Fifty Years of Information Based Control Theory

The 23rd IEEE Control Systems Society
Hendrik W. Bode Lecture

John Baillieul, Keyong Li, Dimitar Baronov, W.S. Wong
Boston University and Chinese University of Hong Kong

Twenty-two years of Bode Lectures

2010 Manfred Morari	1999 Graham C. Goodwin
2009 Peter Caines	1998 J. Boyd Pearson
2008 Christopher I. Byrnes	1997 Edward J. Davison
2007 P.S. Krishnaprasad	1996 Jurgen Ackermann
2006 Arthur J. Krener	1995 Bob Narendra
2005 Pravin Varaiya	1994 Gene Franklin
2004 Tamer Basar	1993 Michael Athans
2003 Lennart Ljung	1992 Brian D.O. Anderson
2002 Eduardo D. Sontag	1991 Petar V. Kokotovic
2001 Alberto Isidori	1990 David Luenberger
2000 Mathukumalli Vidyasagar	1989 Gunther Stein

More than twenty-two years of inspirational interactions are gratefully acknowledged

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D. Liberzon, L. Bushnell, S.K. Mitter, S. Tatikonda, A. Sahai, N. Elia, G.C. Goodwin, R. D'Andrea, B. Sinopoli, C. Langbort, F. Fagnani, S. Zampieri, S.S. Sastry, R.M. Murray I.M.Y Mareels, W. Moran, P. Varaiya, D. Tilbury, T. Samad, J.P. Hespanha, R.M. Murray, J. Doyle, M. Dahleh, P.S. Krishnaprasad, N.C. Martins, P. Antsaklis, T. Basar, R. Middleton, Steve Morse, R. Tempo, Brian Anderson

Outline:

- Setting the stage
- Information and control
- Control with communication constrained feedback channels
- Control systems as communication channels
- The effort required to communicate
- A brief excursion into *control communication complexity*
- The future

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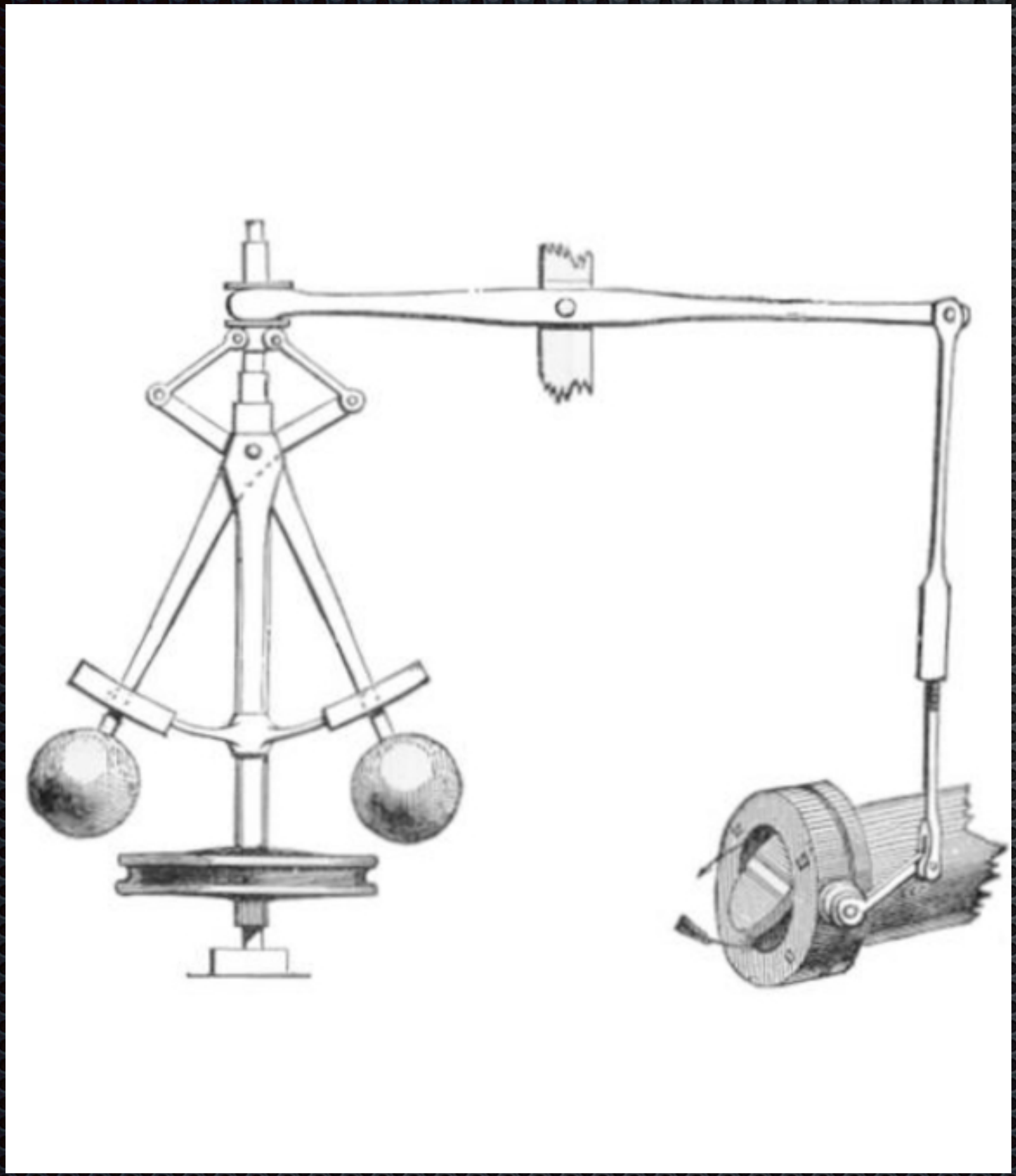
Setting the stage: J.C. Maxwell and Control



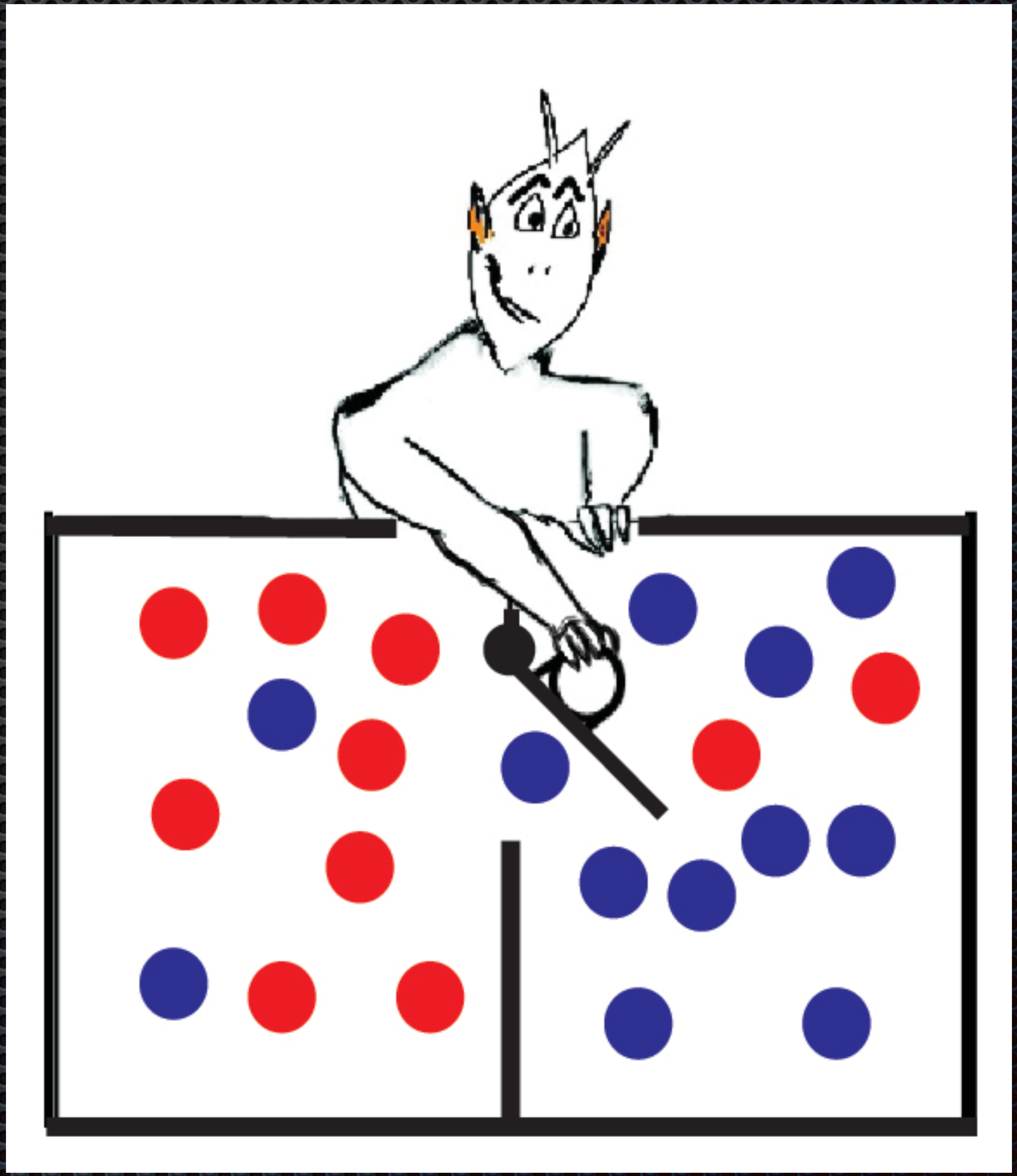
Classical feedback control:

"On Governors,"
Proc. R. Soc. Lond. 1867, Vol. 16: pp. 270-283; doi: 10.1098/rspl.1867.0055

Setting the stage: J.C. Maxwell and Control

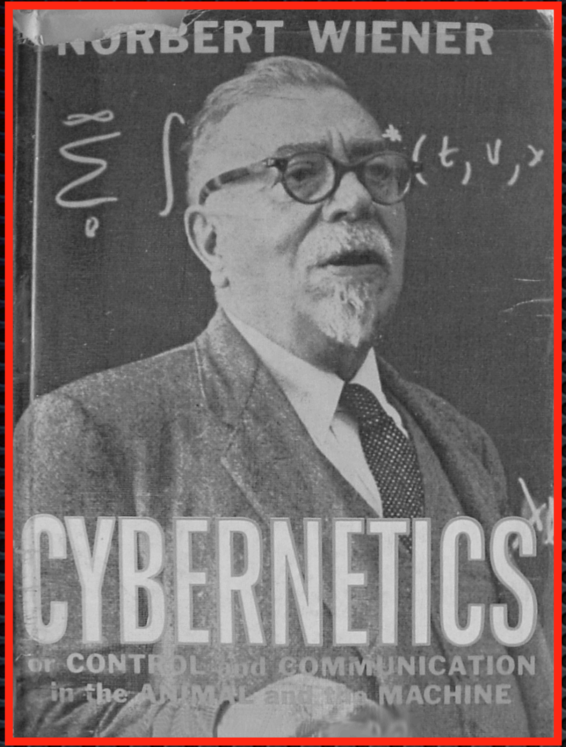
Information based control:

Theory of Heat, Longmans, Green, and Co., London, New York, Bombay, 1902.



Download: http://www.archive.org/details/theoryofheat00maxwrich

1948: The miracle year of information engineering

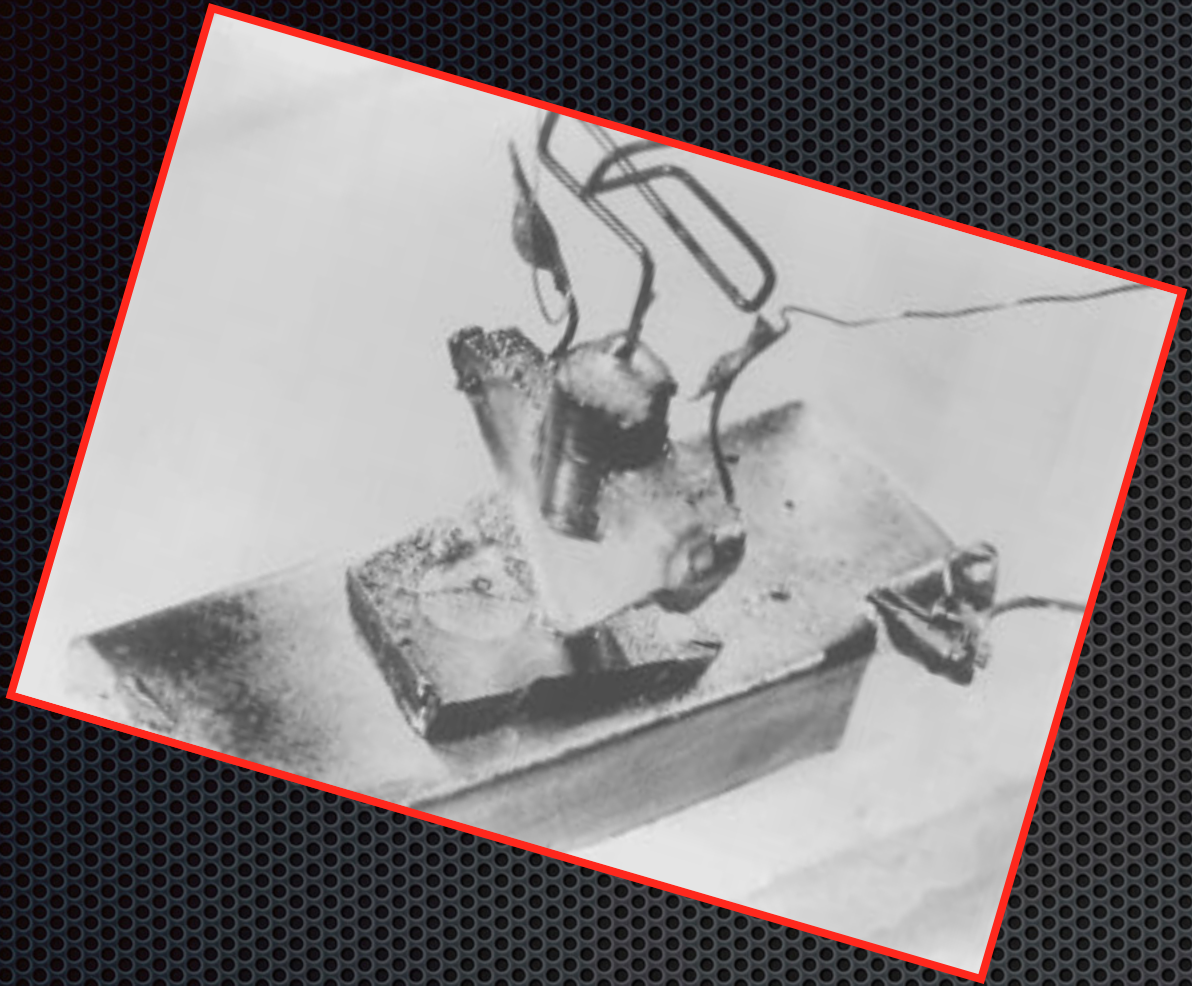


Cybernetics; or Control and Communication in the animal and the machine, Wiley, 1948.

“When I first wrote Cybernetics, the chief obstacles which I found in making my point were that the notions of statistical information and control theory were novel and perhaps even shocking to the established attitudes of the time.”

--Wiener, Preface to the Second Edition

1948: The miracle year of information engineering



"At the suggestion of J. R. Pierce the device was designated the *transistor*; it was disclosed to a meeting of the BTL Research Department Technical Staff in the auditorium at Murray Hill on June 22, 1948."

- W.S. Gorton, "The Genesis of the Transistor," *Proc. of the IEEE*, Vol. 86, No. 1, Jan. 1998, pp. 50-52.

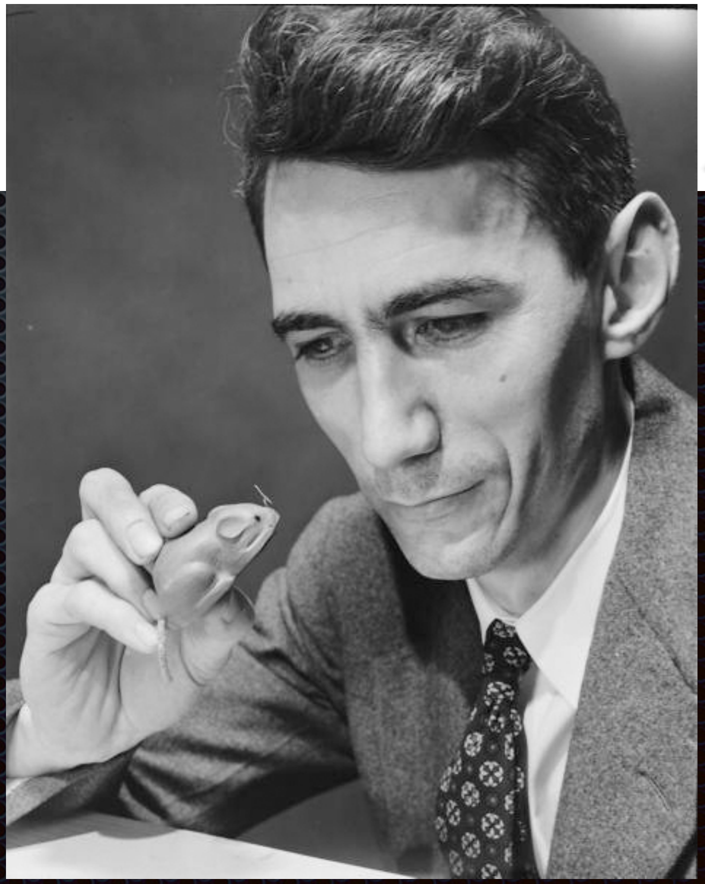
1948: The miracle year of information engineering

The Bell System Technical Journal

Vol. XXVII *July, 1948* *No. 3*

A Mathematical Theory of Communication

By C. E. SHANNON



“The fundamental problem of com-munication is that of reproducing at one point… a message selected at another point.”

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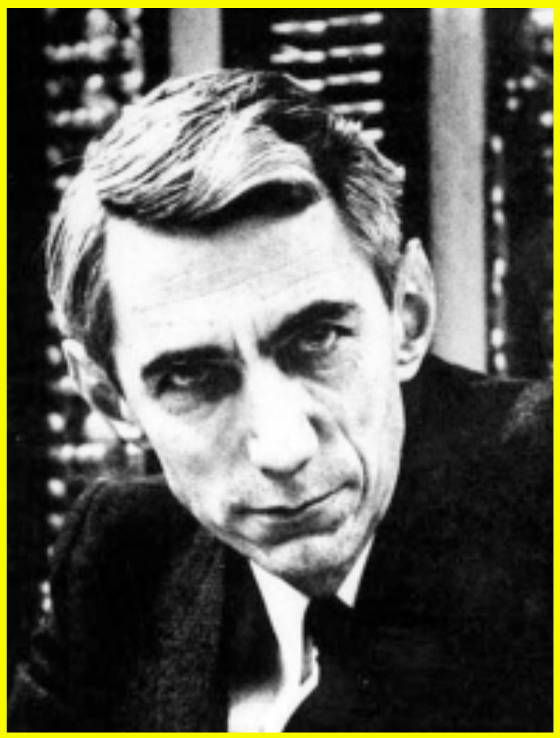
- The future

Early connections:

At its founding, the IRE's Professional Group on Automatic Control stated its Field-of-Interest:

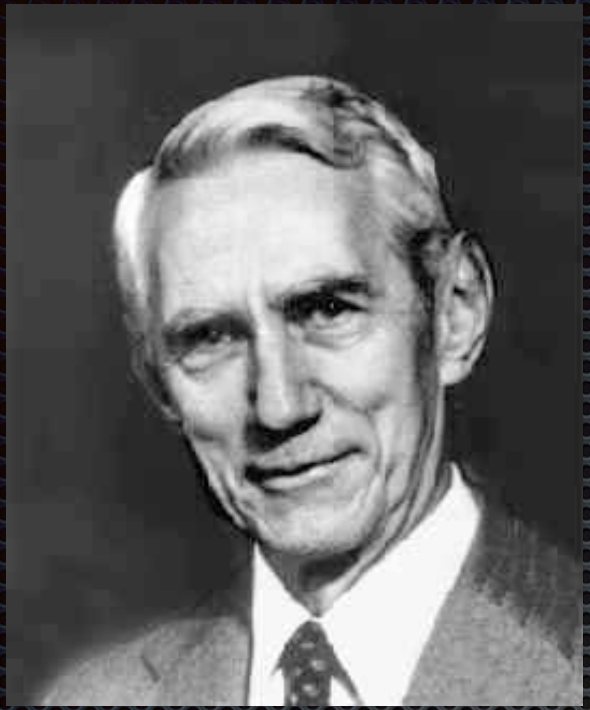
"The field of interest of the Group shall be automatic control systems. It shall encompass the components thereof, such as transducers, data transmission links, computers and control devices and the integration of these components into control systems."

Early connections:



H.W. Bode and C.E. Shannon, "A Simplified Derivation of Linear Least Square Smoothing and Prediction Theory," H.W. Bode and C.E. Shannon, in *Proceedings of the IRE*, Vol. 38:4, pp.417 - 425, DOI: 10.1109/JRPROC.1950.231821

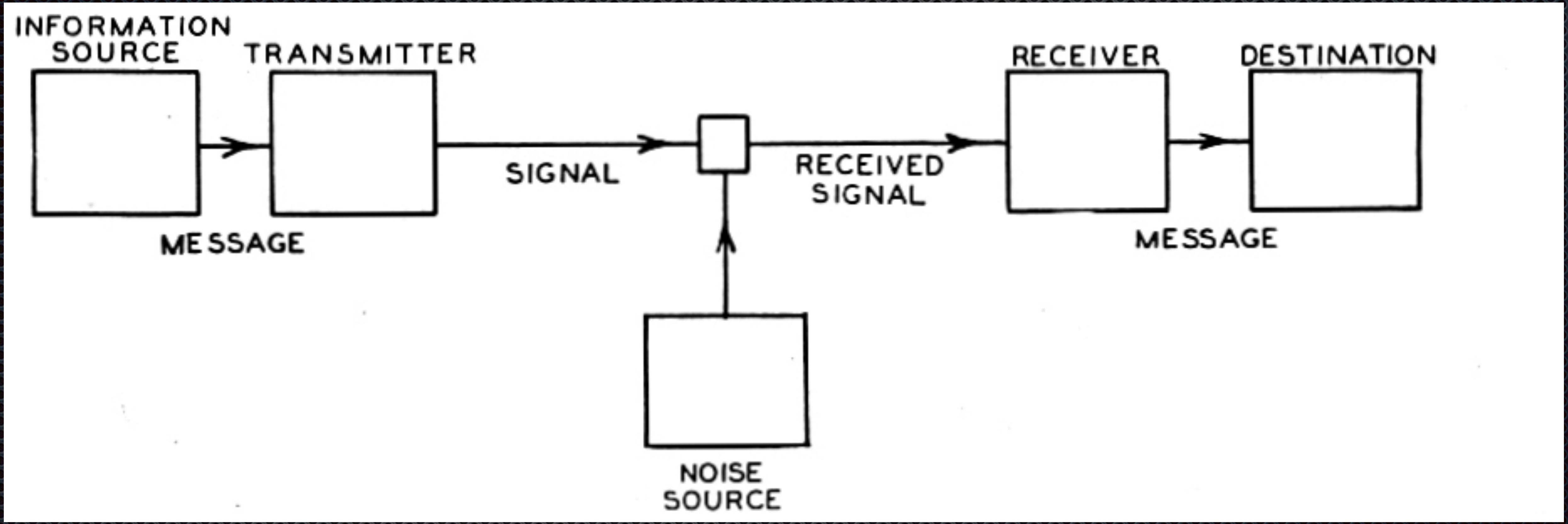
A broad view of communication



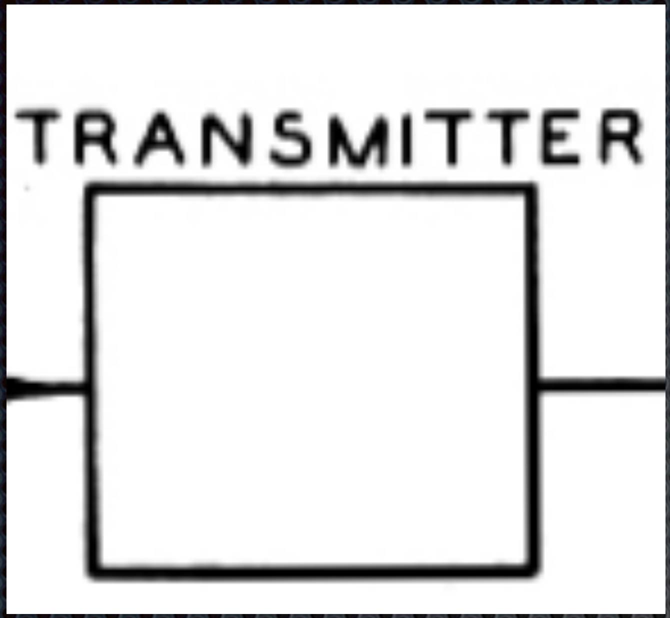
C.E. Shannon and W. Weaver, 1949. *The Mathematical Theory of Communication*, The University of Illinois Press, Urbana.

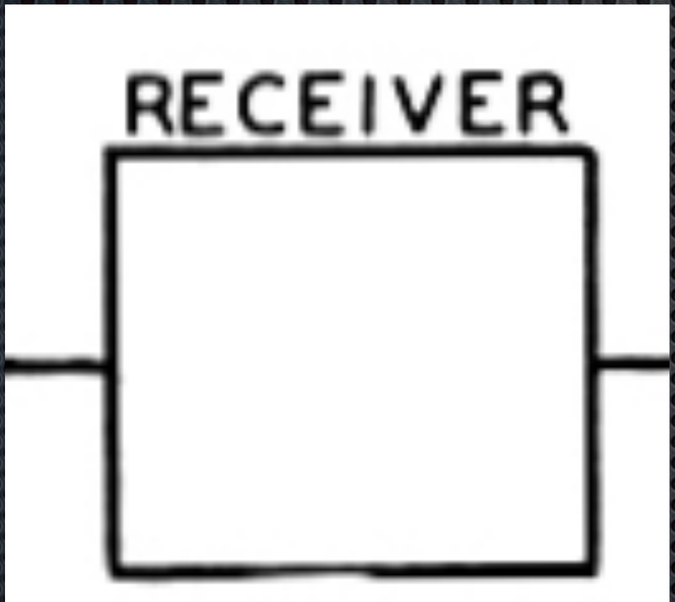
"... communication involves not only written and oral speech, but also music, the pictorial arts, the theatre, the ballet, and in fact all human behavior."

A general communication system - according to Shannon



The Transmitter/Receiver

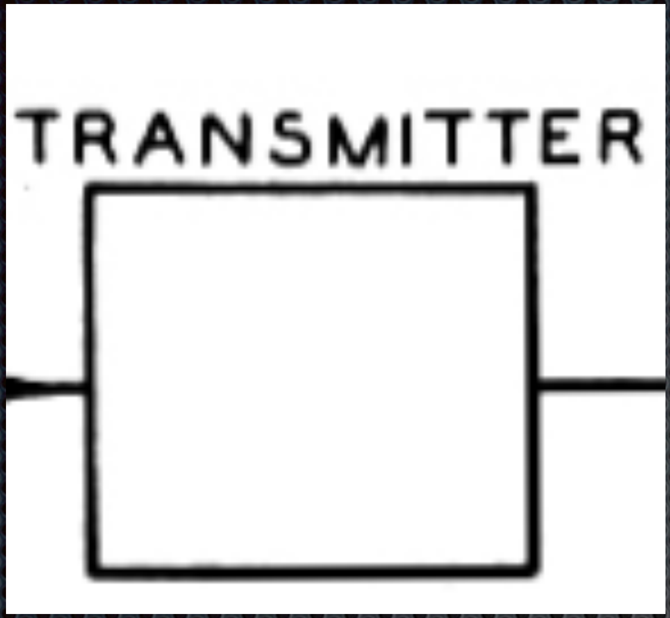


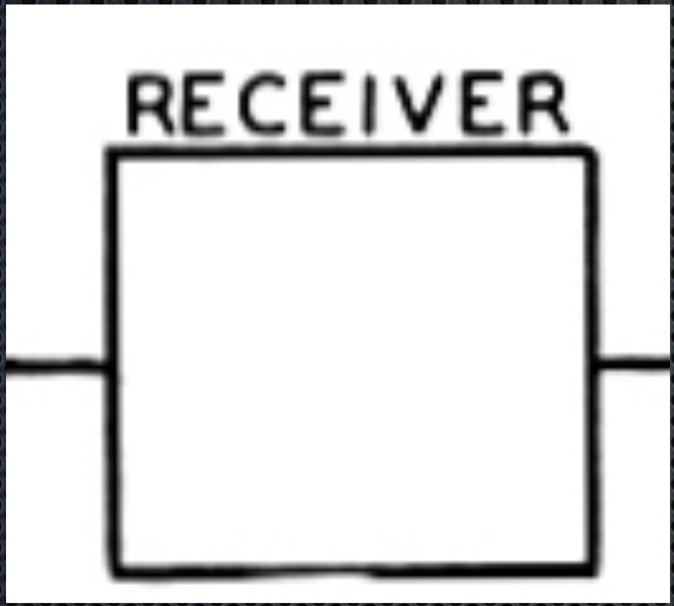


The transmitter and receiver are discrete transducers having a finite number of states. The output is a function of the present state and the present input symbol:

$$y_n = f(x_n, \alpha_n)$$
$$\alpha_{n+1} = g(x_n, \alpha_n)$$

The Transmitter/Receiver





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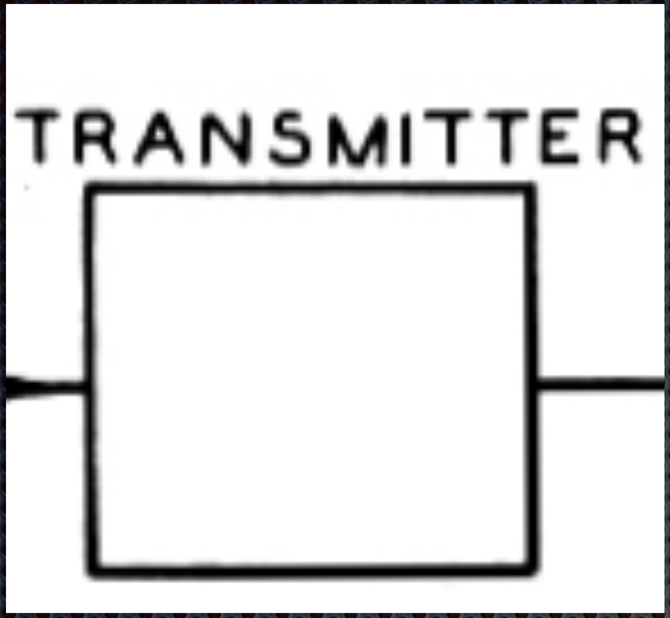
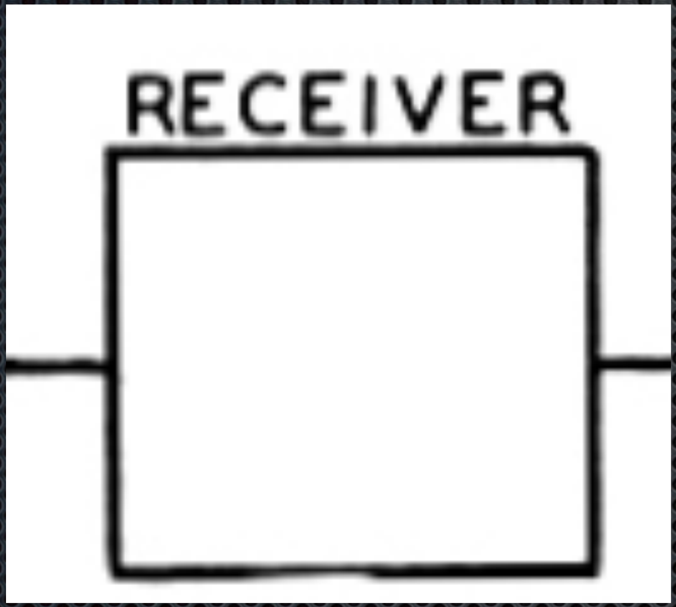


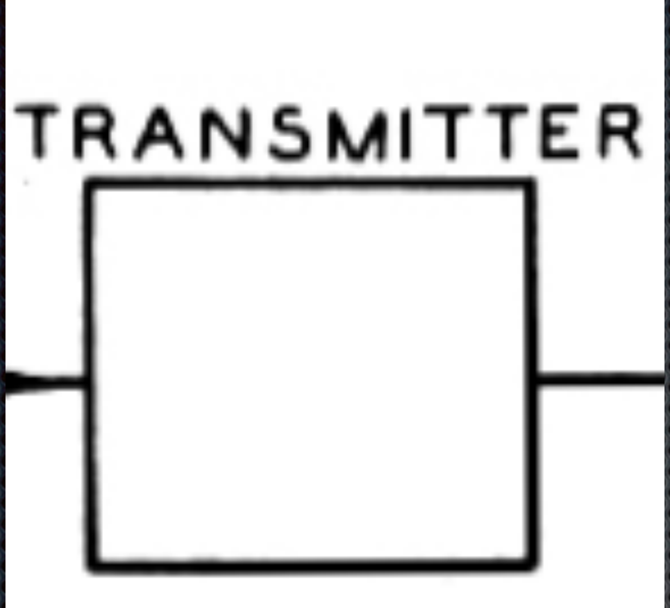


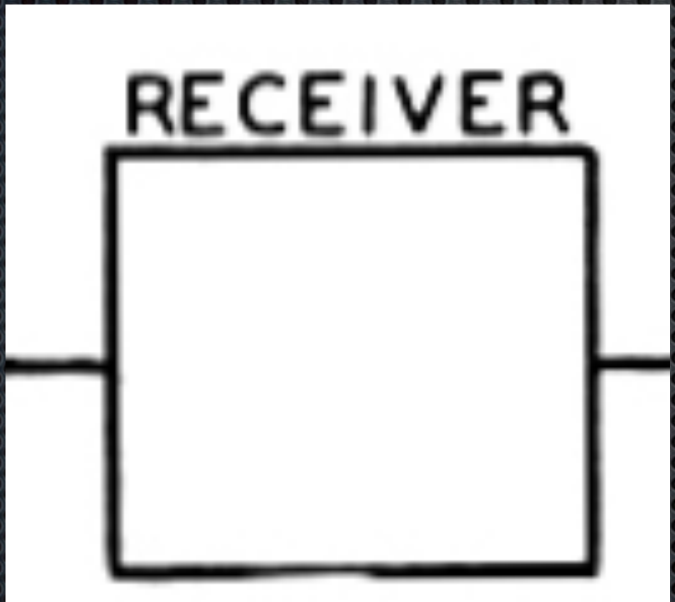
The transmitter and receiver are discrete transducers having a finite number of states. The output is a function of the present state and the present input symbol.

$$x_{n+1} = f(x_n, u_n)$$

$$y_n = h(x_n, u_n)$$

The Transmitter/Receiver





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----C.E. Shannon, "A Mathematical Theory of Communication," The Bell System Technical Journal, XXVII, No. 3, July, 1948, pp. 379-423.

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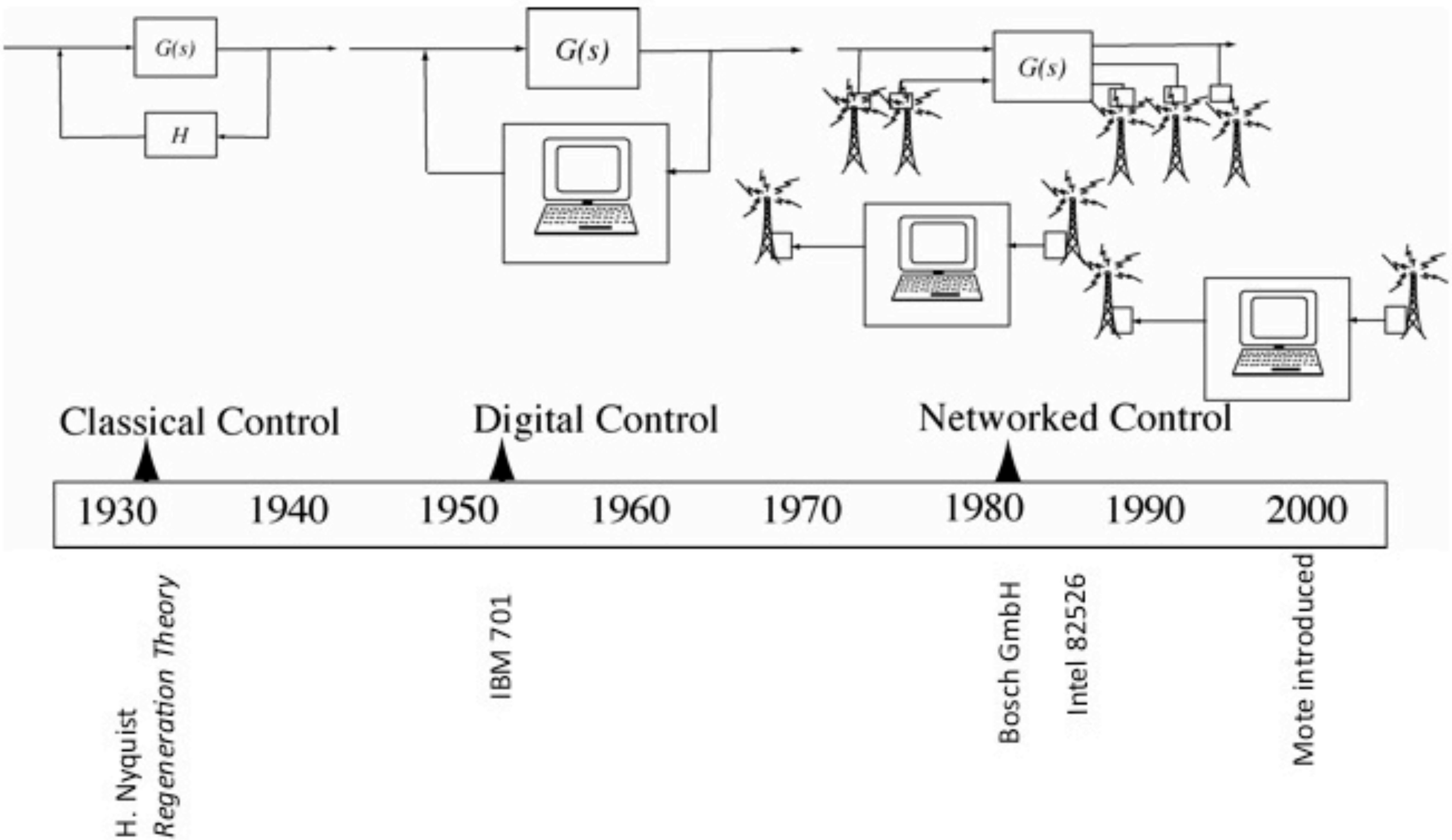
Control with communication



- *Proceedings of the IEEE*, 95:1, 2007, pp. 9-28.

Control with communication constrained feedback channels

D.F. Delchamps, The stabilization of linear systems with quantized feedback, in *Proc. 27th IEEE Conf. Decision and Control*, Dec. 7–9, 1988, pp. 405–410, Digital Object Identifier 10.1109/CDC.1988.194341.

D.F. Delchamps, Controlling the flow of information in feedback systems with measurement quantization, in *Proc. 28th IEEE Conf. Decision Control*, Dec. 13–15, 1989, pp. 2355–2360, Digital Object Identifier 10.1109/CDC.1989.70595.

D.F. Delchamps, Stabilizing a linear system with quantized state feedback, *IEEE Trans. Autom. Control,* vol. 35, no. 8, pp. 916–924, Aug. 1990.

Control with communication constrained feedback channels

W.S. Wong

R.W. Brockett

W.S. Wong & R.W. Brockett, 1995, 1999, "Systems with finite communication bandwidth constraints, II: Stabilization with limited Information Feedback" *IEEE Trans. AC*, May, 1999.

The Data-rate Theorem

G.N. Nair and R.J. Evans, “Stabilization with data-rate-limited feedback: Tightest attainable bounds,” *Sys. Control Lett.*, vol. 41, no. 1, pp. 49–56, Sep. 2000.

The Data-rate Theorem

Theorem: Suppose a system $G(s)$ is controlled using a data-rate constrained feedback channel. Suppose, moreover, G has k right half-plane poles $\lambda_1,\ldots,\lambda_k$. Then there is a critical data-rate

$$R_c = \log_2 e \cdot (\mathrm{Re}(\lambda_1) + \cdots + \mathrm{Re}(\lambda_k))$$

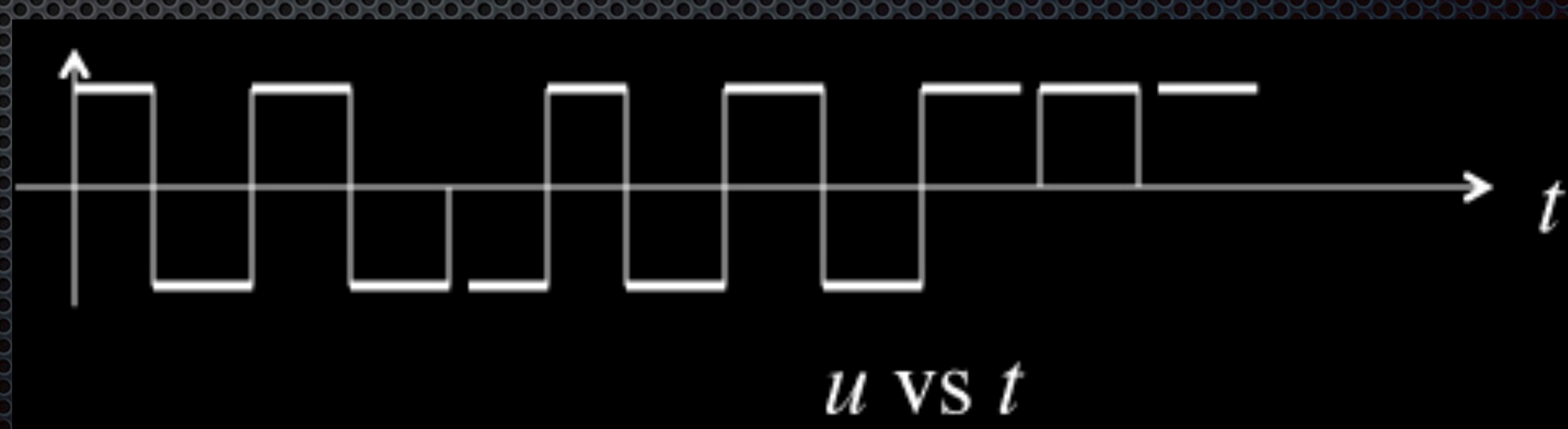
such that the system can be stabilized if and only if the channel capacity $R > R_c$

Nair, Evans, Mitter, Tatikonda, Zampieri, Fagnani, Liberzon, Brockett, John B., K. Li, Savkin, Matveev, ...

The Data-rate Theorem

- Jerk the cart left or right one centimeter
- Under what circumstances can one keep the pendulum upright using this very coarse type of "control?"

Ans: If and only if the jerk rate is fast enough.



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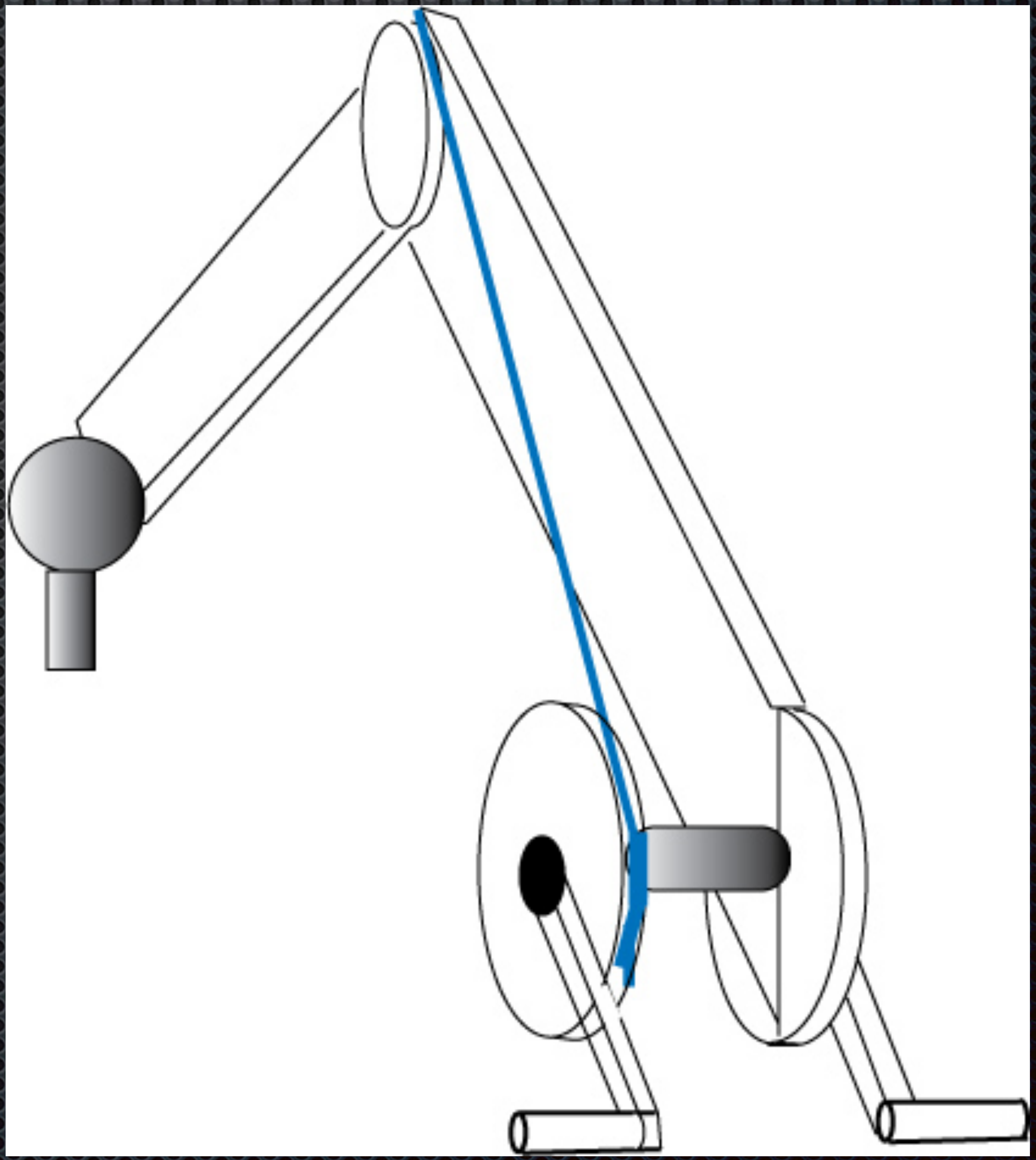
Communicating through collaboration:

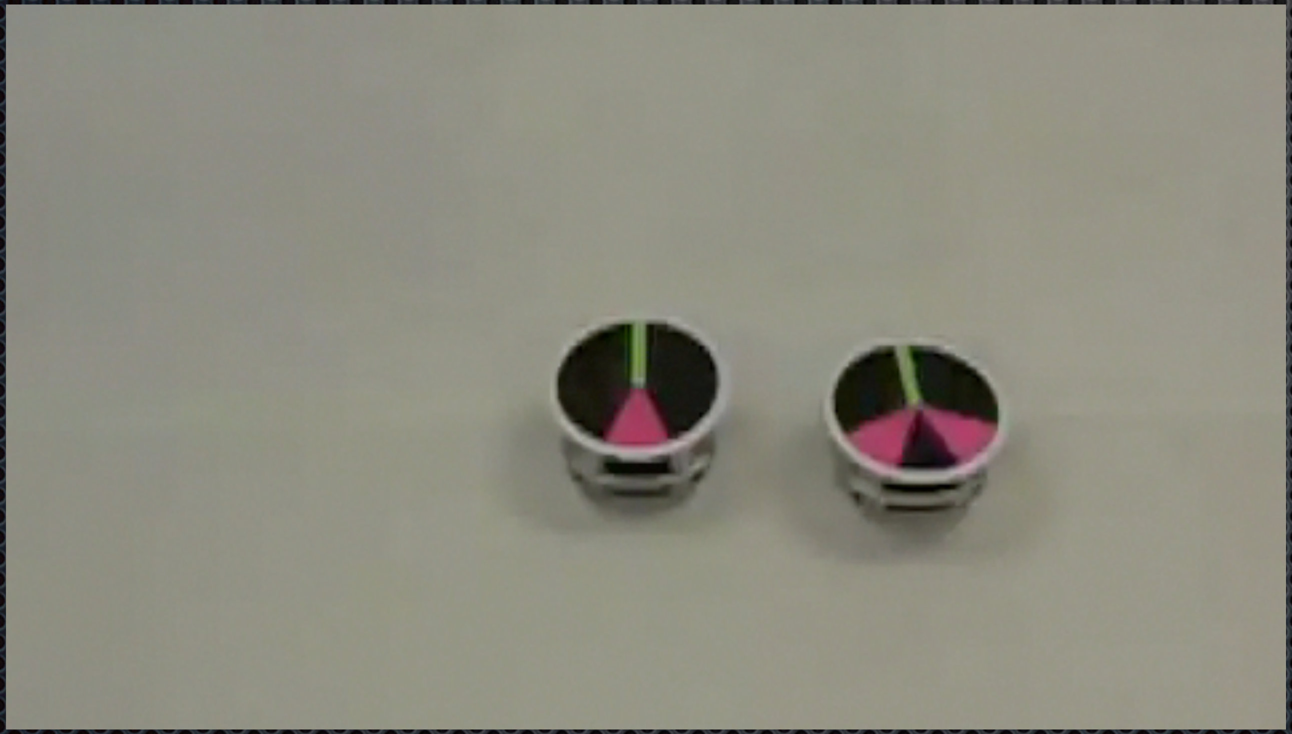


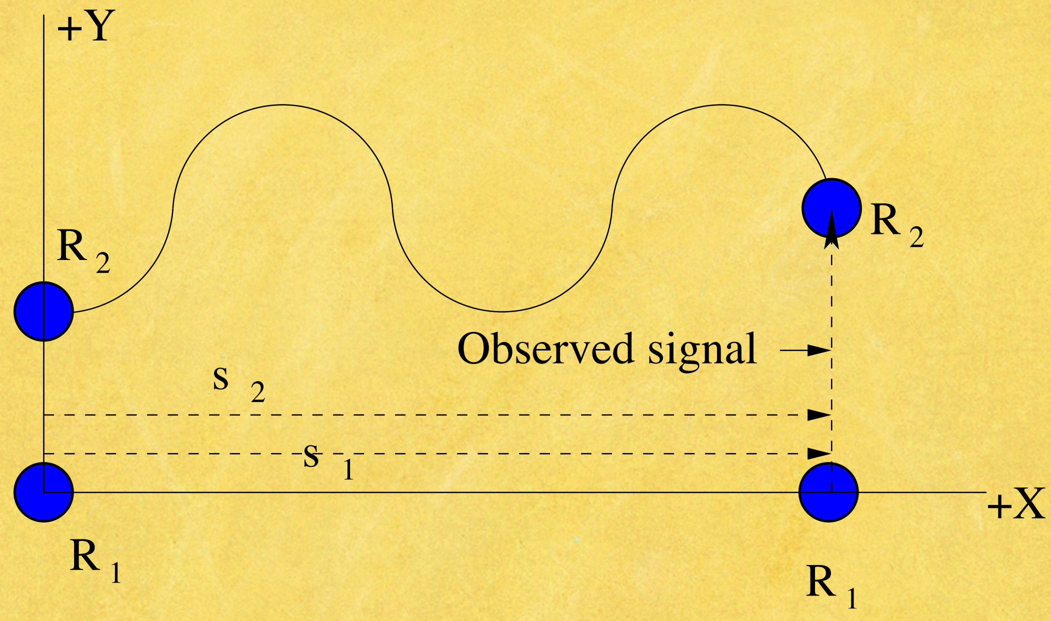
Communicating through action:

Communicating through artistic expression:

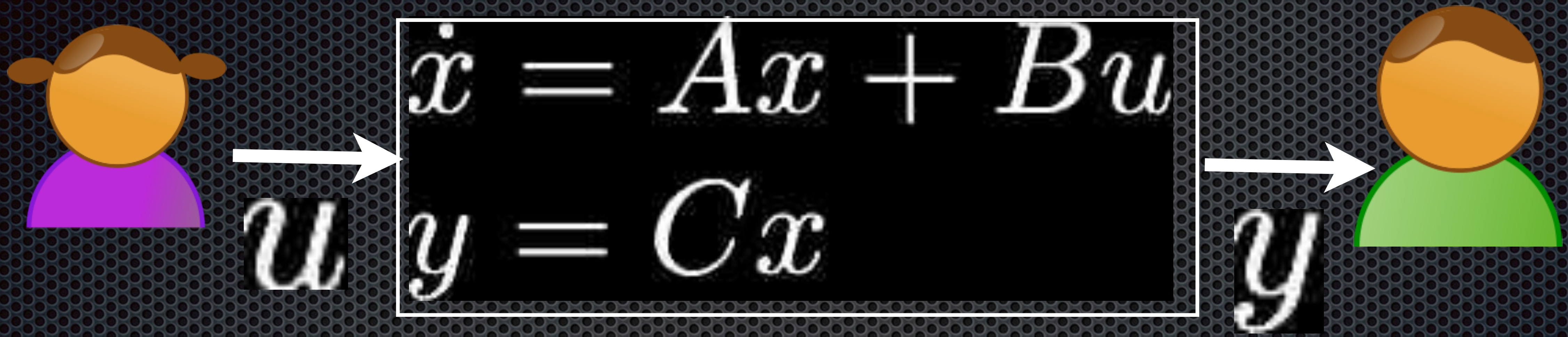
Relative Motion Messaging

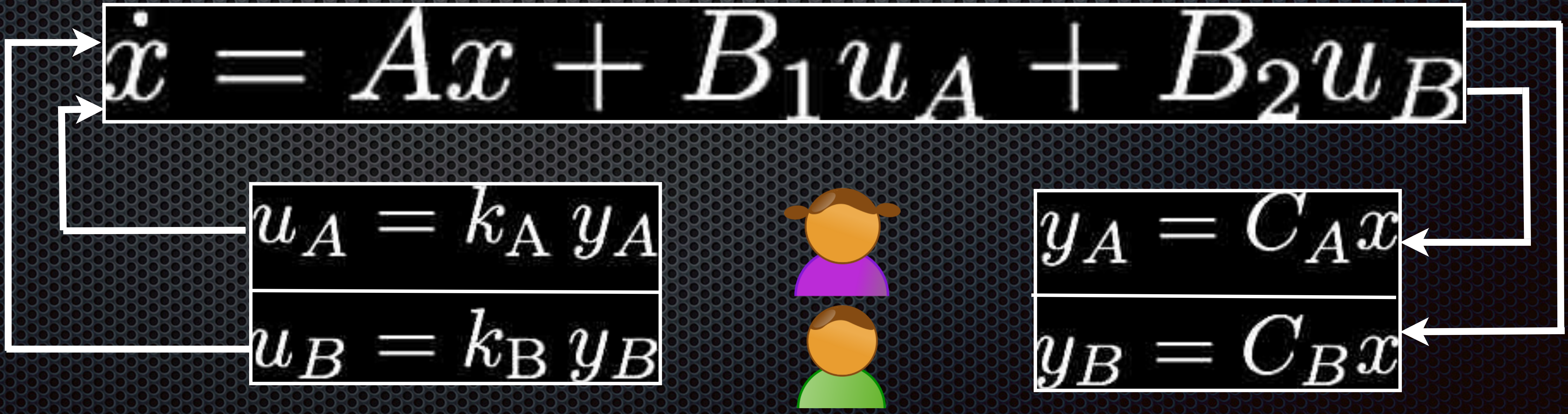
Red robot is gesturing to the blue robot





Can control systems be thought of as communication channels?





Control systems as communication channels - an earlier look

$$\dot{x} = Ax + B_1 u_A + B_2 u_B$$

$$u_A = k_A y_A$$

$$u_B = k_B y_B$$

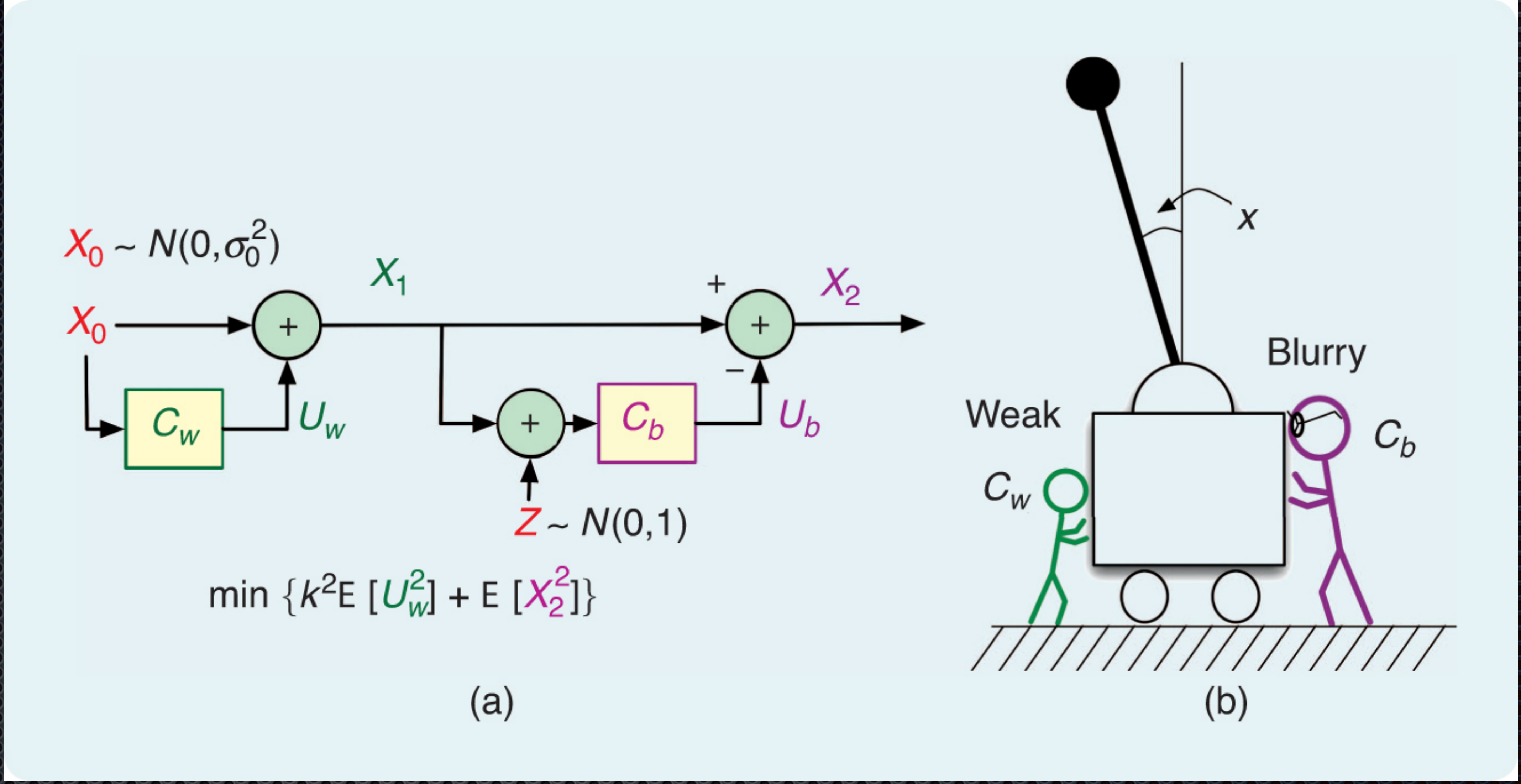
$$y_A = C_A x$$

$$y_B = C_B x$$

- Y.-C. Ho, “Team Decision Theory and Information Structures,” *Proceedings of the IEEE*, 68:6, June, 1980, pp. 644-654.

The Witsenhausen counterexample and the optimality gap

- H.S. Witsenhausen, "A counterexample in stochastic optimum control," *SIAM J. Control*, Vol. 6:1 1968.



- P. Grover & A. Sahai, "Demystifying the Witsenhausen Counterexample," *Control Syst. Mag.*, Dec. 2010.
- T. basar, "Variations on the Theme of the Witsenhausen Counterexample," 47-*th IEEE CDC*, Dec. 2008.

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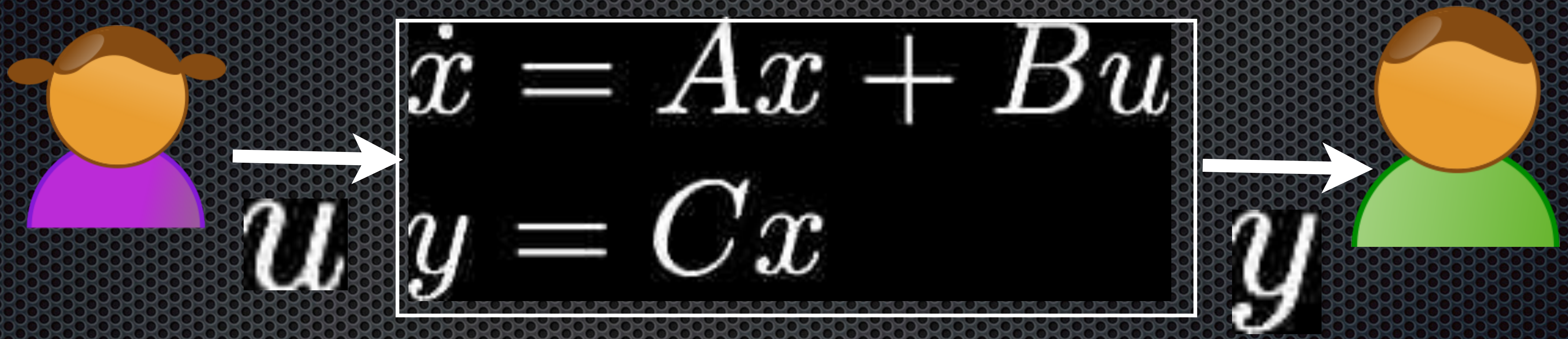
Can control systems be thought of as communication channels?



For point-to-point communication, there is hope.

The Case of Finite Dimensional Linear Systems

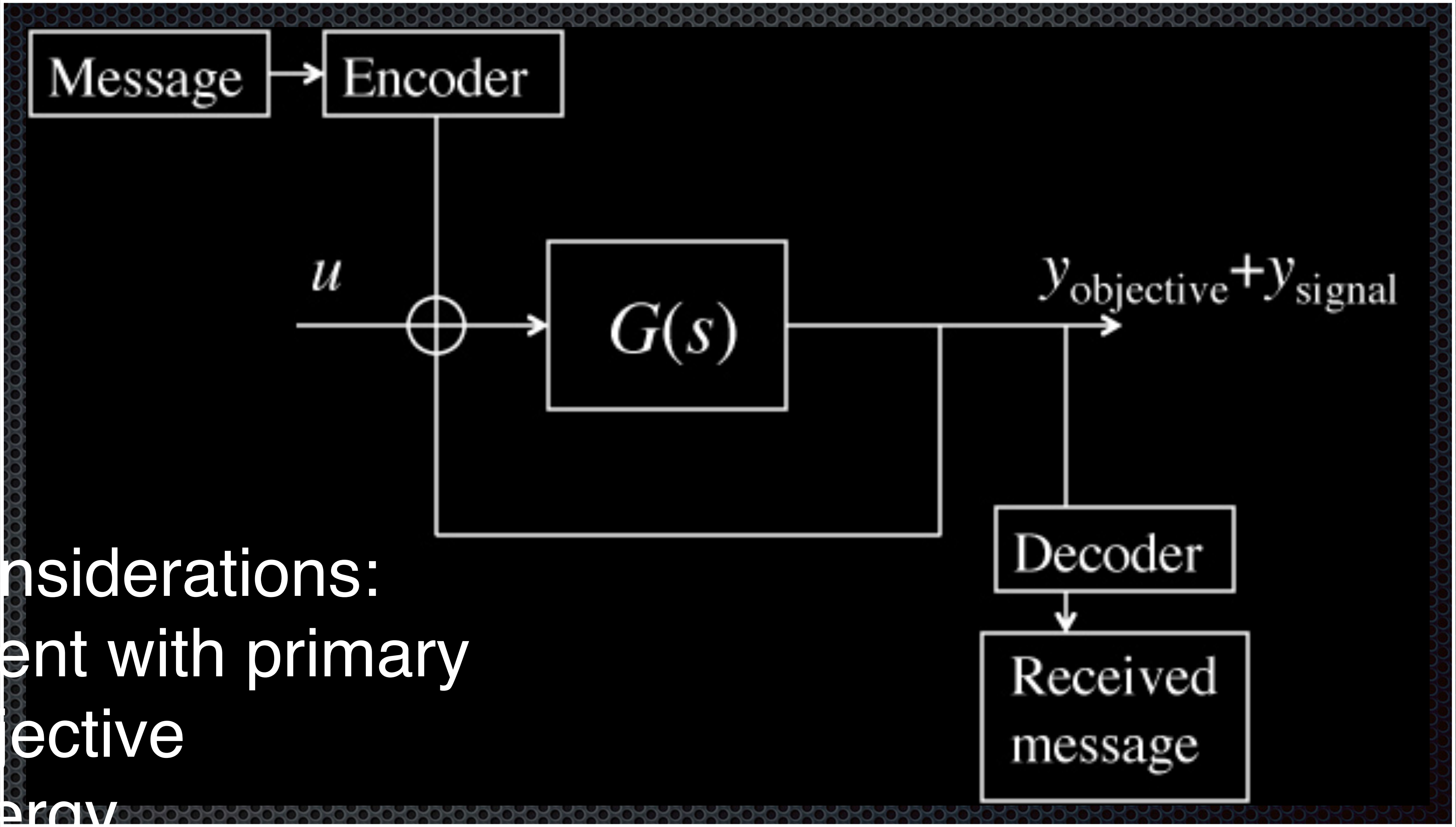
$$L : C^m[0,1] \to \mathbb{R}^n$$

$$u \mapsto L[u]$$

$$L[u] = \int_0^1 e^{A(1-s)} B u(s)\, ds$$

The null-space of L can be used for communication.

Control with Communication



Design considerations:

- Consistent with primary control objective
- Low energy
- Reliability
- Stealth/security

Control Optimized for Communication

$$\mathcal{L}(u) = \int_0^t ce^{A(t-s)} Bu(s)\, ds$$

Find k control inputs u_j such that

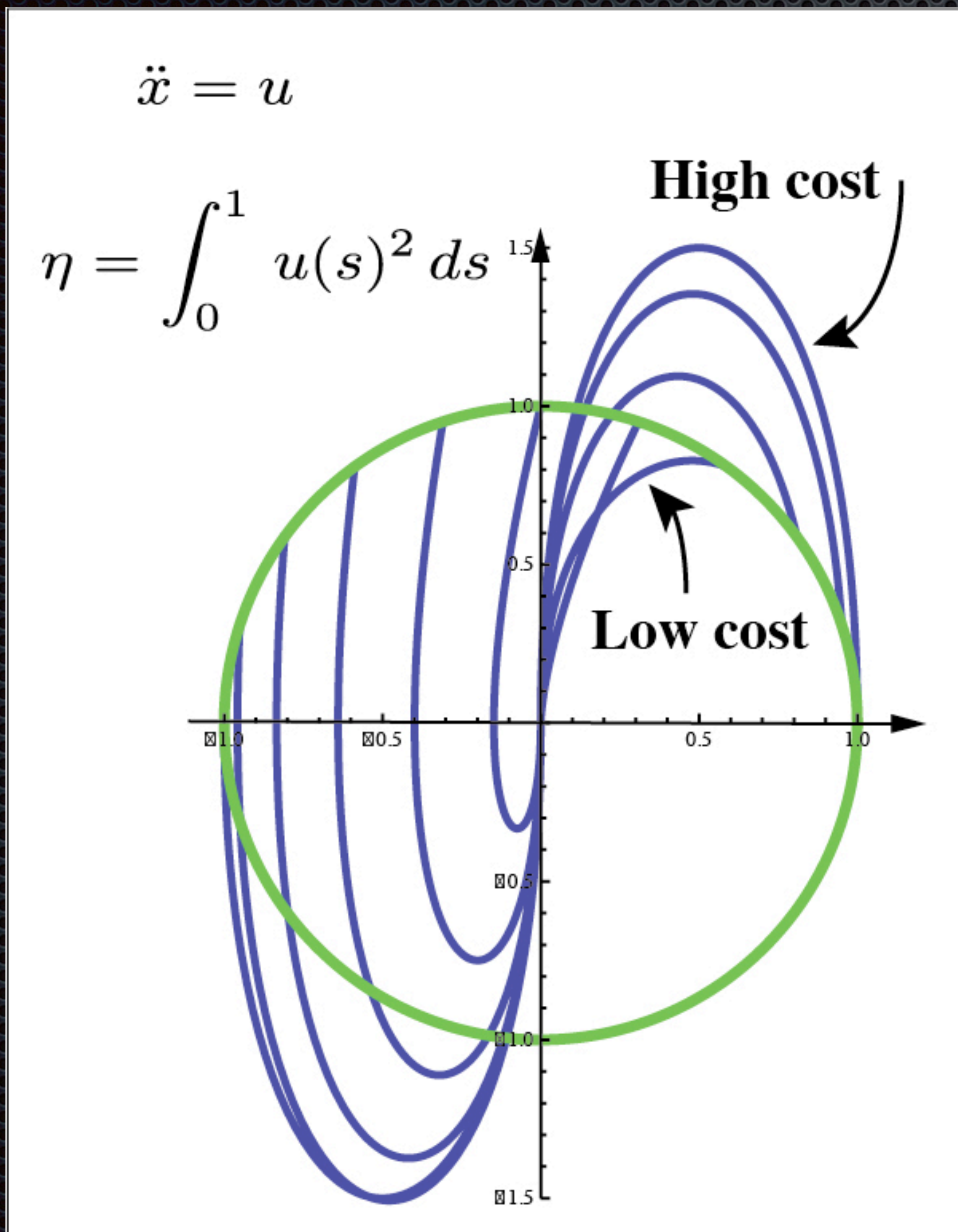
$$\eta = \int_0^1 u_1(t)^2 + \cdots + u_k(t)^2\, dt$$

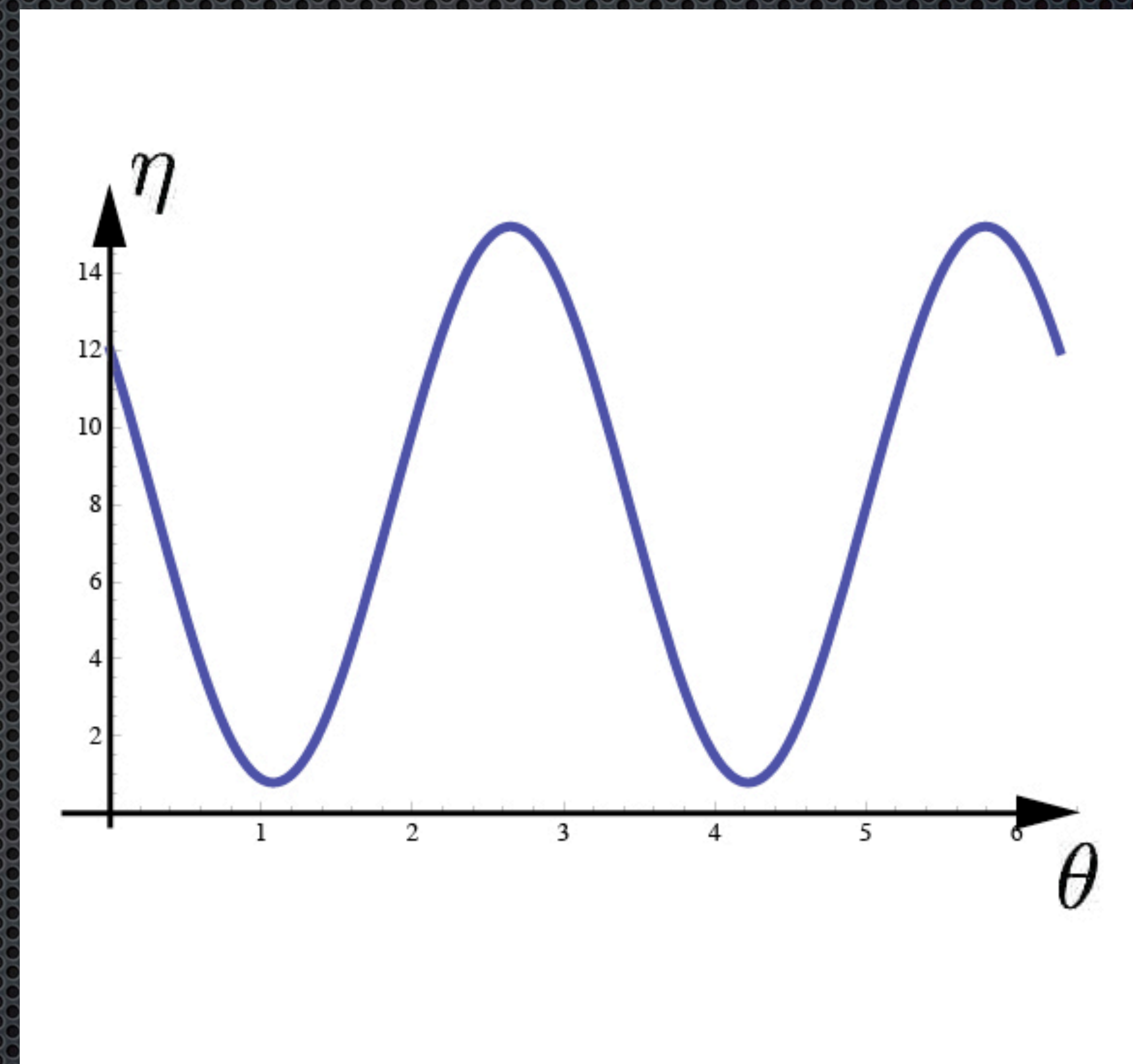
is minimized subject to

$$L[u_j] = \int_0^1 e^{A(1-s)} Bu_j(s)\, ds = x_1 \quad \text{and}$$

$$\sup_{[0,1]} \|\mathcal{L}(u_i) - \mathcal{L}(u_j)\| = h > 0,\ i \neq j$$

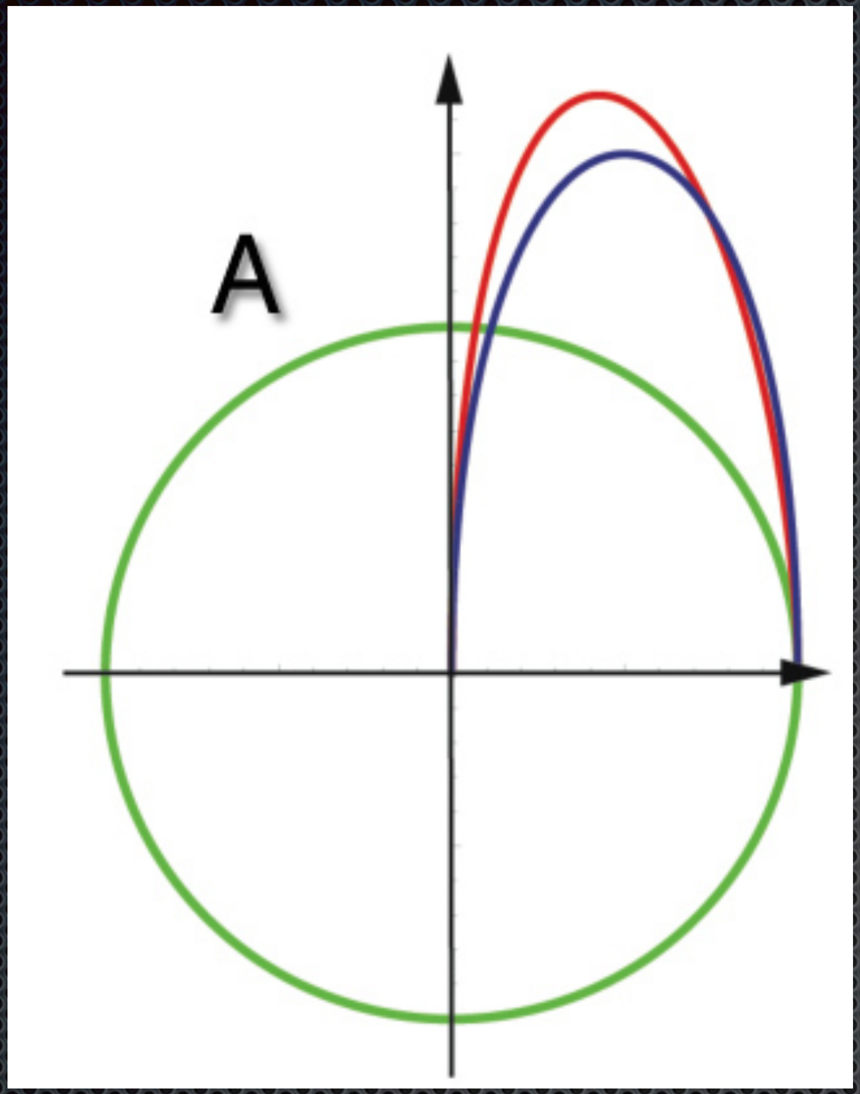
Communication in Context via the Double Integrator



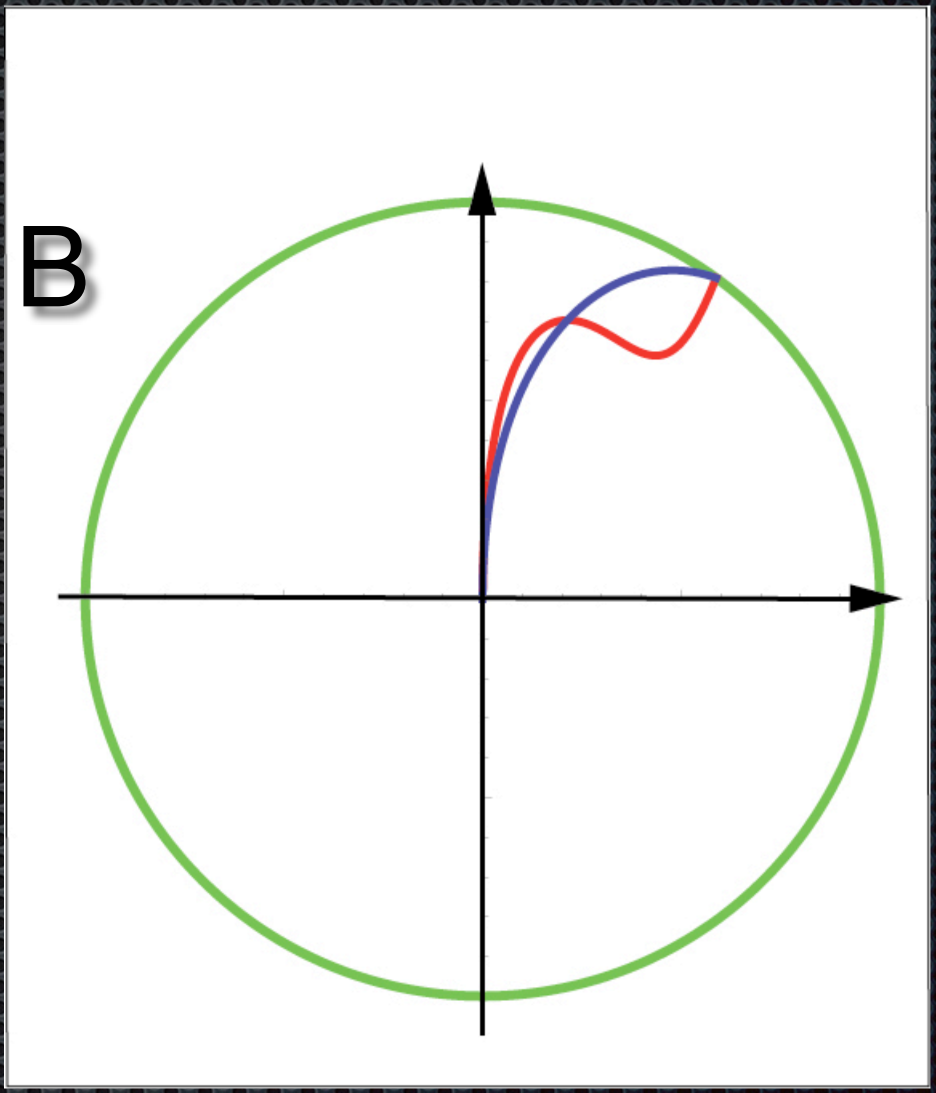


The cost of reaching points unit distance from the origin varies by one order of magnitude.

Communication Cost Dependence on Context



High motion cost (12) increases by 30% in communicating one bit of information.



Low motion cost (1.06) increases by 83% in communicating one bit of information.

An average least squares optimization problem

$$A : \mathbb{R}^n \rightarrow \mathbb{R}^m$$
$$A = m \times n, \quad n \gg m, \quad A \text{ has rank } m$$

Solve

$$Ax = y$$

for

$$x_1, \ldots, x_k$$

such that $\sum x_j^2$ is minimized subject to

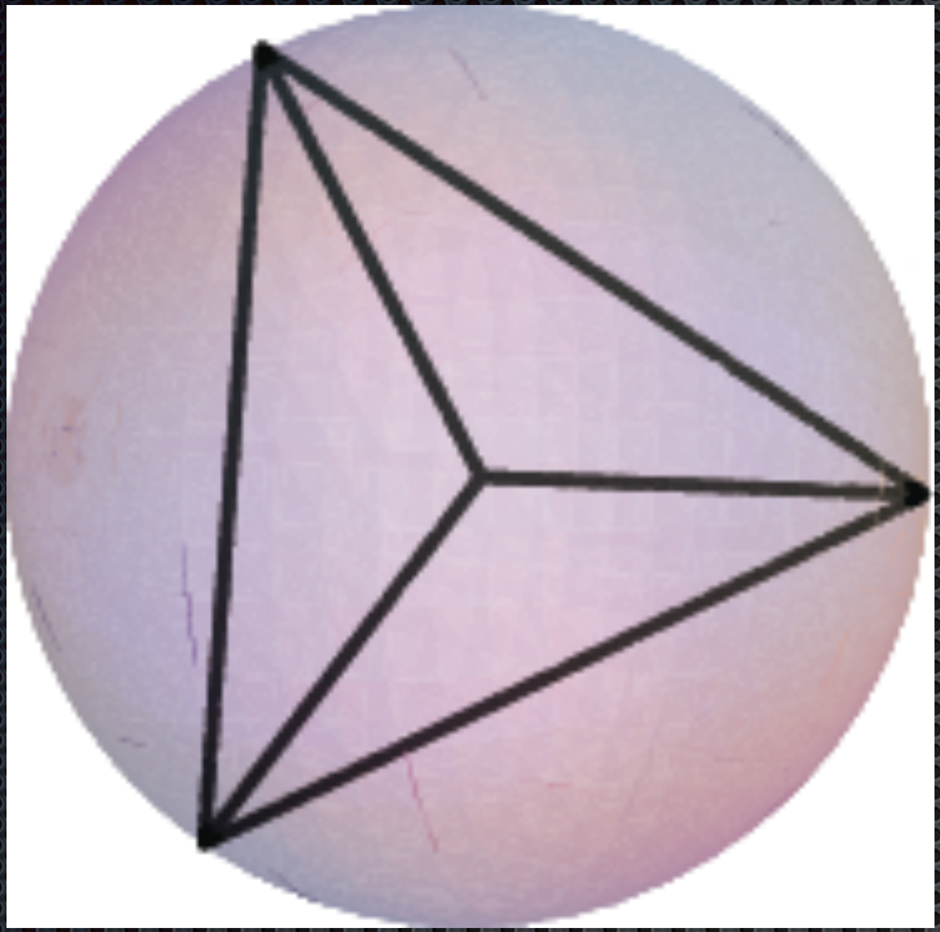
$$\|x_i - x_j\| \geq \epsilon > 0.$$

An average least squares optimization problem

Theorem: Let $\mathcal{N}(A)$ denote the null-space of A and let $S = \{x \in \mathcal{N}(A) \ : \ \|x\| = \left(\frac{k-1}{2k}\right)^{\frac{1}{2}} \epsilon\}$. Let $\vec{n}_1, \ldots, \vec{n}_k \in S$ be the vertices of any $k-1$-simplex. Any solution to the average least squares problem is of the form $x_j = x_0 + \vec{n}_j, \ j = 1, \ldots, k$ for any choice of $\vec{n}_j \in S$, and $x_0 = A^T(AA^T)^{-1}y$. Moreover, the optimal value of the objective function is

$$k\, y^T(AA^T)^{-1}y + \frac{k-1}{2}\epsilon^2.$$

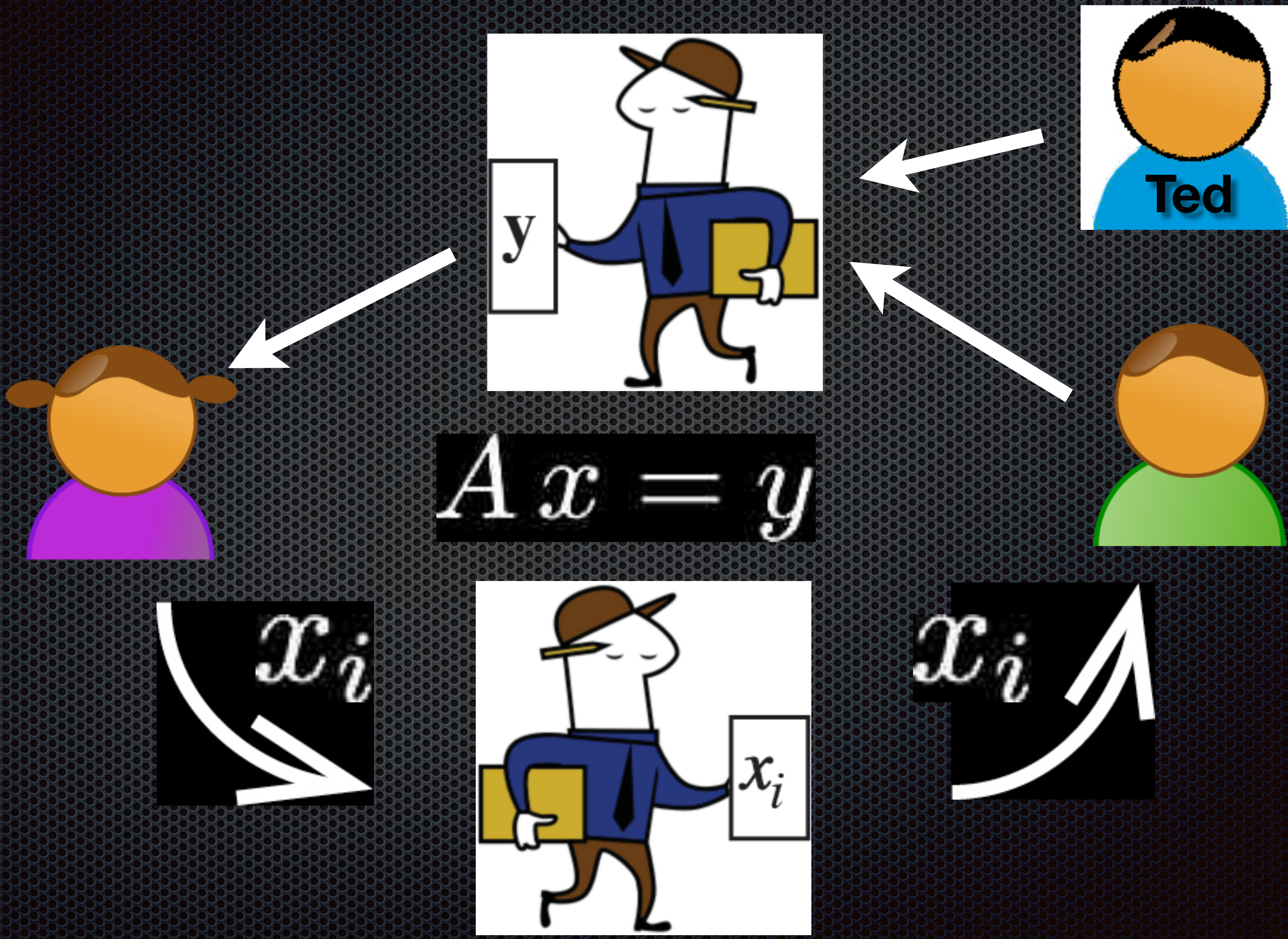
An average least squares optimization problem



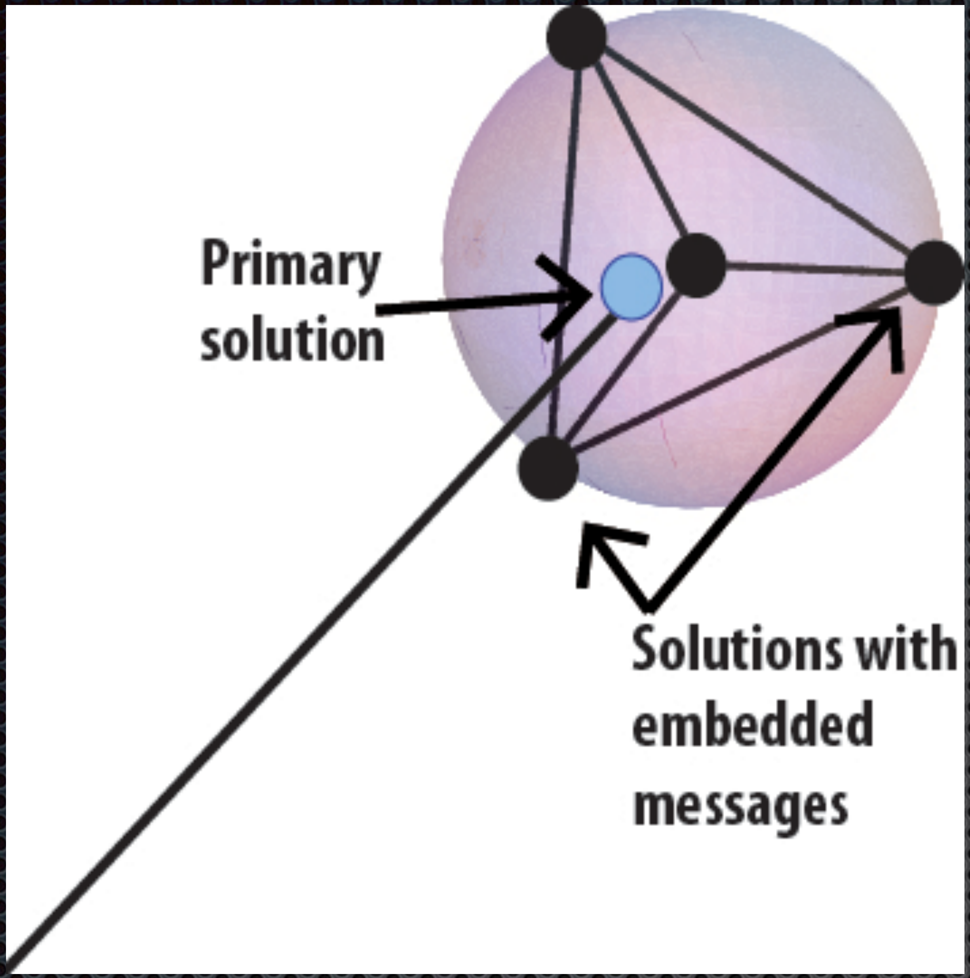
If $\vec{n}_1, \ldots, \vec{n}_k$ are vertices of a *k*-1-simplex centered at 0 with diameter ϵ, the sphere of smallest radius ***r*** that contains the vertices has

$$r = \left(\frac{k-1}{2k}\right)^{\frac{1}{2}} \epsilon.$$

A Linear Operator Communication Game



A Linear Operator Communication Game

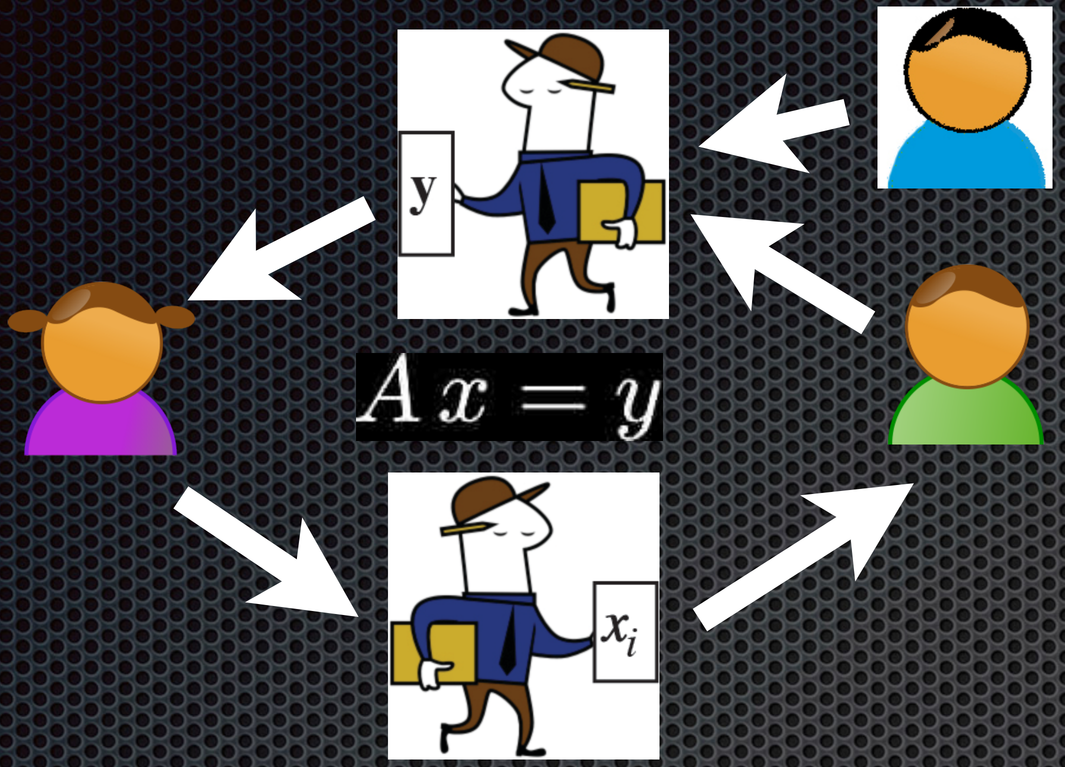


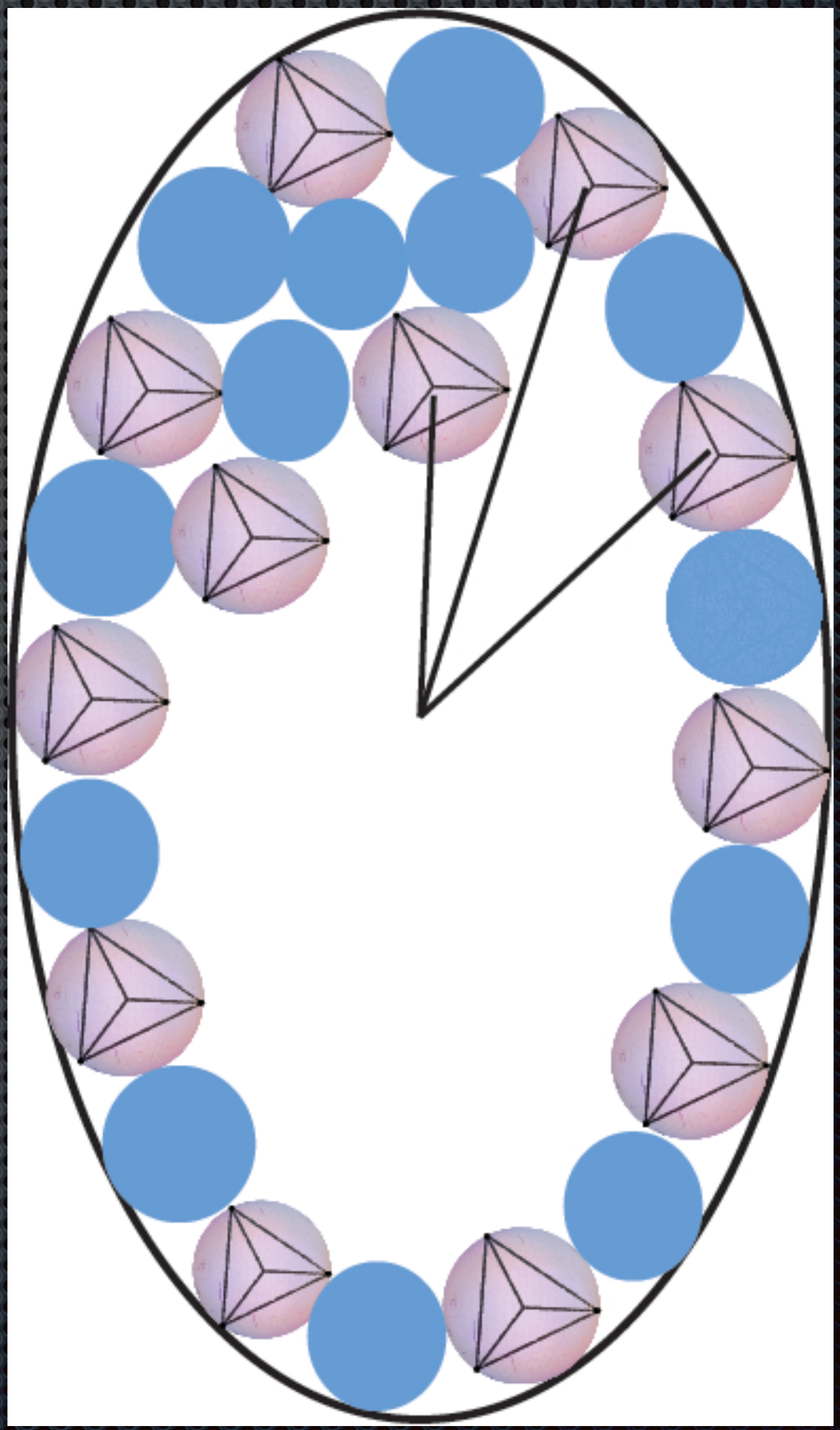
Average cost of embedding one of k ϵ separated “null-space symbols” in solutions of $Ax = y$:

$$y^T(AA^T)^{-1}y + \frac{k-1}{2k}\epsilon^2$$

Primary solution norm **Message symbol norm**

A Linear Operator Communication Game





The asymmetric linear operator channel with norm constraint

$$y^T (AA^T)^{-1} y + \frac{k-1}{2k}\epsilon^2 \leq R$$

has channel capacity equal to the number of spheres located at safe distances from each other in an ellipsoidal region.

A vector space communication problem

Let $\mathcal{U}, \mathcal{V}, \mathcal{W}$ be real vector spaces; $\dim \mathcal{U} \gg \dim \mathcal{V}$;

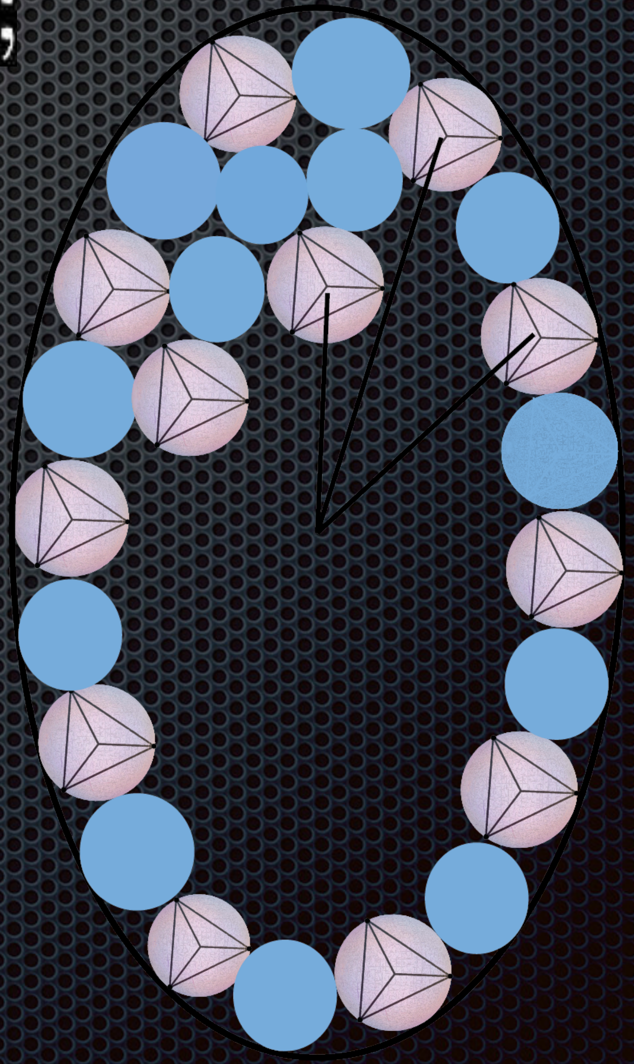
$L : \mathcal{U} \to \mathcal{V}$ (rank $= \dim \mathcal{V}$); $\ell : \mathcal{U} \to \mathcal{W}$;

Find $u_1, \ldots, u_m \in \mathcal{U}$ such that

$$\eta = \sum_{j=1}^{m} \|u_j\|^2$$

is minimized subject to

$$\|\ell u_i - \ell u_j\|_* = \epsilon > 0.$$



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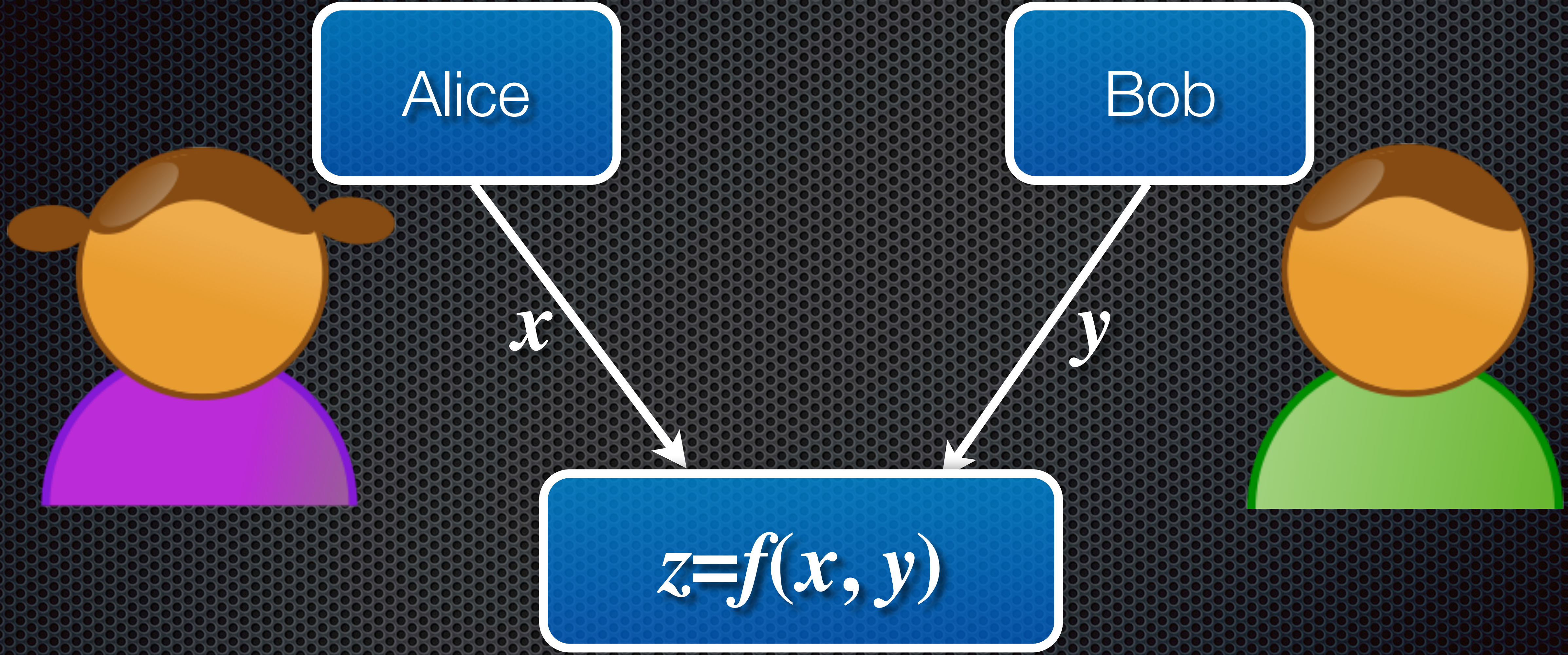
Communication Complexity



A.C. Yao, "Some Complexity Questions Related to Distributive Computing," *Proceedings of the 11-th Ann. ACM Symposium on the Theory of Computing (STOC)*, 1979.

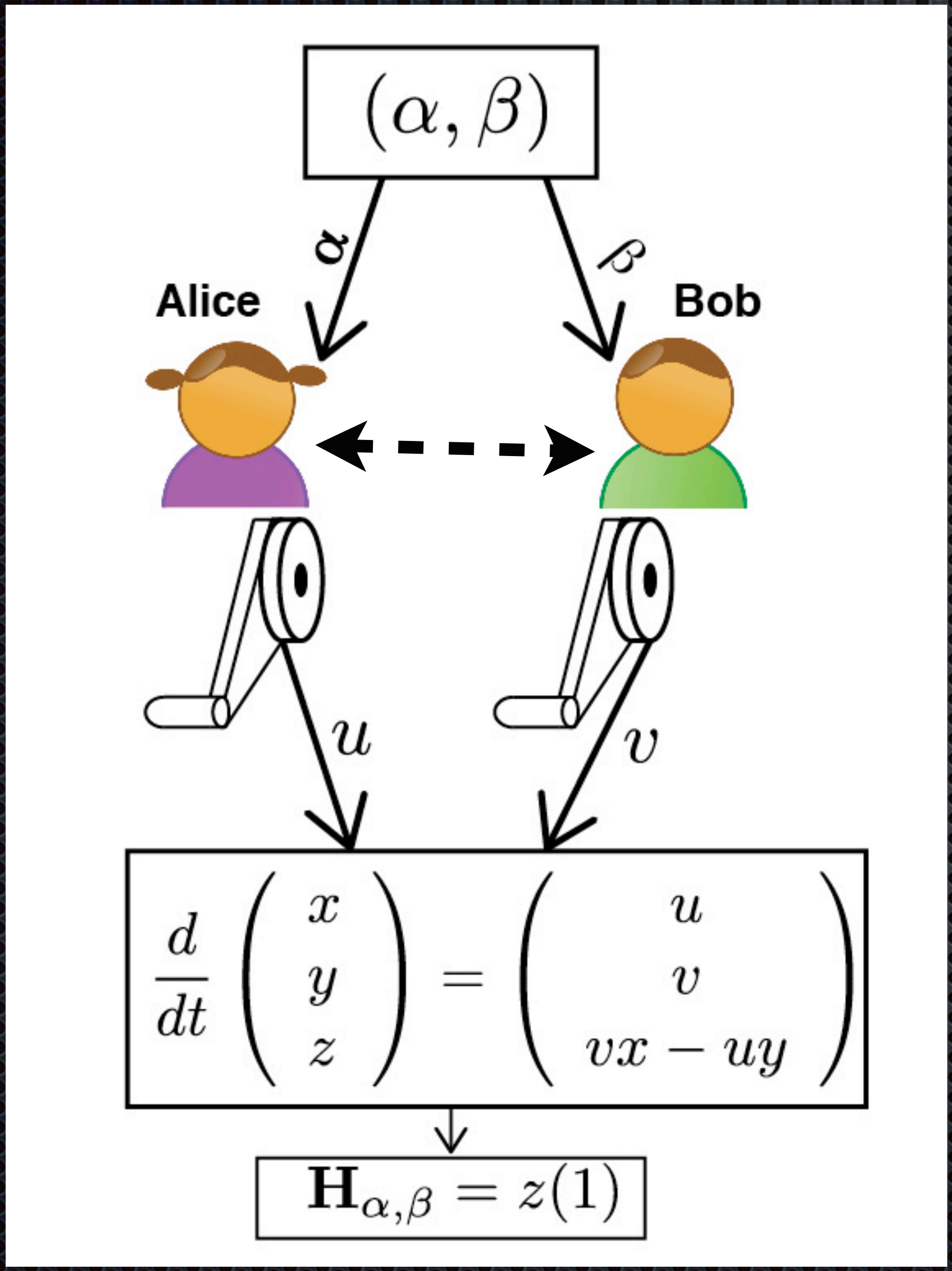
Control Communication Complexity - the two agent model

$$
\begin{cases}
\mathbf{x}_{k+1} = \mathbf{a}(\mathbf{x}_k, u_k, v_k), \quad \mathbf{x}(0) = \mathbf{x}_0 \in \mathbb{R}^N, \\
\mathbf{y}_k^{(A)} = \mathbf{b}_A(\mathbf{x}_k) \in \mathbb{R}^{\ell_A}, \ \mathbf{y}_k^{(B)} = \mathbf{b}_B(\mathbf{x}_k) \in \mathbb{R}^{\ell_B}, \\
u_k = P_k^A(Q_k^{(A)}(\mathbf{b}_A(\mathbf{x}_k)), \alpha), \ v_k = P_k^B(Q_k^{(B)}(\mathbf{b}_B(\mathbf{x}_k)), \beta), \\
\mathbf{z}_k = \mathbf{c}(\mathbf{x}_k) \in \mathbb{R}.
\end{cases}
$$

- $\mathbf{b}_A(\mathbf{x}_k)$ is Alice's observation at the k-th step;
- $\mathbf{b}_B(\mathbf{x}_k)$ is Bob's observation at the k-th step;
- $\mathbf{c}(\mathbf{x}_k)$ is a global system output, observable to Alice, Bob, and possibly other observers as well.

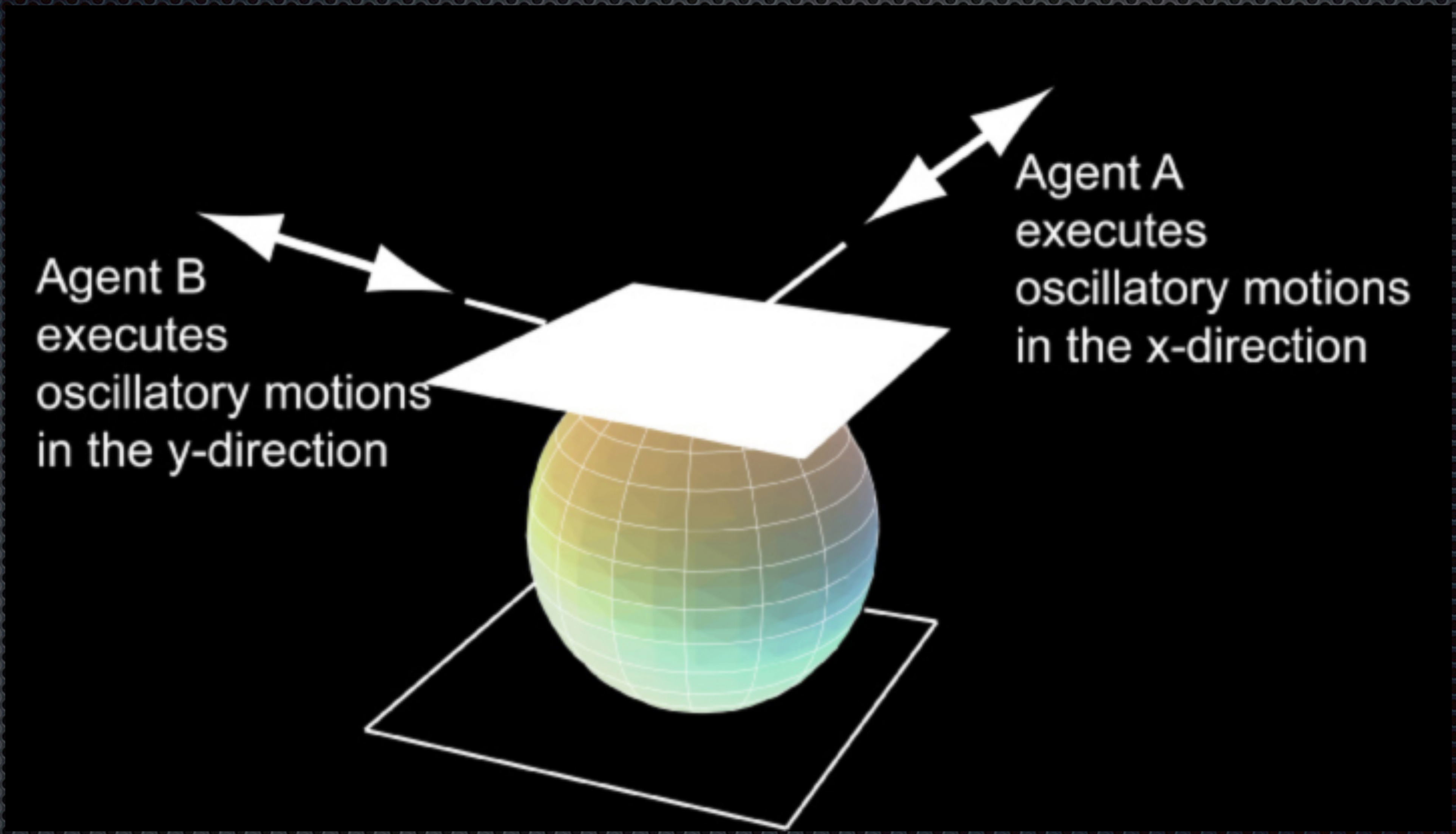
W.S. Wong, "Control Communication Complexity of Distributed Control Systems," *SIAM Journal of Control and Optimization*, 48:3 pp. 1722–1742, 2009.

Control Communication Complexity of Nonlinear Systems



Alice and Bob cooperatively control a dynamical system to compute a function. Each chooses a periodic input from a standard list of periodic inputs.

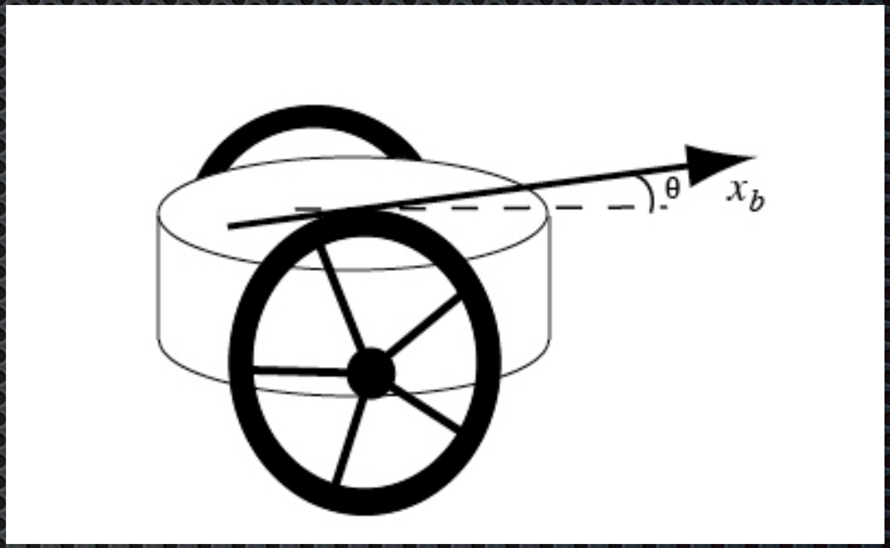
Control Communication Complexity of Nonlinear Systems



"Optimal Pulse Design in Quantum Control: A Unified Computational Method," Jr-Shin Li et al. http://www.pnas.org/cgi/doi/10.1073/pnas.0709640104

Communication Complexity of Dancing Robots

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} v\cos\theta \\ v\sin\theta \\ \omega \end{pmatrix}$$



The Shannon-Weaver diversity index:

$$-\sum_{k=1}^{4} f_k \log_2 f_k$$

Control Communication Complexity of Bilinear Systems

The B.-H. system with inputs for Alice and Bob

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} u_A \\ u_B \\ u_B x - u_A y \end{pmatrix}, \begin{pmatrix} x(0) \\ y(0) \\ z(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

defines an I/O mapping from periodic inputs to *z*(1):

$$\begin{array}{c} F \\ (u, v) \mapsto z(1) \end{array}$$

Control Energy Complexity of Bilinear Systems

A target matrix $\mathbf{H}$ is realized by a single round protocol $\mathcal{P}$ if there exist sets of controls, $\mathcal{U} = \{u_1, \ldots, u_m\}$ and $\mathcal{V} = \{v_1, \ldots, v_n\}$, so that

$$F(u_i, v_j) = H_{ij}.$$

The *control energy* of a protocol is

$$I(\mathcal{U}, \mathcal{V}) = \frac{1}{m} \sum_{i=1}^{m} \int_0^1 u_i^2(t)dt + \frac{1}{n} \sum_{j=1}^{n} \int_0^1 v_j^2(t)dt.$$

Alice's choices **Bob's choices**

Control Energy Complexity

Theorem: Consider a bounded bilinear input-output mapping, F, with a strongly regular matrix representation $\mathbf{F}$. Let $\mathbf{H}$ be an m-by-n target matrix. The control energy complexity of any single round protocol that realizes $\mathbf{H}$ is given by:

$$\hat{C}_F(\mathbf{H}) = \frac{2}{\sqrt{mn}} \sum_{k=1}^{\min(m,n)} \sigma_k(\mathbf{H})/\sigma_k(\mathbf{F}).$$

The Control Energy Value of Information

The difference between the optimal values

$$\hat{C}_F(\mathbf{H}) = \frac{2}{\sqrt{mn}} \sum_{k=1}^{\min(m,n)} \sigma_k(\mathbf{H})/\sigma_k(\mathbf{F})$$

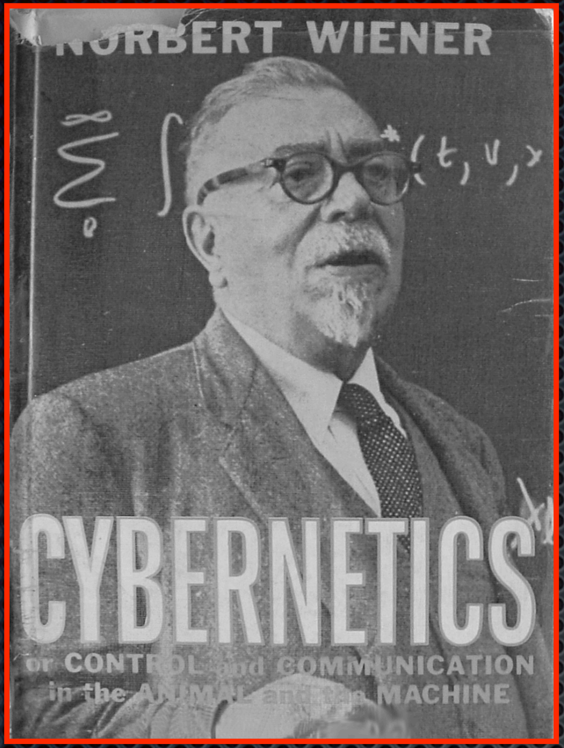
No shared information

$$J(\mathbf{H}) = \frac{2\pi}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} |H_{ij}|$$

Fully shared information

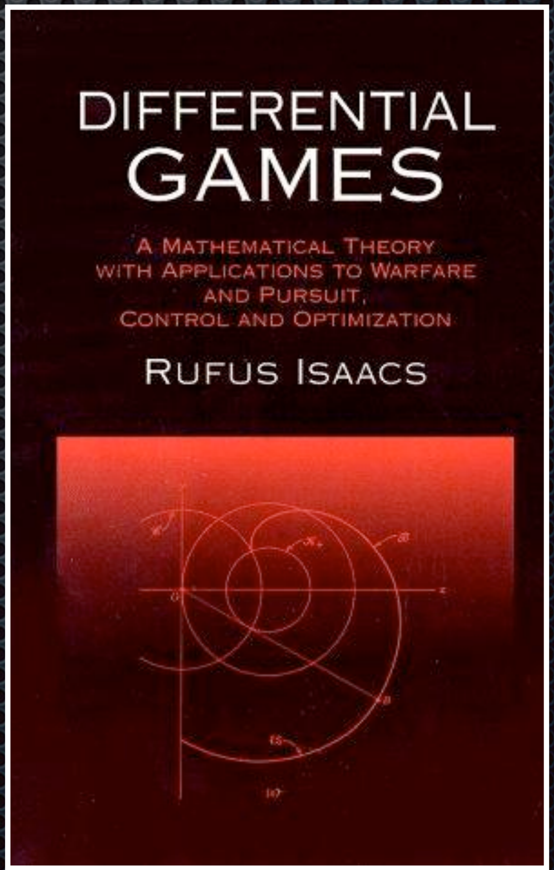
can be used to determine an energy cost of communication.

The Future of Control: Is the thrill gone?



Cybernetics; or Control and Communication in the animal and the machine, Wiley, 1948.

Differential games; a mathematical theory with applications to warfare and pursuit, control and optimization, Wiley, 1965.



Can our community do better than the physicists finding the fundamental limits on the effort needed to compute and communicate?