)} > p, E) = \text{prob}(\beta_0^{\mathsf{comp}} \geq q | \beta_0^{\mathsf{comp}} > p, E)$. This completes the proof of part (ii). ∎

Proof of Proposition 6. We start with part (i). As a first step, we show that for $E = 0, 1$, $\text{prob}(\beta_p^* \geq q | \beta_p^* \geq p, E, \text{cartel wins}) \leq \text{prob}(\beta_0^* \geq q | \beta_0^* \geq p, E, \text{cartel wins})$. In the case of $E = 0$, the result follows from Proposition 5(i). Suppose next that $E = 1$, and consider cost realizations $\mathbf{c}$ such that the cartel wins. By Propositions OB.1 and OB.2, for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$, $\beta_p^*(\mathbf{c}) \leq \beta_0^*(\mathbf{c})$ whenever $\beta_0^*(\mathbf{c}) \geq p$. Therefore, for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$ and all $q > p$, $\text{prob}(\beta_p^* \geq q | \beta_p^* \geq p, E = 1, \text{cartel wins}) \leq \text{prob}(\beta_0^* \geq q | \beta_0^* \geq p, E = 1, \text{cartel wins})$.

[18] Recall that $\mathbf{x}^p(\mathbf{c})$ is the allocation under an optimal equilibrium when the minimum price is p. Therefore, $\mathbf{x}^p(\mathbf{c})$ is such that $x_i^p(\mathbf{c}) = 0$ for all i with $c_i > \beta_p^*(\mathbf{c})$.

It then follows that, for any $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$ and any $q > p$,

$$\begin{aligned}
\text{prob}(\beta_p^* \geq q | \beta_p^* \geq p, \text{cartel wins}) &= \text{prob}(E = 0 | p > 0)\text{prob}(\beta_p^* \geq q | \beta_p^* \geq p, E = 0, \text{cartel wins}) \\
&\quad + \text{prob}(E = 1 | p > 0)\text{prob}(\beta_p^* \geq q | \beta_p^* \geq p, E = 1, \text{cartel wins}) \\
&\leq \text{prob}(E = 0 | p > 0)\text{prob}(\beta_0^* \geq q | \beta_0^* \geq p, E = 0, \text{cartel wins}) \\
&\quad + \text{prob}(E = 1 | p > 0)\text{prob}(\beta_0^* \geq q | \beta_0^* \geq p, E = 1, \text{cartel wins}) \\
&\leq \text{prob}(E = 0 | p = 0)\text{prob}(\beta_0^* \geq q | \beta_0^* \geq p, E = 0, \text{cartel wins}) \\
&\quad + \text{prob}(E = 1 | p = 0)\text{prob}(\beta_0^* \geq q | \beta_0^* \geq p, E = 1, \text{cartel wins}) \\
&= \text{prob}(\beta_0^* \geq q | \beta_0^* \geq p, \text{cartel wins}),
\end{aligned}$$

where the first inequality follows from the arguments in the previous paragraph, and the second inequality follows since $\text{prob}(E = 1 | p = 0) \leq \text{prob}(E = 1 | p > 0)$ (i.e., the probability of entry increases with the minimum price) and since the cartel's winning bid is lower when the entrant participates.

We now turn to part (ii). Consider cost realizations $\mathbf{c}$ such that the entrant wins. By Proposition OB.1, $\beta_0^*(\mathbf{c}) = c_{(e)}$ and $\beta_p^*(\mathbf{c}) = \max\{c_{(e)}, p\}$. Therefore, for all $p > 0$ and all $q > p$, $\text{prob}(\beta_p^* \geq q | \beta_p^* > p, \text{entrant wins}) = \text{prob}(\beta_0^* \geq q | \beta_0^* > p, \text{entrant wins})$. This completes the proof of part (ii). ■

OC Participation by non-performing bidders

The official rationale for introducing minimum prices is that it reduces the incidence of non-performing bidders, i.e. bidders unable to execute the tasks described in the procurement contract. In addition to reducing the cost of procured services, the auctioneer is also interested in reducing the likelihood that a contract is assigned to a non-performing bidder.

The effect of minimum prices can be captured in the framework of Section 4. Non-performing bidders can be modeled as entrants whose cost of production is set to 0. To simplify the analysis, we further assume that the cost of entry of non-performing bidders is equal to 0, and that other bidders are informed of the non-performing status of the entrant. We denote by q the likelihood that a non-performing entrant is present.

It is immediate that the characterization of equilibrium bids given by Proposition OB.1 and the results in Proposition 5 and Proposition 6 continue to hold: they rely only on the bidder-side of the market. Hence the possibility of non-performance does not affect our analysis. We now clarify the effect of minimum bids on non-performance.

Lemma OC.1 (likelihood of non-performance)**.** *Under both competition and collusion, the likelihood that the contract is awarded to a non-performing entrant is equal to* $q \times \mathbb{E}\left[\frac{1}{\sum_{i \in \widehat{N}_t} \mathbf{1}_{c_{i,t} \leq p}}\right]$. *It is decreasing in minimum price* p.

Proof. Since costs are public information across participants, the only equilibrium under competition is such that the equilibrium bid is equal to $\max\{p, c_{(2)}\}$, the maximum between the minimum price and the second lowest cost. Hence the non-performing bidder wins: with probability 1 when all other bidders have a cost of production above p; by tie-breaking when several other bidders have a cost of production below p.

Under collusion, the assumption that non-performing entrants have a cost of entry of 0, and the assumption that their non-performing status is known to other bidders, imply that the cartel is unable to deter entry by non-performing entrants. As a result, when a non-performing entrant is present, cartel members do not bid below their cost of production. Hence, the non-performing entrant wins the contract for the same configuration of costs as in the case of competition. ■

OD Endogenous participation

OD.1 Model

We extend the model in the main text to allow for endogenous participation by cartel members. The main point of the extension is to show that, in an optimal equilibrium, the cartel will actively manage the number of firms that participate at each auction. This allows a cartel to sustain high prices even if it's composed of a large number of firms. We also show that firms can implement the optimal equilibrium by dividing themselves into different sub-cartels.

At each period $t \in \mathbb{N}$, firms in $N = \{1, ..., n\}$ simultaneously choose whether or not to participate in the auction. We let $E_{i,t} \in \{0, 1\}$ denote the entry decision of firm $i \in N$, with $E_{i,t} = 1$ denoting entry.[19] For simplicity, we assume that procurements costs of those firms that enter the market are independently drawn from c.d.f. F with support $[\underline{c}, \overline{c}]$ and density f. We denote by $\tilde{N}_t = \{i \in N : E_{i,t} = 1\}$ the set of firms that participate at period t, and

[19] Note that we assume that all firms in N can participate at every period. The model can be easily extended to allow the set of potential participants to be randomly drawn at each period.

by $\mathbf{c}_t = (c_{i,t})_{i\in\tilde{N}_t}$ the cost realization of all firms in $\tilde{N}_t$. Note that cost vector $\mathbf{c}_t$ contains information about the set participants $\tilde{N}_t$ at period t.

The timing of information and decisions within period t is as follows.

1. Firms $i \in N$ simultaneously make entry decisions $E_{i,t} \in \{0,1\}$. Entry decisions are publicly observed.
2. Production costs $\mathbf{c}_t = (c_{i,t})_{i\in\tilde{N}_t}$ of participating firms are drawn and publicly observed.
3. Participating firms submit public bids $\mathbf{b}_t = (b_{i,t})_{i\in\tilde{N}_t}$ and numbers $\gamma_t = (\gamma_{i,t})_{i\in\tilde{N}_t}$, resulting in allocation $\mathbf{x}_t = (x_{i,t})_{i\in\tilde{N}_t}$.[20]
4. Firms make transfers $T_{i,t}$.

The history among cartel members at the beginning of time t is

$$h_t = \{\mathbf{c}_s, \mathbf{b}_s, \gamma_s, \mathbf{x}_s, \mathbf{T}_s\}_{s=0}^{t-1}.$$

Let $\mathcal{H}^t$ denote the set of period t public histories and $\mathcal{H} = \bigcup_{t\geq 0} \mathcal{H}^t$ denote the set of all histories (note that, for all s, cost vector $\mathbf{c}_s = (c_{i,s})_{i\in\tilde{N}_s}$ contains information about the firms that participate at time s). Our solution concept is subgame perfect equilibrium (SPE), with strategies

$$\sigma_i : h_t \mapsto (E_{i,t}, b_{i,t}(\mathbf{c}_t), \gamma_{i,t}(\mathbf{c}_t), T_{i,t}(\mathbf{c}_t, \mathbf{b}_t, \gamma_t, \mathbf{x}_t))$$

such that entry decisions $E_{i,t}$, bids $(b_{i,t}(\mathbf{c}_t), \gamma_{i,t}(\mathbf{c}_t))$ and transfers $T_{i,t}(\mathbf{c}_t, \mathbf{b}_t, \gamma_t, \mathbf{x}_t)$ can depend on all public data available at the time of decision-making.

OD.2 Optimal collusion

For any SPE σ and any history h_t, we denote by $V(\sigma, h_t)$ the surplus generated by σ under history h_t. As in the main text, we denote by $\overline{V}_p$ the highest surplus that firms can sustain in a SPE. Given a history h_t and a strategy profile σ, we denote by $E(h_t, \sigma)$ and by $\beta(\mathbf{c}_t|h_t, \sigma)$ the entry profile and bidding profile induced by strategy profile σ at history h_t.

Lemma OD.1 (stationarity)**.** *Consider a subgame perfect equilibrium σ that attains $\overline{V}_p$. Equilibrium σ delivers surplus $V(\sigma, h_t) = \overline{V}_p$ after all on-path histories h_t.*

[20] The allocation is determined in the same way as in the main text.

There exists an integer $\tilde{n} \leq n$ and a bidding profile β^* such that, in an equilibrium that attains $\overline{V}_p$, $\tilde{n}$ firms enter and bid according to $\beta(\mathbf{c}_t|h_t, \sigma) = \beta^*(\mathbf{c}_t)$ after all on-path histories h_t.

Proof. The proof is identical to the proof of Lemma 1 and hence omitted. ■

We denote by $\underline{V}_p$ the lowest possible equilibrium payoff for a given firm . Similarly, for any $\tilde{N} \subset N$, we denote $\underline{V}_p^{|\tilde{N}|}$ the lowest equilibrium payoff for a firm starting at a history at which $|\tilde{N}|$ firms chose to participate in the current auction (and before their procurement costs are drawn). Since firms are assumed to be symmetric, $\underline{V}_p$ and $\underline{V}_p^{|\tilde{N}|}$ are the same across firms.

Given a bidding profile (β, γ), let us denote by $\beta^W(\mathbf{c})$ and $\mathbf{x}(\mathbf{c})$ the induced winning bid and allocation profile for realized costs $\mathbf{c} = (c_i)_{i \in \tilde{N}}$.[21] Recall that, for each firm i,

$$\rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c}) \equiv \mathbf{1}_{\beta^W(\mathbf{c})>p} + \frac{\mathbf{1}_{\beta^W(\mathbf{c})=p}}{1 + \sum_{j \in \tilde{N} \setminus \{i\}: x_j(\mathbf{c})>0} \gamma_j(\mathbf{c})}.$$

is a deviator's highest possible chance of winning the contract by attempting to undercut the equilibrium winning bid.

Lemma OD.2 (enforceable bidding and participation). *Entry profile $E \in \{0,1\}^N$ leading to set of participants $\tilde{N} = \{i \in N : E_i = 1\}$, winning bid profile $\beta^W(\mathbf{c})$ and allocation $\mathbf{x}(\mathbf{c})$ are sustainable in SPE if and only if for all $\mathbf{c} = (c_i)_{i \in \tilde{N}}$,*

$$\begin{aligned}&\sum_{i \in \tilde{N}} (\rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c}) - x_i(\mathbf{c})) \left[\beta^W(\mathbf{c}) - c_i\right]^+ + x_i(\mathbf{c}) \left[\beta^W(\mathbf{c}) - c_i\right]^- \\ &\leq \delta(\overline{V}_p - |\tilde{N}|\underline{V}_p) - (n - |\tilde{N}|)\underline{V}_p^{|\tilde{N}|+1}. \end{aligned} \tag{O9}$$

The second term on the right-hand side of (O9) captures the cost of keeping potential participants out of the auction. Indeed, when the set of participants $\tilde{N}$ is a strict subset of N, the cartel has to promise firms that stay out of the auction a payoff at least as large as $\underline{V}_p^{|\tilde{N}|+1}$. Note that when all firms enter the auction (i.e., when $\tilde{N} = N$), obedience constraint (O9) is the same as the obedience constraint in our baseline model (under the assumption of symmetry; i.e., $\underline{V}_{i,p} = \underline{V}_p$ for all i).

[21] Recall that the cost vector $\mathbf{c} = (c_i)_{i \in \tilde{N}}$ contains information about the set of entrants. Hence, $\beta^W(\mathbf{c})$ and $\mathbf{x}(\mathbf{c})$ are allowed to depend on the set of entrants.

Proof. We start with some preliminary observations. Fix an SPE σ and a history h_t. Let E, $\beta(\mathbf{c})$, $\gamma(\mathbf{c})$ and $T(\mathbf{c}, \mathbf{b}, \gamma, \mathbf{x})$ be the entry, bidding and transfer profile that firms use in this equilibrium after history h_t. Let $\beta^W(\mathbf{c})$ and $\mathbf{x}(\mathbf{c})$ be, respectively, the winning bid and the allocation induced by bidding profile $(\beta(\mathbf{c}), \gamma(\mathbf{c}))$. Let $h_{t+1} = h_t \sqcup (\mathbf{c}, \mathbf{b}, \gamma, \mathbf{x}, \mathbf{T})$ be the concatenated history composed of h_t followed by $(\mathbf{c}, \mathbf{b}, \gamma, \mathbf{x}, \mathbf{T})$, and let $\{V(h_{t+1})\}_{i\in N}$ be the vector of continuation payoffs after history h_{t+1}. We let $h_{t+1}(\mathbf{c}) = h_t \sqcup (\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}), \mathbf{T}(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})))$ denote the on-path history that follows h_t when current costs are $\mathbf{c}$. Note that the following inequalities must hold:

(i) for all $i \in N$ such that $E_i = 1$ and $c_i \leq \beta^W(\mathbf{c})$,

$$x_i(\mathbf{c})(\beta^W(\mathbf{c})-c_i)+T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}))+\delta V_i(h_{t+1}(\mathbf{c})) \geq \rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c})(\beta^W(\mathbf{c})-c_i)+\delta \underline{V}_p. \tag{O10}$$

(ii) for all $i \in N$ such that $E_i = 1$ and $c_i > \beta^W(\mathbf{c})$,

$$x_i(\mathbf{c})(\beta^W(\mathbf{c}) - c_i) + T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) + \delta V_i(h_{t+1}(\mathbf{c})) \geq \delta \underline{V}_p. \tag{O11}$$

(iii) for all $i \in N$ such that $E_i = 0$,

$$T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) + \delta V_i(h_{t+1}(\mathbf{c})) \geq \underline{V}_p^{|\tilde{N}|+1}. \tag{O12}$$

(iv) for all $i \in N$,

$$T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) + \delta V_i(h_{t+1}(\mathbf{c})) \geq \delta \underline{V}_p. \tag{O13}$$

Relative to our baseline model, the new constraint is (O12). This inequality must hold since bidder $i \in N$ with $E_i = 0$ can obtain at least $\underline{V}_p^{|\tilde{N}|+1}$ by participating in the current auction rather than staying out.

Conversely, suppose there exists an entry profile E, a winning bid profile $\beta^W(\mathbf{c})$, an allocation $\mathbf{x}(\mathbf{c})$, a transfer profile $\mathbf{T}$ and equilibrium continuation payoffs $\{V_i(h_{t+1}(\mathbf{c}))\}_{i\in N}$ that satisfy inequalities (O10)-(O13) for some $\gamma(\mathbf{c})$ that is consistent with $\mathbf{x}(\mathbf{c})$ (i.e., $\gamma(\mathbf{c})$ is such that $x_i(\mathbf{c}) = \gamma_i(\mathbf{c})/\sum_{j:x_j(\mathbf{c})>0} \gamma_j(\mathbf{c})$ for all i with $x_i(\mathbf{c}) > 0$). Then, $(E, \beta^W, \mathbf{x}, \mathbf{T})$ can be supported in an SPE as follows. Firms in N adopt entry decisions given by E. Let $\tilde{N} = \{i \in N : E_i = 1\}$. For all $\mathbf{c} = (c_i)_{i\in\tilde{N}}$, firms $i \in \tilde{N}$ bid $\beta^W(\mathbf{c})$. Firms $i \in \tilde{N}$ with $x_i(\mathbf{c}) = 0$ choose $\tilde{\gamma}_i(\mathbf{c}) = 0$, and firms $i \in \tilde{N}$ with $x_i(\mathbf{c}) > 0$ choose $\tilde{\gamma}_i(\mathbf{c}) = \gamma_i(\mathbf{c})$. Note that, for all $i \in \tilde{N}$, $x_i(\mathbf{c}) = \tilde{\gamma}_i(\mathbf{c})/\sum_j \tilde{\gamma}_j(\mathbf{c})$ and $\rho_i(\beta^W, \tilde{\gamma}, \mathbf{x})(\mathbf{c}) = \rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c})$. If no firm deviates at the entry and bidding stages, firms make transfers $T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}))$. If no firm deviates at the transfer stage, in the next period firms play an SPE that gives payoff vector $\{V(h_{t+1}(\mathbf{c}))\}_{i\in N}$. If firm $i \notin \tilde{N}$ enters, the cartel reverts to an equilibrium that gives firm i a payoff of $\underline{V}_p^{|\tilde{N}|+1}$; if firm $i \in \tilde{N}$ does not participate, the cartel reverts to an equilibrium that gives bidder i a continuation payoff of $\underline{V}_p$; if a firm $i \in \tilde{N}$ deviates at the bidding stage, there are no transfers and the cartel reverts to an equilibrium that gives firm i a continuation payoff of $\underline{V}_p$; if firm $i \in N$ deviates at the transfer stage, the cartel reverts to an equilibrium that gives firm i a continuation payoff of $\underline{V}_p$ (deviations by more than one firm go unpunished). Since (O10) holds, under this strategy profile no participating firm has an incentive to undercut the winning bid $\beta^W(\mathbf{c})$. Since (O11) holds, no participating firm with $c_i > \beta^W(\mathbf{c})$ and $x_i(\mathbf{c}) > 0$ has an incentive to bid above $\beta^W(\mathbf{c})$ and lose. Moreover, (O10) and (O11) also guarantee that firms $i \in \tilde{N}$ have an incentive to participate. Upward deviations by a firm $i \in \tilde{N}$ with $c_i < \beta^W(\mathbf{c})$ who wins the auction are not profitable since the firm would lose the auction by bidding $b > \beta^W(\mathbf{c})$. Since (O12) holds, firms $i \notin \tilde{N}$ have no incentive to participate. Finally, since (O13) holds, all firms have an incentive to make their required transfers.

We now turn to the proof of Lemma OD.2. Suppose there is an SPE σ and a history h_t at which firms bid according to a bidding profile (β, γ) that induces winning bid $\beta^W(\mathbf{c})$ and allocation $\mathbf{x}(\mathbf{c})$. Since the equilibrium must satisfy (O10)-(O13) for all $\mathbf{c}$,

$$
\begin{aligned}
&\sum_{i\in\tilde{N}} \left\{(\rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c}) - x_i(\mathbf{c}))\left[\beta^W(\mathbf{c}) - c_i\right]^+ + x_i(\mathbf{c})\left[\beta^W(\mathbf{c}) - c_i\right]^-\right\} \\
\leq &\sum_{i\in N} T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) + \delta \sum_{i\in N} V_i(h_{t+1}(\mathbf{c})) - \delta|\tilde{N}|\underline{V}_p - (n - |\tilde{N}|)\underline{V}_p^{|\tilde{N}|+1} \\
\leq &\delta(\overline{V}_p - |\tilde{N}|\underline{V}_p) - (n - |\tilde{N}|)\underline{V}_p^{|\tilde{N}|+1},
\end{aligned}
$$

where we used $\sum_i T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) = 0$ and $\sum_i V_i(h_{t+1}(\mathbf{c})) \leq \overline{V}_p$.

Next, consider an entry profile E, a winning bid profile $\beta^W(\mathbf{c})$ and an allocation $\mathbf{x}(\mathbf{c})$ that satisfy (O9) for all $\mathbf{c} = (c_i)_{i\in\tilde{N}}$ for some $\gamma(\mathbf{c})$ consistent with $\mathbf{x}(\mathbf{c})$ (i.e., such that $x_i(\mathbf{c}) = \gamma_i(\mathbf{c})/\sum_{j:x_j(\mathbf{c})>0}\gamma_j(\mathbf{c})$ for all $i \in \tilde{N}$ with $x_i(\mathbf{c}) > 0$). We now construct an SPE that supports E, $\beta^W(\cdot)$ and $\mathbf{x}(\cdot)$ in the first period. Let $\{V_i\}_{i\in N}$ be an equilibrium payoff vector with $\sum_i V_i = \overline{V}_p$. For each $\mathbf{c} = (c_i)_{i\in\tilde{N}}$ and each $i \in N$, we construct transfers $T_i(\mathbf{c})$ as follows:

$$T_i(\mathbf{c}) = \begin{cases} -\delta(V_i - \underline{V}_p) + (\rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c}) - x_i(\mathbf{c}))(\beta^W(\mathbf{c}) - c_i) + \epsilon(\mathbf{c}) & \text{if } i \in \tilde{N}, c_i \leq \beta^W(\mathbf{c}), \\ -\delta(V_i - \underline{V}_p) - x_i(\mathbf{c})(\beta^W(\mathbf{c}) - c_i) + \epsilon(\mathbf{c}) & \text{if } i \in \tilde{N}, c_i > \beta^W(\mathbf{c}), \\ -\delta V_i + \underline{V}_p^{|\tilde{N}|+1} + \epsilon(\mathbf{c}) & \text{if } i \notin \tilde{N}, \end{cases}$$

where $\epsilon(\mathbf{c}) \geq 0$ is a constant to be determined below. Note that, for all $\mathbf{c}$,

$$\begin{aligned} \sum_{i \in N} T_i(\mathbf{c}) - n\epsilon(\mathbf{c}) = & -\delta(\overline{V}_p - |\tilde{N}|\underline{V}_p) + (n - |\tilde{N}|)\underline{V}_p^{|\tilde{N}|+1} \\ & + \sum_{i \in \tilde{N}} \left\{ (\rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c}) - x_i(\mathbf{c})) \left[\beta^W(\mathbf{c}) - c_i\right]^+ + x_i(\mathbf{c}) \left[\beta^W(\mathbf{c}) - c_i\right]^- \right\} \leq 0, \end{aligned}$$

where the inequality follows since β^W and $\mathbf{x}$ satisfy (O9). We set $\epsilon(\mathbf{c}) \geq 0$ such that transfers are budget balance; i.e., such that $\sum_{i \in N} T_i(\mathbf{c}) = 0$.

The SPE we construct is as follows. At $t = 0$, for each $\mathbf{c}$ all participating firms bid $\beta^W(\mathbf{c})$. Firms $i \in \tilde{N}$ with $x_i(\mathbf{c}) = 0$ choose $\tilde{\gamma}_i(\mathbf{c}) = 0$, and firms $i \in \tilde{N}$ with $x_i(\mathbf{c}) > 0$ choose $\tilde{\gamma}_i(\mathbf{c}) = \gamma_i(\mathbf{c})$. Note that, for all $i \in \tilde{N}$, $x_i(\mathbf{c}) = \tilde{\gamma}_i(\mathbf{c}) / \sum_j \tilde{\gamma}_j(\mathbf{c})$ and $\rho_i(\beta^W, \tilde{\gamma}, \mathbf{x})(\mathbf{c}) = \rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c})$. If no firm deviates at the entry stage nor at the bidding stage, firms exchange transfers $T_i(\mathbf{c})$. If no firm deviates at the transfer stage, from $t = 1$ onwards they play an SPE that supports payoff vector $\{V_i\}$. If firm $i \in N$ deviates either at the bidding stage or at the transfer stage, from $t = 1$ onwards firms play an SPE that gives firm i a payoff $\underline{V}_p$ (if more than one firm deviates, firms punish the lowest indexed firm that deviated). If firm $i \notin \tilde{N}$ deviates at the entry stage and enters, firms revert to an equilibrium that gives firm i a payoff of $\underline{V}_p^{|\tilde{N}|+1}$. If firm $i \in \tilde{N}$ does not enter, firms revert to an equilibrium that gives firm i a payoff of $\underline{V}_p$ starting at $t = 1$. This strategy profile satisfies (O10)-(O13), and so β^W and $\mathbf{x}$ are sustainable in SPE. ■

For each $\tilde{N}$ and each $\mathbf{c} = (c_i)_{i \in \tilde{N}}$, we define

$$b_p^*(\mathbf{c}; \tilde{N}) \equiv \sup \left\{ b \leq r : \sum_{i \in \tilde{N}} (1 - x_i^*(\mathbf{c})) \left[b - c_i\right]^+ \leq \delta(\overline{V}_p - |\tilde{N}|\underline{V}_p) - (n - |\tilde{N}|)\underline{V}_p^{|\tilde{N}|+1} \right\},$$

where $\mathbf{x}^*(\mathbf{c})$ is the efficient allocation (ties broken randomly). Let $\beta_p(\mathbf{c}; \tilde{N}) = \max\{p, b_p^*(\mathbf{c}; \tilde{N})\}$, and let $\mathbf{x}_p(\mathbf{c}) = (x_{i,p})_{i \in \tilde{N}}$ be the most efficient allocation that is consistent with (O9) given $\mathbf{c}$ and the winning bid $\beta_p(\mathbf{c}; \tilde{N})$. Finally, let $\tilde{N}_p^* \in \arg\max_{\tilde{N} \in 2^N} \mathbb{E}[\beta_p^*(\mathbf{c}; \tilde{N}) - \sum_{i \in \tilde{N}} x_{i,p}(\mathbf{c}) c_i]$.

Proposition OD.1. *In any efficient equilibrium, on the equilibrium path, $|\tilde{N}_p^*|$ bidders enter the auction at every period and the winning bid is set equal to $\beta_p^*(\mathbf{c};\tilde{N}_p^*)$. Moreover, the allocation is conditionally efficient: whenever $\beta_p^*(\mathbf{c};\tilde{N}_p^*) > p$, the contract is allocated to the bidder with the lowest procurement cost.*

Proof. By Lemma OD.1, there exists an optimal equilibrium in which, at every on-path history, the same number of firms participate and participating firms use the same bidding profile (β,γ). For each cost vector $\mathbf{c} = (c_i)_{i\in\tilde{N}}$, let $\beta^W(\mathbf{c})$ and $\mathbf{x}(\mathbf{c})$ denote the winning bid and the allocation induced by this bidding profile under cost vector $\mathbf{c}$.

We next show that, if an optimal equilibrium is such that $|\tilde{N}|$ firms participate in the auction at each period along the equilibrium path, then the winning bid must be equal to $\beta_p(\mathbf{c};\tilde{N})$ for all cost vectors $\mathbf{c} = (c_i)_{i\in\tilde{N}}$.

Consider first cost vectors $\mathbf{c}$ such that $b_p^*(\mathbf{c};\tilde{N}) > p$. Towards a contradiction, suppose there exists $\mathbf{c}$ with $\beta^W(\mathbf{c}) \neq b_p^*(\mathbf{c};\tilde{N}) > p$. Since $\mathbf{x}^*(\mathbf{c})$ is the efficient allocation, the procurement cost under allocation $\mathbf{x}(\mathbf{c})$ is at least as large as the procurement cost under allocation $\mathbf{x}^*(\mathbf{c})$. Since bidding profile (β,γ) is optimal, it must be that $\beta^W(\mathbf{c}) > b_p^*(\mathbf{c};\tilde{N}) > p$. Indeed, if $\beta^W(\mathbf{c}) < b_p^*(\mathbf{c};\tilde{N})$, then the cartel would strictly prefer to use a bidding profile that allocates the contract efficiently and has winning bid $b_p^*(\mathbf{c};\tilde{N})$ under cost vector $\mathbf{c}$ than to use bidding profile $(\beta(\mathbf{c}),\gamma(\mathbf{c}))$. By Lemma OD.2, $\beta^W(\mathbf{c})$ and $\mathbf{x}(\mathbf{c})$ must satisfy

$$\begin{aligned}
\delta(\overline{V}_p - |\tilde{N}|\underline{V}_p) - (n-|\tilde{N}|)\underline{V}_p^{|\tilde{N}|+1} &\geq \sum_{i\in\tilde{N}} \left\{(1-x_i(\mathbf{c}))\left[\beta^W(\mathbf{c}) - c_i\right]^+ + x_i(\mathbf{c})\left[\beta^W(\mathbf{c}) - c_i\right]^-\right\} \\
&\geq \sum_{i\in\tilde{N}} (1-x_i^*(\mathbf{c}))\left[\beta^W(\mathbf{c}) - c_i\right]^+,
\end{aligned}$$

which contradicts $\beta^W(\mathbf{c}) > b_p^*(\mathbf{c};\tilde{N}) > p$. Therefore, $\beta^W(\mathbf{c}) = b_p^*(\mathbf{c};\tilde{N})$ for all $\mathbf{c}$ such that $b_p^*(\mathbf{c};\tilde{N}) > p$.

Next, we show that $\beta^W(\mathbf{c}) = p$ for all $\mathbf{c}$ such that $b_p^*(\mathbf{c};\tilde{N}) \leq p$. Towards a contradiction, suppose there exists $\mathbf{c}$ with $b_p^*(\mathbf{c};\tilde{N}) \leq p$ and $\beta^W(\mathbf{c}) > p$. By Lemma OD.2, $\beta^W(\mathbf{c})$ and $\mathbf{x}(\mathbf{c})$ satisfy

$$\begin{aligned}
\delta(\overline{V}_p - |\tilde{N}|\underline{V}_p) - (n-|\tilde{N}|)\underline{V}_p^{|\tilde{N}|+1} &\geq \sum_{i\in\tilde{N}} \left\{(1-x_i(\mathbf{c}))\left[\beta^W(\mathbf{c}) - c_i\right]^+ + x_i(\mathbf{c})\left[\beta^W(\mathbf{c}) - c_i\right]^-\right\} \\
&\geq \sum_{i\in\tilde{N}} (1-x_i^*(\mathbf{c}))\left[\beta^W(\mathbf{c}) - c_i\right]^+,
\end{aligned}$$

which contradicts $\beta^W(\mathbf{c}) > p \geq b_p^*(\mathbf{c};\tilde{N})$. Therefore, $\beta^W(\mathbf{c}) = p$ for all $\mathbf{c}$ such that $b_p^*(\mathbf{c};\tilde{N}) \leq$

p. Combining this with the arguments above, $\beta^W(\mathbf{c}) = \beta_p^*(\mathbf{c};\tilde{N}) = \max\{p, b_p^*(\mathbf{c};\tilde{N})\}$.

The results above show that if in an optimal equilibrium $|\tilde{N}|$ firms participate in the auction at each period along the equilibrium path, then the winning bid is equal to $\beta_p(\mathbf{c};\tilde{N})$ for all cost vectors $\mathbf{c} = (c_i)_{i\in\tilde{N}}$. For any $\tilde{N} \subset N$, winning with $\beta_p(\mathbf{c};\tilde{N})$ and allocation $\mathbf{x}_p(\mathbf{c})$ are sustainable in a SPE.[22] Therefore, in an optimal equilibrium, the number of firms that participate must be equal to $|\tilde{N}_p^*|$ for some $\tilde{N}_p^* \in \arg\max_{\tilde{N}\in 2^N} \mathbb{E}[\beta_p^*(\mathbf{c};\tilde{N}) - \sum_{i\in\tilde{N}} x_{i,p}(\mathbf{c})c_i]$.
■

Proposition OD.1 characterizes entry and bidding behavior of firms in an efficient equilibrium. We note that a large group of firms can achieve the highest surplus $\overline{V}_p$ by dividing themselves into sub-cartels of size $|N_p^*|$. Under such equilibria, firms would coordinate on the auctions at which each subcartel will be active. We note that this type of bidding arrangement is broadly consistent with our data. Indeed, as we show in Appendix OA, the firms that participate frequently in Tsuchiura appear to be organized in smaller subgroups of firms that interact frequently among each other.

Our next result clarifies how minimum prices affect the set of payoffs that firms can sustain in SPE.

Proposition OD.2 (worst case punishment). *(i)* $\underline{V}_0 = 0$, *and* $\underline{V}_p > 0$ *whenever* $p > \underline{c}$; $\forall \tilde{N} \subset N$, $\underline{V}_0^{|\tilde{N}|} = 0$, *and* $\underline{V}_p^{|\tilde{N}|} > \delta\underline{V}_p > 0$ *whenever* $p > \underline{c}$;

(ii) there exists $\overline{p} > \underline{c}$ *such that for all* $p \in [\underline{c}, \overline{p}]$,

$$\delta(\overline{V}_p - |N_p^*|\underline{V}_p) - (n - |N_p^*|)\underline{V}_p^{|N_p^*|+1} < \delta(\overline{V}_0 - |N_0^*|\underline{V}_0) - (n - |N_0^*|)\underline{V}_0^{|N_0^*|+1}.$$

Proof. We first establish part (i). Suppose that $p = 0$. Consider the following entry and bidding profile. All firms in N enter the auction. Then, for all cost realizations $\mathbf{c} = (c_i)_{i\in N}$, all firms $i \in N$ bid $c_{(1)} = \min_{k\in N} c_k$. Firm $i \in N$ chooses $\gamma_i = 1$ if $c_i = c_{(1)}$ and chooses $\gamma_i = 0$ otherwise. Note that this entry and bidding profile constitute an equilibrium of the stage game, and so the infinite repetition of this strategy profile constitutes an SPE. Moreover, this strategy profile gives all players a payoff of 0, so $\underline{V}_0 = 0$.

Consider next a subgame at which $\tilde{N} \subset N$ entered the auction. Consider the following bidding profile: for all $\mathbf{c} = (c_i)_{i\in\tilde{N}}$, all firms $i \in \tilde{N}$ bid $c_{(1)} = \min_{k\in\tilde{N}} c_k$. Firm $i \in \tilde{N}$ chooses $\gamma_i = 1$ if $c_i = c_{(1)}$ and chooses $\gamma_i = 0$ otherwise. Then, regardless of how firms behave,

[22] Recall that $\mathbf{x}_p(\mathbf{c})$ is the most efficient allocation that is consistent with (O9) when the winning bid is $\beta_p(\mathbf{c};\tilde{N})$.

starting from the next period firms play an equilibrium that gives all bidders a payoff of $\underline{V}_0 = 0$. One can check that no firm has an incentive to deviate in the initial period, so this strategy profile constitutes an SPE. Moreover, this strategy profile gives all players a payoff of 0, so $\underline{V}_0^{|\tilde{N}|} = 0$.

Suppose next that $p > \underline{c}$, and note that

$$\underline{V}_p \geq \underline{v}_p \equiv \frac{1}{1-\delta}\mathbb{E}\left[\frac{1}{n}\mathbf{1}_{c_i \leq p}(p - c_i)\right] > 0,$$

where the first inequality follows since $\underline{v}_p$ is the minimax payoff for a firm in an auction with minimum price p. Similarly, note that for all $\tilde{N}$,

$$\underline{V}_p^{|\tilde{N}|} \geq \mathbb{E}\left[\frac{1}{|\tilde{N}|}\mathbf{1}_{c_i \leq p}(p - c_i)\right] + \delta \underline{V}_p.$$

Indeed, firm i can obtain at least $\mathbb{E}\left[\frac{1}{|\tilde{N}|}\mathbf{1}_{c_i \leq p}(p - c_i)\right]$ in an auction in which $|\tilde{N}|$ firms participate; and its continuation value starting the next period must be at least as large as $\delta \underline{V}_p$. Finally, since $\mathbb{E}\left[\frac{1}{|\tilde{N}|}\mathbf{1}_{c_i \leq p}(p - c_i)\right] > 0$ for all $p > \underline{c}$, it follows that $\underline{V}_p^{|\tilde{N}|} > \delta \underline{V}_p$. This establishes part (i).

We now turn to the proof of part (ii). Fix $p > \underline{c}$, and let $|N_p^*|$, $\mathbf{x}^p(\cdot)$ and $\beta_p^*(\cdot) = \max\{p, b_p^*(\cdot)\}$ be, respectively, the number of participants, the allocation, and the winning bid in an optimal equilibrium with minimum price p. The surplus that the cartel generates in an optimal equilibrium under minimum price p is

$$\overline{V}_p = \frac{1}{1-\delta}\mathbb{E}\left[\beta_p^*(\mathbf{c}) - \sum x_i^p(\mathbf{c})c_i \,\middle|\, |N_p^*| \text{ bidders participate}\right],$$

Consider next a setting *without* minimum price, and consider the following strategy profile for the cartel. For all on-path histories, $|N_p^*|$ firms participate in the auction. All participating bidders bid $\beta(\mathbf{c}) = b_p^*(\mathbf{c})$; participating bidder i chooses $\gamma_i(\mathbf{c}) = 1$ if c_i is the lowest cost in $\mathbf{c}$, and $\gamma_i(\mathbf{c}) = 0$ otherwise. Note that the allocation induced by this bidding profile is the efficient allocation $\mathbf{x}^*$. Let $\widehat{V}_p$ be the total payoff that the cartel generates under this entry and bidding profile:

$$\widehat{V}_p = \frac{1}{1-\delta}\mathbb{E}\left[b_p^*(\mathbf{c}) - \sum x_i^*(\mathbf{c})c_i \,\middle|\, |N_p^*| \text{ bidders participate}\right].$$

If no firm deviates at the entry and bidding stages, firms make transfers $T_i(\mathbf{c})$ to be determined below. If no firm deviates at the transfer stage, in the next period firms continue playing the same entry and bidding profile. If a firm who was not suppose to participate in the auction enters, the cartel reverts to an equilibrium that gives firm i a payoff of $\underline{V}_0^{|N_p^*|+1} = 0$; if firm i who was supposed to enter does not participate, the cartel reverts to an equilibrium that gives bidder i a continuation payoff of $\underline{V}_0 = 0$; if a firm i that participates in the auction deviates at the bidding stage, there are no transfers and the cartel reverts to an equilibrium that gives firm i a continuation payoff of $\underline{V}_0 = 0$; if firm $i \in N$ deviates at the transfer stage, the cartel reverts to an equilibrium that gives firm i a continuation payoff of $\underline{V}_0 = 0$ (deviations by more than one firm go unpunished).

Before constructing the transfers $T(\mathbf{c})$, note that

$$\begin{aligned}\overline{V}_p - \widehat{V}_p &= \frac{1}{1-\delta}\mathbb{E}\left[\left(p - b_p^*(\mathbf{c}) - \sum (x_i^p(\mathbf{c}) - x_i^*(\mathbf{c}))c_i\right)\mathbf{1}_{b_p^*(\mathbf{c})<p}\,\middle|\,|N_p^*| \text{ bidders participate}\right] \\ &\leq \frac{1}{1-\delta}\mathbb{E}\left[(p - b_p^*(\mathbf{c}))\mathbf{1}_{b_p^*(\mathbf{c})<p}\,\middle|\,|N_p^*| \text{ bidders participate}\right],\end{aligned}$$

where the first equality follows since $\mathbf{x}^p(\mathbf{c}) = \mathbf{x}^*(\mathbf{c})$ whenever $\beta_p^*(\mathbf{c}) = b_p^*(\mathbf{c}) > p$, and the inequality follows since $\mathbf{x}^*$ is the efficient allocation. Note that $b_p^*(\mathbf{c}) \geq \underline{c} + \Delta$ for some $\Delta > 0$.[23] Let $\overline{p} \equiv \underline{c} + \Delta$. Then, for all $p \in (\underline{c}, \overline{p})$, $b_p^*(\mathbf{c}) \geq p$, and so $\delta\widehat{V}_p \geq \delta\overline{V}_p > \delta(\overline{V}_p - |N_p^*|\underline{V}_p) - (n - |N_p^*|)\underline{V}_p^{|N_p^*|+1}$ (where the last inequality follows from part (i) of the Lemma).

Set $p \in (\underline{c}, \overline{p})$. The transfers we construct are as follows. Let $N_p^* \subset N$ be the set of firms that participate. Then, for all $i \in N$,

$$T_i(\mathbf{c}) = \begin{cases} -\delta\frac{\widehat{V}_p}{n} + (1 - x_i^*(\mathbf{c}))(b_p^*(\mathbf{c}) - c_i) + \epsilon(\mathbf{c}) & \text{if } i \in N_p^*, c_i \leq \beta(\mathbf{c}), \\ -\delta\frac{\widehat{V}_p}{n} + \epsilon(\mathbf{c}) & \text{if } i \notin \tilde{N}_p^*, \end{cases}$$

where $\epsilon(\mathbf{c}) \geq 0$ is a constant to be determined below. Note that, for all $\mathbf{c}$,

$$\begin{aligned}\sum_{i \in N} T_i(\mathbf{c}) - n\epsilon(\mathbf{c}) &= -\,\delta\widehat{V}_p + \sum_{i \in N_p^*}(1 - x_i^*(\mathbf{c}))\left[b_p^*(\mathbf{c}) - c_i\right]^+ \\ &< -\,\delta(\overline{V}_p - |N_p^*|\underline{V}_p) + (n - |N_p^*|)\underline{V}_p^{|N_p^*|+1} + \sum_{i \in N_p^*}(1 - x_i^*(\mathbf{c}))\left[b_p^*(\mathbf{c}) - c_i\right]^+ \leq 0,\end{aligned}$$

where the first inequality follows since $\delta\widehat{V}_p > \delta(\overline{V}_p - |N_p^*|\underline{V}_p) - (n - |N_p^*|)\underline{V}_p^{|N_p^*|+1}$, and the last one follows from the definition of $b_p^*(\mathbf{c})$.

[23] Indeed, $b_p^*(\mathbf{c})$ attains its lowest value equal to when all participating firms have cost $\underline{c}$; this lowest value is $\underline{c} + \frac{1}{|N_p^*|-1}(\delta(\overline{V}_p - |N_p^*|\underline{V}_p) - (n - |\tilde{N}_p^*|)\underline{V}_p^{|\tilde{N}_p^*|+1})$.

One can check that, under this strategy profile, no firm has an incentive to deviate at any stage. Hence, this strategy profile is a SPE, and so $\overline{V}_0 \geq \widehat{V}_p$. Since $\delta\widehat{V}_p > \delta(\overline{V}_p - |N_p^*|\underline{V}_p) - (n-|N_p^*|)\underline{V}_p^{|N_p^*|+1}$, it follows that $\delta\overline{V}_0 > \delta(\overline{V}_p - |N_p^*|\underline{V}_p) - (n-|N_p^*|)\underline{V}_p^{|N_p^*|+1}$.
■

Proposition OD.2 shows that, when entry is endogenous, minimum prices limit the cartel's surplus in two ways. First, as in our baseline model, minimum prices limit the cartel's ability to punish firms that deviate at the bidding stage, thereby reducing the bids that can be sustained in a SPE. Second, minimum prices increase the cost of keeping potential participants out of the auction.

OD.3 Large cartel limit

We now discuss the cartel's ability to sustain high prices at the large cartel limit, i.e. when the number n of cartel members grows large. We first consider the case where minimum prices p are set to 0.

We first consider the case of exogenous participation described in the main text. In this case we assume that $|\widehat{N}_t| \geq \rho n$ for some $\rho \in (0,1)$. The highest sustainable price is determined by condition (1) in the main text. Since pledgeable surplus is bounded above by $\frac{1}{1-\delta}(r - \underline{c})$ (since production costs are bounded below by $\underline{c}$), it must be that the highest sustainable price converges to $\underline{c}$ almost surely as the cartel size n becomes large. As a result expected cartel profits must go to zero as the cartel grows large.

In contrast, when the number of participants is endogenous as in the previous subsection, expected profits are weakly increasing in cartel size. This follows from the fact that when minimum price p is equal to zero the cartel can costlessly control the number of participants in each auction. Since costs are public, any non-equilibrium entrant can be deprived of surplus by setting prices to her cost of production. In formal terms, $\underline{V}_{p=0}^{|\tilde{N}|+1} = 0$ (see Proposition OD.2).

This implies that in the absence of minimum prices, the fact that the number of cartel members in our data is large does not hinder the cartel's ability to sustain high prices. What matters isn't the total size of the cartel, but the number of cartel members participating in each auction. This finding is consistent with our data. While the number of high-frequency participants in our data ranges from 0 to 13 across years, the median number of participants in a given auction is equal to 3. We also note that large cartels are not unheard off in the field of construction. A 2008 press release by the UK's Office of Fair Trading noted that it had filed a case against 112 firms in the construction sector.[24] Reportedly, at least 80 of these firms have admitted engaging in bid-rigging.[25] We also note that firms in this cartel used monetary transfers. Another example of large scale collusion is the Dutch construction cartel, which included approximately 650.[26]

Interestingly, minimum prices also make sustaining cartels with endogenous participation more difficult. It is no longer costless to keep potential participants from entering since $\underline{V}_p^{|\tilde{N}|+1} > 0$ whenever $p > \underline{c}$. As a result, the introduction of minimum prices increases participation by cartel members, making it more difficult to sustain high prices. Table OA.2 shows that this is true in our data. Following the introduction of minimum prices the number of both cartel participants and entrants increases.

OE Measurement Error and Ommited Variable Bias

OE.1 Measurement Error

Proposition 6 requires conditioning on the entrant vs. long-run player status of the winning bidder. In this Appendix we show that Proposition 6 is robust to some forms of measurement error. The main requirement is that no long-run player be wrongly classified as an entrant. This motivates our choice to err on the side of inclusiveness when classifying firms as long-run players in our empirical analysis.

Let $E_W \in \{0, 1\}$ denote the entrant ($E_W = 1$) or long-run player ($E_W = 0$) status of the winning bidder. Proposition 6 establishes that under collusion, there exists $\eta > 0$ such that, for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$ and all $q > p$:

(i) $\text{prob}(\beta_p^* \geq q | \beta_p^* \geq p, E_W = 0) \leq \text{prob}(\beta_0^* \geq q | \beta_0^* \geq p, E_W = 0)$;

(ii) $\text{prob}(\beta_p^* \geq q | \beta_p^* > p, E_W = 1) = \text{prob}(\beta_0^* \geq q | \beta_0^* > p, E_W = 1)$.

Now assume that we only observe a signal $\widehat{E}_W \in \{0, 1\}$ of E_W. In our empirical analysis, $\widehat{E}_W = 0$ if the auction winner is a sufficiently frequent participant. By adjusting the participation-threshold above which a bidder is declared a long-run player, we can trade-off the misclassification of entrants as long-run players, and the misclassification of long-run players as entrants.

[24] http://webarchive.nationalarchives.gov.uk/20140402142426/http://www.oft.gov.uk/news/press/2008/52-08.
[25] https://en.wikipedia.org/wiki/Price_fixing_cases#Construction.
[26] https://www.oecd.org/regreform/sectors/41765075.pdf.

Assumption OE.1. *Assume that*

(i) $prob(E_W|\widehat{E}_W, \beta_W, p) = prob(E_W|\widehat{E}_W)$*;*

(ii) $prob(E_W = 0|\widehat{E}_W = 1) = 0$*.*

Assumption OE.1 states that measurement error is independent of winning bids, and that true long-run players are never classified as entrants.

Proposition OE.1. *If Assumption OE.1 holds, then there exists* $\eta > 0$ *such that, for all* $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$ *and all* $q > p$*:*

(i) $prob(\beta_p^* \geq q|\beta_p^* \geq p, \widehat{E}_W = 0) \leq prob(\beta_0^* \geq q|\beta_0^* \geq p, \widehat{E}_W = 0)$*;*

(ii) $prob(\beta_p^* \geq q|\beta_p^* > p, \widehat{E}_W = 1) = prob(\beta_0^* \geq q|\beta_0^* > p, \widehat{E}_W = 1)$*.*

Proof. Let p be such that Proposition 6 holds. We first establish point (ii). Assume firms are collusive. We have that

$$\begin{aligned}
\text{prob}(\beta_p^* \geq q|\beta_p^* > p, \widehat{E}_W = 1) =& \text{prob}(\beta_p^* \geq q|\beta_p^* > p, E_W = 0)\text{prob}(E_W = 0|\widehat{E}_W = 1) \\
&+ \text{prob}(\beta_p^* \geq q|\beta_p^* > p, E_W = 1)\text{prob}(E_W = 1|\widehat{E}_W = 1) \\
=& \text{prob}(\beta_p^* \geq q|\beta_p^* > p, E_W = 1)\text{prob}(E_W = 1|\widehat{E}_W = 1) \\
=& \text{prob}(\beta_0^* \geq q|\beta_0^* > p, E_W = 1)\text{prob}(E_W = 1|\widehat{E}_W = 1) \\
=& \text{prob}(\beta_0^* \geq q|\beta_0^* > p, \widehat{E}_W = 1)
\end{aligned}$$

where we used the assumption that $\text{prob}(E_W = 0|\widehat{E}_W = 1)$ and Proposition 6 (ii).

Point (i) follows from a similar line of reasoning. We have that

$$\begin{aligned}
\text{prob}(\beta_p^* \geq q|\beta_p^* \geq p, \widehat{E}_W = 0) =& \text{prob}(\beta_p^* \geq q|\beta_p^* \geq p, E_W = 0)\text{prob}(E_W = 0|\widehat{E}_W = 0) \\
&+ \text{prob}(\beta_p^* \geq q|\beta_p^* \geq p, E_W = 1)\text{prob}(E_W = 1|\widehat{E}_W = 0) \\
\leq& \text{prob}(\beta_0^* \geq q|\beta_0^* \geq p, E_W = 0)\text{prob}(E_W = 0|\widehat{E}_W = 0) \\
&+ \text{prob}(\beta_p^* \geq q|\beta_p^* \geq p, E_W = 1)\text{prob}(E_W = 1|\widehat{E}_W = 0)
\end{aligned} \tag{O14}$$

Observe that if $E_W = 1$, then $\beta_p^* > p$ and $\beta_0^* > p$ both imply that $\beta_p^* = \beta_0^*$. Hence

$$\begin{aligned}
\text{prob}(\beta_p^* \geq q|\beta_p^* \geq p, E_W = 1) =& \text{prob}(\beta_p^* \geq q|\beta_p^* > p, E_W = 1)\text{prob}(\beta_p^* > p|\beta_p^* \geq p, E_W = 1) \\
=& \text{prob}(\beta_0^* \geq q|\beta_0^* > p, E_W = 1)\text{prob}(\beta_p^* > p|\beta_p^* \geq p, E_W = 1)
\end{aligned}$$

Since $\beta_0^* = p$ implies $\beta_p^* = p$ it follows that $\text{prob}(\beta_p^* > p|\beta_p^* \geq p, E_W = 1) \leq \text{prob}(\beta_0^* > p|\beta_0^* \geq p, E_W = 1)$. Substituting into (O14), and we get that indeed

$\text{prob}(\beta_p^* \geq q|\beta_p^* \geq p, \widehat{E}_W = 0) \leq \text{prob}(\beta_0^* \geq q|\beta_0^* \geq p, \widehat{E}_W = 0)$. ■

OE.2 Omitted variable bias

If participation is correlated with both unobserved auction characteristics and the introduction of minimum prices, OLS estimates of the impact of minimum prices on winning bids controlling for the number of auction participants will be biased.

Consider the simple linear model of centered winning bids β_W

$$\beta_W = \langle X, \alpha \rangle + \gamma Z + \varepsilon$$

where: centered observable characteristics $X = (\text{min\_price}, N)$ include minimum-price-status and participation; Z is an unobserved auction characteristic correlated with participation. Then the OLS estimator $\widehat{\alpha}$ takes the form

$$\widehat{\alpha} = (X'X)^{-1}X'\beta_W = \alpha + \gamma(X'X)^{-1}X'Z + (X'X)^{-1}X'\varepsilon.$$

Note that we can always change the sign of the omitted variable so that $\gamma > 0$. The free variable is then the correlation between the omitted variable and participation. We assume the omitted variable is uncorrelated to minimum-pricestatus.

We address the possibility of omitted variable bias in two ways. First, we formulate a simple instrumentation strategy using recent past participation for similar auctions as an instrument. Second, in case it cannot be successfully resolved by instrumentation, we discuss the potential sign of this bias.

Instrumentation. One omitted variable of prominent interest that could be taken care of by this strategy is erroneously high reserve prices: if city engineers sometimes overestimate maximum costs, this may jointly lead to more entry and higher prices.

To address this type of bias, we propose to use the number of bidders in previous comparable auctions as an instrument for current participation. This variable is strongly correlated with the current number of bidders and uncorrelated with auction-specific omitted variables – plausibly including erroneously high reserve prices.

Our empirical findings are reported in Table OA.3 of the main Appendix. Our main empirical results continue to hold when we instrument the number of bidders with its lagged value:

- the introduction of a minimum price has a negative effect on winning bids, and
- the effect of the policy change is concentrated on the auctions won by bidders who participate frequently.

Likely sign of the bias. It is useful to evaluate the sign of potential bias absent instrumentation, in the event that the assumptions needed for successful instrumentation do not hold.

Denote $\begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix}$ the coefficients of $(X'X)$. Matrix $(X'X)^{-1}$ takes the form

$$\frac{1}{\sigma_1^2\sigma_2^2 - \sigma_{12}^2} \begin{pmatrix} \sigma_2^2 & -\sigma_{12} \\ -\sigma_{12} & \sigma_1^2 \end{pmatrix}.$$

Since $\sigma_1^2\sigma_2^2 - \sigma_{12}^2 > 0$ (by Cauchy-Schwarz) and $X'Z = \begin{pmatrix} 0 \\ NZ \end{pmatrix}$, it follows that the bias has the sign of $\begin{pmatrix} -\sigma_{12}NZ \\ \sigma_1^2 NZ \end{pmatrix}$.

We are specifically interested in $-\sigma_{12}NZ$: this is the bias in our estimate of the impact of minimum prices on winning bids. Note that the covariance σ_{12} between minimum price status and participation is positive. Hence the bias in our estimate of the impact of minimum prices is: positive if participation is negatively correlated with the omitted variable; negative if participation is positively correlated with the omitted variable.

Subjectively, it seems more plausible that entry will be positively correlated with omitted variables that also increase winning bids. This would be the case if the omitted variable is erroneously high reserve prices. In this case, omitted variable bias would go against our findings.

OF Calibration

Our calibration exercise seeks to gauge the range of plausible treatment effects one may have expected from a model such as ours. As a result we do not seek to estimate costs from bids.

Instead, we consider distribution of costs obtained by deflating winning bids with a fixed markup. This rough assumption lets us get back-of-the-envelope estimates of average and conditional treatment effects.

Equilibrium computation. We start by describing briefly how equilibrium bids can be computed. The optimal bidding behavior described by Proposition 1 is entirely determined by values $\overline{V}_p$ and $(\underline{V}_{i,p})_{i \in N}$. These values are the solution to the usual fixed-point problem: winning bids are a function of equilibrium values, and equilibrium values are a function of winning bids. Solving this fixed point numerically presents no particular difficulty since it's monotone. We illustrate how to proceed in the case in which there is no minimum price. In this case values $\underline{V}_{i,p}$ are equal to 0, and $\overline{V}_{p=0}$ is the only free parameter. For each candidate value $V \geq 0$ and every cost profile $\mathbf{c}$, let

$$\beta_0(\mathbf{c}; V) \equiv \sup \left\{ b \leq r : \sum_{i \in \widehat{N}} (1 - x_i^*(\mathbf{c}))[b - c_i]^+ \leq \delta V \right\}.$$

For every $V \geq 0$, define

$$U_0(V) \equiv \frac{1}{1-\delta} \mathbb{E} \left[\sum_{i \in \widehat{N}} x_i^*(\mathbf{c})(\beta_0(\mathbf{c}; V) - c_i) \right].$$

$U_0(V)$ is the total surplus generated by the optimal enforceable bidding profile when the continuation value is V. U_0 is an increasing function whose largest fixed-point is equal to $\overline{V}_0$, which can be computed as the limit of $(U_0^n(V))_{n \geq 0}$ for any seed value V sufficiently high.

Modeling choices and degrees of freedom. We implement directly the model of Section 4. Our key modeling choices and degrees of freedom are the following:

- We fix the number of cartel bidders to three in each auction. An entrant participates with probability q in the range [.6, .7]. In data from Tsuchiura, on average three cartel members participate in each auction, and bidders labelled as entrants are present in 66% of auctions.
- We keep the firms' yearly discount factor δ_Y as a free parameter in the range [.7, .9]. We note that auctions are not regularly spread out within the year, but rather occur

in batches. This generates an effective discount factor $\delta = \delta_Y^{\frac{D}{365}}$, where D is the average number of days between batches. The mean delay is 19 days.

- We do not estimate a cost distribution from winning bids but investigate treatment effects for not-implausible cost-distributions obtained in the following back of the envelope manner. Given the empirical distribution of winning bids b, we draw 4 independent values $\tilde{c}_i$, $i \in \{1, \cdots, 4\}$ according to distribution $c_i \sim \frac{1}{1+M} b$, where M is a fixed markup taking values in the range [.2,.6]. We then set as costs

$$\forall i \in \{1,2,3\}, \quad c_i = \lambda \frac{\sum_{i=1}^{3} \tilde{c}_i}{3} + (1-\lambda)\tilde{c}_i$$
$$c_4 = \lambda \frac{\sum_{i=1}^{3} \tilde{c}_i}{3} + (1-\lambda)\tilde{c}_4$$

  where λ parametrizes the correlation between the costs of participating cartel members. Given λ, the correlation between the costs of two cartel members is $\lambda^2 + \frac{2}{3}\lambda(1-\lambda)$. Cost c_4 is the entrant's cost if an entrant enters. In our data, correlation between bids is above 99%. We consider values of λ in the range [.95,.99].

  The reserve price r is set at

$$r = (1+m) \times \frac{\sum_{i=1}^{3} c_i}{3}$$

  where m is in the range [.4, .6].

- Minimum prices are a constant ratio of the reserve price. Consistent with our data we set this minimum price ratio in the range [.75, .8].

- We assume that cartel members follow the equilibrium strategies of the model in Section 4.[27] Values are computed by iterating, starting from an upper bound to values.

Findings. For each configuration of the parameters above, we simulate 1000 auctions with and without a minimum price. We compute the percentage change in average winning bids following the introduction of minimum prices for the unconditional distribution of winning bids, and for the conditional distribution of winning bids above the minimum price. We refer to these percentage changes in average procurement costs as the average and conditional treatment effects.

[27]We describe these strategies in detail in Appendix OB.3.

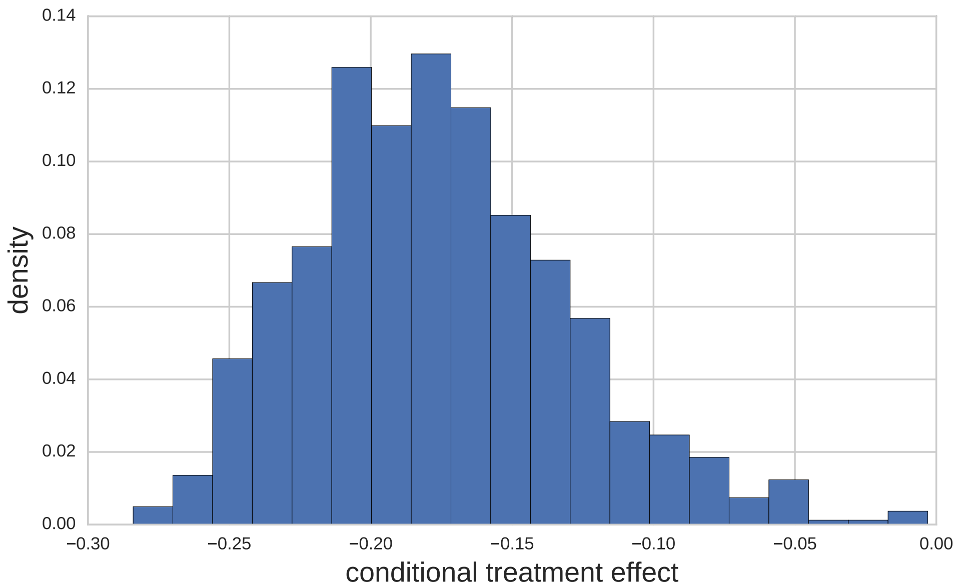

Figure OF.1: Conditional treatment effects.

Figure OF.1 reports the conditional treatment effects for each of the configurations of parameters above.[28] As anticipated, conditional treatment effects are negative. Their range, goes from −28% to −.3% and includes conditional treatment effects of the magnitude we find in our data.

Figure OF.2 reports the unconditional treatment effects for each of the configuration of parameters above. Treatment effects can be negative or positive. Their range, goes from −11% to +11% and includes unconditional treatment effects of the magnitude we find in our data. As Figure OF.3 shows, a key factor in explaining whether the average treatment effect is negative is the minimum price ratio. When it is relatively low, the truncation of the left tail of winning bids does not affect average winning bids much. When it is high, the truncation of the left tail of winning bids cannot be compensated by a drop in the right tail of winning bids.

[28]Therefore the distribution of treatment effects is the one induced by placing a uniform distribution over the product set of parameters we consider.

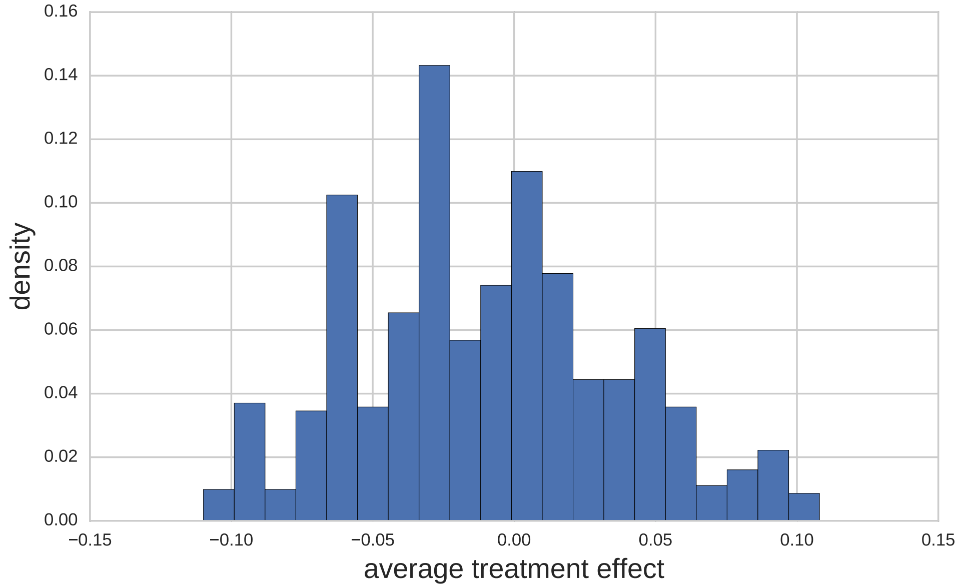

Figure OF.2: Unconditional treatment effects.

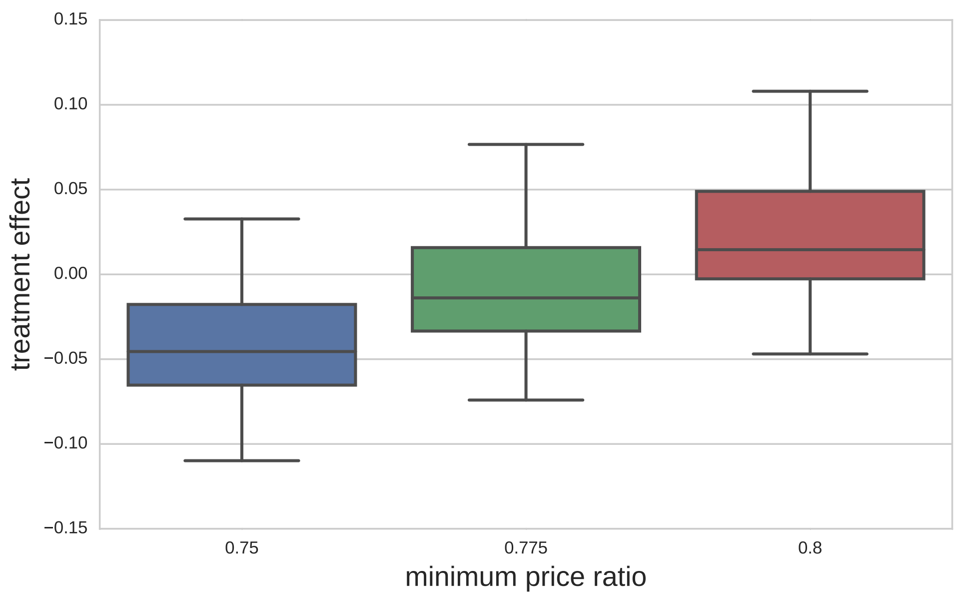

Figure OF.3: Unconditional treatment effect increase with the minimum price ratio.

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