# Productivity, Competition, and Market Outcomes*

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**Abstract**

At the firm-level, productivity is constantly evolving because of the introduction of new technology and innovations. Some of these productivity gains diffuse uniformly across firms, others only spread out in the industry with time. The unequal evolution of productivity impacts the structure of the industry, the more the greater the degree of competition. We analyze the relationship between the distribution of firms' productivity advantages and the distribution of market shares, and show that this relationship is more intense the more competition. We briefly comment on two applications: we show that, because productivity gains, market concentration and inflation can be negatively related, and we give an alternative interpretation to the case for a recent rise of US markups attributed to increased market power.

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## 1. Introduction

This paper discusses how productivity, which results from the innovative activity of firms, impacts market structure and market outcomes. The idea that productivity (or efficiency) impacts the shares of firms is obvious. However, there are at least three related questions that receive much less attention. The first is how the impact of productivity on market shares is linked to the intensity of competition. In fact this is implied by the models that economists have traditionally used to analyze competition, but usually is not the object of attention. The second is how the interaction productivity-competition can determine the distribution of market shares. Innovations and the process of diffusion of new technologies, with the corresponding dynamic development of unequal productivity gains, may determine how the distribution of market shares evolves. The third is how the reallocation of the firm's shares in a market can be one of the keys to explain some market outcomes. This paper deals with these three aspects.

We consider two dimensions of competition. First, products in the market can be homogeneous or differentiated. Differentiated products soften the sharpness of price competition among the firms, because consumers are willing to pay more for their preferred goods. Second, we consider that firms can compete in quantities or prices. Competition in quantities is less aggressive, because prices are affected only indirectly. We hence will use a taxonomy consisting of four situations: competition in prices (or a la Bertrand) with perfectly substitutable products, competition in prices (or a la Bertrand) when products are differentiated, competition in quantities (or a la Cournot) when the products are differentiated, and quantities (or Cournot) competition with perfectly substitutable products. We see this ordering as going from sharper to softer competition.

We start by describing the change in the price or quantity optimally set by a firm when the firm experiences an improvement of productivity. Price competition with perfectly substitutable products determines a radical bound: the firm with a productivity advantage finds optimal to try to gain the entire market at the expense of the rivals. However, the optimal action becomes less aggressive the less competitive is the context. In the other extreme, represented by Cournot, market shares are related to the productivity advantages, but in a more subdued way than when competition is sharper. We then explore formally how the distribution of productivity advantages relates to the distribution of market shares in each continuous game.

There are several motives by which this kind of analysis is worthy. First, it is important to know that the relationship productivity advantages-market shares is a characteristic of all competitive situations. Many times the analyst cannot exactly discern how is the type of competition in the market. May be she is dealing with firms from different markets that compete in different ways. And we may even want to use the relationship in another direction. Just the sharpness by which the market shares are affected by the gains in productivity can provide a guess on how intense is market competition (and hence what type). Second, the distribution of market shares and its evolution over time may be the object of interest because reflects a process of diffusion of an innovation or some new technology, and the inequalities among the firms are the indicator of the process of convergence on productivity.[1] Third, it may be that a few firms have become determinant in the performance of a particular market (employment, profitability...) and the analyst, recognizing this reality, wants to uncover the process by which this has happened.

This paper is related to different strands of literature. Firstly, it draws on the classical theories of oligopoly and their main results. We draw in the excellent and exhaustive account by Vives (1999). This account includes the important Vives (1985)

[1] For example, Jaumandreu and Mullens (2024) detect a wave of labor-augmenting productivity which affects US manufacturing firms and their employment 1980-2000.

result on the relative outcomes of Bertrand and Counot with product differentiation, by which competition in quantities with the same underlying demand produces a less competitive result. What we show in this paper is how the sequence of models mentioned before implies that the effect of an specific productivity advantage can be ranked according to the intensity of competition.

One important classical literature studied the relationships between innovation, competition and market structure, stressing the endogeneity of market structure. Examples of this literature are Dasgupta and Stiglitz (1980), Spence (1984) and Sutton (1991, 1998). The paper Vives (2008) formalizes this tradition in a series of results robust to functional form and according to the conditions of competition (type of product, degree of substitutability, entry...), insisting in the incentives that each form of competition raises for the changes in market structure. Our paper can be positioned in this tradition, focusing on the particular case of productivity advantages (the presumable outcome of innovation and technology) on the market shares of firms. One important characteristic is that we try to develop the consequences of an asymmetric setting: asymmetric gains, different markets shares. In some sense this connects with the old idea of the role of the cost structure on the shares of firms, a topic already present in the earlier critical accounts of the structure-colusion paradigm by Demsetz (1973), Peltzman (1977), and others.

We briefly develop two implications of our analysis for current debates. The first concerns the relationship between concentration and inflation. The second the supposed recent sharp increase of markups, coinciding with a fall of the labor share in cost and revenue, and a concentration of markets.

Ganapati (2021), points out that US manufacturing inflation at the end of the nineties and beginning of the 2000's seems to be empirically unrelated or even negatively related to the changes in the degree of concentration of the markets. The figure that shows the used data suggests that rather it can be a negative relationship with different intercepts. Recent articles as Covarrubias, Gutierrez and Philippon (2020) and Brauning, Fillat, and Joaquim (2023) have challenged this vision, insisting in the relationship between market power and inflation.

Our conjecture is that the industries that have experienced important unequal increases in productivity, have been milder in translating the cost increases to prices because of the competition among firms that has also concentrated the markets. We explain how this is possible and likely, and we confirm empirically that there really are negative relationships between concentration and inflation in the US manufacturing data.

The view of the sharp rise of the US markups originated in the articles by De Loecker, Eckhout and Unger (2020) and Autor, Dorn, Katz, Patterson and Van Reenen (2020). They present the increase in markups, the fall in the labor share, and the concentration of the markets as simultaneous effect of augmenting market power. However, the fall of the labor share with the growth of productivity it is just a distinctive characteristic of labor-augmenting productivity (LAP), when the elasticity of substitution between the inputs is low enough.[2] A key of the markup rise findings is the measurement of the firm-level markups using some estimated elasticity divided by the labor share, which under LAP confounds the increases of the level of efficiency and market power. In addition, this sets the base for an aggregation bias of the firm-level markups, for they must be strongly correlated with the observed market shares.

Our alternative explanation is that an intense growth of LAP has changed the

[2]For recent evidence on the importance of LAP see, for example, Doraszelski and Jaumandreu (2018, 2019), Raval (2019, 2023), Zhang (2019), Demirer (2020), Jaumandreu and Mullens (2024), and Kusaka, Okazaki, Onishi and Wakamori (2024). When the elasticity of substitution is less than one, a consensus among economists, LAP implies a fall of the labor cost share in cost and revenue, an stylized fact repeateadly found in many firm-level data sets.

distribution of market shares and concentrated the markets while produces markup mismeasurements when it is not accounted for. A simple simulation confirms that the observed facts may come from a market with labor-augmenting and Hicksian productivity growth, where market power doesn't increase except if inappropriately measured, at the same time that concentration rises and the labor share falls.

Both the theory and the applications developed in this paper make a strong case for the detailed analysis of the recent growth in productivity, often characterized by important biases towards labor, and the consequences of the technological change via productivity on the firm shares, market structure and concentration. Strong unequal diffusion of productivity is likely to raise temporary firms asymmetries in a way that it is important to understand, in particular to address properly measures of economic policy.

The rest of the paper is organized as follows. The second section establishes the setup. The third analyzes the best response functions and the fourth the distributions of market shares. Sections five and six develop the applications and section seven concludes.

## 2. Setup

### Production and cost

There are $N$ firms, each one producing a product, with production functions $q_j = F(K_j, L_j, M_j)\exp(\omega_j)$, $j = 1, ..., N$, where $K, L$ and $M$ are capital, labor and materials. $F(\cdot)$ is any production function with constant returns to scale. The term $\omega$ measures (percentage) deviations of Hicks-neutral productivity from the productivity of a standard firm for which the term is zero.

The cost function for firm $j$ turns out to be $C(q_j, \omega_j) = c(w)q_j \exp(-\omega_j)$, where $c(\cdot)$ depends on the specification of the product function, and $w$ is the vector of (industry common) input prices. Marginal cost is $C'(\omega_j) = c(w)\exp(-\omega_j)$, which depends negatively of firm's productivity.

To simplify some examples of the text, we will use a first order approximation to marginal cost around the zero value for productivity, $C'(\omega_j) = c(w)(1 - \omega_j)$. Sometimes we write $c$ as a shorthand for $c(w)$.

Many recent papers have emphasized that biased productivity is important, in particular LAP.[3] To avoid to complicate notation, we will not make explicit the presence of this type of productivity and we will simply refer to it in the text when relevant. A justification for this is the following. Suppose that the production function, $q_j = F(K_j, \exp(\omega_L)L_j, M_j)\exp(\omega_{Hj})$, shows both productivities, Hicks-neutral $\omega_H$ and LAP $\omega_L$ say. Cost is $C(q_j, \omega_j) = c(w, \omega_{Lj})q_j \exp(-\omega_{Hj})$, and marginal cost can be linearly approximated as $C'(\omega_j) = c(w, 0)(1 - S_{Lj}\omega_{Lj} - \omega_{Hj}) = c(w, 0)(1 - \omega_j)$ where now $\omega_j = S_{Lj}\omega_{Lj} + \omega_{Hj}$ and $S_{Lj}$ is the share of labor in cost.[4] So we may think of our unique term in productivity as conveying the effects of both productivities.

**Demand side**

Assume first that products are differentiated and the demands for each product are smooth. Demand for product $j$ is $q_j = D_j(p)$, where $p$ is the vector of $N$ prices. We assume that the demands are downward slopping, $\frac{\partial D_j}{\partial p_j} < 0$, and the products gross-substitutes, $\frac{\partial D_j}{\partial p_k} \geq 0$ for $k \neq j$. If the Jacobian of the system has a dominant negative diagonal it can be inverted, giving the inverse demands $p_j = P_j(q)$, with $\frac{\partial P_j}{\partial q_j} < 0$ and $\frac{\partial P_j}{\partial q_k} \leq 0$ for $k \neq j$ (see Vives, 1999). In addition, to simplify things, we assume that all demands are concave.

We also want also to compare the outcomes with product differentiation and without. To do this with two demands that represent a relatively comparable context we consider an homogeneous market with a demand that we call, for product $j$, with "equivalent price-effect" to the system. We will write $Q = D(p)$, where $p$ is here the

[3] See footnote 2.

[4] Notice that $\frac{1}{c}\frac{\partial c}{\partial \omega_L} = \frac{1}{C}\frac{\partial C}{\partial(\frac{W}{\exp(\omega_L)})}(-\frac{W}{\exp(\omega_L)}) = -\frac{WL}{C} = -S_L$.

unique price.[5]

*Definition* The demand for homogeneous goods has equivalent price-effect for good $j$ to the system of differentiated goods if it has a price derivative that equals the effect of the change of $p_j$ on the sum of the differentiated demands $\sum_k D_k(p)$. That is,

$$\frac{\partial Q}{\partial p} = \frac{\partial D_j(p)}{\partial p_j} + \sum_{k \neq j} \frac{\partial D_k(p)}{\partial p_j}.$$

An obvious consequence is that $-\frac{\partial q_j}{\partial p_j} \equiv -\frac{\partial D_j(p)}{\partial p_j} > -\frac{\partial Q}{\partial p}$. Another, less obvious, is given by the following

*Lemma* The demand $Q = D(p)$ with equivalent price-effect for product $j$ to the system $q_j = D_j(p)$ implies $-\frac{\partial p}{\partial Q} > -\frac{\partial p_j}{\partial q_j}$.

*Proof* See Appendix.

The consideration of an homogeneous demand with equivalent price-effect gives us two specific alternative market demands for which we can compare the outcomes resulting from the actions of firm $j$. The price effect in the differentiated demand for firm $j$ is greater in absolute value than in the homogeneous demand, and the lemma establishes that price is less sensitive to quantity than it is in the homogeneous demand.

**Behavior and outcomes**

We are going to consider four situations, that we depict in Table 1. We draw on Vives (1999), who treats exhaustively each one of these situations. Firms may compete alternatively in a market with perfectly sustitutable products or with differentiated products, characterized by the system of demands that we have assumed above. In both situations firms can compete either in prices (Bertrand competition) or in quantities (Cournot competition). Moving clockwise in Table 1 we have the equilibria of Bertrand with homogeneous product, Bertrand with differentiated prod-

[5] We incur in a slight abuse of notation using $p$ sometimes for the unique price and sometimes for the vector of different prices.

uct, Cournot with differentiated product and Cournot with homogeneous product. When we need for clarity we use the shorthands H, B, C, and Q, respectively.

In what follows we briefly describe behavior and outcomes. Product differentiation softens price competition, and quantity competition too. Despite this, a ranking of competitive outcomes among the four situations that includes all cases is not possible. However, the rest of this paper shows that the situations can be clearly ranked according to the impact of productivity advantages on the market shares, and hence on the structure of the market. The most radical impact happens in the top-left corner, when a productivity advantage drives the firm to gain the entire market, and becomes milder as we move clockwise to the outcome of Cournot.

The toughest situation for firms happens when products are perfectly substitutable and firms compete in prices. Let $\omega = \{\omega_1, \omega_2, ..., \omega_N\}$ be the vector of productivities. The unique Nash equilibrium is the firm with the greatest productivity pricing a little below of the marginal cost of the firm with the second greatest productivity (Tirole, 1989; Vives, 1999). Suppose that firm $j$ has the greatest productivity and firm $k$ the second. In this case, $p_j = c(1-\omega_k) - \varepsilon$, $q_j = D(c(1-\omega_k) - \varepsilon)$, $q_k = 0$ and $q_l = 0$ for all $l \neq j, k$. Firm $j$ becomes the only firm in the market ($S_j = 1$), with (approximate) market power $\frac{p_j - C_j'}{p_j} \simeq \frac{\omega_j - \omega_k}{1-\omega_k}$, its relative productivity advantage. If no firm has productivity advantage, the only Nash equilibrium is the competitive outcome of all prices equal to marginal cost.

We locate this radical outcome in the top-left corner of Table 1. The equilibrium is usually known as Bertrand with homogeneous product.

With product differentiation, when firms compete in prices, the first order conditions for profit maximization give $\frac{p_j - C_j'}{p_j} = \frac{1}{\eta_j}$, where $\eta_j = -\frac{p_j}{q_j}\frac{\partial D_j}{\partial p_j}$ is the absolute value of the elasticity of demand for product $j$. And when firms compete in quantities we write, following Vives (1999), $\frac{p_j - C_j'}{p_j} = \varepsilon_j$, where $\varepsilon_j = -\frac{q_j}{p_j}\frac{\partial P_j}{\partial q_j}$ is the absolute value of the elasticity of inverse demand. For a given vector of prices $p$, it happens that $-\frac{\partial P_j}{\partial q_j} \geq \frac{1}{-\frac{\partial D_j}{\partial p_j}}$ and hence $\varepsilon_j^p \geq \frac{1}{\eta_j^p}$ for all $j$. The reason is that the firm is acting at each equilibrium as monopolist with respect to different residual demands, which imply different elasticities. An implication is that, with the same demand system, the market outcome under quantity competition is less competitive, in the sense that the vector of prices is equal or greater than the prices under price competition (see Vives, 1999, p. 156, for the proof). That is, it happens that $p^C \geq p^B$, associated to $\varepsilon_j \geq \frac{1}{\eta_j}$ for all $j$.

Although Bertrand with product differentiation softens price competition, the two market power outcomes of the first row of Table 1 cannot be ranked completely without ambiguity. When firm $j$ chooses the optimal price according to the residual direct demand, the resulting margin may be greater or lesser than its productivity relative advantage, i.e. $\frac{p_j - C_j'}{p_j} = \frac{1}{\eta_j} \gtrless \frac{\omega_j - \omega_k}{1-\omega_k}$, even if all the rest of firms are going to increase their margins with respect to their zero equilibrium margins at Bertrand with homogeneous product. However, in the Cournot equilibrium of the intersection of the second row and second column of Table 1, all firms exercise more market power that in the Bertrand equilibrium with product differentiation.

Let us finally see what happens in the second row of the table, considering as demand for the homogeneous goods a demand that has equivalent price effect for $j$. Competitor $j$ will choose according to the first order conditions $p(Q) + q_j \frac{\partial p}{\partial Q} = C_j'$ if the product is homogeneous, and $p_j(q) + q_j \frac{\partial p_j}{\partial q_j} = C_j'$ if the product is differentiated. At the same price in the homogeneous market as in the differentiated market, $p = p_j$, it can be checked that the Cournot competitor would choose quantities such that elasticities $\frac{q_j}{Q} \frac{Q}{p} \frac{\partial P}{\partial Q} = S_j \varepsilon$ and $\frac{q_j}{p_j} \frac{\partial p_j}{\partial q_j} = \varepsilon_j$ are identical. This implies $q_j^Q < q_j^C$. In this sense, Cournot is less competitive than Cournot with product differentiation.

However, in general, there is no reason for the two scenarios to give the same price. With the demands having the same choke off price, it is clear that the first marginal revenue falls more rapidly with $q_j$ and the choice will imply $q_j^Q < q_j^C$. But we have not made any assumption about the level of the demands and in particular the choke off price. In addition, the heterogeneity of the price effects of firms -with the implication of different homogeneous equivalent price-effect demands- makes the comparisons less straightforward.

## 3. Best response functions

Let us analyze the change of the best response functions with the growth of productivity. We will focus on the change in productivity of one firm while productivity of the rest remains the same. We deal in turn with the three cases with smooth profit functions: Bertrand with product differentiation, B, Cournot with product differentiation, C, and Cournot, Q. In B and C we assume the same demand, in Q an homogeneous demand with equivalent price-effect to the system of B and C.

Let us start with Bertrand with product differentiation. The profit function of the firm $j$ that sets its optimal price as monopolist of the residual demand can be written $\pi_j = p_j q_j(p) - C(q_j(p), \omega_j)$, and the first order condition is $q_j(p) + p_j \frac{\partial q_j(p)}{\partial p_j} - C_j'(\omega_j)\frac{\partial q_j(p)}{\partial p_j} = 0$. Write the first-order condition as optimal response of $j$ to the rest of prices, $p_j = R_j(p_{-j})$

$$q_j(R_j(p_{-j}), p_{-j}) + (R_j(p_{-j}) - C_j'(\omega_j))\frac{\partial q_j(R_j(p_{-j}), p_{-j})}{\partial p_j} = 0.$$

The change in the best response of $j$ due to a change in productivity is

$$\frac{\partial R_j(p_{-j})}{\partial \omega_j} = \frac{\frac{\partial C_j'}{\partial \omega_j}\frac{\partial q_j}{\partial p_j}}{2\frac{\partial q_j}{\partial p_j} + (p_j - C_j')\frac{\partial^2 q_j}{\partial p_j^2}} < 0. \tag{1}$$

With concave demand, the denominator is negative (in fact the denominator coincides with the second order condition for the problem of setting an optimal price). Both terms in the numerator are negative and therefore the product is positive. The sign of the derivative is hence negative.

It follows that an increase in productivity displaces downwards the best response function of the firm, and the firm has incentives to decrease the price. Suppose two moments of time, 1 and 2, say. All the rest equal, a new equilibrium should emerge with a vector of prices $p_2^B \leq p_1^B$. All firms decrease their prices, or at most keep the same. Figure 1 panel B illustrates the displacement and the new equilibrium for the case of two firms, $j$ and $i$.

Let us now discuss Cournot with product differentiation. The profit function of the firm $j$ that sets its optimal quantity as monopolist of the residual demand can be written $\pi_j = p_j(q)q_j - C(q_j, \omega_j)$, and the first order condition is $p_j(q) + q_j \frac{\partial p_j(q)}{\partial q_j} - C_j'(\omega_j) = 0$. Write the first-order condition in terms of the optimal response of $j$ to the rest of quantities, $q_j = R_j(q_{-j})$

$$p_j(R_j(q_{-j}), q_{-j}) + R_j(q_{-j}) \frac{\partial p_j(R_j(q_{-j}), q_{-j})}{\partial q_j} - C_j'(\omega_j) = 0.$$

The change in the reaction function is

$$\frac{\partial R_j(q_{-j})}{\partial \omega_j} = \frac{\frac{\partial C_j'}{\partial \omega_j}}{2\frac{\partial p_j}{\partial q_j} + q_j \frac{\partial^2 p_j}{\partial q_j^2}} > 0. \qquad (2)$$

The denominator is negative for the same reason as before. The numerator is also negative and hence the derivative is positive.

It follows that an increase in productivity displaces upwards the best response function of the firm, and the firm has incentives to increase the quantity put in the market. All the rest equal, a new equilibrium should emerge with more quantity for firm $j$ in detriment of the output of the rivals. Given the usual slope of the reaction functions, total output expands. Figure 1 panel C illustrates the displacement and the new equilibrium for the case of two firms, $j$ and $i$.

Let us finally discuss quantity competition with identical products. Suppose that the firm faces an homogeneous demand with equivalent price-effect for firm $j$. The profit function of Cournot competitor $j$ in a homogeneous product market is $\pi_j =$

$p(Q)q_j - C(q_j, \omega_j)$, where what has changed is that the unique price in the market is the result of the total quantity $Q = \sum_k q_k$ of the perfectly substitutable goods. Firm $j$ chooses quantity optimally according to the first order condition $p(Q) + q_j \frac{\partial p(Q)}{\partial q_j} - C'_j(\omega_j) = 0$. In terms of the best response $q_j = R_j(q_{-j})$ we can write

$$p(R_j(q_{-j}) + \textstyle\sum_{k \neq j} q_k) + R_j(q_{-j}) \frac{\partial p(R_j(q_{-j}) + \sum_{k \neq j} q_k)}{\partial q_j} - C'_j(\omega_j) = 0.$$

The change in the reaction function now is

$$\frac{\partial R_j(\sum_{k \neq j} q_k)}{\partial \omega_j} = \frac{\frac{\partial C'_j}{\partial \omega_j}}{2\frac{\partial p}{\partial q_j} + q_j \frac{\partial^2 p}{\partial q_j^2}} > 0. \tag{3}$$

It follows that an increase in productivity displaces again upwards the best response function of the firm, and the firm has incentives to increase the quantity set in the market. A new equilibrium will imply, as before, more quantity for firm $j$ in detriment of the output of the rivals, and an expanded total output. Figure 2 panel Q illustrates the displacement and the new equilibrium for the case of two firms, $j$ and $i$.

We can summarize what we know as follows. When firm $j$ experiences an increase in its productivity, its best response function moves in the direction of either decreasing the price or expanding output depending the type of market competition (price or quantity). This movements meet the following

*Proposition* 1 The resulting expansion of the output of the firm is greater the more intense is competition: more in Bertrand competition with product differentiation than in Cournot competition with product differentiation, and in Cournot competition with product differentiation than in Cournot with an equivalent price-effect homogeneous demand.

*Proof* We first establish that the absolute value of the denominator of (2) is smaller than the absolute value of the denominator of (3). Then we show that (1) multiplied by $\frac{\partial q_j}{\partial p_j}$ is greater than (2). See Appendix.

## 4. Distribution of market shares

The results of the previous section suggest that, other things equal, market shares should be closely linked to productivity advantages, and that this relationship must be sharper the more intense is price competition. In this section we directly explore the relationship between productivity advantages and market shares. There is one case in which this relationship is straightforward, and is the case of Cournot competition under homogeneous goods. The reason is that the price is common to all firms and hence all advantages are directly conveyed by the quantities. We will see that under this type of competition market shares can be approximated by an expression that is linear in the productivity advantages.

In the other extreme, it lies the radical case of Bertrad competition under homogeneous product. Market shares are so sensitive to price (and hence productivity) differences that a small advantage becomes enough to dominate the entire market. Bertrand and Cournot competition under product differentiation lie in between. Since the product is differentiated and firms price differently, we need to define market shares in therms of sales, $S_j = p_j q_j / \sum_k p_k q_k$. Then productivity advantages both impact the prices and quantities set by firms. There are no easy ways to separate the influence on each variable and the relationships that link market shares and productivity advantages are going to become highly nonlinear. However, the relationships developed in the previous section suggest that the introduction of differentiated prices should help to translate more sharply the advantages to shares than in Cournot with homogeneous product, and more with Bertrand competition than with Cournot competition.

We start describing the shares in Cournot competition with homogeneous product, that we are going to denote by $S^Q$. Then we will characterize the shares with Bertrand with product differentiation and Cournot with product differentiation, respectively.

Finally, we establish a second proposition that ranks the sensitivity of a given share to the productivity advantages under the different competition situations. We will use the notation $S^B$ and $S^C$ for the share corresponding to Bertrand and Cournot under product differentiation.

The first order condition for Cournot competition can be rewritten as

$$p(1 - S_j\varepsilon) = C'_j,$$

which allows to compute Cournot price by aggregation as

$$p = \frac{\overline{C}'}{1 - \varepsilon/N}.$$

where $\overline{C}'$ is the average of marginal costs, $\overline{C} = \frac{1}{N}\sum_k C'_k$. The combination of both expressions gives the relationship between market shares and relative marginal costs

$$S_j^Q = \frac{1}{\varepsilon} - (\frac{1}{\varepsilon} - \frac{1}{N})\frac{C'_j}{\overline{C}'}.$$

The derivative can be written as

$$\frac{\partial S_j^Q}{\partial \omega_j} = (\frac{1}{\varepsilon} - \frac{1}{N})\frac{C'_j}{N\overline{C'}}\frac{N\overline{C'} - C'_j}{N\overline{C'}} > 0, \tag{4}$$

where we use the fact that $\frac{1}{C'_j}\frac{\partial C'_j}{\partial \omega_j} = -1$.

Using the approximation $\frac{C'_j}{\overline{C}'} = 1 + (\frac{C'_j}{\overline{C}'} - 1) \simeq 1 - \frac{\omega_j - \overline{\omega}}{1 - \overline{\omega}}$, where $\overline{\omega} = \frac{1}{N}\sum_k \omega_k$, we can write

$$S_j^Q = \frac{1}{N} + (\frac{1}{\varepsilon} - \frac{1}{N})\frac{\omega_j - \overline{\omega}}{1 - \overline{\omega}}.$$

Under Cournot competition in a perfect substitutes market, market shares diverge if productivity is different across firms, and diverge in direct relationship to the magnitude of productivity advantage of each firm with respect to mean productivity. The impact of productivity is greater the lower is the elasticity of inverse demand (or the greater is the direct elasticity of demand $\eta$)[6], and the larger is the number of firms.

[6] Recall that, with perfect substitutes, $\varepsilon = \frac{1}{\eta}$.

Notice that, under common input prices, for a given number of firms and elasticity of demand, the distribution of productivity $f(\omega)$ completely determines the distribution of shares. A change in the distribution of productivity will be transmitted to the distribution of shares. For example, if during some period of time productivity advantages become asymmetric, we expect market shares to become equally asymmetric and the market become concentrated.

In the case of Bertrand with product differentiation and Cournot with product differentiation we have a different price for each product. First order conditions give

$$\begin{aligned} p_j^B(1-\frac{1}{\eta_j}) &= C_j', \\ p_j^C(1-\varepsilon_j) &= C_j', \end{aligned}$$

respectively. Aggregation produces formulas that depend on all the elasticities and the firm share in costs

$$\begin{aligned} S_j^B &= \frac{1-\frac{1}{\eta}}{1-\frac{1}{\eta_j}}\frac{C_j}{\sum_k C_k}, \\ S_j^C &= \frac{1-\varepsilon}{1-\varepsilon_j}\frac{C_j}{\sum_k C_k}, \end{aligned}$$

where $\frac{1}{\eta} = \sum_k S_k^B \frac{1}{\eta_k}$ and $\varepsilon = \sum_k S_k^C \varepsilon_k$. If all elasticities are equal, market shares exactly coincide with the shares in costs.

Notice that cost shares can be rewritten in terms of relative marginal costs

$$\frac{C_j}{\sum_k C_k} = \frac{C_j' q_j}{\sum_k C_k' q_k} = \frac{S_j^Q C_j'}{\sum_k S_k^Q C_k'},$$

where now we use the notation $S_j^Q$ for the current quantity shares. The weight of marginal costs with the quantity shares introduces an additional complexity. Note that $S_j^Q$ is, as in the Cournot case, and even with equal price elasticities, a function of the entire vector of advantages. Productivity advantages reduce cost of each unit of output but also increase the output amount. How output increases, it depends on the demand relationship and type of competition.

We can approximate the formula for cost shares as $\frac{C_j}{\sum_k C_k} = S_j^Q(1-\omega_j)/(1-\overline{\omega}_Q)$, where $\overline{\omega}_Q = \sum_k S_k^Q \omega_k$. Plugging this expression in the formulas for $S_j^B$ and $S_j^C$ we have the nonlinear analogous of the linear expression for Cournot under homogeneous product.

The expressions for market shares are difficult to manage under arbitrary heterogeneity of firms. However, for a given set of market shares, we can get insights by differentiating directly the market share under Cournot and Bertrand competition (recall that equilibria are associated to different residual demands).

Under Bertrand competition we have

$$S_j^B = \frac{p_j q_j(p)}{\sum_k p_k q_k(p)},$$

and we obtain

$$\frac{\partial S_j^B}{\partial p_j} = -\frac{S_j}{p_j}[(1-S_j)(\eta_j - 1) + \textstyle\sum_{k\neq j} S_k \eta_{kj}],$$

where $\eta_{kj}$ are the cross-price elasticities $\eta_{kj} = \frac{p_j}{q_k}\frac{\partial q_k}{\partial p_j}$. The effect of $\omega_j$ on the share can be now computed as

$$\frac{\partial S_j^B}{\partial \omega_j} = \frac{\partial S_j^B}{\partial p_j}\frac{\partial p_j}{\partial \omega_j} = S_j[(1-S_j)(\eta_j - 1) + \textstyle\sum_{k\neq j} S_k \eta_{kj}] > 0, \tag{5}$$

where we use that $\frac{\partial p_j}{\partial \omega_j} = -p_j$.

Under Cournot competition we have

$$S_j^C = \frac{p_j(q) q_j}{\sum_k p_k(q) q_k},$$

and we obtain

$$\frac{\partial S_j^C}{\partial q_j} = \frac{S_j}{q_j}[(1-S_j)(1-\varepsilon_j) + \textstyle\sum_{k\neq j} S_k \varepsilon_{kj}],$$

where $\varepsilon_{kj}$ are the absolute value of the cross-quantity elasticities $\varepsilon_{kj} = -\frac{q_j}{p_k}\frac{\partial p_k}{\partial q_j}$. The effect of $\omega_j$ on the share can be calculated as

$$\frac{\partial S_j^C}{\partial \omega_j} = \frac{\partial S_j^C}{\partial q_j}\frac{dq_j}{dp_j}\frac{\partial p_j}{\partial \omega_j} = S_j \frac{1}{\varepsilon_j}[(1-S_j)(1-\varepsilon_j) + \textstyle\sum_{k\neq j} S_k \varepsilon_{kj}] > 0, \tag{6}$$

where we use that $\frac{1}{\frac{dp_j}{dq_j}}\frac{\partial p_j}{\partial \omega_j} = \frac{q_j}{\varepsilon_j}$.

We can now establish

*Proposition* 2 Under Bertrand with product differentiation, Cournot with product differentiation and Cournot with homogeneous product, the advantages impact market shares more heavily the greater is price competition, i. e. $\frac{\partial S^B}{\partial \omega_j} > \frac{\partial S^C}{\partial \omega_j} > \frac{\partial S^Q}{\partial \omega_j}$.

*Proof* It amounts to show that derivative (5) is greater than derivative (6), and derivative (6) is greater than derivative (4). See Appendix.

## 5. Concentration and inflation

The relationship between market power and inflation has often worried economists, who have speculated that firms with market power may help to sustain the increase in the prices over time. Although this is an old concern (see, for example, Weiss, 1971), recent papers have revisited it as an explanation for recent trends in the US economy. According to, for example, Covarrubias, Gutierrez and Philippon, 2020, and Brauning, Fillat, and Joaquim, 2023, a presumed rise in concentration and the evolution of prices show the link between inflation, concentration and market power.

However, productivity gains provide a reason of why, in times of technological change, we should rather expect the opposite relationship. Sections 3 and 4 have shown that asymmetric productivity gains may simultaneously determine that the prices fall, or increase less than cost, at the same time that the market becomes concentrated. In this section, we develop with some detail how this happens in a market with Cournot competition. Proposition 2 suggests that this outcome can still be more intensive when price competition is sharper, se we expect it to be a more general phenomenon. We then carry out a simple empirical exercise (in the style of Ganapati, 2020) to illustrate how recent increases in prices have shown negative relationships with increases in concentration by industries.

**Theory**

Let us assume that the productivity advantages $\omega$ of the population of a market are, in a given moment $t$, distributed normal around the mean $\overline{\omega}_t$, i.e. $\omega \sim N(\overline{\omega}_t, \sigma_t^2)$.[7] Take marginal costs as $C' = c(w_t)\exp(-\omega)$. Marginal cost is then a lognormal variable with mean $E(C') = c(w_t)\exp(-(\overline{\omega}_t + \frac{1}{2}\sigma_t^2))$. Suppose that this is a perfect substitutes market where firms behave Cournot. Assume that demand elasticity is constant and the number of firms doesn't change. It follows that the log of the price is related to mean marginal cost as

$$\ln p_t = -\ln(1 - \frac{\varepsilon}{N}) + \ln c(w_t) - (\overline{\omega}_t + \frac{1}{2}\sigma_t^2).$$

Inflation $\Delta \ln p_t$ (the rate of increase over time of the market price) has two determinants. First, the increase in $\Delta \ln c(w_t)$, due to the increase in the prices of the inputs, pushes the price up. Second, the increase in $\Delta(\overline{\omega}_t, + \frac{1}{2}\sigma_t^2)$, because productivity gains, pulls it down. Notice that this second force can operate even with no modifications of the average productivity. It is enough that productivity spreads through the gains in some firms, even if it decreases in others (a mean preserving change in the variance).

An implication of the formula is that, with Cournot exercise of market power, an unequal increase of productivity of the firms will tend to decrease the output price (or to reduce the increase induced by input price inflation).

To see an example of how this happens think of the ordered distribution of productivity advantages $\omega$. Suppose that the upper tail of the distribution, from some arbitrary threshold level of productivity on, have the values systematically increase. We are going to call this type of increase an stochastically dominant productivity

[7] This can be justified, for example, through the central limit theorem. It is usual to assume that productivity advantages of firms evolve as Markovian processes driven by the impact of independent productivity shocks, i.e. $\omega_t = g(\omega_{t-1}) + \xi_t$. As a result, after a period of stationarity, productivity advantages will tend to show a normal distribution.

change (the cumulative distribution function shifts down). We will see a simultaneous increase of $\overline{\omega}_t$ and $\sigma_t^2$.

This kind of change in productivity will affect simultaneously the concentration of the market. The relative marginal cost of a firm can be written as $\frac{C'}{E(C')} = \exp[-(\omega - (\overline{\omega}_t + \frac{1}{2}\sigma^2))] = \exp(-\omega^*)$. We will call $\omega^*$ the relative productive advantage of the firm. The market share of a firm is $S^Q = \frac{1}{\varepsilon} - (\frac{1}{\varepsilon} - \frac{1}{N})\exp(-\omega^*)$. Note that if $\omega^* = 0$ (the productivity advantage equals mean productivity), the share of the firm is just the average market share. Let us discuss what happens when there is a change in the distribution of $\omega^*$.

The share $S^Q$ is a positive monotonic function of $\omega^*$. Call the distribution functions of $\omega^*$ and $S^Q$, $G_1(\omega^*)$ and $F_1(S^Q)$ respectively. Since $S^Q$ is a monotonic transformation of $\omega^*$, the quantiles of $F_1(S^Q)$ are monotonic transformations of the quantiles of $G_1(\omega^*)$. It follows that, if we consider a change of the distribution of $\omega^*$ from $G_1(\omega^*)$ to $G_2(\omega^*)$, where $G_2(\omega^*)$ stochastically dominates $G_1(\omega^*)$, we will have also a new distribution $F_2(S^Q)$ that stochastically dominates $F_1(S^Q)$. A consequence is that

$$CRk_2 - CRk_1 = \int_{Q_S(\tau)}^{1} S^Q dF_2(S^Q) - \int_{Q_S(\tau)}^{1} S^Q dF_1(S^Q) \geq 0,$$

where $CRk$ is the concentration ratio for the first $K$ firms, and $Q_S(\tau)$ is the quantile $\tau = 100\frac{N-K}{N}\%$ in the distribution of $S^Q$.[8]

Summarizing, if the evolution of productivity is such that the relative productive advantages $\omega^*$ do not vary, the distribution of market shares will not change. For example, suppose that all productivities increase in $g$ percentage points, i.e. $\omega_{jt+1} = \omega_{jt} + g$. However, if some firms improve productivities more than others, raising a stochastically dominant new distribution of relative productivity advantages, the market is going to become more concentrated in the sense that concentration ratios are going strictly to increase.

[8] See Hart (1975).

This suggests an important role for processes of innovation and diffusion of technology, where productivity gains are not instantaneous and simultaneous. When some firms experience relatively bigger gains in productivity, and the differences stays for some time, average marginal cost is going to decrease or increase less, moderating price increases. At the same time, the market is going to become concentrated, with an increased share of the most productive firms. This should happen if the market is Cournot competition with homogeneous product, but section 4 suggests that the process of concentration can be even sharper with more price competition.

**Empirics**

We want to relate price variation and changes in concentration. The economic censuses compute concentration ratios ($CR_4$ and $CR_{20}$) by industries of US manufacturing. The data for 2002, 2007, 2012 and 2017 are publicly available for the NAICs classification of industries.[9] We combine these data with the output prices estimated for the manufacturing NBER-CES data base.[10] As we employ the NBER-CES version for NAICS 1997, the use of the concentration ratios without further work matches a decreasing number of industries. However, our objective here is only a quick look and we do not think that this impedes to transmit the main message.[11] We are able to match to the NBER-CES database 468, 466, 298 and 290 industries in the 2002, 2007, 2012 and 2017, respectively.

The average value of the 4-firms concentration ratio is 0.43, and for the 20-firms concentration ratio is about 0.71. Both means show almost no changes over the years, and this despite the changing number of industries for which these means are

[9] See, for example, Kulick and Card (2022). These authors guided me in accessing the census information and graciously facilitated their files to check mines.

[10] See Becker, Gray and Marvakov (2021). This data base aggregates the census results in six-digit manufacturing industries defined according with the NAICS classification of 1997.

[11] Kulick and Card (2022) argue, however, that the omision of industries in not neutral, favoring concentration over time.

computed. Of course, this is compatible with rich movements in both directions of the individual indices across industries. We are able to work with a total of 1039 5-years changes in prices and concentration, after dropping 15 changes in concentration outside the [-0.50, 0.50] interval. We annualize the log changes in prices dividing by 5, and we keep untouched the 5-years differences in concentration. The coefficients of regression can be interpreted as elasticities. If the coefficient shows a value of $X$, we can read it implying an annual price change of $10X\%$ for a $10\%$ change in concentration.

Table 2 summarizes a few results. Column (1) reports the regression of the changes in prices on the change of joint share of the first 4 firms ($\Delta CR_4$) and the change of the share of the next 16 firms ($\Delta CR_{next16} = \Delta CR_{20} - \Delta CR_4$). Concentration of the top four firms tends to raise prices and the share of the next 16 firms tends to decrease them, but the results are not significant.

However, all the important things in concentration happen in the interplay between these two shares (4 and next 16 first firms): in $91.5\%$ of the cases at least one of these shares increases, in $51.9\%$ it is the joint share of the 16 that does, and in $16.2\%$ both of them. We select the more than half of the cases in which the joint share of the next 16 firms increases. Column (2) shows the significant association to price reductions (or more moderate increases). Column (3) makes clear that what happens in these cases with the top firms is nonsignificant.

The data point out to rich patterns of change (only $16\%$ of the changes seem to be stochastically dominant), but there are unequivocal with respect to the association of many increases in concentration with a better behavior of prices. The topic deserves further investigation.

## 6. Market power or efficiency?

The growth of productivity is linked to innovation and technological advances, and recent empirical evidence finds an important role for technological progress that directly affects the marginal productivity of labor or LAP (see footnote 2). This type of technological progress explains the widespread falls in the labor shares in costs and revenue observed in the firms of the US and other advanced economies. In section 2 we have shown that our framework can approximate without modifications the output productivity effect generated by this kind of productivity. In fact, the combination of the idea of unequal gains in LAP generated by technological change and the insights generated by the propositions of sections 3 and 4, give a convincing interpretation of recent facts observed in the US economy and its manufacturing. This interpretation differs radically from the reasonings based on conventional measurements and ideas about market power that have become popular. In this sense, the discussion revives the old questions raised by Peltzman (1977) and others about the separate identification of efficiency and market power.

Two papers claim similar interrelated facts, for US manufacturing and other industries, in a period that approximately spans 1980-2016. De Loecker, Eckhout and Unger (2020) look directly at the evolution of markups. Autor, Dorn, Katz, Patterson and Van Reenen (2020) look at the fall of the labor shares of revenue. For them, labor shares fall because are a decreasing function of markups. These are the joint stylized facts that emerge from both papers as presented by themselves: first, average markup (sum of markups weighted by sales shares) rises while aggregate labor share in income falls; second, there is an important reallocation component in both the rise of the average markup and the fall of the aggregate labor share; third, the process is driven by the biggest firms ("superstar" firms in the second paper); fourth, there is simultaneous concentration of sales.

Let us see how these stylized facts can be alternatively explained by mismeasurements incurred by ignoring the bias in productivity together with the implications suggested by sections 3 and 4 for the dynamics of the distributions resulting from intense unequal technological change.

De Loecker and Warzynski (2012) firm-level markups are typically computed as $\widehat{\mu}_{jt} = \widehat{\beta}_X / S^R_{Xjt}$, where $\widehat{\beta}_X$ is the estimate of the elasticity of a variable input $X$ and $S^R_{Xjt}$ the observed share of the input cost on revenue. It has become customary to use labor. Any rigid estimate of the labor elasticity $\widehat{\beta}_L$ will already create under LAP a positive growth bias because of the evolution downwards of the observed $S^R_{Ljt}$.

In addition, cost minimization implies that, at any moment of time, there is cross-section distribution of the true elasticities $\beta_{Ljt} = \mu_{jt} S^R_{Ljt}$,[12] whose variation is systematically related to the level of LAP via the share of labor. The estimate produced for the average markup is $\widehat{\mu}_t = \sum_j S_{jt} \widehat{\mu}_{jt}$, where $S_{jt}$ are firm market shares. It can be written as

$$\widehat{\mu}_t = \mu_t + \sum_j S_{jt} \left(\frac{\widehat{\beta}_L - \beta_{Ljt}}{\beta_{Ljt}}\right) \mu_{jt},$$

where $\mu_t = \sum_j S_{jt} \mu_{jt}$ is the true average markup.

The second term of the right hand side can be seen as a markup weighted covariance between $S_{jt}$ and $(\widehat{\beta}_L - \beta_{Ljt})$. The term $(\widehat{\beta}_L - \beta_{Ljt})$ depends directly of LAP, because the true elasticity $\beta_{Ljt}$ is smaller the greater is LAP. Under the effects reviewed in sections 3 and 4, the market shares $S_{jt}$ are going to be related with the level of LAP, and more related the greater is price competition. The expected value for the estimate is then the true value plus a (possibly huge) bias, $E(\widehat{\mu}_t) = \mu_t + bias$.[13] A

[12] The first order condition for labor is $MC_{jt} \frac{\partial q_{jt}}{\partial L^*} \exp(\omega_{L_{jt}}) = W_{jt}$, that can be written as $\frac{L^*_{jt}}{q_{jt}} \frac{\partial q_{jt}}{\partial L^*} = \frac{P_{jt}}{MC_{jt}} \frac{W_{jt} L_{jt}}{P_{jt} q_{jt}}$ or $\beta_{Ljt} = \mu_{jt} S^R_{Ljt}$.

[13] The same type of bias should be expected to arise in the measurements of monopsony power that are based on the comparison of relative rigid elasticities and observed input shares, as in Yeh, Macaluso and Hershbein (2022).

huge bias only indicates the importance of LAP and its effects on the distribution of shares. Jaumandreu (2022), using conventional estimates $\widehat{\mu}_{jt} = 0.29/S^R_{Xjt}$, shows the important biases generated in the estimation of an unweighted and weighted average markup with a sample of manufacturing Compustat firms (an average markup close to 1 is multiplied by 1.6 and more than 3, respectively).

The fact of LAP suggests another story. An unequal development of productivity gains is likely to leave markups more or less as they were according to the different competitive environments of firms, to push down the shares of labor cost in variable costs and revenue (as result of the evolution of labor under the displacement/compensation that occurs with an elasticity of substitution less than one), to impulse the growth of the market shares of the firms with productivity gains, more the greater is market competition, and concentrate the market.

A simple proof of the likelihood of this alternative explanation is the following. Let us simulate a sample of firms whose productivity grows through a sequence of Hicks-neutral and labor-augmenting productivity shocks. Firms produce with a three input CES production function with elasticity of substitution $\sigma = 0.7$. The productivity factors $\exp(\omega_H)$ and $\exp(\omega_L)$ multiply respectively the entire input aggregator and the labor input. The productivity terms $\omega_H$ and $\omega_L$ are inhomogeneous autoregressive Markovian processes of parameter 0.8 and normal random innovations. We assume them independent for simplicity. During the 30 periods that we examine, Hicksian productivity grows at an average of 1.2 percentage points and the output effect of labor-augmenting productivity at an average of 2.2 percentage points.

Firms face identical isoelastic demands of elasticity $\eta = 6$ and play Bertrand setting the price for their products by multiplying the short-run marginal cost by a markup $\frac{\eta}{\eta-1} = 1.2$.[14] Because productivity shocks have a random component, firms experi-

[14]The level of capital is optimally adjusted each period according to the demand for the product and current input prices.

ence heterogeneous marginal cost reductions and, since they pass these reductions on prices, they experience different demanded quantities and growth. Firms in fact have started completely identical ten periods before we begin to compute their evolution.

Table 2 summarizes a typical result. The average firm size more than doubles, although employment grows much more moderately. The aggregate labor share falls, and there is an important reallocation component because the firms that experience greater labor augmenting productivity both reduce their labor share and grow more. The concentration as measured by the $CR10$ of sales is big and grows.

Despite firms hold exactly the same market power from the beginning to the end, its measurement with De Loecker and Warzynski (2012) method to compute markups yields an spectacular increase, with an important reallocation component (the difference between the weighted and unweighted means). Markups and their increase are greater in the 90th percentile. The numbers are roughly similar to what the mentioned papers find for the US in the years 1980-2016.

What the exercise suggests is that research should focus on the process of productivity growth and its effects in employment and the shares reallocation. On the evolution of efficiency more than on the effects of market power.

## 7. Concluding Remarks

This paper explores the role of productivity in the determination of the market shares of firms, given different behavior. Our discussion is valid for firms that show two types of productivity advantages over their competitors: Hicks neutral and biased productivity (we focus in the form of LAP). Productivity advantages decrease marginal costs, impacting the shares of firms and hence the structure of the market depending on how is competition. We show that the more competitive is behavior, the stronger is the impact of productivity on the shares.

Price competition with perfectly substitutable products determines a radical bound:

the firm with a productivity advantage finds optimal to fight for the entire market at the expense of the rivals. However, the optimal action becomes less aggressive with product differentiation and quantity competition. In the other extreme, represented by Cournot, market shares are related to the productivity advantages, but in a more subdued way than when competition is sharper. Using oligopoly theory we have shown that, all the rest equal, productivity advantages determine the distribution of market shares and its changes, with an intensity that depends on the game the firms play.

This may seem an obvious consideration, but practical analyses often forget the implications of this link between productivity growth and market structure. We briefly comment two applications of our results. The first is about the relationship between inflation and concentration. We have shown that it can be a negative relationship between concentration and inflation. Milder behaviors of US manufacturing prices during the 2000's are clearly related to the concentration of many industries driven by the firms that are not the highest top of the distribution.

With the second application we raise an alternative explanation to the story that argues a recent increase of markups in US manufacturing induced by market power, accompanied by falls of the labor share in cost and revenue. The measurement of the markups using as denominator the labor share, in a time of intense LAP, confounds the level of efficiency and market power and sets the base for an aggregation bias of the individual-firm markups. A simple simulation confirms that the same results may come from a market with both LAP and Hicksian productivity growth, where market power doesn't increase except if inappropriately measured, at the same time that concentration rises and the labor share falls.

These examples make a strong case for the development of the analysis of the recent growth in productivity, often characterized by important biases towards labor, and the consequences of the technological change via productivity on the firm shares, market structure and concentration. In particular, a strong unequal diffusion of productivity is likely to raise temporary firms asymmetries that it is important to understand and address properly in economic policy.

**Appendix**

*Proof of the Lemma*

Differentiate the identity $p_j = P_j(q(p))$ to get $1 = \frac{\partial p_j}{\partial q_j}\frac{\partial q_j}{\partial p_j} + \sum_{k\neq j}\frac{\partial p_j}{\partial q_k}\frac{\partial q_k}{\partial p_j}$, and hence $1 = -\frac{\partial p_j}{\partial q_j}(-\frac{\partial q_j}{\partial p_j} - \sum_{k\neq j}\theta_{kj}\frac{\partial q_k}{\partial p_j})$, where $\theta_{kj} = \frac{\partial p_j}{\partial q_k}/\frac{\partial p_j}{\partial q_j} < 1$. This implies that $-\frac{\partial p_j}{\partial q_j} < -\frac{1}{\frac{\partial q_j}{\partial p_j}+\sum_{k\neq j}\frac{\partial q_k}{\partial p_j}}$ and hence that $-\frac{\partial p_j}{\partial q_j} < -\frac{1}{\frac{\partial Q}{\partial p}} = -\frac{\partial p}{\partial Q}$. ♦

*Proof of Proposition 1*

Let us first compare (2) and (3), C and Q. The denominators of (2) and (3), written in absolute value are $-2\frac{\partial p_j}{\partial q_j} - q_j\frac{\partial^2 p_j}{\partial q_j^2}$ and $-2\frac{\partial p}{\partial q_j} - q_j\frac{\partial^2 p}{\partial q_j^2}$. On the other hand, for the same price, the first order conditions imply the equality $-q_j^C\frac{\partial p_j}{\partial q_j} = -q_j^Q\frac{\partial p}{\partial q_j}$, and totally differencing it we get $-\frac{\partial p_j}{\partial q_j} - q_j^C\frac{\partial^2 p_j}{\partial q_j^2} = -\frac{\partial p}{\partial q_j} - q_j^Q\frac{\partial^2 p}{\partial q_j^2}$. Adding the result of the Lemma $-\frac{\partial p_j}{\partial q_j} < -\frac{\partial p}{\partial q_j}$ term to term to this expression changes the equality into an inequality, and hence we can see that the absolute value of the denominator of (2) is smaller than the absolute value of the denominator of (3). This shows that the output expansion is greater under Cournot with product differentiation than under Cournot.

According to (1) and (2), B and C, if $\frac{\partial q_j}{\partial p_j}\frac{\partial R_j(p_{-j})}{\partial \omega_j} > \frac{\partial R_j(q_{-j})}{\partial \omega_j}$ the expansion of output implied by the decrease in price under Bertrand with product differentiation will be greater than under Cournot with product differentiation. Dividing by $-\frac{\partial C_j'}{\partial \omega_j}$ and with some manipulation, the left and right hand side of the inequality can be written as $-\frac{\partial q_j}{\partial p_j}/(2 - \frac{1}{\eta_j^2}\frac{(p_j^B)^2}{q_j^B}\frac{\partial^2 q_j}{\partial p_j^2})$ and $-1/\frac{\partial p_j}{\partial q_j}(2 - \frac{1}{\varepsilon_j}\frac{(q_j^C)^2}{p_j^C}\frac{\partial^2 p_j}{\partial q_j^2})$. To compare the parentheses, take the inequality $-\frac{q_j^B}{\frac{\partial q_j}{\partial p_j}} < -q_j^C\frac{\partial p_j}{\partial q_j}$, an implication of the price under Bertrand being smaller than the price under Cournot. Taking the derivative of the left hand side with respect to price and multiplying by $\frac{\partial p_j}{\partial q_j}dq_j$, and differentiating the right hand side with respect to quantity, we get $-\frac{1}{\eta_j^2}\frac{(p_j^B)^2}{q_j^B}\frac{\partial^2 q_j}{\partial p_j^2} < -\frac{1}{\varepsilon_j}\frac{(q_j^C)^2}{p_j^C}\frac{\partial^2 p_j}{\partial q_j^2}$. With concave demand it follows that the first parenthesis is smaller. As we already know that $-\frac{\partial q_j}{\partial p_j} > -1/\frac{\partial p_j}{\partial q_j}$, this makes the expansion of output under Bertrand unequivocally greater.♦

*Proof of Proposition 2*

The inequality between the derivatives (6) and (5) can be established by noting that $[\frac{1}{\varepsilon_j}(1-S_j)(1-\varepsilon_j)+\frac{1}{\varepsilon_j}\sum_{k\neq j} S_k\varepsilon_{kj}] < [(1-S_j)(\eta_j-1)+\sum_{k\neq j} S_k\eta_{kj}]$. The first term of the left hand side is smaller than first term of the right hand side because $\eta_j > \frac{1}{\varepsilon_j}$. The second term of the left hand side can be seen to be smaller than the second term of the right hand side by developing the identity $\frac{\partial \sum_k p_k(q)q_k}{\partial q_j}\frac{\partial q_j}{\partial p_j} = \frac{\partial \sum_k p_k q_k(p)}{\partial p_j}$. It follows that $\sum_{k\neq j} S_k\eta_{kj} > \eta_j \sum_{k\neq j} S_k\varepsilon_{kj}$ and hence also that $\sum_{k\neq j} S_k\eta_{kj} > \frac{1}{\varepsilon_j}\sum_{k\neq j} S_k\varepsilon_{kj}$.

Now we have to establish the inequality between the derivatives (4) and (6), that is, $\frac{1}{\varepsilon}\frac{1}{N}(N-\varepsilon)\frac{C'_j}{N\overline{C'}}\frac{N\overline{C'}-C'_j}{N\overline{C'}} < S_j\frac{1}{\varepsilon_j}(1-S_j)(1-\varepsilon_j+\sum_{k\neq j}\frac{S_k}{1-S_j}\varepsilon_{kj})$. If the homogeneous demand is price-effect equivalent and we are at the same price, $\frac{1}{\varepsilon} = \frac{S_j}{\varepsilon_j}$, $(N-\varepsilon)\frac{C'_j}{N\overline{C'}} = (N-\varepsilon)\frac{1-\varepsilon S_j}{N-\varepsilon} = (1-\varepsilon S_j) = (1-\varepsilon_j)$ and, since $\sum_{k\neq j}\frac{S_k}{1-S_j}\varepsilon_{kj} > 0$, we only have to show that $\frac{1}{N}\frac{N\overline{C'}-C'_j}{N\overline{C'}} \leq 1-S_j$. Using the previous expression for $\frac{C'_j}{N\overline{C'}}$, we can write the left hand side as $1-\frac{(N-1)S_j(NS_j-\varepsilon_j)+(1-\varepsilon_j)S_j}{NS_j(NS_j-\varepsilon_j)}S_j$. The left hand side will be smaller always that the share is not too big, specifically when $S_j < \frac{1}{N}+\frac{N-1}{N}\varepsilon$. ♦

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Table 1:

Competition and Market Power outcomes[a]

| | | Products are: Perfectly Substitutable | Products are: Differentiated |
|---|---|---|---|
| Firms choose | Prices | H: $\frac{p_j - C_j'}{p_j} \simeq \frac{\omega_j - \omega_k}{1-\omega_k}$ | B: $\frac{p_j - C_j'}{p_j} = \frac{1}{\eta_j}$ |
| | Quantities | Q: $\frac{p - C_j'}{p} = S_j \varepsilon$ | C: $\frac{p_j - C_j'}{p_j} = \varepsilon_j$ |

[a] $\eta_j = -\frac{p_j}{q_j}\frac{\partial D_j}{\partial p_j}$, $\varepsilon = -\frac{Q}{p}\frac{\partial P}{\partial Q}$, and $\varepsilon_j = -\frac{q_j}{p_j}\frac{\partial P_j}{\partial q_j}$.

Table 2: Effects of concentration in price, 2002-2017.

Yearly average price change regresed on the variation in concentration ratios.[a]

| Variables | All changes | $\Delta CR_{next16} > 0$ | $\Delta CR_{next16} > 0$ |
|---|---|---|---|
| | (1) | (2) | (3) |
| Constant | 0.028 | 0.033 | 0.034 |
| | (0.001) | (0.002) | (0.002) |
| Dummy 07-12 | -0.006 | -0.008 | -0.009 |
| | (0.002) | (0.003) | (0.003) |
| Dummy 12-17 | -0.026 | -0.029 | -0.029 |
| | (0.002) | (0.003) | (0.003) |
| $\Delta CR_4$ | 0.022 | 0.276 | |
| | (0.020) | (0.277) | |
| $\Delta CR_{next16}$ | -0.039 | -0.110 | -0.142 |
| | (0.027) | (0.048) | (0.036) |
| Observations | 1039 | 548 | 548 |
| $R^2$ | 0.123 | 0.153 | 0.151 |

[a] Computed for subperiods 2002-2007, 2007-2012, and 2012-2017, for 451, 298 and 290 industries respectively.

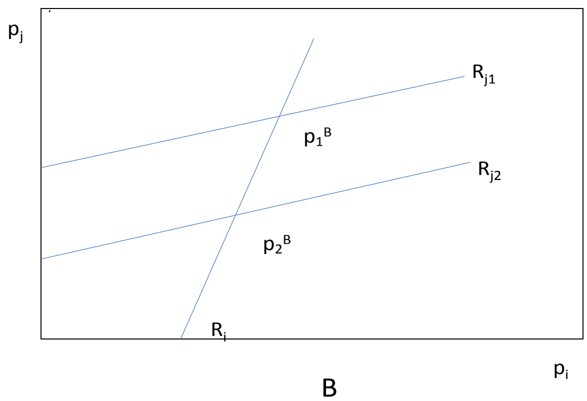
Table 3
Simulation results for a sample of firms with
constant markups and labor-augmenting productivity[a]

| Variable | Simulation[b] Period 1 | Simulation[b] Period 30 | Total change |
|---|---|---|---|
| (1) | (2) | (3) | (4) |
| Average revenue (index) | 1.000 | 2.244 | 1.244 |
| Average employment (index) | 1.000 | 1.649 | 0.649 |
| DLW markups[c]: | | | |
| Weighted mean | 1.376 | 1.901 | 0.525 |
| Unweighted mean | 0.970 | 1.188 | 0.218 |
| 90th weighted percentile | 1.633 | 2.226 | 0.593 |
| Real markup[d] | 1.200 | 1.200 | 0.00 |
| Labor share in revenue[e]: | | | |
| Aggregate labor share | 0.222 | 0.174 | 0.048 |
| Unweighted labor share | 0.315 | 0.281 | 0.034 |
| Concentration of sales (CR10)[f] | 0.282 | 0.341 | 0.059 |

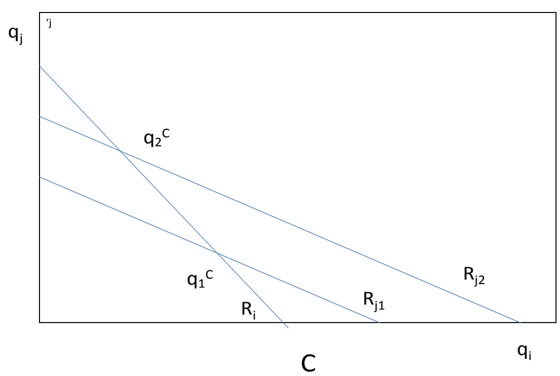
[a] Sample of 1000 firms with identical CES production functions with $\sigma = 0.7$, which produce to serve identical constant elasticity demands of absolute elasticity value $\eta = 6$. Firms experience over time Hicks-neutral and Labor-augmenting productivity shocks and set price with a constant margin over marginal cost.
[b] Firms start equal and we report as Period 1 their values after 10 periods of simulation.
[c] Computed as labor input elasticity divided by the firm's share of labor in revenue. Weighted with revenue weights.
[d] $\frac{\eta}{\eta-1} = \frac{6}{5} = 1.2$.
[e] Cost of labor over revenue. Weighted with revenue weights.
[f] Three years moving averages.

Figure 1

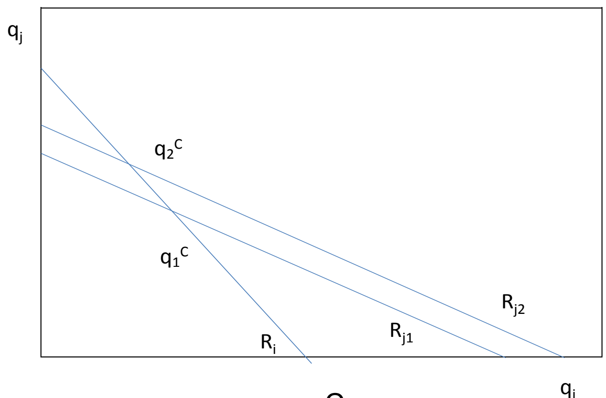
Best response changes with an increase in productivity




B




C

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