Robust Production Function Estimation when there is Market Power*

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Abstract

The production function is an engineering relationship, but recent estimators use firm's optimal choices that depend on market power. Researchers often become puzzled: the estimator dynamic panel (DP), which is robust to market power because it does not use any FOC, often produces unsatisfactory outcomes; the estimators known as OP/LP, which are deemed inconsistent in the presence of market power, typically improve. We prove that the coincidence of DP and OP/LP, except by sampling error, is a necessary condition for consistency, and show how the improvements relate to the production function specification. We derive a novel estimator, robust to arbitrary forms of market power, based on a version of OP/LP that proxies for MC. Using this estimator, we propose a test for market power and a test for the specification, the latter based on the smaller set of assumptions used by DP.

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1 Introduction

What is the relationship between market power and the estimation of the production function? The production function is an engineering relationship that describes how technology relates inputs and output. So, there is no direct relationship. But many recent proposals for the estimation of production functions include the auxiliary use of equations based on the optimization by the firm of some objective that involves the production function. The equations are usually the derivatives of profit with respect to the variable inputs (FOCs). And these FOCs are different when the firm has some market power.

These estimators, born with the work by Olley and Pakes (1996), yearned for validity under any competitive situation. However, most of them have been in practice only justified for situations of perfect competition or almost (Levinsohn and Petrin, 2003; Gandhi, Navarro, and Rivers, 2020, or the way to implement Olley and Pakes, 1996, and Levinsohn and Petrin, 2003, proposed by Ackerberg, Caves, and Frazer, 2015, henceforth ACF). The urgency of having readily available methods to estimate production functions has often implied the neglect of the conditions for applicability of these estimators under imperfect competition.

Take the frequently encountered case of product differentiation. A market with product differentiation shows some market power that arises from the ability and incentives of companies to produce products with different characteristics. [1] Suppose that the empirical researcher accounts for the heterogeneity of characteristics and possibilities of production by means of an additive unobservable in the equation to be estimated. The estimators that use the FOC should still face the implementation in the FOC of the likely heterogeneity of marginal revenues as a consequence of market power.

In the current literature, there are two approaches to the estimation of the production function. They diverge in how they solve the problem of controlling for unobservable productivity.[2]

[1] This situation generates by itself an uncomfortable context for the user of the production function, because products and possibilities of production are distinct across firms. The production function, as a concept of economic theory, was developed as describing a situation in which the characteristics of the unique product are given and the available techniques define the set of production possibilities.

[2] How to treat unobserved productivity has been the dominant worry of researchers since Marschak and Andrews (1944) pointed at the statistical problems created by the

We will call the two approaches "dynamic panel," henceforth DP, and the OP/LP approach. DP was mainly developed by Arellano and Bond (1991) and Blundell and Bond (2000). The OP/LP approach is called this way because it originated in the articles of Olley and Pakes (1996) and Levinsohn and Petrin (2003).

In the DP approach, unobservable productivity is differentiated out after assuming that follows a first order linear Markovian or $AR(1)$ process. The OP/LP approach also assumes that the unobservable follows a Markovian first order process, and models it indistinctly as a nonlinear or linear process. The unobservable is replaced by the inverse of a function representing some optimal observable choice (usually the demand for a variable input) that contains it.[3]

A big advantage of DP is hence that one does not need to use any auxiliary relationship implying behavior. OP/LP assumes instead that some behavioral equation holds. This is often said to free the need for linearity of the Markov process in GMM estimation, but relaxing the linearity is made possible by the complementary assumption that the used FOCs hold with no errors. [4]

A well known empirical paradox, that lies in the center of this paper, is that DP often produces disappointing results while OP/LP, with more assumptions to be met, often produces more reasonable estimates (e.g. in the elasticity of capital and the elasticity of scale). We first explore the theoretical difference between the two estimators and derive what can be learned from their divergence. [5]

We show that both the DP and OP/LP estimators are consistent when

endogeneity of the inputs.

[3]Both methods can be augmented with the specification of fixed effects, but estimation becomes quite demanding from the data.

[4]There are other estimation methods available that can deal with the nonlinearity of a latent variable without using any FOC. Arellano, Blundell, and Bonhomme (2017), for example, show the nonparametric identification of a nonlinear latent process for consumer earnings, and estimate it with quantile techniques; Aguirre, Tapia, and Villacorta (2024) is a first application to production functions.

[5]Modern production function estimation is not the first time that the FOCs have been given a role. Flexible specification of the production function and its dual cost function, started by Diewert (1971) and continued by Christensen, Jorgenson, and Lau (1973), and Caves, Christensen, and Tretheway (1981), raised a role for system estimation of the main equation and share equations; See, for example, Berndt and Wood (1975) and Mc Elroy (1987).

there is perfect competition, but OP/LP is not robust when there is market power. The consistency of a non-robust estimator depends on the detailed assumptions about how the game that firms play in the market is, and their market share consequences. Many symmetry and unwanted restrictions should be set beforehand to ensure the consistency of the OP/LP estimator under market power.

Then, we derive an OP/LP estimator that is consistent under any form of market power. To make OP/LP robust it is enough to proxy marginal cost MC by means of AVC and the short-run elasticity to scale, and specify the process of productivity as linear.[6] We will call it the linear cost-share estimator. Under this specification, it happens that both DP and OP/LP are consistent and should deliver estimates that under market power only differ due to sampling error.

What is the usefulness of this new estimator? This is a relevant question because DP is already consistent. There are at least two uses. First, the new estimator provides a simple way to test the presence of market power, comparing it with the traditional way to compute the OP/LP estimators. An alternative test should be based on testing the FOCs directly, which is possible but not straightforward.

Second, and probably even more important, the estimator naturally provides a test for the specification of the production function and the FOCs, comparing the outcome with the results of DP. The test simply formalizes the idea that both estimators should coincide if there are no problems in the specification of the production function and the FOCs.

Additionally, we clarify how the presence of other unobservables in the FOCs affects the OP/LP estimators, and show how the ACF method of applying OP/LP can soften the effects by projecting the unobservables on observable variables. We also argue that ACF can be a useful procedure applied with the linear cost-share OP/LP estimator.

Two main specification problems, which create this type of unobservables, are the presence of non-neutral productivity and input market power. We integrate these cases in the development of the model and discuss the effects of any other.

Empirical research has recently stressed that productivity is likely to be biased. And there have been contributions on how to apply DP and OP/LP when productivity is non-neutral and affects in particular an input. For

[6] This was first noticed in Doraszelski and Jaumandreu (2019).

example, Doraszelski and Jaumandreu (2018) show how to replace biased productivity from a ratio of FOCs, and Demirer (2025) generalizes the technique. The dominant interest in this field is labor-augmenting productivity, henceforth LAP, presumably very related to the dominant form of current technological progress. In what follows, we systematically take into account the possibility of LAP.

Input market power can be as relevant as product market power, and it affects the first order conditions in a similar way. We also show summarily but systematically how to treat input market power when it is present, and we discuss the way to detect this mispecification if it is binding.

To illustrate the relevance of the problems, the realism of the circumstances, and the working of the procedures we present, we estimate the production function for the sample of US manufacturing Compustat firms used in Jaumandreu and Mullens (2024). It is a sample with more than 5,000 firms and 60,000 observations that is likely to exhibit the more diverse degrees of market power. On the one hand, the test for market power gives the unequivocal answer that market power is present. On the other hand, the estimation by DP and OP/LP diverges when naively applied to a Cobb-Douglas specification, and passes the specification test when applied to its enlargement into a translog with LAP that shows elasticity of substitution less than one and falling labor shares. Applied to this new specification, neither DP nor feasible OP/LP is better. The second test therefore detects that there is something wrong in the specification of the production function and that its flexibilization including LAP solves the problem. We think that this constitutes a reasonable place to start exploration for refinements.

The remainder of the paper is organized as follows. Section 2 comments on the relationship of the paper with the literature. Sections 3 and 4 explain the consistency of DP and OP/LP under perfect competition, put them in a common framework, and derive the new estimator. Section 5 explains the properties of the estimators under market power. Section 6 deals with the failure of the FOCs and section 7 with OP/LP implementation, interpreting ACF. Section 8 develops the test for market power, and section 9 the specification test. Section 10 develops the example with Compustat firms, and section 11 concludes. The five appendices deal with identification, marginal revenue modeling, conduct specification, statistical specification tests, and comment on some additional regressions, respectively.

2 Relation to the literature

The literature on the new estimators for the production function has always been very aware of the need to deal with market power. Olley and Pakes (1996) consider that the firms in the market are playing a dynamic oligopoly game and justify the simplification of the vector of state variables by means of symmetry that includes common input prices. Griliches and Mairesse (1998), writing contemporaneously on the "interesting new approach" of OP, worry if this treatment of the state variables may be ignoring some relevant dimensions as the expectations on the cost of investment. Levinsohn and Petrin (2003) define their setting as a competitive environment, where firms take as given output and input prices, and warn that the model can be generalized to imperfect competition but then it will depend on the specifics of competition.

Ackerberg, Caves, and Frazer (2015), discussing when revenues can replace physical quantities (common output prices), introduce an explicit discussion about the difficulties to invert the demand for an input when the demand for output and/or the supply for an input are downward and upward curves (i.e., there is market power). They warn that, in this situation, even assuming identical curves may be not enough. Gandhi, Navarro, and Rivers (2020) make clear that their model for nonparametric estimation of the production function is developed assuming perfect competition in the output and intermediate markets.[7]

More recently, a few discussions have dealt in one way or another with the ability of the OP/LP framework to address situations with market power. Bond, Hashemi, Kaplan and Traina (2021) stress how the absence of reliable information on firm-level output prices makes difficult the estimation of structural elasticities and hence market power and point at the robustness of the DP approach. Doraszelski and Jaumandreu (2021) develop the biases that affect an OP/LP procedure given the likely presence of correlated unobservable demand heterogeneity. Ackerberg and De Loecker (2024) is a discussion of how to expand the OP/LP estimators to include "sufficient statistics" to account for imperfect competition under behavioral and symmetry assumptions.

In the first place, this paper makes a contribution to these discussions. It

[7]Only online Appendix O6 shows how the model can be applied specifying a parametric CES demand for output, together with the assumption of monopolistic competition, the version that, in fact, many researchers prefer given their needs.

deals with how to construct estimators that are robust to market power, in the sense that they do not depend on the specification of the details of the game the firms play. We provide a useful new estimator and we develop a guide to conduct the specification more than a rule that applies to all sizes and shapes.

A long list of papers has recently stressed that the presence of Hicks neutral productivity should be complemented with the presence of biased productivity, particularly in the form of LAP. See Doraszelski and Jaumandreu (2018, 2019), Raval (2019, 2023), Zhang (2019), Demirer (2025), Jaumandreu and Mullens (2024), Kusaka, Okazaki, Onishi, and Wakamori (2024), and Zhao, Malikov and Kumbhakar (2024).[8]

We add to this literature by uncovering that the mispecification revealed by our estimators, when applied to a sample of Compustat firms, is redressed when we consider an specification that allows shares in cost and elasticities to change from firm to firm and over time.

A recent literature has tried to assess market power in the input markets simultaneously with market power in the product market. See, for example, the papers by Dobbelaere and Mairesse (2013, 2018), Yeh, Macaluso, Hershbein (2022), Rubens (2023), and Azzam, Jaumandreu, and Lopez (2025). The estimators OP/LP are not robust to the presence of unspecified input market power. We show how the tools that we have developed can be applied to the detection of input market power that affects the estimation of the production function and, summarily, how they can be used for consistent estimation under this presence.

In the empirical exercise, we carry out otherwise a pioneering modeling of firm-level different dimensions of productivity in US manufacturing. It confirms the biased technological change that Raval (2019) found with Census of Manufacturing data on plants, and Demirer (2025) with Compustat firms. It provides a rich characterization on the firm dynamics of labor-augmenting productivity (see Jaumandreu and Mullens, 2024), with a flexible production function and subject to the rigor of the specification tests.

[8]A recent literature is exploring the nonparametric estimation of a unique productivity term, freely interacted with the inputs. See Ackerberg, Hahn, and Pan (2023) and Pan (2024).

3 DP and OP/LP under perfect competition

Let us first clarify the properties and relationship between the two estimators in perfect competition. The assumption of perfect competition implies that the price of the output is common for all firms and equals the marginal cost. Firms differ in size, though, because they differ in their marginal cost curves. The usual time and information assumptions are as follows. Firms choose variable inputs labor l and materials m at time t, when productivity becomes their knowledge, but capital k needs time to build and is given as chosen one period before.[9]

Assume a population of firms (we drop the firm and time subscripts). Write the production function in logs as

$$q = f(\mathbf{x}) + \omega + \varepsilon, \tag{1}$$

where $f(\mathbf{x}) = \ln F(\mathbf{x})$, $\mathbf{x} = \{k, l, m\}$ is the capital, labor and materials logs, ω is Hicks-neutral productivity, and ε is an observation error, not autocorrelated and uncorrelated with all variables known at t. Sometimes we will use the notation $q^* = f(\mathbf{x}) + \omega$ for the output without error.

Everything we are going to say is compatible with the presence of LAP. To see this it is enough to suppose that the labor input is $l^* = l + \omega_L$ and LAP ω_L is controlled for observables.[10]

A first order Markov process establishes

$$\omega = g(\omega_{-1}) + \xi, \tag{2}$$

where $g(\cdot)$ is an unknown function and ξ a mean-independent error.

DP

DP assumes that productivity follows the linear Markov process $\omega = \rho\omega_{-1} + \xi$. The implication is that we can "pseudo-differentiate" equation (1) (subtract the lagged equation multiplied by ρ) and unobservable productivity drops

$$q = \rho q_{-1} + f(\mathbf{x}) - \rho f(\mathbf{x}_{-1}) + \xi + \varepsilon - \rho\varepsilon_{-1}. \tag{3}$$

[9]Some researchers treat labor also as predetermined, but this situation is in general less demanding and we will not deal with it in detail.

[10]For example, Doraszelski and Jaumandreu (2018) show that the expression $m - l = cons - \sigma(p_M - w) + (1 - \sigma)\omega_L$, exact in the CES case and a linear approximation for any production function separable in capital, can be used to solve for ω_L. Demirer (2025) generalizes this equation. Zhao, Malikov, and Kumbhakar (2024) show that an equation of this type is possible without separability for the translog specification.

From the point of view of estimation, the inputs of the $\mathbf{x}$ vector that are set at t, when the shock of the Markov process is known, are correlated with ξ and should be instrumented. Take as variable the inputs l and m. If the production function $f(\cdot)$ only requires the estimation of three parameters (additional to the constant), we need four instruments because we have to estimate the extra parameter ρ (which introduces nonlinearity in the model). The model is exactly identified using k, k_{-1}, l_{-1} and m_{-1} as instruments.[11]

It can be assumed that lagged input and output prices are non-correlated with ξ. Then, using them as instruments gets overidentifying restrictions. Cost and firm-demand shifters can be used as additional instruments.

OP/LP

The FOCs of variable inputs are used to replace unobservable productivity.[12]

$$P\frac{\partial F(\mathbf{x})}{\partial X}\exp(\omega) = W_X, \tag{4}$$

where P is the price of the output, $X = L, M$ and $W_X = W, P_M$. Unobserved productivity ω can be obtained by inverting one of these FOC or using a combination of both. A combination of the first order conditions drops one variable input including both input prices in addition to the price of the output. This is the unconditional demand for the input that remains $X = X(K, P, W, P_M, \omega)$. With perfect competition, we expect P to be common between firms, so the only variation of P is over time and can be subsumed in a system of time dummies. But the prices of the inputs are not necessarily the same and must be explicitly included except when equality across firms is assumed.

The model can be extended to the case of input market power by assuming that the input price in the FOC is $W_X^* = W_X(1+\tau)$, where the markdown τ is either an additional parameter to estimate or is controlled for observables.[13]

[11] A more complete discussion on identification is carried out in Appendix A, after presenting OP/LP.

[12] Levinsohn and Petrin (2003) inverted an input demand function, which they simplified assuming common prices; Olley and Pakes (1996) used a nonparametric approximation to the inversion of investment, with price expectations and other state variables common across firms; Many practitioners have tended to suppress prices in all circumstances and, sometimes, add other variables under the argument of controlling for heterogeneity. Our definition and discussion can be seen as taking literally the first proposal.

[13] Rubens (2022); Croft, Luo, Mogstad and Setzler (2025); and Rubens, Wu and Xu

Let us use, for the moment and without loss of generality, only one FOCs (sometimes this has been called using the demand for a variable input conditional on the other; Ackerberg, Caves and Frazer, 2015)[14]

$$\omega = w_X - p - \ln \frac{\partial F(\mathbf{x})}{\partial X}.$$

The assumption that ω follows a first order Markov process allows us to write the production function replacing ω_{-1} by its expression according to the inverse of the conditional input demand

$$q = f(\mathbf{x}) + g(w_{X,-1} - p_{-1} - \ln \frac{\partial F(\mathbf{x}_{-1})}{\partial X_{-1}}) + \xi + \varepsilon. \tag{5}$$

The unknown function $g(\cdot)$ is typically specified by polynomials and the model can be easily estimated in one step using nonlinear GMM.[15]

Note that the derivatives of $F(\mathbf{x})$ will include at most the same parameters as $F(\mathbf{x})$,so $f(\mathbf{x})$ and $\frac{\partial F(\mathbf{x}_{-1})}{\partial X_{-1}}$ are linked by equality restrictions, even if we are dealing with a flexible specification.[16] We face exactly the same problem of endogeneity as before: the variable inputs l and m are correlated with ξ. If we have to estimate four parameters, the variables k, k_{-1}, l_{-1}, and m_{-1} are enough for identification. Prices and shifters can be used as before as additional instruments. As $g(\cdot)$ is usually made up of polynomials, it seems natural to expand the set of instruments with the powers of the instruments.

(2025 a,b) use the production function augmented with a labor supply. Azzam, Jaumandreu, and Lopez (2025) treat τ alternatively as a parameter and a function of observables, and argue that the production function together with the FOCS can identify input market power.

[14]Later we will use the demand for the input conditional on output, which can be obtained using the ratio of FOCs to replace one variable input in the production function by the relationship with the other.

[15]A recent paper showing in practice the advantages of one step GMM estimation is Trunschke and Judd (2024).

[16]Not recognizing this may produce unproductive discussions on identification. It is customary to apply nonparametric estimation with a polynomial specification. See Appendix A for a discussion on identification.

4 A common framework and a novel estimator

DP and OP/LP estimators are presented differently (pseudodifferentiation, replacement of the unobservable by the inverse of an input demand) for pedagogical reasons, but they can be seen under a more common perspective. It happens that both estimators assume a first order Markov process for productivity, and then propose to replace past productivity by an expression in terms of observables.

We will see that this common perspective clarifies many properties and relationships. In discussing the estimators, we will assume for the moment that the specification of the production function $f(\mathbf{x})$ is correct and that the FOCs are met. We relax the assumption on the FOCs in section 6 and the assumption on $f(\mathbf{x})$ in section 9.[17]

We can say that both estimators start by assuming that the production function can be written as

$$q = f(\mathbf{x}) + g(\omega_{-1}) + \xi + \varepsilon, \tag{6}$$

because of the productivity process. Then DP proposes to replace ω_{-1} by $q_{-1} - f(\mathbf{x}_{-1}) - \varepsilon_{-1}$, and OP/LP by $w_{X,-1} - p_{-1} - \ln \frac{\partial F(\mathbf{x}_{-1})}{\partial X_{-1}}$. DP uses the lagged production function, OP/LP the lagged FOC. Accordingly, in what follows we will use the following definitions

DEFINITION 1 The dynamic panel estimator is the application of IV to the equation

$$q = f(\mathbf{x}) + g(q_{-1} - f(\mathbf{x}_{-1}) - \varepsilon_{-1}) + \xi + \varepsilon, \tag{7}$$

with $g(\cdot)$ specified as linear.

DEFINITION 2 The OP/LP estimator is the application of IV to the equation

$$q = f(\mathbf{x}) + g(w_{X,-1} - p_{-1} - \ln \frac{\partial F(\mathbf{x}_{-1})}{\partial X_{-1}}) + \xi + \varepsilon, \tag{8}$$

[17]Note also that we are implicitly assuming that the researcher has the data needed for estimating the production function, which includes the right price indices to deflate revenue and get indices of quantity. The discussion that follows does not add any data need. In particular, the OP/LP estimators can be implemented with no price of output.

with $g(\cdot)$ specified by means of polynomials. We will call linear OP/LP the estimator that only uses a first degree polynomial for $g(\cdot)$.

Revenue-share OP/LP

The expression used by OP/LP can be written in different ways. For example, we will find it useful to employ the revenue-share form, based on the share of expenses in input X in the observed revenue $S_X^R = \frac{W_X X}{PQ}$.

LEMMA 1 The OP/LP estimator can be written in the revenue-share form

$$q = f(\mathbf{x}) + g(q_{-1} - f(\mathbf{x}_{-1}) + \ln \frac{S_{X,-1}^R}{\beta_{X,-1}}) + \xi + \varepsilon, \tag{9}$$

where $\beta_X = \frac{X}{F(\mathbf{x})} \frac{\partial F(\mathbf{x})}{\partial X}$ is the output elasticity of the input X.[18]

Proof

Add and subtract x, and subtract and add q and $f(\mathbf{x}) = \ln F(\mathbf{x})$, to the expression for ω in (8) in current time, then do some reordering. That is, $\omega = w_X - p - \ln \frac{\partial F(\mathbf{x})}{\partial X} = w_X + x - p - q + q - f(\mathbf{x}) - (x - \ln F(\mathbf{x}) + \ln \frac{\partial F(\mathbf{x})}{\partial X}) = q - f(\mathbf{x}) + \ln \frac{S_X^R}{\beta_X}$. ♦

The revenue-share form of the OP/LP estimator makes clear that the OP/LP estimator uses more information than the dynamic panel estimator, and this additional information is included in the term $\ln \frac{S_X^R}{\beta_X}$. Multiplying the first order condition (4) by $\frac{X}{PQ^*}$, using $\frac{Q}{Q^*} = \exp(\varepsilon)$, and taking logs, the first order condition can also be written as $\ln \beta_X = \ln S_X^R + \varepsilon$, and hence the additional term controls for ε in terms of the differences between the share of X in revenue and the specification of the production elasticity of the input.[19]

However, when the model is applied to the data, if the first order condition is not met, the term $\ln \frac{S_X^R}{\beta_X}$. will contain more than ε and the estimates will usually become inconsistent because of a problem of omitted variable (the

[18] It is important to note that we write β_X for notational simplicity, but it should be clear that, in general, it is a function $\beta_X(\cdot)$ of the inputs (and labor-augmenting productivity).

[19] The FOC under perfect competition, in the reordered form $\ln S_X^R = \ln \beta_X - \varepsilon$ is used by Gandhi, Navarro, and Rivers (2020) as the first step of their estimator. Notice that (9) suggests a unique-step form for the estimator. Also suggests that to control for ε, the ACF first stage is not needed.

discussion of this continues in section 6, where we allow for the failure of the FOCs).

Comparing (7) and (9) it turns out that, while the dynamic panel estimator leaves the ε error to become part of the error of the equation, the linear OP/LP estimator controls for ε by means of the difference between the revenue share and the elasticity. This is saying that, under perfect competition, the fulfillment of the first order condition implies that the dynamic panel and the linear OP/LP estimator should only diverge by sampling error.[20]

If $g(\cdot)$ is nonlinear, OP/LP produces a different estimate that comes exclusively from adding nonlinear terms to approximate $g(\cdot)$. DP can be seen as a first order approximation to the productivity process dealt with by OP/LP. As productivity is in practice quite persistent, it should not be expected that this creates a dramatic divergence. Our empirical exercise confirms this (see section 10).

Cost-share OP/LP

We can develop another form of the OP/LP estimator that, instead of using the price, proxies for marginal cost[21]

LEMMA 2 The OP/LP estimator can be written in the cost-share form

$$q = f(\mathbf{x}) + g(q_{-1} - f(\mathbf{x}_{-1}) + \ln \frac{\nu_{-1} S_{X,-1}}{\beta_{X,-1}} - \varepsilon_{-1}) + \xi + \varepsilon, \tag{10}$$

where $\nu = \beta_L + \beta_M$ is the short-run elasticity of scale.[22] The cost-share estimator is only consistent under linearity of $g(\cdot)$.

Proof

Since under competition $p = mc$, by (8) we have $\omega = w_X - mc - \ln \frac{\partial F(\mathbf{x})}{\partial X} = w_X + x - (vc - (q - \varepsilon) - \ln \nu) - f(\mathbf{x}) - (x - \ln F(\mathbf{x}) + \ln \frac{\partial F(\mathbf{x})}{\partial X}) - \varepsilon = q - f(\mathbf{x}) + \ln \frac{\nu S_X}{\beta_X} - \varepsilon$.

In the second equality, we use that $\frac{AVC}{MC} = \nu$, [23] and that $q = q^* + \varepsilon$, which allow us to proxy marginal cost by average variable cost. The price we have

[20]Notice, however, that if (7) and (9) are estimated with the same sample, (9) will produce more efficient estimates because reduces the variance of the error.

[21]This was suggested by Doraszelski and Jaumandreu (2019), page 19.

[22]We again write for simplicity ν, but the short-run elasticity of scale is also in general a function $\nu(\cdot) = \beta_L(\cdot) + \beta_M(\cdot)$. Note, however, that, under homotheticity, $\nu(\cdot)$ becomes a function of Q^* alone.

[23]Adding the FOCs of the variable inputs multiplied by $\frac{X}{Q^*}$ we have $MC\nu = AVC$.

to pay for this is the introduction of the error ε. This error determines that $g(\cdot)$ must be linear for consistency. ♦

The cost-share form of OP/LP encompasses the information added to the dynamic panel estimator in the term $\ln \frac{\nu_{-1} S_{X,-1}}{\beta_{X,-1}}$.[24]

We can establish an important

PROPOSITION 1 Under perfect competition, the dynamic panel estimator and both OP/LP estimators, the linear revenue-share and the linear cost-share OP/LP, should only diverge by sampling error.

Proof
Comparison of (7), (9) and (10) shows that the three estimators can at most diverge due to sampling error. (9) replaces the error $-\varepsilon_{-1}$ in (7) by a term in oservables whose value is $-\varepsilon_{-1}$, and hence will simply be more efficient. (10) adds a term that is zero if the FOC is strictly true. In this case, the estimators would show the same numerical value with the same sample and would only be affected by sampling otherwise. ♦

What makes dynamic panel and linear cost-share estimators particular is that they continue to be consistent in imperfect competition.

5 Market power

When there is market power, under the assumption of short-term profit maximization, the relevant variable in the FOCs is marginal revenue MR instead of P

$$MR \frac{\partial F(x)}{\partial X} \exp(\omega) = W_X.$$

The problem is that MR is, in general, unobservable. At first sight, it seems like we have no alternative than choosing the DP estimation, which does not need this relationship. Let us take a closer look at what the new variable implies.

[24]Notice that we can also pass from one estimator to the other with the chain $\frac{S_X^R}{\beta_X} = \frac{\frac{VC}{PQ}\frac{W_X X}{VC}}{\beta_X} = \frac{\frac{AVC}{MC}\exp(-\varepsilon) S_X}{\beta_X} = \frac{\nu S_X}{\beta_X}\exp(-\varepsilon)$.

There are N firms in a market. We now use firm subindices for the sake of clarity. If firms have market power, the solution of the system of the two variable input FOCs is going to produce for each firm the condition of equilibrium $X_j = X(K_j, MR_j, W_j, P_{Mj}, \omega_j)$,[25] and this is what an OP/LP procedure must now invert to get ω.

Of course, it is possible to obtain expressions in terms of observables by simplifying the cases of behavior and with specific assumptions on symmetry of the firms and their behavior. For example, it is very popular to assume that competition is monopolistic and the elasticity of demand constant and equal for all firms. Under these assumptions $MR_j = P_j(1 - \frac{1}{\eta})$, where η represents the (absolute value) of the demand elasticity.

A discussion of possible behavior restrictions and assumptions of symmetry across oligopoly models is carried out in Ackerberg and De Loecker (2024). They are able to show significant reductions in the information requirements for some cases but, for example, they confirm that there cannot be unobserved characteristics if products are differentiated, as Doraszelski and Jaumandreu (2021) pointed out in relation to the correlated unobserved demand heterogeneity. More in general, tractability needs to assume either common quantities-prices or its aggregability, common input prices, discard unobserved correlated demand heterogeneity, and drop asymmetric behavior (see Appendix B and Appendix C).

The central question is whether it is possible to estimate the production function without taking a position on how competition is. Estimate without having to assume things like whether competition is in prices or quantities, firms either take the rivals actions as given or collude, collusion is either with all or with part of the rivals, some firms have a particular type of advantage or not, and so on. The answer is yes, it is possible.

To see why notice that, in equilibrium, a short-run profit maximizing firm equates marginal revenue and marginal cost, so

$$MR(P, \delta, \mathit{conduct}) = MC(K_j, W_j, P_{Mj}, Q_j^*, \omega_j).$$

On the left hand side, the expression depends on the particular specification of conduct. The right hand side, on the contrary, picks up a specific single value under quite general conditions.[26] It singles out a unique marginal cost

[25] Alternatively, it could be considered the demand conditional in output, which has the symmetric problem of unobservability of the relevant output; see later.

[26] These conditions are basically convexity assumptions on the technology of the firm.

$MC(\cdot)$ for each set of values of the arguments (we have specified possibly varying input prices for the sake of generality).[27] If we have MC, we have what has been called a "sufficient statistic," a variable that contains all the relevant information of the conduct and demand conditions.

An estimator that is robust with respect to market power is an estimator that is consistent regardless of the details of the game that firms play in the market. DP is robust to market power, but not OP/LP. DP is robust to market power because it does not use any FOC which changes when market power replaces competition. OP/LP is not robust because the usual specification, based on the FOCs under market power, needs to model marginal revenue MR. For modeling MR in a tractable way, some particular games and strong symmetry conditions must be assumed.

However, we have shown that there is a feasible OP/LP that is always possible. It consists of proxying MC by AVC and the short-run elasticity of scale, taking into account that this replacement leaves in the expression the error of the production function and the Markov process must be assumed to be linear. This estimator, which we have called the linear cost-share OP/LP estimator, is robust to market power.

To summarize the properties under market power of the estimators concerned, we can formulate the following

PROPOSITION 2 Under market power, when the specification of $f(x)$ is correct, the dynamic panel estimator and the linear cost-share OP/LP estimator when the FOCs hold, are consistent, and their estimates must only differ by sampling error. Instead, the revenue-share OP/LP with the FOC holding, even linear, is generally inconsistent.

Proof

That DP is always consistent under market power follows directly from the fact that the estimator does not need to assume anything about the variable input FOCs.

That the linear cost-share OP/LP estimator is consistent follows from the FOCs under market power with short-run profit maximization. Since under profit maximization $MR = MC$, we have

$$MC \frac{\partial F(x)}{\partial X} \exp(\omega) = W_X,$$

[27]We can even accommodate labor market power and LAP by considering the price $W^* = (1+\tau)W/\exp(\omega_L)$ with the unobservables τ and ω_L replaced.

that are also the conditions for cost minimization of variable cost. Hence, we can use the inversion

$$\omega = w_X - mc - \ln \frac{\partial F(x)}{\partial X} = w_X - vc + q + \ln \nu - \ln \frac{\partial F(x)}{\partial X} - \varepsilon,$$

where in the second equality we use the property $mc = vc - q^* - \ln \nu$. As in lemma 2, we can rewrite this expression lagged as

$$\omega_{-1} = q_{-1} - f(x_{-1}) + \ln \frac{\nu_{-1} S_{X,-1}}{\beta_{X,-1}} - \varepsilon_{-1},$$

where in theory $\ln \frac{\nu_{-1} S_{X,-1}}{\beta_{X,-1}} = 0$. It follows that the dynamic panel estimator and the linear cost-share estimator should only differ by sampling error.

The revenue-share estimator uses P_{-1} instead of MC_{-1}, and therefore we have persistent unobservable $-(p_{-1} - mc_{-1}) = -ln\mu_{-1}$ that cannot be controlled. The variable inputs, chosen at t according to μ, are likely to be correlated with it through the lags of $ln\mu$. ♦

6 Failure of the FOCs

The OP/LP estimators are inconsistent if the FOCs they use do not hold. We are particularly interested in the behavior of the linear cost-share estimator because we want to use it as an estimator robust to market power, but we also briefly include the consequences for the revenue-share estimator.

First order conditions may not hold as in proposition 2 by multiple reasons. The most commonly discussed by researchers are: adjustment costs (see e.g. Bond and Van Reenen, 2007), market power in the input market (see e.g. Manning, 2011), firm optimization errors (see e.g. Marschak and Andrews, 1944), misallocation of inputs (see e.g. Hsieh and Klenow, 2009).

We can add the case of biased technological change (e.g. LAP). However, it is important to take into account that this motive also changes the structure of the production function (we need l^* instead of l).

All of the above circumstances may be represented in the FOCs by the presence of an unobservable. Assume, without loss of generality, that the first order conditions affecting the variable inputs are

$$MC \frac{\partial F(\mathbf{x})}{\partial X} \exp(\omega) = (1 + u_X) W_X, \tag{11}$$

where u_X is an FOC and input-specific unobservable,[28]

When we have these unobservables, the relationship between marginal cost and average variable cost becomes more complex, as we show in the following

LEMMA 3 The relationship between marginal cost and average variable cost is

$$MC = \frac{\theta AVC}{\nu}, \qquad (1)$$

where $AVC = \frac{VC}{Q^*}$ is the average variable cost, recall that $\nu = \beta_L + \beta_M$ is the short-run scale elasticity, and $\theta = 1 + \sum_X S_X u_X$, is a weighted sum of unobservables with weights equal to the cost-shares $S_X = \frac{W_X X}{\sum_X W_X X}$.

Proof

Adding the FOCs of the variable inputs multiplied by $\frac{X}{Q^*}$ we have

$$MC \sum_X \frac{X}{Q^*} \frac{\partial F}{\partial X} \exp(\omega) = \frac{\sum_X (1 + u_X) W_X X}{Q^*} = (1 + \sum_X S_X u_X) \frac{\sum_X W_X X}{Q^*},$$

or

$$MC\nu = \theta AVC,$$

so marginal cost can be written in terms of the average variable cost AVC, the short-term elasticity of scale ν, and θ. ♦

The presence of unobservables in the FOCs modifies the OP/LP estimators of sections 4 and 5. The revenue-share estimator is as follows

$$q = f(\mathbf{x}) + g(q_{-1} - f(\mathbf{x}_{-1}) + \ln \frac{(1 + u_{X,-1}) S^R_{X,-1}}{\beta_{X,-1}}) + \xi + \varepsilon,$$

And, using the lemma, we can develop the form that takes the linear cost-share OP/LP estimator. Notice that, since we are proxying MC by AVC, the estimator is affected by the unobservables in all the FOCs. The form is as follows.

$$q = f(\mathbf{x}) + g(q_{-1} - f(\mathbf{x}_{-1}) + \ln \frac{\nu_{-1} \theta_{X,-1} S_{X,-1}}{\beta_{X,-1}} - \varepsilon_{-1}) + \xi + \varepsilon, \qquad (12)$$

[28] The FOC corresponding to the LAP is usually written as $P \frac{\partial F(\mathbf{x}^*)}{\partial L^*} \exp(\omega_L + \omega) = W_X$, which can also be written as $P \frac{\partial F(\mathbf{x}^*)}{\partial L^*} \exp(\omega) \simeq (1 - \omega_L) W_X$, where $\mathbf{x}^* = \{k, l^*, m\}$.

where $\theta_X = (1 + u_X)/\theta$. If all FOCs for variable inputs hold with no unobservables, $\theta_X = 1$.

Of course, the researcher can model total or partially θ_X if she knows of the presence of some unobservable (e.g. LAP or monopsony power). However, what happens when the equation is specified using the term $\ln \frac{\nu S_X}{\beta_X}$, that is, ignoring the unobservable θ_X?

Multiplying FOC (11) by $\frac{X}{MCQ^*}$ we can write $\frac{X}{Q^*}\frac{\partial F(\mathbf{x})}{\partial X}\exp(\omega) = (1 + u_X)\frac{AVC}{MC}\frac{W_X X}{AVCQ^*}$. It follows that $\beta_X = \nu\theta_X S_x$. This implies that the expression $\ln \frac{\nu S_X}{\beta_X}$ (without θ_X) it will take the value $-\ln \theta_X$ if $\theta_X \neq 1$, and zero if there are no unobservables.

This implies that the linear cost-share OP/LP estimator is inconsistent if there are unaddressed unobservables in the FOCs. The researcher must be aware, and this has consequences in the tests that we are going to develop.

7 Implementing OP/LP

Expressions (9) or (10) are more or less simple to implement according to the specification chosen for the production function. For example, with a Cobb-Douglas production function, they amount to estimation of the production function by pseudo-differences, as the dynamic panel method suggests, including the terms $\rho \ln S^R_{X,-1}$ and $\rho \ln S_{X,-1}$, respectively (see the example of section 8).

However, as mentioned before, this is not the way that OP/LP is usually carried out by researchers. Often, an implicit input demand, assuming a common output price P and common input prices, has been assumed represented by means of the inverted relationship (varying over time) $\omega = h(K, X)$. Substituting then a flexible lagged $h(\cdot)$ for ω_{-1} in the Markov process is an effective way to estimate. See, for example, Wooldridge (2009).

On the negative side, this form may incorrectly omit the output and input prices. However, in general, it has delivered better results for the elasticities than DP. This form grants in fact more flexibility than what is implied by the used production function, and might be picking up problems of specification and improving because of this (e.g. softening the lack of variation of elasticities).

Ackerberg, Caves, and Frazer (2015), henceforth ACF, proposed a form to implement OP/LP, related somehow to the above practice, which has become prevalent. We will use the insights gained with our common framework to provide an interpretation of the ACF OP/LP estimator .

An interpretation of the ACF procedure

The idea of ACF is to regress first output nonparametrically on all inputs and variables relevant to explain the demand for the input used to substitute for unobserved productivity. The goal is to identify separately ε. Let $\mathbf{z}$ be the set of variables on which the output is projected (which we discuss later specifically) with $\mathbf{x} \subset \mathbf{z}$. The first stage of ACF computes

$$\widehat{\phi} = E(q|z) = E(f(\mathbf{x})|\mathbf{z}) + E(\omega|\mathbf{z}) = f(\mathbf{x})+E(\omega|\mathbf{z}).$$

This means that, in the second stage, what ACF carries inside $g(\cdot)$ can be written as

$$\widehat{\phi}_{-1} - f(\mathbf{x}_{-1}) = q_{-1} - f(\mathbf{x}_{-1}) + [E(\omega_{-1}|\mathbf{z}_{-1}) - \omega_{-1}] - \varepsilon_{-1}.$$

Again, as in (12), the difference with respect to the dynamic panel is a term that is only relevant if it contains a bias (the value is not zero). In this case, the potential bias can be thought of as the difference in the projection of lagged productivity on the vector $\mathbf{z}_{-1}$ from the true value of lagged productivity.

Since $E(\omega_{-1}|\mathbf{z}_{-1}) - \omega_{-1} = E(q_{-1} - f(\mathbf{x}_{-1}) + \ln \frac{\nu_{-1}\theta_{X,-1}S_{X,-1}}{\beta_{X,-1}} - \varepsilon_{-1}|\mathbf{z}_{-1}) - \omega_{-1} = E(\ln \nu_{-1} + \ln S_{X,-1} - \ln \beta_{X,-1} + \ln \theta_{X,-1}|\mathbf{z}_{-1})$, everything depends on how the latest expectation is. With input quantities and input prices we can reasonably predict the cost share and the elasticities, but we do not have the unobservable. Consequently, the bias will tend to be zero if $\theta_X = 1$ and $-E(\ln \theta_X|\mathbf{z})$ otherwise. Recall that in the parametric case, the presence of the unobservables imposed on a bias of $-\ln \theta_X$. ACF is likely to smooth out this bias by projecting it on z, since $Var(-\ln \theta_X|z) \leq Var(-\ln \theta_X)$.

From this point of view, ACF estimation using quantities and prices of the variable inputs is a nonparametric approximation to the inclusion of the term in the cost-share. Under a correct specification of $f(\mathbf{x})$, with a linear Markov process, it should also only diverge from DP due to sampling error. In addition, if there is bias induced by unobservables in the FOCs, the ACF estimate is likely to mitigate this bias by reducing its variance.

The ACF procedure can hence be a legitimate way to apply the linear cost-share estimators (we show this in practice in the empirical exercise).

However, if there is market power, it is not correct to include the output price in the first stage of ACF. This corresponds to the demand for the input under perfect competition and is incompatible with consistency in the presence of market power. In the first stage of ACF, instead of P, it should be included S_x, the share of the input in cost.

8 A test for market power

Since applying procedures of estimation that are robust to market power limits the available choices, it is worthy to have a method that allows testing the presence of market power. It turns out that we have developed in section 4 the estimator that can serve to easily construct a test for market power.

The linear cost-share OP/LP estimator is based on approximating MC by AVC and scale elasticity. It is consistent when the firm has market power because it is based on formulas for cost minimization. It is an estimator fully robust to market power in the product market.[29]

On the other hand, a revenue-share OP/LP is not consistent in the presence of market power, even in a linear version, because it is based on the price of the firms, which under market power diverges from marginal cost.

We can test the null of no market power, in which both estimators are consistent, against the alternative of market power, under which the revenue-share estimator is not consistent, while the linear cost-share estimator remains consistent.

In practice, both estimators differ only in the alternative use of the cost or revenue shares of the input and the need to specify the short-run elasticity of scale in the first. Furthermore, if ν is a constant, the difference collapses to only using a different regressor, making it very easy to perform the test with pseudo-differences. Of course, the test can also be applied using an ACF procedure in two steps (see our exercise in section 10)

EXAMPLE Assuming that the production function is Cobb-Douglas, we

[29] And that can be robustified to market power in input markets (and LAP, of course) by means of the model specification. This, if relevant, will avoid from the beginning inconsistencies that we discuss in section 9.

can run the two regressions

$$
\begin{aligned}
q &= \beta_0 + \rho q_{-1} + \beta_K(k - \rho k_{-1}) + \beta_L(l - \rho l_{-1}) + \beta_M(m - \rho m_{-1}) \\
&\quad + \rho \ln S_{X,-1} + e, \\
q &= \beta_0' + \rho q_{-1} + \beta_K(k - \rho k_{-1}) + \beta_L(l - \rho l_{-1}) + \beta_M(m - \rho m_{-1}) \\
&\quad + \rho \ln S^R_{X,-1} + e',
\end{aligned}
$$

where $e = \xi + \varepsilon - \rho\varepsilon_{-1}$, $e' = \xi + \varepsilon$, and we compare the estimates for β_K, β_L and β_M, say. ♦

A Hausman (1978) specification test, or a Durbin-Wu-Hausman test, can be seen as a test of the equality between the parameter estimates under two methods of estimation that are consistent under the null. Following Wooldridge (2010), we set a quadratic form of the differences in the parameters $(\widehat{\beta}_{CS} - \widehat{\beta}_{RS})$ using the inverse of a robust estimate of $Avar[\sqrt{N}(\widehat{\beta}_{CS} - \widehat{\beta}_{RS})] = V_{CS} + V_{RS} - (C + C')$. See Appendix D on the computation of these matrices. Under the null, we have

$$
(\widehat{\beta}_{CS} - \widehat{\beta}_{RS})' Avar[\sqrt{N}(\widehat{\beta}_{CS} - \widehat{\beta}_{RS})]^{-1}(\widehat{\beta}_{CS} - \widehat{\beta}_{RS}) \sim \chi^2(p),
$$

where p degrees of freedom are the number of parameters tested.

A drawback of the test is that we have assumed that the production function is well specified and that the FOCs hold. If this is not the case, the two estimators are inconsistent under the null and under the alternative, although for reasons that are different from market power. Although the test can still be informative under these circumstances, it seems convenient to repeat it again after addressing the specification of the production function and the holding of the FOCs.

9 A specification test

Now we are in a position to transform the linear cost-share estimator into a tool of specification under market power. DP and the linear cost-share OP/LP must coincide under market power, and hence any divergence is informing us of other problems.

To convert the linear cost-share estimator into a specification tool, we need, however, to consider carefully the reasons by which DP and the linear cost-share OP/LP may diverge. We have already seen that a reason is the presence of unobservables in the FOCs (and we have studied how they affect estimation). However, until now we have assumed that $f(\mathbf{x})$ is well specified, with the consequence that under market power and without unobservables in the FOCs, DP and OP/LP must only diverge due to sampling error. Now we also want to include possible errors in the specification of $f(\mathbf{x})$. The production function itself can be not well specified, and this affects both the production function and the FOC.

To take the most relevant example of misspecified production function, think of the case of biased technological progress, LAP say. Now the relevant labor is $l^* = l + \omega_L$, the production function is $f(k, l^*, m)$ and the labor FOC is $MC \frac{\partial F(\mathbf{x}^*)}{\partial L^*} \exp(\omega_L + \omega) = W$. The consistency of the DP estimator fails due to the specification of the production function and the consistency of the OP/LP estimator due to both the specification of the production function and the presence of the unobservable in the labor FOC. Under LAP, DP and OP/LP diverge, and both are inconsistent.

Adopting this broader perspective, the situation under market power can be summarized as follows. On the one hand, both estimators can be inconsistent because the production function is not well specified. On the other hand, if the production function is well specified and the FOC contains no unobservables additional to MC, DP and the linear cost-share OP/LP must only diverge due to sampling error. Hence, a specification test based on the equality of the coefficients of two estimators, this time DP and OP/LP, is available. Under the null both estimators are consistent, and under the alternative either both estimators are inconsistent or only the OP/LP estimator (because only the FOCs fail) is inconsistent.

When there is market power,[30] starting the estimation of the production function with this test is an easy and convenient way to work on the specification. If DP and OP/LP coincide and the test is passed, the researcher has statistical evidence that a necessary condition for consistency is met. Of course, this does not automatically ensure consistency, and hence the specification can be worked on and improved.

However, if the test is not passed (DP and OP/LP diverge) we are sure that one of two things is happening: either the production function specification is wrong or there are unobservables in the FOCs. It is crucial to know this to address the efforts to detect the origin of the problem (produc-

[30] The researcher may have reached this conclusion applying the test for market power developed in the previous section.

tion function specification or FOC unobservables). The problem will not be solved until the test is passed.

Again, the test can be performed by means of pseudo-differences of the implied equations or by means of two stage ACF regressions. The next example does the simplest thing, but in the empirical exercise we do both.

EXAMPLE Assuming that the production function is Cobb-Douglas, we can run the DP and linear cost-share OP/LP regressions

$$\begin{aligned} q &= \beta_0 + \rho q_{-1} + \beta_K(k - \rho k_{-1}) + \beta_L(l - \rho l_{-1}) + \beta_M(m - \rho m_{-1}) + e, \\ q &= \beta_0' + \rho q_{-1} + \beta_K(k - \rho k_{-1}) + \beta_L(l - \rho l_{-1}) + \beta_M(m - \rho m_{-1}) \\ &\quad + \rho \ln S_{X,-1} + e', \end{aligned}$$

where both e and e' have the form $\xi + \varepsilon - \rho\varepsilon_{-1}$, and compare the estimates for β_K, β_L and β_M. ♦

The test can be constructed again by setting a quadratic form of the differences in the parameters $(\widehat{\beta}_{DP} - \widehat{\beta}_{OP/LP})$ and using the inverse of a robust estimate of $Avar[\sqrt{N}(\widehat{\beta}_{DP} - \widehat{\beta}_{OP/LP})] = V_{DP} + V_{OP/LP} - (C + C')$ (see Appendix C). In the null, we have

$$(\widehat{\beta}_{DP} - \widehat{\beta}_{OP/LP})' Avar[\sqrt{N}(\widehat{\beta}_{DP} - \widehat{\beta}_{OP/LP})]^{-1}(\widehat{\beta}_{DP} - \widehat{\beta}_{OP/LP}) \sim \chi^2(p),$$

where p degrees of freedom are the number of parameters tested.

10 Estimating the production function with US firms

In this section, we show with an example how the DP and OP/LP estimators start diverging under standard estimation, and how this divergence reflects at the same time the presence of market power and more general problems of specification. The application of our two tests guides the work of re-specification of the production function, and the two estimators end by coinciding. This coincidence implies that the necessary condition for consistency is met, and we leave the task where it can be continued for refinements.

We estimate the production function for the sample of Compustat US manufacturing firms (1960-2018) used in Jaumandreu and Mullens (2024).

It is a sample of firms belonging to many different manufacturing markets and times, so we expect that they possess various degrees of market power. In estimating the production function of these firms, it is then important to be robust to the exercise of market power.

At first, the DP estimation and a standard ACF implementation of the OP/LP estimator applied to a Cobb-Douglas specification diverge in the estimate of the elasticity of capital and in the assessment of the returns to scale. As is often the case, for the dismay of researchers, DP produces a negative elasticity for capital, and a short-run elasticity of scale well above unity. However, the ACF implementation of OP/LP shows a nicely estimated (small) elasticity of capital and a more moderate short-run elasticity of scale (although not smaller than one as well). As odd as it may sound, this is not a sample-specific phenomenon but a quite typical finding.

A usual interpretation for the DP behavior is that the differentiation of the data exacerbates errors in the measurement of a capital that otherwise is quite persistent over time. However, a little experimentation shows that this explanation is not convincing. For example, the OLS of the Cobb-Douglas in the first differences gives positive coefficients for all inputs (see Appendix E). However, if DP is inconsistent, we have shown that OP/LP cannot be consistent. The result of the estimation raises hence a puzzle: DP cannot be right, but the ACF estimation of OP, which must be equally inconsistent and probably accumulating the inconsistency due to market power, is adding something that improves the estimation.

In what follows, we start by confirming that we are effectively in the presence of market power. Applying the test, we are not able to discard market power, and hence we should stick to the version of the OP/LP estimator that is robust to market power. However, the result of the estimator itself is not satisfactory as we already expected given the performance of DP. Then, we look for reasons related to the production function specification that must be inducing the inconsistency of both estimators. A simple inspection of the labor shares shows that the elasticity of labor must have been falling over time and should be deeply varying across firms, while the Cobb-Douglas specification fails to pick up this characteristic. When labor-augmenting productivity is allowed into the specification, enlarging the Cobb-Douglas to a translog that admits shares and elasticities that are varying, the DP estimator and the OP/LP version robust to market power fully coincide. Our specification test sanctions that.

The conclusion is that the divergence of the estimators was simultaneously detecting the presence of market power and the misspecification of the production function and that the redress of the misspecification allows robust estimators to provide the same answer. We have needed to address both things because both were in the data. The test for market power has limited the estimators for which the comparisons are valid, and the comparisons carried out with robust estimators confirm that the necessary condition for consistency was not met but can be met. This can be a good starting point for the researcher trying to further improve the specification while keeping the equality of the two estimators (after testing again, now presumably with consistency, that market power cannot be discarded).

Exercise details

In what follows, we explain in detail how the above exercise is done.

Table 1 reports the result of the two basic estimators. Column (1) of the table reports the results of applying the DP estimator to a Cobb-Douglas specification. The estimator proceeds as follows. Under the assumption that Hicks-neutral productivity ω_H follows a $AR(1)$ with parameter ρ and innovation ξ, it can be written that

$$\begin{aligned} q_{jt} &= \beta_t + \beta_K k_{jt} + \beta_L l_{jt} + \beta_M m_{jt} \\ &\quad + \rho[q_{jt-1} - \beta_K k_{jt-1} - \beta_L l_{jt-1} - \beta_M m_{jt-1}] + \xi_{jt} + \varepsilon_{jt} - \rho\varepsilon_{jt-1}, \end{aligned} \tag{13}$$

where β_t includes one year that is taken as a base (constant). We estimate this equation by nonlinear GMM using as instruments the constant and time dummies, the input variables $k_{jt}, k_{jt-1}, l_{jt-1}, m_{jt-1}$ and the (real) input prices $w_{jt-1} - p_{jt-1}$ and $p_{Mjt-1} - p_{jt-1}$. This is a fully standard choice of instruments.[31] As we have to estimate (in addition to the constant and time dummies) the four parameters $\beta_K, \beta_L, \beta_M$ and ρ, the instruments provide two overidentifying restrictions.

The result is not nice: the elasticity of capital turns out to be negative, the elasticity of labor very large, and the short-run elasticity of scale ν, the sum of $\beta_L + \beta_M$, above 1.1. Economic theory tells us that it is not realistic that when changing in the short-run the variable factors we encounter increasing returns to scale (although, unfortunately, this is a quite usual result that is reported without further comments).

[31] Even if there is market power, and output price differs from marginal cost, lagged price can be assumed naturally uncorrelated with the next period ξ and observational error ε of the production function. Hence is a legitimate instrument. The price of output scales the input prices.

Column (2) reports the results of computing the OP/LP estimator applied to the Cobb-Douglas specification, implemented by means of a standard ACF method. In a first stage, we regress q_{jt} non parametrically on a constant, time dummies, and using a complete polynomial of order 3 on the five variables $k_{jt}, l_{jt}, m_{jt}, w_{jt} - p_{jt}$ and $p_{Mjt} - p_{jt}$.[32] From the result of this first step we compute the estimate $\widehat{\phi}(k_{jt-1}, l_{jt-1}, m_{jt-1}, w_{jt-1} - p_{jt-1}, p_{Mjt-1} - p_{jt-1})$ that we use in forming the second step equation

$$\begin{aligned} q_{jt} &= \beta_t + \beta_K k_{jt} + \beta_L l_{jt} + \beta_M m_{jt} \\ &\quad + \rho[\widehat{\phi}_{jt-1} - \beta_K k_{jt-1} - \beta_L l_{jt-1} - \beta_M m_{jt-1}] + \xi_{jt} + \varepsilon_{jt}. \end{aligned} \tag{14}$$

As we try to emphasize with the writing, equations (13) and (14) are very close. They only differ, in addition to the error component $-\rho\varepsilon_{jt-1}$, in that the nonparametric estimate $\widehat{\phi}_{jt-1}$ has replaced q_{jt-1}.

If we have thought of the demand for materials to construct the proxy for ω, the application of the analysis of section 4 tells us that

$$\begin{aligned} &\widehat{\phi}_{jt-1} - \beta_K k_{jt-1} - \beta_L l_{jt-1} - \beta_M m_{jt-1} = \\ &\quad q_{jt-1} - \beta_K k_{jt-1} - \beta_L l_{jt-1} - \beta_M m_{jt-1} \\ &\quad + E(\ln \frac{\nu S^R_{Mj,-1}}{\beta_M} | k_{jt-1}, l_{jt-1}, m_{jt-1}, w_{jt-1} - p_{jt-1}, p_{Mjt-1} - p_{jt-1}). \end{aligned}$$

Hence, we interpret the method as if we add in the brackets of the DP estimator the expression corresponding to the nonparametric prediction of the share $S^R_{M,-1}$ (note that β_M and ν are here irrelevant constants). Everything is like in the second stage we were using

$$\begin{aligned} q_{jt} &= \beta'_t + \beta_K k_{jt} + \beta_L l_{jt} + \beta_M m_{jt} \\ &\quad + \rho[q_{jt-1} - \beta_K k_{jt-1} - \beta_L l_{jt-1} - \beta_M m_{jt-1} + \ln \widehat{S}^R_{jt-1}] + \xi_{jt} + \varepsilon_{jt}, \end{aligned}$$

where $\ln \widehat{S}^R_{jt-1}$ represents the empirical expectation. In the second step of the implementation of ACF we use instruments $k_{jt}, k_{jt-1}, l_{jt-1}, m_{jt-1}$ and $\widehat{\phi}_{-1}$, which implies an overidentifying restriction. The use of $\widehat{\phi}_{-1}$ brings as an instrument the (lagged) result of the first stage estimation (including the lagged price).

[32] Notice that this implies the use of the output price dividing the input prices, which is consistent with the input demand under perfect competition .

The results reported in column (2) show the outcome. Revenue share OP/LP helps to rectify two things with respect to the DP results. The capital elasticity becomes significantly positive and the short-term elasticity of the scale decreases.

It is apparent that DP and OP/LP give different answers to the estimation of the production function. This can be attributed to two facts. The first is that we are using an OP/LP estimator that is inconsistent under market power. In the first stage, we use the price of the product (in a specification to replace the unobservable productivity that corresponds to the arguments of the demand for materials) and this use is only correct under perfect competition. Market power implies that price should be replaced by marginal cost. The second is that the simultaneous obvious inconsistency of DP suggests that there is a shortcoming in the specification of the production function that should also affect the production function and the FOC used in OP/LP.

Testing (ex-ante) for market power

The first thing to answer is if we can really disregard the presence of market power. To do this, we performed the market power test, with the results reported in Table 2. In columns (1) and (2) we compute the test using the two parametric [33] versions of OP/LP, that is, adding, respectively, to specification (13) the log of the cost-share of materials and of the revenue-share of materials. The test sharply says that this is not equivalent, rejecting the absence of market power.

However, notice that the regressions for the implementation of the test are already pointing to the researcher that the involved problems are not only market power. Even the simple addition of the log of the cost-share of materials (column (1)) provokes a change in the estimated coefficients that can only be interpreted as revealing the presence of an unobservable strongly correlated with the variable inputs. In fact, we can already guess that this unobservable is LAP by noting that it is positively correlated to materials and negatively to labor.

Columns (3) and (4) report the alternative result of applying the test with the ACF version of OP/LP. The ACF OP/LP estimator when there is perfect competition has already been presented in Table 1, and hence column (4) of Table 2 simply reproduces column (2) of Table 1 for the sake of convenience.

[33] We call them parametric because they use the inverted FOC as it is, as opposed to the projection that ACF does on $\mathbf{z}$.

Column (3) estimates the version of ACF OP/LP that must be consistent in the presence of market power. A direct substitution of ω into the production function leaves the expression

$$q_{jt} = f(x_{jt}) + q^*_{jt} - f(x_{jt}) + \ln \frac{\nu S_{Mjt}}{\beta_M} + \varepsilon_{jt},$$

what suggests to regress q_{jt} non parametrically in the first stage on a constant, time dummies, and using a complete polynomial of order 3 on the four variables k_{jt}, l_{jt}, m_{jt}, and $\ln S_{Mjt}$. In the second step of the ACF implementation we use the instruments $k_{jt}, k_{jt-1}, l_{jt-1}, m_{jt-1}, \widehat{\phi}_{-1}$ and $\ln S_{Mjt-1}$, that implies two overidentifying restrictions. The result of the test is again the rejection of the absence of market power. The value of the test can only be taken as χ^2 with 3 degrees of freedom with probability 1%

Testing for the specification of the production function

Given these results, it seems clear that there is market power, but we also have evidence that market power is not the only problem. The DP estimates do not look consistent and the linear cost-share OP/LP estimator applied to the Cobb-Douglas production function does not produce nice results either. We hence move to change the production function specification. Using the linear cost-share estimator, we again compare the value of the coefficients obtained under DP and OP/LP. The results are reported in Table 3.

Now we estimate a multiproductivity production function. We use the simplest production function that admits LAP, a translog separable in capital and homogeneous of degree ν in labor and materials (we follow Doraszelski and Jaumandreu, 2019).[34] Because it is homogeneous in labor and materials can be written in terms of the log-ratios materials to labor, and these log-ratios exhibit unobserved labor-productivity,

$$q_{jt} = \alpha_0 + \alpha_K k_{jt} + \nu m_{jt} - \alpha_L (m_{jt} - l_{jt} - \omega_{Ljt}) - \frac{1}{2}\alpha (m_{jt} - l_{jt} - \omega_{Ljt})^2 + \omega_{Hjt} + \varepsilon_{jt}.$$

Using the ratio of first order conditions for labor and materials, we derive an expression to be substituted for these ratios, $m_{jt} - l_{jt} - \omega_{Ljt} = -\frac{\alpha_L}{\alpha} + \frac{\nu}{\alpha} S^*_{Ljt}$, with $S^*_{Ljt} = S_{Ljt} - \frac{\alpha}{\nu}\overline{\omega}_L$, and where $\overline{\omega}_L$ is a guess for the mean of

[34] See for more detail Jaumandreu and Mullens (2024). The specification can be conisdered a particular case of Demirer (2025).

labor-augmenting productivity.[35] Hence, the production function becomes a function of observables in which, to control for Hicks-neutral productivity, we can easily apply both the DP estimation procedure and the OP/LP method. Let us see how we compute each estimator and perform the test.

DP

The DP estimator is obtained by applying nonlinear GMM to the equation

$$q_{jt} = \gamma_0 + \beta_t + \alpha_K k_{jt} + \nu m_{jt} - \frac{1}{2}\frac{\nu^2}{\alpha} S_{Ljt}^{*2} \qquad (15)$$
$$+ \rho[q_{jt-1} - \alpha_K k_{jt-1} - \nu m_{jt-1} + \frac{1}{2}\frac{\nu^2}{\alpha} S_{Ljt-1}^{*2}] + \xi_{jt} + \varepsilon_{jt} - \rho\varepsilon_{jt-1},$$

where the expression is written in a similar format to the previous estimators for the sake of comparability. To estimate this more nonlinear equation we enlarge the instruments with the squares of the inputs, and we add the lagged share of labor cost in variable cost (that it is an important part of the translog): $k_{jt}, k_{jt}^2, l_{jt-1}, l_{jt-1}^2, m_{jt-1}$, $m_{jt-1}^2, w_{jt-1} - p_{jt-1}$, $p_{Mjt-1} - p_{jt-1}$,and $S_{L,-1}$. This gives nine instruments and we hence have five overidentifying restrictions.

Parameric OP/LP

We construct the parametric OP/LP version by adding the term $\ln \frac{\nu S_{Mjt}}{\beta_{Mjt}}$ inside the brackets of (15). It is easy to check that in practice it amounts to adding $\ln \frac{S_{Mjt}}{S_{Mjt}+\frac{\overline{\omega}}{\nu}}$. We use the instruments $k_{jt}, k_{jt-1}, l_{jt-1}, m_{jt-1}, w_{jt-1} - p_{jt-1}$, $p_{Mjt-1} - p_{jt-1}, S_{L-1}$ and vc_{-1}, which imply four overidentifying restrictions. We replace the inputs squared by lagged capital, and add the lagged log of variable costs to help the instrumentation of S_{Mjt} (using S_{Mjt-1} would introduce perfect collinearity).

Nonparametric ACF OP/LP

To estimate the nonparametric ACF version of the OP/LP estimator, we first again regress as before q_{jt} non parametrically on the variables k_{jt}, l_{jt}, m_{jt}, and $\ln S_{Mjt}$. This first step gives the estimate $\widehat{\phi}(k_{jt-1}, l_{jt-1}, m_{jt-1}, \ln S_{Mjt-1})$ that we use in forming the second step equation. In the second step, we use the instruments $k_{jt}, k_{jt}^2, k_{jt-1}, l_{jt-1}, l_{jt-1}^2, m_{jt-1}$, $m_{jt-1}^2, \widehat{\phi}_{-1}$ and $S_{Lj,t-1}$, so that we have five overidentifying restrictions.

[35] We use $\overline{\omega}_L = \overline{m} - \overline{l}$. This specification only estimates directly parameter ν, and the elasticities β_L and β_M are determined by the implications $\beta_L = \nu \overline{S}_L$ and $\beta_M = \nu(1 - \overline{S}_L)$.

Test

The results for the new DP and the parametric and nonparametric OP/LP are reported in columns (1), (2) and (3) of Table 3, respectively. They clearly show that now DP and OP/LP give basically the same estimate of the production function (notice that the nonparametric estimation is, however, less efficient and makes the comparison less demanding).

To statistically check that the estimates can be considered the same, we apply specification tests. We construct a quadratic form of the elasticity of capital and the elasticity of scale, using as weight the inverse of a robust estimator of the asymptotic variance of the difference between the coefficients $(\beta_{DP} - \beta_{OP/LP})$, see Appendix D. The tests, reported for the parametric and nonparametric OP/LP at the bottom of columns (2) and (3) respectively, do not reject that the quadratic form is distributed as χ^2 with 2 degrees of freedom and hence tell us that the differences now can be interpreted as coming from sampling error.

Notice that both the elasticity of capital and the short-run elasticity of scale are sensible. The capital elasticity is greater than with the Cobb-Douglas specification, and the scale elasticity estimate is in the range $0.70 - 0.85$, which clearly improves the unrealistic constant returns to scale for the estimation of variable inputs in column (2) of Table 1. No estimator produces a clear better estimate than the other when correctly specified. From now on, the researcher can focus on improving other aspects of the estimation, such as explicitly accounting for the input market power, introducing the effect of the adjustment cost of variable inputs, or experiment with the way to deflate output and materials .

Testing (ex-post) for market power

We finally want to confirm that it has been relevant to use the linear cost-share OP/LP estimator that, in effect, we cannot reject the presence of market power, now with estimators that meet the necessary condition for consistency. To check this, we compare the OP/LP estimator that is consistent under market power (linear cost-share) with the estimator that is based on assuming perfect competition (revenue-share), both in the parametric version and the nonparametric ACF version, by means of the market power test. To construct a parametric OP/LP based on perfect competition, it is enough to add to (15) $vc - r + \ln \frac{S_{Mjt}}{S_{Mjt} + \frac{\varpi}{\nu}}$, that implements revenue-share. To construct a nonparametric estimator based on perfect competition, we go back to the use of the price of the product in the first stage of ACF. Both constructed estimators are reported in columns (1) and (2) of Table 4 respectively. Then we compare column (1) of Table 4 with the estimator of column (1) of Table 2 and column (2) of Table 4 with column (3) of Table 2. The resulting χ^2 with 2 degrees of freedom show a strong rejection of the null of perfect competition.

11 Concluding Remarks

The production function is not affected by market power, but estimators that employ an auxiliary FOC encompassing a derivative of the production function are sensitive to the form of the FOC under market power. DP is an estimator robust to market power because it does not use any FOCs, while the OP/LP approach cannot be generally robust to market power because marginal revenue, present in the FOCs, depends on the details of the game the firms play. However, marginal cost, which equals marginal revenue and summarizes all relevant effects of the firm's strategy and its demand, can be replaced by average variable cost (corrected by the elasticity of scale) plus the uncorrelated error of the production function. Under the linearity of the productivity process, the error does not affect the estimation. This gives an OP/LP estimator that is feasible under market power, which we have called the linear cost-share OP/LP.

The linear cost share OP/LP can be written adding, to the equation used by DP, the (log) difference between the observed variable cost share and the normalized elasticity of the input whose demand is inverted. It can also be implemented nonparametrically as in ACF. In theory, with this addition, we should have exactly the same estimate because if the production function is right, the theoretical value of the expression is zero. If the estimators diverge, either the production function specification is wrong or there is a problem in the FOCs specification. This gives us a test for the specification.

The researcher who wants to estimate the production function under market power may first test if market power is relevant. If market power is relevant, it is convenient to start by estimating both the DP and the feasible OP/LP estimators, and testing the equality between them. The equality still does not ensure that both estimators are consistent, but they meet a necessary condition for consistency. Further work of specification with the two models may warrant consistent estimation under market power.

We have shown how this works with an example of estimation of the production function for a sample of US manufacturing firms. The naive Cobb-Douglas specification of the production function produces, with the DP estimator, a negative elasticity for capital and a too large short-run elasticity of scale. OP/LP is better because there is flexibility, no consistency. Recognizing LAP in addition to Hicks-neutral productivity and allowing it into the specification induces a matching of DP and the robust to market power linear cost-share OP/LP. The specification test is passed.

We have shown that the use of the FOCs together with the production function can be taken advantage of for testing the presence of market power and for improving the specification of the production function itself. Our empirical results suggest that market power can be frequent, that is easy to detect, and its impact in estimation nonnegligible with non robust estimators. The results also suggest the need for more flexible production functions than is normally assumed. The good relative behavior of OP/LP seems more linked to its implicit flexibility than a generality that can only be reached in its robust version.

Appendix A: Identification

Assume the timing and information conditions of the text. The FOC to be used by an OP/LP procedure becomes $\omega = d - \ln \frac{\partial F(\mathbf{x})}{\partial X} = h(d, \mathbf{x}; \theta_1)$, where $d = \ln \frac{W_X}{P}$, and θ_1 represent the parameters of the derivative. The model to be estimated can be written as

$$q = f(\mathbf{x}; c_0, \theta) + g(h(d_{-1}, \mathbf{x}_{-1}; \theta_1); \theta_2) + \xi + \varepsilon,$$

where θ are the parameters of the production function in addition to the constant c_0, and $\theta_1 \subset \theta$. If we are in a strictly nonparametric setting, the vectors θ, θ_1, and θ_2 are infinite-dimensional. If we want to approximate the nonparametric relationship by a flexible form, we are going to use a limited number of parameters. If the procedure for estimation is nonlinear GMM, identification basically depends on having the same or more valid moments than parameters to estimate.

For example, to start with something simple, if the production function is Cobb-Douglas with inputs k, l and m, vector θ has dimension 3, vector θ_1 the same dimension 3 (if we use the demand for m, say), and we may decide to be linear using for $g(\cdot)$ an $AR(1)$ parameter ρ. This gives 4 parameters to estimate in addition to the constant. With instruments k, k_{-1}, l_{-1} and m_{-1} (in addition to the constant), we are exactly identified.

Notice first that this form of estimation avoids any problem of "functional dependence". Combine the FOCs and write the unconditional demands $m = m(d_M, d_L, k, \omega)$ and $l = l(d_M, d_L, k, \omega)$. Inverting these demands we have $\omega = h_M(d_M, d_L, k, m)$ and $\omega = h_L(d_M, d_L, k, l)$. Suppose that we enlarge the number of parameters of $h_M(\cdot)$ carrying out flexible ("nonparametric") estimation to control for ω as proposed by Levinsohn and Petrin (2003) using $q = f(\mathbf{x}) + h_M(d_M, d_L, k, m) + \varepsilon$. Ackerberg, Caves and Frazer (2015, p. 2422-23) argue that $l - E(l|d_M, d_L, k, m)$ is not different from zero because l is an exact function of only (d_M, d_L, k, m) and hence Robinson (1988) procedure of nonparametric control breaks down. This argument does not affect the above procedure because if one uses $q = f(\mathbf{x}) + \rho[h_M(d_{M,-1}, d_{L,-1}, k_{-1}, m_{-1})] + \xi + \varepsilon$, there is no functional dependence and the nonparametric control works fine.

Of course, this works as long as we do not have perfect collinearity between the quantities of the two variable inputs. Bond and Soderbom (2005) argued that if prices are common to all firms at a given point of time, and inputs are perfectly flexible, all variable "inputs are perfectly collinear with the productivity shocks observed by firms" (page 1). That is, it would not be identification for the elasticities of a simple Cobb-Douglas cross section, although adjustment costs can partially alter this. However, the first order conditions allow us to identify the elasticities of the variable inputs from the "share equation," as Doraszelski and Jaumandreu (2013) show in footnote 14.

Gandhi, Navarro, and Rivers (2020), looking for the limits to identification, give a theorem that can be read setting an optimistic minimum: with a unique perfectly flexible input, identification of all elasticities requires time series variation of the ratio of the price of the input to the output price, d_M say. In fact, this variation seems only needed when there are two variable inputs and there are no constant returns to scale (see Trunschke and Judd, 2024, for a Monte Carlo).

These results can be extended for flexible specifications. Assume now that we want to make an approximation of $f(\mathbf{x})$ based on a complete polynomial of order 3 (the most used approximation) and $\mathbf{x}$ has n inputs. The number of parameters in (c_0, θ) is $1+2n+\frac{n(n-1)}{2}+n^2+\frac{n(n-1)(n-2)}{6}$. If $n=3$ this gives us 20 parameters (the first is the constant). If we decide to estimate $g(\cdot)$ also by means of three powers, we have to estimate a total of 23 parameters. Using the constant and vector (k, l_{-1}, m_{-1}) we can form the 20 moments. Adding the new moments that we can form using k_{-1} and interacting with l_{-1} and m_{-1}, we have 10 more. We therefore have 7 overidentifying restrictions. As long as we do not have perfect collinearity between the quantities of two variable inputs, identification is warranted.

Appendix B: Modeling marginal revenue

What is in MR_j? Let us assume for economy of notation that competition, both in quantities and prices, happens with product differentiation.[36] Under product differentiation firm j demand is $Q_j^* = D_j(P, \delta)$, where P is the vector of N prices and δ is a vector of unobserved correlated heterogeneity of demand (observable heterogeneity z can always be easily included). Assume that the demand system can be inverted, $P_j = D_j^{-1}(Q^*, \delta)$, where Q^* is the vector of N quantities. Revenue is $P_j Q_j^*$. If the firm competes in quantities, $MR_j = P_j + Q_j^* \frac{\partial D_j^{-1}}{\partial Q_j^*} + T(P_j, Q_j^*, conduct)$ and if the firm competes in prices,

[36]The discussion can be easily extended to the case of homogeneous product. We take as reference the market power models in Vives (1999).

we can write the implicit $MR_j = P_j + (Q_j^* + T(P_j, Q_j^*, conduct)) \left(\frac{\partial D_j}{\partial P_j}\right)^{-1}$. In both cases $T(\cdot)$ represents a function.

There are two problems implied by these expressions. The first is that they depend in an unspecified way on the market behavior of firms. In the absence of a specific conduct, we have no function of observed variables but only a correspondence. Different $MR(\cdot)$ values can be associated with exactly the same P_j or Q_j^*, depending on the behavior in the market.

The second problem is the presence of Q_j^* and δ in the expressions, and the derivatives $\frac{\partial D_j^{-1}}{\partial Q_j^*}$ and $\frac{\partial D_j}{\partial P_j}$. The variables Q_j^* and δ are non observable (we observe the actual output Q_j). Maybe we can use $Q_j^* = D_j(P, \delta)$ and have $MR(P,\ \delta,\ \mathit{conduct})$, but notice that, in general, P and δ are vectors, the second the vector of N unobserved variables. We can also try using $Q_j^* = R_j(Q_{-j}^*)$ or $P_j = R_j(P_{-j})$, where $R_j(\cdot)$ are the corresponding best response functions. But these reaction functions also include unobservables and are behavior specific.

A compact way to think of the models that are possible is as follows. Start with the output-conditional demand for the variable input $X_j = \widetilde{X}(K_j, W_j, P_{M,j}, Q_j^*, \omega_j)$, that can be written with common input prices $X_j = \widetilde{X}(K_j, Q_j^*, \omega_j)$. Use the (sales) market share of the firm S_j divided by P_j to express the production of the firm as a function of the aggregate sales of the market A, $Q_j^* = \frac{S_j}{P_j} A$. If S_j is only a function of (K_j, ω_j), the demand for the input can be expressed as a time varying function $X_j = \overline{X}(K_j, P_j, \omega_j)$.

This is close to what the standard application of OP/LP assumes to be the arguments of the input demand to be inverted, except for the output price. However, note that we have reached the expression assuming that firms cannot be unequal because of input price differences, suppressing unobserved correlated demand heterogeneity and/or asymmetric behavior. That is, removing efficiency factors other than ω before starting the investigation. In general, we want to avoid this.

Appendix C: Conduct specification

Let us see with an example how MR_j changes with conduct, in this case represented by the type of competition and the unobservable exogenous parameter λ. Let us suppose for simplicity 2 firms (the industry can have, for example, $N/2$ of each type). We remove the asterisk from the quantities Q_j^* for the sake of the economy of notation. We also abstract from the heterogeneity of demand. Firms 1 and 2 have demands

$$\begin{aligned} Q_1 &= P_1^{-\eta} P_2^{\gamma}, \\ Q_2 &= P_2^{-\eta} P_1^{\gamma}, \end{aligned}$$

and costs $C_1(Q_1)$ and $C_2(Q_2)$. Inverse demands are

$$\begin{aligned} P_1 &= Q_1^{-\eta^*} Q_2^{-\gamma^*}, \\ P_2 &= Q_2^{-\eta^*} Q_1^{-\gamma^*}, \end{aligned}$$

where $\eta^* = \frac{\eta}{\eta^2-\gamma^2}$ and $\gamma^* = \frac{\gamma}{\eta^2-\gamma^2}$. Firm i maximizes

$$\pi_i + \lambda\pi_j = P_i Q_i - C_i(Q_i) + \lambda(P_j Q_j - C_j(Q_j)),$$

where λ is an exogenous conduct parameter. If firms compete in prices

$$\begin{aligned} \frac{\partial(\pi_i + \lambda\pi_j)}{\partial p_i} &= Q_i - \eta P_i \frac{Q_i}{P_i} + \eta C_i^{'} \frac{Q_i}{P_i} + \lambda[\gamma P_j \frac{Q_j}{P_i} - \gamma C_j^{'} \frac{Q_j}{P_i}] = \\ &= P_i - \eta(p_i - C_i^{'}) + \lambda\gamma(P_j - C_j^{'}) \frac{Q_j}{Q_i} = 0, \end{aligned}$$

and if they compete in quantities

$$\frac{\partial(\pi_i + \lambda\pi_j)}{\partial Q_i} = P_i - \eta^* Q_i \frac{P_i}{Q_i} - C_i^{'} - \lambda\gamma^* Q_j \frac{P_j}{Q_i} = 0.$$

Using symmetry, it is easy to see that $MR_j = P_j(1 - \frac{1}{\eta(1-\lambda\frac{\gamma}{\eta})}) = C_j^{'}$ under price competition and $MR_j = P_j(1 - \frac{\eta}{\eta^2-\gamma^2}(1+\lambda\frac{\gamma}{\eta})) = C_j^{'}$ under quantity competition. If $\lambda = 1$, both marginal revenues coincide in $P_j\frac{1}{\eta-\gamma}$, the unique total collusive solution. If $\lambda = 0$, Cournot with $P_j(1 - \frac{1}{\eta(1-(\frac{\gamma}{\eta})^2)})$ is less competitive than Bertrand, giving $P_j(1 - \frac{1}{\eta})$. If $0 < \lambda < 1$, the quantity competition is less competitive than the price competition.

Appendix D: Specification tests

A Hausman (1978) specification test, or a Durbin-Wu-Hausman test, can be seen as a test of the equality between the parameter estimates under two methods of estimation that are consistent under the null. The alternative is in our case that one method is inconsistent (market power test) or that either one method or the two are inconsistent (general specification test). Following Wooldridge (2010), we set a quadratic form of the differences in the parameters $(\widehat{\beta}_A - \widehat{\beta}_B)$ using the inverse of a robust estimate of $Avar[\sqrt{N}(\widehat{\beta}_A - \widehat{\beta}_B)] = V_A + V_B - (C + C')$.

Let $i, l = A, B$. To estimate V_i, we use

$$\widehat{V}_i = (\widehat{G}_i' A_{Ni} \widehat{G}_i)^{-1} \widehat{G}_i' A_{Ni} \widehat{\Omega}_i A_{Ni} \widehat{G}_i (\widehat{G}_i' A_{Ni} \widehat{G}_i)^{-1},$$

with $\widehat{G}_i' = N^{-1} \sum_j \frac{\partial (Z_{ij}' \widehat{u}_{ij})}{\partial \beta_i}$, $A_{Ni} = N^{-1} \sum_j Z_{ij}' Z_{ij}$, and $\widehat{\Omega}_i = N^{-1} \sum_j Z_{ij}' \widehat{u}_{ij} \widehat{u}_{ij}' Z_{ij}$. To estimate $\widehat{C}$, we use

$$\widehat{C} = (\widehat{G}_i' A_{Ni} \widehat{G}_i)^{-1} \widehat{G}_i' A_{Ni} \widehat{\Omega}_{il} A_{Nl} \widehat{G}_l (\widehat{G}_l' A_{Nl} \widehat{G}_l)^{-1},$$

where $\widehat{\Omega}_{il} = N^{-1} \sum_j Z_{ij}' \widehat{u}_{ij} \widehat{u}_{lj}' Z_{lj}$. Therefore, $\widehat{Avar}\ (\widehat{\beta}_A - \widehat{\beta}_B) = (\widehat{V}_A + \widehat{V}_B - (\widehat{C} + \widehat{C}'))/N$.

Under the null, we have

$$(\widehat{\beta}_A - \widehat{\beta}_B)' Avar[\sqrt{N}(\widehat{\beta}_A - \widehat{\beta}_B)]^{-1} (\widehat{\beta}_A - \widehat{\beta}_B) \sim \chi^2(p),$$

where p degrees of freedom are the number of parameters tested.

Appendix E: Additional regressions

Table AD reports a few complementary estimates. Column (1) reports the result of carrying out an OLS estimation of the Cobb-Douglas specification in the first differences. Although the capital coefficient is small, it is positive and statistically significant.

Column (2) reports the results of the estimation of a nonlinear nonparametric ACF OP/LP. Recall that this is a theoretically inconsistent estimator under market power. The coefficients modeling the nonlinear productivity process are not separately significant, and the coefficient on capital and the short-run scale elasticity (1.022) are similar to the values obtained with the CD specification.

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Table 1: Standard DP and OP/LP estimation

US manufacturing production function estimation,
firm-level sample, Compustat, 1960-2018.

Parameter	Cobb-Douglas with neutral productivity: DP[a]	Cobb-Douglas with neutral productivity: (ACF) OP/LP[b]
	(1)	(2)
ρ	1.022	1.018
	(0.002)	(0.004)
β_K	-0.023	0.061
	(0.010)	(0.020)
β_L	0.486	0.486
	(0.020)	(0.050)
β_M	0.622	0.532
	(0.014)	(0.024)
Overidentifying restrictions	2	1
Function value	8.043	0.701
No. of firms	5621	5621
No. of observations	65006	65006

[a] Instruments are (in addition of constant and time dummies): $k, k_{-1}, l_{-1}, m_{-1}$, $w_{-1} - p_{-1}, p_{M,-1} - p_{-1}$.

[b] First stage: Constant, time dummies, and nonparametric $\phi(k, l, m, w - p, p_M - p)$. Second stage instruments are (in addition of constant and time dummies): $k, k_{-1}, l_{-1}, m_{-1}, \widehat{\phi}_{-1}$.

Table 2: (Ex-ante) testing for market power

US manufacturing production function estimation,
firm-level sample, Compustat, 1960-2018.

Parameter	Cobb-Douglas with neutral productivity			
	Parametric OP/LP		ACF OP/LP	
	Cost-share[a]	Revenue-share[b]	Cost-share[c]	Revenue-share[d]
	(1)	(2)	(3)	(4)
ρ	1.054	0.964	0.977	1.018
	(0.008)	(0.002)	(0.030)	(0.056)
β_K	0.180	-0.679	0.097	0.061
	(0.025)	(0.137)	(0.101)	(0.020)
β_L	-1.176	-0.424	0.521	0.486
	(0.091)	(0.334)	(0.283)	(0.050)
β_M	2.057	4.337	0.323	0.532
	(0.079)	(0.173)	(0.075)	(0.024)
Market power test				
$\chi^2(df)$		363.581(3)		11.387(3)
$p-value$		0.000		0.010
Overidentifying restrictions	3	3	2	1
Function value	223.199	776.678	12.392	0.701
No. of firms	5621	5621	5621	5621
No. of observations	65006	65006	65006	65006

[a] Instruments are (in addition of constant and time dummies): $k, k_{-1}, l_{-1}, m_{-1}$, $w_{-1} - p_{-1}, p_{M,-1} - p_{-1}, \ln S_{M,-1}$.
[b] Instruments are (in addition of constant and time dummies): $k, k_{-1}, l_{-1}, m_{-1}$, $w_{-1} - p_{-1}, p_{M,-1} - p_{-1}, vc_{-1} - r_{-1} + \ln S_{M,-1}$.
[c] First stage: Constant, time dummies, and nonparametric $\phi(k, l, m, \ln S_M)$.
Second stage instruments are (in addition of constant and time dummies): $k, k_{-1}, l_{-1}, m_{-1}, \widehat{\phi}_{-1}, \ln S_{M,-1}$.
[d] First stage: Constant, time dummies, and nonparametric $\phi(k, l, m, w - p, p_M - p)$.
Second stage instruments are (in addition of constant and time dummies): $k, k_{-1}, l_{-1}, m_{-1}, \widehat{\phi}_{-1}$.

Table 3: Testing the specification

US manufacturing production function estimation,
firm-level sample, Compustat, 1960-2018.

Parameter	Translog multiproductivity		
	DP[a]	Cost-share OP/LP[b]	Cost-share ACF OP/LP[c]
	(1)	(2)	(3)
ρ	1.012	1.012	0.966
	(0.002)	(0.004)	(0.040)
β_K	0.166	0157	0.103
	(0.041)	(0.023)	(0.112)
$\beta_L + \beta_M$	0.700	0.681	0.845
	(0.071)	(0.062)	(0.319)
α	0.089	0.038	0.260
	(0.016)	(0.006)	(0.193)
σ	0.578	0.753	0.367
Specification test			
$\chi^2(df)$		2.031(2)	0.558(2)
$p - value$		0.362	0.757
Overidentifying restrictions	5	4	5
Function value	44.663	113.647	5.783
No. of firms	5621	5621	5621
No. of observations	65006	65006	65006

[a] Instruments are (in addition of constant and time dummies): $k, l_{-1}, m_{-1}, k^2, l_{-1}^2, m_{-1}^2$, $w_{-1} - p_{-1}, p_{M,-1} - p_{-1}, S_{L,-1}$.
[b] Instruments are (in addition of constant and time dummies): $k, k_{-1}, l_{-1}, m_{-1}$, $w_{-1} - p_{-1}, p_{M,-1} - p_{-1}, S_{L,-1}, vc_{-1}$.
[c] First stage: Constant, time dummies, and nonparametric $\phi(k, l, m, \ln S_M)$.
Second stage instruments are (in addition of constant and time dummies): $k, k_{-1}, l_{-1}, m_{-1}$, $k^2, l_{-1}^2, m_{-1}^2, \widehat{\phi}_{-1}, S_{L,-1}$.

Table 4: (Ex-post) testing for market power

US manufacturing production function estimation,
firm-level sample, Compustat, 1960-2018.

Parameter	Translog multiproductivity	
	Parametric OP/LP Revenue-share[a]	ACF OP/LP Revenue-share[b]
	(1)	(2)
ρ	0.969	0.981
	(0.002)	(0.001)
β_K	-1.114	-0.876
	(0.107)	(0.090)
$\beta_L + \beta_M$	4.634	3.626
	(0.293)	(0.241)
α	0.862	10.349
	(0.093)	(7.194)
σ	0.486	0.061
Specification test		
$\chi^2(df)$	234.659(2)	57.851(2)
$p - value$	0.000	0.000
Overidentifying restrictions	4	6
Function value	1892.609	963.820
No. of firms	5621	5621
No. of observations	65006	65006

[a] Instruments are (in addition of constant and time dummies): $k, k_{-1}, l_{-1}, m_{-1}$, $w_{-1} - p_{-1}, p_{M,-1} - p_{-1}, S_{L,-1}, r_{-1} - vc_{-1}$.

[c] First stage: Constant, time dummies, and nonparametric $\phi(k, l, m, w - p, p_M - p)$. Second stage instruments are (in addition of constant and time dummies): $k, k_{-1}, l_{-1}, m_{-1}, k^2, l^2_{-1}, m^2_{-1}$, $\widehat{\phi}_{-1}, S_{L,-1}, r_{-1} - vc_{-1}$.

Table AD: Complementary estimates

US manufacturing production function estimation,
firm-level sample, Compustat, 1960-2018.

Parameter	Cobb-Douglas OLS in differences	Translog multiproductivity Nonlinear nonparametric OP/LP[a]
	(1)	(5)
ρ		0.779
		(0.238)
ρ_2		0.070
		(0.075)
ρ_3		-0.006
		(0.007)
β_K	0.027	0.057
	(0.009)	(0.029)
β_L	0.339	0.506
	(0.003)	(0.079)
β_M	0.555	0.516
	(0.002)	(0.036)
Overidentifying restrictions		1
Function value		0.629
No. of firms	5621	5621
No. of observations	65006	65006

[a] First stage: Constant, time dummies, and nonparametric $\phi(k, l, m, w-p, p_M - p)$.
Second stage instruments are (in addition of constant and time dummies):
$k, k_{-1}, l_{-1}, m_{-1}, \widehat{\phi}_{-1}, \widehat{\phi}^2_{-1}, \widehat{\phi}^3_{-1}$.