

# Productivity, Competition, and Market Outcomes\*

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## Abstract

For firms, productivity is constantly evolving because of the introduction of new technology and innovations. Some of these productivity gains diffuse uniformly across firms, others only spread out in the industry with time. The unequal evolution of productivity impacts the structure of the industry, the more the greater the degree of competition. We analyze the relationship between the distribution of firms' productivity advantages and the distribution of market shares, and show that this relationship is more intense the more competition. We briefly comment on two applications: we show that, because productivity gains, market concentration and inflation can be negatively related, and we give an alternative interpretation to the case for a recent rise of US markups attributed to increased market power.

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## 1. Introduction

This paper discusses how productivity, which results from the innovative activity of firms, impacts market structure and market outcomes. The idea that productivity (or efficiency) impacts the shares of firms is obvious, and it is often explicitly taken into account. However, there are at least two related questions that receive much less attention. The first is how the impact of productivity on market shares is linked to the intensity of competition. In fact this is implied by the models that economists have traditionally used to analyze market outcomes. The second is how the interaction productivity-competition can determine a deep impact in the distribution of markets shares. Innovations and the process of diffusion of new technologies, with the corresponding development of productivity gains, may determine acute changes in the distribution of market shares, even if only temporarily.

These facts are full of implications. As economists involved in the practice of competition policy realize, the advantages of aggregate increases in productivity often come in the hand of a more asymmetric distribution of market shares. More in general, markets subject to intense innovation are likely to experience a pattern of diffusion of productivity gains that influences naturally the evolution of market shares. Markets can become, for example, globally more concentrated, to only grow less asymmetric with the time that diffusion processes need. How sharp is competition may in fact decide how intense will be these processes over time.

An interesting particular case happens with labor-augmenting productivity (LAP), typically less recognizable at first sight than the neutral productivity that affects all inputs. When the diffusion of new techniques implies an important growth of LAP, we must expect simultaneously an increase in productivity (may be not as sharp as in the case of neutral productivity), a decrease of the firm and industry-level shares of labor cost (in both cost and revenue), and a concentration of the markets, with

a dominant role of the firms that are growing more efficient.<sup>1</sup> If we do not measure properly the markups, that are likely to remain more or less stable, the process can be mistakenly taken by an increase in market power (see below).<sup>2</sup>

This paper explores theoretically the role of productivity in the market shares of the firms, under different behaviors. We start by considering some firms as able to produce with productivity advantages over their competitors, and we consider two types of possible advantages: Hicks neutral, that affect simultaneously to all inputs, and LAP, that impact directly the efficiency of the labor input. All productivity advantages decrease marginal costs, but the relationship of quantities and prices to marginal cost depends on how is competition.

We consider two dimensions of competition. First, products in the market can be homogeneous or differentiated. Differentiated products soften the sharpness of price competition among the firms. Second, we consider that firms can compete in quantities or prices. Competition in quantities is less aggressive. Following Vives (1999), we hence use a taxonomy consisting of four situations: competition a la Bertrand with perfectly substitutable products, competition a la Bertrand when products are differentiated, competition a la Cournot when the products are differentiated, and Cournot competition with perfectly substitutable products. We see this ordering as going from sharper to softer competition.

We first analyze the change in the quantity or price optimally set by a firm when the firm experiences an improvement of productivity (either neutral or biased). Price competition with perfectly substitutable products determines a radical bound: the firm with a productivity advantage finds optimal to gain the entire market at the expense of the rivals. However, the optimal action becomes less aggressive the less

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<sup>1</sup>This can perfectly be accompanied by an increase of employment, because the compensation effects of technology (see Jaumandreu and Mullens, 2024).

<sup>2</sup>For an early discussion about the difficulties to separate unequal efficiency from market power see Peltzman (1977).

competitive is the context. In the other extreme, represented by Cournot, market shares are related to the productivity advantages, but in a more subdued way than when competition is sharper. We explore formally how the distribution of productivity advantages relates to the distribution of market shares in each continuous game. A related paper is Vives (2008), that analyzed the relationship between innovation and market structure under different competition models, but mostly using symmetry.

We choose to briefly comment two applications of our results. The first is about the relationship between inflation and market power. A recent article, Ganapati (2020), makes the point that US manufacturing inflation at the end of the nineties and beginning of the 2000's seems to be empirically unrelated or even negatively related to the changes in the degree of concentration of the markets. The figure that shows the used data suggests that rather it can be a negative relationship with different intercepts. Our conjecture is that the industries that have experienced important unequal increases in productivity, have been milder in translating the cost increases to prices because the competition in prices among the firms that have also lead the concentration of the markets. We explain how this is possible and likely.

The second application consists of deriving an alternative explanation to the story that tries to establish a recent increase in concentration and market power, with sharp rise of markups, accompanied by falls of the labor share in revenue. This view originated in the articles by De Loecker, Eckhout and Unger (2020) and Autor, Dorn, Katz, Patterson and Van Reenen (2020). A key of their findings is the measurement of the markups using some estimated elasticity divided by the input share, what confounds the level of efficiency and market power and sets the base for an aggregation bias of the individual-firm markups. A simple simulation shows that the same results may come from a market with labor-augmenting and Hicksian productivity growth, where market power doesn't increase except if inappropriately measured, at the same time that concentration rises and the labor share falls.

Both the theory and the applications make a strong case for the development of the analysis of the recent growth in productivity, often characterized by important biases towards labor, and the consequences of the technological change via productivity on the firm shares, market structure and concentration. Strong unequal diffusion of productivity is likely to raise temporary firms asymmetries that it is important to understand, in particular to address properly measures of economic policy.

The rest of the paper is organized as follows. The second section establishes the setup. The third analyzes the best response functions and the fourth the distributions of market shares. Sections five and six develop the applications and section seven concludes.

## 2. Setup

### Production side

There are  $N$  firms, each one producing a product, with production functions  $q_j = F(K_j, \exp(\omega_{Lj})L_j, M_j \exp(\omega_{Hj}))$ ,  $j = 1, \dots, N$ , where  $K, L$  and  $M$  are capital, labor and materials. The terms  $\omega_H$  and  $\omega_L$  measure (percentage) deviations of Hicks-neutral and LAP from the productivity of a standard firm for which these terms are zero, respectively. We want to emphasize that these two productivities are usually present.  $F(\cdot)$  is any production function with elasticities of substitution less than unit. The production function has constant returns to scale.

The cost function for firm  $j$  turns out to be  $C(q_j, \omega_j) = c(w, \omega_{Lj})q_j \exp(-\omega_{Hj})$ , where  $c(\cdot)$  depends on the specification of the product function,  $w$  is the vector of (industry common) input prices and  $\omega_j = (\omega_{Lj}, \omega_{Hj})$  represents the vector of productivity terms. Marginal cost is  $C'(\omega_j) = c(w, \omega_{Lj}) \exp(-\omega_{Hj})$ , and obviously depends negatively of both productivities.

To simplify some examples, we will use a first order approximation to marginal cost around the zero values for both productivities,  $C'(\omega_j) = c(w, 0)(1 - S_{Lj}\omega_{Lj} - \omega_{Hj})$ ,

where  $S_{Lj}$  is the share of labor in cost.<sup>3</sup> It is convenient to note that equilibrium  $S_{Lj}$  is a decreasing function of LAP (see, for example, Jaumandreu and Mullens, 2024), although the product  $S_{Lj}\omega_{Lj}$  is increasing in LAP.

When we do not need to distinguish between Hicksian and LAP we will use, slightly abusing notation,<sup>4</sup> the expression  $C'(\omega_j) = c(w, 0)(1 - \omega_j)$  where  $\omega_j = S_{Lj}\omega_{Lj} + \omega_{Hj}$  can be read as productivity regardless of its origin. In this case, what is important is that the firms' cost differ just in relative productivity. Sometimes we write  $c$  as a shorthand for  $c(w, 0)$ . In section 5, we use for simplicity the alternative marginal costs simplification  $C' = c(w, 0) \exp(-\omega)$ .

### Demand side

Assume first that products are differentiated and the demands for each product smooth. Demand for product  $j$  is  $q_j = D_j(p)$ , where  $p$  is the vector of  $N$  prices. We assume that the demands are downward slopping,  $\frac{\partial D_j}{\partial p_j} < 0$ , and the products gross-substitutes,  $\frac{\partial D_j}{\partial p_k} \geq 0$  for  $k \neq j$ . The system of demands can be inverted, giving the inverse demands  $p_j = P_j(q)$ , with  $\frac{\partial P_j}{\partial q_j} < 0$  and  $\frac{\partial P_j}{\partial q_k} \leq 0$  for  $k \neq j$  (see Vives (1999) about the conditions for invertibility). In addition, to simplify things, we are going to assume that all demands are concave.

We want also to compare the outcomes with product differentiation and without. To do this we consider an homogeneous market with a demand that we call, for product  $j$ , with "equivalent price-effect" to the system. We will write  $Q = D(p)$ , where  $p$  is here the unique price.<sup>5</sup>

*Definition* The demand for homogeneous goods has equivalent price-effect for good  $j$  to the system of differentiated goods if it has a price derivative that equals the effect

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<sup>3</sup>Notice that the semielasticity  $\frac{1}{c} \frac{\partial c}{\partial \omega_L} = -S_L$ .

<sup>4</sup>We use  $\omega_j$  to denote both a vector and the scalar sum of productivities

<sup>5</sup>We incur here in a new slight abuse of notation using  $p$  sometimes for the unique price and sometimes for a vector of different prices.

of the change of  $p_j$  on the sum of the differentiated demands  $\sum_k D_k(p)$ . That is,

$$\frac{\partial Q}{\partial p} = \frac{\partial D_j(p)}{\partial p_j} + \sum_{k \neq j} \frac{\partial D_k(p)}{\partial p_j}.$$

An obvious consequence is that  $-\frac{\partial q_j}{\partial p_j} \equiv -\frac{\partial D_j(p)}{\partial p_j} > -\frac{\partial Q}{\partial p}$ .

*Lemma* The demand  $Q = D(p)$  with equivalent price-effect for product  $j$  to the system  $q_j = D_j(p)$  implies  $-\frac{\partial p}{\partial Q} > -\frac{\partial p_j}{\partial q_j}$ .

*Proof* See Appendix.

The consideration of an homogeneous demand with equivalent price-effect gives us two specific alternative market demands for which we can compare the outcomes resulting from the actions of firm  $j$ . If the firm plays Cournot, at the same price it will get the same markup with and without product differentiation (see below), what seems a sensible point to start comparisons. While the price effect in the differentiated demand for firm  $j$  is smaller than in the homogeneous demand, the lemma establishes that price it is less sensitive to quantity than it is in the homogeneous demand.

### Behavior

The toughest situation happens when products are perfectly substitutable and firms compete in prices. Let  $\omega = \{\omega_1, \omega_2, \dots, \omega_N\}$  be the vector of productivities, the unique Nash equilibrium is the firm with the greatest productivity pricing a little below of the marginal cost of the firm with the second greatest productivity (Tirole, 1989, p. 211). Suppose that firm  $j$  has the greatest productivity and firm  $k$  the second. In this case,  $p_j = c(1 - \omega_k) - \varepsilon$ ,  $q_j = D(c(1 - \omega_k) - \varepsilon)$ ,  $q_k = 0$  and  $q_l = 0$  for all  $l \neq j, k$ . Firm  $j$  becomes the only firm in the market ( $S_j = 1$ ), with (approximate) market power  $\frac{p_j - C'_j}{p_j} \simeq \frac{\omega_j - \omega_k}{1 - \omega_k}$ , its relative productivity advantage. If no firm has productivity advantage, the only Nash equilibrium is the competitive outcome of all prices equal to marginal cost.

We locate this radical outcome in the top-left corner of Table 1. The equilibrium is usually known as Bertrand with homogeneous products. We then consider,

drawing on Vives (1999), two dimensions that may soften price competition. First, firms can set quantities instead of prices, what is usually called Cournot competition. Second, products can be differentiated, becoming characterized by the system of demands that we have assumed above. Moving clockwise in Table 1 we have the equilibria of Bertrand with differentiated product, Cournot with differentiated product and Cournot with homogeneous product. We will use the notation B, C, and Q, respectively.

With product differentiation, when firms compete in prices the first order conditions for profit maximization give  $\frac{p_j - C'_j}{p_j} = \frac{1}{\eta_j}$ , where  $\eta_j = -\frac{p_j}{q_j} \frac{\partial D_j}{\partial p_j}$  is the absolute value of the elasticity of demand for product  $j$ . And when firms compete in quantities we can write, following Vives (1999),  $\frac{p_j - C'_j}{p_j} = \varepsilon_j$ , where  $\varepsilon_j = -\frac{q_j}{p_j} \frac{\partial P_j}{\partial q_j}$  is the absolute value of the elasticity of inverse demand. For a given vector of prices  $p$ , it happens that  $\varepsilon_j^p \geq \frac{1}{\eta_j^p}$  for all  $j$ . The reason is that the firm is acting at each equilibrium as monopolist with respect to different residual demands, which imply different elasticities. An implication is that, with the same demand system, the market outcome under quantity competition is less competitive, in the sense that the vector of prices is equal or greater than the prices under price competition (see Vives, 1999, p. 156, for the proof). That is, it happens that  $p^C \geq p^B$ , associated to  $\varepsilon_j \geq \frac{1}{\eta_j}$  for all  $j$ .

Hence, in the Cournot equilibrium of the intersection of the second row and second column of Table 1, firms exercise more market power. However, the two outcomes of the first row of Table 1 cannot be ranked completely unambiguously. When firm  $j$  chooses the optimal price according to the residual direct demand, the resulting margin may be greater or lesser than its productivity relative advantage, i.e.  $\frac{p_j - C'_j}{p_j} = \frac{1}{\eta_j} \geq \frac{\omega_j - \omega_k}{1 - \omega_k}$ , even if all the rest of firms increase their margins with respect to their zero equilibrium margins at Bertrand with homogeneous product.

Let us finally now compare the two outcomes of the second row of the table, considering as demand for homogeneous goods a demand that has equivalent price effect for



$j$ . Competitor  $j$  will choose according to the first order conditions  $p(Q) + q_j \frac{\partial p}{\partial Q} = C'_j$  and  $p_j(q) + q_j \frac{\partial p_j}{\partial q_j} = C'_j$ . At the same price in the homogeneous market for firm  $j$  as in the differentiated market,  $p = p_j$ , it can be checked that the Cournot competitor would choose quantities such that elasticities  $\frac{q_j}{Q} \frac{Q}{p} \frac{\partial P}{\partial Q} = S_j \varepsilon$  and  $\frac{q_j}{p_j} \frac{\partial p_j}{\partial q_j} = \varepsilon_j$  are identical. This implies  $q_j^Q < q_j^C$ . In this sense, Cournot is less competitive than Cournot with product differentiation.

But there is no reason for the two scenarios to give the same price. With the same choke off price, it is clear that the first marginal revenue falls more rapidly with  $q_j$  and the choice will imply  $q_j^Q < q_j^C$ . These choices will in general also imply the same price. But we have not made any assumption about the level of the demands and in particular the choke off price. In addition, the heterogeneity of the price effects of firms -with the implication of different homogeneous equivalent price-effect demands- makes the comparisons less straightforward.

Hence, there is no a completely unambiguous ranking of degree of competition over the four outcomes in all conditions. However, we are going to show that the situations can be clearly ranked according to the impact of productivity advantages on the market shares, and hence on the structure of the market. The most radical impact happens in the top-left corner, when a productivity advantage drives the firm to gain the entire market, and becomes milder as we move clockwise to the outcome of Cournot.

### 3. Best response functions

Let us analyze the change of the best response functions with the growth of productivity. We will focus on the change in productivity of one firm while productivity of the rest remains the same. We deal in turn with the three cases with smooth profit functions: Bertrand with product differentiation, Cournot with product differentiation, and Cournot.

The profit function of the firm  $j$  that sets its optimal price as monopolist of the residual demand can be written  $\pi_j = p_j q_j(p) - C(q_j(p), \omega_j)$ , and the first order condition is  $q_j(p) + p_j \frac{\partial q_j(p)}{\partial p_j} - C'_j(\omega_j) \frac{\partial q_j(p)}{\partial p_j} = 0$ . Write the first-order condition as optimal response of  $j$  to the rest of prices,  $p_j = R_j(p_{-j})$

$$q_j(R_j(p_{-j}), p_{-j}) + (R_j(p_{-j}) - C'_j(\omega_j)) \frac{\partial q_j(R_j(p_{-j}), p_{-j})}{\partial p_j} = 0.$$

The change in the best response of  $j$  due to a change in productivity is

$$\frac{\partial R_j(p_{-j})}{\partial \omega_j} = \frac{\frac{\partial C'_j}{\partial \omega_j} \frac{\partial q_j}{\partial p_j}}{2 \frac{\partial q_j}{\partial p_j} + (p_j - C'_j) \frac{\partial^2 q_j}{\partial p_j^2}} < 0. \quad (1)$$

With concave demand, the denominator is negative (in fact it coincides with the second order condition for the problem of setting an optimal price). Both terms in the numerator are negative and therefore the product is positive. The sign of the derivative is hence negative.

It follows that an increase in productivity displaces downwards the best response function of the firm, and the firm has incentives to decrease the price. All the rest equal, a new equilibrium should emerge with a vector of prices  $(p^B)' \leq p^B$ . Figure 1 panel B illustrates the displacement and the new equilibrium for the case of two firms,  $j$  and  $i$ .

The profit function of the firm  $j$  that sets its optimal quantity as monopolist of the residual demand can be written  $\pi_j = p_j(q) q_j - C(q_j, \omega_j)$ , and the first order condition is  $p_j(q) + q_j \frac{\partial p_j(q)}{\partial q_j} - C'_j(\omega_j) = 0$ . Write the first-order condition in terms of the optimal response of  $j$  to the rest of quantities,  $q_j = R_j(q_{-j})$

$$p_j(R_j(q_{-j}), q_{-j}) + R_j(q_{-j}) \frac{\partial p_j(R_j(q_{-j}), q_{-j})}{\partial q_j} - C'_j(\omega_j) = 0.$$

The change in the reaction function is

$$\frac{\partial R_j(q_{-j})}{\partial \omega_j} = \frac{\frac{\partial C'_j}{\partial \omega_j}}{2 \frac{\partial p_j}{\partial q_j} + q_j \frac{\partial^2 p_j}{\partial q_j^2}} > 0. \quad (2)$$

The denominator is negative for the same reason as before. The numerator is also negative and hence the derivative is positive.

It follows that an increase in productivity displaces upwards the best response function of the firm, and the firm has incentives to increase the quantity put in the market. All the rest equal, a new equilibrium should emerge with more quantity for firm  $j$  in detriment of the output of the rivals. Given the usual slope of the reaction functions, total output expands. Figure 1 panel C illustrates the displacement and the new equilibrium for the case of two firms,  $j$  and  $i$ .

Suppose now that the firm faces an homogeneous demand with equivalent price-effect for firm  $j$ . The profit function of Cournot competitor  $j$  in a homogeneous product market is  $\pi_j = p(Q)q_j - C(q_j, \omega_j)$ , where what has changed is that the unique price in the market is the result of the total quantity  $Q = \sum_k q_k$  of the perfectly substitutable goods. Firm  $j$  chooses quantity optimally according to the first order condition  $p(Q) + q_j \frac{\partial p(Q)}{\partial q_j} - C'_j(\omega_j) = 0$ . In terms of the best response  $q_j = R_j(q_{-j})$  we can write

$$p(R_j(q_{-j}) + \sum_{k \neq j} q_k) + R_j(q_{-j}) \frac{\partial p(R_j(q_{-j}) + \sum_{k \neq j} q_k)}{\partial q_j} - C'_j(\omega_j) = 0.$$

The change in the reaction function now is

$$\frac{\partial R_j(\sum_{k \neq j} q_k)}{\partial \omega_j} = \frac{\frac{\partial C'_j}{\partial \omega_j}}{2 \frac{\partial p}{\partial q_j} + q_j \frac{\partial^2 p}{\partial q_j^2}} > 0. \quad (3)$$

It follows that an increase in productivity displaces again upwards the best response function of the firm, and the firm has incentives to increase the quantity set in the market. A new equilibrium will imply, as before, more quantity for firm  $j$  in detriment of the output of the rivals, and an expanded total output. Figure 2 panel Q illustrates the displacement and the new equilibrium for the case of two firms,  $j$  and  $i$ .

We can summarize what we know in the following

*Proposition 1* When firm  $j$  experiences an increase in its productivity, its best response function moves in the direction of either decreasing the price or expanding output depending the type of market competition (price or quantity). The resulting expansion of the output of the firm is greater the more intense is competition (more in Bertrand competition with product differentiation than in Cournot competition with product differentiation, and in Cournot competition with product differentiation than in Cournot with an equivalent homogeneous demand).

*Proof* Let us first compare (2) and (3). The denominators of (2) and (3), written in absolute value are  $-2\frac{\partial p_j}{\partial q_j} - q_j \frac{\partial^2 p_j}{\partial q_j^2}$  and  $-2\frac{\partial p}{\partial q_j} - q_j \frac{\partial^2 p}{\partial q_j^2}$ . On the other hand, for the same price, the first order conditions imply the equality  $-q_j^C \frac{\partial p_j}{\partial q_j} = -q_j^Q \frac{\partial p}{\partial q_j}$ , and totally differencing it we get  $-\frac{\partial p_j}{\partial q_j} - q_j^C \frac{\partial^2 p_j}{\partial q_j^2} = -\frac{\partial p}{\partial q_j} - q_j^Q \frac{\partial^2 p}{\partial q_j^2}$ . Adding  $-\frac{\partial p_j}{\partial q_j} < -\frac{\partial p}{\partial q_j}$  term to term to this expression changes the equality into an inequality, and comparing we can see that the denominator of (2) is smaller in absolute value than the denominator of (3). This shows that the output expansion is greater under Cournot with product differentiation than under Cournot.

According to (1) and (2), if  $\frac{\partial q_j}{\partial p_j} \frac{\partial R_j(p_{-j})}{\partial \omega_j} > \frac{\partial R_j(q_{-j})}{\partial \omega_j}$  the expansion of output implied by the decrease in price under Bertrand with product differentiation will be greater than under Cournot with product differentiation. Dividing by  $-\frac{\partial C'_j}{\partial \omega_j}$  and with some manipulation, the left and right hand side of the inequality can be written as  $-\frac{\partial q_j}{\partial p_j} / (2 - \frac{1}{\eta_j^2} \frac{(p_j^B)^2}{q_j^B} \frac{\partial^2 q_j}{\partial p_j^2})$  and  $-1 / \frac{\partial p_j}{\partial q_j} (2 - \frac{1}{\varepsilon_j} \frac{(q_j^C)^2}{p_j^C} \frac{\partial^2 p_j}{\partial q_j^2})$ . We know that  $\varepsilon_j > \frac{1}{\eta_j}$ , and hence that  $\frac{p_j^C}{q_j^C} \varepsilon_j > 1 / \left( \frac{q_j^B}{p_j^B} \eta_j \right)$ . This implies that the inequality would hold without the parentheses. To compare the parentheses, we can start with the implication of the difference in prices  $-\frac{q_j^B}{\partial q_j} < -q_j^C \frac{\partial p_j}{\partial q_j}$ . Taking the derivative of the left hand side with respect to price and multiplying by  $\frac{\partial p_j}{\partial q_j} dq_j$ , and differentiating the right hand side with respect to quantity, we can finally get  $-\frac{1}{\eta_j^2} \frac{(p_j^B)^2}{q_j^B} \frac{\partial^2 q_j}{\partial p_j^2} < -\frac{1}{\varepsilon_j} \frac{(q_j^C)^2}{p_j^C} \frac{\partial^2 p_j}{\partial q_j^2}$ . With concave demand, this makes the first parenthesis smaller and hence confirms that the expansion of output under Bertrand is greater. ♦

#### 4. Distribution of market shares

The previous results suggest that, other things equal, market shares should be closely linked to productivity advantages, and that this relationship must be sharper the more intense is price competition. Here we first develop the well known relationship between market shares and productivity advantages under Cournot competition. Under this type of competition market shares can be approximated by an expression that is linear in the productivity advantages.

In the case of Cournot, price is the same for all firms and market shares are identical in sales and in quantities. When the product is differentiated and firms price differently, we need to define market shares in terms of sales,  $S_j = p_j q_j / \sum_k p_k q_k$ . Productivity advantages both impact the prices and quantities set by firms. There are no easy ways to separate the influence on each variable and the expressions that link market shares and productivity advantages are nonlinear. We explore the impact of productivity advantages on these market shares and compare with the case of Cournot competition. We will use the notation  $S^Q$ ,  $S^C$ , and  $S^B$  for the share under Cournot, Cournot with product differentiation, and Bertrand with product differentiation, respectively.

The first order condition for Cournot competition can be rewritten as

$$p(1 - S_j \varepsilon) = C'_j,$$

which allows to compute Cournot price by aggregation as

$$p = \frac{\bar{C}'}{1 - \varepsilon/N}.$$

where  $\bar{C}'$  is the average of marginal costs,  $\bar{C}' = \frac{1}{N} \sum_k C'_k$ . The combination of both expressions gives the relationship between market shares and relative marginal costs

$$S_j^Q = \frac{1}{\varepsilon} - \left(\frac{1}{\varepsilon} - \frac{1}{N}\right) \frac{C'_j}{\bar{C}'}$$

The derivative can be written as

$$\frac{\partial S_j^Q}{\partial \omega_j} = \left(\frac{1}{\varepsilon} - \frac{1}{N}\right) \frac{C_j'}{NC'} \frac{N\bar{C}' - C_j'}{NC'} > 0, \quad (4)$$

where we use the fact that  $\frac{1}{C_j'} \frac{\partial C_j'}{\partial \omega_j} = -1$ .

Using the approximation  $\frac{C_j'}{\bar{C}'} = 1 + \left(\frac{C_j'}{\bar{C}'} - 1\right) \simeq 1 - \frac{\omega_j - \bar{\omega}}{1 - \bar{\omega}}$ , where  $\bar{\omega} = \frac{1}{N} \sum_k \omega_k$ , we can write

$$S_j^Q = \frac{1}{N} + \left(\frac{1}{\varepsilon} - \frac{1}{N}\right) \frac{\omega_j - \bar{\omega}}{1 - \bar{\omega}}.$$

Under Cournot competition in a perfect substitutes market, market shares diverge if productivity is different across firms, and diverge in direct relationship to the magnitude of productivity advantage of each firm with respect to mean productivity. The impact of productivity is greater the lower is the elasticity of inverse demand (or the greater is the direct elasticity of demand  $\eta$ )<sup>6</sup>, and the larger is the number of firms.

Notice that, under common input prices, for a given number of firms and elasticity of demand, the distribution of productivity  $f(\omega)$  completely determines the distribution of shares. Specifically, shares are distributed with density

$$f_S(S^Q) = f\left(\frac{S^Q - \frac{1}{N}}{\frac{1}{\varepsilon} - \frac{1}{N}}\right) \left(\frac{1}{\varepsilon} - \frac{1}{N}\right),$$

where  $f(\cdot)$  is the distribution of productivity advantages  $\omega$ .

A change in the distribution of productivity will be transmitted to the distribution of shares. For example, if during some period of time productivity advantages become asymmetric, we expect market shares to become equally asymmetric and the market become concentrated.

In the case of Bertrand with product differentiation and Cournot with product differentiation we have a different price for each product. First order conditions give

$$\begin{aligned} p_j^B \left(1 - \frac{1}{\eta_j}\right) &= C_j', \\ p_j^C (1 - \varepsilon_j) &= C_j', \end{aligned}$$

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<sup>6</sup>Recall that, with perfect substitutes,  $\varepsilon = \frac{1}{\eta}$ .

respectively. Aggregation produces formulas that depend on all the elasticities and the firm share in costs

$$S_j^B = \frac{1 - \frac{1}{\eta}}{1 - \frac{1}{\eta_j}} \frac{C_j}{\sum_k C_k},$$

$$S_j^C = \frac{1 - \varepsilon}{1 - \varepsilon_j} \frac{C_j}{\sum_k C_k},$$

where  $\frac{1}{\eta} = \sum_k S_k^B \frac{1}{\eta_k}$  and  $\varepsilon = \sum_k S_k^C \varepsilon_k$ . If all elasticities are equal, market shares exactly coincide with the shares in costs.

Notice that cost shares can be rewritten in terms of relative marginal costs

$$\frac{C_j}{\sum_k C_k} = \frac{C'_j q_j}{\sum_k C'_k q_k} = \frac{S_j^Q C'_j}{\sum_k S_k^Q C'_k},$$

where we use the notation  $S_j^Q$  for the current quantity shares. The weight of marginal costs with the quantity shares introduces an additional complexity. Note that  $S_j^Q$  is, as in the Cournot case, and even with equal price elasticities, a function of the entire vector of advantages. Productivity advantages reduce cost of each unit of output but also increase the output amount. How output increases, it depends on the demand relationship and type of competition.

We can approximate the formula for cost shares as  $\frac{C_j}{\sum_k C_k} = S_j^Q (1 - \omega_j) / (1 - \bar{\omega}_Q)$ , where  $\bar{\omega}_Q = \sum_k S_k^Q \omega_k$ . Plugging this expression in the formulas for  $S_j^B$  and  $S_j^C$  it becomes clear the more complex character of the relationship between market shares and advantages.

However, for a given set of market shares, we can get insights differentiating directly the market share under Cournot and Bertrand competition (recall that equilibria are associated to different residual demands).

Under Bertrand competition we have

$$S_j^B = \frac{p_j q_j(p)}{\sum_k p_k q_k(p)},$$

and we obtain

$$\frac{\partial S_j^B}{\partial p_j} = -\frac{S_j}{p_j}[(1 - S_j)(\eta_j - 1) + \sum_{k \neq j} S_k \eta_{kj}],$$

where  $\eta_{kj}$  are the cross-price elasticities  $\eta_{kj} = \frac{p_j}{q_k} \frac{\partial q_k}{\partial p_j}$ . The effect of  $\omega_j$  on the share can be now computed as

$$\frac{\partial S_j^B}{\partial \omega_j} = \frac{\partial S_j^B}{\partial p_j} \frac{\partial p_j}{\partial \omega_j} = S_j[(1 - S_j)(\eta_j - 1) + \sum_{k \neq j} S_k \eta_{kj}] > 0. \quad (5)$$

Under Cournot competition we have

$$S_j^C = \frac{p_j(q)q_j}{\sum_k p_k(q)q_k},$$

and we obtain

$$\frac{\partial S_j^C}{\partial q_j} = \frac{S_j}{q_j}[(1 - S_j)(1 - \varepsilon_j) + \sum_{k \neq j} S_k \varepsilon_{kj}],$$

where  $\varepsilon_{kj}$  are the absolute value of the cross-quantity elasticities  $\varepsilon_{kj} = -\frac{q_j}{p_k} \frac{\partial p_k}{\partial q_j}$ . The effect of  $\omega_j$  on the share can be calculated as

$$\frac{\partial S_j^C}{\partial \omega_j} = \frac{\partial S_j^C}{\partial q_j} \frac{dq_j}{dp_j} \frac{\partial p_j}{\partial \omega_j} = S_j \frac{1}{\varepsilon_j} [(1 - S_j)(1 - \varepsilon_j) + \sum_{k \neq j} S_k \varepsilon_{kj}] > 0. \quad (6)$$

We can now establish

*Proposition 2* Under Bertrand with product differentiation, Cournot with product differentiation and Cournot, the advantages impact market shares more heavily the greater is competition, i. e.  $\frac{\partial S^B}{\partial \omega_j} > \frac{\partial S^C}{\partial \omega_j} > \frac{\partial S^Q}{\partial \omega_j}$ .

*Proof* The inequality between the derivatives (6) and (5) can be established by noting that  $[\frac{1}{\varepsilon_j}(1 - S_j)(1 - \varepsilon_j) + \frac{1}{\varepsilon_j} \sum_{k \neq j} S_k \varepsilon_{kj}] < [(1 - S_j)(\eta_j - 1) + \sum_{k \neq j} S_k \eta_{kj}]$ . That the first term of the left hand side is smaller than first term of the right hand side follows from  $\varepsilon_j > \frac{1}{\eta_j}$ , since it implies  $\eta_j - 1 > \frac{1}{\varepsilon_j} - 1$ . That the second term of the left hand side is smaller than the second term of the right hand side can be seen by developing the identity  $\frac{\partial \sum_k p_k(q)q_k}{\partial q_j} \frac{\partial q_j}{\partial p_j} = \frac{\partial \sum_k p_k q_k(p)}{\partial p_j}$ . It follows that  $\sum_{k \neq j} S_k \eta_{kj} > \eta_j \sum_{k \neq j} S_k \varepsilon_{kj}$  and hence also that  $\sum_{k \neq j} S_k \eta_{kj} > \frac{1}{\varepsilon_j} \sum_{k \neq j} S_k \varepsilon_{kj}$ .



Now we want to establish the inequality between the derivatives (4) and (6), that is,  $\frac{1}{\varepsilon} \frac{1}{N} (N - \varepsilon) \frac{C'_j}{NC'} \frac{N\bar{C}' - C'_j}{NC'} < S_j \frac{1}{\varepsilon_j} (1 - S_j) (1 - \varepsilon_j + \sum_{k \neq j} \frac{S_k}{1 - S_j} \varepsilon_{kj})$ . If the homogeneous demand is price-effect equivalent and we are at the same price,  $\frac{1}{\varepsilon} = \frac{S_j}{\varepsilon_j}$ ,  $(N - \varepsilon) \frac{C'_j}{NC'} = (N - \varepsilon) \frac{1 - \varepsilon S_j}{N - \varepsilon} = (1 - \varepsilon S_j) = (1 - \varepsilon_j)$  and, since  $\sum_{k \neq j} \frac{S_k}{1 - S_j} \varepsilon_{kj} > 0$ , we only have to show that  $\frac{1}{N} \frac{N\bar{C}' - C'_j}{NC'} \leq 1 - S_j$ . Using the previous expression for  $\frac{C'_j}{NC'}$ , we can write the left hand side as  $1 - \frac{(N-1)S_j(NS_j - \varepsilon_j) + (1 - \varepsilon_j)S_j}{NS_j(NS_j - \varepsilon_j)} S_j$ . The left hand side will be smaller always that the share is not too big, specifically when  $S_j < \frac{1}{N} + \frac{N-1}{N} \varepsilon$ .  $\blacklozenge$

## 5. Inflation and market power

The relationship between market power and inflation has often worried economists, who have speculated that firms with market power may help to sustain the increase in the prices over time. However, the spread of productivity gains provides an example of why we can also have the opposite relationship. Productivity may simultaneously determine that the prices fall, or increase less than cost, at the same time that the market becomes concentrated. In this section we first develop how this can happen in a market with Cournot competition. It follows that the model could be extended to the other cases. We then show that recent increases in prices have tended to show by industries a negative relationship with increases in concentration.

### Theory

Let us assume that productivity advantages  $\omega$  of the population of a market are, in a given moment  $t$ , distributed normal around the mean  $\bar{\omega}_t$ , i.e.  $\omega \sim N(\bar{\omega}_t, \sigma^2)$ . This can be justified, for example, through the central limit theorem. As it is usual to assume, take the productivity advantages of firms resulting from Markovian processes driven by the impact of independent productivity shocks, i.e.  $\omega_t = g(\omega_{t-1}) + \xi_t$ . Productivity advantages will tend to have a normal distribution.

Approximate now marginal costs, for simplicity, as  $C' = c(w, 0) \exp(-\omega)$ . Marginal

cost becomes a lognormal variable with expectation  $E(C') = c \exp(-(\bar{\omega}_t + \frac{1}{2}\sigma^2))$ .

Suppose that this is a perfect substitutes market where firms behave Cournot. It follows that the log of the price is related to mean marginal cost as

$$\ln p = -\ln(1 - \frac{\varepsilon}{N}) + \ln c - (\bar{\omega}_t + \frac{1}{2}\sigma^2).$$

Suppose that demand elasticity is constant and the number of firms doesn't change. Inflation (the increase over time of the market price) has two determinants. First, the increase in  $c(w, 0)$ , due to the increase in the prices of the inputs, pushes the price up. Second, the decrease in marginal cost, because productivity gains, pulls it down. Notice that this second force can operate even with no modifications of the average productivity. It is enough that productivity spreads through the gains in some firms, even if it decreases in others (a mean preserving change in the variance).

The consequence is that, with Cournot exercise of market power, the increase or the spread of productivity of the firms -affecting or not average productivity-, will tend to decrease the output price (or to reduce the increase induced by input price inflation).

Productivity will affect simultaneously the concentration of the market. The relative marginal cost of a firm can be written as  $\frac{C'}{E(C')} = \exp[-(\omega - (\bar{\omega}_t + \frac{1}{2}\sigma^2))] = \exp(-\omega^*)$ . We will call  $\omega^*$  to the relative productive advantage of the firm. The market share of a firm is  $S^Q = \frac{1}{\varepsilon} - (\frac{1}{\varepsilon} - \frac{1}{N}) \exp(-\omega^*)$ . Note that if  $\omega^* = 0$  (the productivity advantage equals mean productivity), the share of the firm is just the average market share. Let us discuss what happens when there is a change in the distribution of  $\omega^*$ .

The share  $S^Q$  is a positive monotonic function of  $\omega^*$ . Call the distribution functions of  $\omega^*$  and  $S^Q$ ,  $G(\omega^*)$  and  $F(S^Q)$  respectively. Since  $S^Q$  is a monotonic transformation of  $\omega^*$ , the quantiles of  $F(S^Q)$  are monotonic transformations of the quantiles of  $G(\omega)$ . It follows that, if we consider a change of the distribution of  $\omega$  from  $G(\omega)$  to  $G'(\omega)$ , where  $G'(\omega)$  stochastically dominates  $G(\omega)$ , we will have also a new distribution

$F'(S^Q)$  that stochastically dominates  $F(S^Q)$ . A consequence is that

$$CR'_k - CR_k = \int_{Q_S(\tau)}^1 S^Q dF'(S^Q) - \int_{Q_S(\tau)}^1 S^Q dF(S^Q) \geq 0,$$

where  $CR_K$  is the concentration ratio for the first  $K$  firms, and  $Q_S(\tau)$  is the quantile  $\tau = 100\frac{N-K}{N}\%$  in the distribution of  $S^Q$ .<sup>7</sup>

Summarizing, if the evolution of productivity is such that the relative productive advantages  $\omega^*$  do not vary, the distribution of market shares will not change. For example, suppose that all productivities increase in  $g$  percentage points, i.e.  $\omega' = \omega + g$ . However, if some firms improve productivities more than others, raising a stochastically dominant new distribution of relative productivity advantages, the market is going to become more concentrated in the sense that some concentration ratios are going strictly to increase.

### **Empirics**

We want to relate price variation and changes in concentration. The economic censuses compute concentration ratios by industries of US manufacturing. The data for 2002, 2007, 2012 and 2017 are publicly available for the NAICS classification of industries.<sup>8</sup> We combine these data with the output prices estimated for the manufacturing NBER-CES data base.<sup>9</sup> As we employ the NBER-CES version for NAICS 1997, the use of the concentration ratios without further work matches a decreasing number of industries. However, our objective here is only a quick look and we do not think that this impedes to transmit the main message.<sup>10</sup> We are able to match to the NBER-CES database 468, 466, 298 and 290 industries of the different census years,

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<sup>7</sup>See Hart (1975).

<sup>8</sup>See, for example, Kulick and Card (2022). These authors guided me in accessing the census information and graciously facilitated their files to check mines.

<sup>9</sup>See Becker, Gray and Marvakov (2021). This data base aggregates the census results in six-digit manufacturing industries defined according with the NAICS classification of 1997.

<sup>10</sup>Kulick and Card (2022) argue, however, that the omission of industries is not neutral, favoring concentration over time.

respectively.

The average value of the 4-firms concentration ratio is 0.43, and for the 20-firms concentration ratio about 0.71. Both means show almost no changes over the years, and this despite the changing number of industries with which these means are computed. Of course, this is compatible with rich movements in both directions of the individual indices across industries. We are able to work with a total of 1039 5-years changes in prices and concentration, after dropping 15 changes in concentration outside the  $[-0.50, 0.50]$  interval. We annualize the log changes in prices dividing by 5, and we keep untouched the 5-years differences in concentration. The coefficients of regression can be interpreted as elasticities and read as a  $10X$  annual percentage change in the price for a 10% change in concentration.

Table 2 summarizes a few results. Column (1) reports the regression of the changes in prices on the change of joint share of the first 4 firms ( $\Delta CR_4$ ) and the change of the share of the next 16 firms ( $\Delta CR_{next16} = \Delta CR_{20} - \Delta CR_4$ ). The results are mixed and not fully significant. Concentration of the top four firms tends to raise prices but the share of the next 16 firms tends to decrease them.

However, all the important things in concentration happen in the interplay between these two shares: in 91.5% of the cases at least one of these shares increases, in 51.9% is the joint share of the 16 that does, and in 16% both of them. We hence select the more than half of the cases in which the joint share of the next 16 firms increases. Column (2) shows the significant association to price reductions (or more moderate increases) of the increases of the joint share of the next 16 firms. Column (3) makes clear that what happens in these cases with the top firms is nonsignificant.

The data point out to rich patterns of change (only 16% of the changes seem to be stochastically dominant), but there are unequivocal with respect to the association of many increases in concentration with a better behavior of prices. The topic deserves further investigation.

## 6. Market power and efficiency

Two papers claim similar interrelated facts, for manufacturing and other industries, in a period that approximately spans 1980-2016. De Loecker, Eckhout and Unger (2020) look directly at the evolution of markups in the US. Autor, Dorn, Katz, Patterson and Van Reenen (2020) look at the fall of the labor shares of revenue in the US and other OECD countries. For them, labor shares fall because are a decreasing function of markups. These are the joint stylized facts that emerge from both papers: first, average markup (sum of markups weighted by sales shares) rises while aggregate labor share in income falls; second, there is an important reallocation component in both the rise of the average markup and the fall of the aggregate labor share; third, the process is driven by the biggest firms ("superstar" firms in the second paper); fourth, there is simultaneously concentration of sales.

Let us simulate a sample of firms whose productivity grows through a sequence of Hicks-neutral and labor-augmenting productivity shocks. Firms produce with a three input CES production function with elasticity of substitution  $\sigma = 0.7$ . The productivity factors  $\exp(\omega_H)$  and  $\exp(\omega_L)$  multiply respectively the entire input aggregator and the labor input. The productivity terms  $\omega_H$  and  $\omega_L$  are inhomogeneous autoregressive Markovian processes of parameter 0.8 and normal random innovations. We assume them independent for simplicity. During the 30 periods that we examine, Hicksian productivity grows at an average of 1.2 percentage points and the output effect of labor-augmenting productivity at an average of 2.2 percentage points.

Firms face identical isoelastic demands of elasticity  $\eta = 6$  and play Bertrand setting the price for their products by multiplying the short-run marginal cost by a markup  $\frac{\eta}{\eta-1} = 1.2$ .<sup>11</sup> Because productivity shocks have a random component, firms experi-

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<sup>11</sup>The level of capital is optimally adjusted each period according to the demand for the product and current input prices.

ence heterogeneous marginal cost reductions and, since they pass these reductions on prices, they experience different demanded quantities and growth. Firms in fact have started completely identical ten periods before we begin to compute their evolution.

Table 2 summarizes a typical result. The average firm size more than doubles, but employment grows much more moderately. The aggregate labor share falls, but there is an important reallocation component because the firms that experience greater labor augmenting productivity both reduce their labor share and grow more. The concentration as measured by the *CR10* of sales is big and grows.

Despite firms hold exactly the same market power from the beginning to the end, its measurement with De Loecker and Warzynski (2012) method to compute markups yields an spectacular increase, with an important reallocation component (the difference between the weighted and unweighted means).<sup>12</sup> Markups and their increase are greater in the 90th percentile. The numbers are roughly similar to what the mentioned papers find for the US in the years 1980-2016.

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<sup>12</sup>Individual De Loecker and Warzynski (2012) markups are typically computed as  $\hat{\mu}_{jt} = \hat{\beta}_X / S_{Xjt}^R$ , where  $\hat{\beta}_X$  is the estimate of the elasticity of a variable input  $X$  and  $S_{Xjt}^R$  the observed share of the input cost on revenue. Any rigid estimate  $\hat{\beta}_X$  will create a positive growth bias because of the evolution downwards of  $S_{Xjt}^R$  under LAP. In addition, cost minimization implies that, at any moment of time, there is cross-section distribution of elasticities  $\beta_{Xjt}$ , whose variation is going to be systematic under labor-augmenting productivity (Doraszelski and Jaumandreu (2019)). The estimate for the average markup is  $\hat{\mu}_t = \sum_j w_{jt} \hat{\mu}_{jt}$ , where  $w_{jt} = R_{jt} / \sum_j R_{jt}$  are revenue weights. Using labor shares of variable cost it can be written

$$\hat{\mu}_t = \mu_t + \sum_j \frac{w_{jt}}{S_{Ljt}} (\hat{\beta}_L - \beta_{Ljt}) \mu_{jt},$$

where  $S_{Ljt}$  is labor share in variable cost. The expectation of the second term of the right hand side is likely to introce huge positive biases.

## 7. Concluding Remarks

This paper explores the role of productivity in the determination of the market shares of firms, given different behavior. Our discussion is valid for firms that show two types of productivity advantages over their competitors: Hicks neutral and biased productivity in the form of LAP. Productivity advantages decrease marginal costs, impacting the shares of firms and hence the structure of the market depending on how is competition. We show that the more competitive is behavior, the stronger is the impact of productivity on the shares.

Price competition with perfectly substitutable products determines a radical bound: the firm with a productivity advantage finds optimal to fight for the entire market at the expense of the rivals. However, the optimal action becomes less aggressive with product differentiation and quantity competition. In the other extreme, represented by Cournot, market shares are related to the productivity advantages, but in a more subdued way than when competition is sharper. We have shown that, all the rest equal, productivity advantages determine the distribution of market shares and its changes, with an intensity that depends on the game the firms play.

This may seem an obvious consideration, but practical analyses seem to often forget the implications of this link between productivity growth and market structure and concentration. We briefly comment two applications of our results. The first is about the relationship between inflation and market power. We have shown that it can be a negative relationship between inflation and concentration and that, in fact, milder behavior of US manufacturing prices during the 2000's are clearly related to the concentration of many industries driven by the firms that are not the highest top of the distribution.

With the second application we raise an alternative explanation to the story that argues a recent increase of concentration and market power in US manufacturing

and other sectors, with sharp rise of markups, accompanied by falls of the labor share in revenue. A key of these findings is the measurement of the markups using as denominator the labor share, what confounds the level of efficiency and market power and sets the base for an aggregation bias of the individual-firm markups. A simple simulation shows that the same results may come from a market with both LAP and Hicksian productivity growth, where market power doesn't increase except if inappropriately measured, at the same time that concentration rises and the labor share falls.

These examples make a strong case for the development of the analysis of the recent growth in productivity, often characterized by important biases towards labor, and the consequences of the technological change via productivity on the firm shares, market structure and concentration. In particular, a strong unequal diffusion of productivity is likely to raise temporary firms asymmetries that it is important to understand and address properly in economic policy.



## Appendix

*Proof* Use the definition to write  $-\frac{\partial Q}{\partial p} = -\frac{\partial q_j}{\partial p_j}(1 + \sum_{k \neq j} \frac{\partial q_k}{\partial p_j} / \frac{\partial q_j}{\partial p_j})$ . On the other hand, differentiate the identity  $p_j = P_j(q(p))$  to get  $1 = \frac{\partial p_j}{\partial q_j} \frac{\partial q_j}{\partial p_j} + \sum_{k \neq j} \frac{\partial p_j}{\partial q_k} \frac{\partial q_k}{\partial p_j}$ , and hence  $-\frac{\partial q_j}{\partial p_j} = \frac{1 - \sum_{k \neq j} \frac{\partial p_j}{\partial q_k} \frac{\partial q_k}{\partial p_j}}{-\frac{\partial p_j}{\partial q_j}}$ . It follows that  $\frac{1}{-\frac{\partial p}{\partial Q}} = \frac{(1 - \sum_{k \neq j} \frac{\partial p_j}{\partial q_k} \frac{\partial q_k}{\partial p_j})(1 + \sum_{k \neq j} \frac{\partial q_k}{\partial p_j} / \frac{\partial q_j}{\partial p_j})}{-\frac{\partial p_j}{\partial q_j}}$  or  $\frac{-\frac{\partial p_j}{\partial q_j}}{-\frac{\partial p}{\partial Q}} \leq 1$  if the product of the numerator is equal or less than one. Equivalently, we need  $\sum_{k \neq j} \left(-\frac{\partial p_j}{\partial q_k}\right) \frac{\partial q_k}{\partial p_j} < \sum_{k \neq j} \left(-\frac{1}{\frac{\partial q_j}{\partial p_j} + \sum_{k \neq j} \frac{\partial q_k}{\partial p_j}}\right) \frac{\partial q_k}{\partial p_j}$ , and a sufficient condition is that the weights of the left are smaller than the weights of the right. Use the previous differentiation to write  $1 = -\frac{\partial p_j}{\partial q_j} \left(-\frac{\partial q_j}{\partial p_j} - \sum_{k \neq j} \theta_{kj} \frac{\partial q_k}{\partial p_j}\right)$ , where  $\theta_{kj} = \frac{\partial p_j}{\partial q_k} / \frac{\partial p_j}{\partial q_j} < 1$ . This implies that  $-\frac{\partial p_j}{\partial q_j} < -\frac{1}{\frac{\partial q_j}{\partial p_j} + \sum_{k \neq j} \frac{\partial q_k}{\partial p_j}}$  and hence that  $-\frac{\partial p_j}{\partial q_k} < -\frac{1}{\frac{\partial q_j}{\partial p_j} + \sum_{k \neq j} \frac{\partial q_k}{\partial p_j}}$ . ♦

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Table 1:

Competition and Market Power outcomes<sup>a</sup>

Products are

		Perfectly Substitutable	Differentiated
Firms choose	Prices	B: $\frac{p_j - C'_j}{p_j} \simeq \frac{\omega_j - \omega_k}{1 - \omega_k}$	B with PD: $\frac{p_j - C'_j}{p_j} = \frac{1}{\eta_j}$
	Quantities	C: $\frac{p - C'_j}{p} = S_j \varepsilon$	C with PD: $\frac{p_j - C'_j}{p_j} = \varepsilon_j$

<sup>a</sup>  $\eta_j = -\frac{p_j}{q_j} \frac{\partial D_j}{\partial p_j}$ ,  $\varepsilon = -\frac{Q}{p} \frac{\partial P}{\partial Q}$ , and  $\varepsilon_j = -\frac{q_j}{p_j} \frac{\partial P_j}{\partial q_j}$ .

Table 2: Effects of concentration in price, 2002-2017.

Yearly average price change regressed on the variation in concentration ratios.<sup>a</sup>

Variables	All changes (1)	$\Delta CR_{next16} > 0$ (2)	$\Delta CR_{next16} > 0$ (3)
Constant	0.028 (0.001)	0.033 (0.002)	0.034 (0.002)
Dummy 07-12	-0.006 (0.002)	-0.008 (0.003)	-0.009 (0.003)
Dummy 12-17	-0.006 (0.002)	-0.029 (0.003)	-0.029 (0.003)
$\Delta CR_4$	0.022 (0.020)	0.276 (0.277)	
$\Delta CR_{next16}$	-0.039 (0.027)	-0.110 (0.048)	-0.142 (0.036)
Observations	1039	548	548
$R^2$	0.123	0.153	0.151

<sup>a</sup> Computed for subperiods 2002-2007, 2007-2012, and 2012-2017, for 451, 298 and 290 industries respectively.

Table 3  
Increasing market power as an artifact of improper measurement  
(Simulation results for a sample of firms with constant markups but labor-augmenting productivity)<sup>a</sup>

Variable	Simulation <sup>b</sup>		Total change
	Period 1	Period 30	
(1)	(2)	(3)	(4)
Average revenue (index)	1.000	2.244	1.244
Average employment (index)	1.000	1.649	0.649
DLW markups <sup>c</sup> :			
Weighted mean	1.376	1.901	0.525
Unweighted mean	0.970	1.188	0.218
90th weighted percentile	1.633	2.226	0.593
Real markup <sup>d</sup>	1.200	1.200	0.00
Labor share in revenue <sup>e</sup> :			
Aggregate labor share	0.222	0.174	0.048
Unweighted labor share	0.315	0.281	0.034
Concentration of sales (CR10) <sup>f</sup>	0.282	0.341	0.059

<sup>a</sup> Sample of 1000 firms with identical CES production functions with  $\sigma = 0.7$ , which produce to serve identical constant elasticity demands of absolute elasticity value  $\eta = 6$ . Firms experience over time Hicks-neutral and Labor-augmenting productivity shocks and set price with a constant margin over marginal cost.

<sup>b</sup> Firms start equal and we report as Period 1 their values after 10 periods of simulation.

<sup>c</sup> Computed as labor input elasticity divided by the firm's share of labor in revenue. Weighted with revenue weights.

<sup>d</sup>  $\frac{\eta}{\eta-1} = \frac{6}{5} = 1.2$ .

<sup>e</sup> Cost of labor over revenue. Weighted with revenue weights.

<sup>f</sup> Three years moving averages.

Figure 1

Best response changes with an increase in productivity

