Pooling and Tranching under Belief Disagreement

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Abstract

We study optimal security design when issuer and market participants disagree about the characteristics of the underlying asset. We show that pooling and tranching assets can be preferable to selling optimal securities backed by individual assets: pooling can be a response to belief disagreement between issuer and investors; tranching allows the issuer to exploit belief disagreement among investors. Moreover, differences in beliefs can make pooling and tranching complements; asymmetric information alone cannot.

Keywords: Disagreement, Security Design, Optimism, Overconfidence, Pooling, Tranching, Behavioral Finance

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1 Introduction

Which securities does a firm optimally issue when firm and market participants agree to disagree about the firm's cash-flow distribution? Existing literature, reviewed below, has used belief disagreement to explain capital structure choices, investment decisions, the choice of debt maturities, or the emergence of intermediaries. Complementing this literature, we study how differences in beliefs between issuer and markets, and between market participants, influence a firm's optimal security design in the sense of Allen and Gale (1988); i.e., in a model that imposes only minimal restrictions on the shape of the contract. We show that belief disagreement can generate various commonly observed financial contracts, can generate interactions between pooling and tranching, and – by contrast to asymmetric information – has the power to make predictions about which securities firms *issue* rather than which securities firms retain.[1] A direct implication is that asymmetric information alone is insufficient to generate an interaction between pooling and tranching.

We consider an issuer who owns an asset that will pay uncertain cash-flows at a future date. To raise capital, the issuer designs a security which is backed by the cash flows of one or multiple assets. Following DeMarzo and Duffie (1999), we assume the issuer discounts future cash-flows more than the market.[2] We allow for different types of investors in the market, who may differ in their beliefs about the assets' cash-flow distribution. Our main assumption is that the issuer is more optimistic than market participants. The issuer's problem is to design monotonic securities (one for each type of investor) backed by the underlying assets

[1] There are various asymmetric information based theories, e.g. Boot and Thakor (1993) and Friewald et al. (2016), in which firms issue multiple securities, but ingredients other than asymmetric information are employed, such as the effective exclusion of particular investors from the market at particular times. Our paper shows that these other ingredients are necessary; in other words, asymmetric information alone cannot generate a rationale for multiple tranches.

[2] The assumption that the issuer discounts future cash-flows at a higher rate than the market is a metaphor, for example for a situation in which the issuer has some profitable investment opportunity. Also, the assumption will hold if the issuer faces credit constraints or, as in the case of financial entities, minimum-capital requirements.

to maximize her expected payoff, which is given by the sum of the market prices of the securities she sells and the expected discounted value of retained cash flows.

Our analysis delivers four main results. First, we provide conditions under which it is optimal for the issuer to sell different tranches to the different types of investors. Second, we show that selling a security backed by a pool of several underlying assets can be strictly preferred to selling individual asset-backed securities. Third, we provide conditions under which pooling and tranching are complements. Fourth, we show that informational models of security design without disagreement only yield predictions about the shape of the aggregate security the firm issues to the market; in other words, they predict which security the firm retains. Informational theories don't yield predictions about how the cash-flows that firms sell to the market are tranched.

The intuition behind the optimality of tranching is simple, and related to Garmaise (2001): when there are differences in beliefs among investors, it is optimal for the issuer to design multiple securities, targeted to the different investor types. We show that, under certain conditions, the issuer finds it optimal to retain the most junior tranche.[3] In the special case in which all investors in the market share the same beliefs, we show that, under standard conditions, the optimal security is debt. The intuition for this result is closely related to earlier studies on capital structure choice and investment amid disagreement, (e.g., De Meza and Southey (1996); Heaton (2002); Hackbarth (2008)): the issuer finds it optimal to only sell cash-flows in the left tail of the cash-flow distribution, which the market values relatively more, and to retain the right tail of the distribution, which the issuer values relatively more.

For our results on pooling, we consider an issuer who owns two underlying assets.[4] We

[3]The latter prediction depends on the assumption that the issuer is more optimistic than the market. The prediction would be reversed in a (perhaps trivial) twist on the model that involves allowing some investors to be more optimistic than the issuer.

[4]For simplicity we focus on the case of two assets, although the results extend to the case in which the issuer owns several assets. The fact that disagreement makes pooling and tranching optimal even for a small

start by assuming that there is a single type of investor in the market. In this setting, we show that an optimistic issuer may strictly prefer to sell a security backed by the pool of assets than to sell individual asset-backed securities. Intuitively, while outside investors might be pessimistic about the probability of an individual asset delivering high profits, they will typically be relatively less pessimistic about the event that at least one of several assets pays off a high return. As a result, an issuer who owns multiple assets may find it strictly optimal to sell a "senior" security backed by the pool of assets. The following example illustrates the logic of this result.

Example 1. Consider first an issuer who owns a single asset, which can either pay a return of 1 or a return of 0. The market believes that the probability of the asset paying off is $\frac{1}{3}$; the issuer believes in an upside probability of $\frac{2}{3}$. The issuer discounts future cash-flows with a factor of 0.6, whereas the market does not discount. The market is therefore willing to pay $\frac{1}{3}$ for the asset. Since the asset is worth $\frac{2}{3} \cdot 0.6 = 0.4$ to the issuer, she retains it.

Consider now an issuer who owns two of these assets with *iid* returns. The issuer's payoff from retaining the two assets is 0.8, which is strictly larger than her payoff from selling two individual securities, each backed by an asset. Suppose instead that the issuer sells a "senior" security backed by the pool of assets that pays 1 if at least one asset pays off and zero otherwise. Investors are willing to pay $1 - \left(\frac{2}{3}\right)^2 = \frac{5}{9}$ for the security, while the issuer assigns to it a value of $\left(1 - \left(\frac{1}{3}\right)^2\right) \cdot 0.6 = \frac{8}{15} < \frac{5}{9}$. Because the issuer retains a cash-flow of 1 in the event that both assets pay off, her expected payoff from selling this security is $\frac{5}{9} + \left(\frac{2}{3}\right)^2 0.6 \approx 0.822$.

We note that differences in beliefs between the issuer and the market are crucial for pooling to be optimal in this setting. Because issuer discounts future cash-flows more than the market, under homogenous beliefs the issuer always finds it optimal to sell the entire firm, and so

number of assets is one dimension of distinction from asymmetric information theories of security design.

she is indifferent between pooling her assets or selling them as separate concerns.

To study the interaction between pooling and tranching, we consider the case of an issuer who owns two assets but faces different types of investors in the market. We impose restrictions on primitives such that an issuer who does not pool the assets finds it optimal not to tranche, and such that an issuer who does not tranche finds it optimal not to pool. We show that, under these conditions, it can still be strictly optimal for the issuer to pool and tranche – in other words, we provide conditions under which pooling and tranching are complements.

Finally, we show that belief disagreement is a necessary ingredient to obtain predictions about how firms' cash-flows are tranched in the market. We consider a general model of security design in which the issuer has private information about the asset's cash-flow distribution. There are different types of investors, who are differentially informed about the type of asset that the issuer owns. In contrast to our baseline model, we assume that all players (issuer and investors) share a common prior. Building on the insights of Aumann (1976), we show that if the issuer sells multiple tranches, then all investors will agree on the value they assign to each tranche. As a result, the issuer is indifferent between selling multiple tranches to different investors, or combining all cash-flows into a single tranche. A direct implication is that pooling and tranching can't be complements due to asymmetric information between issuer and investors.

The paper proceeds as follows. We discuss the related literature in Section 2. Section 3 introduces the basic model and derives the optimality of tranching. Sections 4 presents the results on pooling and its interaction with tranching. Section 5 shows that models of security design without disagreement don't have predictions about how firms' cash-flows are tranched. Section 6 concludes. All proofs are in the Appendix.

2 Related Literature

The idea that belief disagreement can shape security design goes back at least to Modigliani and Miller (1958), who write "Grounds for preferring one type of financial structure to another still exist within the framework of our model. If the owners of a firm discovered a major investment opportunity which they felt would yield much more than [the market's discount rate], they might well prefer not to finance it via common stock. A better course would be to finance the project initially with debt. Still another possibility might be to [issue] a convertible debenture." (excerpts from p. 292) Our paper offers a formal investigation into the role of disagreement in optimal design of securities.

Many papers have invoked differences in beliefs to explain stylized facts of entrepreneurship as well as corporate investment, financing, payout and capital structure choices.[5] By contrast, we allow for a less restrictive state space and/or contracting space, and study the question of which security is optimal under these general conditions.[6]

Our paper also relates to Garmaise (2001), who shows that tranching can be optimal in a model in which there is disagreement among investors and in which security prices are determined through a first price auction. Also related is Coval and Thakor (2005), who show that rational actors can arise to intermediate between optimistic entrepreneurs and pessimistic investors, issuing safe debt and retaining a mezzanine tranche of the projects they finance (see also Gennaioli et al., 2013).[7]

Our paper complements asymmetric information theories of security design, and in par-

[5]See, for instance, De Meza and Southey (1996); Boot et al. (2006, 2008); Landier and Thesmar (2009); Malmendier et al. (2011); Boot and Thakor (2011); Bayar et al. (2011); Thakor and Whited (2011); Huang and Thakor (2013); Adam et al. (2014); Bayar et al. (forthcoming). See also Simsek (2013), who studies how differences in beliefs among investors affect asset prices in the presence of collateral constraints.

[6]Yang (2013) shows that limited channel capacity can render debt and pooling optimal.

[7]Our paper is also related to a literature on corporate financial choices amid an ambiguity-averse pool of investors (e.g. Dicks and Fulghieri, 2015), because ambiguity aversion on behalf of the market collapses to disagreement between issuer and market. Lee and Rajan (2017) study optimal security design when both issuer *and* market are ambiguity averse. Malenko and Tsoy (2018) study optimal security design when a privately informed issuer faces an ambiguity-averse investor.

ticular the work of DeMarzo (2005).[8] We add to this literature by providing a model that rationalizes multiple tranches, and in which pooling and tranching can be complements. Moreover, we clarify why differences in beliefs (or differences in preferences) are a necessary ingredient to obtain predictions about how the cash-flows that firms' sell to the market are tranched.[9]

Lastly, our theory makes no use of moral hazard as a driver of the optimal security as in Admati and Pfleiderer (1994); Bergemann and Hege (1998); Schmidt (2003); Winton and Yerramilli (2008); Antic (2014); Hébert (2014).

3 Basic Model

3.1 Payoffs, Beliefs, and Objectives

At date $t = 0$, an issuer owns a risky asset yielding state-contingent payoffs at date $t = 1$. For now we treat this as a single asset – Section 4 considers a setting with several assets. Let $S = \{1, .., K\}$ and $\{X_s\}_{s \in S}$ be the possible cash-flow realizations at $t = 1$: the asset pays an amount $X_s \in \mathbb{R}_+$ if $s \in S$ is realized. We order S so that $X_1 < X_2 < ... < X_K$. With little loss of generality, we assume that there exists $\Delta > 0$ such that $X_s - X_{s-1} = \Delta$ for all $s \geq 1$, and we let $X_0 = 0$.

Let π^I be the probability distribution over S that represents the issuer's beliefs. Market participants have different beliefs about the cash-flow distribution of the underlying asset than the issuer. In particular, we assume that there are two types of investors in the market,

[8]Main contributions in this literature include Myers and Majluf (1984); Noe (1988); Innes (1990); Nachman and Noe (1990); Gorton and Pennacchi (1990); Stein (1992); Nachman and Noe (1994); Manove and Padilla (1999); Inderst and Mueller (2006); Axelson (2007); Fulghieri et al. (2013).

[9]Our model may help explain in ways consistent with the empirical evidence on issuers' relatively optimistic beliefs (Cheng et al., 2014) and the pro-cyclical nature of belief disagreement (see e.g. Chen et al., 2002; Scheinkman and Xiong, 2003; Hong and Stein, 2007) why pooling and tranching appear to coincide in the time series and cross-section (Fender and Mitchell, 2005; Coval et al., 2009; Stein, 2010; Chernenko et al., 2014; Fuster and Vickery, 2014).

$\tau = t_1, t_2$.[10] For $j = 1, 2$, let π^j be the probability distribution over S representing the beliefs of investors of type t_j. We assume that the issuer is more optimistic than both types of investors: for $j = 1, 2$, π^I first-order stochastically dominates π^j.

The issuer designs securities $(F^1, F^2) \in \mathbb{R}_+^K$ backed by the cash-flows $X = \{X_s\}_{s \in S}$ to sell in the market. Thus, securities (F^1, F^2) must be such that $0 \leq F_s^1 + F_s^2 \leq X_s$ for all $s \in S$. Following DeMarzo and Duffie (1999) we assume that the issuer discounts retained cash-flows at a rate that is higher than the market rate (which we normalize to 1). Thus, the issuer attaches a value of $\delta \sum_{s \in S} \pi_s^I (X_s - F_s^1 - F_s^2)$ to retained cash-flows, where $\delta \in (0, 1)$ is the issuer's discount rate. Altogether, the expected payoff of an issuer who sells securities (F^1, F^2) at prices p^1 and p^2 is $p^1 + p^2 + \delta \sum_{s \in S} \pi_s^I (X_s - F_s^1 - F_s^2)$.

The price that investors of type t_j are willing to pay for security F is $p^j(F) := \sum_s \pi_s^j F_s$. For any security F, let $p(F) = \max\{p^1(F), p^2(F)\}$ be the highest price that market participants are willing to pay for F. Overall, the issuer's payoff from selling (F^1, F^2) is

$$U(F^1, F^2) := p(F^1) + p(F^2) + \delta \sum_{s \in S} \pi_s^I \left(X_s - F_s^1 - F_s^2\right).$$

As is standard in the literature on optimal security design (e.g. DeMarzo and Duffie, 1999), we assume the issuer is restricted to sell securities that are monotonic.[11]

Definition 1. Securities F^1 and F^2 are *monotonic* if F_s^1 and F_s^2 are increasing in s and if $X_s - F_s^1 - F_s^2$ is increasing in s.

[10] The assumption that there are two types of investors is for simplicity; all of our results extend to the case with $n \geq 1$ types of investors. In particular, the number of tranches will depend on the number of types of investors.

[11] As is well known, this assumption can be microfounded with a moral hazard problem. To avoid high payments implied by a non-increasing security, the issuer could easily inflate cash-flows, e.g., by borrowing privately, and thus decrease payments to the investor.

Let $\mathcal{F}$ be the set of feasible securities

$$\mathcal{F} := \left\{F^1, F^2 \in \mathbb{R}_+^K : 0 \leq F_s^1 + F_s^2 \leq X_s \forall s \in S \text{ and } F^1 \text{ and } F^2 \text{ are monotonic}\right\}.$$

The issuer's problem is to find the securities (F^1, F^2) that solve

$$\sup_{(F^1,F^2)\in\mathcal{F}} U\left(F^1, F^2\right). \tag{1}$$

3.2 Optimal Security Design with Divergent Beliefs

In this section we present the solution to problem (1). We introduce additional notation before presenting our results. For any $s \in S$, let $A_s := \{s, s+1, ..., K\}$ be the event that the asset yields cash-flows weakly larger than X_s. For all $s \in S$, let $\pi^I(A_s) := \sum_{s'\geq s} \pi^I_{s'}$ and $\pi^j(A_s) := \sum_{s'\geq s} \pi^j_{s'}$ be, respectively, the probability that the issuer and investors of type t_j assign to A_s. Since π^I first-order stochastically dominates π^1 and π^2, $\pi^I(A_s) \geq \pi^j(A_s)$ for all $s \in S$ and for $j = 1, 2$.

Lemma 1. *Let (F^1, F^2) be a solution to (1). Then, there exists $(\hat{F}^1, \hat{F}^2) \in \mathcal{F}$ with $U(\hat{F}^1, \hat{F}^2) = U(F^1, F^2)$ such that, for $j = 1, 2$, $p(\hat{F}^j) = p^j(\hat{F}^j)$.*

By Lemma 1, it is without loss of optimality to consider solutions (F^1, F^2) to (1) such that, for $j = 1, 2$, security F^j is bought by investors of type t_j.

The following result characterizes the optimal securities. In what follows, for $j = 1, 2$, we use $-j$ to denote the investors of type $t_i \neq t_j$.

Proposition 1. *The optimal securities (F^1, F^2) satisfy: $F_1^1 + F_1^2 = X_1$, and for $j = 1, 2$ and* *for all $s \in S \setminus \{1\}$,*

$$F_s^j = \begin{cases} F_{s-1}^j + X_s - X_{s-1} & \text{if } \pi^j(A_s) > \max\left\{\pi^{-j}(A_s), \delta\pi^I(A_s)\right\}, \\ F_{s-1}^j & \text{if } \pi^j(A_s) \leq \max\left\{\pi^{-j}(A_s), \delta\pi^I(A_s)\right\}. \end{cases}$$

The key value that determines the shape of the optimal securities (F^1, F^2) at each s is the difference between $\delta\pi^I(A_s)$ and $\max\{\pi^1(A_s), \pi^2(A_s)\}$. Intuitively, when $\delta\pi^I(A_s) < \max\{\pi^1(A_s), \pi^2(A_s)\}$, market participants value cash-flows weakly above X_s more than the issuer does. Thus, the optimal securities (F^1, F^2) pay the largest possible amount (subject to monotonicity constraints) at profit level X_s; i.e., $F_s^1 + F_s^2 = F_{s-1}^1 + F_{s-1}^2 + X_s - X_{s-1}$. In contrast, if $\delta\pi^I(A_s) \geq \max\{\pi^1(A_s), \pi^2(A_s)\}$, the optimal securities pay the least possible amount (again, subject to monotonicity constraints) at profit level X_s; i.e., $F_s^1 + F_s^2 = F_{s-1}^1 + F_{s-1}^2$.

Corollary 1. *Suppose that there exists $s_1, s_2 \in S, s_1 < s_2$, such that*

(i) $\pi^1(A_s) > \max\left\{\pi^2(A_s), \delta\pi^I(A_s)\right\}$ *if and only if* $s \leq s_1$, *and*

(ii) $\pi^2(A_s) > \max\left\{\pi^1(A_s), \delta\pi^I(A_s)\right\}$ *if and only if* $s \in (s_1, s_2]$.

Then, the optimal securities are $F_s^1 = \min\{X_s, X_{s_1}\}$ *and* $F_s^2 = \min\{X_s - F_s^1, X_{s_2} - X_{s_1}\}$.

Under the conditions in Corollary 1, the issuer sells a senior tranche F^1, which is bought by investors of type t_1, and a mezzanine tranche F^2, which is bought by investors of type t_2. The issuer only retains the most junior cash-flows $X_s - X_{s_2}$ at profit levels larger than X_{s_2}. The mezzanine tranche can be interpreted as preferred equity or junior debt.

3.3 Single Investor

A special case of the model is one in which there is effectively a single investor in the market. To formalize this, suppose that $\pi^1 = \pi^2$, so all investors share the same beliefs. We use the convention that $F_0 = X_0 = 0$ for any security F.

Corollary 2. *Suppose that $\pi^1 = \pi^2$. Then, it is optimal to sell (F^1, F^2) with $F_s^2 = 0$ for all s, and*

$$\forall s \in S, \quad F_s^1 = \begin{cases} F_{s-1}^1 + X_s - X_{s-1} & \text{if } \pi^1(A_s) > \delta\pi^I(A_s), \\ F_{s-1}^1 & \text{if } \pi^1(A_s) \leq \delta\pi^I(A_s). \end{cases} \tag{2}$$

Corollary 2 characterizes the optimal security in the case in which all investors share the same beliefs. When $\frac{\pi^1(A_s)}{\delta\pi^I(A_s)}$ is decreasing in s, it is optimal to sell a debt contract with face value X_{s^*}, where $s^* = \min\left\{s \in S : \pi^1(A_s) \leq \delta\pi^I(A_s)\right\}$. Holding δ fixed, the face value of debt X_{s^*} depends on how different the beliefs of the issuer and market are. When the market is extremely pessimistic, the firm issues only risk-free debt. (Once that option is exhausted, it stops issuance altogether, as we show in Section 6.) By contrast, the issuer sells the entire cash-flow stream when belief disagreement is small.

This prediction is consistent with the timing of securities issuances to meet market sentiment (e.g., Marsh (1982); Baker and Wurgler (2002), and in particular Dittmar and Thakor (2007)), and contrasts the predictions of many theories of security design based on asymmetric information. Most prominently, the traditional "pecking order" hypothesis holds that firms issue equity only as a "last resort" (e.g., Myers, 1984); i.e., only the worst firms that have run out of other options issue equity.[12]

[12] This feature is not unique to our model, however. Informational theories of security design in which equity is not a last resort include Boot and Thakor (1993), Axelson (2007), Fulghieri et al. (2013) and Malenko and Tsoy (2018).

We note that there is some empirical evidence in line with our model's predictions. For instance, Frank and Goyal (2003) and Fama and French (2005) show that firms issue equity predominantly when not in financial distress. Farre-Mensa (2015) analyses firms that are hit with negative cash-flow shocks and thus face a need to issue securities (a decrease in δ in our model), and shows that firms whose stock is overvalued issue equity, whereas undervalued firms issue debt. Similar in spirit, Erel et al. (2011) and McLean and Zhao (2014) find that equity issuance is cyclical and higher amid positive investor sentiment, whereas firms turn to issuing safer securities during market downturns.

4 Pooling and Tranching

In this section, we consider an issuer who owns several assets. We establish two main results. First, we show that an issuer who has more optimistic beliefs than the market can strictly benefit from pooling different assets and designing a security backed by the cash-flows generated by the pool. Second, we show that when there are different types of investors in the market, pooling and tranching can be complements.

4.1 General Framework

Consider an issuer who owns two assets, X^1 and X^2, with *iid* returns.[13] Let $S = \{1, ..., K\}$ and let $\{X_s\}_{s \in S}$ be the possible cash-flow realizations of asset X^a, $a = 1, 2$. We continue to assume that $X_1 < X_2 < ... < X_K$, and that $X_s - X_{s-1} = \Delta > 0$ for all $s \geq 1$ (with $X_0 = 0$).

As in Section 3, we let π^I be the distribution over S representing the beliefs of the issuer, and π^1 and π^2 be the probability distributions over S representing the beliefs of investors of type t_1 and t_2. For $j = 1, 2$, π^I first-order stochastically dominates π^j. The issuer discounts

[13] We focus on the case of two assets for simplicity. The results can be extended to the case of $n > 2$ assets.

future profits at rate $\delta < 1$, whereas the market discounts future profits at rate 1.

The timing of events is as follows. First, the issuer publicly decides whether to pool the assets or not. Then, she designs securities optimally.

Separate assets. Suppose the issuer chooses not to pool her assets. For each asset X^a, $a = 1, 2$, recall that $\mathcal{F}$ is the set of securities (F^1, F^2) backed by X^a that are monotonic. For any $(F^1, F^2) \in \mathcal{F}$, we let $U_{X^a}(F^1, F^2)$ be the profits that the issuer obtains from selling securities (F^1, F^2) (calculated as in Section 3). Then, an issuer who doesn't pool the assets solves the following problem for each asset $a = 1, 2$:

$$\sup_{(F^1,F^2)\in\mathcal{F}} U_{X^a}(F^1, F^2).$$

The solution to this problem is characterized by Proposition 1.

Pooled assets. If the issuer pools the assets, she designs securities (F^1, F^2) backed by the asset pool $Y = X^1 + X^2$. For $j = 1, 2$ and any $s, s' \in S^2$, let $F^j_{s,s'}$ be the payoff of security F^j when asset 1's realized return is X_s and asset 2's realized return is $X_{s'}$.[14] We restrict the issuer to sell securities (F^1, F^2) that satisfy the following monotonicity requirements:

Definition 2. Say that securities F^1 and F^2 backed by $Y = X^1 + X^2$ are Y-*monotonic* if:

(i) for $j = 1, 2$, $F^j_{s,s'}$ is increasing in s and s';

(ii) $X_s + X_{s'} - (F^1_{s,s'} + F^2_{s,s'})$ is increasing in s and s'.

These monotonicity restrictions assume it is difficult for the issuer to manipulate profits across assets. For example, the issuer may face legal constraints that make it difficult for her to transfer profits from one asset to another.

[14] By the same arguments as in Lemma 1, it is without loss of generality to restriction attention to securities (F^1, F^2) such that, for $j = 1, 2$, investors of type j buy security F^j.

Let $\mathcal{F}_Y$ be the set of feasible securities:

$$\mathcal{F}_Y := \left\{F^1, F^2 \in \mathbb{R}_+^{K\times K} : F^1_{s,s'} + F^2_{s,s'} \in [0, X_s + X_{s'}]\forall s, s' \text{ and } \left(F^1, F^2\right) \text{ are } Y\text{-monotonic}\right\}.$$

The price that type j market participants are willing to pay for security F is $p^j(F) := \sum_{s\in S}\sum_{s'\in S}\pi^j_s\pi^j_{s'}F_{s,s'}$. The issuer's profits from pooling the assets and selling securities $(F^1, F^2) \in \mathcal{F}(Y)$ are

$$U_Y(F^1, F^2) = p(F^1) + p(F^2) + \delta\sum_{s\in S}\sum_{s'\in S}\pi^I_s\pi^I_{s'}(X_s + X_{s'} - F^1_{s,s'} - F^2_{s,s'}),$$

where, for any security F, $p(F) = \max_{j=1,2} p^j(F)$ is the highest price that investors are willing to pay for F. The problem of an issuer who pools the asset is then

$$\sup_{(F^1,F^2)\in\mathcal{F}_Y} U_Y(F^1, F^2).$$

4.2 Single Investor

We start by considering the case with one single investor (i.e., $\pi^1 = \pi^2$). The following example, which generalizes Example 1 in the Introduction, illustrates why pooling can be strictly optimal.

Example 2. Suppose the issuer has two assets, X^1 and X^2. Each of the assets can produce cash-flows in $\{X_1, X_2\}$, with $X_2 > X_1 > 0$. Let $\pi^I \in (0, 1)$ and $\pi \in (0, 1)$ be, respectively, the probability the issuer and market assigns to the asset yielding cash-flows X_1. The issuer is more optimistic than the market, so $\pi^I < \pi$. We further assume that $\delta(1 - \pi^I) > 1 - \pi$; that is, the issuer values high cash-flows more than the market does.

Consider first the problem of an issuer who does not pool the assets. By Proposition 1, for each asset X^a, an optimal security F has $F_1 = F_2 = X_1$ (since $\delta(1 - \pi^I) > 1 - \pi$). The issuer's profits from selling the securities separately are $2X_1 + 2\delta(1-\pi^I)(X_2 - X_1)$.

Suppose instead that the issuer pools the two assets and sells a single security backed by the pool. Let $Y = X^1 + X^2$ and $F_Y = \min\{Y, X_1 + X_2\}$; that is, security F_Y pays $2X_1$ if both assets yield a return of X_1, and pays $X_1 + X_2$ if one of the two assets yields a return of X_2. The market-price of security F_Y is

$$p(F_Y) = \pi^2 2X_1 + (1-\pi^2)(X_1 + X_2) = 2X_1 + (1-\pi^2)(X_2 - X_1),$$

and the issuer's payoff from selling F_Y is $2X_1 + (1 - \pi^2 + \delta(1-\pi^I)^2)(X_2 - X_1)$. The issuer strictly prefers to pool the assets and sell security F_Y if $\pi < \sqrt{1-\delta(1-(\pi^I)^2)}$. Therefore, for $\pi \in \left(1-\delta(1-\pi^I), \sqrt{1-\delta(1-(\pi^I)^2)}\right)$, pooling is strictly optimal.

Intuitively, the market is relatively less pessimistic about the event that one of the two assets yields a cash-flow of X_2. By pooling the two assets, the issuer is able to design a security that pays off a high return precisely when this event occurs.

We now present a general result for the case in which there is a single type of investor in the market. Recall that $A_s = \{s, s+1, ..., K\}$ is the event that an asset pays weakly more than X_s.

Assumption 1. *There exists $k \in S \backslash \{K\}$ such that $\pi^1(A_s) \geq \delta \pi^I(A_s)$ if and only if $s \leq k$.*

Under Assumption 1, if the issuer sells the assets as separate concerns it is optimal for her to issue two securities $\hat{F}_s = \min\{X_s, X_k\}$. The issuer's profits from doing so are $2U_{X^a}(\hat{F})$.

Let

$$\mathcal{F}_Y^1 := \left\{(F^1, F^2) \in \mathcal{F}_Y : F_s^2 = 0 \text{ for all } s \in S\right\},$$

denote the set of feasible securities when the issuer pools the assets and designs a single security to investors with beliefs π^1 (recall that $\pi_1 = \pi_2$). The seller's profits from pooling in this case are $\sup_{F \in \mathcal{F}_Y^1} U_Y(F)$.

Proposition 2. *Suppose that Assumption 1 holds, and that there exists* $\hat{s}, \hat{s}' \in S$ *with* $\hat{s}' > k$ *and* $\hat{s} \in \{1\} \cup (k, \hat{s}']$*, such that*

$$\pi^1(A_{\hat{s}'})(2\pi^1(A_{\hat{s}}) - \pi^1(A_{\hat{s}'})) > \delta\pi^I(A_{\hat{s}'})(2\pi^I(A_{\hat{s}}) - \pi^I(A_{\hat{s}'})). \tag{3}$$

Then, pooling is strictly optimal: $\sup_{F \in \mathcal{F}_Y^1} U_Y(F) > 2U_{X^a}(\hat{F})$.

To gain intuition for Proposition 2, let $F^S \in \mathcal{F}_Y^1$ denote the security that corresponds to selling $\hat{F}_s = \min\{X_s, X_k\}$ separately on each individual asset; i.e., $F^S_{s,s'} = \min\{X_s, X_k\} + \min\{X_{s'}, X_k\}$. Suppose there exists $\hat{s}$ and $\hat{s}'$ satisfying inequality (3), and let

$$\hat{S} := \left\{(s, s') \in S^2 : s \geq \hat{s} \text{ and } s' \geq \hat{s}' \text{ or } s \geq \hat{s}' \text{ and } s' \geq \hat{s}\right\}.$$

Note that for $i = 1, I$,

$$\begin{aligned} Pr_i(s, s' \in \hat{S}) &= Pr_i(X^a \geq X_{\hat{s}'} \text{ and } X^{a'} \geq X_{\hat{s}} \text{ for } a, a' = 1, 2) \\ &= \pi^i(A_{\hat{s}'})(2\pi^i(A_{\hat{s}}) - \pi^i(A_{\hat{s}'})) \end{aligned}$$

is the probability that, under beliefs π^i, one asset yields returns larger than $X_{\hat{s}'}$ and the other asset yields returns larger than $X_{\hat{s}}$. Inequality (3) implies that the market values returns in $\hat{S}$ more than the issuer does. Therefore, the issuer strictly benefits from pooling the assets and selling security F^P with

$$F^P_{s,s'} = \begin{cases} F^S_{s,s'} & \text{if } s, s' \notin \hat{S}, \\ F^S_{s,s'} + \Delta & \text{otherwise,} \end{cases}$$

Our next result provides sufficient conditions under which there are no gains from pooling.[15]

Proposition 3. *Suppose Assumption 1 holds, and that,*

$$\forall s, s' \leq k, \quad \pi^1(A_s)\pi^1(A_{s'}) > \delta\pi^I(A_s)\pi^I(A_{s'}), \tag{4}$$

$$\forall s \in S, \forall s' > k, \quad \pi_s^1\pi^1(A_{s'}) < \delta\pi_s^I\pi^I(A_{s'}). \tag{5}$$

Then, there are no gains from pooling: $\sup_{F\in\mathcal{F}_Y^1} U_Y(F) = 2U_{X^a}(\hat{F})$.

Conditions (4) and (5) essentially imply that differences in beliefs are not too large. Indeed, the following (stronger) condition implies both (4) and (5):

$$\forall s \in S, \quad \max_{s'\leq k} \frac{\delta\pi^I(A_{s'})}{\pi^1(A_{s'})} < \frac{\pi_s^1}{\pi_s^I} < \min_{s'>k} \frac{\delta\pi^I(A_{s'})}{\pi^1(A_{s'})}.$$

4.3 Multiple Investors

We now consider a setting in which there are two types of investors. The main goal of the section is to show that belief disagreement among investors can make pooling and tranching complements.

Assumption 2. *There exists $k, k' \in S$, with $k' \geq k$, such that*

(i) $\pi^1(A_{s'}) > \pi^2(A_{s'})$ *for all* $s' \in (1, k']$ *and* $\pi^1(A_{s'}) < \pi^2(A_{s'})$ *for* $s' > k'$.

(ii) $\pi^1(A_{s'}) > \delta\pi^I(A_{s'})$ *for all* $s' \in (1, k]$ *and* $\delta\pi^I(A_{s'}) \geq \max\{\pi^1(A_{s'}), \pi^2(A_{s'})\}$ *for all* $s' > k$.

Assumption 2(i) implies that the c.d.f.'s of the investors' beliefs cross at exactly one point. When the two types of investors assign the same value to the underlying assets ($\sum_s \pi_s^1 X_s^a = \sum \pi_s^2 X_s^a$), Assumption 2(i) implies that π^1 second-order stochastically dominates π^2.

[15] It can be checked that the conditions in Proposition 2 cannot hold when the conditions in Proposition 3 are satisfied.

Assumption 2(ii) implies that an issuer who sells the two assets separately (i.e., an issuer who does not pool her assets) finds it optimal to not tranche the assets: for each asset $a = 1, 2$, she will sell a single security F^a targeted to investors with beliefs π^1. Indeed, by Proposition 1, under Assumption 2(ii) the optimal securities $(F^{1,a}, F^{2,a})$ when selling assets $(X^a)_{a=1,2}$ separately are given by $F_s^{1,a} = \min\{X_k, X_s\}$ and $F_s^{2,a} = 0$ for all s and for $a = 1, 2$. The issuer's profits from selling the two assets separately are then $2U_{X^a}(F^{1,a}, F^{2,a})$.

For $j = 1, 2$, let U_Y^j denote the issuers payoffs from pooling the assets and designing a single security for investors of type j.

Assumption 3. *(i) π_1 satisfies conditions (4) and (5),*

(ii) $U_Y^1 > U_Y^2$.

Assumption 3 implies that if the issuer does not tranche, then there are no gains from pooling. Indeed, by Proposition 3, under Assumption 3 (i) there are no gains from pooling if the issuer only sells securities designed for investors of type π^1. Moreover, Assumption 3 (ii) says that doing this is more profitable than pooling and selling a security designed for investors of type t_2. Lemma 2 in the Appendix provides conditions on the model's primitives that guarantee that $U_Y^1 > U_Y^2$.

Taken together, Assumptions 2 and 3 imply that an issuer who does not tranche does not benefit from pooling, and an issuer who does not pool does not benefit from tranching. The next proposition summarizes these results.

Proposition 4. *Suppose Assumptions 2 and 3 hold. Then,*

1. *If the issuer does not tranche the asset, there are no gains from pooling.*

2. *If the issuer does not pool the assets, there are no gains from tranching.*

Our next result shows that, under these conditions, the issuer might still find it strictly optimal to pool the assets: by doing so, she can profit from selling an additional tranche to investors with beliefs π^2.

Proposition 5. *Suppose Assumptions 2 and 3 holds, and that there exists $\hat{s}, \hat{s}' \in S$ with $\hat{s}' > k'$ and $\hat{s} \in \{1\} \cup (k, \hat{s}']$, such that*

$$\pi^2(A_{\hat{s}'})(2\pi^2(A_{\hat{s}}) - \pi^2(A_{\hat{s}'})) > \delta\pi^I(A_{\hat{s}'})(2\pi^I(A_{\hat{s}}) - \pi^I(A_{\hat{s}'})). \tag{6}$$

Then, it is strictly optimal to pool and tranche: $\sup_{(F^1,F^2)\in\mathcal{F}_Y} U_Y(F^1, F^2) > 2U_{X^a}(F^{1,a}, F^{2,a})$.

The results above show that, under certain conditions, pooling and tranching are complements: while neither pooling nor tranching are beneficial on their own, the issuer finds it strictly optimal to pool the assets and then tranche. The intuition is similar to Proposition 2. Indeed, inequality (6) implies that investors with beliefs π^2 value returns in $\hat{S} := \Big\{(s, s') \in S^2 : s \geq \hat{s} \text{ and } s' \geq \hat{s'} \text{ or } s \geq \hat{s}' \text{ and } s' \geq \hat{s}\Big\}$ more than the issuer does. Therefore, under these conditions, the issuer gains from pooling the assets, selling security $F^1_{s,s'} = \min\{X_s, X_k\} + \min\{X_{s'}, X_k\}$ to investors with beliefs π^1, and selling security F^2 with $F^2_{s,s'} = \Delta \times \mathbf{1}_{(s,s')\in\hat{S}}$ to investors with beliefs π^2.

5 Asymmetric information with a common prior

In this section, we show that informational models of security design in which all agents share a common prior only yield predictions about the total cash-flows that firms will sell to the market – they don't yield predictions about how these cash-flows will be split among multiple tranches.

We consider the following variation of our model of Section 3. An issuer owns one asset X. Let $S = \{1, ..., K\}$ and let $\{X_s\}_{s\in S}$ be the possible cash-flow realizations of asset X, with $X_s - X_{s-1} = \Delta > 0$ for all $s \geq 1$.[16] There are two types of investors, $\tau = t_1, t_2$.

[16]For conciseness, we focus on the case in which the issuer owns a single asset. However, our main result of this Section (Proposition 6) continues to hold if the issuer owns multiple assets.

Let $\Omega \subset \mathbb{R}$ be the set of possible states of the world; Ω may be discrete or continuous. Conditional on the state being $\omega \in \Omega$, all players commonly believe that the returns of asset X are distributed according to distribution $\pi^\omega = \{\pi_s^\omega\}_{s \in S}$. We assume that, for all $\omega > \omega'$, π^ω first-order stochastically dominates $\pi^{\omega'}$. In contrast to our model in Sections 3 and 4, all players (i.e., issuer, and both types of investors) share a common prior H over Ω.[17]

We assume that players are differentially informed about the state of the world. In particular, we assume that the issuer perfectly observes state ω, while investors get imperfect signals about ω. Let P^1 and P^2 be two (possibly different) partitions of Ω, representing the information that each type of investor receives: under state $\omega \in \Omega$, investors of type t_i learn that the state is in $P^i(\omega) \subset \Omega$, where $P^i(\omega)$ is the element of partition P^i containing ω.

The timing of the game is as follows. First, the issuer learns perfectly the realization of state ω, while investors of type t_i learn the element $P^i(\omega)$ of partition P^i that ω lies in. Then, the issuer designs monotonic securities $(F^1, F^2) \in \mathcal{F}$ backed by X, which are publicly observed. Finally, prices are determined and are publicly observed, and the securities are traded in the market.

The issuer's payoff from selling securities $\mathbf{F} = (F^1, F^2) \in \mathcal{F}$ at prices $\mathbf{p} = (p_1, p_2)$ at state ω is

$$U(\mathbf{F}, \mathbf{p}; \omega) = p_1 + p_2 + \delta \sum_s \pi_s^\omega (X_s - F_s^1 - F_s^2).$$

Let $\sigma : \Omega \mapsto \mathcal{F}$ be a strategy for the Issuer, and let $p : \Omega \times \mathcal{F} \mapsto \mathbb{R}^2$ be a *pricing function*; i.e. p gives prices for each possible securities $\mathbf{F} = (F_1, F_2) \in \mathcal{F}$ at each possible state, with $p_j(\omega, \mathbf{F})$ the price of security F_j at state ω.[18] Let $\hat{P}^i_{\sigma,p}$ denote the information partition of investors of type i given (σ, p), which includes the original signal these investors observed,

[17] Note that the model in Sections 3 and 4 can be thought of as a setting in which the different players (issuer and investors) have different doctrinaire beliefs over Ω – and hence have different beliefs about the cash-flow distribution.

[18] Note that the equilibrium price of a given security might depend on the state, since investors' information changes with ω.

the information that is conveyed by the issuer's choice of securities (F^1, F^2) through σ, and the information conveyed by market prices. Then, the payoff of investors of type t_i from buying security F^j at price $\hat{p}$ at state ω is $\mathbb{E}[F^j|\hat{P}^i_{\sigma,p}(\omega)] - \hat{p}$.

For each strategy $\sigma : \Omega \to \mathcal{F}$, we let $\sigma(\omega) = (F^1_\sigma(\omega), F^2_\sigma(\omega))$ be the securities that the issuer issues at state ω under strategy σ.

Definition 3. A Perfect Bayesian equilibrium (PBE) is given by a strategy $\sigma : \Omega \mapsto \mathcal{F}$ of the issuer and a pricing function $p : \Omega \times \mathcal{F} \mapsto \mathbb{R}^2$, such that:

(i) for all on-path securities $\sigma(\omega) = (F^1_\sigma(\omega), F^2_\sigma(\omega))$ and for $j = 1, 2$,

$$p_j(\omega, \sigma(\omega)) = \max\left\{\mathbb{E}[F^j_\sigma(\omega)|\hat{P}^1_{\sigma,p}(\omega)], \mathbb{E}[F^j_\sigma(\omega)|\hat{P}^2_{\sigma,p}(\omega)]\right\}$$

(ii) for all $\omega \in \Omega$ and all $\mathbf{F} \in \mathcal{F}$, $\sigma(\omega) = (F^1_\sigma(\omega), F^2_\sigma(\omega))$ must be such that

$$U(\sigma(\omega), p(\omega, \sigma(\omega)); \omega) \geq U(\mathbf{F}, p(\omega, \mathbf{F}); \omega).$$

Condition (i) states that, on the equilibrium path, security prices must be consistent with investor's beliefs. Condition (ii) states that, at every state, the securities chosen by the issuer must be optimal. Note that, with this formulation, investors' off-path beliefs are summarized by pricing function $p(\omega, \mathbf{F})$.[19]

Definition 4. PBE (σ, p) is a *tranching* equilibrium if there exists $\omega \in \Omega$ such that (i) $F^j_\sigma(\omega) \neq 0$ for $j = 1, 2$, and (ii) $p_1(\omega, \sigma(\omega)) = \mathbb{E}[F^1_\sigma(\omega)|\hat{P}^i_{\sigma,p}(\omega)]$ and $p_2(\omega, \sigma(\omega)) = \mathbb{E}[F^2_\sigma(\omega)|\hat{P}^{i'}_{\sigma,p}(\omega)]$ for $i, i' = 1, 2, i \neq i'$. Otherwise, PBE (σ, p) is a *non-tranching* equilibrium.

[19] We could add the restriction that, for all ω and all $\mathbf{F} = (F^1, F^2)$,

$$p_j(\omega, \mathbf{F}) \in \left[\inf_{\omega' \in \Omega} \sum_s \pi^{\omega'}_s F^j_s, \sup_{\omega' \in \Omega} \sum_s \pi^{\omega'}_s F^j_s\right] \text{ for } j = 1, 2,$$

to guarantee that off-path prices are consistent with some investors' beliefs. Our results don't change if we add this restriction.

In words, in a tranching equilibrium there exists a state ω at which the issuer issues two non-zero securities, and at which each security is bought by a different type of investor.

The next proposition shows that, in any tranching equilibrium, both types of investors are willing to pay the same price for each security at every state ω at which each security is bought by a different type of investor.

Proposition 6. *Fix a tranching equilibrium (σ, p). For all states $\omega \in \Omega$ such that (i) $F_\sigma^j(\omega) \neq 0$ for $j = 1, 2$, and (ii) $p_1(\omega, \sigma(\omega)) = \mathbb{E}[F_\sigma^1(\omega)|\hat{P}_{\sigma,p}^i(\omega)]$ and $p_2(\omega, \sigma(\omega)) = \mathbb{E}[F_\sigma^2(\omega)|\hat{P}_{\sigma,p}^{i'}(\omega)]$ for $i, i' = 1, 2, i \neq i'$, we have $\mathbb{E}[F_\sigma^j(\omega)|\hat{P}_{\sigma,p}^1(\omega)] = \mathbb{E}[F_\sigma^j(\omega)|\hat{P}_{\sigma,p}^2(\omega)]$ for $j = 1, 2$.*

The intuition (and proof) of Proposition 6 is closely related to the result by Aumann (1976) that agents who share a common prior and who receive differential information cannot agree to disagree. Since market prices are publicly observed, at any state $\omega \in \Omega$ it is common knowledge among all investors that $p_j(\omega, \sigma(\omega)) = \mathbb{E}[F_\sigma^j(\omega)|\hat{P}_\sigma^i(\omega)]$ for some group of investors $\tau = t_i$. Since all investors share a common prior, after observing market prices investors of type $\tau' \neq \tau$ must have the same beliefs over Ω as investors of type τ; and hence must assign the same value to security $F_\sigma^j(\omega)$.

By Proposition 6, at any tranching equilibrium, both types of investors attach the same value to the different tranches, and so the issuer is indifferent as to how she tranches the assets. (Also, investors will not want to further tranch the security they purchased from the firm.) An implication is that informational theories of security design yield predictions on the total cash-flows that firms sell to the market, but not how these cash-flows are split among multiple tranches.[20] To obtain predictions about the different tranches that a firm will issue, the model needs to incorporate differences in beliefs (or in preferences) among investors.

[20]Indeed, it can be shown that, for any tranching equilibrium (σ, p), there exists a non-tranching equilibrium (σ', p') that gives the issuer the same profits as (σ, p).

6 Extensions

Our model admits several natural extensions. In this section we briefly outline a few of them.

Pre-existing debt. Consider the problem of an issuer who has senior debt outstanding that is backed by the cash-flows that her asset will generate, and who is considering to issue a new security backed by the remaining cash-flows. For simplicity, we assume that there is a single investor type.

Suppose the issuer has debt outstanding with face value $D < X_K$. The issuer's goal is to design a security $F \in \mathbb{R}_+^K$ to sell to the market, with F backed by the remaining cash-flows; i.e., for all s, F satisfies $0 \leq F_s \leq X_s - \min\{X_s, D\}$. As before, we restrict the issuer to design monotonic securities; that is, securities F such that F_s and $X_s - F_s - \min\{X_s, D\}$ are increasing in s. Let $\mathcal{F}_D$ denote the set of feasible securities: $\mathcal{F}_D := \{F \in \mathbb{R}^K : 0 \leq F_s \leq X_s - \min\{X_s, D\} \forall s \in S$ and F_s and $X_s - F_s - \min\{X_s, D\}$ are increasing in $s\}$.

The issuer's problem is $\sup_{F \in \mathcal{F}_D} U_D(F)$, where for any $F \in \mathcal{F}_D$,

$$U_D(F) := \sum_{s \in S} \pi_s^1 F_s + \delta \sum_{s \in S} \pi_s^I (X_s - \min\{X_s, D\} - F_s).$$

Let $s_D = \max\{s \in S : X_s \leq D\}$. For simplicity, suppose that $X_{s_D} = D$.

Proposition 7. *Suppose the issuer already has debt outstanding with face value D. Then, the optimal security is described by*

$$\forall s \in S, \quad F_s = \begin{cases} 0 & \textit{if } s \leq s_D \\ F_{s-1} + X_s - X_{s-1} & \textit{if } \pi^1(A_s) \geq \delta \pi^I(A_s) \textit{ and } s > s_D, \\ F_{s-1} & \textit{if } \pi^1(A_s) < \delta \pi^I(A_s) \textit{ and } s > s_D. \end{cases} \tag{7}$$

Proposition 7 shows that the firm in our model may stop the issuance of all securities when it becomes over-levered, and is thus similar to the underinvestment result in Heaton (2002). This prediction contrasts with that of informational theories of security design as well as with tradeoff models, in which the firm may start to issue equity instead of debt when it has preexisting debt. Our model's prediction is supported by empirical evidence in Erel et al. (2011), who show that low market sentiment can lead firms not only to stop equity issuances but to not access credit markets at all.

Correlated assets and disagreement on correlations. We now briefly discuss the possibility of having assets with correlated returns, and of having disagreement about the correlation of these assets between issuer and market. For simplicity, we focus on the case in which there is a single investor.

Consider first the case in which the underlying assets' returns are not *iid*. In the Appendix we consider a simple setting with two assets, each of which can yield two possible returns X_1 and X_2, as in Example 2 – the only difference is that we allow these returns to be correlated. Consistent with the time-series variation in the issuance of asset-backed securities discussed above, we show that pooling remains optimal as long as the correlation between the underlying assets is not too high relative to the disagreement in beliefs.

Second, our model assumes that issuer and market disagree about the return distribution of each of the underlying assets, but agree about the correlation between these assets (i.e., they agree that returns are *iid*). Disagreement about the correlation in the assets' return can strengthen the investor's incentives for pooling. To see this, consider again the setting in Example 2. Suppose that the market believes that the two assets are *iid*, while the issuer believes that the two assets are perfectly correlated. Assume again that $\pi > 1-\delta(1-\pi^I)$, so that the optimal security backed by asset X^a has $F_s = X_1$ for $s = 1, 2$. The issuer's profits from selling the securities separately are given by $2X_1+2\delta(1-\pi^I)(X_2-X_1)$, while her payoff from selling security $F_Y = \min\{Y, X_1 + X_2\}$ now is $2X_1 + (1 - \pi^2 + \delta(1 - \pi^I))(X_2 - X_1)$. Pooling is strictly optimal whenever $\pi \in (1 - \delta(1 - \pi^I), \sqrt{1 - \delta(1 - \pi^I)})$.

7 Conclusion

This paper offers a simple but broadly applicable theory of security design based on the premise that issuer and market openly disagree about the asset's cash-flow distribution. We show that an issuer may strictly prefer to sell securities backed by a pool of assets (instead of issuing one security for each asset). In addition, when there is disagreement among investors, the issuer optimally sells different tranches to the market. We further show that differences in beliefs can make pooling and tranching complements. Finally, we argue that informational theories of security design in which all agents are risk-neutral and share a common prior don't yield predictions on how the issuer will tranche the cash-flows she sells to the market. To obtain such predictions – and an interaction between pooling and tranching – one needs to incorporate differences in beliefs, or in preferences.

A Proofs

Proofs of Section 3

Proof of Lemma 1. Let $(F^1, F^2) \in \mathcal{F}$ be a solution to the issuer's problem. Note that the lemma clearly holds if the two securities (F^1, F^2) are bought by different types of investors. If the two securities (F^1, F^2) are bought by investors of type t_i, then the issuer's payoff from selling security (F^1, F^2) is

$$U(F^1, F^2) = p^i(F^1) + p^i(F^2) + \delta \sum_{s\in S} \pi_s^I (X_s - F_s^1 - F_s^2)$$
$$= \sum_{s\in S} \pi_s^i (F_s^1 + F_s^2) + \delta \sum_{s\in S} \pi_s^I (X_s - F_s^1 - F_s^2).$$

Consider the pair of securities $(\tilde{F}^1, \tilde{F}^2)$ with $\tilde{F}_s^i = F_s^1 + F_s^2$ for all s and $\tilde{F}_s^j = 0$ for all s. Since investors of type t_i buy the two securities F^1, F^2, it must be that $p^i(F^j) \geq p^{-i}(F^j)$ for $j = 1, 2$. Note that $p^i(\tilde{F}^i) = p^i(F^1) + p^i(F^2)$ and $p^{-i}(\tilde{F}^i) = p^{-i}(F^1) + p^{-i}(F^2)$. Hence, $p(\tilde{F}^i) = p^i(\tilde{F}^i)$. Moreover, $p^j(\tilde{F^{-i}}) = 0$ for $j = 1, 2$, and $p(\tilde{F}^{-i}) = p^{-i}(\tilde{F}^{-i})$. Finally, note that

$$U(\tilde{F}^1, \tilde{F}^2) = p^i(\tilde{F}^i) + \delta \sum_{s\in S} \pi_s^I (X_s - F_s^1 - F_s^2)$$
$$= \sum_{s\in S} \pi_s^i (F_s^1 + F_S^2) + \delta \sum_{s\in S} \pi_s^I (X_s - F_s^1 - F_s^2) = U(F^1, F^2).$$

□

Proof of Proposition 1. Fix a pair of securities $(F^1, F^2) \in \mathcal{F}$ such that, for $j = 1, 2$, security F^j is bought by investors of type t_j. The issuer's payoff from selling this pair of securities is

$$\begin{aligned} U(F^1, F^2) &= \sum_{s=1}^K \pi_s^1 F_s^1 + \sum_{s=1}^K \pi_s^2 F_s^2 + \delta \sum_{s=1}^K \pi_s^I (X_s - F_s^1 - F_s^2) \\ &= (F_1^1 + F_1^2)(1 - \delta) + \sum_{s=2}^K \left(\pi^1(A_s) - \delta \pi^I(A_s)\right) \left(F_s^1 - F_{s-1}^1\right) \\ &\quad + \sum_{s=2}^K \left(\pi^2(A_s) - \delta \pi^I(A_s)\right) \left(F_s^2 - F_{s-1}^2\right) + \delta \sum_{s=1}^K \pi_s^I X_s. \end{aligned} \tag{8}$$

Note that any pair of securities $(F^1, F^2) \in \mathcal{F}$ must be such that: (i) $F_1^1 + F_1^2 \in [0, X_1]$, (ii) for all $s > 1$, $F^1_s + F^2_s \in [F^1_{s-1} + F^2_{s-1}, F^1_{s-1} + F^2_{s-1} + X_s - X_{s-1}]$ and (iii) for $i = 1, 2$, $F^i_s \geq F^i_{s-1}$. From equation (8), it is optimal for the issuer to set $F^1_1 + F^2_1 = X_1$. Moreover, for $s \in S \setminus \{1\}$ and $j = 1, 2$, it is optimal to set $F^j_s = F^j_{s-1} + X_s - X_{s-1}$ if $\pi^j(A_s) > \max\left\{\delta\pi^I(A_s), \pi^{-j}(A_s)\right\}$, and to set $F^j_s = F^j_{s-1}$ otherwise.

□

Proofs of Section 4

Proof of Proposition 2. By Proposition 1, under Assumption 1 the optimal security backed by a single asset X^a is $F = \min\{X_s, X_k\}$. Note that selling two individual securities F, each backed by one of the assets, is the same as selling security $F^S \in \mathcal{F}_{Y^1}$ with

$$F^S_{s,s'} = \begin{cases} X_s + X_{s'} & \text{if } s, s' \leq k, \\ X_k + X_{s'} & \text{if } s > k, s' \leq k, \\ X_s + X_k & \text{if } s \leq k, s' > k, \\ 2X_k & \text{if } s > k, s' > k. \end{cases}$$

Suppose there exists exists $\hat{s}, \hat{s}' \in S$ with $\hat{s}' > k$ and with $\hat{s} \in \{1\} \cup (k, \hat{s}']$, such that $\pi^1(A_{\hat{s}'})(2\pi^1(A_{\hat{s}}) - \pi^1(A_{\hat{s'}})) > \delta\pi^I(A_{\hat{s}})(2\pi^I(A_{\hat{s}'}) - \pi^I(A_{\hat{s}}))$. Define

$$\hat{S} := \left\{(s, s') \in S^2 : s \geq \hat{s} \text{ and } s' \geq \hat{s'} \text{ or } s \geq \hat{s}' \text{ and } s' \geq \hat{s}\right\},$$

and consider security F^P with

$$F^P_{s,s'} = \begin{cases} F^S_{s,s'} & \text{if } s, s' \notin \hat{S}, \\ F^S_{s,s'} + \Delta = F^S_{s,s'} + X_{\hat{s}} - X_{\hat{s}-1} & \text{otherwise.} \end{cases}$$

We first show that security F^P satisfies the monotonicity requirements; i.e. that $F^P \in \mathcal{F}^1_Y$.

Clearly, for all s, s', $F^P_{s,s'} \in [0, X_s + X_{s'}]$. Moreover, since $F^S_{s,s'}$ is increasing in s and s', it follows that $F^P_{s,s'}$ is also increasing in s and s'. We now show that $X_s + X_{s'} - F^P_{s,s'}$ is increasing in s; the proof that $X_s + X_{s'} - F^P_{s,s'}$ is increasing in s' is symmetric and omitted.

Consider first s, s' such that s, s' and $s-1, s'$ both belong to $\hat{S}$. In this case,

$$
\begin{aligned}
&\left(X_s + X_{s'} - F^P_{s,s'}\right) - \left(X_{s-1} + X_{s'} - F^P_{s-1,s'}\right) \\
=&\left(X_s + X_{s'} - F^S_{s,s'} - \Delta\right) - \left(X_{s-1} + X_{s'} - F^S_{s-1,s'} - \Delta\right) \geq 0,
\end{aligned}
$$

where the inequality follows since $F^S_{s,s'}$ is monotonic. Consider next s, s' such that both s, s' and $s-1, s'$ don't belong to $\hat{S}$. Then,

$$
\begin{aligned}
&\left(X_s + X_{s'} - F^P_{s,s'}\right) - \left(X_{s-1} + X_{s'} - F^P_{s-1,s'}\right) \\
=&\left(X_s + X_{s'} - F^S_{s,s'}\right) - \left(X_{s-1} + X_{s'} - F^S_{s,s'}\right) \geq 0.
\end{aligned}
$$

Lastly, consider (s, s') such that $(s, s') \in \hat{S}$ and $(s-1, s') \notin \hat{S}$. There are two cases to consider: $\hat{s} = 1$ or $\hat{s} > k$. Suppose first that $\hat{s} = 1$. Note that, for all $s \geq 2$, $(s-1, s') \in \hat{S}$ whenever $(s, s') \in \hat{S}$. Hence, if $(s, s') \in \hat{S}$ and $(s-1, s') \notin \hat{S}$, it must be that $s = 1$, in which case monotonicity is trivially satisfied (since $s - 1 = 0 \notin S$).

Consider next the case with $\hat{s} > k$. Note that $\hat{s} > k$, together with $\hat{s}' > k$, implies that $F^S_{s,s'} = X_k + X_k = F^S_{s-1,s'}$ for all $(s, s') \in \hat{S}$. Then,

$$
\begin{aligned}
&\left(X_s + X_{s'} - F^P_{s,s'}\right) - \left(X_{s-1} + X_{s'} - F^P_{s-1,s'}\right) \\
=&\left(X_s + X_{s'} - 2X_k - \Delta\right) - \left(X_{s-1} + X_{s'} - 2X_k\right) \\
=&X_s - X_{s-1} - \Delta = 0.
\end{aligned}
$$

Hence, F^P belongs to $\mathcal{F}^1_Y$.

Note next that, for any beliefs π over S,

$$\begin{aligned}\sum_s \sum_{s'} \pi_s \pi_{s'} (F^P_{s,s'} - F^S_{s,s'}) &= \sum_{s=\hat{s}}^{K} \pi_s \sum_{s'=\hat{s}'}^{K} \pi_{s'} \Delta + \sum_{s=\hat{s}'}^{K} \pi_s \sum_{s'=\hat{s}}^{\hat{s}'-1} \pi_{s'} \Delta \\ &= \Delta \left[\pi(A_{\hat{s}}) \pi(A_{\hat{s}'}) + \pi(A_{\hat{s}'})(\pi(A_{\hat{s}}) - \pi(A_{\hat{s}'})) \right] \\ &= \Delta \left[\pi(A_{\hat{s}'})(2\pi(A_{\hat{s}}) - \pi(A_{\hat{s}'})) \right]. \end{aligned} \tag{9}$$

Therefore,

$$\begin{aligned} U_Y(F^P) - 2U_{X^a}(F^S) &= U_Y(F^P) - U_Y(F^S) \\ &= p_Y(F^P) - p_Y(F^S) + \delta \sum_s \sum_{s'} \pi^I_s \pi^I_{s'} (F^S_{s,s'} - F^P_{s,s'}) \\ &= \sum_s \sum_{s'} \pi^1_s \pi^1_{s'} (F^P_{s,s} - F^S_{s,s'}) + \delta \sum_s \sum_{s'} \pi^I_s \pi^I_{s'} (F^S_{s,s'} - F^P_{s,s'}) \\ &= \left(\pi^1(A_{\hat{s}'})(2\pi^1(A_{\hat{s}}) - \pi^1(A_{\hat{s}'})) - \delta \pi^I(A_{\hat{s}'})(2\pi^I(A_{\hat{s}}) - \pi^I(A_{\hat{s}'})) \right) \Delta > 0, \end{aligned}$$

where we used equation (9) and the inequality in the statement of the proposition. □

Proof of Proposition 3. Let F^1 be the optimal security when the issuer pools the two assets and sells a single security to market participants with beliefs π^1. Since the two assets have iid returns, it is without loss to assume that F^1 is symmetric: $F^1_{s,s'} = F^1_{s',s}$ for all $s, s' \in S$.[21] To establish the result, we show that $F^1 = F^S$, where F^S is the security that the issuer will

[21] To see why, suppose the seller finds it optimal to sell a security F^1 that is not symmetric. Let $\hat{F}^1$ be the security such that, for all $s, s' \in S$, $\hat{F}^1_{s,s'} = F^1_{s',s}$. Since the two assets have iid returns, securities F^1 and $\hat{F}^1$ yield the same profits to the issuer. Since F^1 satisfies the monotonicity requirements, so does $\hat{F}^1$. Let G be a security such that, for all s, s', $G_{s,s'} = \frac{1}{2}(F^1_{s,s'} + \hat{F}^1_{s,s'})$. Note that security G is symmetric, satisfies the monotonicity requirements, and gives the same profits to the issuer as security F_1.

effective sell if she were to sell two individual securities, each backed by its own asset:

$$F^S_{s,s'} = \begin{cases} X_s + X_{s'} & \text{if } s, s' \leq k, \\ X_k + X_{s'} & \text{if } s > k, s' \leq k, \\ X_s + X_k & \text{if } s \leq k, s' > k, \\ 2X_k & \text{if } s > k, s' > k. \end{cases}$$

We start by showing that $F^1_{s,s'} = F^S_{s,s'}$ for all s, s' such that $s \leq k$ and $s' \leq k$. Towards a contradiction, suppose not. Let $\hat{s} = \min\left\{s \leq k : F^1_{s,s'} < X_s + X_{s'} \text{ for some } s' \leq k\right\}$, and let $\hat{s}' = \min\left\{s' \leq k : F^1_{\hat{s},s'} < X_{\hat{s}} + X_{s'}\right\}$. Note that $F^1_{\hat{s},\hat{s}'} = X_{\hat{s}} + X_{\hat{s}'} - \epsilon$ for some $\epsilon > 0$. Consider security $\tilde{F}$, with

$$\tilde{F}_{s,s'} = \begin{cases} F^1_{s,s'} & \text{if } s < \hat{s} \text{ or } s' < \hat{s}' \\ F^1_{s,s'} + \epsilon & \text{otherwise.} \end{cases}$$

We now show that $\tilde{F} \in \mathcal{F}^1_Y$. Since $F^1_{s,s'}$ is increasing in s and s', then $\tilde{F}_{s,s'}$ is also increasing in s and s'. Moreover, monotonicity of F^1 implies that $\tilde{F}_{s,s'} \in [0, X_s + X_{s'}]$ for all s, s'.[22] We now show that $X_s + X_{s'} - \tilde{F}_{s,s'}$ is increasing in s; the proof that $X_s + X_{s'} - \tilde{F}_{s,s'}$ is increasing in s' is symmetric and omitted. Consider first (s, s') with $s < \hat{s}$, so that $\tilde{F}_{s,s'} = F^1_{s,s'}$ and $\tilde{F}_{s-1,s'} = F^1_{s-1,s'}$. Monotonicity of F^1 then implies $X_s + X_{s'} - \tilde{F}_{s,s'} \geq X_{s-1} + X_{s'} - \tilde{F}_{s-1,s'}$. Consider next (s, s') with $s > \hat{s}$, so $\tilde{F}_{s,s'} = F^1_{s,s'} + \epsilon$ and $\tilde{F}_{s-1,s'} = F^1_{s-1,s'} + \epsilon$. Since F^1 is monotonic, $X_s + X_{s'} - \tilde{F}_{s,s'} \geq X_{s-1} + X_{s'} - \tilde{F}_{s-1,s'}$. Finally, consider (s, s') with $s = \hat{s}$. If $s' < \hat{s}'$, then $\tilde{F}_{s,s'} = F^1_{s,s'}$ and $\tilde{F}_{s-1,s'} = F^1_{s-1,s'}$, and so $X_s + X_{s'} - \tilde{F}_{s,s'} \geq X_{s-1} + X_{s'} - \tilde{F}_{s-1,s'}$ by monotonicity of F^1.

Suppose next that $s' \geq \hat{s}'$ (and continue to assume $s = \hat{s}$). Then, $\tilde{F}_{s,s'} = F^1_{s,s'} + \epsilon$ and $\tilde{F}_{s-1,s'} = F^1_{s-1,s'} = X_{s-1} + X_{s'}$, where the last equality follows since $s = \hat{s} =$

[22] Indeed, since $X_s + X_{s'} - F^1_{s,s'}$ is increasing in s, s' and since $F^1_{\hat{s},\hat{s}'} = X_{\hat{s}} + X_{\hat{s}'} - \epsilon$, it follows that $X_{\hat{s}} + X_{\hat{s}'} - F^1_{s,s'} \geq \epsilon$ for all $(s, s') \geq (\hat{s}, \hat{s}')$.

$\min\left\{s \le k : F^1_{s,s'} < X_s + X_{s'} \text{ for some } s' \le k\right\}$. Then,

$$\begin{aligned}
&(X_s + X_{s'} - \tilde{F}_{s,s'}) - (X_{s-1} + X_{s'} - \tilde{F}_{s-1,s'}) \\
=&X_s + X_{s'} - F^1_{s,s'} - \epsilon
\end{aligned} \tag{10}$$

Since F^1 is monotonic, it must be that

$$\begin{aligned}
X_s + X_{s'} - F^1_{s,s'} &\ge X_s + X_{\hat{s}'} - F^1_{s,\hat{s}'} \\
&= X_{\hat{s}} + X_{\hat{s}'} - F^1_{\hat{s},\hat{s}'} = \epsilon
\end{aligned}$$

where the first inequality follows since $s' \ge \hat{s}'$, and the equality follows since $s = \hat{s}$. Using this in (10), it follows that $X_s + X_{s'} - \tilde{F}_{s,s'} \ge X_{s-1} + X_{s'} - \tilde{F}_{s-1,s'}$. Hence, $\tilde{F} \in \mathcal{F}^1_Y$.

For any beliefs π,

$$\sum_s \sum_{s'} \pi_s \pi_{s'} (\tilde{F}_{s,s'} - F^1_{s,s'}) = \sum_{s=\hat{s}}^K \pi_s \sum_{s'=\hat{s}'}^K \pi_{s'} \epsilon = \epsilon \pi(A_{\hat{s}}) \pi(A_{\hat{s}'}). \tag{11}$$

Note then that

$$\begin{aligned}
U_Y(\tilde{F}) - U_Y(F^1) &= p_Y(\tilde{F}) - p_Y(F^1) + \delta \sum_s \sum_{s'} \pi^I_s \pi^I_{s'} (F^1_{s,s'} - \tilde{F}_{s,s'}) \\
&= \sum_s \sum_{s'} \pi^1_s \pi^1_{s'} (\tilde{F}_{s,s'} - F^1_{s,s'}) + \delta \sum_s \sum_{s'} \pi^I_s \pi^I_{s'} (F^1_{s,s'} - \tilde{F}_{s,s'}) \\
&= \epsilon \left[\pi^1(A_{\hat{s}}) \pi^1(A_{\hat{s}'}) - \delta \pi^I(A_{\hat{s}}) \pi^I(A_{\hat{s}'}) \right],
\end{aligned}$$

where we used equation (11). By inequality (4),

$$\pi^1(A_{\hat{s}}) \pi^1(A_{\hat{s}'}) > \delta \pi^I(A_{\hat{s}}) \pi^I(A_{\hat{s}'}),$$

and so $U_Y(\tilde{F}) > U_Y(F^1)$. But this cannot be, since security F^1 was optimal. Hence, it must be that $F^1_{s,s'} = F^S_{s,s'}$ for all $(s, s') \leq (k, k)$.

Next we show that $F^1_{s,s'} = F^S_{s,s'}$ for all s, s' with $s > k$ or $s' > k$. Since $F^1_{s,s'} = F^S_{s,s'} = X_s + X_{s'}$ for all s, s' with $s \leq k$ and $s' \leq k$, by monotonicity it must be that that $F^1_{s,s'} \geq \min\{X_s, X_k\} + \min\{X_{s'}, X_k\} = F^S_{s,s'}$ for all s, s' with $s > k$ or $s' > k$.

Towards a contradiction, suppose that there is s, s' with $s > k$ or $s' > k$ such that $F^1_{s,s'} > F^S_{s,s'}$. Let $\hat{s} := \min\{s \in S : F^1_{s,s'} > F^S_{s,s'}$ for some $s'\}$, and let $\hat{s}' := \min\{s' : F^1_{\hat{s},s'} > F^S_{\hat{s},s'}\}$. Let $\epsilon = F^1_{\hat{s},\hat{s}'} - F^S_{\hat{s},\hat{s}'} > 0$. Note first that, since F^1 and F^S are symmetric, it must be that $\hat{s} \leq \hat{s}'$.[23] It then follows that $\hat{s}' > k$; otherwise $k \geq \hat{s}' \geq \hat{s}$, and so $F^1_{s,s'} = F^S_{s,s'}$. Note further that, since $F^1 \in \mathcal{F}^1_Y$, it must be that $F^1_{s,s'} - F^S_{s,s'} \geq \epsilon$ for all s, s' with $s = \hat{s}$ and $s' \geq \hat{s}'$ or $s \geq \hat{s}'$ and $s' = \hat{s}$.[24]

Recall that $\epsilon = F^1_{\hat{s},\hat{s}'} - F^S_{\hat{s},\hat{s}'} > 0$, and let $\hat{F}^1$ be a security given by

$$\hat{F}^1_{s,s'} = \begin{cases} F^1_{s,s'} - \epsilon & \text{if } s = \hat{s}, s' \geq \hat{s}' \text{ or } s \geq \hat{s}', s' = \hat{s} \\ F^1_{s,s'} & \text{otherwise.} \end{cases}$$

While security $\hat{F}^1$ may not satisfy all the monotonicity requirements, it holds that $\hat{F}^1_{s,s'}$ is increasing in s and s'. Indeed, consider $\hat{F}^1_{s,s'} - \hat{F}^1_{s-1,s'}$ for some s, s'. Note that, unless $s \in \{\hat{s}, \hat{s}+1\}$ and $s' \geq \hat{s}'$ or $s' = \hat{s}$ and $s \geq \hat{s}'$, $\hat{F}^1_{s,s'} - \hat{F}^1_{s-1,s'} = F^1_{s,s'} - F^1_{s-1,s'} \geq 0$ (since F^1 is monotonic). Consider s, s' with $s = \hat{s}+1$ and $s \geq \hat{s}'$, and note that $\hat{F}^1_{s,s'} - \hat{F}^1_{s-1,s'} = F^1_{s,s'} - (F^1_{s-1,s'} - \epsilon) \geq 0$ (since F^1 is monotonic). Consider next s, s' with $s = \hat{s}$ and $s' \geq \hat{s}'$. By definition of $\hat{s}$, it must be that $\hat{F}^1_{s-1,s'} = F^1_{s-1,s'} = F^S_{s-1,s'}$. Moreover, we showed above that $F^1_{\hat{s},s'} \geq F^S_{\hat{s},s'} + \epsilon$ for all $s' \geq \hat{s}'$. Hence, $\hat{F}^1_{\hat{s},s'} - \hat{F}^1_{\hat{s}-1,s'} = F^1_{\hat{s},s'} - \epsilon - F^S_{\hat{s}-1,s'} \geq F^S_{\hat{s},s'} - F^S_{\hat{s}-1,s'} \geq 0$,

[23]To see why, suppose by contradiction that $\hat{s} > \hat{s}'$. Since $F^1_{\hat{s},\hat{s}'} > F^S_{\hat{s},\hat{s}'}$, by symmetry of F^1 and F^S it must be that $F^1_{\hat{s}',\hat{s}} > F^S_{\hat{s}',\hat{s}}$. Since $\hat{s} > \hat{s}'$, this contradicts the fact that $\hat{s} := \min\{s : F^1_{s,s'} > F^S_{s,s'}$ for some $s'\}$. Therefore, it must be that $\hat{s}' \geq \hat{s}$.

[24]Indeed, by monotonicity of F^1, $F^1_{\hat{s},s'} \geq F^1_{\hat{s},\hat{s}'}$ for all $s' \geq \hat{s}'$. Since $\hat{s}' > k$, it follows that $F^S_{\hat{s},s'} = F^S_{\hat{s},\hat{s}'}$ for all $s' \geq \hat{s}'$, and so $F^1_{\hat{s},s'} - F^S_{\hat{s},s'} \geq F^1_{\hat{s},\hat{s}'} - F^S_{\hat{s},\hat{s}'} = \epsilon$ for all $s' \geq \hat{s}'$. By symmetry of F^1 and F^S, $F^1_{s',\hat{s}} - F^S_{s',\hat{s}} \geq \epsilon$ for all $s' \geq \hat{s}'$.

where the last inequality follows since $F^S_{s,s'}$ is increasing in s, s'. Lastly, consider s, s' with $s' = \hat{s}$ and $s \geq \hat{s}'$. If $s \geq \hat{s}' + 1$, then $\hat{F}^1_{s,s'} - \hat{F}^1_{s-1,s'} = F^1_{s,s'} - \epsilon - (F^1_{s-1,s'} - \epsilon) \geq 0$. If $s = \hat{s}'$, then $\hat{F}^1_{s,s'} - \hat{F}^1_{s-1,s'} = \hat{F}^1_{\hat{s}',\hat{s}} - \hat{F}^1_{\hat{s}'-1,\hat{s}} = F^1_{\hat{s}',\hat{s}} - \epsilon - F^1_{\hat{s}'-1,\hat{s}} = F^S_{\hat{s}',\hat{s}} - F^S_{\hat{s}',\hat{s}} \geq 0$. Hence $\hat{F}^1_{s,s'}$ is increasing in s. A symmetric argument establishes that $\hat{F}^1_{s,s'}$ is increasing in s'.

Recall that $\hat{s} \leq \hat{s}'$, and that $\hat{s}' > k$. If $\hat{s} < \hat{s}'$, then for any beliefs π,

$$\sum_s \sum_{s'} \pi_s \pi_{s'} (\hat{F}^1_{s,s'} - F^1_{s,s'}) = -2\pi_{\hat{s}} \pi(A_{\hat{s}'})\epsilon.$$

Hence,

$$\begin{aligned} U_Y(\hat{F}^1) - U_Y(F^1) &= p_Y(\hat{F}) - p_Y(F^1) + \delta \sum_s \sum_{s'} \pi^I_s \pi^I_{s'} (F^1_{s,s'} - \hat{F}_{s,s'}) \\ &= \sum_s \sum_{s'} \pi^1_s \pi^1_{s'} (\hat{F}_{s,s'} - F^1_{s,s'}) + \delta \sum_s \sum_{s'} \pi^I_s \pi^I_{s'} (F^1_{s,s'} - \hat{F}_{s,s'}) \\ &= -2\epsilon \left[\pi^1_{\hat{s}} \pi^1(A_{\hat{s}'}) - \delta \pi^I_{\hat{s}} \pi(A^I_{\hat{s}'})\right] > 0, \end{aligned}$$

where the strict inequality uses (5) and $\hat{s}' > k$. If instead $\hat{s} = \hat{s}'$, then for any beliefs π,

$$\sum_s \sum_{s'} \pi_s \pi_{s'} (\hat{F}^1_{s,s'} - F^1_{s,s'}) = -\epsilon \pi_{\hat{s}} (\pi(A_{\hat{s}'}) + \pi(A_{\hat{s}'+1})),$$

and so

$$\begin{aligned} U_Y(\hat{F}^1) - U_Y(F^1) &= p_Y(\hat{F}) - p_Y(F^1) + \delta \sum_s \sum_{s'} \pi^I_s \pi^I_{s'} (F^1_{s,s'} - \hat{F}_{s,s'}) \\ &= \sum_s \sum_{s'} \pi^1_s \pi^1_{s'} (\hat{F}_{s,s'} - F^1_{s,s'}) + \delta \sum_s \sum_{s'} \pi^I_s \pi^I_{s'} (F^1_{s,s'} - \hat{F}_{s,s'}) \\ &= -\epsilon \left[\pi^1_{\hat{s}} (\pi^1(A_{\hat{s}'}) + \pi^1(A_{\hat{s}'+1})) - \delta \pi^I_{\hat{s}} (\pi(A^I_{\hat{s}'}) + \pi(A^I_{\hat{s}'+1}))\right] > 0, \end{aligned}$$

where the strict inequality again follows from (5) and $\hat{s}' > k$. Hence, in either case, security

$\hat{F}^1$ gives the issuer a strictly larger payoff than security F^1.

Recall that $F^1_{s,s'} - F^S_{s,s'} \geq 0$ for all s, s', with equality if $(s, s') \leq (k, k)$, and $F^1_{s,s'} - F^S_{s,s'} \geq \epsilon$ for all s, s' with $s = \hat{s}$ and $s' \geq \hat{s}'$ or $s \geq \hat{s}'$ and $s' = \hat{s}$. Hence, for all s, s', $\hat{F}^1_{s,s'} \geq F^S_{s,s'}$, with equality if $(s, s') \leq (k, k)$. If $\hat{F}^1 = F^S$, then we've reached a contradiction, because $\hat{F}^1$ yields a strictly larger payoff than F^1 (which is assumed to be optimal), and since $F^S \in \mathcal{F}^1_Y$.

Suppose then that $\hat{F}^1 \neq F^S$. Hence, there exists s, s', with either $s > k$ or $s' > k$, such that $\hat{F}^1_{s,s'} > F^S_{s,s'}$. Let $\hat{s}_1 := \min\{s \in S : \hat{F}^1_{s,s'} > F^S_{s,s'}$ for some $s'\}$, and let $\hat{s}'_1 := \min\{s' : \hat{F}^1_{\hat{s}_1,s'} > F^S_{\hat{s},s'}\}$. Let $\epsilon_1 = \hat{F}^1_{\hat{s}_1,\hat{s}'_1} - F^S_{\hat{s}_1,\hat{s}'_1} > 0$. Note first that $\hat{s}'_1 \geq \hat{s}_1$, and so $\hat{s}'_1 > k$.[25] Note further that, since $\hat{F}^1_{s,s'}$ is increasing in s, s', it must be that $\hat{F}^1_{s,s'} - F^S_{s,s'} \geq \epsilon_1$ for all s, s' with $s = \hat{s}_1$ and $s' \geq \hat{s}'_1$ or $s \geq \hat{s}'_1$ and $s' = \hat{s}_1$.[26] Let $\hat{F}^2$ be a security given by

$$\hat{F}^2_{s,s'} = \begin{cases} \hat{F}^1_{s,s'} - \epsilon_1 & \text{if } s = \hat{s}_1, s' \geq \hat{s}'_1 \text{ or } s \geq \hat{s}'_1, s' = \hat{s}_1 \\ \hat{F}^1_{s,s'} & \text{otherwise.} \end{cases}$$

While $\hat{F}^2$ may not satisfy all the monotonicity requirements, $\hat{F}^2_{s,s'}$ is increasing in s and in s' – the arguments establishing this are exactly the same as the arguments we used to establish that $\hat{F}^1_{s,s'}$ is increasing in s and s'. Moreover, since $\hat{F}^1_{s,s'} \geq F^S_{s,s'}$ for all s, s', and since $\hat{F}^1_{s,s'} - F^S_{s,s'} \geq \epsilon_1$ for all s, s' with $s = \hat{s}_1$ and $s' \geq \hat{s}'_1$ or $s \geq \hat{s}'_1$ and $s' = \hat{s}_1$, it follows that $\hat{F}^2_{s,s'} \geq F^S_{s,s'}$ for all s, s'.

Repeating the same arguments as above, and using inequality (5) in the statement of the proposition, we can show that $U_Y(\hat{F}^2) > U_Y(\hat{F}^1)$, and so $U_Y(\hat{F}^2) > U_Y(F^1)$. If $\hat{F}^2 = F^S$, then we've reached a contradiction, because $F^S \in \mathcal{F}^1_Y$. Otherwise, we can repeat the same

[25] The argument as to why $\hat{s}'_1 \geq \hat{s}_1$ follows the steps as in footnote 23. If $\hat{s}'_1 \leq k$, then $\hat{s}_1 \leq k$, and so $\hat{F}^1_{\hat{s},\hat{s}'} = F^S_{\hat{s},\hat{s}'}$, a contradiction.

[26] The proof of this statement follows from the same arguments as those in footnote 24, using the fact that $\hat{F}^1_{s,s'}$ is increasing in s, s' and that $\hat{F}^1$ is symmetric.

process, defining a new security $\hat{F}^3$ given by

$$\hat{F}^3_{s,s'} = \begin{cases} \hat{F}^2_{s,s'} - \epsilon_2 & \text{if } s = \hat{s}_2, s' \geq \hat{s}_2' \text{ or } s \geq \hat{s}_2', s' = \hat{s}_2 \\ \hat{F}^2_{s,s'} & \text{otherwise.} \end{cases},$$

where $\hat{s}_2 := \min\{s \in S : \hat{F}^2_{s,s'} > F^S_{s,s'} \text{ for some } s'\}$, $\hat{s}_2' := \min\{s' : \hat{F}^2_{\hat{s}_2,s'} > F^S_{\hat{s},s'}\}$ and $\epsilon_2 := \hat{F}^2_{\hat{s}_2,\hat{s}_2'} - F^S_{\hat{s}_2,\hat{s}_2'}$. Since S is finite, eventually this process will converge to a security $\hat{F}^n = F^S$, which gives the issuer a strictly larger payoff than F^1, which cannot be. Hence, when (5) holds, the optimal security is F^S, and so the issuer does not benefit from pooling the assets. □

Lemma 2. *Suppose that π^1 satisfies Assumption 1, and that:*

$$\sum_{s\in S}\left[(\pi_s^1 - \delta\pi_s^I)(\min\{s,k\}+1) + \sum_{s'=2}^{k}(\pi_s^1\pi^1(A_{s'}) - \delta\pi_s^I\pi^I(A_{s'}))\right]$$
$$> \sum_{s\in S}\left[(\pi_s^2 - \delta\pi_s^I)\mathbf{1}_{\pi_s^2>\delta\pi_s^I}(s+1) + \sum_{s'=2}^{k}(\pi_s^2\pi^2(A_{s'}) - \delta\pi_s^I\pi^I(A_{s'}))\mathbf{1}_{\pi_s^2\pi^2(A_{s'})>\delta\pi_s^I\pi^I(A_{s'})}\right].$$

Then, $U_Y^1 > U_Y^2$.

Proof. Under Assumption 1, an issuer who does not pool the assets and who designs a security exclusively for investors of type t_1 finds it optimal to sell security F^1 with $F_s^1 = \min\{X_k, X_s\}$. Note then that, if the issuer pools the asset, her profits from designing a single security for investors of type t_1 is at least as large as what she gets if she sells security F^S with $F^S_{s,s'} = \min\{X_k, X_s\} + \min\{X_k, X_{s'}\}$. Let $\mathbb{E}_{\pi^I}[X] := \sum_{s\in S}\pi_s^I X_s$, and for all $s \in S$, let

$F^S_{s,0} = 0$. Then,

$$\begin{aligned}
U^1_Y &\geq \sum_{s\in S}\sum_{s'\in S}\pi^1_s\pi^1_{s'}F^S_{s,s'} + \delta\sum_{s\in S}\sum_{s'\in S}\pi^I_s\pi^I_{s'}(X_s + X_{s'} - F^S_{s,s'}) \\
&= \sum_{s\in S}\sum_{s'\in S}(\pi^1_s\pi^1_{s'} - \delta\pi^I_s\pi^I_{s'})F^S_{s,s'} + \delta 2\mathbb{E}_{\pi^I}[X] \\
&= \sum_{s\in S}\left[(\pi^1_s - \delta\pi^I_s)F^S_{s,1} + \sum_{s'\geq 2}(\pi^1_s\pi^1(A_{s'}) - \delta\pi^I_s\pi^I(A_{s'}))(F^S_{s,s'} - F^S_{s,s'-1})\right] + \delta 2\mathbb{E}_{\pi^I}[X] \\
&= \Delta\sum_{s\in S}\left[(\pi^1_s - \delta\pi^I_s)(\min\{s,k\}+1) + \sum_{s'=2}^{k}(\pi^1_s\pi^1(A_{s'}) - \delta\pi^I_s\pi^I(A_{s'}))\right] + \delta 2\mathbb{E}_{\pi^I}[X]
\end{aligned}$$

where the last equality follows since $F^S_{s,1} = \min\{X_s, X_k\} + X_1 = \Delta(\min\{s,k\}+1)$ and since, for all $s' \geq 2$, $F^S_{s,s'} - F^S_{s,s'-1} = \Delta \times \mathbf{1}_{s'\leq k}$.

Consider next an issuer who designs a security to be sold exclusively to investors of type t_2. For any monotonic security F backed by the pool of assets $Y = X_1 + X_2$, the issuer's payoff is

$$\begin{aligned}
&\sum_{s\in S}\sum_{s'\in S}\pi^2_s\pi^2_{s'}F_{s,s'} + \delta\sum_{s\in S}\sum_{s'\in S}\pi^I_s\pi^I_{s'}(X_s + X_{s'} - F_{s,s'}) \\
=&\sum_{s\in S}\sum_{s'\in S}(\pi^2_s\pi^2_{s'} - \delta\pi^I_s\pi^I_{s'})F_{s,s'} + \delta 2\mathbb{E}_{\pi^I}[X] \\
=&\sum_{s\in S}\left[(\pi^2_s - \delta\pi^I_s)F_{s,1} + \sum_{s'\geq 2}(\pi^2_s\pi^2(A_{s'}) - \delta\pi^I_s\pi^I(A_{s'}))(F_{s,s'} - F_{s,s'-1})\right] + \delta 2\mathbb{E}_{\pi^I}[X] \\
\leq&\Delta\sum_{s\in S}\left[(\pi^2_s - \delta\pi^I_s)\mathbf{1}_{\pi^2_s > \delta\pi^I_s}(s+1) + \sum_{s'=2}^{k}(\pi^2_s\pi^2(A_{s'}) - \delta\pi^I_s\pi^I(A_{s'}))\mathbf{1}_{\pi^2_s\pi^2(A_{s'}) > \delta\pi^I_s\pi^I(A_{s'})}\right] \\
&\quad + \delta 2\mathbb{E}_{\pi^I}[X],
\end{aligned}$$

where the inequality follows since $F_{s,1} \in [0, X_s + X_1] = [0, \Delta(s+1)]$ and since, for all s, s' with $s' \geq 2$, monotonicity requires that $F_{s,s'} - F_{s,s'-1} \in [0, X_{s'} - X_{s'-1}] = [0, \Delta]$. Since the inequality above holds for any monotonic security F, it follows that

$$U_Y^2 \leq \Delta \sum_{s \in S} \left[(\pi_s^2 - \delta \pi_s^I) \mathbf{1}_{\pi_s^2 > \delta \pi_s^I} (s+1) + \sum_{s'=2}^{k} (\pi_s^2 \pi^2(A_{s'}) - \delta \pi_s^I \pi^I(A_{s'})) \mathbf{1}_{\pi_s^2 \pi^2(A_{s'}) > \delta \pi_s^I \pi^I(A_{s'})} \right] + \delta 2 \mathbb{E}_{\pi^I}[X].$$

Combining the inequalities above, we get

$$\begin{aligned} U_Y^1 - U_Y^2 \geq\ & \Delta \sum_{s \in S} \left[(\pi_s^1 - \delta \pi_s^I)(\min\{s, k\} + 1) + \sum_{s'=2}^{k} (\pi_s^1 \pi^1(A_{s'}) - \delta \pi_s^I \pi^I(A_{s'})) \right] \\ & - \Delta \sum_{s \in S} \left[(\pi_s^2 - \delta \pi_s^I) \mathbf{1}_{\pi_s^2 > \delta \pi_s^I} (s+1) + \sum_{s'=2}^{k} (\pi_s^2 \pi^2(A_{s'}) - \delta \pi_s^I \pi^I(A_{s'})) \mathbf{1}_{\pi_s^2 \pi^2(A_{s'}) > \delta \pi_s^I \pi^I(A_{s'})} \right] > 0, \end{aligned}$$

where we used the inequality in the statement of the Lemma. □

Proof of Proposition 5. By the arguments in the main text, under Assumption 2 the optimal securities $(F^{1,i}, F^{2,i})$ backed by a single asset X^i are $F_s^{1,i} = \min\{X_s, X_k\}$ and $F_s^{2,i} = 0$. Note that selling two securities $F^{1,i}$, each backed by one of the assets, is the same as selling security $\tilde{F}^1 \in \mathcal{F}_Y$ such that

$$\tilde{F}^1_{s,s'} = \begin{cases} X_s + X_{s'} & \text{if } s, s' \leq k, \\ X_k + X_{s'} & \text{if } s > k, s' \leq k, \\ X_s + X_k & \text{if } s \leq k, s' > k, \\ 2X_k & \text{if } s > k, s' > k. \end{cases}$$

Let $\hat{S} := \left\{ (s, s') \in S^2 : s \geq \hat{s} \text{ and } s' \geq \hat{s}' \text{or } s \geq \hat{s}' \text{ and } s' \geq \hat{s} \right\}$. Suppose the issuer pools the assets and sells securities $\left(\tilde{F}^1, \tilde{F}^2\right)$, with $\tilde{F}_1$ as above, and

$$\tilde{F}^2_{s,s'} = \begin{cases} 0 & \text{if } s, s' \notin \hat{S}, \\ X_{\hat{s}} - X_{\hat{s}-1} = \Delta & \text{otherwise.} \end{cases}$$

We now show that $\left(\tilde{F}^1, \tilde{F}^2\right) \in \mathcal{F}_Y$. Note first that $\tilde{F}^1_{s,s'}$ and $\tilde{F}^2_{s,s'}$ are both increasing in s and s'. Let $\hat{F}_{s,s'} = \tilde{F}^1_{s,s'} + \tilde{F}^2_{s,s'}$. Clearly, for all s, s', $\hat{F}_{s,s'} \in [0, X_s + X_{s'}]$. Thus, to show that $\left(\tilde{F}^1, \tilde{F}^2\right) \in \mathcal{F}_Y$, we need to show that $X_s + X_{s'} - \hat{F}_{s,s'}$ is increasing in s and s'.

Consider first s, s' such that s, s' and $s-1, s'$ both belong to $\hat{S}$. In this case,

$$\begin{aligned}
&\left(X_s + X_{s'} - \hat{F}_{s,s'}\right) - \left(X_{s-1} + X_{s'} - \hat{F}_{s-1,s'}\right) \\
=&\left(X_s + X_{s'} - \tilde{F}^1_{s,s'} - \Delta\right) - \left(X_{s-1} + X_{s'} - \tilde{F}^1_{s-1,s'} - \Delta\right) \geq 0,
\end{aligned}$$

where the inequality follows since $\tilde{F}^1_{s,s'}$ is monotonic. Consider next s, s' such that both s, s' and $s-1, s'$ don't belong to $\hat{S}$. Then,

$$\begin{aligned}
&\left(X_s + X_{s'} - \hat{F}_{s,s'}\right) - \left(X_{s-1} + X_{s'} - \hat{F}_{s-1,s'}\right) \\
=&\left(X_s + X_{s'} - \tilde{F}^1_{s,s'}\right) - \left(X_{s-1} + X_{s'} - \tilde{F}^1_{s,s'}\right) \geq 0.
\end{aligned}$$

Lastly, consider (s, s') such that $(s, s') \in \hat{S}$ and $(s-1, s') \notin \hat{S}$. There are two cases to consider: $\hat{s} = 1$ or $\hat{s} > k$. Suppose first that $\hat{s} = 1$. Note that, for all $s \geq 2$, $(s-1, s') \in \hat{S}$ whenever $(s, s') \in \hat{S}$. Hence, if $(s, s') \in \hat{S}$ and $(s-1, s') \notin \hat{S}$, it must be that $s = 1$, in which case monotonicity is trivially satisfied (since $s - 1 = 0 \notin S$).

Consider next the case with $\hat{s} > k$. Note that $\hat{s} > k$, together with $\hat{s}' > k$, implies that $\tilde{F}^1_{s,s'} = X_k + X_k = \tilde{F}^1_{s-1,s'}$ for all $(s, s') \in \hat{S}$. Then,

$$\begin{aligned}
&\left(X_s + X_{s'} - \hat{F}_{s,s'}\right) - \left(X_{s-1} + X_{s'} - \hat{F}_{s-1,s'}\right) \\
=&\,(X_s + X_{s'} - 2X_k - \Delta) - (X_{s-1} + X_{s'} - 2X_k) \\
=&X_s - X_{s-1} - \Delta = 0.
\end{aligned}$$

This shows that $X_s + X_{s'} - \hat{F}_{s,s'}$ is increasing in s. A symmetric argument can be used to show that $X_s + X_{s'} - \hat{F}_{s,s'}$ is increasing in s'. Hence, $\left(\tilde{F}^1, \tilde{F}^2\right) \in \mathcal{F}_Y$.

Note that, for any beliefs π,

$$\begin{aligned} \sum_s \sum_{s'} \pi_s \pi_{s'} \tilde{F}^2_{s,s'} &= \sum_{s=\hat{s}}^{K} \pi_s \sum_{s'=\hat{s}'}^{K} \pi_{s'} \Delta + \sum_{s=\hat{s}'}^{K} \pi_s \sum_{s'=\hat{s}}^{\hat{s}'-1} \pi_{s'} \Delta \\ &= \Delta \left[\pi(A_{\hat{s}}) \pi(A_{\hat{s}'}) + \pi(A_{\hat{s}'})(\pi(A_{\hat{s}}) - \pi(A_{\hat{s}'})) \right] \\ &= \Delta \left[\pi(A_{\hat{s}'})(2\pi(A_{\hat{s}}) - \pi(A_{\hat{s}'})) \right]. \end{aligned} \tag{12}$$

The issuer's payoff from selling the two assets as separate concerns, issuing for each asset X^a securities $(F^{1,a}, F^{2,a})$ with $F^{1,a}_s = \min\{X_s, X_k\}$ and $F^{2,a}_s = 0$, is equal to

$$2U_{X^a}(F^{1,a}, F^{2,a}) = \sum_s \sum_{s'} \pi^1_s \pi^1_{s'} \tilde{F}^1_{s,s'} + \delta \sum_s \sum_{s'} \pi^I_s \pi^I_{s'} (X_s + X_{s'} - \tilde{F}^1_{s,s'}). \tag{13}$$

On the other hand, the payoff that the issuer gets from pooling the assets and selling securities $\left(\tilde{F}^1, \tilde{F}^2\right) \in \mathcal{F}_Y$ is

$$U_Y\left(\tilde{F}^1, \tilde{F}^2\right) = \sum_s \sum_{s'} \pi^1_s \pi^1_{s'} \tilde{F}^1_{s,s'} + \sum_s \sum_{s'} \pi^2_s \pi^2_{s'} \tilde{F}^2_{s,s'} + \delta \sum_s \sum_{s'} \pi^I_s \pi^I_{s'} (X_s + X_{s'} - \tilde{F}^1_{s,s'} - \tilde{F}^2_{s,s'}). \tag{14}$$

Comparing (13) and (14), it follows that

$$\begin{aligned} U_Y\left(\tilde{F}^1, \tilde{F}^2\right) - 2U_{X^a}(F^{1,a}, F^{2,a}) &= \sum_s \sum_{s'} \pi^2_s \pi^2_{s'} \tilde{F}^2_{s,s'} - \delta \sum_s \sum_{s'} \pi^I_s \pi^I_{s'} \tilde{F}^2_{s,s'} \\ &= (\pi^2(A_{\hat{s}'})(2\pi^2(A_{\hat{s}}) - \pi^2(A_{\hat{s}'})) - \delta \pi^I(A_{\hat{s}'})(2\pi^I(A_{\hat{s}}) - \pi^I(A_{\hat{s}'})))\Delta > 0, \end{aligned}$$

where the second equality follows from (12) and the strict inequality follows from the assumption in the statement of the Proposition. □

Proofs of Section 5

Proof of Proposition 6. Fix a tranching equilibrium (σ, p), and let $\hat{P}^1_{\sigma,p}, \hat{P}^2_{\sigma,p}$ be the equilibrium informational partitions of investors of type t_1 and t_2, respectively. Let $\hat{P} = \hat{P}^1_{\sigma,p} \wedge \hat{P}^2_{\sigma,p}$ be the meet of these two partitions (i.e., $\hat{P}$ is the finest common coarsening of $\hat{P}^1_{\sigma,p}$ and $\hat{P}^2_{\sigma,p}$).

Consider a state $\omega \in \Omega$ with $\sigma(\omega) = (F^1(\omega), F^2(\omega))$ such that $F^1(\omega) \neq 0$, $F^2(\omega) \neq 0$ and $p_1(\omega, \sigma(\omega)) = \mathbb{E}[F^1(\omega)|\hat{P}^i_{\sigma,p}(\omega)]$ and $p_2(\omega, \sigma(\omega)) = \mathbb{E}[F^2(\omega)|\hat{P}^{i'}_{\sigma,p}(\omega)]$ for $i, i' = 1, 2, i \neq i'$. Let $\hat{P}(\omega)$ be the element of partition $\hat{P}$ that contains ω, and note that $\hat{P}(\omega) = \cup_n P^i_n$, where each P^i_n is an element of $\hat{P}^i_{\sigma,p}$. Let E denote the event $p_1 = p_1(\omega, \sigma(\omega)) = \mathbb{E}[F^1(\omega)|\hat{P}^i_{\sigma,p}(\omega)]$ and $p_2 = p_2(\omega, \sigma(\omega)) = \mathbb{E}[F^2(\omega)|\hat{P}^i_{\sigma,p}(\omega)]$. Since prices are public, event E is common knowledge at state ω, and so $\hat{P}(\omega) \subset E$. This implies that the price of security $F^1(\omega)$ is constant and given by $p_1(\omega, \sigma(\omega)) = [F^1(\omega)|\hat{P}^i_{\sigma,p}(\omega)]$ throughout $\hat{P}(\omega)$; and so the beliefs of investors of type t_i are constant and equal to some $q^{i,\omega} \in \Delta(\Omega)$ throughout $\hat{P}(\omega)$.[27]

For any event B and any n, $q^{1,\omega}(B) = H(B \cap P^j_n)/H(P^j_n) \iff q^{1,\omega}(B)H(P^j_n) = H(B \cap P^j_n)$. Summing over all n, $q^{1,\omega}(B)H(\hat{P}(\omega)) = H(B \cap \hat{P}(\omega))$. By a symmetric argument, $q^{2,\omega}(B)H(\hat{P}(\omega)) = H(B \cap \hat{P}(\omega))$, and so $q^{1,\omega}(B) = q^{2,\omega}(B)$. That is, the two groups of investors share the same beliefs at ω. As a result, for $j = 1, 2$, $\mathbb{E}[F^j(\omega)|\hat{P}^1_{\sigma,p}(\omega)] = \mathbb{E}[F^j(\omega)|\hat{P}^2_{\sigma,p}(\omega)]$. □

[27]Recall that $\Delta(\Omega)$ is the set of distributions over Ω.

Proofs of Section 6

Proof of Proposition 7. The proof uses arguments similar to those in the proof of Proposition 1. For any security $F \in \mathcal{F}_D$, the issuer's payoff is

$$\begin{aligned} U(F) &= \sum_{s=1}^{K} \pi_s^M F_s + \delta \sum_{s=1}^{K} \pi_s^I (X_s - \min\{X_s, D\} - F_s) \\ &= F_1(1-\delta) + \sum_{s=2}^{K} \left(\pi^M(A_s) - \delta \pi^I(A_s)\right)(F_s - F_{s-1}) + \delta \sum_{s=1}^{K} \pi_s^I (X_s - \min\{X_s, D\}), \end{aligned} \tag{15}$$

Note that any security $F \in \mathcal{F}_D$ must be such that $F_s = 0$ and for all $s \leq s_D$, $F_s \in [F_{s-1}, F_{s-1} + X_s - X_{s-1}]$ for all $s > s_D$ and $F_s \geq F_{s-1}$ for all s. Moreover, any security that satisfies these conditions belongs to $\mathcal{F}_D$. From equation (15), for any $s > s_D$ it is optimal to set $F_s = F_{s-1} + X_s - X_{s-1}$ if $\pi^M(A_s) \geq \delta \pi^I(A_s)$, and to set $F_s = F_{s-1}$ if $\pi^M(A_s) < \delta \pi^I(A_s)$. □

Correlated assets and disagreement on correlation. We extend the example of section 4.2 to allow for non-zero correlation between the assets to be securitized. As in section 4.2, suppose the issuer owns two assets, X^1 and X^2, each of which can generate a return in $\{X_1, X_2\}$ (with $X_1 < X_2$). In contrast to section 4.2, suppose that the returns of assets X^1 and X^2 are correlated. Let $sk \in \hat{S} = \{11, 12, 21, 22\}$ denote the event that asset 1's return is X_s and asset 2's return is X_k. The beliefs of the issuer and market over the set of possible return realizations are, respectively, $\hat{\pi}^I$ and $\hat{\pi}^M$. For $j = I, M$, $\hat{\pi}^j_{sk}$ denotes the probability that j assigns to the event sk. We assume that the assets are symmetric, so that $\hat{\pi}^j_{12} = \hat{\pi}^j_{21}$ for $j = I, M$. The *iid* case of section 4.2 is the special case with $\hat{\pi}^j_{sk} = \pi^j_s \pi^j_k$ for $j = I, M$ and for all $sk \in \hat{S}$.

Suppose first that the issuer sells two individual securities, each backed by an asset. It can be shown that an optimal security F has $F_1 = X_1$ and $F_2 \in [F_1, X_2]$. The price that the market is willing to pay for security F is $p(F) = X_1(\hat{\pi}^M_{11} + \hat{\pi}^M_{12}) + F_2(\hat{\pi}^M_{21} + \hat{\pi}^M_{22})$; the issuer's payoff from selling this security is

$$p(F) + \delta(X_2 - F_2)(\hat{\pi}^M_{21} + \hat{\pi}^M_{22}) = X_1(\hat{\pi}^M_{11} + \hat{\pi}^M_{12}) + F_2(\hat{\pi}^M_{21} + \hat{\pi}^M_{22}) + \delta(X_2 - F_2)(\hat{\pi}^I_{21} + \hat{\pi}^I_{22}). \quad (16)$$

The issuer finds it optimal to set $F_2 = X_1$ if $\delta(\hat{\pi}^I_{21} + \hat{\pi}^I_{22}) > \hat{\pi}^M_{21} + \hat{\pi}^M_{22}$ and $F_2 = X_2$ if $\delta(\hat{\pi}^I_{21} + \hat{\pi}^I_{22}) \leq \hat{\pi}^M_{21} + \hat{\pi}^M_{22}$. In what follows we maintain the assumption that $\delta(\hat{\pi}^I_{21} + \hat{\pi}^I_{22}) > \hat{\pi}^M_{21} + \hat{\pi}^M_{22}$, so that an issuer who sells individual securities F^1 and F^2, each backed respectively by asset X^1 and X^2, finds it optimal to set $F^1_s = F^2_s = X_1$ for $s = 1, 2$.

Suppose next that the issuer pools the two assets and sells a single security backed by cash-flows $Y = X^1 + X^2$. Consider a security $F_Y = \min\{Y, X_1 + X_2\}$. The price that the market is willing to pay for security F_Y is $p(F_Y) = \hat{\pi}^M_{11} 2X_1 + (1 - \hat{\pi}^M_{11})(X_1 + X_2)$, and the issuer's payoff from selling this security is

$$p(F_Y) + \delta\hat{\pi}^I_{22}(X_2 - X_1) = \hat{\pi}^M_{11} 2X_1 + (1 - \hat{\pi}^M_{11})(X_1 + X_2) + \delta\hat{\pi}^I_{22}(X_2 - X_1). \quad (17)$$

Comparing (16) and (17), the issuer strictly prefers selling security F_Y backed by the pool of assets than selling the two individual securities $F^1_s = F^2_s = X_1$ for $s = 1, 2$ if and only if $2\hat{\pi}^M_{21} + \hat{\pi}^M_{22} = 1 - \hat{\pi}^M_{11} > \delta(1 - \pi^I_{11}) = \delta(2\hat{\pi}^I_{21} + \hat{\pi}^I_{22})$. Combining this with $\delta(\hat{\pi}^I_{21} + \hat{\pi}^I_{22}) > \hat{\pi}^M_{21} + \hat{\pi}^M_{22}$, the issuer strictly prefers to pool the assets and sell security F_Y if

$$\hat{\pi}^M_{11} \in \left(1 - \delta(\hat{\pi}^I_{21} + \hat{\pi}^I_{22}) - \hat{\pi}^M_{21}, 1 - \delta(2\hat{\pi}^I_{21} + \hat{\pi}^I_{22})\right). \quad (18)$$

If the issuer and the market both perceive the asset to be perfectly correlated (so that $\hat{\pi}^j_{21} = 0$ for $j = I, M$), the condition in (18) can never be satisfied, and hence pooling does not obtain.

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