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(11/19/24)

ENG ME/EC/SE 501:

Exercises (Set 7) (Due 11/26/24)

1. Consider the following single input system

$$\dot{x} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u.$$

(a) Show that the uncontrolled system (i.e. $u \equiv 0$) is unstable.

(b) Where should you put the poles to ensure that the 2% settling time is less than 1 second and the overshoot is less than 1%. Recall that the 2% setting time is approximately $4/(\zeta\omega_0)$ and overshoot is $\exp(-\pi\zeta/\sqrt{1-\zeta^2})$.

(c) Design a feedback controller to achieve your specifications.

2. Consider the system

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} s & 0 & 0 \\ 0 & t & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} u \qquad \text{(S)}$$

$$y = x_1 + x_2 + x_3.$$

(i) State necessary and sufficient conditions on the system parameters for (S) to be observable.

(ii) Design a full-state observer,

$$\dot{z} = Az + E(y - Cz) + Bu,$$

such that the error components $z_i - x_i$ approach 0 like e^{-t}.

3. Suppose that the partitioned system

$$\begin{pmatrix} \dot{w}(t) \\ \dot{y}(t) \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} w(t) \\ y(t) \end{pmatrix}$$

with output $y(t)$ is observable. Show that $\{A_{11}, A_{21}\}$ is an observable pair.