Online Appendix:
The Value of Privacy in Cartels: An Analysis of the Inner Workings of a Bidding Ring

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Abstract

This Online Appendix to "The Value of Privacy in Cartels: An Analysis of the Inner Workings of a Bidding Ring" provides proofs, extensions, robustness checks, and excerpts from the original ruling and their translations. Section OA collects all proofs that are not in the main Appendix. Section OB provides an extension of our baseline model with bid preparation costs and procurement costs that are increasing in backlog. Section OC presents robustness checks to our empirical exercise. Section OD collects excerpts from the original ruling and their translations.

KEYWORDS: procurement, collusion, bidding ring, cartel, privacy.

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OA Proofs of Corollary 1 and Propositions 3 and 4

Proof of Corollary 1. From Proposition 2, we have that $\underline{b}(r) = \min\{r, c(r) + \delta^{n-1}\overline{W}\}$, $\overline{b}(r) = \min\{r, c(r) + \delta^{n-1}(1+\delta)\overline{W}\}$ and $m(r) \equiv \overline{b}(r) - \underline{b}(r) = \max\{0, \min\{r - c(r) - \delta^{n-1}\overline{W}, \delta^n\overline{W}\}\}$, which are all increasing in r.

Let $r_1 = \inf\{r : r \geq c(r) + \delta^{n-1}\overline{W}\}$ and $r_2 = \inf\{r : r \geq c(r) + \delta^{n-1}\overline{W}(1+\delta)\}$. Then, we have that

$$\mathbb{E}_{F^*(\cdot;r)}[b] = \begin{cases} r & \text{if } r \leq r_1 \\ c(r) + \delta^{n-1}\overline{W}\left[1 + \log\left(\frac{r-c(r)}{\delta^{n-1}\overline{W}}\right)\right] & \text{if } r \in (r_1, r_2] \\ c(r) + \delta^{n-1}\overline{W}\left[1 + \log(1+\delta)\right] & \text{if } r > r_2, \end{cases}$$

which is also increasing in r.

Lastly, we show that $\overline{b}(r) - \mathbb{E}_{F^*(\cdot;r)}[b]$ is increasing in r. Note that $\overline{b}(r) - \mathbb{E}_{F^*(\cdot;r)}[b] = 0$ for all $r \leq r_1$. For all $r \in (r_1, r_2]$, we have that

$$\begin{aligned} \overline{b}(r) - \mathbb{E}_{F^*(\cdot;r)}[b] &= r - c(r) - \delta^{n-1}\overline{W}\left[1 + \log\left(\frac{r - c(r)}{\delta^{n-1}\overline{W}}\right)\right] \\ \implies \frac{\partial \overline{b}(r) - \mathbb{E}_{F^*(\cdot;r)}[b]}{\partial r} &= (1 - c'(r))\left(1 - \frac{\delta^{n-1}\overline{W}}{r - c(r)}\right) > 0, \end{aligned}$$

where the inequality follows since $r - c(r)$ is increasing and $\delta^{n-1}\overline{W} < r - c(r)$ for all $r > r_1$. Lastly, for all $r > r_2$, we have that

$$\overline{b}(r) - \mathbb{E}_{F^*(\cdot;r)}[b] = \delta^{n-1}\overline{W}(\delta - \log(1+\delta)),$$

which is constant in r. Hence, $\overline{b}(r) - \mathbb{E}_{F^*(\cdot;r)}[b]$ is increasing in r. This completes the proof.

■

Proof of Proposition 3. Suppose only the winning bid and the winner's identity are made public. Hence, the period-t public history is now $h_t^0 = (b_s^w, i_s^w, r_s)_{s<t}$, where b_s^w is the winning bid and i_s^w is the winner's identity at time s. Similarly, the mediator's period-t history is now

$h_{M,t} = (\widehat{\mathbf{b}}_s, b_s^w, i_s^w, r_s)_{s<t} \sqcup r_t$, and bidder i's period-t history is $h_{i,t} = (\widehat{b}_{i,s}, b_{i,s}, b_s^w, i_s^w, r_s)_{s<t} \sqcup (r_t, \widehat{b}_{i,t})$.

Consider first optimal equilibria in Σ_{ck}. Let $\overline{V}^w$ denote the largest payoff that the designated winner at $t = 0$ obtains under an equilibrium in Σ_{ck}. By the same arguments as in the proof of Proposition 1, we have that $\overline{V}_{\mathsf{ck}} \leq \sum_i \delta^{i-1}\overline{V}^w$.

Fix $(\sigma, \mu) \in \Sigma_{\mathsf{ck}}$, and let $b(r_0)$ denote the winning bid at $t = 0$ under σ. Note that $b(r_0) \leq c(r_0)+\delta^{n-1}\overline{V}^w$. Indeed, the continuation payoff of the loser who will be winner at period $n-1$ can't be larger than $\delta^{n-1}\overline{V}^w$. If $b(r_0) > c(r_0)+\delta^{n-1}\overline{V}^w$, this bidder would find it profitable to undercut $b(r_0)$. Since $b(r_0)$ must be lower than r_0, $b(r_0) \leq \min\{r_0, c(r_0)+\delta^{n-1}\overline{V}^w\}$. Hence, we have that $\overline{V}^w \leq \mathbb{E}_{F_r}[\min\{r, c(r)+\delta^{n-1}\overline{V}^w\}]+\delta^n\overline{V}^w$, or $\overline{V}^w \leq \frac{1}{1-\delta^n}\mathbb{E}_{F_r}[\min\{r, c(r)+\delta^{n-1}\overline{V}^w\}]$. Hence, $\overline{V}^w \leq \overline{W}_{\mathsf{ck}}$, where $\overline{W}_{\mathsf{ck}}$ is the largest solution to equation (5) in the paper's Appendix.

To see that $\overline{V}^w = \overline{W}_{\mathsf{ck}}$, consider the following strategy profile. Along the equilibrium path, at each period t bidder i with $(t + 1) \bmod n = i$ bids $b_{i,t} = \min\{r_t, c(r_t) + \delta^{n-1}\overline{W}_{\mathsf{ck}}\}$. All other bidders bid $\min\{r_t, c(r_t) + \delta^{n-1}\overline{W}_{\mathsf{ck}}\} + \epsilon$ for some $\epsilon \in (0, \delta^n\overline{W}_{\mathsf{ck}})$. If the winning bid at any time t is different from $\min\{r_t, c(r_t) + \delta^{n-1}\overline{W}_{\mathsf{ck}}\}$, or if the winner's identity is different from i such that $(t + 1) \bmod n = i$, play reverts to Bertrand-Nash.[1] Note that no player gains by deviating. Indeed, a designated loser who will be the winner in $k \leq n - 1$ periods obtains a payoff of $\delta^k\overline{W}_{\mathsf{ck}}$ by placing a losing bid. This bidder would earn $\min\{r_t - c(r_t), \delta^{n-1}\overline{W}_{\mathsf{ck}}\} \leq \delta^k\overline{W}_{\mathsf{ck}}$ by undercutting the winning bid, and would earn $\frac{1}{2}(\min\{r_t - c(r_t), \delta^{n-1}\overline{W}_{\mathsf{ck}}\} + \delta 0) + \frac{1}{2}\delta^k\overline{W}_{\mathsf{ck}} \leq \delta^k\overline{W}_{\mathsf{ck}}$ by placing a bid equal to $\min\{r_t, c(r_t) + \delta^{n-1}\overline{W}_{\mathsf{ck}}\}$. Moreover, the designated winner does not gain by increasing her bid either: if $c(r_t) + \delta^{n-1}\overline{W}_{\mathsf{ck}} < r_t$, her payoff from bidding $b' \in (c(r_t) + \delta^{n-1}\overline{W}_{\mathsf{ck}}, c(r_t) + \delta^{n-1}\overline{W}_{\mathsf{ck}} + \epsilon)$ is at most $\delta^{n-1}\overline{W}_{\mathsf{ck}} + \epsilon + \delta \times 0 < \delta^{n-1}\overline{W}_{\mathsf{ck}} + \delta^n\overline{W}_{\mathsf{ck}}$, while her payoff from following her strategy is $\delta^{n-1}\overline{W}_{\mathsf{ck}} + \delta^n\overline{W}_{\mathsf{ck}}$. Hence, this strategy profile is an equilibrium in Σ_{ck}, giving the winner at $t = 0$ an expected discounted payoff of $\overline{W}_{\mathsf{ck}}$. Therefore, we have that $\overline{V}^w = \overline{W}_{\mathsf{ck}}$. Since $\overline{V}_{\mathsf{ck}} \leq \sum_i \delta^{i-1}\overline{V}^w = \sum_i \delta^{i-1}\overline{W}_{\mathsf{ck}}$, this equilibrium attains $\overline{V}_{\mathsf{ck}}$.

Consider next optimal equilibria in Σ. Let $\hat{V}^w$ denote the largest expected continuation

[1] As in the proof of Proposition 1, we don't specify the mediator's strategy or bidders' beliefs, since under the proposed strategy profile bidders' bids depend only on the public history.

payoff that the designated winner at $t = 0$ can obtain under an equilibrium in Σ, and note that $\overline{V} \leq \sum_i \delta^{i-1}\hat{V}^w$.

Fix $(\sigma, \mu) \in \Sigma$, and let $F(b; r_0)$ be the c.d.f. from which the winning bid at $t = 0$ is drawn under σ. Let $\bar{b}$ denote the highest point in the support of $F(b; r_0)$. Then, for the designated loser who will be winner at period $t = n - 1$ not to want to deviate, it must be that

$$\forall b < \bar{b}, \quad (1 - F(b; r_0))(b - c(r) + \delta \times 0) + F(b; r_0)\delta^{n-1}\hat{V}^w \leq \delta^{n-1}\hat{V}^w$$
$$\iff b \leq c(r) + \delta^{n-1}\hat{V}^w, \tag{O1}$$

where the inequality follows since $\delta^{n-1}\hat{V}^w$ is the largest expected continuation payoff that the designated loser who will be winner at $t = n-1$ can obtain under a bid rotation equilibrium, and since the deviation of this bidder goes undetected if she doesn't win the auction.

Since (O1) holds for all $b < \bar{b}$, it follows that $\bar{b} \leq c(r) + \delta^{n-1}\hat{V}^w$. Since $\bar{b} \leq r_0$, we have that $\hat{V}^w \leq \mathbb{E}_{F_r}[\min\{r, c(r) + \delta^{n-1}\hat{V}^w\}] + \delta^n\hat{V}^w$, or $\hat{V}^w \leq \frac{1}{1-\delta^n}\mathbb{E}_{F_r}[\min\{r, c(r) + \delta^{n-1}\hat{V}^w\}]$. Hence, we have that $\hat{V}^w \leq \overline{W}_{\mathsf{ck}}$, where $\overline{W}_{\mathsf{ck}}$ is the largest $W \geq 0$ that solves equation (5) in the paper's Appendix. Hence, $\overline{V} \leq \sum_i \delta^{i-1}\overline{W}_{\mathsf{ck}} = \overline{V}_{\mathsf{ck}}$. Since $\overline{V} \geq \overline{V}_{\mathsf{ck}}$ (since $\Sigma_{\mathsf{ck}} \subset \Sigma$), it follows that $\overline{V} = \overline{V}_{\mathsf{ck}}$. ■

Proof of Proposition 4. We first characterize equilibria in Σ^*_{ck} attaining V^*_{ck}. Fix an equilibrium $(\sigma, \mu) \in \Sigma^*_{\mathsf{ck}}$. Let $b(r_0)$ denote the winning bid at time $t = 0$ under (σ, μ), and let $\{V_j(r_0)\}_{j \in N}$ be the bidders' on-path expected continuation payoffs under (σ, μ). Then, for all $j \in N$, we must have

$$x_j(b(r_0) - c(r_0)) + \delta V_j(r_0) \geq b(r_0) - c(r_0) \iff (1 - x_j)(b(r_0) - c(r_0)) \leq \delta V_j(r_0), \tag{O2}$$

where $x_j \in [0, 1]$ is the probability with which j wins the auction at time $t = 0$ when the reserve price is r_0. Since (O2) holds for all $j \in N$, and since $\sum_j x_j \leq 1$, we have that

$$(n - 1)(b(r_0) - c(r_0)) \leq \sum_{j \in N}(1 - x_j)(b(r_0) - c(r_0)) \leq \delta \sum_{j \in N} V_j(r_0) \leq \delta V^*_{\mathsf{ck}}, \tag{O3}$$

where the last inequality follows from the definition of V^*_{ck}. Let V denote the cartel's expected payoff at the start of the game under (σ, μ). Then, (O3) implies

$$V \leq \mathbb{E}_{F_r}\left[\min\left\{r - c(r), \frac{1}{n-1}\delta V^*_{\mathsf{ck}}\right\}\right] + \delta V^*_{\mathsf{ck}}.$$

Since the inequality above holds for all $(\sigma, \mu) \in \Sigma^*_{\mathsf{ck}}$,

$$\begin{aligned} & V^*_{\mathsf{ck}} \leq \mathbb{E}_{F_r}\left[\min\left\{r - c(r), \frac{1}{n-1}\delta V^*_{\mathsf{ck}}\right\}\right] + \delta V^*_{\mathsf{ck}} \\ \Longleftrightarrow\ & V^*_{\mathsf{ck}} \leq \frac{1}{1-\delta}\mathbb{E}_{F_r}\left[\min\left\{r - c(r), \frac{1}{n-1}\delta V^*_{\mathsf{ck}}\right\}\right]. \end{aligned}$$

Let W^* denote the highest value of $V \geq 0$ solving

$$V = \frac{1}{1-\delta}\mathbb{E}_{F_r}\left[\min\left\{r - c(r), \frac{1}{n-1}\delta V\right\}\right],$$

Note that $V^*_{\mathsf{ck}} \leq W^*$. We now show that $V^*_{\mathsf{ck}} = W^*$. Consider a strategy profile such that, at all on path histories, all bidders place bid $b_t = \min\left\{r_t, c(r_t) + \frac{1}{n-1}\delta W^*\right\}$, and they each win with probability $1/n$. Any deviation is punished by Nash reversion. One can check that this strategy profile is indeed an equilibrium with common-knowledge bids, and gives the cartel a payoff of W^*. Hence, $V^*_{\mathsf{ck}} = W^*$.

Note that equilibria attaining V^*_{ck} achieve perfect collusion when $\overline{r} \leq c(\overline{r}) + \frac{1}{n-1}\delta V^*_{\mathsf{ck}}$. Hence, $V^* = V^*_{\mathsf{ck}}$ when $\overline{r} \leq c(\overline{r}) + \frac{1}{n-1}\delta V^*_{\mathsf{ck}}$. We now show that $V^* > V^*_{\mathsf{ck}}$ when $\overline{r} > c(\overline{r}) + \frac{1}{n-1}\delta V^*_{\mathsf{ck}}$.

Suppose $\overline{r} > c(\overline{r}) + \frac{1}{n-1}\delta V^*_{\mathsf{ck}}$. Consider the following strategy profile. At $t = 0$, if $r_0 \leq c(r_0) + \frac{1}{n-1}\delta V^*_{\mathsf{ck}}$, the mediator recommends each bidder to bid $\widehat{b}_i = r_0$. If $r_0 > c(r_0) + \frac{1}{n-1}\delta V^*_{\mathsf{ck}}$, the mediator draws each bidder i's recommendation independently from a distribution that places probability $\gamma = \left(\frac{1}{n-1}\frac{\delta V^*_{\mathsf{ck}}}{r_0 - c(r_0)}\right)^{\frac{1}{n-1}}$ on $\widehat{b}_i = r_0$, and places probability $1-\gamma$ on $\widehat{b}_i = c(r_0) + \frac{1}{n-1}\delta V^*_{\mathsf{ck}}$. If all bidders follow the mediator's recommendation, then from $t = 1$ onwards they play an equilibrium attaining V^*_{ck} (which gives each bidder a discounted continuation payoff of $\delta V^*_{\mathsf{ck}}/n$). Otherwise, if a bidder deviates at $t = 0$, from $t = 1$ onwards the mediator sends bidding recommendations $\widehat{b}_{i,t} = c(r_t)$ to all players, and players follow these recommendations (i.e., they play Bertrand-Nash).

Clearly, the sum of players' payoffs under this strategy profile is strictly larger than V^*_{ck}.[2] We now show that this strategy profile (along with beliefs μ derived from the mediator's strategy) is an equilibrium. Note first that no bidder has an incentive to deviate at $t = 0$ when $r_0 \le c(\overline{r}) + \frac{1}{n-1}\delta V^*_{\mathsf{ck}}$, and no bidder has an incentive to deviate at any $t > 0$.

Consider next $r_0 > c(\overline{r}) + \frac{1}{n-1}\delta V^*_{\mathsf{ck}}$. If bidder i gets bid recommendation $\widehat{b}_i = r_0$ and follows this recommendation, she obtains a payoff equal to

$$\mathsf{prob}(i\ wins|\widehat{b}_i = r_0) \times (r_0 - c(r_0)) + \delta\frac{1}{n}V^*_{\mathsf{ck}} = \gamma^{n-1}\frac{1}{n}(r_0 - c(r_0)) + \delta\frac{1}{n}V^*_{\mathsf{ck}} = \frac{1}{n-1}\delta V^*_{\mathsf{ck}}.$$

Indeed, she wins with probability $1/n$ if all of her opponents are recommended to bid r_0, and wins with probability 0 if at least one of her opponents is recommended to bid $c(r_0)+\frac{1}{n-1}\delta V^*_{\mathsf{ck}}$. The optimal deviation is either to undercut bid r_0, or to undercut bid $c(r_0) + \frac{1}{n-1}\delta V^*_{\mathsf{ck}}$. Her payoff from undercutting r_0 is $\gamma^{n-1}(r_0 - c(r_0)) + \delta 0 = \frac{1}{n-1}\delta V^*_{\mathsf{ck}}$, while her payoff from undercutting $c(r_0) + \frac{1}{n-1}\delta V^*_{\mathsf{ck}}$ is $\frac{1}{n-1}\delta V^*_{\mathsf{ck}} + \delta 0$. Hence, bidders don't gain from deviating at $t = 0$ when recommended to bid $\widehat{b}_i = r_0$.

If instead bidder i gets recommendation $\widehat{b}_i = c(r_0) + \frac{1}{n-1}\delta V^*_{\mathsf{ck}}$, by following her recommendation she obtains a payoff equal to

$$\mathsf{prob}\left(i\ wins|\widehat{b}_i = c(r_0) + \frac{1}{n-1}\delta V^*_{\mathsf{ck}}\right) \times \frac{1}{n-1}\delta V^*_{\mathsf{ck}} + \delta\frac{1}{n}V^*_{\mathsf{ck}}.$$

Note that since the strategy profile is symmetric, each bidder wins with probability $1/n$. Hence, we have that

$$\frac{1}{n} = \gamma \times \mathsf{prob}\left(i\ wins|\widehat{b}_i = r_0\right) + (1-\gamma) \times \mathsf{prob}\left(i\ wins|\widehat{b}_i = c(r_0) + \frac{1}{n-1}\delta V^*_{\mathsf{ck}}\right).$$

[2] Indeed, whenever $r_0 > c(r_0) + \frac{1}{n-1}\delta V^*_{\mathsf{ck}}$, the winning bid under this strategy profile is equal to r_0 with probability γ^n and is equal to $c(r_0) + \frac{1}{n-1}\delta V^*_{\mathsf{ck}}$ with probability $1-\gamma^n$. Instead, under an equilibrium in Σ^*_{ck} attaining V^*_{ck}, the winning bid is equal to $c(r_0) + \frac{1}{n-1}\delta V^*_{\mathsf{ck}}$ with probability 1 whenever $r_0 > c(r_0) + \frac{1}{n-1}\delta V^*_{\mathsf{ck}}$.

Using $\mathsf{prob}(i\ wins|\widehat{b}_i = r_0) = \frac{1}{n}\gamma^{n-1}$, we get that

$$\mathsf{prob}\left(i\ wins|\widehat{b}_i = c(r_0) + \frac{1}{n-1}\delta V^*_{\mathsf{ck}}\right) = \frac{1}{n}\frac{1-\gamma^n}{1-\gamma}.$$

Therefore, the payoff that bidder i gets by following recommendation $\widehat{b}_i = c(r_0) + \frac{1}{n-1}\delta V^*_{\mathsf{ck}}$ is equal to

$$\frac{1}{n-1}\delta V^*_{\mathsf{ck}}\frac{1}{n}\frac{1-\gamma^n}{1-\gamma} + \delta\frac{1}{n}V^*_{\mathsf{ck}} = \frac{1}{n-1}\delta V^*_{\mathsf{ck}}\left(\frac{n-1}{n} + \frac{1}{n}\frac{1-\gamma^n}{1-\gamma}\right) > \frac{1}{n-1}\delta V^*_{\mathsf{ck}},$$

where the strict inequality follows since $\frac{1-\gamma^n}{1-\gamma} > 1$ (which, in turn, follows since $\gamma < 1$). Undercutting bid $c(r_0) + \frac{1}{n-1}\delta V^*_{\mathsf{ck}}$ gives her a payoff of $\frac{1}{n-1}\delta V^*_{\mathsf{ck}} + \delta 0$, while undercutting bid r_0 gives her a payoff $\gamma^{n-1}(r_0 - c(r_0)) + \delta 0 = \frac{1}{n-1}\delta V^*_{\mathsf{ck}}$. Hence, bidders don't gain from deviating at $t = 0$ when recommended to bid $\widehat{b}_i = c(r_0) + \frac{1}{n-1}\delta V^*_{\mathsf{ck}}$, and so this strategy profile (along with beliefs μ derived from the mediator's strategy) is an equilibrium.

The arguments above show that $V^* > V^*_{\mathsf{ck}} \iff \overline{r} > c(\overline{r}) + \frac{1}{n-1}\delta V^*_{\mathsf{ck}}$. We complete the proof by showing that $\overline{r} > c(\overline{r}) + \frac{1}{n-1}\delta V^*_{\mathsf{ck}} \iff \overline{r} > c(\overline{r}) + \frac{1}{n-1}\frac{\delta}{1-\delta}\mathbb{E}_{F_r}[r - c(r)]$. Since $V^*_{\mathsf{ck}} \leq \frac{1}{1-\delta}\mathbb{E}_{F_r}[r - c(r)]$, $\overline{r} > c(\overline{r}) + \frac{1}{n-1}\frac{\delta}{1-\delta}\mathbb{E}_{F_r}[r - c(r)] \implies \overline{r} > c(\overline{r}) + \frac{1}{n-1}\delta V^*_{\mathsf{ck}}$. Next, we show that $c(\overline{r}) + \frac{1}{n-1}\frac{\delta}{1-\delta}\mathbb{E}_{F_r}[r - c(r)] \geq \overline{r} \implies c(\overline{r}) + \frac{1}{n-1}\delta \overline{V}^*_{\mathsf{ck}} \geq \overline{r}$ (the contrapositive of $c(\overline{r}) + \frac{1}{n-1}\delta \overline{V}^*_{\mathsf{ck}} < \overline{r} \implies c(\overline{r}) + \frac{1}{n-1}\frac{\delta}{1-\delta}\mathbb{E}_{F_r}[r - c(r)] < \overline{r}$).

Suppose $c(\overline{r}) + \frac{1}{n-1}\frac{\delta}{1-\delta}\mathbb{E}_{F_r}[r - c(r)] \geq \overline{r}$ and consider the following strategy profile. On the equilibrium path, at each time t each bidder i submits bid $b_{i,t} = r_t$. Defections are punished by Nash reversion. At each period t, the expected payoff of each bidder i under this strategy profile is

$$\frac{1}{n}(r_t - c(r_t)) + \frac{1}{n}\frac{\delta}{1-\delta}\mathbb{E}_{F_r}[r - c(r)],$$

while her payoff from undercutting r_t is

$$r_t - c(r_t) + \delta 0 \leq \frac{1}{n}(r_t - c(r_t)) + \frac{1}{n}\frac{\delta}{1-\delta}\mathbb{E}_{F_r}[r - c(r)],$$

where the inequality follows since $c(\overline{r}) + \frac{1}{n-1}\frac{\delta}{1-\delta}\mathbb{E}_{F_r}[r - c(r)] \geq \overline{r}$ and since $r - c(r)$ is increasing in r. Hence, this strategy profile is an equilibrium, and so $\overline{V}^*_{\mathsf{ck}} = \frac{\mathbb{E}_{F_r}[r-c(r)]}{1-\delta}$ whenever $c(\overline{r})+\frac{1}{n-1}\frac{\delta}{1-\delta}\mathbb{E}_{F_r}[r-c(r)] \geq \overline{r}$. Therefore, $c(\overline{r})+\frac{1}{n-1}\frac{\delta}{1-\delta}\mathbb{E}_{F_r}[r-c(r)] \geq \overline{r} \Longrightarrow c(\overline{r})+\frac{1}{n-1}\delta\overline{V}^*_{\mathsf{ck}} \geq \overline{r}$. This completes the proof. ■

OB A Richer Model

In this Appendix we study a richer model in which firms face increasing marginal costs and bid preparation costs. We show that, in this richer model, bid rotation can be optimal. To simplify the exposition, we assume that there are two bidders; i.e., $N = \{1, 2\}$.

As in the model in Section 2, bidders in $N = \{1, 2\}$ participate at each time $t \in \mathbb{N}_0$ in a first-price procurement auction with public reserve price r_t. Reserve price r_t is drawn i.i.d. over time from c.d.f. F_r, with density f_r and support $[\underline{r}, \overline{r}]$ with $0 < \underline{r} < \overline{r} < \infty$. Firms' costs are increasing in their backlog: at each time $t \geq 1$, the bidder who won the auction at $t-1$ has procurement cost $\overline{c}(r_t)$, while the bidder who lost the auction at $t-1$ has procurement cost $\underline{c}(r_t) \leq \overline{c}(r_t)$. At $t = 0$, bidder 1 has cost $\underline{c}(r_0)$ and bidder 2 has cost $\overline{c}(r_0)$. We assume that $\underline{c}(\cdot)$, $\overline{c}(\cdot)$, $r - \underline{c}(r)$ and $r - \overline{c}(r)$ are all increasing in r, with $\underline{r} - \overline{c}(\underline{r}) > 0$.

The rules of the auction at each time t are as follows. Each bidder $i \in N$ submits bid $b_i \in \mathbb{R}_0$, and the firm with the lowest bid wins. In case of ties, we assume that the lowest cost bidder wins the auction. This tie-breaking rule greatly simplifies the exposition, but is not needed: our results below continue to hold if ties are broken randomly.[3]

Assumption O1 (increasing marginal costs)**.** *There exists $\Delta > 0$ such that, for all $r \in$ supp F_r, $\overline{c}(r) - \underline{c}(r) \geq \Delta$.*

In addition, we assume that firms face strictly positive bid preparation costs when submitting a bid that has positive probability of winning.

[3] As we argue below, with this tie-breaking rule the stage game of the repeated game has an equilibrium delivering zero payoff to both bidders. If instead we assumed random tie-breaking, for any $\epsilon > 0$ small the stage game would have an equilibrium giving the lowest cost firm a payoff of ϵ and the other firm a payoff of zero – see footnote 4.

Definition O1. *Fix a strategy profile σ and beliefs μ. We say that bid b is a* serious bid *for firm i at history $h_{i,t}$ under (σ, μ) if $\textsf{prob}_{\sigma,\mu}(i \text{ wins } | h_{i,t}, b_{i,t} = b) > 0$.*

Assumption O2 (bid preparation costs)**.** *Fix a strategy profile σ and beliefs μ. Bidder i incurs a bid preparation cost $\kappa > 0$ from submitting bid b at history $h_{i,t}$ if and only if b is a serious bid. Cost $\kappa > 0$ is such that, for all $r \in \textsf{supp}\, F_r$, $r - \overline{c}(r) - \kappa > 0$.*

Hence, bidders incur bid preparation costs if and only if their bids are serious.

Recall that Σ and Σ^* are, respectively, the set of bid rotation equilibria and the set of all equilibria. Recall also that

$$\overline{V} = \sup_{(\sigma,\mu)\in\Sigma} \sum_{i\in N} V_i(\sigma, \mu), \text{ and } V^* = \sup_{(\sigma,\mu)\in\Sigma^*} \sum_{i\in N} V_i(\sigma, \mu)$$

Since $\Sigma \subset \Sigma^*$, we have that $V^* \geq \overline{V}$.

Consider first our stage game, with firm i having procurement cost $\underline{c}(r)$ and firm $j \neq i$ having procurement cost $\overline{c}(r) > \underline{c}(r)$. We note that this stage game admits a Nash equilibrium in which both firms submit a bid equal to $\underline{c}(r) + \kappa$, and (because of our tie-breaking rule) the low cost bidder wins with probability 1. Hence, only the low cost bidder's bid is serious. Note that both firms earn a payoff of zero under this equilibrium.[4]

We first show that, for large δ, every optimal equilibrium is a bid rotation equilibrium.

Proposition O1. *Suppose Assumptions O1 and O2 hold. There exists $\delta_{\textsf{opt}} < 1$ such that, for all $\delta \in [\delta_{\textsf{opt}}, 1)$, $\overline{V} = V^*$.*

Proof. Consider the following strategy profile. Along the path of play, at each $t \in \mathbb{N}_0$ the bidder with the lowest cost bids r_t, and the other bidder bids $r_t + \eta$ for some $\eta > 0$. If at any period t a bidder deviates, then from $t + 1$ onwards bidders play at each period a static

[4]If instead we assumed random tie-breaking, for any $\epsilon \in (0, \Delta)$ the stage game would admit a Nash equilibrium in which the low cost firm earns a payoff of ϵ, and the other firm earns a payoff of zero. Under this equilibrium, firm i submits a serious bid equal to $b_i = \underline{c}(r) + \kappa + \epsilon$, and firm j randomizes her (non-serious) bid uniformly over the open interval $(b_i, b_i + \eta)$ for some $\eta > 0$ (note that b_j is always strictly above b_i). One can check that, when $\eta > 0$ is small enough, this constitutes a Nash equilibrium: player i does not gain by increasing her bid above b_i, as this would substantially lower her likelihood of winning, and bidder j does not gain from undercutting b_i, as this would lead to negative payoffs.

Nash equilibrium that gives both players a payoff of zero (by our discussion above, such an equilibrium always exists). We now show that there exists $\delta_{\mathsf{opt}} < 1$ such that this strategy profile is an equilibrium for all $\delta \geq \delta_{\mathsf{opt}}$.

The continuation payoff that the lowest cost bidder gets at time t (before learning r_t) under this strategy profile is $W_{\mathsf{opt}} \equiv \frac{1}{1-\delta^2}\mathbb{E}_{F_r}[r - \underline{c}(r) - \kappa]$. The sum of firms' payoffs under this strategy profile is $(1+\delta)W_{\mathsf{opt}} = \frac{1}{1-\delta}\mathbb{E}_{F_r}[r - \underline{c}(r) - \kappa]$, which is an upper bound to total payoffs under any equilibrium in Σ^*.

Note that, at any on-path history $h^0_{t+} = h^0_t \sqcup r_t$, the bidder with the lowest cost does not gain from deviating. The bidder with the highest cost gets δW_{opt} from playing her equilibrium action, and gets at $r_t - \overline{c}(r_t) - \kappa$ from undercutting the winning bid (she gets $r_t - \overline{c}(r_t) - \kappa$ at t, and gets 0 from then on).

Let $\delta_{\mathsf{opt}} < 1$ be the value of δ such that $\delta W_{\mathsf{opt}} = \overline{r} - \overline{c}(\overline{r}) - \kappa$ (since $\overline{r} < \infty$, such a δ_{opt} exists). Then, for all $\delta \geq \delta_{\mathsf{opt}}$, we have that $\delta W_{\mathsf{opt}} \geq \overline{r} - \overline{c}(\overline{r}) - \kappa$, so the highest cost bidder does not have an incentive to defect either. Hence, for $\delta \geq \delta_{\mathsf{opt}}$, this strategy profile is a bid rotation equilibrium. Since this strategy profile achieves optimal cartel payoffs $\frac{1}{1-\delta}\mathbb{E}_{F_r}[r - \underline{c}(r) - \kappa]$, we have that $\overline{V} = V^*$ for all $\delta \geq \delta_{\mathsf{opt}}$. □

Let W_{br} be the largest value of $W \geq 0$ solving

$$W = \frac{1}{1-\delta^2}\mathbb{E}_{F_r}[\min\{r - \underline{c}(r) - \kappa, \overline{c}(r) + \delta W - \underline{c}(r)\}].$$

Note that W_{br} is continuous and increasing in δ, with $W_{\mathsf{br}} \nearrow W_{\mathsf{opt}}$ as $\delta \nearrow \delta_{\mathsf{opt}}$.[5]

Fix $\delta > 0$, and consider the following strategy profile. Along the path of play, at each history $h^0_{t+} = h^0_t \sqcup r_t$ the bidder with the lowest cost bids $b_t = \min\{r_t, \overline{c}(r_t) + \delta W_{\mathsf{br}} + \kappa\}$, and the other bidder bids $b_t + \eta$, with $\eta \in (0, \delta^2 W_{\mathsf{br}})$. Following a deviation, bidders play a continuation equilibrium in which, at each time t, both bidders earn 0. The continuation payoff that the lowest cost bidder gets at time t (before learning r_t) under this strategy profile is W_{br}.

[5]For all $\delta \geq \delta_{\mathsf{opt}}$, we have $\delta W_{\mathsf{opt}} \geq \overline{r} - \overline{c}(\overline{r}) - \kappa$, and so $\min\{r - \underline{c}(r) - \kappa, \overline{c}(r) + \delta W_{\mathsf{opt}} - \underline{c}(r)\} = r - \underline{c}(r) - \kappa$ for all $r \in \mathsf{supp}\, F_r$. Hence, for all $\delta \geq \delta_{\mathsf{opt}}$, $W_{\mathsf{br}} = W_{\mathsf{opt}}$.

Note that this strategy profile is a bid rotation equilibrium. Indeed, at any on-path history $h^0_{t+} = h^0_t \sqcup r_t$, the designated loser gets δW_{br} from playing according to her strategy, and gets at most δW_{br} from undercutting the winning bid. The designated winner cannot gain by deviating either. Cartel profits under this equilibrium are $V_{\mathsf{br}} \equiv (1+\delta)W_{\mathsf{br}}$.

For any $\delta < \delta_{\mathsf{opt}}$, let $r(\delta) = \sup\{r \in [\underline{r}, \overline{r}] : \overline{c}(r) + \delta W_{\mathsf{br}} + \kappa \geq r\}$. Note that, under the bid rotation equilibrium we constructed above, the winning bid is equal to the reserve price for all $r \leq r(\delta)$. Define $\tau(\delta) = \min\{t \in \mathbb{N}_0 : r_t > r(\delta)\}$ to be the first time the reserve price falls above $r(\delta)$. Since $W_{\mathsf{br}} \nearrow W_{\mathsf{opt}}$ as $\delta \nearrow \delta_{\mathsf{opt}}$, and since $\delta W_{\mathsf{opt}} \geq \overline{r} - \overline{c}(\overline{r}) - \kappa$ for all $\delta \geq \delta_{\mathsf{opt}}$, it follows that stopping time $\tau(\delta)$ diverges to $+\infty$ as $\delta \nearrow \delta_{\mathsf{opt}}$.

Let $\hat{\delta} < \delta_{\mathsf{opt}}$ be such that, for all $\delta \in (\hat{\delta}, \delta_{\mathsf{opt}})$,[6]

$$\min\{\kappa, \Delta\} > \delta(1+\delta)(W_{\mathsf{opt}} - W_{\mathsf{br}}). \tag{O4}$$

The following result shows that, for $\delta \in (\hat{\delta}, \delta_{\mathsf{opt}})$, any optimal equilibrium has firms rotating at all times $t < \tau(\delta)$. The proof of the result uses the equilibrium we constructed above.

Proposition O2. *Suppose Assumptions O1 and O2 hold. For all $\delta \in (\hat{\delta}, \delta_{\mathsf{opt}})$, there exists a stopping time $\tau(\delta)$ such that in any equilibrium attaining V^* the lowest cost firm wins the auction with probability 1 at all times $t < \tau(\delta)$.*

Stopping time $\tau(\delta)$ is such that $\lim_{\delta \nearrow \delta_{\mathsf{opt}}} \tau(\delta) = +\infty$.

Proof. Fix $\delta \in (\hat{\delta}, \delta_{\mathsf{opt}})$. We start by showing that, in any equilibrium attaining V^*, only one firm submits a serious bid at all on-path public histories h^0_{t+} with $t < \tau(\delta)$. To see why, fix an equilibrium under which both firms submit serious bids at some public history h^0_{t+} with $t < \tau(\delta)$. The sum of firms' continuation profits at h^0_{t+} under this equilibrium are bounded above by

$$r_t - \underline{c}(r_t) - 2\kappa + \frac{\delta}{1-\delta}\mathbb{E}_{F_r}[r - \underline{c}(r) - \kappa] = r_t - \underline{c}(r_t) - 2\kappa + \delta(1+\delta)W_{\mathsf{opt}}. \tag{O5}$$

This follows since cartel flow profits at h^0_{t+} when both firms submit serious bids are bounded

[6] Note that such $\hat{\delta}$ exists, since W_{br} converges to W_{opt} as $\delta \nearrow \delta_{\mathsf{opt}}$.

by $r_t - \underline{c}(r_t) - 2\kappa$, and since cartel continuation profits are bounded by $\frac{\delta}{1-\delta}\mathbb{E}_{F_r}[r - \underline{c}(r) - \kappa]$.

Consider the bid rotation equilibrium attaining profits $V_{\mathsf{br}} = (1+\delta)W_{\mathsf{br}}$ described above. If firms play according to this equilibrium from history h^0_{t+} onwards, they obtain a total continuation payoff equal to

$$\min\{r_t - \underline{c}(r_t) - \kappa, \delta W_{\mathsf{br}} + \overline{c}(r_t) - \underline{c}(r_t)\} + \delta(1+\delta)W_{\mathsf{br}} = r_t - \underline{c}(r_t) - \kappa + \delta(1+\delta)W_{\mathsf{br}}, \quad \text{(O6)}$$

where the equality follows since $t < \tau(\delta)$, and so $\delta W_{\mathsf{br}} \geq r_t - \overline{c}(r_t) - \kappa$. Note that (O6) is strictly larger than (O5) (by equation (O4)). This implies that that firms' profits under the original equilibrium at the start of the game are lower than V_{br}: indeed, the equilibrium attaining V_{br} has optimal cartel flow profits at all periods prior to t (since $t < \tau(\delta)$, so prices are always equal to the reserve price at $s < t$, and the lowest cost bidder procures), and attains strictly higher continuation profits at period t. Hence, in any equilibrium attaining optimal cartel profits V^*, only one firm submits a serious bid at all on-path public histories h^0_{t+} with $t < \tau(\delta)$.

Continue to assume $\delta \in (\hat{\delta}, \delta_{\mathsf{opt}})$. We now show that, in any equilibrium attaining V^*, the lowest cost firm wins the auction with probability 1 at all on-path public histories h^0_{t+} with $t < \tau(\delta)$. To see why, fix an equilibrium under which only one firm submits a serious bid at each on-path public history h^0_{t+} with $t < \tau(\delta)$ (by the arguments above, any equilibrium attaining V^* has this property). Suppose that, under this equilibrium, there exists some on-path public history h^0_{t+} with $t < \tau(\delta)$ such that the firm with the largest cost is the one that submits a serious bid. The sum of firms' continuation profits at h^0_{t+} under this equilibrium are then bounded above by

$$r_t - \overline{c}(r_t) - \kappa + \frac{\delta}{1-\delta}\mathbb{E}_{F_r}[r - \underline{c}(r) - \kappa] = r_t - \overline{c}(r_t) - \kappa + \delta(1+\delta)W_{\mathsf{opt}}. \quad \text{(O7)}$$

Consider instead the equilibrium attaining profits V_{br} described above. If firms play according to this equilibrium from history h^0_{t+} onwards, they obtain a total continuation payoff given by (O6), which is strictly larger than continuation profits in (O7) by the inequality in (O4) and by Assumption O1. This implies that firms' profits under the original equilibrium at the start of the game are strictly lower than V_{br}: indeed, the equilibrium attaining V_{br} has optimal cartel flow profits at all periods prior to t (since $t < \tau(\delta)$, so prices are always equal to the reserve price at $s < t$, and the lowest cost bidder procures), and attains strictly higher continuation profits at period t. Hence, in any equilibrium attaining V^*, only the firm with the lowest cost submits a serious bid at each on-path public history h^0_{t+} with $t < \tau(\delta)$.

Finally, by our arguments above, $\tau(\delta)$ diverges to $+\infty$ as $\delta \nearrow \delta_{\mathsf{opt}}$. This completes the proof. $\square$

We end this Appendix by noting that incentive compatibility constraints bind whenever $\delta < \delta_{\mathsf{opt}}$: indeed, cartel profits are bounded away from $\frac{1}{1-\delta}\mathbb{E}[r - \underline{c}(r) - \kappa]$ for discount factors below δ_{opt}. By using arguments similar to those in the main text, one can show that for $\delta \in (\hat{\delta}, \delta_{\mathsf{opt}})$, a cartel strictly benefits from having a mediator send randomized bidding recommendations, to relax IC constraints. Put differently, for all $\delta \in (\hat{\delta}, \delta_{\mathsf{opt}})$, $V^*_{\mathsf{ck}} < V^*$.

OC Robustness Checks

This Appendix presents calibration results for different values of the cost parameter, c_0, and the discount factor, δ. We also present calibration results from an alternative assumption on the beliefs of the bidders in which we treat the Obara Residences auction as a one-time unexpected realization.

Robustness to c_0 and δ. In the calibration exercise we report in the main text, we set the cost parameter, c_0, to 0.75 and the discount factor, δ, to 0.9. Table 1 presents the calibration results for alternative values of the cost parameter and the discount factor. In particular, we vary the cost parameter from 0.65 to 0.85 and the discount factor from 0.88 to 0.92. The first two numbers in the top row of each cell correspond to $\overline{V}_{\mathsf{ck}}$ and $\overline{V}$, the total cartel surplus under common-knowledge bids and that without common-knowledge bids. For example, at $c_0 = 0.65$ and $\delta = 0.88$, we find that $\overline{V}_{\mathsf{ck}} = 498$ million yen and $\overline{V} = 788$ million yen. The two numbers in the bottom row correspond to the range of winning bids predicted for the

		δ				
		0.88	0.89	0.90	0.91	0.92
	0.65	$498, 788$ $[505, 580]$	$601, 958$ $[523, 616]$	$738, 1148$ $[543, 656]$	$927, 1424$ $[572, 657]$	$1170, 1788$ $[611, 657]$
	0.70	$434, 675$ $[527, 591]$	$520, 812$ $[541, 620]$	$639, 984$ $[559, 656]$	$795, 1225$ $[585, 657]$	$1003, 1538$ $[618, 657]$
costs	0.75	$354, 563$ $[548, 602]$	$438, 682$ $[561, 627]$	$525, 820$ $[575, 656]$	$662, 1026$ $[597, 657]$	$836, 1287$ $[625, 657]$
	0.80	$289, 450$ $[570, 613]$	$357, 552$ $[581, 634]$	$426, 656$ $[592, 656]$	$530, 811$ $[608, 657]$	$668, 1036$ $[632, 657]$
	0.85	$225, 338$ $[592, 624]$	$260, 406$ $[599, 639]$	$328, 492$ $[608, 657]$	$397, 612$ $[621, 657]$	$501, 769$ $[638, 657]$

Table 1: Cartel Surplus and Range of Winning Bids for Obara Residences Auction, reported in millions of yen.

Obara Residences auction in the optimal equilibrium without common-knowledge bids. For example, with $c_0 = 0.65$ and $\delta = 0.88$, the range of winning bids that can be supported is between 505 and 580 million yen.

Although the values of $\overline{V}$ and $\overline{V}_{\mathsf{ck}}$ change significantly as we vary c_0 and δ, we find that the ratio $\overline{V}/\overline{V}_{\mathsf{ck}}$ remains relatively stable across parameter values, ranging from 1.5 to 1.6. In other words, the increase in cartel surplus from using mediation relative to common-knowledge bids is about 50%, regardless of the cost and discount parameters. Hence, our result regarding the value of randomized bids relative to common-knowledge bids is robust to the choice of parameter values.

In terms of the range of predicted winning bid for the Obara Residences auction, we find that the realized winning bid falls above the predicted range at discount factor 0.88. For discount factors 0.89 to 0.92, there exist cost parameters such that the realized winning bid falls within the predicted range. In particular, at $\delta = 0.90$ and 0.91, the winning bid falls within the predicted range for all cost parameters between 0.65 and 0.85.

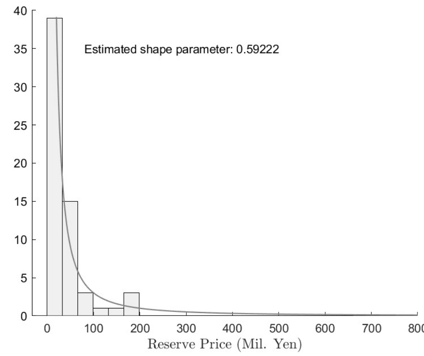

Figure 1: The bar graph corresponds to the histogram of the reserve and the curve corresponds to the estimated Pareto distribution. Obara Residences auction is removed from the sample.

Robustness to Removing Obara Residences from Estimating Sample. Obara Residences auction was somewhat unusual among auctions the cartel bid on because Tier B firms would not have been invited to bid on it without active lobbying by Mr. Kitagawa. It may then be more appropriate to treat this auction as a one-off aberrant realization and exclude this auction when estimating the distribution of the reserve price, F_r. Below, we present calibration results by estimating F_r without including the Obara Residences auction.

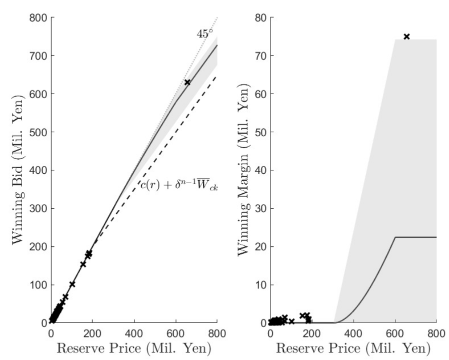

Figure 2: The left panel illustrates the relationship between the winning bid and the reserve price. The shaded region corresponds to the range of winning bids under an optimal equilibrium that attains $\overline{V}$. The dashed line corresponds to the equilibrium winning bid under common-knowledge bids. The actual realization of the winning bids are marked with **x**s. The right panel plots the winning margin (as a percent of the reserve price) against the reserve price for an optimal equilibrium that attains $\overline{V}$. The actual realizations are marked with **x**s.

Figure 1 plots the histogram of the reserve price and the density function of the estimated Pareto distribution where we have removed the Obara Residences auction from the sample.[7] Figure 2 illustrates the relationship between the reserve price and the winning bid in the left panel and the relationship between the reserve price and the winning margin in the right panel for $c_0 = 0.75$ and $\delta = 0.9$. The left panel of the figure shows that the optimal winning bid under common-knowledge bids is equal to the reserve price for r less than 198 million yen and it is less than the reserve for r higher than 198 million yen. The left panel also shows that the winning bid in the optimal equilibrium with mediation is degenerate when r is less than 304 million yen. Above 304 million yen, the winning bid follows a non-degenerate distribution. At the calibrated parameters, $\overline{V}_{\mathsf{ck}}$ is 491.8 million yen and $\overline{V}$ is 754.1 million yen.

The right panel of Figure 2 plots the winning margin against the reserve price. At the calibrated parameters, we find that the realized winning margin for the Obara residences is slightly bigger than the predicted range. However, an increase in $\overline{V}$ of about 2.1% is enough to rationalize the observed winning margin.

OD Excerpts From the Ruling

This Appendix collects some key passages from the original ruling of the case against the Kumatori cartel. For each passage, we provide a translation prepared by the authors, as well as a translation done by an online software.

[7] We also considered estimating an exponential distribution to fit the reserve price distribution. In that case, we find that the cartel would benefit from randomization for auctions with reserve prices larger than 165 million yen. The Kumatori cartel participated in four auctions with a reserve price above 165 million yen.

（1）町営大原住宅工事は，１期から3 期に分かれており，平成1 6 年頃，その第1 期工事をA ランク業者である浅沼組が落札した。北川は，第2期及び第3期工事に関しては本件組合や地元業者に落札させたいと考え，第１期工事が着工した平成16年秋頃から，熊取町長や総務部長らに対し，大原住宅第2期工事はBランクの建設業者が入札に参加できるよう何度も陳情を行った。

Authors’ Translation:
The construction of municipal Obara housing project is divided into three phases, and around 2004, Asanuma-gumi, an A tier contractor, won the work for the first phase. Kitagawa wanted the cooperative and local contractors to work on the second and third phases of the project, and around the fall of 2004, as the first phase of construction was starting, repeatedly petitioned the mayor and the chief of the town’s general affairs department to let B-tiered contractors to bid on the second phase.

Google Translate:
The Municipal Ohara Housing Project is divided into phases 1 to 3, and around 2004, Asanuma-gumi, an A-ranked contractor, won the bid for the first phase of construction. Kitagawa wanted the association and local contractors to make bids for the second and third stages of construction. For the second phase construction, we petitioned many times so that B-ranking contractors could participate in the bidding.

Notes from the authors. Google translate performs poorly for this passage. Another translator, DeepL, did a better job of translating as follows:

The construction of the Ohara municipal housing project was divided into three phases, and around 2004, the bid for the first phase was won by Asanuma-gumi, an A-rated contractor. Kitagawa wanted the union and local contractors to win the bid for the second and third phases of construction, and from the fall of 2004, when construction of the first phase began, he repeatedly petitioned the mayor of Kumatori Town and the general manager to allow B-rank contractors to participate in the bidding for the second phase of Ohara Housing construction.

（2）平成1８年4 月頃，それまでB ランクの建設業者が入札に参加することについて消極的な態度を取っていた総務部長が，方針を変更して積極的な態度を示したため，北川は，大原住宅第2 期工事においてBランク業者が入札に参加できる可能性が高まったと考え，その頃から本件組合の会合等においてその旨の発言をするようになった。

Authors’ Translation:
Around April 2006, the head of the town’s general affairs department, who until then, had taken a negative stance towards letting B tier bidders bid on the auction, changed his attitude and took a positive stance. Kitagawa, thinking that the likelihood of Tier B firms being able to bid for the second phase of the Obara residences increased, began making remarks to that effect at meetings of the cooperative.

Google Translate:
Around April 2006, the director of the general affairs department, who until then had taken a passive attitude toward B-rank construction companies participating in bidding, changed his policy and took a more positive attitude. Thinking that the possibility of B-rank contractors participating in the bidding for the second stage of housing construction had increased, he began to make remarks to that effect at meetings of the Association.

Notes from the authors. In the last sentence of Google translate, “he” refers to Kitagawa.

（3）北川は， 同年6月下旬頃に開催された本件組合の会合において，参加した組合員に対し，「大原住宅第2期工事については，正式にBランク業者でやることになった」,「この件は組合預かりにしてほしい」，「現場説明会で配布される設計図書等は説明会が終わり次第回収する」などと説明した。北川の上記説明は，大原住宅第2 期工事において談合を行い，今勝建設に落札させてほしいこと， 積算に必要な設計図書等の回収は談合破りを防ぐためであることを趣旨としており，上記会合に参加した組合員もその趣旨で理解し，異議を述べる者はいなかった。

Authors’ Translation:
At a meeting of the cooperative held around the end of June of the same year, Kitagawa told the participating members, “It has been officially decided that the second phase of the Obara residences will be done by B-tiered firms”, “The cooperative will take care of this one”, and “Building plans that will be distributed at the on-site briefing will be collected at the end of the briefing”. By these statements, Kitagawa meant that the cooperative will collude on the auction for the second phase of the Obara residences, he wants to let Imakatsu construction win the auction, collecting the building plans at the on-site briefing is for the purpose of preventing defection. Attendees at the meeting understood Kitagawa’s statements as such, and no one objected.

Google Translate:
At a meeting of the association held around the end of June of the same year, Kitagawa told the participating members, ``It has been officially decided that the second stage construction of the Ohara housing will be done by a B-rank contractor.'' We would like the matter to be left in the care of the union," he said, adding, "We will collect the design documents distributed at the on-site briefings as soon as the briefings are over." The purpose of Kitagawa's explanation above is to hold bid-rigging for the second stage construction of Ohara Housing, and to let Kamachi Construction win the bid. None of the union members who attended the above meeting understood the purpose and raised any objections.

Notes from the authors. Kamachi Construction is supposed to be Imakatsu Construction. Also, the last sentence should be translated as

> All of the union members who attended the above meeting understood the purpose and none raised any objections.

（4）同年8 月3 日， 熊取町役場から， 阪南土木工業， 高田組， 成公建設，今勝建設及び山本工務店の5 業者に対し， 大原住宅第2 期工事の入札参加の指名が通知された。

Authors’ Translation:
On August 3 of the same year, the Town office notified five firms, Hannan Construction, Takada Gumi, Seiko Construction, Imakatsu Construction , and Yamamoto Construction, that they are invited to bid on the second phase of the Obara residences construction.

Google Translate:
On August 3 of the same year, the Kumatori Town Hall notified Hannan Civil Engineering, Takada Gumi, Seiko Construction, Imakatsu Construction, and Yamamoto Construction to participate in the bidding for the Ohara Housing Phase 2 construction.

（5）北川は， 同日頃，西尾に対し，設計金額の推定，ボーリング，入札参加各社の参考内訳書の作成，入札金額の決定，入札金額の指示等を依頼する趣旨で，「今回頼むで。」と述べた。西尾は，北川の上記発言の意味を理解し，今勝建設が大原住宅第2 期工事を落札するために必要な段取りを行うことを了承した。

Authors' Translation:
Around the same day, Kitagawa told Nishio "I'm counting on you for this one," by which Kitagawa meant that he was asking Nishio to come up with a prediction of the engineering estimate, conduct "boring" [1], create cost breakdowns for each bidding firm, decide on what to bid, and instruct bidders the amount each bidder should bid. Nishio understood what Kitagawa meant, and agreed to take necessary steps for Imakatsu Construction to win the second phase of the Obara residences project.

Google Translate:
Around the same day, Kitagawa asked Nishio to estimate the design cost, conduct drilling, prepare a reference breakdown for each company participating in the bid, determine the bid price, and give instructions on the amount of the donation. said. Nishio understood the meaning of Kitagawa's statement above and agreed to make necessary arrangements for Imakatsu Construction to win the bid for the second phase construction of Ohara Housing.

（6）北川は，同月4 日，全員協議会の開催後に，大原住宅第2期工事の入札に指名された5業者を小会議室に集め，上記5業者に対し，現場説明会の後で設計図書等の資料を回収する旨を再度伝えた。
Authors' Translation
On the 4th of the same month, after the all-members meeting of the cooperative, Kitagawa gathered the 5 contractors invited to bid on the second phase of the Obara residences auction into a small meeting room. He then reiterated to the 5 bidders that the documents such as the detailed building plans would be collected from them after the on-site briefing.

Google Translate:
On the 4th of the same month, after the all-members council was held, Kitagawa gathered the five contractors who had been nominated to bid for the second stage construction of Ohara Housing in a small meeting room, and gave the above five contractors design documents, etc. after a site briefing. I told them again that I would collect the materials.

Notes from the authors. Google translate performs poorly for this passage. Below is a better translation by DeepL:

On the 4th of the same month, after the all-member council meeting, Kitagawa gathered the five contractors nominated to bid on the Ohara Housing Phase 2 construction project in a small conference room and again informed the above five contractors that he would collect the design documents and other materials after the on-site briefing.

[1] Typically, the designated bidder would visit the town office and seek out the engineering estimate prior to bidding in order to gain information about the engineering estimate (and hence the reserve price), an activity that the bidders called "boring". See Criminal case. In the Kumatori case, Kitagawa asked Nishio to do the "boring".

（7）同月7 日，大原住宅第2期工事に関する現場説明会が開催された。現場説明会において配布された設計図書等の積算に必要な書類は，現場説明会の直後，北川及び西尾の指示により，本件組合の事務責任者である？？？が今勝建設を除く各指名業者から回収した（ なお，上記設計図書等については，入札前日の同月2 1日， 北川の指示により，？？？が各指名業者に返還した。）。

Authors' Translation:
On the 7th of the same month, an on-site briefing for the second phase of the Obara residences auction was held. Right after the on-site briefing, ???, the person in charge of administration of the cooperative, at the instruction of Kitagawa and Nishio, collected the documents necessary to estimate costs, such as the building plans from each of the invited bidders except from Imakatsu Construction. (??? returned the above building plans etc. to the invited bidders on the 21st of the same month, the day before the auction, at the instruction of Kitagawa.)

Google Translate:
On the 7th of the same month, a briefing session was held on the site of the Ohara Housing Phase 2 construction. Documents necessary for cost estimation such as design documents distributed at the on-site briefing were given by Kitagawa and Nishio's instructions immediately after the on-site briefing. ? ? (In addition, on the 21st of the same month, the day before the bidding, ??? returned the above design documents, etc. to each of the nominated contractors under Kitagawa's instructions.).

Notes from the authors. The name of the person who collected the documents at the on-site briefing and later returned them to the bidders is redacted. A more accurate translation of the original document by DeepL is as follows:

On the 7th of the same month, a site briefing regarding the Ohara Housing Phase 2 construction was held. Immediately after the briefing, ???, who was in charge of the affairs of the association, collected the documents necessary for the cost estimation, such as the design drawings distributed at the briefing, from each of the nominated contractors except Imakatsu Construction, at the direction of Kitagawa and Nishio (The above design drawings were returned by ??? to each nominated contractor on February 21, the day before the bidding, at the direction of Kitagawa). (In addition, ??? returned the above-mentioned design drawings, etc. to each of the nominated contractors at the direction of Kitagawa on February 21, the day before the bidding.)

（8）西尾は，現場説明会の後で，熊取町建築課職員である???から大原住宅第2 期工事の設計金額を聞き出したり，同課を訪れてボーリングを行ったりするなどして，設計金額を6億8000万円台と推定した。

Author's Translation:
Nishio, after the on-site briefing, elicited the engineering estimate for the second phase of the Obara residences project from ???, an official at the Kumatori town construction section as well as visited the construction section for "boring"[2], and projected the engineering estimate to be around 680 million yen.

Google Translate:
After the on-site briefing session, Nishio found out from ???, a staff member of the Kumatori Town Construction Section, the design cost for the Ohara Housing Phase 2 construction, and visited the section to conduct a drilling. The design cost was estimated at around 680 million yen.

[2] See footnote 1.

Notes from the authors. The name of the staff at the Kumatori Town is redacted.

（9）北川と西尾は，同月21日頃，今勝建設事務所において会談し，今勝建設の入札金額を6億3000万円とすること，他の入札業者に対しては，今勝建設の入札金額を推測されないよう，いずれも7億円以上の入札金額を指示することを決めた。また，北川は，同日，？？？に対し，今勝建設の参考内訳書を渡した上，今勝建設の入札金額を指示した。さらに，北川は，今勝建設の落札を確実なものとするため，熊取町総務部の課長と連絡を取り，予定価格に対する最低制限価格の割合が83%であることを聞き出した。

Authors' Translation"
Kitagawa and Nishio held talks at the office of Imakatsu Construction on the 21st of the same month, and decided that Imatatsu Consruction's bid would be 630 million and, for the other bidders, decided to instruct them to bid an amount above 700 million yen in order to make it difficult for them to guess Imakatsu's bid. On the same day, Kitagawa handed to ??? Imakatsu's cost breakdown and indicated the bid amount for Imakatsu. Moreover, in order to make it even more certain that Imakatsu will win the auction, Kitagawa contacted the section chief of the general affairs department of Kumatori town and managed to elicit from the official that the lowest acceptable bid would be 83% of the reserve price.

Google Translate:
Kitagawa and Nishio held talks at the Imakachi Construction Office around the 21st of the same month, and decided that the bid amount for Imakachi Construction would be 630 million yen. In order to prevent speculation, it was decided to specify a bid amount of 700 million yen or more. Also, on the same day, Kitagawa said, ? ? ? In addition to handing over a reference itemized breakdown of Imamatsu Construction, I instructed the bidding amount for Imamatsu Construction. In addition, Kitagawa contacted the manager of the general affairs department of Kumatori Town to ensure that he won the bid for Kamachi Construction, and learned that the ratio of the minimum price limit to the planned price was 83%.

Notes from the authors. A slightly more accurate translation of the original document by DeepL is as follows:

Around the 21st of the same month, Kitagawa and Nishio met at the Imakatsu Construction Office and decided to set the bid amount of Imakatsu Construction at 630 million yen and to instruct other bidders that their bids should all be 700 million yen or more so that no one would guess the amount of Imakatsu Construction's entrance fee. On the same day, Kitagawa also instructed ? Kitagawa gave Imakatsu Construction a reference translation and instructed Imakatsu Construction the amount of its bid. Furthermore, in order to ensure that Imakatsu Construction would win the bid, Kitagawa contacted the section chief of the Kumatori Town General Affairs Department and obtained information that the ratio of the minimum price limit to the estimated price was 83%.

（10）西尾は，同日，今勝建設以外の指名業者4者（山本工務店の？？？，高田組の？？？，阪南土木工業の？？？，成公建設の？？？）に連絡をし，入札当日に西尾が参考内訳書と入札金額の指示を渡すこと，参考内訳書の表紙に会社の記名判を押して持参してほしいこと等を伝えた。

Authors' Translation:

On the same day, Nioshio contacted the four invited bidders aside from Imakatsu Construction (??? At Yamamoto Construction, ??? at Takada Gumi, ??? at Hannnan Construction ??? at Seiko Construction) and told them that Nishio would hand to the them the cost breakdown document and instruct them on the amount to bid. Nishio also told them that he wanted them to bring the cover page of the cost breakdown with a company seal stamped on.

Google Translate:

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On the same day, Nishio contacted four designated contractors other than
Imakatsu Construction (??? of Yamamoto Construction,??? of Takada Gumi,??? of
Hannan Civil Engineering,??? of Seiko Construction). On the day of the
bidding, I told them that Nishio would hand over the reference breakdown and
instructions on the bid price, and that they should bring the company's name
stamp on the cover of the reference breakdown.
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Notes from the authors. A slightly more accurate translation of the last part of the original document should be as follows:

On the day of the bidding, Nishio told them that he would hand over the document with the cost estimates and instruct them on how to bid, and that they should bring the cover of the cost estimates document with the company's seal stamped on it.

（11）同月22日，大原住宅第2期工事に係る入札が行われた。入札には，今勝建設の？？？，山本工務店の？？？，高田組の？？？，阪南土木工業の？？？，成公建設の？？？がそれぞれ参加した。西尾は，入札会場である熊取町役場付近に赴き，今勝建設以外の指名業者の各担当者から，記名判を押した参考内訳書の1枚目を受け取り，予め積算金額を記載した参考内訳書の2枚目をホッチキスで留めた上，指名業者の各担当者に渡した。また，西尾は，各担当者に対し，それぞれ7 億円以上の金額が記載された付箋を見せ，入札金額を指示した。今勝建設を除く指名業者の各担当者は，西尾から受け取った参考内訳書を熊取町に提出し，また，西尾から見せられた付箋に記載された金額と同額かそれよりもやや高い額で入札した。その結果，今勝建設が6億3000万円，山本工務店が7億0500万円，高田組が7億0700万円，阪南土木工業が7億1000万円，成公建設が7億2300万円で入札し，今勝建設が落札した。なお，大原住宅第2期工事の設計価格は6億8647万2000円，予定価格は6億5700万円，最低制限価格は5億4900万円で、あった。

Authors’ Translation:

On the 22nd of the same month, bidding for the second phase of the Obrara residences project was held. ??? from Imakatsu Construction, ??? from Yamamoto Construction, ??? from Takada Gumi, ??? from Hannann Construction and ??? from Seiko Construction participated in the auction. Nishio went to the vicinity of the Kumatori town office, which is where the auction took place, received the cover sheet of the cost breakdown document with the company seal from the representatives of each of the invited bidders aside from Imaktsu, stapled on the prefilled cost breakdown document, and handed them back. Also, Nishio showed to each of the representatives a slip of paper with an amount greater than 700 million yen written on it and indicated to them the amount they should bid. Representatives of the invited bidders aside from Imakatsu all submitted to the town of Kumatori the cost breakdown document handed over to them by Nishio, and they bid an amount equal to, or slightly higher than the amount written on the slip of paper shown by Nishio. As a result, Imakatsu Construction bid 630 million yen, Yamamoto Construction bid 705 million, Takada Gumi bid 707 million, Hannan Construction bid 710 million, Seiko Construction bid 723 million yen, and Imakatsu Construction won the auction. The engineering estimate of the second phase construction of Obara residences was 686million and 472thousand yen, the reserve price was 657 million yen, and the lowest acceptable bid was 549 million yen.

Google Translate:

On the 22nd of the same month, a tender was held for the second phase construction of Ohara Housing. XXX from Imamachi Construction, XXX from Yamamoto Komuten, XXX from Takada Gumi, and XXX from Seiko Construction participated in the tender. Nishio went to the vicinity of the Kumatori Town Hall, where the bidding was held, and received the first copy of the reference breakdown with a name stamp from each person in charge of the nominated contractors other than Imamachi Construction. The second page of the book was stapled and handed over to each person in charge of the appointed contractor. In addition, Nishio showed each person in charge a sticky note with an amount of 700 million yen or more written on it, and instructed them on the bid amount. Each person in charge of the nominated contractors, excluding Imamachi Construction, submitted the reference breakdown received from Nishio to Kumatori Town, and also paid an amount equal to or slightly higher than the amount indicated on the sticky note shown by Nishio. made a bid. As a result, Imamachi Construction was 630 million yen, Yamamoto Komuten was 705 million yen, Takada Gumi was 707 million yen, Hannan Civil Engineering was 710 million yen, and Seiko Construction was 723 million yen. The bid was made in yen, and Kamachi Construction won the bid. The design price for the Ohara Housing Phase 2 construction was 686,472,000 yen, the planned price was 657 million yen, and the minimum limit price was 549 million yen.

Notes from the authors. The representatives of the firms who came to the auction to bid are redacted from the ruling. Google translates 今勝 as Imamachi, but it is Imakatsu.

（12）落札予定業者（チャンピオン）を誰にするかは，本件組合内部の取り決めによって星取り表を作成し，本件組合加入業者の持ち回りで決めていた。星取り表は，本件組合加入業者が集まって作成していたが，最終的な調整は西尾が行っていた。

Authors’ Translation:

The decision of who will be the designated winner (champion) was determined by taking turns amongst the members of the cooperative, and according to a win-loss table created in accordance with the internal rules of the cooperative. The win-loss table was created collectively by the members of the cooperative, but final adjustments were made by Nishio.

Google Translate:

As for who was to be the successful bidder (champion), a star ranking table was prepared according to the agreement within the Association, and it was decided by the members of the Association in turn. The star chart was created by the members of the union, but Nishio made the final adjustments.

Notes from the authors. 星取り表 is translated as star ranking table, but it should be translated as a scoring sheet that keeps track of who wins what.

（13）チャンピオンに決まった業者は，設計図書に基づく積算を行い，また，熊取町役場に赴いてボーリングを行うなどして設計金額を予想した。チャンピオン以外の指名業者は，自ら積算することはなく，ボーリングに行くこともなかった。

Authors' Translation:
The champion, or the designated winner, would estimate costs of construction based on the building plan, as well as predict the engineer's estimate by visiting the Kumatori town office and conducting "boring"[3]. Bidders other than the designated bidder would not undertake cost estimates on their own, nor would they conduct "boring".

Google Translate:
`The contractor who was selected as the champion estimated the cost of the design by estimating the cost based on the design documents and by going to the Kumatori town hall and doing some boring. Non-champion nominated vendors never scored themselves and never went bowling.`

Notes from the authors. Google translates ボーリング as "boring" and "bowling". Here, the word ボーリング is used to express the act of asking the officials information about the estimated cost of the project.

（14）？？？は，大原住宅第2期工事以外においても，北川の指示により，設計図書等を指名業者から回収したことがあった。

Authors' Translation:
??? had collected the building plans from the invited bidders on occasions prior to the second phase of the Obara Residences project at Kitagawa's instructions.

Google Translate:
`In addition to the Ohara Housing Phase 2 Construction, ??? has also collected design documents, etc. from designated contractors under Kitagawa's instructions.`

（15）チャンピオンは，予想した設計金額に基づき，他の指名業者が入札時に役場に提出する参考内訳書の原案を作成したり，他の指名業者に対して入札金額の指示を出したりしていた。チャンピオンが作成した他の指名業者の参考内訳書の原案と入札金額の指示は，入札日の1日か2日前の午前中に，本件組合事務所でチャンピオンが各指名業者に渡すことが通例となっていた。指定の時間に来られなかった指名業者については，？？？がチャンピオンから参考内訳書を預かつて代わりに渡すなどしていた。

Authors' translation:
The champion, based on predictions of the engineer's estimate, would draft the cost breakdown document that other invited bidders would submit to the town at the time of bidding, as well as instruct other invited bidders the amount that they should bid. It was customary for the champion to hand over the draft of the cost breakdown and give instructions on the amount other bidders should bid on the morning of one or two days prior to the date of the auction, at the office of the Cooperative. For invited bidders who could not come at the designated time, ??? would receive the draft of the cost breakdown from the champion and deliver it to them.

[3] See footnote 1.

Google Translate:

Based on the estimated design price, the champion created a draft of a reference breakdown for other nominated contractors to submit to the government office at the time of bidding, and gave instructions to the other nominated contractors on the amount of the donation. It is customary for the champion to hand over the original draft of the reference breakdown of the other nominated contractors prepared by the champion and the instructions on the amount of the donation to each nominated contractor at the union office in the morning one or two days before the bid date. It was For the nominated contractors who could not come at the designated time, XXX kept a reference breakdown from the champion and gave it to them instead.

Notes from the authors. Here, "donation" should be replaced with "bid" and union should be replaced with cooperative.

（16）上記（ウ）の指名競争入札を含め，平成19年12月1日から平成20年3月31日までに実施された指名競争入札（18件）の平均落札率は76. 7%，平成20年度に実施された指名競争入札（48件）の平均落札率は80. 9%であった

Authors' Translation"

Including the above auction with invited bidders in (c) above, the average winning bid in the (18) auctions with invited bidders let between December 1, 2007 to March 31, 2008, was 76.7%. The average winning bid in the (48) auctions with invited bidders let in fiscal year 2008, was 80.9%.

Google Translate:

Including the designated competitive bidding in (c) above, the average successful bid rate for the designated competitive bidding (18 cases) conducted from December 1, 2007 to March 31, 2008 was 76.7%. The average successful bid rate of the designated competitive bidding (48 cases) conducted in the fiscal year was 80.9%