The Value of Privacy in Cartels: An Analysis of the Inner Workings of a Bidding Ring

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Abstract

We study how incentive constraints can be relaxed by randomization in a repeated-game setting. Our study is motivated by the workings of a detected bidding cartel that operated in Kumatori, Japan that adopted a protocol of keeping the winning bid secret from the designated losers when defection was a concern. Keeping the winning bid secret makes accurately undercutting the winning bid more difficult and makes defection less attractive as potential defectors risk not winning the auction even if they deviate. We formalize these ideas in the context of a repeated-game setting and show that a cartel can attain higher payoffs by having the preselected winner randomize its bid and keep it secret from other members. Calibration of the model to the bid data of the cartel suggests that randomization may increase firms profits by about 56%.

Keywords: procurement, collusion, bidding ring, cartel, privacy.

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1 Introduction

In this paper, we explore how randomization can relax incentive compatibility (IC) constraints in a repeated-game setting when defection is a concern. We first document, through a detailed case study of a detected bidding ring participating in procurement auctions, how the ring responded to the threat of defection by strategically hiding the bid of the designated winner from the designated losers. In particular, the designated losers of the ring were instructed what to bid, but were deliberately kept in the dark as to what the bid of the designated winner would be. Not knowing the winning bid makes it difficult for a potential defector to correctly estimate, and undercut the winning bid. This makes defection less attractive, as potential defectors risk not winning the auction even if they deviate. We formalize these ideas by showing that randomization of the winning bid expands the set of available payoffs to a bidding ring that participates repeatedly in procurement auctions. Using non-deterministic, or random, bidding strategies make it possible to keep the winning bid secret. We explore quantitatively the importance of randomization by calibrating our theoretical model to the bidding data of the cartel.

Our analysis is motivated by a detailed case study of a bidding ring in Kumatori, Japan that was investigated and subsequently brought to trial. From the court documents, we identify features of the cartel that allow us to gain an understanding of the constraints the cartel faced and the protocols adopted by the cartel to overcome them. In particular, the cartel faced a stream of projects of varying sizes, some of which were very large, and hence potentially very profitable, making defection a concern. Prior to each auction, a designated winner was selected based on a bid rotation scheme. In order to counter defection by designated losers, the cartel devised a protocol in which the losing bidders would be instructed how they should bid, but were kept in the dark as to how the designated winner would bid. Moreover, the winning bid was occasionally set substantially lower than what the losing bidders were instructed to bid, making it difficult for the designated losers to correctly predict and undercut the winning bid. Much of the communication between the bidders was mediated by an intermediary.

While it is intuitive that keeping the designated winner's bid secret can relax IC constraints and achieve efficiency, this idea contrasts with existing results that show that optimal cartel bidding requires the losing bids to be placed within a very small margin of the winning bid. For example, Marshall and Marx (2007, 2012) and Chassang and Ortner (2019) show that the losing bidders should bid within a very close margin of the winning bid in order to incentivize the winning bidder not to deviate. In these papers, it is important that the designated losers know the winner's bid.

In order to formalize the idea that secrecy relaxes IC constraints, and to understand how it relates to existing work, we construct a model of a bidding ring that repeatedly participates in first-price procurement auctions. The size of the project auctioned off is drawn i.i.d. each period, and firms share the same costs. Consistent with our empirical application, we assume that the cartel has access to a mediator, and allocates contracts through a bid rotation scheme (although both of these assumptions can be dispensed with). We say that an equilibrium has *common-knowledge bids* if firms' bids depend deterministically on the public history.

Under the cartel's optimal bid rotation equilibrium with common-knowledge bids, winning bids are determined by either the losers' IC constraints or the reserve price. Whenever the project is sufficiently large, the winning bid is set to satisfy the IC constraints with equality, and the losing bidders are instructed to bid within a very small margin of the winning bid. These results are similar to those in Marshall and Marx (2007, 2012) and Chassang and Ortner (2019).

Our main result shows that a cartel may strictly gain from bidding schemes without common-knowledge bids. Under the optimal bid rotation equilibrium, the winner's bid is drawn randomly from a non-degenerate distribution F when the reserve price is sufficiently large. Non-winners are instructed to place a losing bid just above the support of F. Distribution F has the property that non-winners are indifferent between placing a losing bid and deviating by placing a bid in the support of F. By keeping losing firms uninformed, such a bidding scheme relaxes incentive constraints and yields larger profits for the cartel. Note that, for any deviating bid by a designated loser that is in the support of F, the deviator faces the prospect of certain punishment (since all bids become public ex-post) but a less-than-certain prospect of outbidding the designated bidder. We show that the value of randomized bids extends beyond bid-rotation equilibria: in general, a cartel's overall optimal equilibrium involves bidding schemes without common-knowledge bids.

A key implication of the optimal equilibria of our model is that the cartel is more likely to use randomized bidding schemes for large, and hence potentially more profitable projects. Moreover, optimal randomized bidding schemes tend to exhibit a sizable gap between the winning bid and the second lowest bid; i.e., there is significant amount of money left on the table, ex-post. These predictions are different from those of Marshall and Marx (2007, 2012) and borne out in the bidding pattern of the cartel in our case study.

In order to evaluate quantitatively the importance of randomized bidding, we calibrate our model to the bidding data from our case study. We find that randomizing the winning bid increases expected cartel profits by as much as 56% compared to the case with no randomization. Our results suggest that the gains from randomization are not insignificant, but instead, are of first-order importance. We also show that the calibrated model can replicate features of cartel bidding patterns such as the winning margin and how they vary with the reserve price, suggesting that, although parsimonious, our model captures key salient features of the cartel.

We see the main contribution of the paper as highlighting the practical value of randomization and privacy in relaxing IC constraints in naturally occurring settings. While the idea that randomization can help relax IC constraints appears broadly in theoretical work on mechanism design and repeated games, empirical evidence on the use of such randomization has largely been limited to audits, such as tax audits and police traffic stops. Our case study offers a concrete instance in which randomization is used to relax IC constraints outside of audits. Our analysis suggests that this idea is broadly useful in positive theories of behavior.

More specific to the antitrust context, our analysis sheds light on the role that transparency plays in sustaining a successful collusive scheme. It is well known from the theory of repeated games that transparency allows colluders to better coordinate and monitor each others' actions. Recently, however, Sugaya and Wolitzky (2018) identify a potential drawback of transparency: they show that transparency may hinder collusion by enabling potential defectors to devise more profitable deviations. Our case study provides an actual example of how privacy helps cartels. By keeping its bid secret from other bidders, the pre-selected winner makes it harder for potential defectors to profitably deviate.

Lastly, the paper also has implications for the information that an auctioneer facing collusive bidders should make public after each auction. In particular, while randomized bids can be profitable for a cartel when all bids are public, we show that such strategies are no longer beneficial when only the winning bid is public. As a result, only revealing the winning bid can alleviate collusion and lead to strictly lower prices. Intuitively, if only the winning bid is public, deviations are only detected when the defector wins the auction. This contrasts with settings in which all bids are public, in which defectors can be detected and punished even when they don't win.

Related literature. Previous papers have highlighted how randomization and privacy may help relax incentive constraints. Rahman and Obara (2010) and Rahman (2012) show that randomized messages can help provide incentives by making it easier to identify deviators. Jehiel (2015) establishes conditions under which a principal may benefit from keeping her agent in the dark about payoff relevant parameters. Ederer et al. (2018) show that randomized contracts may help prevent gaming by agents. Ortner and Chassang (2018) and Chassang and Miquel (2019) illustrate how randomized incentive schemes may help deter collusion by introducing contracting frictions among collusive parties. Within repeated-game settings, Kandori (2003), Kandori and Obara (2006), and Rahman (2014) show how randomization may allow arbitrarily patient players to sustain larger equilibrium payoffs.[1]

Our work also closely relates to previous work on how cartels bid in a repeated-game setting. There is a large theoretical literature on this topic exploring issues such as monitoring (e.g., Green and Porter, 1984, Skrzypacz and Hopenhayn, 2004, Sugaya and Wolitzky, 2018),

[1]Relatedly, Bernheim and Madsen (2017) show that a cartel's optimal stationary collusive scheme may involve randomization.

efficiency and private costs (e.g., Athey and Bagwell, 2001, Athey et al., 2004, Athey and Bagwell, 2008) and demand shocks (e.g., Rotemberg and Saloner, 1986, Haltiwanger and Harrington, 1991). Our baseline model has features that are related to monitoring and demand shocks, but abstracts away from private costs. Our work also relates to prior papers studying how communication may help sustain collusion (e.g., Compte, 1998, Kandori and Matsushima, 1998, Harrington and Skrzypacz, 2011, Rahman, 2014, Awaya and Krishna, 2016).

The empirical portion of our work is closely related to studies that test whether or not the price patterns implied by models of collusion are borne out in the data. Porter (1983) and Ellison (1994) study pricing by the Joint Executive Committee to test for the models of Green and Porter (1984) and Rotemberg and Saloner (1986). Levenstein (1997) studies the bromine cartel and finds evidence consistent with the model of Green and Porter (1984). Borenstein and Shepard (1996) study the retail gasoline market to test for the model of Haltiwanger and Harrington (1991). Wang (2009) also studies the retail gasoline market and finds evidence consistent with the equilibrium of Maskin and Tirole (1988). Wang (2009) is also one of the first papers to document evidence of mixed strategy equilibria for non zero-sum games outside of the lab. Earlier work that documents the use of mixed strategies for zero-sum games include Walker and Wooders (2001) and Chiappori et al. (2002).

Our paper is also related to the literature on detecting instances of collusion. Early seminal work includes Hendricks and Porter (1988), Baldwin et al. (1997) and Porter and Zona (1993, 1999). More recent work includes Bajari and Ye (2003), Abrantes-Metz et al. (2006), Athey et al. (2011), Conley and Decarolis (2016), Schurter (2017), Kawai and Nakabayashi (2018), Chassang et al. (2020) and Kawai et al. (2020).[2]

Lastly, our results relate to the literature on information design (Kamenica and Gentzkow, 2011, Bergemann and Morris, 2019). In particular, our results illustrate how a mediator

[2]Other related work includes Pesendorfer (2000), who studies bidding rings with and without side-payments, and Asker (2010), who studies knockout auctions among cartel members. Ohashi (2009) and Chassang and Ortner (2019) document how changes in auction design can affect the ability of bidders to sustain collusion. Clark et al. (2018) analyze the breakdown of a cartel and its implications on prices. For surveys of the literature, see Porter (2005) and Harrington (2008).

can help players in a repeated game achieve larger equilibrium payoffs by using random bidding recommendations. Moreover, the indifference condition characterizing the optimal bid distribution in our model has analogs in Roesler and Szentes (2017) and Condorelli and Szentes (2020), who study information acquisition by a buyer, Perez-Richet and Skreta (2018), who study test design, and Ortner and Chassang (2018), who study the design of anti-corruption schemes.

2 Bid-rigging in the Town of Kumatori

This section provides a description of the internal organization of a bidding ring that operated in the town of Kumatori that motivates our analysis. Kumatori is a town in Japan, located near Osaka, with a population of a little over 40,000.

2.1 Background

Auctions for construction projects in Kumatori. The town of Kumatori uses auctions to allocate construction projects that are estimated to cost above 1.3 million yen, or about 13 thousand dollars. The auction format is first-price sealed bid with a secret reserve price, although the reserve price was not binding in any of the auctions that we study.

An important feature of the auctions is that participation is by invitation only. The town maintains a list of qualified contractors and invites a subset of firms from the list to bid. The town maintains separate lists for project categories as well as for different project sizes within a category. For example, for building construction, the town maintains four mutually exclusive lists of contractors (Tier A through Tier D). The town typically invites Tier A firms to bid on the largest projects, Tier B firms to bid on the next largest projects, and so on, essentially segmenting the market by project size. All of Tier A firms are headquartered outside of Kumatori and are invited to bid on exceptionally large projects. Tier B firms and below are local firms typically headquartered within the town. Most of the Tier B and C firms were members of the Kumatori Contractors Cooperative, a trade association that consisted of a little more than 20 mid to small-size contractors in Kumatori. The members of this cooperative were found to be colluding.

Bidding ring in the town of Kumatori. Reports of police investigation of the members of the Kumatori Contractors Cooperative for bid-rigging first appeared in the news on October 12, 2007. In addition to the contractors in Kumatori, more than 20 town officials, including the town mayor, were questioned by the police. In November 2007, four individuals were indicted for bid-rigging. The criminal charges focused on the defendants' involvement in a bid-rigging arrangement in a single auction that took place on August 22, 2006 that Imakatsu Construction won, an auction for rebuilding a public housing complex (Obara Residences). The defendants included Mr. Kitagawa, the owner of Imakatsu Construction and director of the Kumatori Contractors Cooperative; his son, who was an employee of Imakatsu Construction; Mr. Nishio, the vice-director of the Cooperative; and Mr. Takano, an employee of the Cooperative. While Mr. Nishio and Mr. Takano were not participants of the auction, they mediated much of the communication between Mr. Kitagawa and the other participants of the auction. For example, Mr. Nishio gave out the instructions to other bidders on how they should bid. All four defendants were found guilty in trial in March 2008. Mr. Kitagawa was sentenced to prison for 18 months, Mr. Nishio to 14 months, and the other defendants to 10 months.

Although the criminal case focused on the defendants' involvement in a bid-rigging arrangement in the Obara Residences auction alone, the court ruling of the case made it clear that the Obara Residences auction was not an isolated incident, and that there were many other individuals who actively participated in the bid-rigging scheme.[3] In particular, the court stated that the members of the Kumatori Contractors Cooperative allocated the projects according to a predetermined order to even out the work of each contractor. While none of the town officials were formally charged, the court ruling also stated that the designated winner of the cartel would approach the town officials to seek out the engineering

[3]Our description of the cartel in this paragraph is taken from page 2 and 3 of Ruling H19 (WA) No. 6418, Osaka District Court.

estimate.

In response to the ruling of the criminal case, the town of Kumatori withheld part of its payment to Imakatsu Construction for work that had been completed on the Obara residences in order to off-set liability claims. However, the mayor and town officials showed little interest in pursuing claims for damages incurred on other auctions. This inaction led some of the residents of Kumatori to file suit against the mayor asking the court to order the mayor to pursue claims against 23 firms, all members of the Kumatori Contractors Cooperative, for damages incurred on other auctions. The District Court of Osaka ruled in favor of the plaintiffs, ordering the town to pursue claims against the bidders in the amount of about 375 million yen, or about 3.75 million dollars. The mayor appealed the ruling, but the verdict was upheld by the Osaka High Court with relatively minor modifications.

2.2 Obara Residences Auction

Among the auctions that the cartel bid on, the Obara residences auction was unique because of the large value of the project. We now discuss how, despite the town's policy of segmenting the market by project size, the members of the Kumatori Contractors Cooperative were invited to bid in the auction. We also draw on the court rulings to provide a detailed qualitative description of the way in which bid-rigging was carried out for the Obara Residences auction. The workings of the cartel motivate our theoretical analysis in Section 4.

The town of Kumatori started planning for the rebuilding of the Obara Residence Complex, an ageing public housing project, around year 2000 according to the minutes of the meeting of the town council.[4] The new residence complex would consist of three separate buildings. Construction of the first and smallest of the three buildings was put to an auction in April 2004 to Tier A firms. The winning bid was 363 million yen, or about 3.6 million dollars. The winner of the auction was Asanuma Corporation, a contractor headquartered in the city of Osaka with annual sales of about 2 billion dollars.

In the Fall of 2004, Mr. Kitagawa, the director of the Kumatori Contractors Cooperative

[4]Minutes of the meeting of the town countil to discuss the budget, March 2001, page 49.

and owner of Imakatsu Construction began lobbying the town's mayor and the head of the town's general affairs department to let Tier B firms bid on the second and third components of the Obara Residences Complex.[5] Mr. Kitagawa was an important supporter of the mayor. Despite the repeated lobbying by Mr. Kitagawa, the head of the department was reluctant to let Tier B firms bid on the Obara residences project initially, according to court documents. However, the head of the department started to warm towards the idea around April 2006.[6]

According to the Osaka District Court ruling, Mr. Kitagawa of Imakatsu Construction started to believe, around April of 2006, that Tier B firms would be invited to bid on the second part of the Obara residences project and started mentioning the project at the meetings of the Kumatori Contractors' Cooperative. In a meeting of the Cooperative in late June 2006, Mr. Kitagawa stated that he wanted others to let his firm, Imakatsu Construction, win the auction. He also told the members that he would be collecting, from each of the invited bidders, the detailed project plan that the town distributes at the on-site briefing. This was understood by the members of the Cooperative as a preventative measure to make defection more difficult by making cost estimates for other firms harder.[7]

On August 3, 2006, the town office invited five contractors to bid on the housing complex project, including Imakatsu Construction. All invited bidders were Tier B firms and members of the Cooperative. On or around this day, Mr. Kitagawa asked Mr. Nishio to help him with the operation and to obtain confidential information about the project from the town. Mr. Nishio was the vice director of the Cooperative and a senior managing director of Nishinuki Construction at the time. Nishinuki Construction was not one of the invited bidders.

On August 4, 2006, after the general meeting of the Cooperative, Mr. Kitagawa met with the four other bidders that were invited to bid on the Obara Residences project and repeated his intention to collect the project plans from them after the on-site briefing.

The town of Kumatori held an on-site briefing for the Obara residences project for the five invited firms on August 7. At the on-site briefing, the town distributed the detailed

[5]Our description of the cartel in this paragraph are taken from page 17 of Ruling H21 (Gyo-U) No. 99, Osaka District Court.

[6]Last paragraph, page 17 of the Osaka District Court ruling.

[7]Page 18 of the Osaka District Court ruling (Ruling H21 Gyo-U, No. 99).

project plan as well as other documents required to estimate costs. These documents were collected immediately after the briefing from all bidders other than Imakatsu by an employee of the Cooperative. The plans and the documents were not returned to the bidders until August 21, the day before the auction. During this time, Imakatsu Construction was the only firm that had access to the documents required to estimate costs: Other bidders did not have access to the documents that would make cost estimates possible. After the on-site briefing, Mr. Nishio met with an official of the Buildings Division of the town to obtain information about the engineering estimate.[8]

On August 21, one day before the day of the auction, Mr. Kitagawa and Mr. Nishio met at the office of Imakatsu Construction and decided on a bid of 630 million yen for Imakatsu Construction and bids of above 700 million yen for all other bidders. According to the court ruling, they decided to set the losing bids to be above 700 million yen with the explicit intent of making it hard for the other bidders to guess the bid of Imakatsu.[9]

Also on the same day, Mr. Nishio contacted the four other invited bidders of the auction. He told them that he would hand them the document containing each firm's break-down of the estimated costs on the day of the auction (a bid would need to be accompanied by this document for it to be considered valid). Mr. Nishio also told the bidders that he would give instructions on how much to bid on the day of the auction. According to the court ruling, Mr. Nishio's decision to hand the documents and give instructions on bids on the day of the auction (as opposed to two days prior to the day of the auction, as was customary for the bidding ring) was to prevent defection given the large size of the project.[10]

The auction for the public housing complex was held at the town office on August 22 2006. Mr. Nishio went to the town office and stapled together the documents containing the cost break-down of each bidder with the cover page brought by the representatives of the firms. Mr. Nishio also indicated the amount that each bidder should bid by showing a slip of paper with a number above 700 million yen. Importantly, the bid of Imakatsu Construction

[8]Page 19 of the Osaka District Court ruling (Ruling H21 Gyo-U, No. 99).
[9]Page 19 of the Osaka District Court ruling (Ruling H21 Gyo-U, No. 99).
[10]Page 4 of the Osaka High Court ruling (Ruling H24 Gyo-Ko, No. 101).

was kept secret to the other bidders. The representatives of the invited bidders bid the same or slightly above the amount shown on the slip of paper. As a result, Imakatsu Construction won the auction with a bid of 630 million yen. The other bids were 705 million yen, 707 million yen, 710 million yen and 723 million yen, respectively.

3 Data and Summary Statistics

In this section, we briefly discuss the data and the key features of the bidding pattern that we seek to capture in our model.

3.1 Data

Our primary data source is the bid data submitted to court by the residents of Kumatori who sued the mayor for failing to pursue damages against the bidding ring. The residents claimed that the town was owed money for a subset of the auctions won by the bidding ring from April 2003 to October 2007. We have information on all of the bids, date of the auction, reserve price, and the identity of the winner (but not the identity of the losing bidders) for those auctions. A limitation of this dataset is that it does not include auctions for which the plaintiffs did not pursue damages even if a cartel member won the auction. Moreover, lettings that were awarded to non-colluding firms are not included.

We complement our primary data source with auction data that were collected by the town of Kumatori. This dataset covers all auctions let by the town between April 2006 and December 2009. Because the second dataset includes the universe of auctions let by the town during this period, the second dataset is a superset of the first during the period the two datasets overlap, i.e., between April 2006 and October 2007. The second dataset contains information on all bids, identity of the bidders, date of the auction and the reserve price.

Table 1 reports the summary statistics of the two datasets. The top panel reports the summary statistics without conditioning on the tier of the participants and the bottom panel

	Data from Plaintiffs		Data from Kumatori
	All periods	2006.4 - 2007.10	2006.4 - 2007.10
	(1)	(2)	(3)
All Tiers			
Reserve price (mil. Yen)	28.93	39.65	31.33
	(58.48)	(105.07)	(79.97)
Lowest bid (mil. Yen)	28.06	38.27	29.21
	(56.44)	(100.65)	(75.67)
Lowest bid/Reserve	0.967	0.964	0.928
	(0.011)	(0.011)	(0.068)
#Bidders	10.41	8.00	9.41
	(2.76)	(2.77)	(2.68)
Sample size	158	39	79
Tier B Auctions			
Reserve price (mil. Yen)	47.80	166.83	158.63
	(87.97)	(244.92)	(224.63)
Lowest bid (mil. Yen)	46.46	161.17	152.69
	(84.79)	(234.60)	(215.33)
Lowest bid/Reserve	0.969	0.971	0.965
	(0.011)	(0.015)	(0.020)
#Bidders	10.84	8.33	8.57
	(2.78)	(1.63)	(1.62)
Sample size	63	6	7

Table 1: Sample Statistics – Auctions. Columns (1) and (2) correspond to the sample of auctions in the plaintiffs dataset and column (3) corresponds to those in the dataset we obtained from the town. Column (2) corresponds to a subset of the auctions in the plaintiffs dataset that were let between April 2006 and October 2007. The bottom panel corresponds to Tier B auctions. Standard errors are reported in parenthesis.

focuses on auctions in which Tier B firms were invited to bid. As we discussed in section 2.1, the town of Kumatori segments the market by the size of the project and invites bidders from a particular tier to bid on a given auction for almost all auctions. We are specifically interested in Tier B firms because the participants of the Obara residence auction were Tier B firms and these are the firms we know the most about from the court documents. All of the Tier B firms were members of the bidding ring.

The first two columns of Table 1 correspond to the dataset we obtained from the plaintiffs. Column (1) reports the summary statistics for all of the auctions in that sample (top panel) and those for Tier B auctions (bottom panel). Column (2) reports the summary statistics for the subset of the auctions that were let after April 2006, which is when the dataset we obtained from the town begins. We find that the average reserve price is 28.93 million yen (about $290,000) in the top panel of column (1) while it is significantly higher, at 39.65 million yen in the top panel of column (2). The difference in the mean reserve price between column (1) and column (2) is explained by the fact that column (2) includes the Obara residence auction. The reserve price of the Obara residences auction was about 17 times larger than the other auctions. This auction raises the average reserve price and the winning bid much more in column (2) given the smaller sample size in column (2). We also find that the reserve price and the bids in the bottom panel are larger than in the top panel, reflecting the fact that Tier B auctions are generally larger than Tier C auctions. Note that the data from the plaintiffs only include Tier B and Tier C auctions. The average winning bid is about 96-7% of the reserve price.

Column (3) reports the sample statistics for the dataset that we obtained from the town of Kumatori. There are 79 auctions that were let by the town between the beginning of the sample (April 2006) and the breakdown of the cartel in October 2007. Because the set of auctions in the town data is not a selected sample, the data include auctions in which the cartel bidders did not participate (e.g., Tier A and Tier D auctions) as well as those in which

Name	(1) Revenue	(2) #Won	(3) Ohara
Imakatsu Construction	759.45	6	●
Nakabayashi Construction	488.65	12	
Nishio Gumi	454.40	8	
Tokushin construction	416.39	6	
Takada Gumi	396.25	9	○
Yamamoto Construction	195.60	7	○
Nakajima Kougyou	159.80	5	
Hannan Construction	109.70	4	○
Seiko Construction	48.23	7	○

Table 2: Sample Statistics – Tier B firms.

the cartel bidders participated but are not included in the plaintiffs dataset. Comparing the sample sizes between columns (2) and (3) in the top panel, we find that about half (39/79) of the auctions are included in the first dataset during the period of overlap. If we focus on the bottom panel, we find that the sample size is 6 in column (2) and 7 in column (3). The fact that 6 out of 7 auctions let during the period of overlap are contained in the first dataset suggests that sample selection issues are unlikely to be too severe for Tier B auctions.

Table 2 reports the summary statistics of the 9 contractors that were on Tier B.[11] Column (1) reports the total amount awarded to each of the firms during the sample period. The award amount varies from a high of about 760 million yen to a low of about 50 million yen. The number of auctions awarded is reported in column (2). Column (3) reports whether or not a firm was invited to the Obara residences auction. White circles correspond to those that were invited to bid in the auction and a black circle corresponds to the winner of the auction.

[11]The sample of auctions used for Table 2 include all of the 63 Tier B auctions from the first dataset and one Tier B auction from the town data.

3.2 Size of Auctions and Winning Margin

The Obara residences auction was unusually large compared to other auctions that Tier B firms were invited to bid on. The left Panel of Figure 1 plots the reserve price of Tier B auctions in our data. The horizontal axis is the calendar date and the vertical axis is the reserve price of the auction. The vertical dotted line corresponds to the start of the town data. The figure shows that, except for the Obara residence auction that took place on August of 2006 (corresponding to the dark circle), Tier B firms were invited to bid on auctions with reserve prices below 200 million yen. The average reserve price during this period excluding the Obara residences auction is about 39.1 million yen. The Obara residences auction was about 17 times the average size of projects on which these bidders were invited to bid.

The right panel of Figure 1 plots the difference between the lowest and the second lowest bids as a fraction of the reserve price for these auctions. The figure shows that the winning margin is always less than 4%, except for the Obara residences auction (corresponding to the dark circle). The average winning margin for lettings excluding Obara Residences is about 0.93%. The margin for Obara Residences, on the other hand, is 11.4%. The bidding patterns suggest that the cartel members kept the winning margin small for all projects except for the Obara residences project. The next section explores theoretically in the context of a repeated-game how these bidding patterns can be explained as an optimal cartel response when IC constraints of the designated losers bind.

4 Theory

4.1 Model

Consider a repeated game in which, in each period $t \in \mathbb{N}$, a buyer procures a project from firms $i = 1, 2$. To simplify the exposition, we assume that the buyer uses a first-price auction

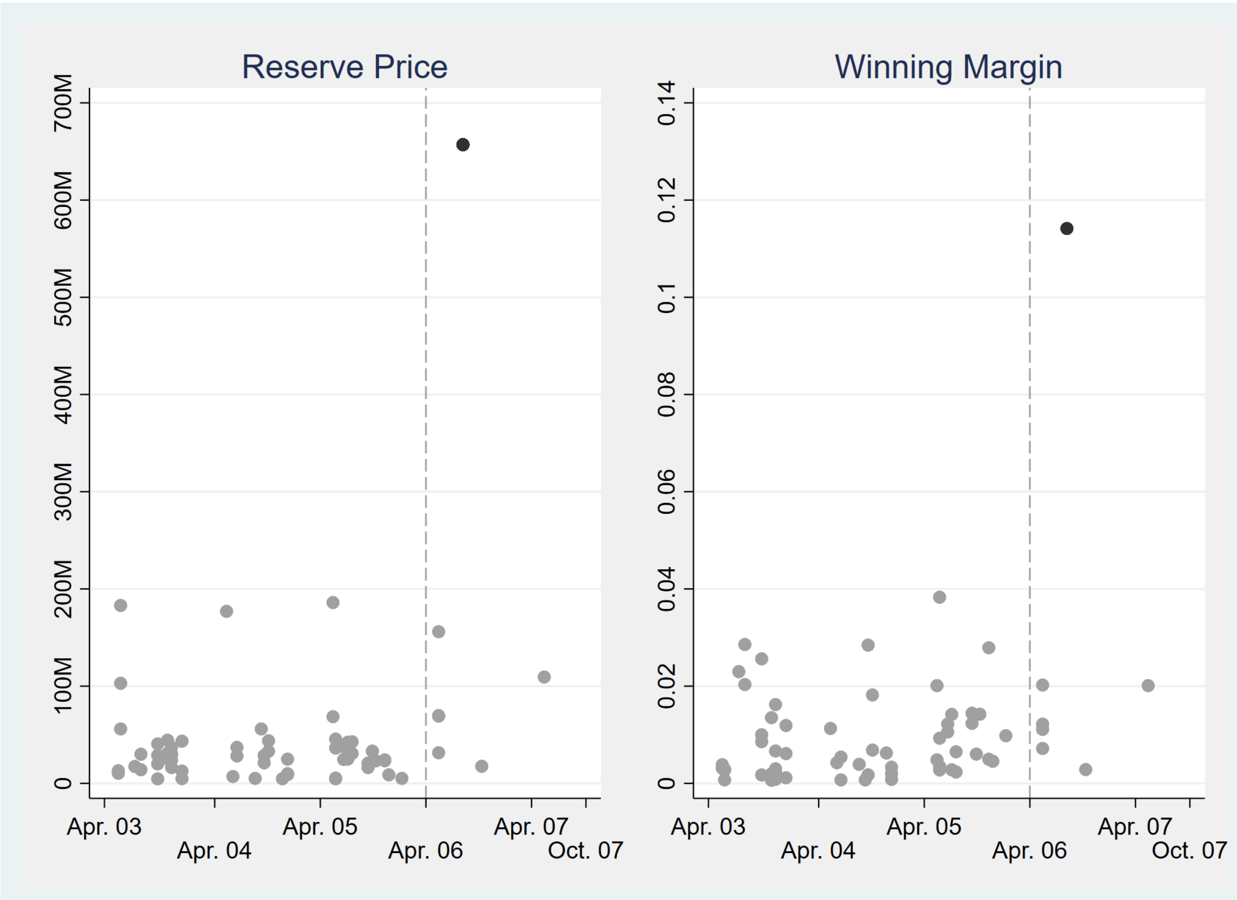

Figure 1: Reserve Price of Auctions (Left Panel) and Difference between Winning Bid and Second Lowest Bid (Right Panel). Left panel of the Figure plots the reserve price of auctions in which collusive firms in Tier B were invited to bid. The right panel plots the difference between the winning bid and the lowest losing bid, as a fraction of the reserve price.

with a public reserve price. Our results generalize to the case of $n > 2$ bidders (see the Online Appendix), and to auctions with secret reserve prices.

In each period t, firms $i = 1, 2$ share the same procurement cost $c \geq 0$, which we normalize to $c = 0$. We abstract away from the issue of private costs and efficient allocation.[12] Public reserve price r_t is drawn i.i.d. over time from distribution F_r with support $[\underline{r}, \overline{r}]$, with $\overline{r} > \underline{r} \geq 0$. After observing r_t, firms submit public bids $\mathbf{b}_t = (b_{i,t})_{i=1,2}$. This yields allocation $\mathbf{x}_t = (x_{i,t})_{i=1,2} \in [0,1]^2$ such that: if $b_{j,t} > b_{i,t}$ then $x_{i,t} = 1$; if $b_{j,t} < b_{i,t}$ then $x_{i,t} = 0$. In the case of ties, we follow Athey and Bagwell (2001) and Chassang and Ortner (2019) and let bidders jointly determine the allocation. Formally, we allow bidders $i = 1, 2$ to simultaneously and publicly pick numbers $\gamma_{i,t} \in [0,1]$. When bids are tied, the allocation to bidder i is $x_{i,t} = \frac{\gamma_{i,t}}{\gamma_{i,t}+\gamma_{j,t}}$.[13] Firm i's profits in period t are $x_{i,t}(b_{i,t} - c) = x_{i,t} b_{i,t}$. Firms share a common discount factor $\delta < 1$.

We make two observations about our model. First, the assumption that bidders can break ties allows us to formalize having one bidder bidding "just above" her opponent, placing essentially the same bid and losing with probability 1: bidder i may bid $(b_i, \gamma_i) = (b, 1)$ and bidder $j \neq i$ may bid $(b_j, \gamma_j) = (b, 0)$. We note that this assumption can be dispensed with if bidders were restricted to place bids on a finite grid.

Second, we assume that costs are zero, which implies that the reserve price is a measure of the profitability of the project. In practice, costs are likely to be roughly proportional to the reserve price: $c = c_0 \times r$ for some constant $c_0 \in (0,1)$. Note that the case with costs proportional to the reserve price is isomorphic to our model with zero costs.[14] All of our results hold as long as the profitability of a project increases in r.

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[12] Our decision to abstract away from private costs reflects the fact that the designated losers in the Kumatori cartel did not even have the opportunity to estimate costs in the Obara residences auction because the plans were taken away from them.

[13] If $\gamma_{i,t} = \gamma_{j,t} = 0$, then $x_{i,t} = x_{j,t} = 1/2$.

[14] One can simply think of $(1 - c_0) \times r$ as the reserve price in our model.

Mediation. Consistent with our application, we assume that firms have access to a mediator.[15] In each period t, the mediator observes the history of past reserve prices and bids, as well as current reserve price r_t. Prior to bidding, the mediator privately sends recommended bids $(\widehat{b}_{i,t}, \widehat{\gamma}_{i,t})$ to firms $i = 1, 2$. Recommendations $(\widehat{b}_{i,t}, \widehat{\gamma}_{i,t})_{i=1,2}$ may depend on the history of past reserve prices and bids, current reserve price r_t, and the history of past recommendations.[16]

Solution concepts. A period-t history for bidder i,

$$h_{i,t} = (r_s, \widehat{b}_{i,s}, \widehat{\gamma}_{i,s}, \mathbf{b}_s, \boldsymbol{\gamma}_s)_{s<t} \sqcup (r_t, \widehat{b}_{i,t}, \widehat{\gamma}_{i,t}),$$

records past reserve prices $(r_s)_{s<t}$, mediator's recommendations $(\widehat{b}_{i,s}, \widehat{\gamma}_{i,s})_{s<t}$, and realized bids $(\mathbf{b}_s, \boldsymbol{\gamma}_s)_{s<t}$, as well as current reserve price r_t and mediator's recommendation $(\widehat{b}_{i,t}, \widehat{\gamma}_{i,t})$. A pure strategy $\sigma_i : h_{i,t} \mapsto (b_{i,t}, \gamma_{i,t})$ for bidder i maps bidder i histories to bids. Our solution concept is weak Perfect Bayesian Equilibrium, which we simply refer to as equilibrium. The period-t public history is $h_t^0 = (r_s, \mathbf{b}_s, \boldsymbol{\gamma}_s)_{s<t}$.

The next subsection focuses on bid rotation equilibria, which corresponds to the bidding scheme used by the bidding ring in Kumatori.[17,18] Subsection 4.3 extends the analysis beyond bid rotation equilibria.

Definition 1. *We say that equilibrium $\sigma = (\sigma_i)_{i=1,2}$ is a* bid rotation equilibrium *if there exists $i, j = 1, 2$, $i \neq j$, such that for all on-path public histories h_t^0, $\mathbb{E}_\sigma[x_{i,t}|h_t^0] = 1$ if t is*

[15] Our results go through without a mediator by allowing for communication between bidders, see remark in footnote 23.

[16] See Appendix A for a formal description of the mediator's strategies. See also Sugaya and Wolitzky (2017) for a detailed exposition of repeated games with a mediator.

[17] The ruling of the Osaka District Court states that the members of the Kumatori Contractors Cooperative allocated projects according to a predermined order to even out the work of each contractor.

[18] We note that bid rotation equilibria would be optimal under a natural generalization of our model, in which firms have (publicly observed) costs that are increasing in their backlog. Indeed, suppose that the procurement cost of last period's winner is $\overline{c} > 0$, and the cost of last period's loser is $\underline{c} \in [0, \overline{c})$. If $\overline{c} - \underline{c}$ is large enough, then the cartel's optimal equilibrium involves bid rotation.

even and $\mathbb{E}_\sigma[x_{j,t}|h_t^0] = 1$ *if* t *is odd.*[19]

We now define common-knowledge bids.

Definition 2. *We say that equilibrium* $\sigma = (\sigma_i)_{i=1,2}$ *has* common-knowledge bids *if, for* $i = 1, 2$ *and for all histories* $h_{i,t}$, $\sigma_i(h_{i,t})$ *is a pure action and depends only on* h_t^0 *and* r_t.

We note that equilibria with common-knowledge bids correspond to pure strategy equilibria of the game without the mediator.

4.2 Bid Rotation Equilibria

Let Σ denote the set of bid rotation equilibria, and let $\Sigma_{\mathsf{ck}} \subset \Sigma$ denote the set of bid rotation equilibria with common-knowledge bids. For each equilibrium σ and $i = 1, 2$, let $V_i(\sigma)$ denote firm i's expected discounted payoff at the start of the game under σ.

Define

$$\overline{V}_{\mathsf{ck}} \equiv \sup_{\sigma \in \Sigma_{\mathsf{ck}}} \sum_{i=1,2} V_i(\sigma), \text{ and}$$
$$\overline{V} \equiv \sup_{\sigma \in \Sigma} \sum_{i=1,2} V_i(\sigma),$$

to be, respectively, the cartel's largest payoffs under an equilibrium in Σ_{ck} and Σ. Since $\Sigma_{\mathsf{ck}} \subset \Sigma$, we have $\overline{V} \geq \overline{V}_{\mathsf{ck}}$.

Equilibria with common-knowledge bids. Our first result characterizes optimal equilibria in Σ_{ck}. These equilibria take the following intuitive form. The designated bidder bids the reserve price, r_t, or the expected continuation payoff of the designated loser, whichever is

[19] Formally, an equilibrium of this game is given by a strategy profile $(\sigma_M, \sigma_1, \sigma_2)$, where σ_M is the mediator's strategy and $(\sigma_i)_{i=1,2}$ are the bidders' strategies, and bidders' beliefs $\mu = (\mu_1, \mu_2)$ about the messages sent by the mediator; see Appendix A for details. To economize on notation, in the main text we denote an equilibrium simply by the bidders' strategies $\sigma = (\sigma_i)_{i=1,2}$.

less. The designated loser bids a marginally losing bid.[20] Deviations are punished by Nash reversion. Because the designated loser becomes the designated winner next period under bid rotation, if we let $\overline{W}_{\mathsf{ck}}$ denote the expected continuation value of the designated winner, the expected continuation value of the designated loser is $\delta\overline{W}_{\mathsf{ck}}$, and we have $\overline{V}_{\mathsf{ck}} = \overline{W}_{\mathsf{ck}} + \delta\overline{W}_{\mathsf{ck}}$. As we show in Appendix B, continuation value $\overline{W}_{\mathsf{ck}}$ is the largest solution $W \geq 0$ to the following equation:

$$W = \frac{1}{1-\delta^2}\mathbb{E}_{F_r}[\min\{r, \delta W\}]. \tag{1}$$

Proposition 1 formalizes this discussion. All proofs are collected in Appendix B.

Proposition 1. *On the equilibrium path, the bidding strategy in any $\sigma \in \Sigma_{\mathsf{ck}}$ that attains $\overline{V}_{\mathsf{ck}}$ is such that, in all periods t, the winning bid is given by the minimum between r_t and $\delta\overline{W}_{\mathsf{ck}}$, where $\delta\overline{W}_{\mathsf{ck}}$ is the expected continuation payoff of a designated loser under any equilibrium that attains $\overline{V}_{\mathsf{ck}}$.*

In these equilibria, the losing bid is marginally higher than the winning bid, similar to the equilibria in Marshall and Marx (2007, 2012) and Chassang and Ortner (2019). Moreover, note that the winning bid, as a fraction of the reserve price r_t, becomes lower than 1 as r_t becomes larger. This feature of the equilibria is similar to that in Rotemberg and Saloner (1986).

Equilibria without common-knowledge bids. Our next result characterizes optimal equilibria in Σ, and establishes when $\overline{V} > \overline{V}_{\mathsf{ck}}$. We describe the optimal equilibrium in words before stating the results in the form of a proposition.

Let $\overline{W}$ be the value function of the designated winner under an equilibrium that attains $\overline{V}$. Because the designated loser becomes the designated winner next period, the expected continuation value of the losing bidder is $\delta\overline{W}$.[21] In any optimal equilibrium in Σ, along

[20]More precisely, the designated loser bids the same bid b as the designated winner but chooses $\gamma = 0$.
[21]We show in Appendix B that optimal equilibria in Σ are stationary.

the path of play the mediator recommends bid b to the winner, with b drawn from c.d.f. $F^*(\cdot; r_t)$. The distribution $F^*(\cdot; r_t)$ is degenerate at r_t if $r_t \leq \delta\overline{W}$ and is non-degenerate otherwise. The loser is recommended to bid marginally above $\bar{b}$, where $\bar{b}$ is the largest point in the support of $F^*(\cdot; r_t)$.[22] If either bidder deviates, the mediator sends bidding recommendations $(b_i, \gamma_i) = (0, 1)$ to $i = 1, 2$ from the next period onwards, and players adhere to this recommendation; i.e., they play Bertrand-Nash. Note that, while deviations by the loser are publicly observed, deviations by the winner may only be detected by the mediator (since bidding recommendations are private). The mediator's messages following a deviation provide the winner with incentives to follow the recommended bid.

Next, we show how we derive the distribution $F^*(\cdot; r)$. Recall that the loser's discounted continuation payoff is $\delta\overline{W}$. Suppose that the winning bid at time t is drawn from c.d.f. F_t. Let $\bar{b}$ and $\underline{b}$ denote, respectively, the largest and smallest points in the support of F_t. For the loser not to have an incentive to deviate and place a bid $b < \bar{b}$, F_t must satisfy:

$$\forall b < \bar{b}, \quad (1 - F_t(b))b \leq \delta\overline{W} \iff F_t(b) \geq 1 - \frac{\delta\overline{W}}{b}. \tag{2}$$

Equation (2) implies $\underline{b} \leq \delta\overline{W}$.

Consider now the incentives of the predetermined winner. If the mediator recommends bid $b < \bar{b}$, the winner can increase its bid to $\bar{b} - \epsilon \approx \bar{b}$ and still win the auction. For the winner to have incentives to follow the mediator's recommendation, we must have

$$\underline{b} + \delta^2\overline{W} \geq \bar{b}, \tag{3}$$

where the inequality follows since the winner's equilibrium continuation payoff is $\delta^2\overline{W}$. Since $\underline{b} \leq \delta\overline{W}$ (by equation (2)), inequality (3) gives us $\bar{b} \leq \delta(1+\delta)\overline{W}$. Distribution $F^*(\cdot; r)$ is the highest distribution (in terms of f.o.s.d.) with $\bar{b} \leq r$ satisfying (2) and (3). When $\delta\overline{W} \geq r$,

[22]Formally, the designated loser is recommended to bid $(\bar{b}, \gamma = 0)$.

$F^*(\cdot; r)$ puts all its mass at r. When $\delta\overline{W} < r$, $F^*(\cdot; r)$ is given by:

$$F^*(b; r) = \begin{cases} 0 \text{ if } b < \delta\overline{W}, \\ 1 - \frac{\delta\overline{W}}{b} \text{ if } b \in [\delta\overline{W}, \min\{r, \delta(1+\delta)\overline{W}\}), \\ 1 \text{ if } b \geq \min\{r, \delta(1+\delta)\overline{W}\}. \end{cases}$$

We note two features of distribution $F^*(\cdot; r)$. First, this c.d.f. has a mass point at $\min\{r, \delta(1+\delta)\overline{W}\}$. Second, $F^*(\cdot; r)$ has the property that a designated loser is indifferent between submitting a losing bid marginally above $\bar{b}$, or placing any bid $b < \bar{b}$ in the support of F^*.

The following proposition formalizes this discussion:

Proposition 2. *On the equilibrium path, the bidding strategy in any $\sigma \in \Sigma$ that attains $\overline{V}$ is such that, in all periods t, the winning bid is drawn from c.d.f. $F^*(\cdot; r_t)$. Distribution $F^*(\cdot; r_t)$ is degenerate at r_t if $r_t \leq \delta\overline{W}$ and is non-degenerate otherwise, where $\delta\overline{W}$ is the expected continuation payoff of a designated loser under any equilibrium that attains $\overline{V}$.*

Moreover, $\overline{V} > \overline{V}_{\mathsf{ck}}$ if and only if $\overline{r} > \delta\overline{W}_{\mathsf{ck}}$.

The first key takeaway of the results in this section is that a cartel strictly benefits from strategies without common-knowledge bids when the largest point in the support of F_r is high enough; i.e., $\overline{r} > \delta\overline{W}_{\mathsf{ck}}$. The next section shows, through a calibration of our model, that the gains from randomization can be significant. Randomization makes the winning bid secret to the designated losers and helps relax IC constraints. The second takeaway is that when the reserve price is high enough, the winning bid is drawn from a non-degenerate distribution and the expected winning margin is positive in any equilibrium that attains $\overline{V}$. The value of randomization is positive only when the reserve price is high and not when it is low. Appendix B shows how $\overline{W}$ is pinned down.

We note that the mediator plays two roles in a bid-rotation equilibrium attaining $\overline{V}$: she sends bidding recommendations to each firm, and acts as a whistleblower against potential defections by the designated winner. One potential concern with such bidding schemes is that the mediator might have an incentive to lie about a deviation by the designated winner, especially if she has an interest in sustaining collusion in the future. When there are multiple bidders, and not all of them participate in every auction, this concern can be ameliorated by adjusting the equilibrium so that following a deviation, collusion breaks down only at auctions in which the defector participates. In our application, the mediator (Mr. Nishio) was the owner of a firm which belonged to a different tier than the firms participating in the Obara Residence auction, and thus had limited participation overlap with these firms.[23]

Lastly, we note that the results above extend to settings in which projects are allocated according to a predetermined order. Indeed, for any equilibrium with common-knowledge bids such that there is a predetermined winner at each on-path history, we can use the same arguments as in Proposition 2 to construct an equilibrium without common-knowledge bids that delivers greater profits for the cartel.

4.3 Beyond Bid Rotation Equilibria

This subsection extends our analysis beyond bid-rotation equilibria. Let Σ^* denote the set of all equilibria of the repeated game, and let $\Sigma^*_{\mathsf{ck}} \subset \Sigma^*$ denote the set of equilibria with common-knowledge bids. Recall that, for any equilibrium σ, we denote by $V_i(\sigma)$ firm i's expected discounted payoff under σ at the start of the game.

[23]Alternatively, we can dispense with the mediator completely if we allow for communication between the bidders and assume that bidders have access to a hash function. With access to a hash function, it becomes possible for the designated winner to commit to randomize its bid from F^* without a mediator. To see this, consider the following protocol: (i). The designated winner and loser each independently draw a random variable from Uniform[0,1], U^w and U^l. (ii). The winner and loser declare the hash values of the random variables, $H(U^w)$ and $H(U^l)$ without disclosing the original values. (iii). The loser reveals U^l. (iv). The winner computes Z, the fractional part of $U^w + U^l$, i.e., $Z = U^w + U^l$ or $Z = U^w + U^l - 1$. (v). The winner bids $F^{*-1}(Z)$, where F^{*-1} is the inverse of F^*. (vi). The winner reveals the original value U^w. Note that Z is distributed uniform, so the winner's bid $F^{*-1}(Z)$ has distribution F^*. Also, the designated winner has no incentive to change the value of U^w at stage (i) because Z is uniform regardless of the realization of U^w. Finally, the designated loser can verify ex-post that U^w produces hash value $H(U^w)$. We thank Shunya Noda for pointing this out.

Define

$$V_{\mathsf{ck}}^* \equiv \sup_{\sigma \in \Sigma_{\mathsf{ck}}^*} \sum_{i=1,2} V_i(\sigma), \text{ and}$$

$$V^* \equiv \sup_{\sigma \in \Sigma^*} \sum_{i=1,2} V_i(\sigma)$$

to be, respectively, the highest cartel payoff that can be attained by an equilibrium in Σ_{ck}^* and Σ^*. Since $\Sigma_{\mathsf{ck}}^* \subset \Sigma^*$, we again have $V^* \geq V_{\mathsf{ck}}^*$.

Our next result shows that bidding schemes without common-knowledge bids are strictly beneficial whenever $\overline{r} > \delta V_{\mathsf{ck}}^*$.

Proposition 3. *$V^* > V_{ck}^*$ if and only if $\overline{r} > \delta V_{ck}^*$.*

The proof of Proposition 3 consists of two parts. In the first part, we characterize equilibria in Σ_{ck}^* that attain V_{ck}^*. Under such equilibria, both bidders place the same bid at every period, and win the auction with equal probability. Each bidder then earns a continuation payoff equal to $\delta V_{\mathsf{ck}}^*/2$ after every on-path history. To deter bidders from defecting, bid b_t placed at time t must be such that $b_t/2 + \delta V_{\mathsf{ck}}^*/2 \geq b_t$, or $b_t \leq \delta V_{\mathsf{ck}}^*$. Hence, bid b_t is the minimum between r_t and δV_{ck}^*. Note that equilibria in Σ_{ck}^* attaining V_{ck}^* achieve perfect collusion whenever $\overline{r} \leq \delta V_{\mathsf{ck}}^*$, and so $V_{\mathsf{ck}}^* = V^*$ in this case.

The second part of the proof of Proposition 3 shows that when $\overline{r} > \delta V_{\mathsf{ck}}^*$, cartel members can attain profits strictly larger than V_{ck}^* by having the mediator send random bid recommendations to both bidders. In particular, we use the following scheme to prove Proposition 3. If $r_0 \leq \delta V_{\mathsf{ck}}^*$, the mediator recommends each bidder to bid r_0 at $t = 0$. If instead $r_0 > \delta V_{\mathsf{ck}}^*$, the mediator sends i.i.d. recommendations to each firm, recommending them to bid r_0 with probability $\mu \in (0, 1)$ and δV_{ck}^* with the complement probability. Deviations are punished by Nash reversion. From $t = 1$ onwards, players play the equilibrium attaining V_{ck}^*. Clearly, this bidding scheme generates profits strictly larger than V_{ck}^*. The proof of Proposition 3 shows that, for an appropriately chosen μ, bidders don't have an incentive to defect.

5 Calibrating the Model to the Data

We now assess quantitatively the value of using randomized strategies relative to common-knowledge bids. In order to do so, we calibrate our model of Section 4.2 (bid rotation equilibria) and simulate equilibrium bidding behavior with and without common-knowledge bids under the actual realizations of the reserve price in the Kumatori data.

The two key parameters that we calibrate are the distribution, F_r, and the discount factor, δ. In order to calibrate F_r, we need a measure of profitability for each project since F_r in the model corresponds to the distribution of the profitability of the projects (recall that we assume $c = 0$). In this calibration, we assume that the costs of construction are proportional to the reserve price as $c = c_0 \times r$, or the profitability of each auction is $(1-c_0)\times r$. In particular, we set c_0 to be 0.75 and the profit margin to be 0.25. We note that 25% is the mid-point of the estimated range of excess cartel profits for Japanese public procurement auctions reported in McMillan (1991).[24] Our calibrated parameter is also consistent with the fact that the average winning bid fell to 76.7% of the reserve price for procurement auctions let by the town of Kumatori in fiscal year 2007 after the cartel was prosecuted. Once we set the profit margin to be 0.25, we can simply multiply the actual realizations of the reserve price in the data by 0.25 to obtain the profitability of each auction. Figure 2 plots the histogram of 0.25×(reserve price) for the Tier B auctions in our sample. We then fit a Pareto distribution to this empirical distribution by Maximum Likelihood. Figure 2 also plots the density of the fitted Pareto distribution.[25] We use the fitted Pareto distribution when computing the equilibrium outcomes.

[24]McMillan (1991) estimates excess cartel profits to be between 16% to 33% of the reserve price for public procurement auctions in Japan.

[25]The estimated parameters are 1.1675×10^6 for the scale parameter and 0.0575 for the shape parameter.

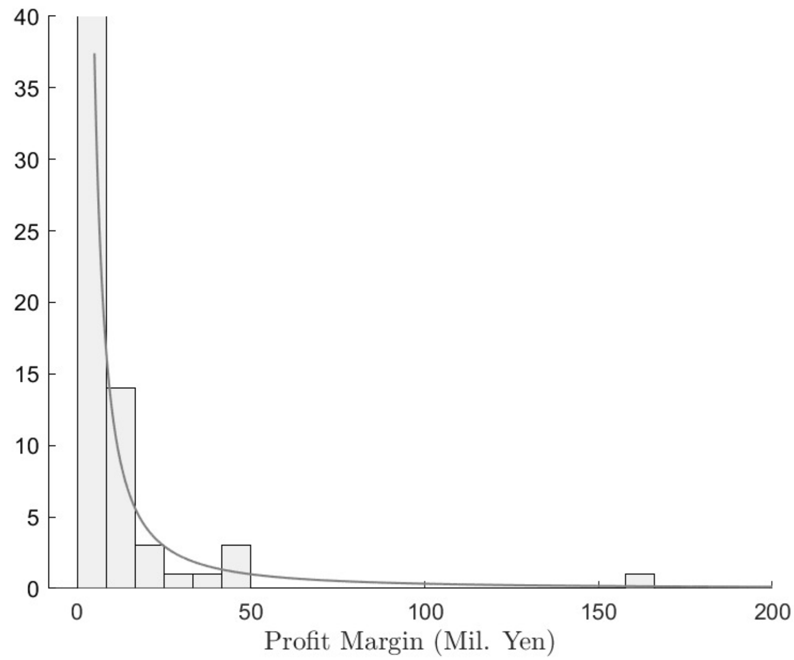

Figure 2: The bar graph corresponds to the histogram of $0.25r$, and the curve corresponds to the estimated Pareto distribution.

The second parameter that we calibrate is the discount factor between auctions. We calibrate it by setting the annual discount to be 0.9 and adjust it by the number of Tier B auctions per year.[26] Our calibrated parameter is approximately 0.9765.

Once we calibrate F_r and δ and specify the number of bidders, n, we can compute $\overline{V}_{\mathsf{ck}}$, $\overline{V}$ and the associated bid distribution $F^*(\cdot; r)$ using expressions (12), (14) and (13) in Appendix C, which generalizes the results in Section 4.2 to settings with $n > 2$ bidders. We set $n = 9$, which is the number of Tier B firms. Under the calibrated parameters, we estimate $\overline{V}_{\mathsf{ck}}$ to be 525 million yen, or about 5 million dollars, and we estimate $\overline{V}$ to be about 820 million yen, or about 8 million dollars. Our first observation is that the gain from using bidding schemes without common-knowledge bids is substantial. In our calibration, the gain is about 56%.

Figure 3 illustrates, for each realization of the reserve price, the range of possible winning

[26] In particular, we use the average number of auctions computed using the data obtained from Kumatori (2006.4 to 2007.10).

margins (left panel) and the range of possible winning bids (right panel) under the equilibrium that achieves $\overline{V}$. In this figure, we take the actual realizations of the reserve price (as opposed to 25% of the realizations) to be the horizontal axis in both of the panels. Note that when the reserve price is relatively small, the equilibrium that attains $\overline{V}$ prescribes that both the winning bid and the losing bids be at the reserve price. This implies that the winning margin is 0. For relatively large realizations of the reserve price, however, the winning bid is a non-degenerate distribution. The range of possible winning margins is given by the width of the support of $F^*(\cdot|r)$. This is depicted by the shaded area that begins at around 350 million yen. We have also drawn in the average winning margin, $\mathbb{E}[b^2 - b^1]$, by a solid curve in the left panel. Because the upper bound of $F^*(\cdot;r)$ is bounded above by $\delta(1+\delta)\overline{W}$, the range of possible winning margin does not increase above a certain point. With our calibrated parameters, the winning margin is capped at around 80 million yen. The **x**'s in the figure correspond to the plot of the winning margin and the reserve price for each auction in our sample.

The right panel of Figure 3 illustrates the relationship between the reserve price and the winning bid for the equilibria that attain $\overline{V}_{\mathsf{ck}}$ and $\overline{V}$, respectively.[27] Under common-knowledge bids, the wining bid is given by $\min\{r, c_0 r + \delta\overline{W}_{\mathsf{ck}}\}$, where $c_0 = 0.75$. When the reserve price exceeds about 215 million yen, the latter term becomes larger than the former. We depict the winning bid under common-knowledge bids by the dashed line in the figure: note that the dashed line has slope $c_0 = 0.75$ when the reserve price is above about 215 million yen.

Under the equilibrium that attains $\overline{V}$, the winning bid is equal to r as long as $(1-c_0)r$ is lower than $\delta\overline{W}$. Because $\delta\overline{W} > \delta\overline{W}_{\mathsf{ck}}$, the winning bid under $\overline{V}$ is equal to the reserve price even for reserve prices above 215 million yen. We depict the expected winning bid under $\overline{V}$

[27]We compute the winning bids by first simulating the equilibrium bids using F_r calibrated based on 25% of the actual reserve prices. We then add 0.75 of the reserve price back to the equilibrium bids.

by a solid curve and the range of possible winning bids by the shaded area. The winning bid under $\overline{V}$ is deterministic until the reserve price is above 350 million yen. The vertical distance between the solid and the dotted curves corresponds to the average gain in profits from using mixed strategies. The actual reserve prices and the winning bids of the auctions in the sample are illustrated as **x**'s.

The calibration results suggest that our model of cartel bidding with a mediator can explain the data quite well. The calibrated model predicts zero winning margin when the reserve price is relatively low, and predicts that the expected winning margin is positive when the reserve price is set equal to that of the Obara residences auction. The realized winning margin in the Obara residences auction is within the range predicted by the theory. We also find that the winning bid lies within the range predicted by the theory. These results suggest that our model can quantitatively explain the bidding patterns documented in Section 3.2.

6 Discussion

This paper highlights the practical value of keeping the winning bid private in relaxing IC constraints. We also offer a case study in which privacy helps a cartel obtain higher profits.

We conclude the paper with discussions of (i) our model's implications for the price-variance screen for collusion, (ii) how our model may help explain puzzling bidding patterns documented elsewhere, and (iii) implications for the information that the auctioneer should make public after each auction.

Price-variance screen. Our model predicts that when the project is relatively small (i.e., r_t is small), the winning bid measured as a fraction of the reserve price ($\frac{\min_i b_{i,t}}{r_t}$) will be close to 1 and the money left on the table will be close to zero. This implies that the time-series variance in the winning bids, $\frac{\min_i b_{i,t}}{r_t}$, will be close to zero and the within-auction variance

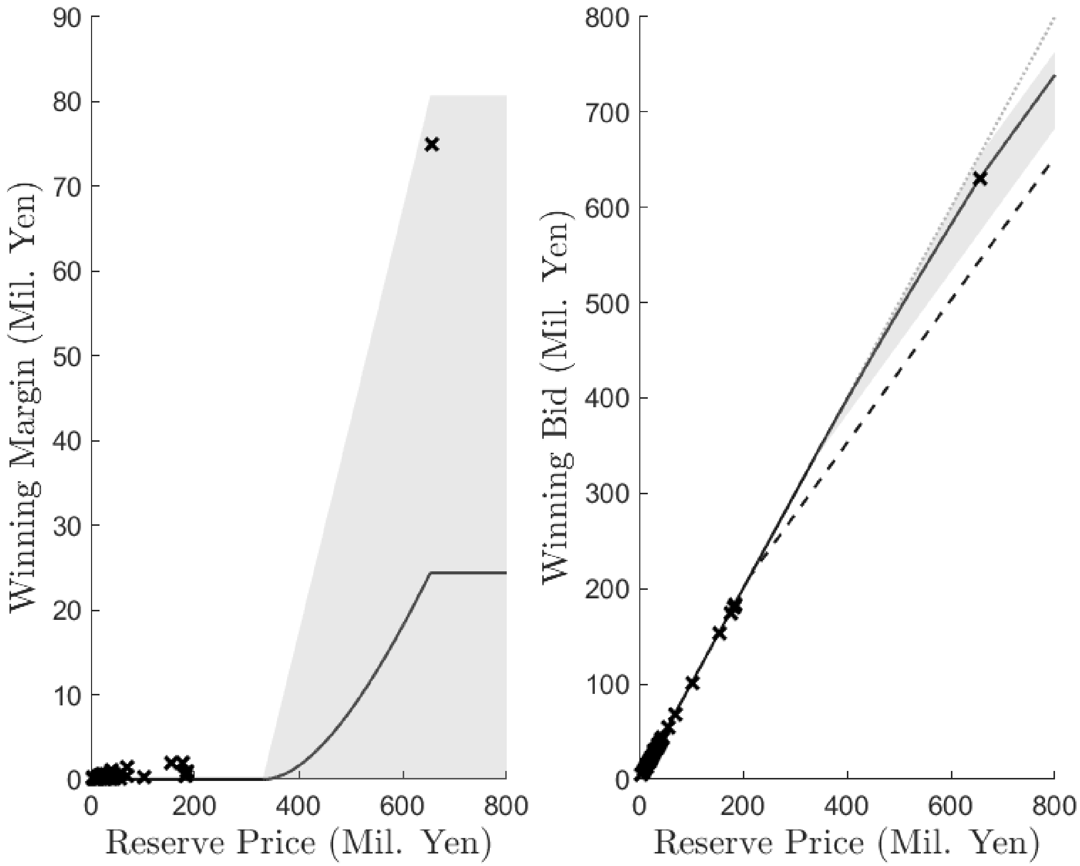

Figure 3: The left plots the winning margin against the reserve price for an optimal equilibrium that attains $\overline{V}$. The actual realization of the reserve and winning margin are marked with **x**s. The right panel illustrates the relationship between the winning bid and the reserve price. The shaded region corresponds to the range of winning bids under an optimal equilibrium that attains $\overline{V}$. The dashed line corresponds to the equilibrium winning bid under common-knowledge bids. The actual winning bids are marked with **x**s.

of bids will also be close to zero. These results are consistent with the premise of the price-variance screen. However, when projects are relatively large, our model predicts that there will be considerable time-series variance in the winning bids that results from randomization. Moreover, the within-auction variance of bids will not be close to zero. Hence, increases in the variance of bids should not be taken as failure of collusion when those increases are associated with increases in project sizes.

Explaining other bidding patters. A distinct feature of the equilibrium in Proposition 2 is that the winner has a static incentive to raise its bid whenever the mediator recommends a bid below $\overline{b} = \sup_b \mathsf{supp}\, F^*(\cdot; r)$. This feature is present in the bidding data analyzed in Chassang et al. (2020).[28] Chassang et al. (2020) study the sample distribution of normalized bid differences $\Delta_{i,t} \equiv (b_{i,t} - \wedge b_{-i,t})/r_t$ for procurement auctions let by the Ministry of Land, Infrastructure and Transportation of Japan, and by cities in Ibaraki prefecture and the Tohoku region (bid data from Kumatori are not included in the data analyzed by Chassang et al. (2020)). That paper documents a missing mass in the distribution of Δ around $\Delta = 0$. In other words, winning bids tend to be isolated. This implies that upward deviations by winners are profitable, consistent with the collusive scheme in Proposition 2.

What information should the auctioneer make public? Our model assumes that all submitted bids are made public after each auction, consistent with how the town of Kumatori operates. We note that this is crucial for randomized bidding schemes to be profitable. To see why, consider instead a setting in which only the bid of the winning firm is made public. Suppose that the cartel uses bid rotation to allocate contracts, and that the winning bid at some period t is drawn from c.d.f. F_t. Then, for the designated loser not to want to deviate,

[28] Tóth et al. (2014), Imhof et al. (2016) and Clark et al. (2020) document similar patterns. Clark et al. (2020) offer an explanation based on the bidders' desire to leave some margin of error.

it must be that

$$\forall b \in \mathsf{supp}\, F_t, \quad (1 - F_t(b))(b + \delta \times 0) + F_t(b)\delta V^l \leq \delta V^l$$
$$\iff b \leq \delta V^l,$$

where δV^l denotes the loser's expected continuation payoff. Hence, only winning bids below the loser's continuation payoff are sustainable, just as when bids are common-knowledge. Intuitively, when only winning bids are public, a deviation by a designated loser can only be detected if she wins the auction. In contrast, when all bids are made public, a defector can get punished even if she doesn't win.

Appendix

A Mediation and Solution Concept

This appendix formally describes the strategies of the mediator and our solution concept.

Mediator's histories and strategies. A period-t history for the mediator,

$$h_{M,t} = \left(r_s, \widehat{\mathbf{b}}_s, \widehat{\boldsymbol{\gamma}}_s, \mathbf{b}_s, \boldsymbol{\gamma}_s\right)_{s<t} \sqcup r_t,$$

records all previous reserve prices, bidding recommendations and realized bids, as well as current reserve price. Let H_M denote the set of all possible histories of the mediator. A strategy $\sigma_M : H_M \to \Delta(\mathbb{R}_+^2 \times [0,1]^2)$ for the mediator maps mediator histories to a distribution over bidding recommendations.

Equilibrium notion. Let $H_{i,t}$ denote the set of period-t histories of player i, and let $H_t = H_{1,t} \times H_{2,t}$. Our solution concept is weak Perfect Bayesian Equilibrium (σ, μ), where $\sigma = (\sigma_M, \sigma_1, \sigma_2)$ is a strategy profile, and $\mu = (\mu_1, \mu_2)$ are bidders' beliefs about their opponent's history; i.e., $\mu_i : h_{i,t} \mapsto \mu_i(h_{i,t})$, where $\mu_i(h_{i,t}) \in \Delta(H_t)$ is a belief over H_t with the property that $\mathsf{supp}\,\mu_i(h_{i,t}) \subseteq \{h_{i,t}\} \times H_{-i,t}$ (i.e., bidder i's beliefs assign probability 1 to her own history $h_{i,t}$). In a weak Perfect Bayesian Equilibrium (σ, μ), bidders' strategies (σ_1, σ_2) must be sequentially rational at every history, and, for every on-path history $h_{i,t}$, $\mu_i(h_{i,t})$ must be consistent with the mediator's strategy σ_M. The mediator is assumed to be a disinterested third party, with no incentives.

B Proofs

Proof of Proposition 1. Fix an equilibrium $\sigma \in \Sigma_{\mathsf{ck}}$. Let V^w and V^l denote, respectively, the expected discounted payoff that the designated winner and loser at $t = 0$ obtain under σ. Let $\overline{V}^w$ be the highest payoff that the designated winner at $t = 0$ can obtain under an equilibrium in Σ_{ck}. Since $V^w \leq \overline{V}^w$ and $V^l \leq \delta\overline{V}^w$ (since the loser at $t = 0$ is the winner at $t = 1$), $V^w + V^l \leq (1 + \delta)\overline{V}^w$. Since this inequality holds for all $\sigma \in \Sigma_{\mathsf{ck}}$, $\overline{V}_{\mathsf{ck}} \leq (1 + \delta)\overline{V}^w$.

Let $b(r_0)$ be the winning bid at time $t = 0$ under σ. Note that $b(r_0) \leq \delta\overline{V}^w$. Indeed, the continuation payoff of the loser at $t = 0$ can't be larger $\delta\overline{V}^w$ (since the loser at $t = 0$ is the winner at $t = 1$). If $b(r_0) > \delta\overline{V}^w$, the loser would have a strict incentive to undercut $b(r_0)$. Since $b(r_0)$ must be lower than r_0, $b(r_0) \leq \min\{r_0, \delta\overline{V}^w\}$. Hence, $V^w \leq \mathbb{E}_{F_r}[\min\{r_0, \delta\overline{V}^w\}] + \delta^2\overline{V}^w$. Since the inequality holds for all $\sigma \in \Sigma_{\mathsf{ck}}$,

$$\overline{V}^w \leq \frac{1}{1 - \delta^2}\mathbb{E}_{F_r}[\min\{r_0, \delta\overline{V}^w\}].$$

Let $\overline{W}_{\mathsf{ck}}$ be the largest $W \geq 0$ solving

$$W = \frac{1}{1-\delta^2}\mathbb{E}_{F_r}[\min\{r, \delta W\}].$$

Note that $\overline{V}^w \leq \overline{W}_{\mathsf{ck}}$.[29] We now show that $\overline{V}^w = \overline{W}_{\mathsf{ck}}$. Consider the following strategy profile. Along the equilibrium path, in each period t the designated winner bids $\min\{r_0, \delta\overline{W}_{\mathsf{ck}}\}$ and $\gamma = 1$, and the designated loser bids $\min\{r_0, \delta\overline{W}_{\mathsf{ck}}\}$ and $\gamma = 0$. Deviations are punished with Nash reversion. This strategy profile is an equilibrium in Σ_{ck} giving the winner at $t = 0$ an expected discounted payoff of $\overline{W}_{\mathsf{ck}}$. Hence, $\overline{V}^w = \overline{W}_{\mathsf{ck}}$. Since $\overline{V}_{\mathsf{ck}} \leq (1+\delta)\overline{W}_{\mathsf{ck}}$, this equilibrium attains $\overline{V}_{\mathsf{ck}}$. ■

For each reserve price r and each value $W \geq 0$, let $F(\cdot; r, W)$ be the c.d.f. given as follows. If $\delta W \geq r$, $F(\cdot; r, W)$ puts all its mass at r. If $\delta W < r$, $F(\cdot; r, W)$ is given by:

$$F(b; r, W) = \begin{cases} 0 \text{ if } b < \delta W, \\ 1 - \frac{\delta W}{b} \text{ if } b \in [\delta W, \min\{r, \delta(1+\delta)W\}), \\ 1 \text{ if } b \geq \min\{r, \delta(1+\delta)W\}. \end{cases} \tag{4}$$

Proof of Proposition 2. Fix an equilibrium $\sigma \in \Sigma$. Let V^w and V^l denote, respectively, the expected discounted payoff of the designated winner and loser at $t = 0$ under σ. Let $\hat{V}^w$ be the highest payoff that the winner at time $t = 0$ can obtain under an equilibrium in Σ. By the same arguments as in the proof of Proposition 1, $\overline{V} \leq (1+\delta)\hat{V}^w$.

Let $F(b; r_0)$ be the c.d.f. from which the winning bid at $t = 0$ is drawn under σ. Let $\overline{b} = \sup_b \mathsf{supp}\, F(\cdot; r_0)$ and $\underline{b} = \inf_b \mathsf{supp}\, F(\cdot; r_0)$. Note that the designated loser at $t = 0$ must place a bid weakly higher than $\overline{b}$ under σ; otherwise, it would win with positive probability.

[29]Since the right-hand side of (1) is bounded by $\overline{r}/(1-\delta^2)$, $W > \frac{1}{1-\delta^2}\mathbb{E}_{F_r}[\min\{r, \delta W\}]$ for all $W > \overline{W}_{\mathsf{ck}}$.

Moreover, we must have

$$\forall b < \bar{b}, \qquad (1 - F(b; r_0))b \le \delta \hat{V}^w, \tag{5}$$

$$\forall b \in \mathsf{supp}\, F(\cdot; r_0), \qquad \bar{b} \le b + \delta^2 \hat{V}^w. \tag{6}$$

If inequality (5) didn't hold for some $b < \bar{b}$, the loser would have a strict incentive to bid b and win the auction with probability $1 - F(b; r_0)$. If inequality (6) didn't hold for $b \in \mathsf{supp}\, F(\cdot; r_0)$, the winner would have an incentive to bid $\bar{b} - \epsilon$ with $\epsilon \approx 0$ instead of $b < \bar{b}$. Condition (5) implies

$$\underline{b} \le \delta \hat{V}^w \text{ and } \forall b < \bar{b}, F_b(b; r_0) \ge 1 - \delta \frac{\hat{V}^w}{b}. \tag{7}$$

Condition (6), together with $\bar{b} \le r_0$, implies

$$\bar{b} \le \min\{r_0, \underline{b} + \delta^2 \hat{V}^w\}. \tag{8}$$

Consider the problem of finding the c.d.f. F that maximizes expected winning bid $\int b dF$, subject to (7) and (8). When $\delta \hat{V}^w \ge r_0$, the c.d.f. F that solves this problem puts all its mass at r_0. When $\delta \hat{V}^w < r_0$, the c.d.f. F that solves this problem is given by (4), with $r = r_0$ and $W = \hat{V}^w$. Therefore, $V^w \le \mathbb{E}_{F_r}\left[\int b dF(b; r, \hat{V}^w)\right] + \delta^2 \hat{V}^w$. Since the inequality holds for all $\sigma \in \Sigma$,

$$\hat{V}^w \le \frac{1}{1 - \delta^2} \mathbb{E}_{F_r}\left[\int b dF(b; r, \hat{V}^w)\right].$$

Let $\overline{W} \ge 0$ be the largest solution to

$$W = \frac{1}{1 - \delta^2} \mathbb{E}_{F_r}\left[\int b dF(b; r, W)\right], \tag{9}$$

and note that $\hat{V}^w \leq \overline{W}$.[30] For each r, let $F^*(\cdot; r)$ be the c.d.f. given by $F^*(\cdot; r) = F(\cdot; r, \overline{W})$.

We now show that $\hat{V}^w = \overline{W}$. Consider the following strategy profile. Along the equilibrium path, in each period t, after observing r_t the mediator sends bidding recommendation $(\widehat{b}_i, \widehat{\gamma}_i) = (b, 1)$ to the designated winner, with b drawn from c.d.f. $F^*(\cdot; r_t) = F(\cdot; r_t, \overline{W})$; and sends bidding recommendation $(\widehat{b}_j, \widehat{\gamma}_j) = (\bar{b}_t, 0)$ to the designated loser, with $\bar{b}_t = \sup_b \mathsf{supp}\, F^*(\cdot; r_t)$. Deviations from the mediator's recommendations are punished with Nash reversion.[31] One can check that this strategy profile is an equilibrium in Σ, and gives the designated winner at $t = 0$ a payoff equal to $\overline{W}$. Hence, $\hat{V}^w = \overline{W}$. Since $\overline{V} \leq (1 + \delta)\hat{V}^w$, this equilibrium also attains $\overline{V}$.

Finally, note that $\int b dF(b; r, W) \geq \min\{r, \delta W\}$ for each r, W, with strict inequality whenever $\delta W < r$.[32] Hence, $\overline{W} \geq \overline{W}_{\mathsf{ck}}$, with strict inequality whenever $\delta\overline{W}_{\mathsf{ck}} < \overline{r}$. Hence, when $\overline{r} > \delta\overline{W}_{\mathsf{ck}}$, we have $(1 + \delta)\overline{W} = \overline{V} > \overline{V}_{\mathsf{ck}} = (1 + \delta)\overline{W}_{\mathsf{ck}}$. ■

Proof of Proposition 3. We first characterize equilibria in Σ^*_{ck} attaining V^*_{ck}. We start by showing that, when looking for an equilibrium in Σ^*_{ck} attaining V^*_{ck}, it is without loss to consider equilibria under which, at all on path histories, both bidders place the same bid and win the auction with probability 1/2. Consider $\sigma \in \Sigma^*_{\mathsf{ck}}$ such that there exists a public history h^0_t under which some bidder i wins the auction with probability $x_{i,t} > 1/2$. Let b_t denote the winning bid under σ at history h^0_t, and let $\{V_j\}_{j=1,2}$ be the bidders' on-path

[30] Since the right-hand side of (9) is bounded, $W > \frac{1}{1-\delta^2}\mathbb{E}_{F_r}\left[\int b dF(b; r, W)\right]$ for all $W > \overline{W}$.

[31] Formally, following a deviation, the mediator sends bidding recommendations $(b_i, \gamma_i) = (0, 1)$, which both bidders follow.

[32] When $\delta W \geq r$, $r = \int b dF(b; r, W)$. When $\delta W < r$,

$$\delta W < \int b dF(b; r, W) = \begin{cases} \delta W[1 + \ln(1 + \delta)] \text{ if } \delta(1 + \delta)W < r, \\ \delta W\left[1 + \ln(r) - \ln(\delta W)\right] \text{ if } \delta(1 + \delta)W \geq r. \end{cases}$$

expected continuation payoffs under σ after history h_t^0. Then, for $j = 1, 2$, we must have

$$x_{j,t}b_t + \delta V_j \geq b_t \Longleftrightarrow (1 - x_{j,t})b_t \leq \delta V_j. \tag{10}$$

Indeed, if (10) does not hold, bidder j would have an incentive to undercut the winning bid b_t. Since (10) holds for $j = 1, 2$, we have that

$$b_t = \sum_{j=1,2}(1 - x_{j,t})b_t \leq \delta \sum_{j=1,2} V_j. \tag{11}$$

Consider changing σ such that, at history h_t^0, both bidders bid b_t and each bidder wins the auction with probability 1/2. If no player deviates, then starting at time $t+1$ players play an equilibrium that gives each player a continuation payoff of $V = \frac{1}{2}\sum_{j=1,2} V_j$.[33] By (11), this modified strategy profile is also an equilibrium with common knowledge bids. Moreover, the cartel attains the same total payoffs than under the original strategy profile σ.

Let $\sigma \in \Sigma_{\mathsf{ck}}^*$ be an equilibrium that attains V_{ck}^* under which, along the equilibrium path, at each period both bidders place the same bid and win the auction with probability 1/2. Hence, each firm obtains an expected discounted payoff of $\frac{1}{2}V_{\mathsf{ck}}^*$ at the start of the game under σ. We now show that σ is stationary: under σ, along the equilibrium path, at each period t firms get an expected continuation payoff of $\frac{1}{2}V_{\mathsf{ck}}^*$. Suppose not. Hence, there exists an on-path public history h_t^0 such that, under σ, firms get an expected continuation payoff of $V < \frac{1}{2}V_{\mathsf{ck}}^*$ at time $t+1$. Consider changing strategy profile σ so that, after history h_t^0, if no player deviates at time t, firms play a continuation equilibrium that delivers payoffs $\frac{1}{2}V_{\mathsf{ck}}^* > V$ to each firm $i = 1, 2$ starting at $t+1$. This relaxes firms' incentive constraints (10) at time t, and increases their total payoff.

Consider equilibrium $\sigma \in \Sigma_{\mathsf{ck}}^*$ attaining V_{ck}^* under which, along the equilibrium path,

[33] Since the game is symmetric across bidders, and since the game admits an equilibrium with payoffs (V_1, V_2), the game also admits an equilibrium with payoffs (V_2, V_1). Since bidders have access to a public randomization device, the game also admits an equilibrium in which each player obtains payoff V.

at each period all bidders place the same bid and win the auction with probability 1/2. Consider a public history h_t^0 and a reserve price r_t. Since σ is stationary and attains V^*_{ck}, by inequality (11) we must have $b_t = \min\{r_t, \delta V^*_{\mathsf{ck}}\}$. Hence, $V^*_{\mathsf{ck}} \leq \mathbb{E}_{F_r}[\min\{r_t, \delta V^*_{\mathsf{ck}}\}] + \delta V^*_{\mathsf{ck}}$, and so

$$V^*_{\mathsf{ck}} \leq \frac{1}{1-\delta}\mathbb{E}_{F_r}[\min\{r, \delta V^*_{\mathsf{ck}}\}].$$

Let W^* denote the highest value of V solving

$$V = \frac{1}{1-\delta}\mathbb{E}_{F_r}[\min\{r, \delta V\}],$$

Note that $V^*_{\mathsf{ck}} \leq W^*$. Finally, to see that $V^*_{\mathsf{ck}} = W^*$ consider a strategy profile σ such that, at all on path histories, both bidders place bid $(b_t, \gamma_t) = (\min\{r_t, \delta W^*\}, 1)$, and they each win with probability 1/2. Any deviation is punished by Nash reversion. One can check that σ is indeed an equilibrium with common-knowledge bids, and gives the cartel a payoff of W^*.

Note that an equilibrium attaining V^*_{ck} achieves perfect collusion when $\overline{r} \leq \delta V^*_{\mathsf{ck}}$. Hence, we have $V^* = V^*_{\mathsf{ck}}$ when $\overline{r} \leq \delta V^*_{\mathsf{ck}}$. We now show that $V^* > V^*_{\mathsf{ck}}$ when $\overline{r} > \delta V^*_{\mathsf{ck}}$.

Suppose $\overline{r} > \delta V^*_{\mathsf{ck}}$. Consider the following strategy profile σ. At $t = 0$, if $r_0 \leq \delta V^*_{\mathsf{ck}}$, the mediator recommends each bidder to bid $(\widehat{b}_i, \widehat{\gamma}_i) = (r_0, 1)$. If $r_0 > \delta V^*_{\mathsf{ck}}$, the mediator draws each bidder i's recommendation independently from a distribution that places probability $\mu = \delta V^*_{\mathsf{ck}}/r_0$ on $(\widehat{b}_i, \widehat{\gamma}_i) = (r_0, 1)$, and places probability $1 - \mu$ on $(\widehat{b}_i, \widehat{\gamma}_i) = (\delta V^*_{\mathsf{ck}}, 1)$. If both bidders follow the mediator's recommendation, then from $t = 1$ onwards they play an equilibrium attaining V^*_{ck} (which gives each bidder a discounted continuation payoff of $\delta V^*_{\mathsf{ck}}/2$). Otherwise, if either bidder deviates at $t = 0$, from $t = 1$ onwards the mediator sends bidding recommendations $(\widehat{b}_i, \widehat{\gamma}_i) = (0, 1)$ to both players, and players follow these recommendations (i.e., they play Bertrand-Nash).

Clearly, $\sum_{i=1,2} V_i(\sigma, h_0) > V^*_{\mathsf{ck}}$.[34] We now show that σ is an equilibrium. Clearly, no

[34] Indeed, whenever $r_0 > \delta V^*_{\mathsf{ck}}$, the winning bid under σ is equal to r_0 with probability μ^2 and is equal to

bidder has an incentive to deviate at $t = 0$ when $r_0 \leq \delta V^*_{\mathsf{ck}}$, and no bidder has an incentive to deviate at any time $t > 0$.

Consider next $r_0 > \delta V^*_{\mathsf{ck}}$. If bidder i gets bid recommendation $(\widehat{b}_i, \widehat{\gamma}_i) = (r_0, 1)$, she obtains a payoff equal to $\mu\frac{1}{2}r_0 + \delta V^*_{\mathsf{ck}}/2 = \delta V^*_{\mathsf{ck}}$: she wins with probability 1/2 if her opponent is recommended to bid r_0, and wins with probability 0 if her opponent is recommended to bid δV^*_{ck}. The optimal deviation is either to undercut bid r_0, or to undercut bid δV^*_{ck}. Her payoff from undercutting r_0 is $\mu r_0 + \delta 0 = \delta V^*_{\mathsf{ck}}$, while her payoff from undercutting δV^*_{ck} is $\delta V^*_{\mathsf{ck}} + \delta 0$. Hence, bidders don't gain from deviating at $t = 0$ when recommended to bid $(\widehat{b}_i, \widehat{\gamma}_i) = (r_0, 1)$.

If instead bidder i gets recommendation $(\widehat{b}_i, \widehat{\gamma}_i) = (\delta V^*_{\mathsf{ck}}, 1)$, she obtains a payoff equal to $\delta V^*_{\mathsf{ck}}\left(\mu + \frac{1-\mu}{2}\right) + \delta V^*_{\mathsf{ck}}/2 = \delta V^*_{\mathsf{ck}}\left(1 + \frac{\mu}{2}\right)$: she wins with probability 1 if her opponent is recommended to bid r_0, and with probability 1/2 if her opponent is recommended to bid δV^*_{ck}. Undercutting bid δV^*_{ck} gives her a payoff of $\delta V^*_{\mathsf{ck}} + \delta 0$, while undercutting bid r_0 gives her a payoff $\mu r_0 + \delta 0 = \delta V^*_{\mathsf{ck}}$. Hence, bidders don't gain from deviating at $t = 0$ when recommended to bid $(\widehat{b}_i, \widehat{\gamma}_i) = (\delta V^*_{\mathsf{ck}}, 1)$, and so σ is an equilibrium. ■

C Online Appendix: Bid Rotation Equilibria with $n > 2$ Bidders

To simplify the exposition, our analysis in the main text assumes two bidders. This Appendix shows how to extend the results in Section 4.2 to settings with $n > 2$ bidders. Let $N = \{1, ..., n\}$ denote the set of bidders. We start by extending definition 1 to settings with $n > 2$.

Definition C.1. *We say that equilibrium $\sigma = (\sigma_i)_{i \in N}$ is a* bid rotation equilibrium *if there exists a permutation $\Pi : N \to N$, such that for all $i \in N$ and all on-path public histories h^0_t,*

δV^*_{ck} with probability $1 - \mu^2$. Instead, under an equilibrium in Σ^*_{ck} attaining V^*_{ck}, the winning bid is equal to δV^*_{ck} with probability 1 whenever $r_0 > \delta V^*_{\mathsf{ck}}$.

$\mathbb{E}_\sigma[x_{i,t}|h_t^0] = 1$ *if* $(t+1) \mod n = \Pi(i)$ *and* $\mathbb{E}_\sigma[x_{i,t}|h_t^0] = 0$ *otherwise.*

Let Σ denote the set of bid rotation equilibria, and $\Sigma_{\mathsf{ck}} \subset \Sigma$ denote the set of bid rotation equilibria with common-knowledge bids. Recall that, for each equilibrium σ and $i \in N$, $V_i(\sigma)$ denotes firm i's expected discounted payoff at the start of the game under σ. As in the main text, $\overline{V}_{\mathsf{ck}}$ and $\overline{V}$ denote, respectively, the cartel's largest payoffs under an equilibrium in Σ_{ck} and Σ.

The next result generalizes Proposition 1 to settings with $n > 2$ bidders.

Proposition C.1. *On the equilibrium path, the bidding strategy in any $\sigma \in \Sigma_{\mathsf{ck}}$ that attains $\overline{V}_{\mathsf{ck}}$ is such that, in all periods t, the winning bid is given by the minimum between r_t and $\delta^{n-1}\overline{W}_{\mathsf{ck}}$, where $\overline{W}_{\mathsf{ck}}$ is the expected continuation payoff of a designated winner under any equilibrium that attains $\overline{V}_{\mathsf{ck}}$.*

The proof of Proposition C.1 is essentially the same as the proof of Proposition 1, and hence omitted. The key difference is that now the bidder who won last period (and has to wait for $n-1$ periods to win again) is the one who has the largest incentive to defect. To deter this bidder from defecting, the winning bid must be weakly below $\delta^{n-1}\overline{W}_{\mathsf{ck}}$. Note that $\overline{W}_{\mathsf{ck}}$ and $\overline{V}_{\mathsf{ck}}$ solve:

$$\overline{W}_{\mathsf{ck}} = \frac{1}{1-\delta^n}\mathbb{E}_{F_r}[\min\{r_t, \delta^{n-1}\overline{W}_{\mathsf{ck}}\}] \text{ and } \overline{V}_{\mathsf{ck}} = \sum_{k=0}^{n-1} \delta^k \overline{W}_{\mathsf{ck}}. \tag{12}$$

We now generalize Proposition 2 to the case with $n > 2$ bidders. Let $\overline{W}$ be the expected continuation payoff of a designated winner under any equilibrium that attains $\overline{V}$. Let c.d.f. $F^*(\cdot; r)$ be such that: (i) if $\delta^{n-1}\overline{W} \geq r$, $F^*(\cdot; r)$ puts all its mass at r; (ii) if $\delta^{n-1}\overline{W} < r$,

$F^*(\cdot; r)$ is given by:

$$F^*(b; r) = \begin{cases} 0 \text{ if } b < \delta^{n-1}\overline{W}, \\ 1 - \frac{\delta^{n-1}\overline{W}}{b} \text{ if } b \in [\delta\overline{W}, \min\{r, \delta^{n-1}(1+\delta)\overline{W}\}), \\ 1 \text{ if } b \geq \min\{r, \delta^{n-1}(1+\delta)\overline{W}\}. \end{cases} \tag{13}$$

Proposition C.2. *On the equilibrium path, the bidding strategy in any $\sigma \in \Sigma$ that attains $\overline{V}$ is such that, in all periods t, the winning bid is drawn from c.d.f. $F^*(\cdot; r_t)$. Distribution $F^*(\cdot; r_t)$ is degenerate at r_t if $r_t \leq \delta^{n-1}\overline{W}$ and is non-degenerate otherwise, where $\overline{W}$ is the expected continuation payoff of a designated winner under any equilibrium that attains $\overline{V}$.*

Moreover, $\overline{V} > \overline{V}_{ck}$ if and only if $\overline{r} > \delta^{n-1}\overline{W}_{ck}$.

We omit the proof of Proposition C.2, since it is essentially the same as the proof of Proposition 2. Again, the difference is that now the bidder who won last period is the one who has the largest incentive to defect. Distribution $F^*(\cdot; r)$ is the largest distribution (in terms of f.o.s.d.) that deters the winner last period from undercutting the winning bid, while also providing incentives to the designated winner not to increase her bid when she is recommended to place a low bid.

Note that values $\overline{W}$ and $\overline{V}$ now solve:

$$\overline{W} = \frac{1}{1-\delta^n}\mathbb{E}_{F_r}\left[\int b dF^*(b; r)\right] \text{ and } \overline{V} = \sum_{k=0}^{n-1} \delta^k \overline{W}. \tag{14}$$

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