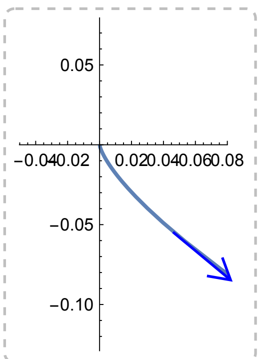
Nyquist Example

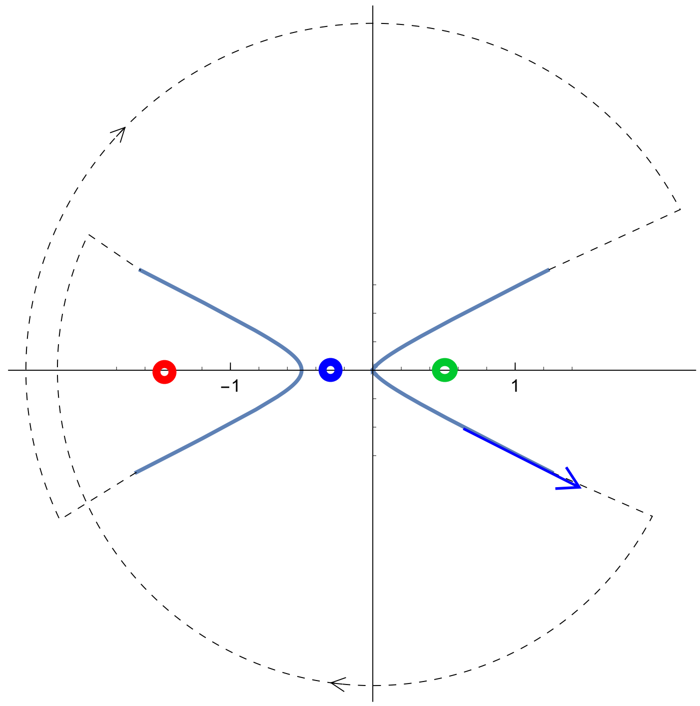
This example illustrates some of the complexity in interpreting Nyquist plots. Consider

$$g[s] = \frac{1}{s^3-2*s^2+s-2}.$$

The denominator can be factored: (-2+s) $(1+s^2)$, and thus g[s] has a single right halfplane pole. The easiest way to plot the Nyquist locus -- that would be plotting g[$i\omega$] in the complex plane letting ω run from -∞ to +∞ -- is to make the denominator real, giving the parametric representation $\left(\frac{2}{-4+3\,\omega^2+\omega^4}, \frac{\omega}{-4+3\,\omega^2+\omega^4}\right)$. The first element of the ordered pair is the real part of g[iω] and the second element is the imaginary part. As the value is increased from -∞, the Nyquist locus moves from (0,0) into the lower right quadrant of the plane:



It continues and eventually traces the complete Nyquist locus:



Obviously, for more than the first order example we gave in class, it is much easier to use software, but this example illustrates how clever people had to be in the early days. Now the theorem states that since g[s] has a single pole in the rhp, 1+kg[s] will have 1+ρ pole in the right halfplane, where ρ = number of times the Nyquist plot encircles -1/k. We consider three possible placements for -1/k -- the red dot, the blue dot, and the green dot. These correspond to -1/k < -1/2, -1/2 < -1/k < 0, and -1/k>0. The encriclement is 2-times for -1/k < -1/2, meaning that for these values of k, (0<k<2) there are three (3) rhp poles. For -1/2<-1/k<0, there is one encirclement, meaning there are two (2) rhp poles. There are no encirclements of the green dot, and hence g has only a single pole in the rhp.

Free Hand Nyquist Plot

$$g(s) = \frac{1}{s^3 - 2s^2 + s - 2}$$