Robust Production Function Estimation when there is Market Power

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Abstract

The production function is an input/output technical relationship, not affected by market power. However, recent estimators make auxiliary use of optimal choices of the firm that depend on market power. In this context, marginal cost encompasses all relevant information on the firm's strategic actions and heterogeneity of demand. For being robust, it is enough to specify the estimator in terms of the average variable cost and the ratio AVC/MC or short-run elasticity of scale. This also reveals that the estimators known as "dynamic panel" and "OP/LP" are closely related. We derive two specification tests.

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1 Introduction

What is the relationship between production function estimation and market power? The production function is a technical relationship, that describes how technology relates inputs and output. So there is no direct relationship. But many recent proposals for the estimation of production functions include the auxiliary use of equations based on the optimization of some objective that involves the production function. Mainly the FOCs corresponding to profit maximization. And these FOCs are different when the firm has some market power.

Many production function estimators have been developed for situations of perfect competition or almost (Levinsohn and Petrin, 2003; Gandhi, Navarro and Rivers, 2020, or the way to implement these estimators proposed by Ackerberg, Caves and Frazer, 2015; see later on the case of the starting work by Olley and Pakes, 1996). The urgency to have readily available methods to estimate production functions has often implied the neglect of the conditions for applicability of these estimators under imperfect competition.

One particular case is product differentiation. A market with product differentiation is a market with some market power that emerges from the ability and incentives of the firms to produce products with different characteristics. This situation generates by itself an uncomfortable context for the user of the production function. The production function, as a concept of economic theory, was developed as describing a situation in which the characteristics of the unique product are given and the available techniques define the set of production possibilities. Products and possibilities are now distinct across firms.

How to introduce the heterogeneity of characteristics and possibilities that emerge with differentiated products? Suppose that the empirical researcher contents herself with the allowance for heterogeneity in the production function that gives the inclusion of an additive random deviation in the equation to be estimated. The market power still implies that some estimators cannot be applied ignoring the consequences in the FOCs.

To study what to do when there is market power it is convenient to start with the usual practices. In the current literature there are two approaches to the estimation of the production function. They diverge in how they solve the problem of controlling for unobservable productivity. How to treat unobserved productivity has been the dominant worry of researchers since Marschak and Andrews (1944) pointed at the statistical problems created by the endogeneity of the inputs.

We will call the two approaches "dynamic panel," henceforth DP, developed in Arellano and Bond (1991) and Blundell and Bond (2000), and the OP/LP approach, called in this way because it was originated in the articles of Olley and Pakes (1996) and Levinsohn and Petrin (2003). In the first approach, unobservable productivity is differentiated out after assuming that follows a first order linear Markovian or $AR(1)$ process. The second approach also assumes that the unobservable follows a Markovian first order process, but this process needs not to be restricted to be linear. The unobservable is replaced by the inverse of a function representing some optimal observable choice that contains it. A big advantage of DP is hence that doesn't need to use any auxiliary relationship implying behavior. OP/LP frees the linearity form of the Markov process, at the price of assuming that some behavioral equation holds.

However, a well known empirical paradox is that DP often produces disappointing results while OP/LP, with more conditions to be met, often produces more reasonable estimates (e.g. in the elasticity of capital). In this paper we explore the theoretical difference between the two estimators and derive what can be learned from their divergence.

The conclusion is that both the DP and OP/LP estimators are consistent when there is perfect competition, but OP/LP is not robust when there is market power. The consistency of a non-robust estimator depends on the detailed assumptions about how the game that firms play in the market is, and their market share consequences. A lot of symmetry and unwanted restrictions should be set beforehand to ensure consistency.

We then derive an OP/LP estimator that is consistent under any form of market power. To be robust it is enough to replace MR/MC by AVC and the short-run elasticity to scale, and specify the process of productivity as linear. Under this specification, it happens that both DP and OP/LP are consistent and should deliver estimates that only differ due to sampling error.

This suggests an specification test: the coefficients of both models should be equal under the null of consistency of both estimators, while they will differ under the alternative of either both estimators, or only OP/LP, being inconsistent. The first variant of the alternative can happen because the production function is wrongly specified, the second because what is wrong is the specification of the FOC on which OP/LP bases the demand for an input.

The researcher may want to confirm the presence of market power, and this can be done in the same context carrying out an additional specification test. OP/LP can be specified using the price of the firms or using the "proxied" marginal cost. Under the null of no market power both estimators should only differ due to sampling error, under the alternative of market power only the OP/LP based on proxying MC is consistent.

To illustrate the relevance of the problem, the realism of the circumstances, and the working of the procedures that we employ, we estimate the production function for the sample of US manufacturing Compustat firms used in Jaumandreu and Mullens (2024). It is a sample with more that 5,000 firms and 60,000 observations that are likely to exhibit the more diverse degrees of market power. The estimation by DP and OP/LP diverges when naively applied to a Cobb-Douglas specification, and passes the specification test when applied to its enlargement into a translog with labor-augmenting productivity that shows elasticity of substitution less than one and falling labor shares. Applied to this new specification, neither DP nor the feasible OP/LP are better. We think that this constitutes a reasonable place where to start the exploration for refinements.

Current approaches have treated intensively how to deal with Hicks-neutral productivity, that affects all inputs in the same way. However, empirical research has recently stressed that productivity is likely to be biased. And there have been contributions on how to apply DP and OP/LP when productivity is non-neutral and affects in particular an specific input. For example, Doraszelski and Jaumandreu (2018, 2019), show how to replace biased productivity from a ratio of FOCs. The dominant interest in this field is labor-augmenting productivity, presumably very related to the dominant form of current technological progress. In what follows, we systematically take into account the possibility of labor-augmenting productivity. The previous paragraph already shows that, in practice, it solves a problem of specification of the production function.

Input market power can be as relevant as product market power, and it affects the first order conditions as well as product market power. We also show summarily but systematically how to treat input market power when it is present, and we discuss the way to detect this mispecification if it is binding.

The rest of the paper is organized as follows. Section 2 comments on the relation of the paper to the literature. Sections 3 and 4 explain the consistency of DP and OP/LP under perfect competition and how the specification can be tested using these two models. Section 5 studies the consequences of facing market power for an OP/LP type of estimator. Section 6 develops the way to replace MC so that the estimator is robust to market power. Section 7 develops the example with the sample of Compustat firms, and section 8 concludes. Four appendices deal with identification, conduct specification, statistical specification tests, and additional regressions and test, respectively.

2 Relation to the literature

The literature on the new estimators for the production function, sometimes called structural, has always been very conscious of the need to deal with market power. Olley and Pakes (1996) consider that the firms in the market are playing a dynamic oligopoly game and justify the simplification of the vector of state variables by means of symmetry that includes common input prices. Griliches and Mairesse (1996), writing contemporaneously on the "interesting new approach" of OP, worry if this treatment of the state variables may be ignoring some relevant dimensions as the expectations on the cost of investment. Levinsohn and Petrin (2003) define their setting as a competitive environment, where firms take as given output and input prices, and warn that the model can be generalized to imperfect competition but then it will depend on the specifics of competition.

Ackerberg, Caves and Frazer (2015), discussing when revenues can replace physical quantities (e.g. common output prices), introduces an explicit discussion about the difficulties to invert the demand for an input when the demand for output and/or the supply for an input are downward and upward curves respectively (i.e. there is market power). They warn that, in this situation, even assuming identical curves may be not enough. Gandhi, Navarro and Rivers (2020) make clear that their model for non parametric estimation of the production function is developed assuming perfect competition in the output and intermediate markets. However, they show in Appendix O6 how the model can be applied specifying a parametric CES demand for output, together with the assumption of monopolistic competition, a model that many researchers prefer.

More recently, a few discussions have dealt in one way or another with the ability of the OP/LP framework to address the situations with market power. Bond, Hashemi, Kaplan and Traina (2021) stress how the absence of reliable information on firm-level output prices makes difficult the estimation of structural elasticities and hence market power, and stress the robustness of the DP approach. Doraszelski and Jaumandreu (2021) develops the biases that affect an OP/LP procedure given the likely presence of correlated unobservable demand heterogeneity. Ackerberg and De Loecker (2024) is a discussion on how to expand the OP/LP estimators to include "sufficient statistics" to account for imperfect competition under behavioral and symmetry assumptions.

This paper makes, in the first place, a contribution to these discussions. It deals about how to construct estimators that are robust to market power, in the sense that they do not depend on the specification of the details of the game the firms play. This possibility builds on proxying MC as the variable that accounts for the result of the firm-level strategic interactions and heterogeneity of demand, and makes the need for other variables redundant. We show that OP/LP is nonrobust to market power, but also that there is a feasible (linear) OP/LP that can avoid this difficulty. This estimator uses more information than DP, and the divergence with DP can be used as a test of specification. Also it is possible to use the different versions of the feasible OP/LP as a test for the presence for market power. The result of our discussion is then rather a way to conduct the specification more than a particular estimator that fits all sizes and shapes.

When the estimation of the production function goes wrong, the researcher can have a feeling on what the problem is by looking to the results of the optimal choices of the firms. For example, the input cost shares. However, testing the production function specification cannot be done if the estimation is not dealing properly with market power, the same that trying to assess market power with a production function that is wrongly specified it is likely to be uninformative. There is no simple way to separate the problems.

A long list of papers have recently stressed that the presence of Hicks neutral productivity should be complemented with the presence of biased productivity, particularly in the form of labor-augmenting productivity. See mainly Doraszelski and Jaumandreu (2018), Raval (2019, 2023), Zhang (2019), Demirer (2022), Jaumandreu and Mullens (2024), Kusaka, Okazaki, Onishi and Wakamori (2024).[1] We add to this literature by uncovering that the mispecification re-

[1] A recent literature is exploring the nonparametric estimation of a unique productivity term, freely interacted with the inputs. See Ackerberg, Hahn and Pan (2023) and Pan (2024).

vealed by our estimators and procedure to specify, when applied to sample of Compustat firms, is redressed when we consider an specification that allows shares in cost and elasticities to change from firm to firm and over time. We could not have formally assessed that this was the specification problem without estimators robust to market power, and we had never got the coincidence of the estimators without the change in the specification of productivity in the production function. Our addition is a pioneering modeling of firm-level different dimensions of productivity in US manufacturing, that confirms the biased technological change that Raval (2019) found with Census of Manufacturing data on plants. It provides a rich chacterization on the firm dynamics of labor-augmenting productivity (see Jaumandreu and Mullens, 2024), with a flexible production function and subject to the rigor of the specification tests.

A recent literature has stressed that market power in the input markets can be as relevant as market power in the product market. See, for example, the papers by Dobbelaere and Mairesse (2013, 2018), Yeh, Macaluso, Hershbein (2022), Azzam, Jaumandreu and Lopez (2022) and Rubens (2023). The estimator OP/LP is not robust to the presence of input market power. Given the initial character of this paper, we only focus marginally on this topic. However, we show how the tools that we have developed can be applied to the detection of input market power affecting the estimation of the production function and, summarily, how they can be used for consistent estimation under this presence.

3 DP and OP/LP under perfect competition

Let us first clarify the relationships between the two estimators under perfect competition. The assumption of perfect competition implies that the price of the output is common for all firms and equals marginal cost. Firms differ in size because differ in their marginal cost curves, though. The usual time and information assumptions are as follows. Firms choose the variable inputs l and m at t, when unobserved productivity becomes their knowledge, but capital needs time to build and is given as chosen one period before.

Notice that everything that we are going to say is compatible with the presence of labor-augmenting productivity. To see this it is enough to suppose that the labor input is $l^{*} = l + \omega_L$ and that labor-augmenting productivity ω_L has been controlled for observables.

Assume a population of firms (we drop firm and time subscripts). Write the production function in logs as

$$q = f(x) + \omega + \varepsilon, \tag{1}$$

where $f(x) = \ln F(x)$, $x = \{k, l, m\}$ is the logs of capital, labor and materials, ω is Hicks-neutral productivity, and ε is an error of observation, not autocorrelated and uncorrelated with all variables known at t. Sometimes we will use the notation $q^* = f(x) + \omega$ for the output without error.

A first order Markov process establishes

$$\omega = g(\omega_{-1}) + \xi. \tag{2}$$

DP assumes that productivity follows the linear Markov process $\omega = \rho\omega_{-1} + \xi$. The implication is that we can "pseudo-differentiate" equation (1) (subtract the lagged equation multiplied by ρ) and unobservable productivity drops

$$q = \rho q_{-1} + f(x) - \rho f(x_{-1}) + \xi + \varepsilon - \rho\varepsilon_{-1}. \tag{3}$$

From the point of view of estimation, the inputs of x vector that are set at t, when the shock of the Markov process is known, are correlated with ξ and should be instrumented. These are the variable inputs l and m. If the production function $f(\cdot)$ only requires the estimation of three parameters (additional to the constant), we need four instruments because we have to estimate the extra parameter ρ (that introduces nonlinearity in the model). The model is exactly identified using k, k_{-1}, l_{-1} and m_{-1} as instruments.

It can be assumed that lagged input and output prices, w_{-1}, $p_{M,-1}$ and p_{-1}, are non-correlated with ξ. Then, using them as instruments gets overidentifying restrictions. Cost and firm-demand shifters can be used as additional instruments.

OP/LP is based on the first order conditions for the variable inputs. These have the form.

$$P\frac{\partial F(x)}{\partial X}\exp(\omega) = W_X,$$

where P is the price of the output, $X = L, M$ and $W_X = W, P_M$. Unobserved productivity ω can be obtained by inverting one of these FOC or using the combination of both. A combination of the first order conditions drops one variable input including both input prices in addition to the price of the output (this is the unconditional input demand). With perfect competition we expect P to be common across firms, so the only variation of P is over time and can be subsumed in a system of time dummies. But the price (s) of the input (s) is (are) not necessarily the same and they must be explicitly included except when equality across firms is assumed.

The model can be extended to the case of input market power by assuming that the relevant input price is $W_X^* = W_X(1+\tau)$, where the markdown τ is either an additional parameter to estimate or is controlled for observables.[2]

Let us use without loss of generality only one FOC (sometimes this has been called to use the demand for a variable input conditional on the other)[3]

$$\omega = w_X - p - \ln \frac{\partial F(x)}{\partial X}.$$

The assumption that ω follows a general first order Markov process allows us to write the production function replacing ω_{-1} by its expression according to the inverse of the conditional input demand

$$q = f(x) + g(w_{X,-1} - p_{-1} - \ln \frac{\partial F(x_{-1})}{\partial X_{-1}}) + \xi + \varepsilon. \tag{4}$$

The unknown function $g(\cdot)$ is typically specified by means of polynomials and the model easily estimated in one step by nonlinear GMM. Note that the derivatives of $F(x)$ will include at most the same parameters as $F(x)$, so $f(x)$ and $\frac{\partial F(x_{-1})}{\partial X_{-1}}$ are linked by equality restrictions, even if we are dealing with a flexible specification.[4] See Appendix A for a discussion on identification. We face exactly the same problem of endogeneity as before: the variable inputs l and m are correlated with ξ. If we have to estimate four parameters, variables k, k_{-1}, l_{-1}, and m_{-1} are enough for identification. As $g(\cdot)$ is usually made of polynomials, it seems natural to enlarge the instrument set with powers of the instruments. Prices and shifters can be used as before as additional instruments.

[2]See, for example, the treatment of τ as parameter in Azzam, Jaumandreu and Lopez (2022).

[3]Later we will use the demand for the input conditional on output, that can be obtained using the ratio of FOCs to replace one variable input in the production function by the relationship with the other.

[4]Not recognizing this may produce unproductive discussions on identification. It is customary to apply nonparametric estimation with a polynomial specification.

4 DP and OP/LP in a common framework

Both DP and OP/LP estimators are presented differently for pedagogical reasons (pseudodifferentiation, replacement by the inverse of an input demand), but they can be seen under a more common perspective. It happens that both estimators assume a first order Markov process for productivity, and then propose to replace past productivity by an expression in terms of observables. We can say that both estimators start by assuming that the production function can be written as

$$q = f(x) + g(\omega_{-1}) + \xi + \varepsilon, \tag{5}$$

because of the process of productivity. Then DP proposes to replace ω_{-1} by $q_{-1} - f(x_{-1}) - \varepsilon_{-1}$, and OP/LP by $w_{X,-1} - p_{-1} - \ln \frac{\partial F(x_{-1})}{\partial X_{-1}}$. DP uses the lagged production function, OP/LP the lagged FOC.

However, call β_X to the production elasticity of input X and S_X its share in variable costs. We use β_X for notational simplicity, but it should be clear that in general it is a function $\beta_X(\cdot)$ of the inputs and labor-augmenting productivity and so is the short-run elasticity of scale $\nu(\cdot) = \beta_L(\cdot) + \beta_M(\cdot)$.[5] It is easy to see that the OP/LP expression can be equivalently written as $q_{-1} - f(x_{-1}) + \ln \frac{\nu_{-1} S_{X,-1}}{\beta_{X,-1}} - \varepsilon_{-1}$.[6] Both estimators are hence nested in the expression

$$q = f(x) + g(q_{-1} - f(x_{-1}) + \ln \frac{\nu_{-1} S_{X,-1}}{\beta_{X,-1}} - \varepsilon_{-1}) + \xi + \varepsilon. \tag{6}$$

From this equation, DP can be obtained by assuming a linear $g(\cdot)$ of parameter ρ and dropping the term that includes lagged S_X. The term in ε_{-1} doesn't create any problem because $\rho\varepsilon_{-1}$ it is a zero mean error that goes to the composite error of the equation.

Notice that general OP/LP can circumvent the problem of the unobservable ε_{-1} inside $g(\cdot)$ by using an expression of the FOC that doesn't contain it. But, if $g(\cdot)$ is linear, it can equivalently be applied using (6). The reason by which the expression inside $g(\cdot)$ in equation (6) can be written without

[5]However we know that, under homotheticity, $\nu(\cdot)$ becomes a function of Q^* alone.

[6]Since $p = mc$ and $mc = avc - \ln \nu$, $w_{X,-1} - p_{-1} - \ln \frac{\partial F(x_{-1})}{\partial X_{-1}} =$
$\ln \nu_{-1} + w_{X,-1} + \ln X_{-1} - avc_{-1} - q^*_{-1} + q_{-1} - f(x_{-1}) + f(x_{-1}) - \ln X_{-1} - \ln \frac{\partial F(x_{-1})}{\partial X_{-1}} - \varepsilon_{-1} =$
$\ln\nu_{-1} + \ln S_{X,-1} + q_{-1} - f(x_{-1}) - \ln \beta_{X,-1} - \varepsilon_{-1}$.

error, as in equation (4), is because $p = mc$. Market power will definitely break this possibility. You may have noticed that (6) is based on replacing mc by avc and hence the error ε is the price that we should pay (see section 5).

When applied with data, the OP/LP estimator is employing more information than the observables used by DP. To be completely precise, it adds the information conveyed in the expression $\ln \frac{S_X}{\beta_X/\nu}$ lagged. If, for example, only S_X depends on data, it simply adds the observed labor share in variable cost minus the (estimation-irrelevant) constant specification that has been given to β_X/ν.

Notice that in this context there are at least three practical ways to carry out OP/LP. The first is applying equation (4) giving the corresponding parametric form to the derivative $\ln \frac{\partial F(x_{-1})}{\partial X_{-1}}$. The second is adopting the parametric specification corresponding to equation (6). The third is the popular form proposed by ACF. Assume that $E(\ln \frac{\nu S_X}{\beta_X}|x) \neq 0$. The first stage of ACF, trying to control for ε, computes

$$\widehat{\phi} = E(f(x) + \omega + \ln \frac{\nu S_X}{\beta_X}|x) = E(q|x) + E(\ln \frac{\nu S_X}{\beta_X}|x).$$

This means that, in the second stage, what we carry inside the $g(\cdot)$ is

$$\begin{aligned}
\widehat{\phi}_{-1} - f(x_{-1}) &= q_{-1} - f(x_{-1}) - [q_{-1} - E(q_{-1}|x_{-1})] + E(\ln \frac{\nu_{-1} S_{X,-1}}{\beta_{X,-1}}|x_{-1}) \\
&= q_{-1} - f(x_{-1}) + E(\ln \frac{\nu_{-1} S_{X,-1}}{\beta_{X,-1}}|x_{-1}) - \varepsilon_{-1}.
\end{aligned}$$

Notice that we have lagged ε in the expression. Everything is like instead of specifying the parametric form in the term of the share of (6) we are using the projection of the share and elasticities on x_{-1}. Later, we will call this the nonparametric (ACF) OP/LP estimator.

Equation (6) is full of implications. Lets suppose that $f(x)$ is well specified, noticing that this implies that β_X is also the true elasticity. First, if $g(\cdot)$ is linear, the DP and OP/LP estimators should produce in theory exactly the same estimates. The reason is that they are numerically equivalent except for the inclusion of a term that, under the correctness of the specification, should be equal to zero. That is $\ln \frac{S_{L,-1}}{\beta_{X,-1}/\nu_{-1}} = 0$.

Second, if $g(\cdot)$ is nonlinear, the different estimate produced by OP/LP comes exclusively from adding nonlinear terms to approximate $g(\cdot)$. If $f(x)$

is well specified, DP can be seen as a first order approximation to the productivity process dealt with by OP/LP. As productivity is in practice quite persistent, it would be surprising that this creates a dramatic divergence. This implies that any major divergence between the DP and OP/LP estimators is likely to come from the difference between the specification of the normalized elasticity $\frac{\beta_X}{\nu}$ and the observed share in cost S_X.

In summary, under competition both DP and OP/LP estimators are consistent and, if the production function specification is right, they should differ only in the effect of thr OP/LP nonlinear modeling of the productivity process. However, if the OP/LP estimator is specified using a linear Markov process, they should only diverge due to sampling error. This is regardless of how the OP/LP estimator is computed: parametrically, writing in one or another way the used FOC or FOCs, or nonparametrically, with an ACF first stage that regresses output on all inputs and variables relevant to explain the demand for the input used to substitute for unobserved productivity.

However, OP/LP uses more information than DP. If DP and the linear OP/LP diverge, either the production function is wrongly specified or the first order condition is not met. A leading case for the first situation is the presence of labor-augmenting productivity (or other forms of biased technical change), but there are reasons by which the share can be varying differently that specified that do not directly affect the production function (adjustment cost in the inputs, input market power...). In the first case both estimators will be inconsistent, in the second only the OP/LP estimator is inconsistent.

All this gives a nice way to proceed in the analysis of the specification. Firts, a test of specification comparing the results of estimating (4) and (6) under linear Markov is a test of the relevance of market power, since the estimation should only differ due to sampling error under the null of perfect competition and significantly differ when $p \neq mc$.Notice that, in general, a nice way to perform the test will be to add an estimate of $-\ln \mu$ inside the parenthesis of (6), where μ is the price-marginal cost ratio $\frac{P}{MC}$.[7]

Second, a test of specification of (6) under linear Markov including or not the term in $\ln \frac{\nu_{-1} S_{X,-1}}{\beta_{X,-1}}$ (or its projection) is a test of the null of consistency of DP and OP/LP against the inconsistecy of either both or OP/LP.

All the above assumes perfect competition in the product market, it is time to switch to the presence of market power.

[7]Note that we can base an approximation on $\frac{P}{MC} = \frac{\nu P}{AVC} = \frac{\nu R}{VC} \exp(-\varepsilon)$.

5 Estimating under market power

When there is market power, under the assumption of short-run profit maximization, the relevant variable in the FOCs is MR instead of P

$$MR\frac{\partial F(x)}{\partial X}\exp(\omega) = W_X.$$

The problem is that MR is, in general, unobservable. At first sight it seems like we have no more alternative than choosing DP estimation, that doesn't need this relationship. Let us take a closer look at what the new variable implies.

Let us now use firm subindices for clarity. There are N firms in a market. Under competition, LP proposed to use the unconditional demand for a variable input that we can get from solving the system of FOCs of the firm , that is $X_j = X(K_j, P, W, P_M, \omega_j)$. Assuming a common output price P, and with common input prices, it can be written as the (time varying) $X_j = X(K_j, \omega_j)$ relationship. If the firms have market power the solution of the system of first order conditions is going to produce the condition of equilibrium $X_j = X(K_j, MR_j, \omega_j)$, possibly also including varying input prices.

What is in MR_j? Let us assume for economy of notation that competition, both in quantities and prices, happens with product differentiation.[8] Under product differentiation firm j demand is $Q_j^* = D_j(P, \delta)$, where P is the vector of N prices and δ is a vector of unobserved correlated heterogeneity of demand (observable heterogeneity z can always be easily included). Assume that the demand system can be inverted, $P_j = D_j^{-1}(Q^*, \delta)$, where Q^* is the vector of quantities. Revenue is $P_j Q_j^*$. If the firm competes in quantities, $MR_j = P_j + Q_j^* \frac{\partial D_j^{-1}}{\partial Q_j^*} + T(P_j, Q_j^*, conduct)$ and, if the firm competes in prices, we can write the implicit $MR_j = P_j + (Q_j^* + T(P_j, Q_j^*, conduct))\left(\frac{\partial D_j}{\partial P_j}\right)^{-1}$. In both cases $T(\cdot)$ represents a function.

There are two problems implied by these expressions. The first is that they depend in an unspecified way on the market behavior of firms. In the absence of an specific market conduct, we do not have a function of observed variables but only a correspondence. Different $MR(\cdot)$ values can be associated to exactly the same P_j or Q_j^*, depending on behavior in the market. See Appendix B for a simple example.

[8] The discussion can be easily extended to the case of homogeneous product. We take as reference the models in Vives (1999).

The second problem is the presence of the expressions on Q_j^*, δ, and the derivatives $\frac{\partial D_j^{-1}}{\partial Q_j^*}$ and $\frac{\partial D_j}{\partial P_j}$. Q_j^* and δ are non observable. Maybe we can use $Q_j^* = D_j(P, \delta)$ and have $MR(P,\ \delta,\ \mathit{conduct})$, but notice that in general P and δ are vectors, the second the vector of N unobserved variables. We can try also using $Q_j^* = R_j(Q_{-j}^*)$ or $P_j = R_j(P_{-j})$, where $R_j(\cdot)$ are the corresponding best response functions. But these reaction functions also include unobservables and are behavior specific.

Of course simplifying the cases of behavior, and with specific assumptions on symmetry of firms, it is possible to obtain expressions in terms of observables. For example, it is very popular to assume that competition is monopolistic and the elasticity of demand constant and equal for all firms. Under these assumptions $MR_j = P_j(1 - \frac{1}{\eta})$, where η represents the (absolute value) of the elasticity of demand. A discussion of possible behavior restrictions and assumptions of symmetry across oligopoly models is carried out in Ackerberg and De Loecker (2024). They are able to reduce significantly the information requirements but, for example, they confirm that there cannot be unobserved characteristics if products are differentiated, as Doraszelski and Jaumandreu (2021) pointed out in relation to the correlated unobserved demand heterogeneity.

A compact way to think of the possible models is the following. Start with the output-conditional demand for the variable input $X = \widetilde{X}(K_j, W, P_M, Q_j^*, \omega_j)$, that with common input prices can be written $X = \widetilde{X}(K_j, Q_j^*, \omega_j)$. Use the (sales) market share of the firm S_j divided by P_j to express the production of the firm as a function of the market aggregate sales, $Q_j^* = \frac{S_j}{P_j}A$. If S_j is only a function of (K_j, ω_j), the demand for the input can be expressed in the market as a time varying function $X_j = \overline{X}(K_j, P_j, \omega_j)$. Notice that this implies to assume that firms cannot be unequal because input price differences, unobserved correlated demand heterogeneity, and/or asymmetric behavior. It is like discarding other factors of efficiency other than ω before starting the investigation. In general we want to avoid this.

The central question is whether it is possible to estimate the production function without taking a stance on how is competition. Estimate without having to assume things like whether competition is in prices or quantities, firms either take the rivals actions as given or collude, some firms have a particular type of advantages or not, collusion is either with all or with part of the rivals,...and so on. The answer is yes, it is possible.

To see why notice that, in equilibrium, a short-run profit maximizing firm equates marginal revenue and marginal cost, so

$$MR(P, \delta, conduct) = MC(K_j, W_j, P_{Mj}, Q_j^*, \omega_j).$$

On the left hand side, the expression depends on the particular specification of conduct. The right hand side, on the contrary, picks up a specific single value under quite general conditions.[9] It singles out a unique marginal cost $MC(\cdot)$ for each set of values of the arguments (we have specified possibly varying input prices for the sake of generality). We can even accommodate labor-augmenting productivity by considering the price $W^* = W/\exp(\omega_L)$ with the unobservable replaced. We can also accommodate input market power. If we have MC, we have what has been called a "sufficient statistic," a variable that contains all the relevant information of the conduct and demand conditions.

6 Replacing MC by AVC

The FOCs for cost minimization of variable cost can be written[10]

$$MC \frac{\partial F(x)}{\partial X} \exp(\omega) = W_X,$$

Adding them up, each one multiplied by the amount of the input, we get an expression that links marginal cost and average variable cost

$$\begin{aligned} MC \sum X \frac{\partial F}{\partial X} \exp(\omega) &= \sum W_X X, \\ mc + \ln \nu &= vc - q^*. \end{aligned}$$

The link is the short-run elasticity of scale $\nu = \frac{\partial \ln F}{\partial l} + \frac{\partial \ln F}{\partial m}$, that was already present in the specification as a dimension of the production function. This means that one can treat marginal cost as known up to the output at which average variable cost has to be measured (q^*) minus the (log of) the specification of ν.

[9] These conditions are basically convexity assumptions on the technology of the firm.

[10] The expression can also be interpreted as the FOC for profit maximization with MR replaced by MC.

To estimate by an OP/LP procedure we can use the inversion

$$\omega = w_X - mc - \ln \frac{\partial F(x)}{\partial X} = w_X - vc + q + \ln \nu - \ln \frac{\partial F(x)}{\partial X} - \varepsilon.$$

Not surprisingly, we can rewrite this expression as $q_{-1} - f(x_{-1}) + \ln \frac{\nu_{-1} S_{L,-1}}{\beta_{X,-1}} - \varepsilon_{-1}$, exactly as in (6). In perfect competition it was the price that measured marginal cost, now under market power we have directly specified marginal cost.

The conclusion is that, if we want to estimate using the OP/LP method, we need to replace marginal cost by the observables variable cost and output, and specify the short-run elasticity of scale. This collapses to an expression that simply adds the term $\ln \frac{\nu_{-1} S_{X,-1}}{\beta_{X,-1}}$ to the DP specification. The expression $\ln \frac{\nu_{-1} S_{X,-1}}{\beta_{X,-1}}$ can be more or less complex depending on the specification of the production function and hence the elasticities β_X and ν. We may like an ACF implementation of the estimator, that avoids to write the expression by using its nonparametric projection.

The problem is that now we cannot get rid of the unobservable ε inside the function $g(\cdot)$. This means that consistent estimation is in general impossible or very involved. However, there is a simple solution readily available: adopt a linear $g(\cdot)$, with what we have an OP/LP that is "feasible" under market power.

However, it is worthy to estimate OP/LP with this specification under market power when we can use DP? The answer is clearly yes. The estimator can significantly diverge from the DP estimator, and in general to give more plausible results. The reason is that OP/LP encompasses in the estimation the divergence between the data and the specification of the production function picked up through the FOC. When this mismatch is important, the difference between the estimators DP and OP/LP is detecting mispecification. Hence, the estimation of OP/LP is a tool for specification.

If the researcher obtains DP and feasible OP/LP estimates that basically coincide, she can proceed with the confidence that the production function and the first order conditions are met. If the researcher gets a significant divergence between the two estimators, it can be either that the specification of the production function is wrong or that the FOCs are not met. As mentioned before, a leading case of the first problem is the presence of biased technological change (that changes the production function itself and determines a variation of the used FOC). Frequent cases of the second problem are the presence of adjustment costs, and the presence of market power in some input market or markets. These latest mispecifications affect only the FOC.

A reasonable agreement between the feasible OP/LP and DP is a necessary condition for the consistency of both estimators. The researcher should be able to get coincidence of the estimators, to test it, and to interpret how it has been reached. In particular looking at how the cost share has been modelled. Even then, still some work may be needed to improve the estimation, because the estimators are equal but not necessarily consistent.[11]

In summary, DP is robust to market power but OP/LP is not. A robust estimator is an estimator that is consistent whatever are the details of the game that firms play in the market. For OP/LP being consistent, some particular games and strong symmetry conditions must be assumed. However, there is a feasible OP/LP that always is possible. It consists of replacing MC by AVC and the short-run elasticity of scale, taking into account that this replacement leaves unavoidably the error of the production function and the Markov process must be assumed to be linear.

The feasible OP/LP can be specified either by extending DP with the term $\ln \frac{\nu_{-1} S_{L,-1}}{\beta_{X,-1}}$ or, in the ACF manner, including all variables relevant to the demand for the input (e.g. input prices) in a first nonparametric step. Under market power, if the production function is well specified, DP and the feasible OP/LP must only diverge due to sampling error. Hence, an specification test based on the equality of the coefficients is again available. Under the null both estimators are consistent, under the alternative either both estimators or OP/LP (because the FOCs fail) are inconsistent. When there is market power, starting the estimation of the production function with this test is an easy and convenient way to work on the specification. One may want also to check if the presence of market power can be rejected using the specification test explained in section 4.

[11]It could be the case, for example, that the error in an input deflator is making inconsistent the estimation, although the first order condition is not detecting any specific problem in the share of the input, inducing both estimators to give a similar outcome. Further improvement of the estimates can be reached by trying different deflators until an equal and better specification of both models is reached.

7 The equality of DP and feasible OP/LP as a test for the specification

In this section we show with an example how the DP and OP/LP estimators can start diverging, and come close when the specification fits better the data. We estimate the production function for the sample of Compustat US manufacturing firms (1960-2018) used in Jaumandreu and Mullens (2024). It is a sample of firms belonging to many different markets and times, so we should expect that they possess various degrees of market power. It is then important to be robust to the exercise of market power.

The DP estimation and the ACF implementation of (feasible) OP/LP with a Cobb-Douglas specification diverge in the capital elasticity estimate and in the assessment of the returns to scale. As it is often the case, for dismay of researchers, DP produces a negative elasticity for capital, and a short-run elasticity of scale well above unity. However, the ACF implementation of OP/LP shows up a nicely estimated elasticity of capital and a more moderated short-run elasticity of scale (although non smaller than one as well). As odd as it may sound, this is not a sample-specific phenomenon but a quite typical finding.

A usual interpretation for the DP behavior is that the differentiation of the data exacerbates errors in measurement of an otherwise quite persistent capital. However, a little experimentation shows that the result is very sensitive to the specification. For example, OLS of the Cobb-Douglas in first differences gives positive coefficients (see Appendix D). If DP is inconsistent because the production function specification, sections 2 and 3 have shown that OP/LP cannot be consistent. Recall that we are using the feasible version to ensure robustness in the presence of market power, and feasible OP/LP basically adds to the DP specification a term that should be zero.

Hence we need to consider that some reason, related to specification, must be determining the inconsistency of both estimators. A simple inspection of the labor shares shows that the elasticity of labor must have been falling over time and the Cobb-Douglas specification fails in picking up this characteristic. When labor-augmenting productivity is allowed into the specification, by enlarging the Cobb-Douglas to a translog with elasticity of substitution less than unit that admits falling labor shares, both estimators coincide.

The conclusion is that the divergence of the estimators was detecting the mispecification of the production function, and that the redressement of this mispecification allows the estimators to provide the same answer. Notice that feasible OP/LP has been useful to fix the consistency of DP and hence to assess the consistency of both estimators in a new specification. This can be a good starting point for the researcher try to further improve the specification keeping the equality of the two estimators.

In what follows, we explain in detail how the above exercise is done.

Column (1) of the Table reports the results of applying the DP estimator to the Cobb-Douglas specification. The estimator proceeds as follows. Under the assumption that Hicks-neutral productivity ω_H follows an $AR(1)$ of parameter ρ and innovation ξ, it can be written that

$$\begin{aligned} q_{jt} &= \beta_t + \beta_K k_{jt} + \beta_L l_{jt} + \beta_M m_{jt} \\ &\quad + \rho[q_{jt-1} - \beta_K k_{jt-1} - \beta_L l_{jt-1} - \beta_M m_{jt-1}] + \xi_{jt} + \varepsilon_{jt} - \rho\varepsilon_{jt-1}. \end{aligned} \tag{7}$$

We estimate this equation by nonlinear GMM using as instruments the (constant and) time dummies, the input variables $k_{jt}, k_{jt-1}, l_{jt-1}, m_{jt-1}$ and the (real) input prices $w_{jt-1} - p_{jt-1}$ and $p_{Mjt-1} - p_{jt-1}$. This is a fully standard choice of instruments. As we have to estimate (in addition to the constant and time dummies) the four parameters $\beta_K, \beta_L, \beta_M$ and ρ, the instruments provide two overidentifying restrictions.

The result is not nice: the elasticity of capital turns out to be negative, the elasticity of labor very large and the short-run elasticity of scale, the sum of $\beta_L + \beta_M$, above 1.1. Economic theory tells us that it is not realistic that when changing in the short-run the variable factors we encounter increasing returns to scale (although unfortunately this is a quite usual result that is reported without further comments).

Column (2) reports the results of computing the feasible OP/LP, implemented by means of the ACF method, applied to the Cobb-Douglas. In a first stage, we regress q_{jt} non parametrically (using a complete polynomial of order 3) on the five variables $k_{jt}, l_{jt}, m_{jt}, w_{jt} - p_{jt}$ and $p_{Mjt} - p_{jt}$. From the result of this first step we compute the estimate $\widehat{\phi}(k_{jt-1}, l_{jt-1}, m_{jt-1}, w_{jt-1} - p_{jt-1}$ $p_{Mjt-1} - p_{jt-1})$ that we use in forming the second step equation

$$\begin{aligned} q_{jt} &= \beta_t + \beta_K k_{jt} + \beta_L l_{jt} + \beta_M m_{jt} \\ &\quad + \rho[\widehat{\phi}_{jt-1} - \beta_K k_{jt-1} - \beta_L l_{jt-1} - \beta_M m_{jt-1}] + \xi_{jt} + \varepsilon_{jt}. \end{aligned} \tag{8}$$

As we try to emphasize with our notation, equations (7) and (8) are very close. They only differ, in addition to the component of the error $-\rho\varepsilon_{jt-1}$, in that the nonparametric estimate $\widehat{\phi}_{jt-1}$ has replaced q_{jt-1}.

If we have decided to use the demand for materials to construct the proxy for ω, the application of the analysis of section 4 tells us that

$$\begin{aligned} \widehat{\phi}_{jt-1} - \beta_K k_{jt-1} - \beta_L l_{jt-1} - \beta_M m_{jt-1} &= \\ q_{jt-1} - \beta_K k_{jt-1} - \beta_L l_{jt-1} - \beta_M m_{jt-1} & \\ +E(\ln \frac{\nu S_{Mj,-1}}{\beta_M} | k_{jt-1}, l_{jt-1}, m_{jt-1}, w_{jt-1} - p_{jt-1}, p_{Mjt-1} - p_{jt-1}) - \varepsilon_{jt-1}. & \end{aligned}$$

Hence, everything is like we were adding into the bracket of the DP estimator the expression corresponding to the nonparametric prediction of the share $S_{M,-1}$. In the second step of the ACF implementation we use the instruments $k_{jt}, k_{jt-1}, l_{jt-1}, m_{jt-1}$ and $\widehat{\phi}_{-1}$, what implies one overidentifying restriction.

The results reported in column (2) show the outcome. The addition (the nonparametric prediction of the share) helps to redress two things with respect to the DP results. The elasticity of capital becomes positive, and the short-run elasticity of scale falls.

We obviate in the table the parametric OP/LP estimator, the estimator that simply introduces the share $\ln S_M$ in the regression (you can check it in Appendix D). The reason is that the estimation is always likely to diverge less when we include the nonparametric prediction of the share. At this stage, what we are interested is wether DP and OP/LP should be taking as producing the same estimate. So it is enough to employ the popular nonparametric OP/LP estimator since it already shows the difference.

It is apparent that DP and OP/LP are giving different answers to the estimation of the production function. The researcher may be puzzled why two theoretically consistent estimators give very different results. The explanation that we develop next is that both estimates are in fact inconsistent because there is a shortcoming in the specification of the production function.

Columns (3) and (4) change the basic specification of the production function. Now we estimate a multiproductivity production function. We use the simplest production function that admits labor-augmenting productivity, a translog separable in capital and homogeneous of degree ν in labor and materials (we follow Doraszelski and Jaumandreu, 2018).[12] Because it is homogeneous in labor and materials it depends on the log-ratios materials to labor, but these log-ratios exhibit unobserved labor-productivity,

$$q_{jt} = \alpha_0 + \alpha_K k_{jt} + \nu m_{jt} - \alpha_L (m_{jt} - l_{jt} - \omega_{Ljt}) - \frac{1}{2} \alpha (m_{jt} - l_{jt} - \omega_{Ljt})^2 + \omega_{Hjt} + \varepsilon_{jt}.$$

[12] See for more detail Jaumandreu and Mullens (2024).

Using the ratio of first order conditions for labor and materials, we derive an expression to be substituted for these ratios, $m_{jt} - l_{jt} - \omega_{Ljt} = -\frac{\alpha_L}{\alpha} + \frac{\nu}{\alpha} S^*_{Ljt}$, with $S^*_{Ljt} = S_{Ljt} - \frac{\alpha}{\nu}\overline{\omega}_L$, and where $\overline{\omega}_L$ is a guess for the mean of labor-augmenting productivity.[13] Hence, the production function becomes a function of observables in which, to control for Hicks-neutral productivity, we can apply easily both the DP estimation procedure and the OP/LP method.

The DP estimator is obtained by applying nonlinear GMM to the equation

$$\begin{aligned} q_{jt} &= \gamma_0 + \beta_t + \alpha_K k_{jt} + \nu m_{jt} - \frac{1}{2}\frac{\nu^2}{\alpha} S^{*2}_{Ljt} \qquad (9) \\ &\quad + \rho[q_{jt-1} - \alpha_K k_{jt-1} - \nu m_{jt-1} + \frac{1}{2}\frac{\nu^2}{\alpha} S^{*2}_{Ljt-1}] + \xi_{jt} + \varepsilon_{jt} - \rho\varepsilon_{jt-1}, \end{aligned}$$

where the expression is written in a similar format to the previous estimators for the sake of comparability. To estimate this more nonlinear equation we enlarge the instruments with the squares of the inputs and input prices, and we add the lagged share of labor cost in variable cost: $k_{jt}, k^2_{jt}, l_{jt-1}, l^2_{jt-1}, m_{jt-1}, m^2_{jt-1}, w_{jt-1} - p_{jt-1}, (w_{jt-1} - p_{jt-1})^2$, $p_{Mjt-1} - p_{jt-1}, (p_{Mjt-1} - p_{jt-1})^2$ and $S_{L,-1}$. This gives eleven instruments and we hence have seven overidentifying restrictions.

To estimate the nonparametric ACF version of the OP/LP estimator we first again regress q_{jt} non parametrically on the variables $k_{jt}, l_{jt}, m_{jt}, w_{jt} - p_{jt}$ and $p_{Mjt} - p_{jt}$. From the result of this first step we compute the estimate $\widehat{\phi}(k_{jt-1}, l_{jt-1}, m_{jt-1}, w_{jt-1} - p_{jt-1}\ p_{Mjt-1} - p_{jt-1})$ that we use it in forming the second step equation. In the second step we use the instruments $k_{jt}, l_{jt-1}, m_{jt-1}, \widehat{\phi}_{-1}$ and $S_{Lj,t-1}$, so that we have one overidentifying restriction.

The results for the new DP and OP/LP, reported in columns (3) and (4) respectively, clearly indicate that now they give basically the same estimate of the production function.

To check statistically that this is the case we apply an specification test. We construct a quadratic form of the elasticity of capital and the elasticity of scale, using as weight the inverse of a robust estimator of the asymptotic variance of the difference between the coefficients $(\beta_{DP} - \beta_{OP/LP})$, see Appendix C. The test doesn't reject that the quadratic form is distributed as a χ^2 with 2 degrees of freedom, and hence that the differences now can be interpreted as coming from sampling error.

[13] We use $\overline{\omega}_L = \overline{m} - \overline{l}$. This specification only estimates directly parameter ν, and the elasticities β_L and β_M are determined by the implications $\beta_L = \nu\overline{S}_L$ and $\beta_M = \nu(1-\overline{S}_L)$.

We want to confirm that it has been relevant to use the feasible OP/LP estimator, that in effect we cannot reject the presence of market power. To check this we compare the two parametric versions of the feasible OP/LP by means of an specification test. On the one hand, the estimator that proxies MC as in equation (6). On the other the estimator that uses prices, as in equation (4), presuming that is equal to MC. It can be shown that, in the case of the translog specification, this simply amounts to include in the first estimator the ratio price/average variable cost of the firms. The resulting $\chi^2(2)$ shows a strong rejection of the null of perfect competition (see Appendix D).

Appendix D also shows the result of computing the nonlinear version of OP/LP with the translog multiproductivity. Although the coefficients of capital and short-run elasticity of scale are not crazy (are quite similar to the estimation with the Cobb-Douglas), they are clearly different from our common result applying DP and OP/LP to the translog multiproductivity specification.

Notice that in columns (3) and (4) both the elasticity of capital and the short-run elasticity of scale are sensible. The elasticity of capital is greater than with the Cobb-Douglas specification, and the estimate of the elasticity of scale is in the range $0.79 - 0.83$, what clearly improves the unrealistic constant returns to scale for the variable inputs estimation of column (2). No estimator produces a clear better estimation than the other when rightly specified. From here on, the researcher can focus on improving other aspects of the estimation, as accounting for input market power, introduce the effect of adjustment cost of the variable inputs, or experiment with the way to deflate output and materials.

8 Concluding Remarks

The production function is not affected by market power, but the estimators that employ an auxiliary FOC that includes a derivative of the production function are sensitive to the form of the FOC under market power. DP is an estimator robust to market power because doesn't use any FOC. However, the OP/LP approach cannot be generally robust to market power because marginal cost, that summarizes all relevant effects of the strategy of the firm and its demand, can at most be replaced by average variable cost plus the uncorrelated error that characterizes the production function. Linearity of the productivity process, however, doesn't allow the error to affect the estimation. This gives an OP/LP that is feasible under market power.

A feasible OP/LP adds the (log) difference between the observed share and the normalized elasticity of the input whose demand is inverted to the equation used by DP. In theory, with this addition we should have exactly the same estimate because, if the production function is right, the theoretical value of the expression is zero. If the estimators diverge, either the production function specification is wrong or there is a problem in the specification of the FOC. This gives us a test for the specification.

The researcher who wants to estimate the production function under market power may first test if market power is relevant by using the expressions of the feasible OP/LP either using output prices or proxying for marginal cost. If market power is relevant, it is convenient to start by estimating both the DP and the feasible OP/LP estimators, and testing the equality between them. This still doesn't ensure that both estimators are consistent, although we know that they are meeting the necessary condition for consistency (of being equal). Further work of specification with the two models can warrant consistent estimation under market power.

We have shown how this works with an example of estimation of the production function for a sample of US manufacturing firms. The naive Cobb-Douglas specification of the production function produces a negative elasticity for capital and a too large short-run elasticity of scale with the DP estimator. Recognizing labor-augmenting productivity in addition Hicks-neutral productivity, and allowing it into the specification, induces a matching of DP and the feasible OP/LP under market power that passes the specification test.

We have some extensions in mind. It would be nice to explore less restrictive production functions in the context of robustness to market power. A first objective would be estimating the translog allowing for the variation of the short-run elasticity of scale. A complementary possibility would be relaxing the separability of capital, something that has been shown parametrically possible under the translog specification by Zhao, Malikov and Kumbhakar (2024). Another, more ambitious extension, would be the estimation of a fully nonparametric production function with a flexible specification of the input-biased productivities. We leave these extensions for the continuing research.

Appendix A: Identification

Assume the timing and information conditions of the text, and suppose (worst case) that the price of the variable inputs and the price of the output are constant. The FOC to be used by an OP/LP procedure becomes $\omega_{-1} = c_1 - \ln \frac{\partial F(x_{-1})}{\partial X_{-1}} = h(x_{-1}; c_1, \theta_1)$, where c_1 and θ_1 represent the constant and the parameters of the derivative. The model to be estimated can be written as

$$q = f(x; c_0, \theta) + g(h(x_{-1}; c_1, \theta_1); \theta_2) + \xi + \varepsilon,$$

where θ are the parameters of the production function and $\theta_1 \subset \theta$. If we are in a strictly nonparametric setting, θ, θ_1, and θ_2 are infinite-dimensional. If we want to approximate the nonparametric relationship by a flexible form, the ideal procedure for estimation is nonlinear GMM (l and m are correlated with ξ) and identification basically depends on the relationship between parameters and moments.

Assume that we want to make an approximation to $f(x)$ based on a complete polynomial of order 3 (the most used approximation) and x has n inputs. The number of parameters in (c_0, θ) is $1+2n+\frac{n(n-1)}{2}+n^2+\frac{n(n-1)(n-2)}{6}$. If $n = 3$ this gives us 20 parameters (the first is the constant). If we decide to estimate $g(\cdot)$ also by means of three powers, we have to estimate a total of 23 parameters. Using the constant and the vector (k, l_{-1}, m_{-1}) we can form 20 moments. Adding the new moments that we can form using k_{-1} and interacting it with l_{-1} and m_{-1} we have 10 more. We hence have 7 overidentifying restrictions.

As long as we do not have perfect collinearity between the quantities of two inputs we should be fine identifying the production function.

Appendix B: Conduct specification

Let us suppose for simplicity 2 firms (the industry can have, for example, $N/2$ of each type). We drop the asterisk from the quantities Q_j^* for economy of notation. We also abstract from heterogeneity of demand. Firms 1 and 2 have demands

$$\begin{aligned} Q_1 &= P_1^{-\eta} P_2^{\gamma}, \\ Q_2 &= P_2^{-\eta} P_1^{\gamma}, \end{aligned}$$

and costs $C_1(Q_1)$ and $C_2(Q_2)$. Inverse demands are

$$\begin{aligned} P_1 &= Q_1^{-\eta^*} Q_2^{-\gamma^*}, \\ P_2 &= Q_2^{-\eta^*} Q_1^{-\gamma^*}, \end{aligned}$$

where $\eta^* = \frac{\eta}{\eta^2 - \gamma^2}$ and $\gamma^* = \frac{\gamma}{\eta^2 - \gamma^2}$. Firm i maximizes

$$\pi_i + \lambda \pi_j = P_i Q_i - C_i(Q_i) + \lambda(P_j Q_j - C_j(Q_j)),$$

where λ is an exogenous conduct parameter. If firms compete in prices

$$\begin{aligned} \frac{\partial(\pi_i + \lambda\pi_j)}{\partial p_i} &= Q_i - \eta P_i \frac{Q_i}{P_i} + \eta C_i^{'} \frac{Q_i}{P_i} + \lambda[\gamma P_j \frac{Q_j}{P_i} - \gamma C_j^{'} \frac{Q_j}{P_i}] = \\ &= P_i - \eta(p_i - C_i^{'}) + \lambda\gamma(P_j - C_j^{'}) \frac{Q_j}{Q_i} = 0, \end{aligned}$$

and if they compote in quantities

$$\frac{\partial(\pi_i + \lambda\pi_j)}{\partial Q_i} = P_i - \eta^* Q_i \frac{P_i}{Q_i} - C_i^{'} - \lambda\gamma^* Q_j \frac{P_j}{Q_i} = 0.$$

Using symmetry, is easy to see that $MR_j = P_j(1 - \frac{1}{\eta(1-\lambda\frac{\gamma}{\eta})}) = C_j^{'}$ under price competition and $MR_j = P_j(1 - \frac{\eta}{\eta^2-\gamma^2}(1 + \lambda\frac{\gamma}{\eta})) = C_j^{'}$ under quantity competition. If $\lambda = 1$, both marginal revenues coincide in $P_j\frac{1}{\eta-\gamma}$, the unique total collusive solution. If $\lambda = 0$, Cournot with $P_j(1 - \frac{1}{\eta(1-(\frac{\gamma}{\eta})^2)})$ is less competitive than Bertrand, which gives $P_j(1 - \frac{1}{\eta})$. If $0 < \lambda < 1$, quantity competition is less competitive than price competition.

The point is how MR_j changes with conduct, in this case represented by the unobservable exogenous parameter λ.

Appendix C: Specification tests

A Hausman (1978) specification test, or a Durbin-Wu-Hausman test, can be seen as a test of the equality between the parameter estimates under two methods of estimation that are consistent under the null. The alternative is in our case that either one method or the two are inconsistent. Following Wooldridge (2010), we set a quadratic form of the differences in the parameters $(\widehat{\beta}_{DP} - \widehat{\beta}_{OP/LP})$ using the inverse of a robust estimate of $Avar[\sqrt{N}(\widehat{\beta}_{DP} - \widehat{\beta}_{OP/LP})] = V_{DP} + V_{OP/LP} - (C + C')$.

Let $i, l = DP, OP/LP$. To estimate V_i, we use

$$\widehat{V}_i = (\widehat{G}_i' A_{Ni} \widehat{G}_i)^{-1} \widehat{G}_i' A_{Ni} \widehat{\Omega}_i A_{Ni} \widehat{G}_i (\widehat{G}_i' A_{Ni} \widehat{G}_i)^{-1},$$

with $\widehat{G}_i' = N^{-1} \sum_j \frac{\partial (Z_{ij}' \widehat{u}_{ij})}{\partial \beta_i}$, $A_{Ni} = N^{-1} \sum_j Z_{ij}' Z_{ij}$, and $\widehat{\Omega}_i = N^{-1} \sum_j Z_{ij}' \widehat{u}_{ij} \widehat{u}_{ij}' Z_{ij}$. To estimate $\widehat{C}$, we use

$$\widehat{C} = (\widehat{G}_i' A_{Ni} \widehat{G}_i)^{-1} \widehat{G}_i' A_{Ni} \widehat{\Omega}_{il} A_{Nl} \widehat{G}_l (\widehat{G}_l' A_{Nl} \widehat{G}_l)^{-1},$$

where $\widehat{\Omega}_{il} = N^{-1} \sum_j Z_{ij}' \widehat{u}_{ij} \widehat{u}_{lj}' Z_{lj}$. Hence $\widehat{Avar}\ (\widehat{\beta}_{DP} - \widehat{\beta}_{OP/LP}) = (\widehat{V}_{DP} + \widehat{V}_{OP/LP} - (\widehat{C} + \widehat{C}'))/N$.

Under the null, we have

$$(\widehat{\beta}_{DP} - \widehat{\beta}_{OP/LP})' Avar[\sqrt{N}(\widehat{\beta}_{DP} - \widehat{\beta}_{OP/LP})]^{-1} (\widehat{\beta}_{DP} - \widehat{\beta}_{OP/LP}) \sim \chi^2(p),$$

where the p degrees of freedom are the number of parameters being tested.

Appendix D: Additional regressions and test

Table AD reports a few complementary estimates and test. Column (1) reports the result of carrying out an OLS estimation of the Cobb-Douglas specification in first differences. Although the coefficient on capital is small, it is positive and statistically significant.

Column (2) shows the result of computing the parametric OP/LP specification with a Cobb-Douglas production function. Although the coefficient on capital tends to raise, the log of S_M (materials share in variable cost) minus an (implicit) constant is determining residuals highly negatively correlated with labor and positively with materials. This is in fact a sign of the presence of the non accounted labor-augmenting productivity.

In column (3) it can be appreciated how much the regression changes with the translog multiproductivity specification estimated by a parametric OP/LP based on proxying MC, and in column (4) how much the result is perturbed by the introduction of the price (implemented by the addition of the log of the price/average variable cost ratio). The null hypothesis of perfect competition is strongly rejected.

Column (5) reports the results of the estimation of a nonlinear nonparametric OP/LP. Recall that this is a theoretically inconsistent estimator. Although the coefficients modeling the nonlinear productivity process are sensible, the coeffient on capital and the short-run elasticity of scale (1.041) ressemble the values obtained with the CD specification. The elasticity of substitution is also very low.

References

Ackerberg, D., K. Caves and G. Frazer (2015), "Structural Identification of Production Functions," *Econometrica*, 6, 2411-2451.

Ackerberg, D., and J. De Loecker (2024), "Production Function Identification Under Imperfect Competition," mimeo, University Texas Austin.

Ackerberg, D., J. Hahn, and Q. Pan (2023), "Nonparametric Identification Using Timing and Information Set Assumptions with an Application to Non-Hicks Neutral Productivity Shocks," University of Texas-Austin.

Arellano, M. and S. Bond (1991), "Some Tests of Specification for Panel Data: Monte Carlo Evidence and an Application to Employment Equations," *Review of Economic Studies,* 58, 277-297.

Azzam, Jaumandreu and Lopez (2022), "Input and Output Market Power with Non-neutral Productivity: Livestock and Labor in Meatpacking," Boston University.

Blundell, R. and S. Bond (2000), "GMM Estimation with Persistent Panel Data: An Application to Production Functions," *Econometric Reviews* 19, 321-340.

Bond, S., A. Hashemi, G. Kaplan and P. Zack (2023), "Some unpleasant markup arithmetic: Production function elasticities and their estimation from production data," *Journal of Monetary Economics,* 121, 1-14.

Demirer, M. (2022), "Production function Estimation with Factor-Augmenting Technology: An Application to Markups," MIT.

Dobbelaere, S. and J. Mairesse (2013), "Panel Data Estimation of the Production Function and Production and Labor Market Imperfections," *Journal of Applied Econometrics,* 28, 1-46.

Dobbelaere, S. and J. Mairesse (2018), "Comparing micro-evidence on rent shring from two different econometric models," *Labour Economics*, 52, 18-26.

Doraszelski, U. and J. Jaumandreu (2013), "R&D and Productivity: Estimating Endogenous Productivity," *Review of Economic Studies,* 80, 1338-1383.

Doraszelski, U. and J. Jaumandreu (2018), "Measuring the Bias of Technological Change," *Journal of Political Economy,* 126, 1027-1084.

Doraszelski, U. and J. Jaumandreu (2019), "Using Cost Minimization to Estimate Markups," CEPR Discussion Paper 14114.

Doraszelski, U. and J. Jaumandreu (2021), "Reexamining the De Loecker and Warzynski (2012) Method for Estimating Markups," CEPR Dscussion Paper 16027.

Gandhi, A., S. Navarro and D. Rivers (2020), "On the Identification of Gross Output Production Functions," *Journal of Political Economy,* 128, 2973-3016.

Griliches, Z. and J. Mairesse (1998), "Production Functions: The Search for Identification," in *Econometrics and Economic Theory in th Twentieth Century: The Ragnar Frish Centennial Symposium,* S. Strom, ed. Cambridge University Press

Hausman, J. (1978), "Specification Tests in Econometrics," *Econometrica*, 46, 1251-1271.

Jaumandreu, J. and I. Mullens (2024), "Technology and US Manufacturing Employment (Lessons from the estimation of a microeconomic multiproductivity production function)," Boston University.

Kusaka, S., T. Okazaki, K. Onishi and N. Wakamori (2024), "New Technology Diffusion: Implications for Monopsony Power, Markups, and Labor Share," Yale University.

Levinsohn, J and A. Petrin (2003), "Estimating Production Functions Using Inputs to Control for Unobservables," *Review of Economic Studies,* 70, 317-341.

Marschak, J. and W.Andrews (1944), Random simultaneous equations and the theory of production," *Econometrica,* 143-205.

Olley, S. and A. Pakes (1996), "The Dynamics of Productivity in the Telecommunications Equipment Industry," *Econometrica*, 64, 1263-1298.

Pan (2024), "Identification of Gross Output functions with a Nonseparable Productivity Shock," Shandong University.

Raval, D. (2019), "The Micro Elasticity of Substitution and Non-neutral Technology," *Rand Journal of Economics*, 50, 147-167.

Raval, D. (2023), "Testing the Production Approach to Markup Estimation," *Review of Economic Studies,* 90, 2592-2611.

Rubens, M. (2022), "Market Structure, Oligopsony Power, and Productivity," *American Economic Review,* 113, 2382-2410.

Vives, X. (1999), *Oligopoly Pricing: Old Ideas and New Tools*, Harvard University Press.

Wooldridge, J. (2010), *Econometric Analysis of Cross Section and Panel Data,* MIT Press.

Yeh, C., C. Macaluso and B. Hershbein (2022), "Monopsony in the US Labor Market," *American Economic Review*, 112, 7, 2099-2138.

Zhang, H. (2019), "Non-neutral Technology, Firm Heterogeneity, and Labor Demand," *Journal od Development Economics*, 140, 145-168.

Zhao, S. , E. Malikov and S. Kumbhakar (2024), "On the estimation of Nonneutral Production Technologies without Separability," Binghamton University.

Table: The equality of DP and OP/LP estimators as a test for the specification

US manufacturing production function estimation,
firm-level sample, Compustat, 1960-2018.

	Cobb-Douglas with neutral productiсity		Translog multiproductivity	
Parameter	DP	Nonparametric (ACF) OP/LP	DP	Nonparametric (ACF) OP/LP
	(1)	(2)	(3)	(4)
ρ	1.022	1.018	1.019	1.014
	(0.002)	(0.004)	(0.002)	(0.005)
β_K	-0.023	0.061	0.144	0.166
	(0.010)	(0.020)	(0.041)	(0.076)
β_L	0.486	0.486	0.226	0.239
	(0.020)	(0.050)	(0.064)	(0.109)
β_M	0.622	0.532	0.561	0.594
	(0.014)	(0.024)	(0.064)	(0.109)
α			0.111	0.153
			(0.016)	(0.037)
σ			0.553	0.489
Specification test				
$\chi^2(df)$			5.356 (2)	
$p-value$			0.069	
Overidentifying restrictions	2	1	7	1
Function value	8.043	0.701	59.669	4.476
No. of firms	5621	5621	5621	5621
No. of observations	65006	65006	65006	65006

Table AD: Complementary estimates and test

US manufacturing production function estimation,
firm-level sample, Compustat, 1960-2018.

Parameter	Cobb-Douglas		Translog multiproductivity		
	OLS in differences	Parametric OP/LP	Test of maket power: Marginal cost	Test of maket power: Price	Nonlinear nonparametric OP/LP
	(1)	(2)	(3)	(4)	(5)
ρ		0.986	1.035	0.958	0.545
		(0.004)	(0.004)	(0.004)	(0.201)
ρ_2					0.245
					(0.090)
ρ_3					-0.032
					(0.012)
β_K	0.027	0.104	0.170	-0.421	0.052
	(0.009)	(0.068)	(0.044)	(0.112)	(0.113)
β_L	0.339	-1.569	0.238	0.732	0.299
	(0.003)	(0.182)	(0.055)	(0.255)	(0.156)
β_M	0.555	2.916	0.593	1.825	0.742
	(0.002)	(0.102)	(0.055)	(0.255)	(0.156)
α			0.056	0.337	0.243
			(0.006)	(0.054)	(0.066)
σ					0.431
Specification test					
$\chi^2(df)$			66.566		
$p-value$			0.000		
Overidentifying restrictions		2	7	7	1
Function value		99.349	130.980	971.789	0.180
No. of firms	5621		5621	5621	5621
No. of observations	65006		65006	65006	65006