Delays and Partial Agreements in Multi-Issue Bargaining*

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Abstract

We model a situation in which two players bargain over two issues (pies), one of which can only be resolved at a future date. We find that if the players value the issues asymmetrically (one player considers the existing issue more important than the future one, while the other player has the opposite valuation) then they may delay agreement on the first issue until the second one is finally on the table. If we allow for partial agreements, then the players never leave an issue completely unresolved. They either reach a partial agreement on the first issue, and wait for the second one to emerge before completing the agreement; or they reach complete agreements on each of the issues at their earliest possible dates.

JEL Classification Codes: C73, C78

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1 Introduction

The non-cooperative theory of bargaining, introduced by Stahl (1972) and Rubinstein (1982), deals with situations in which two players seek an agreement on how to divide a given surplus. Once an agreement is reached, each player receives his share and the game ends with the two players never interacting again in the future. However, as Schelling (1960) noted many years ago, in many real life situations the bargaining parties negotiate over not one but multiple issues. While bargaining over an issue today, the players may, for instance, know that in the future they will come to the negotiating table again to bargain over a new issue altogether.

In this paper, we build a model to capture such a situation. We assume that there are two pies, one of which is already on the table.[1] The other pie can only be consumed starting at a future date. The assumption that the second pie is available only in the future may be interpreted as a physical constraint on the environment, or may capture the idea that some issues are not yet ripe for discourse. We assume that the players have opposite valuations for the two pies: one player values the first pie more than the second, while the other player values the second pie more than the first. The players can either bargain over the two pies as they arrive, or they can wait for the second pie to arrive so as to bargain over the two pies simultaneously. We first consider a model in which the players are constrained to make *complete offers*: that is, each pie must either be completely consumed or remain completely unconsumed. We show that players may wait for the second pie to arrive before coming to an agreement on the first.

Although our setup is highly stylized, there are many real life situations that have its structure. Consider the following example. An economics department has two openings in the job market—one for a political economist and one open search. Suppose that a candidate appears that is attractive to both the political economy faculty and those who specialize in theory, but more so to the political economists. To hire her, the political economists need the support of the theorists, and would like to extend her an offer under the *quid pro quo* understanding that when an excellent theorist who does some political economy arrives on the market, they will support the theorists in their bid to make that candidate an offer. But the political economists cannot commit to supporting the theorists in any of their future bids. If the department makes an early offer to the existing candidate, then the political economy faculty can afford to be obstinate in the future and seek another of their own for the open search. Because of the commitment

[1] We use the terms "pie," "issue" and "surplus" interchangeably.

problem, the theorists may want to wait and see if the candidate that they are looking for appears on the market in the same year. If she does, then the department could make the two offers simultaneously. But if they wait too long, the department suffers the risk that the existing candidate will take a job at a competing institution. In other words, delay is costly.

The example provides some insight into the role of commitment problems in producing delays in bargaining situations. If the players in our model come to an early agreement on the first pie (and immediately consume the benefits of this agreement), then the subgame that starts when the second pie arrives is a standard bargaining game *a la* Rubinstein. Consequently, the players will split the second pie almost evenly, and this division will be independent of how they value the second pie with respect to the first. On the other hand, if the players delay an agreement on the existing pie until the second one arrives, then each player will receive most of the pie he values more. Therefore, if the discounting costs are not too large, then the players will delay an agreement on the first pie so as to bargain over the two pies simultaneously. However, if the discounting costs are large, then the players will reach an immediate agreement.

After analyzing the case of complete offers, we consider a model that is otherwise the same, except that players are able to make *partial offers*. In this case, the players can reach agreements in which they consume any fraction of the pies that are on the table, leaving the remainder for future consumption. We find that in this setting, the players may come to a partial agreement on the first pie, completing the agreement only when the second one arrives. More precisely, for some parameter values the players will consume some (but not all) of the first pie in the initial period, and the remainder of it in the period in which the second pie arrives. Therefore, even the partial offers case features a form of delay; but, more importantly, it provides a framework for understanding situations in which we may observe partial agreements.

The intuition for the partial offers case is as follows. Players in our model face the following tradeoff: they can either reach a complete agreement on the first pie in the initial period and not incur any cost of delay, or they can delay reaching a complete agreement on the first pie until the arrival of the second pie and thus achieve an efficient allocation. In the complete offers case, players resolve this tradeoff by either reaching a complete agreement on the first pie at the start of the game (and avoiding the costs of delay altogether) or by delaying all consumption of the first pie until the arrival of the second. In the case of partial offers, players may be able to obtain a larger payoff by reaching a partial agreement over the first pie in the first period, and completing this agreement once the second pie arrives.

We stress that if the players could commit to an agreement on the second issue before it is on the table (for instance, if they could write enforceable contracts), then they would always be able to implement an efficient allocation. However, in the absence of a commitment mechanism, any early agreement that the players may reach on the second issue will be violated with certainty once it is finally on the table. Therefore, when players lack commitment, inefficiencies arise naturally as a result of the timing of the game (the fact that the second pie is not yet on the negotiating table) and the assumption that the players attach different values to each of the pies. In particular, if the players value the two pies the same, then they always reach complete and immediate agreements on both pies. But if there is an asymmetry in valuations the players may delay reaching a complete agreement on the first issue until the second one is finally on the table.

Related literature. Our paper is related primarily to the literature on bargaining impasses. Kennan and Wilson (1993) survey the literature on bargaining delays in models with incomplete information. Merlo and Wilson (1995, 1998) consider a stochastic bargaining model in which players may delay an agreement as they wait for the pie to increase in size. Abreu and Gul (2000) show that delays may arise when players can build a reputation for being irrational. Yildiz (2004) shows that players may delay an agreement if they hold optimistic beliefs about their future bargaining power and they update these beliefs as the game proceeds.[2] Finally, Compte and Jehiel (2004) introduce a bargaining model with complete information and history-dependent outside options, and show that parties will make gradual concessions until reaching an agreement.

Because of the presence of the second issue, our paper is also related to the literatures on repeated bargaining and multi-issue bargaining. Muthoo (1995) develops a model of repeated bargaining in which, unlike our setting, a new issue emerges only after players have reached an agreement on the existing one. Fershtman (1990) studies a multi-issue bargaining game in which players bargain over the issues sequentially, and in which agreements are implemented after all issues have been settled. He shows that the agenda (i.e., the order in which issues are negotiated) affects the outcome of the game, and that the outcome might be inefficient if players have conflicting valuations over the issues at stake. Busch and Hortsmann (1997) also study the effect of the bargaining agenda on allocations. Their model also features two pies, and players can reach an agreement on

[2]See also Babcock and Loewenstein (1997) for some empirical evidence on delays and optimism.

the second pie only after agreeing on the first. Our model differs from these papers in two respects. First, in our model the second pie may arrive before the players reach an agreement on the first one. Second, and more importantly, we are concerned with bargaining delays and partial agreements rather than the effect of the agenda on the allocation of the surplus.[3]

Finally, our paper is also related to the literature on the hold-up problem (Grout 1984; Grossman and Hart 1986). As in the hold-up problem, the source of inefficiency in our model arises from the lack of commitment. In particular, players in our model may delay agreement over the first pie since they cannot commit to any allocation of the second pie until it finally arrives.

2 The Model

There are two players $i = 1, 2$ and two pies X and Y. Time is discrete and indexed by $t = 0, 1, 2, ...$ If x_{it} and y_{it} are the shares of pies X and Y consumed by player i in period t, then the players' payoffs at period t are

$$u_1(x_{1t}, y_{1t}) = x_{1t} + ry_{1t}$$
$$u_2(x_{2t}, y_{2t}) = rx_{2t} + y_{2t}$$

where $0 < r \leq 1$ is player 1's marginal rate of substitution between pies and $\infty > 1/r \geq 1$ is the corresponding quantity for player 2.[4] A *consumption path* is denoted $\{((x_{1t}, x_{2t}), (y_{1t}, y_{2t}))\}_{t=0}^{\infty}$. Player i's preferences over consumption paths are represented by $\sum_{t\geq 0} \delta^t u_i(x_{it}, y_{it})$, where $\delta < 1$ is a common discount factor.

In every period, each of the two players is recognized with probability $1/2$ to be the *proposer*. The proposer then offers nonnegative consumption shares $((x_{1t}, x_{2t}), (y_{1t}, y_{2t}))$. The other player, the *responder*, will then either accept or reject the offer. If the offer is accepted, then the proposed shares are consumed and the period ends; if the offer is rejected, then the players consume 0 of each pie and the period ends.

The only restriction on offers is that they be feasible, and feasibility is state-contingent. In each period the state is given by (j, s) where $j = 1, 2$ is the identity of the proposer and s determines the fractions of pies X and Y that can be consumed in that period.[5]

[3] Inderst (1998) and In and Serrano (2003, 2004) study multi-issue bargaining settings in which the agenda is endogenous.

[4] The assumption that players have diametrically opposite valuations is for simplicity. Indeed, our qualitative results would continue to hold if we instead assumed more generally that $u_1(x_{1t}, y_{1t}) = x_{1t} + r_1 y_{1t}$ and $u_2(x_{2t}, y_{2t}) = r_2 x_{2t} + y_{2t}$, with $r_1, r_2 \in (0, 1)$.

[5] We sometimes abuse terminology and refer only to component s as the state.

We assume that pie X exists beginning in period 0, and thus part or all of it may be consumed starting in the first period. On the other hand, pie Y arrives stochastically: if it has not arrived by period t, then it arrives at the beginning of period $t+1$ with probability $p < 1$.[6] Obviously, no fraction of pie Y can be consumed before it arrives.

Let $\lambda_t = 1 - \sum_{\tau<t}(x_{1\tau} + x_{2\tau})$ denote the fraction of pie X not yet consumed by period t, with the convention that $\lambda_0 = 1$. We will later argue that in any subgame in which pie Y has already arrived, the players will come to an agreement over all of pie Y and all of the fraction of pie X that has not yet been consumed (and thus the game will effectively end). In other words, if pie Y arrives in period t, and λ_t is the fraction of pie X not yet consumed by that period, then in equilibrium the players will consume the total shares $x_{1t} + x_{2t} = \lambda_t$ and $y_{1t} + y_{2t} = 1$ in period t. This implies that the only relevant periods are those up to (and including) the period at which pie Y arrives. For such periods t, the state s can be written either as $\lambda_t X$ if pie Y has not yet arrived, or $\lambda_t XY$ if pie Y arrived in period t. Feasibility of offers in a given state $s = \lambda X, \lambda XY$ then requires that[7]

$$\begin{aligned} & x_{1t} + x_{2t} \leq \lambda \text{ and} \\ & y_{1t} + y_{2t} \leq \psi = \begin{cases} 0 & \text{if } s = \lambda X \\ 1 & \text{if } s = \lambda XY. \end{cases} \end{aligned} \tag{1}$$

Consider states (j, s) with $s = \lambda X$ or λXY. If $s = 0X$, then we say that the players are *waiting* for pie Y to arrive (having already consumed all of pie X). We say that the players reach a *complete agreement* at state (j, s), $s \neq 0X$, if they consume $x_{1t} + x_{2t} = \lambda$ and $y_{1t} + y_{2t} = \psi$. We say that the players *delay* at state (j, s), $s \neq 0X$, if they consume $x_{1t} + x_{2t} = 0$ and $y_{1t} + y_{2t} = 0$. We say that the players reach a *partial agreement* at state (j, s) in all other cases, i.e. if they do not delay, are not waiting, or do not reach a complete agreement.

Our equilibrium concept is subgame perfect equilibrium (SPE). In the next section, we characterize the SPE of the game with complete offers; i.e., the game in which proposals must satisfy (1) with both inequality signs replaced by equalities. Section 4 characterizes the SPE of the game in which the constraints in (1) are satisfied as stated.

[6] The assumption that the second pie arrives stochastically is not crucial for our results. For instance, the qualitative insights of the model would continue to hold if instead we assumed that the second pie arrives at some fixed date in the future.

[7] Formally, offers are denoted $((x_{1t}, x_{2t}), (y_{1t}, y_{2t}))$ but in states $s = \lambda X$ it is redundant to specify 0-consumption of pie Y; therefore, in these states we write offers simply as (x_{1t}, x_{2t}).

3 Complete Offers

In this section, all offers must be complete. In other words, we assume that the feasibility constraints are given by (1) with the two inequalities replaced by equalities. This assumption further simplifies the states: in each period, we have $s \in \{0X, 0XY, 1X, 1XY\}$.

Theorem 1. *(i) There are unique SPE payoffs. (ii) In all SPE, the players reach complete agreements in all subgames beginning in states $0XY$ and $1XY$. (iii) There exists a function $\phi(p, \delta, r)$, strictly increasing in p and δ and strictly decreasing in r, such that for all subgames beginning in state $1X$, and all SPE, the players reach a complete agreement when $\phi(p, \delta, r) < 0$ and they delay when $\phi(p, \delta, r) > 0$.*[8]

Proof. (i) & (ii) Standard arguments can be used to show that for all subgames beginning in states of the form $(j, 0XY)$ and $(j, 1XY)$, the SPE payoffs are unique, and the players reach complete agreements in all such states. (Appendix A computes the equilibrium offers as a function of the parameters for these states.) Let $v_i(j, s)$ be the equilibrium payoff to player i in state (j, s), $s = 0XY$, $1XY$, and let $w_i(s) = \frac{1}{2}v_i(1, s) + \frac{1}{2}v_i(2, s)$. If $s = 0X$, players are waiting for pie Y to arrive, so

$$w_i(0X) = \delta\left(pw_i(0XY) + (1-p)w_i(0X)\right) \Rightarrow w_i(0X) = \alpha w_i(0XY) \tag{2}$$

where $\alpha \equiv \delta p/(1 - \delta(1-p))$. This implies that SPE payoffs in states of the form $(j, 0X)$ are also unique. Finally, in Appendix B, we adapt the proof of Shaked and Sutton (1984) to show that SPE payoffs in states of the form $(j, 1X)$ are also unique.

(iii) Since SPE payoffs are unique, we can let $v_i(j, s)$ denote the unique equilibrium payoff for player i in state (j, s). It is also useful to define

$$w_i(s) = \frac{1}{2}v_i(1, s) + \frac{1}{2}v_i(2, s) \qquad \text{and} \qquad W(s) = rw_1(s) + w_2(s) \tag{3}$$

for all $i = 1, 2$, $s = \lambda X, \lambda XY$ and $\lambda \in \{0, 1\}$. The quantity $w_i(s)$ is player i's *ex ante* expected equilibrium payoff for state s, and $W(s)$ can be interpreted as the total (normalized) equilibrium payoff for state s.

Now suppose that when $s = 1X$, there exists an offer that, if accepted, would give both players larger payoffs than the payoffs that they would receive from an offer being rejected. In other words, suppose that there exists a number $x \in [0, 1]$ such that

$$\begin{aligned} x + \delta\left(pw_1(0XY) + (1-p)w_1(0X)\right) &> \delta\left(pw_1(1XY) + (1-p)w_1(1X)\right) \\ r(1-x) + \delta\left(pw_2(0XY) + (1-p)w_1(0X)\right) &> \delta\left(pw_2(1XY) + (1-p)w_2(1X)\right) \end{aligned} \tag{4}$$

[8] The omitted case, $\phi(p, \delta, r) = 0$, corresponds to a knife-edge set of parameters for which there are SPE supporting both delay and complete agreement at state $1X$.

Then, it would have to be the case that there is complete agreement at state $1X$; otherwise, if there is delay, then the proposer could deviate to make an offer that the responder would have to accept, and would also make the proposer strictly better off.

Combining the inequalities in (4) and using the definition of $W(\cdot)$ in (3), such a number $x \in [0,1]$ exists if and only if

$$r + \delta p W(0XY) + \delta(1-p)W(0X) > \delta p W(1XY) + \delta(1-p)W(1X). \tag{5}$$

Next, note that if there is complete agreement at state $s = 1X$ then

$$\begin{aligned} W(1X) &= rw_1(1X) + w_2(1X) = r + rw_1(0X) + w_2(0X) \\ &= r + W(0X) = r + \alpha W(0XY) \end{aligned} \tag{6}$$

where the last equality follows from (2). Substituting (6) into (5) and rearranging gives

$$r + \alpha W(0XY) > \alpha W(1XY). \tag{7}$$

In Appendix A, we explicitly compute $W(\lambda XY)$ (where $\lambda \in \{0,1\}$) as a function of the parameters δ and r. Substituting these expressions in (7) and rearranging gives

$$\phi(p,\delta,r) \equiv p - \frac{2r(1-\delta)(2-\delta+r\delta)}{\delta^2(1-r)(1+r^2)} < 0. \tag{8}$$

Therefore, if (8) is satisfied, then in all SPE the players must reach a complete agreement whenever $s = 1X$.

Conversely, suppose that there is no number $x \in [0,1]$ that simultaneously satisfies the inequalities in (4) even when both inequality signs are replaced by weak inequalities. In this case, there must be delay at state $s = 1X$, since there is no division of pie X that gives both players a payoff larger than what they would get by delaying. This implies $W(1X) = \delta(pW(1XY) + (1-p)W(1X))$, so $W(1X) = \alpha W(1XY)$. Since we know from (2) that $W(0X) = \alpha W(0XY)$, we can substitute these expressions into (5) (with reverse inequality) and rearrange to get $\phi(p,\delta,r) > 0$. Therefore, the players delay at state $s = 1X$ if $\phi(p,\delta,r) > 0$. $\square$

The idea behind the proof of part (iii) in Theorem 1 is as follows. If $\phi(p,\delta,r) < 0$, then there exists a number $x \in [0,1]$ that satisfies the two inequalities in (4). But this means that whoever is the proposer in state $1X$ can find an offer that gives him *and* the responder payoffs that are *strictly* larger than their respective continuation values. Consequently, there can be no equilibrium with delay at state $1X$; and, therefore, there must be complete agreement. On the other hand, if $\phi(p, \delta, r) > 0$, then there is no possible division of pie X that satisfies both of the inequalities in (4) even when the two strict inequality signs are replaced by weak inequalities. In words, the proposer cannot find an offer that is both incentive compatible for him to make, and incentive compatible for the responder to accept. As a result, there must be delay at state $1X$. Note that $\lim_{r \to 0} \phi(p, \delta, r) = p > 0$, so there exists a non-empty set of parameters under which players delay whenever $s = 1X$.

The proof of Theorem 1 shows that whether or not there is agreement in state $1X$ does not depend on the identity of the proposer. Indeed, the proposer (regardless of his identity) will be able to find an offer that is (strictly) incentive compatible for both players only if inequality (5) holds. Intuitively, players will come to an immediate agreement at $t = 0$ only if there exists a division of the first surplus that gives each of them a payoff that is at least as large as the payoff they would get by waiting until pie Y arrives; and whether such a division of the pie X exists is independent of the identity of the proposer.

The function $\phi(p, \delta, r)$ is strictly increasing in δ and p and strictly decreasing in r. This means that players are more likely to delay an agreement when δ and/or p are large, and are less likely to delay an agreement when r is large. The intuition behind these comparative statics is as follows. The payoff that players get from delaying an agreement until pie Y arrives is increasing in both δ and p: a larger δ implies a lower cost of delay, while a larger p means that players expect pie Y to arrive earlier. Therefore, an increase in either of these parameters "tightens" the inequality in (5), and makes it harder for players to reach an early agreement on pie X. On the other hand, an increase in r decreases the efficiency gains from delaying until the arrival of pie Y and bargaining over the two pies jointly, since now the differences in the players' valuations are smaller. As a result, inequality (5) becomes "looser," making it easier for the players to reach an immediate agreement.

We stress that the equilibrium outcome of the bargaining game is always inefficient, either because players delay, or because immediate agreements on each of the issues involve inefficient allocations of the pies. Indeed, if the players reach an immediate agreement on pie X, then they both consume a positive fraction of each pie, while an efficient allocation must have one player consuming all of the pie he values more.

Finally, the results in Theorem 1 can be generalized to settings in which pies X and Y are of different size. For instance, suppose that pie Y is of size $\rho > 0$, and normalize X to be of size 1. By arguments similar to those in the proof of Theorem 1, in this setting there exists a function $\tilde{\phi}(p, \delta, r, \rho)$, increasing p, δ and ρ and decreasing in r, such that players delay at state $1XY$ whenever $\tilde{\phi}(p, \delta, r, \rho) > 0$. That is, the set of parameters for which there is delay is increasing in the size of the pie Y.

4 Partial Offers

We now suppose that all proposals must satisfy the feasibility constraints in (1) as stated. That is, in this section we consider the bargaining game in which players can make partial agreements over existing pies, and leave part of any pie for future consumption. In this setup the possible states in any period t are $s = \lambda X$, λXY with $\lambda \in [0, 1]$. The first result of this section shows that the states λXY are indeed terminal.

Lemma 1. *In every SPE, the players reach a complete agreement when the state is λXY with $\lambda \in [0, 1]$. Moreover, the SPE payoffs for these states are unique.*

Proof. See Appendix C.1 □

Lemma 1 shows that in all SPE, players will always reach a complete agreement at states λXY with $\lambda \in [0, 1]$. At such states, proposer j will make a feasible offer $((x_1^j, x_2^j), (y_1^j, y_2^j))$ that maximizes his total payoff, subject to the constraint that the offer is acceptable to player i:

$$\max_{x_1^j, x_2^j, y_1^j, y_2^j \geq 0} u_j\left(x_j^j, y_j^j\right) \text{ subject to} \tag{9}$$
$$u_i\left(x_i^j, y_i^j\right) \geq \delta\left(\frac{1}{2}u_i\left(x_i^j, y_i^j\right) + \frac{1}{2}u_i\left(x_i^i, y_i^i\right)\right)$$
$$x_1^j + x_2^j \leq \lambda,\ y_1^j + y_2^j \leq 1$$

The offers that solve (9) for $j = 1, 2$ depend on the fraction λ of pie X that remains. When λ is close to 1, player 2's offer will be such that he will keep all of pie Y for himself, since he can get player 1 to accept his proposal by offering him most of what remains of pie X. On the other hand, when λ is very low any offer that gives player 2 the entirety of pie Y will be rejected by player 1, since such an offer will give player 1 at most a fraction λ of pie X. In this case, player 2 must offer player 1 a fraction of pie Y in order for player 1 to accept the offer. In Appendix A, we show that the the threshold $\hat{\lambda}(\delta, r) = \frac{r\delta}{2-\delta} \in (0, 1)$ defines the critical point at which player 2 must offer some of pie Y to player 1. Note that at states λX with $\lambda \leq \hat{\lambda}(\delta, r)$ player 2 has a strong incentive to delay until Y arrives, since any further consumption of pie X substantially reduces his payoff in states in which pie Y has arrived. In contrast, when $\lambda > \hat{\lambda}(\delta, r)$ player 2's incentive to delay until pie Y arrives is weaker.

Let $\hat{\phi}(p, \delta, r) = p - \frac{1-\delta}{\delta}\frac{2r^2}{1-r^2}$, and note that $\hat{\phi}(p, \delta, r)$ is strictly increasing in p and δ and strictly decreasing in r.

Theorem 2. *(i) There are unique SPE payoffs. (ii) For any subgame beginning in state λX, we have the following:*[9]

1. *if $\hat{\phi}(p, \delta, r) < 0$, then in all SPE the players reach a complete agreement, and*

2. *if $\hat{\phi}(p, \delta, r) > 0$, then in all SPE the players reach a partial agreement (consuming $x_1 + x_2 = \lambda - \hat{\lambda}(\delta, r)$) when $\lambda > \hat{\lambda}(\delta, r)$, and they delay when $\lambda \leq \hat{\lambda}(\delta, r)$.*

Proof. See Appendix C.2 □

Unlike the case of complete offers (Section 3), when players can make partial offers, the unique equilibrium always involves some form of agreement (either complete or partial) in the initial round of negotiations. Indeed, at any state λX with $\lambda > \hat{\lambda}(\delta, r)$ the players reach a partial agreement on pie X if $\hat{\phi}(p, \delta, r) > 0$ and then delay until Y arrives; and they reach a complete agreement on pie X if $\hat{\phi}(p, \delta, r) < 0$.

To understand the intuition behind Theorem 2, first consider the case in which $\hat{\phi}(p, \delta, r) > 0$ and the state is λX with $\lambda \leq \hat{\lambda}(\delta, r)$. Lemma 2 in the Appendix shows that players will always delay at these states. The intuition behind this is as follows. Note first that $\hat{\phi}(p, \delta, r) > 0$ implies that the discounting costs are small relative to the efficiency gains from bargaining over the two pies together. Moreover, when $\lambda \leq \hat{\lambda}(\delta, r)$, consuming pie X (either partially or in its entirety) has a large cost in terms of a less efficient allocation once pie Y arrives. In this case, the costs of consuming pie X outweigh the benefits; as a result, at these states there is no offer involving positive consumption of pie X that is incentive compatible for both players, so there must be delay.

Next, consider states λX with $\lambda > \hat{\lambda}(\delta, r)$. Lemma 3 in the Appendix shows that if $\hat{\phi}(p, \delta, r) > 0$, then in every SPE, the proposer will make an offer to consume a total $\lambda - \hat{\lambda}(\delta, r)$ of pie X. Intuitively, at states λX with $\lambda > \hat{\lambda}(\delta, r)$ the total marginal benefit of consuming an additional slice of pie X outweighs the cost of implementing a less efficient allocation of the pies in the period that pie Y arrives; hence, in this case players must consume a fraction $\lambda - \hat{\lambda}(\delta, r)$ of pie X, leaving a fraction $\hat{\lambda}(\delta, r)$ to negotiate

[9] As with the function $\phi(p, \delta, r)$ in Theorem 1, the case where $\hat{\phi}(p, \delta, r) = 0$ is a knife-edge case that supports equilibria with both partial and complete agreements.

together with pie Y in the period in which it arrives. Lemma 4 uses this result to derive bounds on the players' supremum and infimum SPE payoffs. Lemma 5 then uses these bounds, and an argument similar to those in Shaked and Sutton (1984), to show that all SPE are payoff equivalent.

Finally, consider the case where $\hat{\phi}(p, \delta, r) < 0$, so that delaying the consumption of pie X until pie Y arrives is costly relative to the efficiency gains accrued from bargaining over the two pies simultaneously. One can show that, in this case, the proposer will find it optimal to make an offer in which the players consume all of the remainder of pie X at any state λX, regardless of whether $\lambda > \hat{\lambda}(\delta, r)$ or $\lambda \leq \hat{\lambda}(\delta, r)$. Using this observation, one can again derive bounds on the players' supremum and infimum SPE payoffs and apply arguments similar to those in Shaked and Sutton (1984) to show that all SPE are payoff equivalent.

The threshold $\hat{\phi}(p, \delta, r)$ is comparable to the threshold $\phi(p, \delta, r)$ that we derived for the case of complete offers. According to Theorem 2, the players will reach a partial agreement on pie X at the beginning of the game if $\hat{\phi}(p, \delta, r) > 0$, and they will reach a complete agreement if $\hat{\phi}(p, \delta, r) < 0$. Similarly, when offers are restricted to be complete, Theorem 1 shows that the players will delay an agreement on pie X if $\phi(p, \delta, r) > 0$, and they will reach a complete agreement if $\phi(p, \delta, r) < 0$. One can show that $\phi(p, \delta, r) < \hat{\phi}(p, \delta, r)$ for all $0 < p, \delta, r < 1$. In other words, there is a region of the parameter space in which the players would come to a complete agreement on pie X if restricted to make complete offers, but would reach only a partial agreement if they had the option. In this sense, the added flexibility from partial offers may in fact hinder the chances of reaching a complete agreement at the start of the game.

As in the complete offers case, the results of Theorem 2 can also be generalized to settings in which the two pies have different sizes. Let pie X be of size 1 and pie Y be of size $\rho > 0$. In this case, the agreement that players reach at states λXY will again depend on the fraction λ of pie X that remains to be consumed. In particular, there exists $\tilde{\lambda}(\delta, r, \rho)$, increasing in δ, r and ρ, such that player 2 offers a positive share of pie Y to player 1 whenever he is proposer at states λXY if $\lambda < \tilde{\lambda}(\delta, r, \rho)$, but offers a zero share of pie Y to player 1 if $\lambda \geq \tilde{\lambda}(\delta, r, \rho)$. In this case, if delaying the consumption of pie X until Y arrives is costly, the players will reach a partial agreement on X at the start of the game, consuming a fraction $1 - \tilde{\lambda}(\delta, r, \rho)$, and they will then complete this agreement in the period in which pie Y arrives. Since $\tilde{\lambda}(\delta, r, \rho)$ is increasing in ρ, the fraction of pie X that players consume at the beginning of the game is decreasing in the size of pie Y.

Appendix

A. Equilibrium Offers for States λXY

In this section we compute the offers players make in equilibrium at states λXY. This results will be used both in the case of complete offers (Section 3) and the case of partial offers (Section 4.)

Let $s = \lambda XY$. (In the case of Section 3, $\lambda \in \{0, 1\}$, whereas in Section 4 we may have any $\lambda \in [0, 1]$.) Denote by $\left(\left(x_1^j, x_2^j\right), \left(y_1^j, y_2^j\right)\right)$ the consumption shares proposed by player j when he is the proposer. These offers solve the following problems

$$\begin{aligned} \max_{x_1^1, x_2^1, y_1^1, y_2^1 \geq 0} \quad & x_1^1 + r y_1^1 \text{ subject to} \\ r x_2^1 + y_2^1 \geq \delta & \left(\frac{1}{2}\left(r x_2^1 + y_2^1\right) + \frac{1}{2}\left(r x_2^2 + y_2^2\right)\right) \\ & x_1^1 + x_2^1 \leq \lambda, \ y_1^1 + y_2^1 \leq 1 \end{aligned} \tag{A1}$$

$$\begin{aligned} \max_{x_1^2, x_2^2, y_1^2, y_2^2 \geq 0} \quad & r x_2^2 + y_2^2 \text{ subject to} \\ x_1^2 + r y_1^2 \geq \delta & \left(\frac{1}{2}\left(x_1^1 + r y_1^1\right) + \frac{1}{2}\left(x_1^2 + r y_1^2\right)\right) \\ & x_1^2 + x_2^2 \leq \lambda, \ y_1^2 + y_2^2 \leq 1 \end{aligned} \tag{A2}$$

Problem (A1) says that player 1's proposal at state λXY must maximize his payoff subject to being feasible and to being acceptable to player 2. Problem (A2) says that player 2's proposal at state λXY must maximize his payoff subject to being feasible and to being acceptable to player 1. The solutions to these problems are as follows:

If $\lambda \geq \hat{\lambda}(\delta, r) = \dfrac{r\delta}{2-\delta}$, then

$$\begin{aligned} \left(\left(x_1^1, x_2^1\right), \left(y_1^1, y_2^1\right)\right) &= \left((\lambda, 0), \left(\frac{2(1-\delta)(2-\delta(1+r\lambda))}{4(1-\delta)+\delta^2(1-r^2)}, 1 - \frac{2(1-\delta)(2-\delta(1+r\lambda))}{4(1-\delta)+\delta^2(1-r^2)}\right)\right) \\ \left(\left(x_1^2, x_2^2\right), \left(y_1^2, y_2^2\right)\right) &= \left(\left(\lambda - \frac{2(1-\delta)(\lambda(2-\delta)-r\delta)}{4(1-\delta)+\delta^2(1-r^2)}, \frac{2(1-\delta)(\lambda(2-\delta)-r\delta)}{4(1-\delta)+\delta^2(1-r^2)}\right), (0, 1)\right) \end{aligned} \tag{A3}$$

If, on the other hand, $\lambda \leq \hat{\lambda}(\delta, r)$ then

$$\begin{aligned} \left(\left(x_1^1, x_2^1\right), \left(y_1^1, y_2^1\right)\right) &= \left((\lambda, 0), \left(1 - \frac{\delta(r+\lambda)}{2r}, \frac{\delta(r+\lambda)}{2r}\right)\right) \\ \left(\left(x_1^2, x_2^2\right), \left(y_1^2, y_2^2\right)\right) &= \left((\lambda, 0), \left(\frac{r\delta - \lambda(2-\delta)}{2r}, 1 - \frac{r\delta - \lambda(2-\delta)}{2r}\right)\right) \end{aligned} \tag{A4}$$

The players' payoffs at state $(j, \lambda XY)$ are then given by $v_1(j, \lambda XY) = x_1^j + ry_1^j$ and $v_2(j, \lambda XY) = rx_2^j + y_2^j$, and their ex ante expected payoffs are $w_i(\lambda XY) = \frac{1}{2}v_i(1, \lambda XY) + \frac{1}{2}v_i(2, \lambda XY)$, $i = 1, 2$. From (A3) and (A4) we calculate

$$\begin{aligned} w_1(\lambda XY) &= \begin{cases} \frac{2r(1-\delta)+\lambda(2-\delta-r^2\delta)}{4(1-\delta)+\delta^2(1-r^2)} & \text{if } \lambda > \hat{\lambda}(\delta, r) \\ \frac{r+\lambda}{2} & \text{if } \lambda \leq \hat{\lambda}(\delta, r) \end{cases} \\ w_2(\lambda XY) &= \begin{cases} \frac{(2-\delta-r^2\delta)+2\lambda r(1-\delta)}{4(1-\delta)+\delta^2(1-r^2)} & \text{if } \lambda > \hat{\lambda}(\delta, r) \\ \frac{r+\lambda}{2r} & \text{if } \lambda \leq \hat{\lambda}(\delta, r). \end{cases} \end{aligned} \tag{A5}$$

From these, we can further compute the *total (normalized) payoff*

$$W(\lambda XY) = rw_1(\lambda XY) + w_2(\lambda XY) = \begin{cases} \frac{2r(1-\delta)(r+\lambda)+(1+r\lambda)(2-\delta-r^2\delta)}{4(1-\delta)+\delta^2(1-r^2)} & \text{if } \lambda > \hat{\lambda}(\delta, r) \\ \frac{(1+r^2)(r+\lambda)}{2r} & \text{if } \lambda \leq \hat{\lambda}(\delta, r). \end{cases} \tag{A6}$$

Finally, for results proven in Appendix C, it will be useful to note that

$$W((\lambda - \kappa)XY) = \begin{cases} W(\lambda XY) - \xi\kappa & \text{if } \hat{\lambda}(\delta, r) \leq \lambda - \kappa \leq \lambda \leq 1 \\ W(\lambda XY) - \zeta\kappa & \text{if } 0 \leq \lambda - \kappa \leq \lambda \leq \hat{\lambda}(\delta, r). \end{cases} \tag{A7}$$

where $\xi \equiv \frac{r(2-\delta-r^2\delta)+2r(1-\delta)}{4(1-\delta)+\delta^2(1-r^2)}$ and $\zeta \equiv \frac{r}{2} + \frac{1}{2r}$.

B. Uniqueness of Equilibrium Payoffs in States $s = 1X$

Let $\overline{v}_i(j, 1X)$ and $\underline{v}_i(j, 1X)$ denote the supremum and infimum SPE payoffs of player i in state $(j, 1X)$, and define $\overline{w}_i(1X) = \frac{1}{2}\overline{v}_i(1, 1X) + \frac{1}{2}\overline{v}_i(2, 1X)$ and $\underline{w}_i(1X) = \frac{1}{2}\underline{v}_i(1, 1X) + \frac{1}{2}\underline{v}_i(2, 1X)$. To show that $\overline{v}_i(j, 1X) = \underline{v}_i(j, 1X)$ it suffices to show that $\overline{w}_i(1X) = \underline{w}_i(1X)$. At state $(1, 1X)$, player 2's payoff from accepting an offer $(1 - x, x)$ is $rx + w_2(0X)$. However, player 2 can guarantee himself a payoff $\delta(pw_2(1XY) + (1-p)\underline{w}_2(1X))$ by rejecting the offer. Thus, player 2 will only accept the offer if

$$x \geq \frac{\delta}{r}\left(pw_2(1XY) + (1-p)\underline{w}_2(1X)\right) - \frac{w_2(0X)}{r}. \tag{B1}$$

Player 1's payoff from making the offer $(1 - x, x)$ is $1 - x + w_1(0X)$, provided player 2 accepts. But player 1 could also make an offer that player 2 rejects, in which case he gets a continuation payoff of at most $\delta\left(pw_1(1XY) + (1-p)\overline{w}_1(1X)\right)$. Using this together with (B1), it follows that

$$\overline{v}_1(1, 1X) \leq \max\left\{ \begin{array}{l} 1 + \frac{w_2(0X)}{r} - \frac{\delta}{r}\left(pw_2(1XY) + (1-p)\underline{w}_2(1X)\right) + w_1(0X), \\ \delta\left(pw_1(1XY) + (1-p)\overline{w}_1(1X)\right) \end{array} \right\} \tag{B2}$$

On the other hand, at state $(2,1X)$ it must be that

$$\overline{v}_1(2,1X) \leq \delta\left(pw_1\left(1XY\right) + (1-p)\,\overline{w}_1(1X)\right). \tag{B3}$$

Combining (B2) and (B3) yields

$$\begin{aligned}\overline{w}_1\left(1X\right) \leq \frac{1}{2}\max &\left\{\begin{array}{l} 1 + \frac{w_2(0X)}{r} - \frac{\delta}{r}\left(pw_2\left(1XY\right) + (1-p)\,\underline{w}_2\left(1X\right)\right) + w_1\left(0X\right), \\ \delta\left(pw_1\left(1XY\right) + (1-p)\,\overline{w}_1(1X)\right) \end{array}\right\} \\ &+ \frac{1}{2}\delta\left(pw_1\left(1XY\right) + (1-p)\,\overline{w}_1(1X)\right) \end{aligned} \tag{B4}$$

Using similar arguments, we get

$$\begin{aligned}\underline{w}_1\left(1X\right) \geq \frac{1}{2}\max &\left\{\begin{array}{l} 1 + \frac{w_2(0X)}{r} - \frac{\delta}{r}\left(pw_2\left(1XY\right) + (1-p)\,\overline{w}_2\left(1X\right)\right) + w_1\left(0X\right), \\ \delta\left(pw_1\left(1XY\right) + (1-p)\,\underline{w}_1\left(1X\right)\right) \end{array}\right\} \\ &+ \frac{1}{2}\delta\left(pw_1\left(1XY\right) + (1-p)\,\underline{w}_1(1X)\right) \end{aligned} \tag{B5}$$

Finally, subtracting (B5) from (B4) we get

$$\overline{w}_1\left(1X\right) - \underline{w}_1\left(1X\right) \leq \frac{\delta\left(1-p\right)}{2-\delta\left(1-p\right)}\max\left\{\frac{1}{r}\left(\overline{w}_2\left(1X\right) - \underline{w}_2\left(1X\right)\right), \left(\overline{w}_1(1X) - \underline{w}_1\left(1X\right)\right)\right\}. \tag{B6}$$

A symmetric argument for player 2 establishes that

$$\overline{w}_2\left(1X\right) - \underline{w}_2\left(1X\right) \leq \frac{\delta\left(1-p\right)}{2-\delta\left(1-p\right)}\max\left\{r\left(\overline{w}_1\left(1X\right) - \underline{w}_1\left(1X\right)\right), \left(\overline{w}_2\left(1X\right) - \underline{w}_2\left(1X\right)\right)\right\}. \tag{B7}$$

There are two possible cases:

1. $r\left(\overline{w}_1\left(1X\right) - \underline{w}_1\left(1X\right)\right) \geq \overline{w}_2\left(1X\right) - \underline{w}_2\left(1X\right)$, and
2. $r\left(\overline{w}_1\left(1X\right) - \underline{w}_1\left(1X\right)\right) \leq \overline{w}_2\left(1X\right) - \underline{w}_2\left(1X\right)$.

In case (1), from (B6) we have $\overline{w}_1\left(1X\right) - \underline{w}_1\left(1X\right) \leq \frac{\delta(1-p)}{2-\delta(1-p)}\left(\overline{w}_1(1X) - \underline{w}_1\left(1X\right)\right)$, so $\overline{w}_1(1X) = \underline{w}_1\left(1X\right)$. Since $r(\overline{w}_1\left(1X\right) - \underline{w}_1\left(1X\right)) \geq \overline{w}_2\left(1X\right) - \underline{w}_2\left(1X\right) \geq 0$ and $r > 0$, this implies $\overline{w}_2\left(1X\right) - \underline{w}_2\left(1X\right) = 0$. In case (2), from (B7) we have $\overline{w}_2\left(1X\right) - \underline{w}_2\left(1X\right) \leq \frac{\delta(1-p)}{2-\delta(1-p)}\left(\overline{w}_2\left(1X\right) - \underline{w}_2\left(1X\right)\right)$, so $\overline{w}_2\left(1X\right) = \underline{w}_2\left(1X\right)$. Since $\overline{w}_2\left(1X\right) - \underline{w}_2\left(1X\right) \geq r(\overline{w}_1\left(1X\right) - \underline{w}_1\left(1X\right)) \geq 0$ and $r > 0$ we have $\overline{w}_1\left(1X\right) - \underline{w}_1\left(1X\right) = 0$. We conclude that $\overline{w}_i\left(1X\right) = \underline{w}_i\left(1X\right)$, so $\overline{v}_i(j,1X) = \underline{v}_i(j,1X)$, $i = 1,2$, $j = 1,2$. Hence, all SPE are payoff equivalent. $\square$

C. Proofs for Section 4

C.1. Proof of Lemma 1

Suppose $s = \lambda XY$. Let $\underline{v}_i(j, \lambda XY)$ and $\overline{v}_i(j, \lambda XY)$ be the infimum and supremum SPE payoffs to player $i = 1, 2$ when the state is $(j, \lambda XY)$, $j = 1, 2$, and let $\underline{w}_i(\lambda XY) = \frac{1}{2}\underline{v}_i(1, \lambda XY) + \frac{1}{2}\underline{v}_i(2, \lambda XY)$ and $\overline{w}_i(\lambda XY) = \frac{1}{2}\overline{v}_i(1, \lambda XY) + \frac{1}{2}\overline{v}_i(2, \lambda XY)$. To prove Lemma 1, let $i \neq j$ and consider first player j's problem of choosing an offer that maximizes his payoff, subject to the constraint that player i's payoff from accepting this offer equals $\delta\overline{w}_i(\lambda XY)$. Since there is positive discounting, at the solution to this problem player j will make an offer such that he and player i consume all λ of pie X and all of pie Y. The payoff that player j receives when this offer is accepted is a lower bound on his SPE payoff, since player i must always accept such an offer. Similarly, consider player j's problem of choosing an offer that maximizes his payoff, subject to the constraint that player i's payoff from accepting this offer equals $\delta\underline{w}_i(\lambda XY)$. Again, at the solution to this problem, player j makes an offer such that he and player i consume all λ of pie X and all of pie Y. In this case, the payoff that player j gets when this offer is accepted is an upper bound on his SPE payoff, since this is the worst offer that player i could possibly accept. Using these bounds on payoffs, one can apply arguments similar to those in Shaked and Sutton (1984) to show that SPE payoffs at states $s = \lambda XY$ are unique, and that these payoffs can be attained by a strategy profile in which the proposer always makes an offer to consume all λ of pie X and all of pie Y. Finally, note that these unique SPE payoffs are the same payoffs that would arise at states $s = \lambda XY$ in a game in which players are constrained to make complete offers (as in Section 3). $\square$

C.2. Proof of Theorem 2

The proof is organized as follows. We first consider the case where $\hat{\phi}(p, \delta, r) = p - \frac{1-\delta}{\delta}\frac{2r^2}{1-r^2} > 0$. Lemma 2 shows that players delay in states λX with $\lambda \leq \hat{\lambda}(\delta, r)$ if $\hat{\phi}(p, \delta, r) > 0$. Lemma 3 then shows that at states λX with $\lambda > \hat{\lambda}(\delta, r)$, the proposer finds it optimal to make an offer to consume a total $\lambda - \hat{\lambda}(\delta, r)$ of pie X. Lemma 4 uses these results to derive upper and lower bounds on the players' SPE payoffs at states λX with $\lambda > \hat{\lambda}(\delta, r)$. Using these bounds, Lemma 5 adapts the arguments in Shaked and Sutton (1984) to show that SPE payoffs are unique at any state λX with $\lambda > \hat{\lambda}(\delta, r)$. Finally, Lemma 6 provides a sketch of the argument for the case where $\hat{\phi}(p, \delta, r) < 0$. In what follows, we write $\hat{\phi}$ and $\hat{\lambda}$ in place of $\hat{\phi}(p, \delta, r)$ and $\hat{\lambda}(\delta, r)$.

We now introduce some more notation. Let the state be λX and let $t = 0$ denote the period in which this state is reached. A *consumption plan* is a sequence $\{(x_{1t}, x_{2t})\}_0^\infty$, with the interpretation that x_{it} is the share of pie X that player i consumes in period t conditional on the event that pie Y has not arrived. Define the *consumption sequence* $\{\mu_t\}_0^\infty$ associated with the consumption plan $\{(x_{1t}, x_{2t})\}_0^\infty$ by $\mu_t = x_{1t} + x_{2t}$ for all t. Let the *total normalized payoff* from consumption plan $\{(x_{1t}, x_{2t})\}_0^\infty$ be

$$U\left(\{(x_{1t}, x_{2t})\}_0^\infty\right) \equiv \mathbb{E}\left[r \sum_{t=0}^{\infty} \delta^t u_1\left(x_{1t}, y_{1t}\right) + \sum_{t=0}^{\infty} \delta^t u_2\left(x_{2t}, y_{2t}\right)\right],$$

where the x_{it}'s are given by the consumption plan $\{(x_{1t}, x_{2t})\}_0^\infty$ until pie Y arrives, and are determined in equilibrium along with the y_{it}'s when pie Y arrives. If pie Y has not arrived by period $t-1$ it arrives in period t with probability p and players come to an agreement over all of what is left of pie X and all of pie Y (by Lemma 1); with probability $1-p$ pie Y does not arrive in period t, so players consume x_{1t} and x_{2t} as determined by the consumption plan. For any period t prior to the arrival of pie Y, we have $ru_1(x_{1t}, y_{1t}) + u_2(x_{2t}, y_{2t}) = r\mu_t$. On the other hand, if pie Y arrives in period $t > 0$ then $ru_1(x_{1t}, y_{1t}) + u_2(x_{2t}, y_{2t}) = W(\lambda_t XY)$, where $\lambda_t = 1 - \sum_{\tau=0}^{t-1} \mu_\tau$ is the fraction of pie X left in period t and $W(\cdot XY)$ is given by (A6). Therefore, we have

$$U\left(\{(x_{1t}, x_{2t})\}_0^\infty\right) = r \sum_{t=0}^{\infty} \delta^t (1-p)^t \mu_t + \delta p \sum_{t=0}^{\infty} \left[\delta^t (1-p)^t W\left(\left(1 - \sum_{\tau=0}^{t} \mu_\tau\right) XY\right)\right]. \tag{C1}$$

Lemma 2. *Let $\hat{\phi} > 0$. Then in every SPE the players delay in states λX with $\lambda \leq \hat{\lambda}$.*

Proof. Fix an equilibrium strategy profile. Let $\{(x_{1t}, x_{2t})\}$ be the consumption plan associated with this strategy profile, and let $\{\mu_t\}$ be the associated consumption sequence. Assume for the sake of contradiction that $\lambda \geq \mu_0 > 0$. The total normalized payoff from this consumption plan is

$$\begin{aligned}
U(\{(x_{1t}, x_{2t})\}) &= r \sum_{t=0}^{\infty} \delta^t (1-p)^t \mu_t + \delta p \sum_{t=0}^{\infty} \left[\delta^t (1-p)^t W\left(\left(\lambda - \sum_{\tau=0}^{t} \mu_\tau\right) XY\right)\right] \\
&= r \sum_{t=0}^{\infty} \delta^t (1-p)^t \mu_t + \delta p \sum_{t=0}^{\infty} \left[\delta^t (1-p)^t \left(W\left(\lambda XY\right) - \zeta \sum_{\tau=0}^{t} \mu_\tau\right)\right] \\
&= (r - \alpha\zeta) \sum_{t=0}^{\infty} \delta^t (1-p)^t \mu_t + \alpha W(\lambda XY) < \alpha W(\lambda XY)
\end{aligned}$$

where the second equality follows from (A7) and from the fact that $\delta p \sum_{t=0}^{\infty} \delta^t (1-p)^t = \delta p / (1 - \delta(1-p)) = \alpha$; the third equality follows from adding terms; and the inequality follows since it is easy to verify that $\hat{\phi} > 0$ if and only if $r - \alpha\zeta < 0$. Therefore, the total payoff from the strategy profile results in a payoff strictly smaller than $\alpha W(\lambda XY)$, which is the total payoff of delaying all consumption until pie Y arrives. This means that at least one of the two players is getting a payoff strictly lower than the payoff he would get if there were delay. Since that player can unilaterally deviate to generate delay (by either rejecting all proposals or making offers that the other player will reject), it cannot be that the players consume $\mu_0 > 0$ in states λX when $\lambda \leq \hat{\lambda}$. □

Lemma 3. *Let $\hat{\phi} > 0$ and $s = \lambda X$ with $\lambda > \hat{\lambda}$. Consider proposer j's problem of choosing an offer that maximizes his discounted payoff subject to the constraint that the responder's discounted payoff be equal to $w_i \leq u_i(\lambda - \hat{\lambda}, 0) + \alpha w_i(\hat{\lambda} XY)$, where $w_i(\hat{\lambda} XY)$ is given by (A5). At the solution to this problem, the proposer makes an offer such that he and the responder consume a total fraction $\lambda - \hat{\lambda}$ of pie X.*

Proof. We prove Lemma 3 for $j = 1$ and $i = 2$. The proof for $j = 2$ and $i = 1$ is symmetric and omitted. Suppose player 1 makes an offer (x_1, x_2) with $x_1 + x_2 = \lambda - \hat{\lambda}$. Player 2's discounted payoff from accepting this offer $rx_2 + \alpha w_2(\hat{\lambda} XY)$, since Lemma 2 implies that after such an offer is accepted the players will delay until pie Y arrives. Let x_2 satisfy $rx_2 + \alpha w_2(\hat{\lambda} XY) = w_2$. Then, player 1's payoff from this offer is $(\lambda - \hat{\lambda} - \frac{1}{r} w_2) + \alpha(\frac{1}{r} w_2(\hat{\lambda} XY) + w_1(\hat{\lambda} XY))$. Multiplying this quantity by r, player 1's *normalized* payoff from player 2 accepting this offer is

$$r(\lambda - \hat{\lambda}) - w_2 + \alpha(w_2(\hat{\lambda} XY) + r w_1(\hat{\lambda} XY)) = \underbrace{r(\lambda - \hat{\lambda}) + \alpha W(\hat{\lambda} XY)}_{a} - w_2.$$

On the other hand, suppose player 1 makes any other offer that gives player 2 a discounted payoff of w_2, and which leads to a consumption plan $\{(x_{1t}, x_{2t})\}$ with associated consumption sequence $\{\mu_t\}$. The total normalized payoff from this consumption plan is given by equation (C1) above, and player 1's *normalized* payoff from this offer is

$$\underbrace{r \sum_{t=0}^{\infty} \delta^t (1-p)^t \mu_t + \delta p \sum_{t=0}^{\infty} \left[\delta^t (1-p)^t W\left(\left(\lambda - \sum_{\tau=0}^{t} \mu_\tau \right) XY \right) \right]}_{b} - w_2,$$

since player 1's offer must give player 2 a continuation payoff equal to w_2. Thus, to establish the Lemma it suffices to show that $a > b$. There are two possibilities: (1)

$\sum_{t=0}^{\infty} \mu_t \leq \lambda - \hat{\lambda}$, or (2) there exists $t' \geq 0$ such that $\sum_{t=0}^{t'} \mu_t > \lambda - \hat{\lambda}$ (and $\sum_{t=0}^{\tau} \mu_t \leq \lambda - \hat{\lambda}$ for all $\tau < t'$).

Consider first case (1). In this case,

$$
\begin{aligned}
b &= r \sum_{t=0}^{\infty} \delta^t (1-p)^t \mu_t + \delta p \sum_{t=0}^{\infty} \left[\delta^t (1-p)^t \left(W\left(\lambda XY\right) - \xi \sum_{\tau=0}^{t} \mu_\tau \right) \right] \\
&= r \sum_{t=0}^{\infty} \delta^t (1-p)^t \mu_t + \alpha W\left(\lambda XY\right) - \alpha \xi \sum_{t=0}^{\infty} \delta^t (1-p)^t \mu_t \\
&= (r - \alpha \xi) \sum_{t=0}^{\infty} \delta^t (1-p)^t \mu_t + \alpha W\left(\lambda XY\right),
\end{aligned}
$$

where the first equality follows from (A7) and from $\sum_{t=0}^{\infty} \mu_t \leq \lambda - \hat{\lambda}$, the second one follows from adding terms and the third one follows from rearranging. Therefore,

$$
\begin{aligned}
b - a &= (r - \alpha \xi) \sum_{t=0}^{\infty} \delta^t (1-p)^t \mu_t + \alpha W(\lambda XY) - r(\lambda - \hat{\lambda}) - \alpha W(\hat{\lambda} XY) \\
&< -\alpha \xi (\lambda - \hat{\lambda}) + \alpha W\left(\lambda XY\right) - \alpha W(\hat{\lambda} XY) = -\alpha \xi (\lambda - \hat{\lambda}) + \alpha \xi (\lambda - \hat{\lambda}) = 0
\end{aligned}
$$

where the inequality follows from the fact that $\sum_{t=0}^{\infty} \delta^t (1-p)^t \mu_t < \sum_{t=0}^{\infty} \mu_t \leq \lambda - \hat{\lambda}$, and the equality that follows is a consequence of (A7).

Consider next case (2). By Lemma 2, it must be that $\mu_t = 0$ for all $t > t'$, since for any such t the state would be $\lambda' X$ with $\lambda' < \hat{\lambda}$ (and hence, by Lemma 2, players will delay). Let $\{\tilde{\mu}_t\}$ be such that $\tilde{\mu}_t = \mu_t$ for all $t \neq t'$, $\tilde{\mu}_{t'} = \lambda - \hat{\lambda} - \sum_{t=0}^{t'-1} \mu_t$. Note that this implies that $\sum_{t=0}^{t'} \tilde{\mu}_t = \sum_{t=0}^{\infty} \tilde{\mu}_t = \lambda - \hat{\lambda}$. Let $\tilde{b}$ be the same expression than b, but with $\tilde{\mu}_t$ replacing μ_t. Since $\sum_{t=0}^{\infty} \tilde{\mu}_{t'} = \lambda - \hat{\lambda}$, the arguments above imply that $a > \tilde{b}$. Note that $\lambda - \sum_{t=0}^{t'} \mu_t = \hat{\lambda} - (\mu_t - \tilde{\mu}_{t'})$. Thus, it follows that

$$
\begin{aligned}
\frac{\tilde{b} - b}{\delta^{t'} (1-p)^{t'}} &= r\left(\tilde{\mu}_{t'} - \mu_{t'}\right) + \alpha \left[W\left(\left(\lambda - \sum_{\tau=0}^{t'} \tilde{\mu}_\tau \right) XY \right) - W\left(\left(\lambda - \sum_{\tau=0}^{t'} \mu_\tau \right) XY \right) \right] \\
&= r\left(\tilde{\mu}_{t'} - \mu_{t'}\right) + \alpha \left[W\left(\hat{\lambda} XY \right) - W\left(\left(\hat{\lambda} - (\mu_t - \tilde{\mu}_{t'}) \right) XY \right) \right] \\
&= r\left(\tilde{\mu}_{t'} - \mu_{t'}\right) - \alpha \zeta \left(\tilde{\mu}_{t'} - \mu_t\right) > 0,
\end{aligned}
$$

where the first equality follows the fact that $\tilde{\mu}_t = \mu_t$ for all $t < t'$ and from the fact that $\tilde{\mu}_t = \mu_t = 0$ for all $t > t'$; the second equality follows from $\sum_{t=0}^{t'} \tilde{\mu}_t = \lambda - \hat{\lambda}$ and from $\lambda - \sum_{t=0}^{t'} \mu_t = \hat{\lambda} - (\mu_t - \tilde{\mu}_{t'})$; the third follows from equation (A7); and the inequality follows since $\tilde{\mu}_{t'} < \mu_{t'}$ and since $r - \alpha \zeta < 0$ whenever $\hat{\phi} > 0$. It then follows that $a > b$. $\square$

Lemma 4. *Let* $\underline{v}_i(j, \lambda X)$ *and* $\overline{v}_i(j, \lambda X)$ *be the infimum and supremum SPE payoffs for player* i *when the state is* $(j, \lambda X)$, *and define* $\overline{w}_i(\lambda X) = \frac{1}{2}\overline{v}_i(1, \lambda X) + \frac{1}{2}\overline{v}_i(2, \lambda X)$ *and* $\underline{w}_i(\lambda X) = \frac{1}{2}\underline{v}_i(1, \lambda X) + \frac{1}{2}\underline{v}_i(2, \lambda X)$. *If* $\lambda > \hat{\lambda}$ *and* $\hat{\phi} > 0$, *then we have*

1. $\underline{v}_i(j, \lambda X) \geq \delta(pw_i(\lambda XY) + (1-p)\underline{w}_i(\lambda X))$ *for* $i, j = 1, 2, j \neq i$

2. $\overline{v}_i(j, \lambda X) \leq \delta(pw_i(\lambda XY) + (1-p)\overline{w}_i(\lambda X))$ *for* $i, j = 1, 2, j \neq i$

3. $r\underline{v}_1(1, \lambda X) \geq r(\lambda - \hat{\lambda}) + \alpha W(\hat{\lambda}XY) - \delta(pw_2(\lambda XY) + (1-p)\overline{w}_2(\lambda X))$

4. $r\overline{v}_1(1, \lambda X) \leq r(\lambda - \hat{\lambda}) + \alpha W(\hat{\lambda}XY) - \delta(pw_2(\lambda XY) + (1-p)\underline{w}_2(\lambda X))$

5. $\underline{v}_2(2, \lambda X) \geq r(\lambda - \hat{\lambda}) + \alpha W(\hat{\lambda}XY) - r\delta(pw_1(\lambda XY) + (1-p)\overline{w}_1(\lambda X))$

6. $\overline{v}_2(2, \lambda X) \leq r(\lambda - \hat{\lambda}) + \alpha W(\hat{\lambda}XY) - r\delta(pw_1(\lambda XY) + (1-p)\underline{w}_1(\lambda X))$

where $w_i(\cdot XY)$, $i = 1, 2$, *are the functions defined in (A5).*

Proof. Claims (1) and (2) are immediate, and the arguments for (5) and (6) are the same as the arguments for (3) and (4). We therefore only prove (3) and (4).

We start by proving (3). Suppose the state is λX with $\lambda > \hat{\lambda}$. Suppose player 1 offers $(\lambda - \hat{\lambda} - \overline{x}_2, \overline{x}_2)$ where $0 \leq \overline{x}_2 \leq \lambda - \hat{\lambda}$. (Note that this offer leaves fraction $\hat{\lambda}$ of pie X for future consumption.) By Lemma 2, if player 2 accepts this offer then players delay an agreement on the remainder of pie X until pie Y arrives. Therefore, the payoff to player 2 from accepting this offer is $r\overline{x}_2 + \alpha w_2(\hat{\lambda}XY)$. Note that if $\overline{x}_2$ solves

$$r\overline{x}_2 + \alpha w_2(\hat{\lambda}XY) = \delta\left(pw_2(\lambda XY) + (1-p)\overline{w}_2(\lambda X)\right)$$

then player 2 will accept the offer $(\lambda - \hat{\lambda} - \overline{x}_2, \overline{x}_2)$. Importantly, one can show that $\overline{x}_2 < \lambda - \hat{\lambda}$, so that $(\lambda - \hat{\lambda} - \overline{x}_2, \overline{x}_2)$ is in fact a feasible offer. The payoff player 1 gets if this offer is accepted is $\lambda - \hat{\lambda} - \overline{x}_2 + \alpha w_1(\hat{\lambda}XY)$. Since player 2 always accepts $(\lambda - \hat{\lambda} - \overline{x}_2, \overline{x}_2)$, it must be that

$$\begin{aligned} r\underline{v}_1(1, \lambda X) &\geq r(\lambda - \hat{\lambda} - \overline{x}_2 + \alpha w_1(\hat{\lambda}XY)) \\ &= r(\lambda - \hat{\lambda}) - \delta(pw_2(\lambda XY) + (1-p)\overline{w}_2(\lambda X)) + \alpha(rw_1(\hat{\lambda}XY) + w_2(\hat{\lambda}XY)) \end{aligned}$$

which establishes (3) since $rw_1(\hat{\lambda}XY) + w_2(\hat{\lambda}XY) = W(\hat{\lambda}XY)$.

Next, we prove (4). To show this, consider player 1's problem of making an offer that maximizes his discounted payoff subject to the constraint that player 2's payoff is equal to $w_2 = \delta(pw_2(\lambda XY) + (1-p)\underline{w}_2(\lambda X))$. By definition, such an offer is the worst offer that player 2 would ever accept. Moreover, note that $w_2 \leq r\overline{x}_2 + \alpha w_2(\hat{\lambda}XY) < r(\lambda - \hat{\lambda}) + \alpha w_2(\hat{\lambda}XY)$, since $\underline{w}_2(\lambda X) \leq \overline{w}_2(\lambda X)$ and $\overline{x}_2 < \lambda - \hat{\lambda}$. By Lemma 3, the solution to this problem is for player 1 to make an offer $(\lambda - \hat{\lambda} - \underline{x}_2, \underline{x}_2)$ with $r\underline{x}_2 + \alpha w_2(\hat{\lambda}XY) = \delta(pw_2(\lambda XY) + (1-p)\underline{w}_2(\lambda X))$. The payoff player 1 gets if this offer is accepted is $\lambda - \hat{\lambda} - \underline{x}_2 + \alpha w_1(\hat{\lambda}XY)$. Therefore,

$$\begin{aligned} r\overline{v}_1(1, \lambda X) &\leq r(\lambda - \hat{\lambda} - \underline{x}_2 + \alpha w_1(\hat{\lambda}XY)) \\ &= r(\lambda - \hat{\lambda}) - \delta(pw_2(\lambda XY) + (1-p)\underline{w}_2(\lambda X)) + \alpha(rw_1(\hat{\lambda}XY) + w_2(\hat{\lambda}XY)) \end{aligned}$$

which establishes (4) since $rw_1(\hat{\lambda}XY) + w_2(\hat{\lambda}XY) = W(\hat{\lambda}XY)$. □

Lemma 5. *Let $\hat{\phi} > 0$ and $\lambda > \hat{\lambda}$. For each $j = 1, 2$, all SPE starting at state $(j, \lambda X)$ are payoff equivalent. Moreover, in every SPE the players reach a partial agreement, consuming a total fraction $\lambda - \hat{\lambda}$ of pie X.*

Proof. The inequalities stated in Lemma 4 imply

$$\begin{aligned} r\overline{w}_1(\lambda X) \leq &\frac{1}{2}\left(r(\lambda - \hat{\lambda}) + \alpha W(\hat{\lambda}XY) - \delta\left(pw_2(\lambda XY) + (1-p)\underline{w}_2(\lambda X)\right)\right) \\ &+ \frac{1}{2} r\delta\left(pw_1(\lambda XY) + (1-p)\overline{w}_1(\lambda X)\right) \\ r\underline{w}_1(\lambda X) \geq &\frac{1}{2}\left(r(\lambda - \hat{\lambda}) + \alpha W(\hat{\lambda}XY) - \delta\left(pw_2(\lambda XY) + (1-p)\overline{w}_2(\lambda X)\right)\right) \\ &+ \frac{1}{2} r\delta\left(pw_1(\lambda XY) + (1-p)\underline{w}_1(\lambda X)\right). \end{aligned}$$

These in turn imply

$$r\left(\overline{w}_1(\lambda X) - \underline{w}_1(\lambda X)\right) \leq \frac{\delta(1-p)\left(\overline{w}_2(\lambda X) - \underline{w}_2(\lambda X)\right)}{2 - \delta(1-p)}. \tag{C2}$$

Similarly, for player 2 we get

$$\overline{w}_2(\lambda X) - \underline{w}_2(\lambda X) \leq \frac{r\delta(1-p)\left(\overline{w}_1(\lambda X) - \underline{w}_1(\lambda X)\right)}{2 - \delta(1-p)}. \tag{C3}$$

Combining (C2) and (C3) yields

$$\overline{w}_1(\lambda X) - \underline{w}_1(\lambda X) \leq \left(\frac{\delta\,(1-p)}{2 - \delta(1-p)}\right)^2 \left(\overline{w}_1(\lambda X) - \underline{w}_1(\lambda X)\right),$$

which implies $\overline{w}_1(\lambda X) = \underline{w}_1(\lambda X) = w_1(\lambda X)$. Then, (C3) implies $\overline{w}_2(\lambda X) = \underline{w}_2(\lambda X) \equiv w_2(\lambda X)$, so $\overline{v}_i(j, \lambda X) = \underline{v}_i(j, \lambda X) \equiv v_i(j, \lambda X)$ for $i, j = 1, 2$, and the SPE payoffs are unique. Substituting back, we find that for $i \neq j$

$$\begin{aligned} v_i(j, \lambda X) &= \delta\left(pw_i(\lambda XY) + (1-p)w_i(\lambda X)\right) \\ rv_1(1, \lambda X) &= r(\lambda - \hat{\lambda}) + \alpha W(\hat{\lambda} XY) - v_2(1, \lambda X) \\ v_2(2, \lambda X) &= r(\lambda - \hat{\lambda}) + \alpha W(\hat{\lambda} XY) - rv_1(2, \lambda X). \end{aligned}$$

Finally, note that these payoffs can only be supported by a strategy profile in which the proposer offers to consume a total fraction $\lambda - \hat{\lambda}$ of pie X and the responder accepts. □

Lemma 6. *Let $\hat{\phi} < 0$ and $\lambda \in [0, 1]$. For each $j = 1, 2$, all SPE starting at state $(j, \lambda X)$ are payoff equivalent. Moreover, in every SPE the players reach a complete agreement.*

Proof. **(sketch)** Let the state be $s = \lambda X$. Using arguments similar to those in Lemma 3, one can show that when $\hat{\phi} < 0$ the proposer will find it optimal to make an offer such that the players consume all λ of pie X, regardless of whether $\lambda > \hat{\lambda}$ or $\lambda \leq \hat{\lambda}$. Because players always make offers over all λ of pie X, we can again find bounds for $\underline{v}_i(j, \lambda X)$ and $\overline{v}_i(j, \lambda X)$ as in Lemma 4, and then use an argument similar to the one in Lemma 5 to establish the uniqueness of equilibrium payoffs. □

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