A Model of the Convenience Yields in On-the-run Treasuries

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October 2003

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Abstract

The convenience yield differential between on- and off-the-run Treasury securities with identical maturities has two components. A non-cyclical component may arise due to the higher illiquidity of off-the-run bonds. Also, trading in the market for the next issue often causes cyclical shortages of the on-the-runs. When this occurs, owners of the on-the-run bond can earn riskless profits by borrowing at a special repo rate while lending at the prevailing risk free market rate. This second component of the convenience yield, induced by the auction, is cyclical.

We first show that special repo rates and the convenience yield are jointly cyclical over the auction cycle. The patterns are statistically significant and pervasive. Repo specials are highest around the announcement day and disappear by the issue day. The off- minus on-the-run yield spread is highest at the beginning of the cycle and collapses near its end, consistent with a decreasing present value of profits over a decreasing horizon. Second, we develop a first no-arbitrage continuous-time model, with both interest and special repo rates stochastic, that prices the on-the-run bonds that command this convenience yield. A simple implementation of the model can generate yields consistent with the evidence.

1 Introduction

A *convenience yield*, a common concept in the valuation of commodity futures, is the benefit from storage of the physical commodity, but not from holding the futures contract on the commodity. Storage may provide profit opportunities in case of temporary shortages in the commodity. When shortages occur, the spot price for the commodity is high, generating a convenience yield. This high spot price is still consistent with no arbitrage, because shortages imply that the underlying commodity cannot be borrowed to execute a short sale.

This paper documents and provides a first no-arbitrage pricing model for the convenience yield of on-the-run over off-the-run Treasury securities. This yield has two components: a "non-seasonal" general liquidity risk differential and a "seasonal" component induced by the auction cycle. Current issues, i.e., *on-the-run*, are inventoried by government bond dealers and, consequently actively traded, they are often used for hedging or speculation. This higher trading activity implies a liquidity based pricing premium relative to the off-the-run bonds. This is the first, non-seasonal component of the convenience yield. In addition, on-the-run securities may be in excess demand due to their being used to hedge short positions in the when-issued market. Their owners can then borrow cash at a *special* rate in the overnight repo market, using their on-the-runs as collateral. The special rate can be much lower than the prevailing *general* repo rate for loans with similar term and risk. These owners can earn riskless profits by borrowing at this special repo rate and lending at the general rate. This imparts extra value, the auction induced convenience yield, to the on-the-run bond, see Duffie (1996a).

Shortages in on-the-run Treasuries frequently happen before the auction date, as follows. Prices often fall at Treasury auctions to accommodate the auctioned supply.[1] Speculators short sell the not-yet auctioned Treasury in a forward market, the *when-issued* market, hoping to buy it back at a lower price in the auction. To hedge the interest rate risk inherent in this transaction, they may want to buy existing on-the-run Treasuries, the closest substitutes. Combined with the reduction in the "floating" supply of the existing on-the-run Treasuries due to their seasoning (being outstanding for several months), this increased demand may create shortages for the on-the-run Treasuries before the auction, and specials in the repo market.[2] This appears to be the most common type of shortage, see Sundaresan (1994). Shortages after the auction date due to corner and short squeeze manipulations, also occur albeit infrequently.

First, we examine the cyclicality of specialnees, the spread between overnight general repo rate and special repo rates for on-the-run collaterals, from 1987 to 1997. An event-study around the auction cycle shows that cyclicality is pervasive across auctions, statistically significant, and strongest for the quarterly cycles.[3] Second, we show the strong cyclicality

[1] This discount may be related to a winner's curse problem faced by some bidders. A large literature discusses this issue. See Cammack (1991), and Malvey and Archibald (1998) for a review.

[2] A view is that the seasoned issue is largely held by buy-and-hold investors who may be unwilling to lend it, thus causing a contraction in the supply of collateral.

[3] Sundaresan (1994) reports average repo spreads and specialness for 4 days around the auction day until

of the spread of *just* off-the-run minus on-the-run bond yields, around the auction cycle.[4] The difference, highest early in the cycle, collapses at the end, as would a present value of expected benefits over a decreasing horizon. This is the intuition of our pricing model.

To see the difference between the two above-mentioned components of the convenience yield, consider the following three riskless zero coupon bonds with maturity T. Let $p(t,T)$ be the time t price of an on-the-run bond with an auction induced convenience yield trading in a liquid market. Let $\hat{P}(t,T)$ be the price of a conceptual bond with no convenience yield also trading in a liquid market. $\hat{P}(t,T)$ is synthetically constructed, for example using put-call parity and liquid stock index options maturing at T. Finally, let $P(t,T)$ be the price of an off-the-run bond with no convenience yield trading in an illiquid market. As the auction induced convenience yield is non-negative, $p(t,T) > \hat{P}(t,T)$ is possible. As the liquidity discount due to illiquid markets is non-negative, $\hat{P}(t,T) > P(t,T)$ is possible. There is, however, no arbitrage opportunity if $p(t,T)$ cannot be short sold or $P(t,T)$ cannot be bought, without excessive transaction costs. The inability to short sell p(t,T) is due to the temporary shortages in the on-the-run security discussed above. The inability to buy $P(t,T)$ is due to its not being inventoried by government bond dealers. We restrict ourselves to a no transaction cost model, capturing the special by these trading constraints.

The model developed provides an explicit valuation of the convenience yield, $p(t,T) > P(t,T)$. It prices the expected riskless profit opportunity available in the repo market, only to the owners of on-the-run Treasuries. We use the no-arbitrage pricing paradigm to explicitly model the link between the special repo rate and an endogenous convenience yield. Because we are mainly interested in pricing Treasury securities, we formulate our model under the martingale pricing measure. Consequently, risk premia relating to the Treasury auction cycle and the repurchase agreement markets do not enter the analysis. Nonetheless, the martingale pricing measure must necessarily preserve all events that occur with probability one under the empirical measure. As noted earlier, the special repo rate applies only during the auction cycle as long as the bond is on-the-run. The repo rate process we utilize for valuation reflects this empirical regularity, a probability one event.

In related work, Grinblatt (1999) adds an exogenous convenience yield for holding Treasuries, due to unspecified liquidity considerations. He does not provide a mechanism by which the convenience yield accrues as cash flows to the owner of the on-the-run bond. Duffie's (1996a) structural model shows how specialness can arise endogenously due to rational shorting activity in the when-issued market, and is a motivation for our work. In his model, see proposition 1 p. 511 in a one period setup, on-the-run bonds have lower yields than off-the-runs, the correct ranking. However, his assumption of perfect knowledge of future rates over the remainder of the auction cycle does not produce an implementable pricing model.[5] Jarrow and Turnbull (1997) provide general arbitrage-free frameworks for

1991. Keane (1996) plot average patterns, and Fisher and Gilles (1996) average term repos. Jordan and Jordan (1997) report spreads around 161 basis points between 1991 and 1992.

[4] Duffee (1996) conjectures that one common component in the variation of Treasuries may be due to the convenience yield. This is possible as several auction cycles coincide.

[5] Fisher and Gilles (1996) and Keane (1996), also prorate the benefit from a repo special discount over the

pricing Treasuries. However, they do not model the convenience yield and special repo rate process. Fisher and Gilles (1996) discuss no-arbitrage restrictions on the term structure of repo spreads. Krishnamurty (2001) studies specific equilibrium scenarios consistent with our no-arbitrage framework. While this provides intuition on the specific scenario, the risk-neutral framework is robust to the underlying equilibrium situation and can be implemented without specifying it.

Our model for pricing Treasury securities captures illiquidities - in the Treasury securities market. We exploit the existence of a liquid parallel market that prices this illiquidity risk, the repo market, to use the arbitrage-free pricing paradigm and sidestep risk premia. As such, our technique for modelling illiquidity can apply to any instruments with this parallel market structure, e.g., publicly traded corporate debt. The only modification needed is to formulate a process analogous to the special repo rate to reflect the security's primary and secondary market mechanisms.

The next section reviews the repo and Treasury markets. Section 3 documents the common cyclical patterns of repo specialness and on- vs off-the-run yield spreads. Section 4 develops the pricing model of Treasury securities with an endogenous convenience yield. We first develop the continuous time model and then show how to implement it. Section 5 concludes and discusses other potential uses of this new model.

2 The Repo Market

Repurchase agreements, i.e., repos, are contracts, where two parties swap a financial security used as collateral, for cash. The repo trader typically sells a Treasury or similar security to the reverse repo trader with an agreement to repurchase it on a future *buy-back* day. The seller in fact borrows funds from the buyer with an interest rate r agreed upon initiation of the contract. The most common contract in the U.S. repo market, the standard repurchase agreement, is structured as follows.[6] At initiation the seller transfers the collateral security to the buyer in exchange for its market value $\$S$ in cash.[7] On the buy-back date, d days later, the seller recovers the collateral and pays the buyer $\$B = S(1 + r\frac{d}{360})$. Repo maturities range from overnight to several months. Maturities below one month have the highest trading volume. Overnight repos are the most common in the U.S. market.

Repos can be seen as a form of collateralized loan, but they serve a dual function. First, the seller borrows cash and views the buyer as a lender. Second, the buyer buys the collateral at repo initiation and sees the seller as a seller of the collateral. Traders wanting to

remaining days in the auction cycle. As Keane notes, this implies perfect knowledge and does not constitute a pricing model.

[6] This and other, less common, types of repo contracts are described in further detail in Stigum (1989), Short and Schneider (1994), Corrigan et al. (2000), Sundaresan (1994).

[7] If credit risk and collateral volatility matter, buyers may require a *haircut* from sellers, a margin to protect them from unfavorable price moves in the collateral if the sellers default. With a x% haircut, the seller receives only $(1 - x/100)S$ at initiation.

short-sell Treasury securities often borrow the securities through a reverse repo agreement.

Treasuries are normally grouped together as general collateral and obtain the same general repo rate. The general rate is not a function of the maturity of the collateral. Rather, it is nearly identical to the Federal funds rate, akin to a very short rate. 90-95% of Treasuries trade at the general repo rate, see Sundaresan (1994).

U.S. Treasury securities are issued via periodic auctions. Typically, three and six-month bills are auctioned every Monday and issued the following Thursday. Two and five-year notes are auctioned monthly, typically settling on the last business day of the month. Three to thirty-year securities have been issued mostly quarterly.[8] A specific *special* repo rate, often much lower than the general rate, is tied to each *on-the-run* (last auctioned) issue, which then loses most of its specialness when it goes off-the run.[9] With a longer auction cycle, the benefit of specialness accrues for a longer period of time. Hence, the length and the stage of the auction cycle, not the maturity of the collateral, determine the seasonal component of the convenience yield accruing to the owner of an on-the-run security.

Specials occur mostly as follows. Before the auction day, potential bidders and speculators may trade in the *when-issued* market. There is evidence of a significant primary market price effect in the auction market, possibly linked to a winner's curse facing some bidders, see Cammack (1991).[10] Speculators may attempt to lock in the profit due to this auction price effect. Namely, they short-sell the future issue in the when-issued market, in anticipation that its price will be lower at the auction. Then, they may use the outstanding on-the-run issue to hedge the interest rate risk inherent in this position. Combined with the reduction in the "floating" supply of the existing on-the-run Treasuries due to their seasoning (being outstanding for several months), this may cause the on-the-run Treasury security to be in short supply prior to the auction.

Off-the-run Treasuries of similar maturity are not used for such transactions because of the illiquidities (or large bid-ask spreads) that arise from dealers not inventorying them. While these illiquidity costs are not actual shortages, off-the-run trading does not seem to be a preferred medium for such transactions. These illiquidities can cause the off-the-runs to sell at a discount relative to the on-the-run securities, especially in times of increased market uncertainties, see Lowenstein (2000). This discount is independent of the auction cycle.

In contrast, the common practice of shorting Treasuries mostly involves reverse repos due to their low transactions cost. A dealer, to short sell the security, borrows it from another dealer or institution through a reverse repo agreement. This high demand by the

[8]These schedules change. Quad-weekly one-year bill auctions were discontinued in 2001. The U.S. Treasury reduced the thirty-year bond schedule from quarterly to semiannual in 1993, and increased it to three per year in 1996. Ten-year bonds, usually issued quarterly, were issued six times in 1996. The 5-year changed between monthly and quarterly. Three, four, and seven-year notes were discontinued in 1998, 1990, and 1993 respectively.

[9]First-off-the-run bonds can also trade as special collateral. Our model concentrates on the specialness of on-the-run relative to off-the-run issues.

[10]From 1992 to 1994, the U.S. Treasury offered two-year and five-year notes through uniform price Dutch auctions on an experimental basis presumably to mitigate the winner's curse problem.

short-seller causes the security to be on special on the repo market. Then, dealers with long positions can borrow cash at a special repo rate by lending their security through a repo.

Sundaresan (1994) indicates that specialness can also occur after the auction, albeit infrequently, because of short squeezes. In this scenario, dealers with short positions in when-issued market will need to cover these positions at the auction. Other dealers, anticipating this, may try to corner the auctioned supply. Then unable to cover their position at the auction, the short dealers try to cover it in the secondary market. This may cause shortages in the recently auctioned on-the-run and an accompanying special repo rates. Jegadeesh (1993) discusses the 1991 Salomon Brothers corner of the 2-year note. See also Cornell and Shapiro (1989), Chatterjea and Jarrow (1998), and Cherian and Jarrow (1995) for discussions.

3 Evidence

There is some evidence on the average cyclicality of special repo rates. Sundaresan (1994) reports average special rates before 1992, for 4 days around the auction date. Keane (1996) plots average spreads for some proprietary data. For a longer calendar period and a wide range of bonds, we find that the cyclicality is strong, statistically significant and pervasive across bonds and auctions. There is also evidence on the generic on- versus off-the-run yield spread. For example, Warga (1992) documents yield spreads between on- and off-the-run notes of similar maturities, relating them to illiquidity. Here, we document the *joint cyclicality* of 1) the spread between general and special repo rates and 2) the auction-induced convenience yield in on-the-run Treasuries.

We use daily quotes for the general overnight repo rate and each special repo rate tied to a specific on-the-run 5, 10 and 30 year bonds, from March 1987 to May 1997. We also use daily secondary market quotes for on-the-run and *one*-off-the-run bonds, those just gone off-the-run.[11] Unlike older bonds, often used in the literature, the one-off is not very illiquid, only suffering from being taken off the dealers' inventories. The one-off-the run allows us to concentrate on the cyclical component of the convenience yield.[12]

[11] Market makers often keep a book of repo trades as they are on both sides. That is, they lend cash at the reverse repo (bid) rate and borrow cash at the repo (ask) rate, earning the spread between the higher bid and the lower ask. The bid-ask spread is relatively small. Our data are ask rates. We thank the Fixed Income Trading Research Group of Merrill Lynch for supplying the data.

[12] Krishnamurty (2001) documents the discount of older 30-year bonds over their one-off-the-run counterpart. That discount is linked to the non-cyclical component of the convenience yield. Another advantage of the one-off-the-run is that its coupon yield is nearly identical to the on-the-run. This matters as we use the quotes and the coupon yields to create two zero coupon daily term structures. We obtained the coupon yields from the treasury Web site and assumed a flat forward term structure between adjacent maturities to compute the zero curves.

3.1 Cycles in Repo Spread

Consider first the repo spread for Treasuries of different maturities. We compute the spread as the difference between the general rate r_0 and the special rate R for a given Treasury issue. A time series plot of a daily repo spread, e.g., Duffie (1996), shows that it is a random variable and cannot be assumed to be deterministic. Beyond this, time series plots are not very informative on the patterns of repo spreads.

Instead we plot the repo spreads in event time. The event day (day 0) can be the announcement or the issue (settlement) day, bracketing the period of when-issued trading that may see the strongest repo specials. As the number of days between announcement and issue days varies a bit with the auction, one can not sharply label both on one same plot. For brevity, we only present plots with the issue day as day 0. Plots using announcement day as day 0 provide nearly identical patterns. For brevity, we also restrict ourselves to the 5, 10 and 30 year bonds.

Each day of the sample period is then allocated to its position relative to day 0. This produces a cross-section - number of auction cycles collected, of repo-spread for each day in event time. For each such event day, we compute the cross-sectional first and third quartiles, median, mean and a confidence band for the mean of 1.96 standard errors of the mean.

Figure 1 plots these statistics with the issue day as event day. The 10, 30, and 5-year issues with quarterly or longer cycles exhibit remarkably similar and strongly cyclical patterns. The average spread peaks around 10 days before the issue day. It collapses suddenly in one or two days immediately preceding the issue day. The trough clearly occurs on the issue day. After a number of days at the trough, it then builds up slowly to the next peak. Plots, not shown here, with the announcement day chosen as event day, are similar but confirm that the peak spread indeed occurs around the announcement day.

The average peak is high, above 2% for the 10 year, and 1.5% for the 30 year. The difference between the peak and the low of the spread is between 1% and 1.5%. The peaks for the 5 year issues with a monthly auction cycle. However, a test strongly rejects the null hypothesis of no difference in spread through the cycle for all issues, as the 1.96 standard error band around the average shows.

The median spread is below the average spread, showing that the distribution of specialness across auctions is skewed. However, the median and the lower quartile Q_1 are also cyclical, showing that the average pattern is not driven by a few exceptional options, but is indeed pervasive across auctions. Finally, note that even at its lowest, the repo spread is not zero.

For the longer, say quarterly, cycles, the repo spread can be seen as having two regimes. First, from 20 days before until announcement day, the spread is large. Second, over the ten days or so between announcement and issue day, the spread drops very sharply to its lowest value. The pattern is pervasive across auctions and statistically significant.

3.2 Joint cycles in convenience yields and repo spreads

If one of the two components of the convenience yield is due to the auction cycle, one should be able to observe a cyclicality in on-the-run yields. As is well known, the autocorrelation function of bond yields, is mostly driven by their non-stationarity. The random walk component prevents the detection of other patterns such as cyclicality. The daily spread between the on- and one-off-the-run yields is however stationary. Autocorrelation function plots of yield spreads do reveal cycles in the autocorrelation functions of the yield spreads for most maturities with periods consistent with the length of their auction cycle, see Cherian et al. (2002). This is consistent with the hypothesis that the auction-induced cycle in repo spreads is transmitted to the yield spread.

Here we document directly the link between the repo special spread and the convenience yield spread over the auction cycle. We obtained simultaneous data on both convenience yields and repo spreads from 1993 to 1995 for the 10 and 30 year bonds, corresponding to 9 and 6 auction cycles respectively.[13] The one-off-the-run bond was on-the-run during the previous auction cycle, it is the most liquid of all off-the-run's. Figure 2 shows, in event time around the issue day, the average convenience yield (top plot) and repo spreads (bottom plot) together. The bottom plots are not exactly those of Figure 1 for the same bonds, because the sample periods differ.

The patterns are obvious despite the small number of auctions used. The repo special stays high until nearly the issue date. Then it collapses, to rise again 20 to 30 days later. However, the notion that the repo spread "rises" may just be an artefact of the averaging across auctions of a two regime process where the high regime starts at slightly different days for each auction. The convenience yield, off- minus on-the-run yield, rises sharply around the issue date and stays high for about 30 days. It then decreases to reach its lowest value just before the end of the cycle.

This pattern is consistent with the view that the convenience yield results from the present value of the repo special rate, over the remaining life of the cycle. Toward the end of the cycle, the present value of benefits may decrease with the number of days in the cycle as the repo spread is expected to remain constant. So, why doesn't the yield spread, highest at the beginning of the cycle, start decreasing on day 0. Here, the decrease in number of days left in the cycle may be compensated by an increasing expected benefit per day. This can happen if the agents expect latter days of the cycle to carry a higher repo spread than the early days.

4 A Model to Price the Convenience Yield

The model prices Treasuries with a two-component convenience yield. The first reflects the higher liquidity of on-the-run versus off-the-run Treasuries. The second is the auction

[13] The 30 year bond was on a 6 month auction cycle after 1993.

induced convenience yield when on-the-run Treasuries exhibit temporary shortages and their owners can benefit from special repo rates. In contrast, off-the-run Treasury securities of identical maturity with no auction induced convenience yield, may not be in short supply but trade in the relatively illiquid off-the-run market. We also consider synthetic zero-coupon bonds of similar maturities which can be constructed using put-call parity with long stock index options. These are not affected by the auction mechanism. Hence they do not have an auction induced convenience yield, but can be argued to trade in a liquid market.

Keeping with the notation introduced on page 2, we denote the off-the-run by $P(t,T)$, the on-the-run by $p(t,T)$, and the synthetic with no auction-induced convenience yield by $\hat{P}(t,T)$. All are default-free, zero-coupon bonds paying a dollar at time T. The no-arbitrage relationships among these prices are

$$p(t,T) \geq \hat{P}(t,T) \geq P(t,T), \quad \text{for all } t \leq T. \tag{1}$$

These inequalities cannot be arbitraged away because $p(t,T)$, in short supply, cannot be sold short, and $P(t,T)$, not inventoried by goverment bond dealers cannot be readily purchased. In contrast, the reverse inequalities cannot exist as $\hat{P}(t,T)$ can be shorted and the convenience yield is non-negative. As in Jarrow and Turnbull (1997], the second inequality in (1) implies the existence of a liquidity adjustment term $\gamma(t,T) \geq 0$, such that

$$\hat{P}(t,T) = e^{\gamma(t,T)} P(t,T). \tag{2}$$

$\gamma(t,T)$ captures the illiquidity related, non auction-induced convenience yield. The remainder of this section develops the pricing model for the cyclical auction-induced component of the convenience yield. Section A develops the model. Section B discusses implementation.

4.1 Pricing the auction-induced convenience yield

We first introduce risk neutral stochastic processes for the short rate $r(t)$ and the repo factor $\theta_\tau(t)$, defined below. We use the result of Harrison and Kreps (1979) which states that under mild technical conditions there exists an equivalent martingale measure Q such that the discounted sum of prices and cumulative dividends of traded securities are martingales. It can be shown that if the economy is arbitrage free, such a Q exists. To avoid unnecessary mathematics, the analysis proceeds under Q.

Consider an auction cycle, the time elapsed between successive issue dates of a $\tau-$year bond, $[T_0, T_1]$. The on-the-run bond is issued at T_0 with a maturity $\tau = T - T_0$. Let $R_\tau(t)$ denote the special repo rate available if the $\tau-$year on-the-run Treasury is used as collateral in a repo transaction. We write $R_\tau(t)$ as

$$R_\tau(t) = \theta_\tau(t) r(t), \quad \text{with} \quad \theta_\tau(t) \leq 1 \;\; \forall t \in [T_0, T_1], \quad \text{and} \quad \theta_\tau(t) = 1 \;\; \forall t \notin [T_0, T_1]. \tag{3}$$

$\theta_\tau(t)$ reflects the fact, seen in the empirical section, that each issue τ experiences specialness according to its specific auction cycle.

Effectively, the only significant assumptions in (3) are that θ must be below 1 to avoid arbitrage violations, and specialness only occurs during the auction cycle. Both assumptions, observed in the empirical section, occur with probability one under the empirical measure.[14] As probability one events, these properties must be preserved under the equivalent risk-neutral probability measure and reflected in pricing.

Although very rare, $\theta_\tau(t)$ can conceptually be negative in case of a short squeeze for the τ-year on-the-run Treasury. This may happen right after T_0. Then, a negative repo rates may occur because a short trader are willing to lend money at negative rates in order to guarantee delivery against a short position. As this is extraordinarily rare, the numerical implementation in the next section imposes a lower bound of zero on $\theta_\tau(t)$.

The auction effect on $\theta_\tau(t)$ is the strongest toward the end of the cycle, between the announcement date and T_1, the next issue date. This is when the on-the-run security may be in short supply and the effect on the repo spread most severe.[15] After T_1, the on-the-run becomes off-the-run and the cycle repeats itself for the new on-the-run issue.

The owner of an on-the-run Treasury can borrow in the repo market at the special rate, $R_\tau(t)$ while lending at the general rate $r(t)$. This yields a risk-free profit $y(t)$ equal to $p(t,T)\ (r(t) - R_\tau(t))$, the auction-induced convenience yield per unit time. Duffie (1996a) uses this strategy for a different purpose, assuming that future rates are known with certainty.

As in HJM (1992) and others, we define a money market account as

$$B(t) = e^{\int_0^t r(s)ds}. \tag{4}$$

Under mild technical conditions, see Harrison and Kreps (1979), there exists an equivalent martingale measure Q under which the discounted sum of accumulated profits and prices of both p and P are martingales. This implies that the on-the-run bond $p(t,T)$ is equivalent to the synthetically constructed bond $\hat{P}(t,T)$ plus the present value of the cash flows $y(s)$ over the remaining life of the auction cycle. After the auction cycle, $p(t,T) = \hat{P}(t,T)$ for $t > T_1$. Let E_t denote the conditional expectation under Q at time t. It follows that

$$\hat{P}(t,T) = E_t\left[e^{\int_t^T r(u)du}\right], \tag{5}$$

$$p(t,T) = \hat{P}(t,T) + B(t)\ E_t \int_t^{T_1} \frac{y(s)}{B(s)}ds \qquad t \in [T_0, T_1]. \tag{6}$$

Expression (5) is the standard martingale representation for bond prices. It only holds for the liquid off-the-run bond. Expression (6) relates this off-the-run bond price to the on-the-run bond price when the latter can not be shorted. The second component on the right hand side of (6) is the value accruing because $p(t,T)$ can not be shorted.

[14] While occasional data points violate this condition, they are most likely due to non synchronous general and special quotes. The quotes do not have time stamps.

[15] The auction announcement date runs from 5 to 8 days prior to the auction date itself. The issue or settlement date usually runs from 1 to 7 days after the auction date. "When-issued" trading takes place between the auction announcement and issue dates.

From a micro-perspective, (6) is consistent with no generally available arbitrage opportunities. Recall that both p and $\hat{P}$ trade in liquid markets. From a utility perspective, economic equilibrium would require arbitrageurs to have a strictly positive marginal utility with a binding wealth constraint, a corner solution. The arbitrageurs would like to short more Treasuries, but are unable to. No improvement in utility is possible. The same is true for individuals not holding the Treasuries, except that they have zero holdings in equilibrium.[16]

Now substitute $B(t)$ with (4) and $R_\tau(s)$ with (3). Also recall that $y(t) = p(t,T)(r(t) - R_\tau(t))$. Expression (6) becomes

$$p(t,T) = \hat{P}(t,T) + B(t)\ E_t \int_t^{T_1} e^{-\int_0^s r(u)\,du}\ p(s,T)\ r(s)(1-\theta_\tau(s))\ ds. \tag{7}$$

Expression (7) gives an implicit equation for the on-the-run bond price because p(t,s) appears both on the left and right sides of this expression. Assuming only that both $r(t)$ and $\theta_\tau(t)$ are continuous sample path processes, we show in the appendix that an explicit solution for the on-the-run bond price is given by:

$$p(t,T) \ = \ E_t\left[\hat{P}(T_1,T)\ e^{-\int_t^{T_1}\theta_\tau(u)r(u)\,du}\right] = E_t\left[\hat{P}(T_1,T)\ e^{-\int_t^{T_1}R_\tau(u)\,du}\right] \tag{8}$$

First, note that this result is quite general as both θ_τ and $r(t)$ can have processes with, for example, stochastic drifts and diffusion. Second, expression (8) has a nice intuitive interpretation. The price of the on-the-run bond, $p(t,T)$, is a discounted value of the liquid off-the-run, which it will *almost* become, at time T_1. However, the discount rate is less than the spot rate due to the auction-induced convenience yield ($\theta_\tau(u) \leq 1$). Hence, the on-the-run price $p(t,T)$ exceeds the liquid off-the-run $\hat{P}(t,T)$. In fact, the discount rate over the auction cycle in (9) below is the special repo rate, giving a very simple and elegant result.

In summary, the relationship between $p(t,T)$ the on-the-run, and $P(t,T)$ the illiquid off-the-run bond prices combines expressions (2) and (8).

$$p(t,T) \ = \ E_t\left[e^{\gamma(T_1,T)}P(T_1,T)\ e^{-\int_t^{T_1}R_\tau(u)\,du}\right] \tag{9}$$

Expression (9) is best understood by contrasting with (5), the price of the liquid off-the-run bond. The on-the-run bond is equal to the discounted expectation of the illiquid off-the-run bond which it will become at time T_1, with two differences. First, as seen above, the discount rate over the auction cycle is the special repo rate R_τ. Second, the fact that this liquid bond will become an *illiquid* security at T_1 adds the additional convenience yield represented by $\gamma(T_1,T)$, possibly a random variable when seen from t. Both of these convenience yields, in conjunction, make the on-the-run bond more valuable than the illiquid off-the-run bond. The results in (8) and (9), obtained with the equivalent martingale technique are conveniently implemented by the use of trees which we now discuss.

[16]Brennan and Schwartz (1985) point out that financial and real assets with convenience yields are held by arbitrageurs who derive the most convenience yield from holding them in a marginal utility sense. In equilibrium, if the arbitrageurs are perfectly competitive, the convenience yield of holding an additional unit of the asset will adjust to equality across arbitrageurs.

4.2 Implementation

The purpose of this section is to demonstrate a simple implementation of the auction-induced convenience yield model in equation (9) and show that it can generate bond yields consistent with the cyclical patterns observed in Figures 1 and 2. This illustrative implementation is based upon a one-factor Gaussian term structure model and a one-factor special repo rate process discussed in subsection B1. More general stochastic processes are also discussed.

4.2.1 Assumptions

Assumption 1: Off-the-run spot rate process

The spot rate of interest $r(t)$ is also the forward rate for immediate delivery associated with $\hat{P}(t,T)$, a liquid Treasury without an auction-induced convenience yield. Consequently, we assume in the model that the general repo rate is equal to the spot rate $r(t)$. We use the mean-reverting extended Vasicek (1977) model where $r(t)$ follows:

$$dr(t) = \lambda[\bar{r}(t) - r(t)]dt + \sigma dw_1(t), \tag{10}$$

where $w_1(t)$ is a standard Brownian motion process under the equivalent martingale measure Q. λ is the constant rate of reversion. $\bar{r}(t)$ is the deterministic long-run spot rate. σ is constant. This is a one-factor model but multiple factors could easily be incorporated into the subsequent structure. To prevent arbitrage given an initial forward rate curve, $\bar{r}(t)$ can be shown to be

$$\bar{r}(t) = -\frac{\partial \ln P(0,t)}{\partial t} + \left[-\frac{\partial^2 \ln P(0,t)}{\partial t^2} + \frac{\sigma^2(1-e^{-2\lambda t})}{2\lambda}\right]/\lambda, \tag{11}$$

and the arbitrage-free evolution of the spot rate process is given by

$$r(t) = -\frac{\partial \ln P(0,t)}{\partial t} + \frac{1}{2}\left(\frac{\sigma}{\lambda}\right)^2\left[1-e^{-\lambda t}\right]^2 + \int_0^t \sigma e^{-\lambda(t-u)} dW_1(u).$$

Given this initial forward rate curve, the liquid off-the-run bond price is known to have an analytical solution [17]

$$\hat{P}(t,T) = F(t,T)e^{-G(t,T)r(t)}. \tag{12}$$

Assumption 2: Special Repo Rate Process

We now specify the process for $\theta_\tau(t)$ in expression (3) under the risk-neutral measure Q. The process selected captures the two empirical regularities noted in our discussion following

[17] $F(t,T) = \frac{P(0,T)}{P(0,t)} \exp\left\{-G(t,T)\frac{\partial \log P(0,t)}{\partial t} - \sigma^2(e^{-\lambda T} - e^{-\lambda t})^2(e^{2\lambda t}-1)/(4\lambda^3)\right\}$, $G(t,T) = (1 - e^{-\lambda(T-t)})/\lambda$. See for example Hull and White (1990) and Heath, Jarrow, and Morton (1992).

expression (3). First, θ is below 1 to avoid arbitrage violations, and second, specialness only occurs during the auction cycle. Both assumptions, occur with probability one under the empirical measure, and therefore will have no impact on the specification of the risk premium. For $t \in [T_0, T_1]$, let $d\theta$ follow the process

$$d\theta_\tau(t) = \sigma_\theta \theta_\tau(t)(1 - \theta_\tau(t))dw_2(t), \tag{13}$$

where σ_θ is a strictly positive constant, w_2 is a standard Brownian motion process under Q. This deterministic form of heteroskedasticity specified allows the variance to vanish as θ nears its boundaries. It is easily verified that $\theta_\tau(t) \in [0, 1]$ if $\theta_\tau(T_0) \in [0, 1]$.[18] The initial point $\theta_\tau(T_0)$ reflects the specialness of the τ-year on-the-run Treasury on the issue day. In many circumstances it will be unity, but it could be less than 1 in presence of short-squeezes. $d\theta_\tau$ in (13) has no drift in this illustration. A drift term can easily be introduced. However, the absence of drift in the risk-neutral measure does not preclude the existence of a drift under the empirical measure if there is a risk premium associated with the specialness risk.[19] As our interest here is the arbitrage free pricing of the on-the-run Treasury bonds, we need not concern ourselves with risk premiums. Despite the simplicity of its process, $\theta_\tau(t)$ cannot be expressed as an explicit function of $w_2(t)$.

We assume that w_1 and w_2 are independent. This is reasonable as θ_τ relates to auction-cycle specific variables such as short demand, hedging or speculative, in the cash market relative to the issue size, and does not depend on the forward rate process of the shortable Treasuries. If desired, one could easily extend the model with correlated Brownian motions. While bringing little conceptual or theoretical modification, the implementation would require a more complex tree construction.

4.2.2 Numerical Implementation

The result in (8) shows that the on-the-run price can be computed as the expectation of the product of 1) the synthetic price $P(T_1, T)$ at the expiration of the cycle and 2) a discounting factor based on $R_\theta(t)$ the special rate.[20] For example, if the risk neutral

[18] For simplicity, expression (13) has absorbing barriers at the boundaries. Alternatively, we could have used reflecting barriers, appropriately modifying the numerical procedure discussed below. While a reflecting barrier at 0 may appear more realistic, a reflecting barrier at 1 is less so. It can be argued that an absorbing barrier at 1, implying no further specials, is reasonable. Given the conceptual limitations of either absorbing and reflecting barriers, we choose the absorbing barrier for its simplicity.

[19] Letting $\tilde{w}_2(t)$ be the Brownian motion under the empirical measure, then $d\tilde{w}_2(t) = -\phi(t)dt + dw_2(t)$, where $\phi(t)$ is the risk premium. Substitution gives the θ process under the empirical measure.

[20] Note also that an alternative representation of expression (8) as a partial differential equation is possible. Since $p(t, T) \equiv v(t, r(t), \theta_\tau(t))$, by Ito's lemma, the PDE for v is:

$$v_t + v_r \; \lambda \; [\bar{r}(t) - r] + \frac{1}{2} v_{r,r} \; \sigma^2 + \frac{1}{2} \; v_{\theta,\theta} \; \sigma_\theta^2 \theta^2 (1 - \theta)^2 = \theta r V.$$

A Feynman-Kac solution by finite differences can be implemented for this PDE. See appendix E of Duffie (1996b) for a general discussion of these formulations.

anticipation is of no special at all until the end of the cycle, the integral in (8) is a mere discounting factor involving the short rate $r(t)$.

To compute this expectation, we need to use a bivariate trinomial recombining tree. The tree-building process is based on Hull and White (1994, 1996) and Hull (2000). It is augmented to include the stochastic auction-induced convenience yield. First, we construct two independent one-factor trees, describing the short rate $r(t)$ and the specialness process $\theta(t)$. Second, we combined them into a trinomial recombining tree. The tree intervals are equally divided until the issue date T_1, when the specialness disappears.

To build the 2 independent one-factor trees, the trinomial branching process must satisfy 3 conditions. First, in the limit it is consistent with the continuous time processes used for r and θ. This ensures the convergence of the numerical solution to the solution of the original problem. Second, the branching probabilities for each process are chosen so as to match the expected return and variance of the change in r and θ over the branch's time interval. Third, branching probabilities must be positive and sum to one.

Pricing proceeds as follows. First, we calculate terminal values, at T_1, of the liquid on-the-run bond of maturity T using the analytic solution available for the extended Vasicek zero coupon bond prices in (5). Second, given these time T_1 terminal values of the on-the-run bond, its initial price is determined by backward substitution, rolling back through the 2-factor interest rate and convenience yield trinomial tree.

We now illustrate some features of the model with a numerical implementation. We use 70 time steps to price two-year to ten-year on-the-run discount bonds. We choose a zero drift for simplicity. We use the following required parameters. Let $T_0 = 0$. Next, we assume an initial upward sloping term structure of annualized riskless yields derived from $P(0,T)$ with rates for maturities from 2 to 10 years rising from 6% to 8% in 25 basis points increments. For the instantaneous spot rate process r, we use the parameters $r(0) = 0.05, \sigma = 0.014, \lambda = 0.1$. For θ, we use $\theta(0) = 0.5, \sigma_\theta = 0.01$. These parameters are realistic given actual data.

Figure 3 plots the yield spreads between the synthetic $\hat{P}(t,T)$ and the on-the-run $p(t,T)$. The purpose is to demonstrate three features of the model. First, for a given cycle length, the cyclical, auction-induced convenience yield is stronger for shorter maturities. This is apparent in the dashed line which represents $p(t,T)$ when all auctions have the same cycle length of 1 year. Clearly, the shorter maturities exhibit the larger yield differences. Second, as expected from (8), for a given maturity an increase in cycle length increases the value of the convenience yield. The dotted line shows $p(t,T)$ if issues with more than 5 years to maturity have a cycle length 1.5 times longer. As the dotted line shows, this increase in cycle lowers the yield. The longer the auction cycle, the larger the convenience rents earned by the on-the-run bonds, and hence the larger the yield spread. This result is not surprising in our arbitrage-free pricing framework. Finally, the dotted line demonstrates a third feature of the cyclical convenience yield. Different auction cycle lengths can cause non convexities in the term structure even if the synthetic or off-the-run term structures are convex.

The pricing relationship (8) ties the on-the-run to the synthetic prices while (2) relates the synthetic and the off-the-run via $\hat{P}(t,T) = e^{\gamma(t,T)}P(t,T)$. In yield terms, $\text{yld}(P(t,T)) =$

$\text{yld}(\hat{P}(t,T)) + \frac{\gamma(t,T)}{T-t}$. The liquidity risk adjustment generates an additional wedge between on-the-run and illiquid off-the-run bond yields. For example, if $\gamma(t,T) = \gamma_0 + \gamma_1 (T-t)$ for γ_0, γ_1 positive constants, the additional yield differential is $\gamma_1 + \gamma_0/(T-t)$. It can clearly affect both the level and slope of the illiquid off-the-run yield curve in comparison to the synthetic bond's yield curve. Given the model for the auction induced convenience yield, one can easily calibrate γ_0, γ_1 to match observed yield differentials between p(t,T) and $P(t,T)$.

5 Conclusions

This paper first documents the joint cyclicality of (1) the difference between general and special repo rates and (2) auction induced convenience yields in Treasury securities, using repo spread data, on-the-run and just off-the-run Treasury prices. While average repo specialness is already reported, we show that its cyclicality is statistically significant and pervasive across bonds and auctions for a large sample period. We also show that the auction induced convenience yield is positive and cyclical, only affects the on-the-run Treasuries, and is consistent with the pricing of the benefits accruing to the owner of the on-the-run over the remaining life of the auction.

Then, for subsequent analysis, the paper develops a dynamic model of the evolution of Treasury bond prices, including both types of convenience yields. Intuitively, the endogenous auction-induced convenience yield prices the expected benefits to the owner of the on-the-run Treasury over the remaining days of the auction cycle. Exploiting the repo market that effectively prices the illiquidity, our pricing model can use a pure no-arbitrage framework and sidestep issues of risk premia and specific equilibrium situations. The risk-neutral framework allows us to model only the features of specialness that occur with probability 1 around the auction cycle. The model is quite general, imposing almost no structure on the types of stochastic processes utilized, e.g. the number of factors, mean-reversion, etc. It extends the Heath-Jarrow-Morton model to allow for these regular, auction induced, market imperfections.

A numerical computation illustrates the feasibility of implementation for a practical example with a two-factor interest rate model, where the general repo rate follows a one-factor extended Vasicek evolution and the special repo rate is an additional stochastic factor. The general repo rate process is calibrated to market prices, while the special repo rate process is varied to study the sensitivity of on-the-run/off-the-run yield differences to these variations. It is seen herein that under certain conditions, the auction induced market imperfections can have a significant impact on Treasury on-the-run/off-the-run yield spreads.

This model opens avenues for further research, used for pricing of on-the-run and off-the-run bonds. It may also help us better understand various market practices and empirical regularities that have been observed in fixed income markets, and remain explained with the standard term structure theories alone. For example, it may explain the observed "trading profits" in violation of put-call parity, where riskless returns are measured without including a convenience yield. Furthermore, it is currently an open question whether the TED spread (between the Treasury and Eurodollar curves) is mainly attributable to the convenience yield of Treasury instruments or the default risk of Eurodollar deposits. The model provides a method for answering this question, and a new method for fitting the Eurodollar term structure using Treasury data. These auction induced convenience yields may also explain the differences observed between prices of almost identical maturity Treasury bills and Treasury strips, e.g., Grinblatt and Longstaff (2000). Lastly, the model may enable one to obtain more precise estimates of the marginal income tax rate implicit in interest rate spreads between U.S. Treasury securities and municipal bonds.

Appendix A: Proof of the main result in (8)

Duffie, Schroder and Skiadas (1996), page 1082, show the following result. Consider a filtered probability space $\Omega, F, (F_t : t \geq 0), P)$ satisfying the usual conditions. X is F_T-measurable and satisfies $E(|X^p|) < \infty$ for some $p > 1$. r_t is an F_t adapted stochastic process. $\psi : \Omega \times [0, \infty) \leftarrow R$ is an F_t adapted stochastic process. Let $\mathbf{S}$ be the space of semimartingales S such that $E[(sup_t|S_t|)^p)] < \infty$. Then there exists an unique V in $\mathbf{S}$ that satisfies

$$V_t = E\left[\int_t^T (-\psi_u V_u - r_u V_u)du + X|F_t\right]. \tag{14}$$

Further, if V is predictable, e.g., continuous, then

$$V_t = E\left[e^{-\int_t^T(\psi_u + r_u)du} X|F_t\right]. \tag{15}$$

We apply this result with the following identifications. $T = T_1, r_t = r(t), \psi_u + r(u) = -r(u)(1 - \theta_\tau(u)), V_t = p(t,T)e^{-\int_0^t r(u)du}, X = \hat{P}(T_1,T)e^{-\int_0^{T_1} r(u)du}$, and $E[.|F_t] = E_t[.]$. Substitution of these identifications into (14) gives

$$p(t,T)e^{-\int_0^t r(u)du} = E_t\left[\int_t^{T_1} p(s,T)e^{-\int_0^s r(u)du} r(s)(1 - \theta_\tau(s))ds + \hat{P}(T_1,T)e^{-\int_0^{T_1} r(u)du}\right].$$

This becomes our intermediate result (7) after multiplying by $B(t) \equiv e^{\int_0^t r(u)du}$ and noting that $\hat{P}(t,T) = E_t\left[\hat{P}(T_1,T)e^{-\int_t^{T_1} r(u)du}\right]$

As $V_t = p(t,T)e^{-\int_0^t r(u)du}$ is continuous in our case, we can also use the result in (15). Substitution of the identifications and again multiplying by B(t) yields our result in (8):

$$p(t,T) = E_t\left[e^{-\int_t^{T_1}(-r(u)(1-\theta_\tau(u)))du}\hat{P}(T_1,T)e^{-\int_t^{T_1} r(u)du}\right].$$

Appendix B: Transition density for $\theta_\tau(t)$

The transition density for the process θ_τ may be useful for an estimation of the parameters of the process for θ. A proof of the following result is available on request. Denote $p(t,x,y)$ the transition density of a homogeneous Markov process X_t. That is,

$$\Pr(X_{s+t} \in A \mid X_s = x) = \int_A p(t,x,y)dy.$$

If X_t follows the process $dX_t = \mu(X_t)dt + \sigma(X_t)dW_t$, it can be shown that

$$p(\tau,x,y) = \frac{1}{\sqrt{4\pi\tau}}\sqrt{\frac{x(1-x)}{y^3(1-y)^3}}\exp\left(-\frac{1}{4\tau}[\ln\frac{y}{1-y} - \ln\frac{x}{1-x}]^2 - \frac{\tau}{4}\right)1_{(0,1)}(y),$$

where $\tau = t\frac{\sigma^2}{2}$. That is, p depends on σ and t only through τ.

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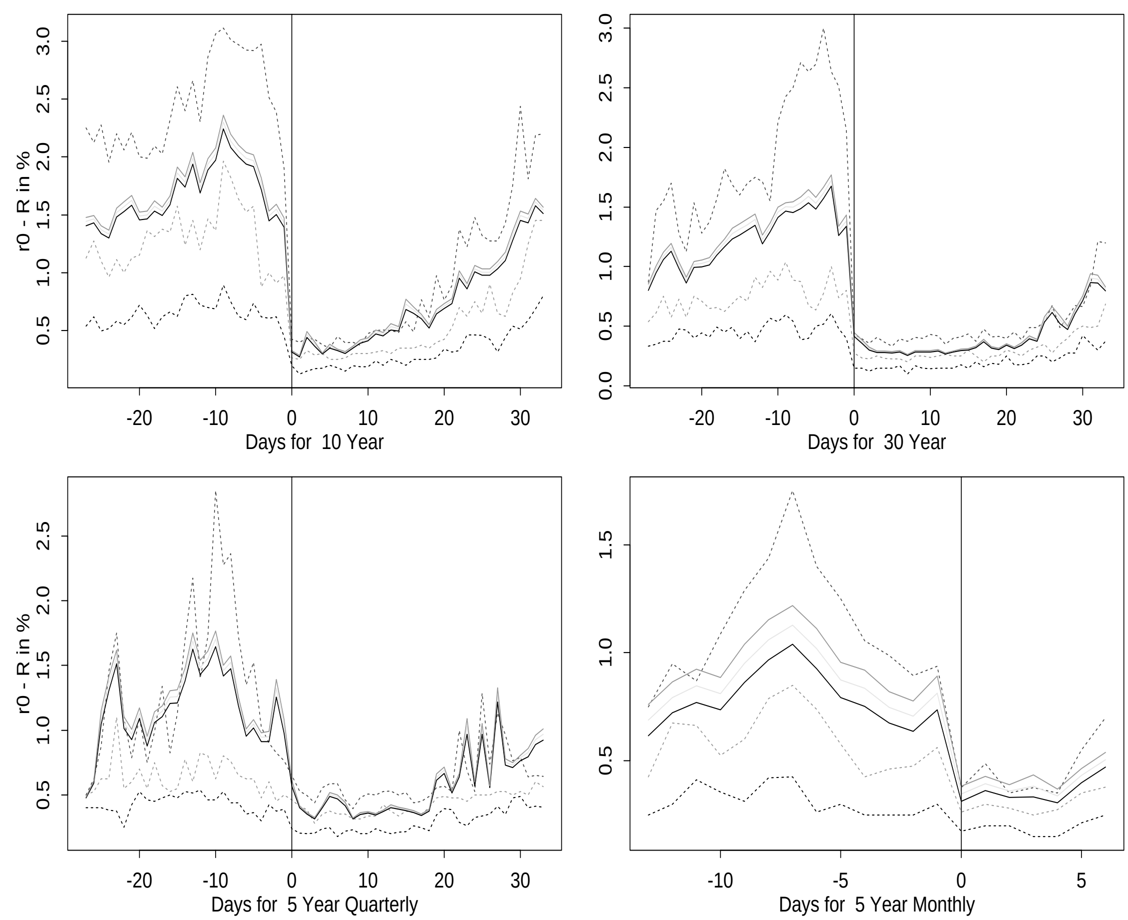

Figure 1: Annualized daily repo spread around issue day: 5, 10, and 30 year bonds
Solid lines: mean +/- 1.96 std.dev. Dotted lines: Q1,Q2,Q3. r_0 - R: General minus special rate. The vertical line indicates a change in the identity of the on-the-run bond issue on day 0.

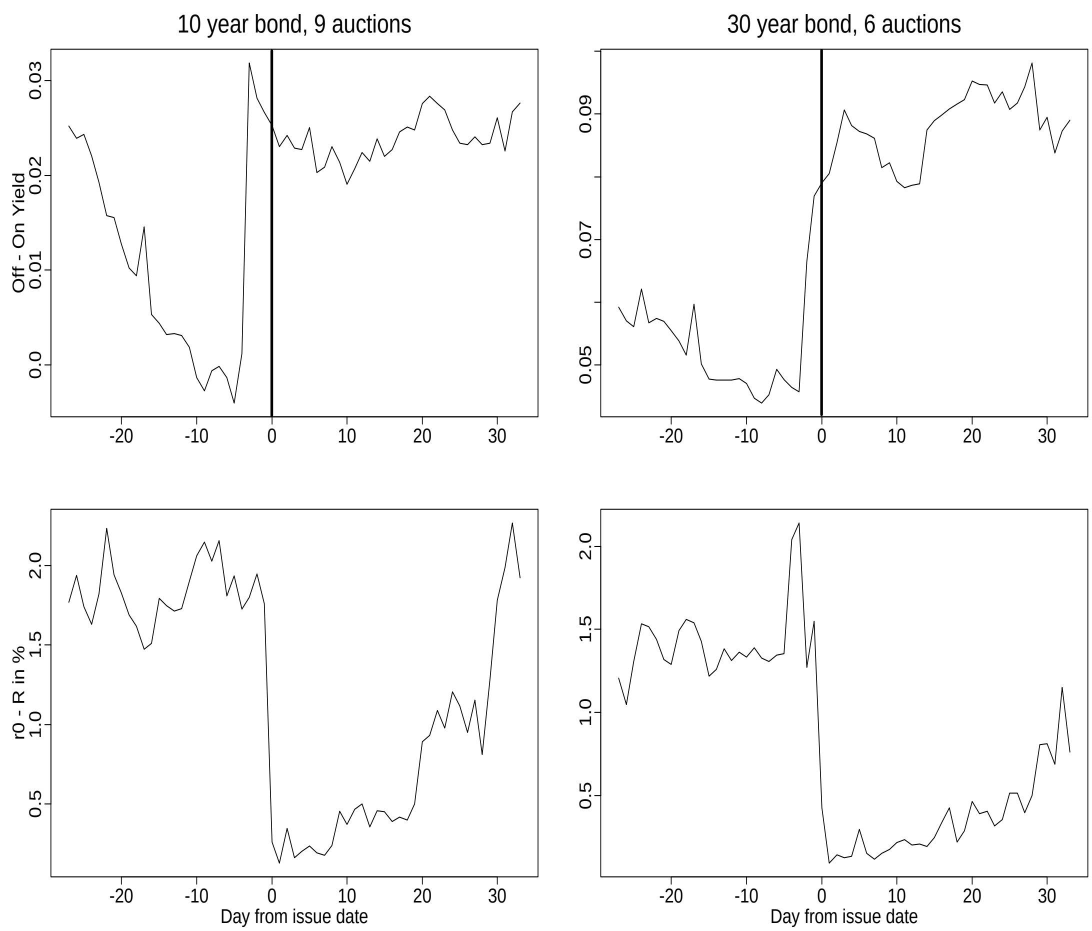

Figure 2: Annualized yield and repo spreads around issue day, auction cycles 1993 to 1995 Average spreads across auctions. The vertical line indicates a change in the identity of the on-the-run bond issue on day 0.

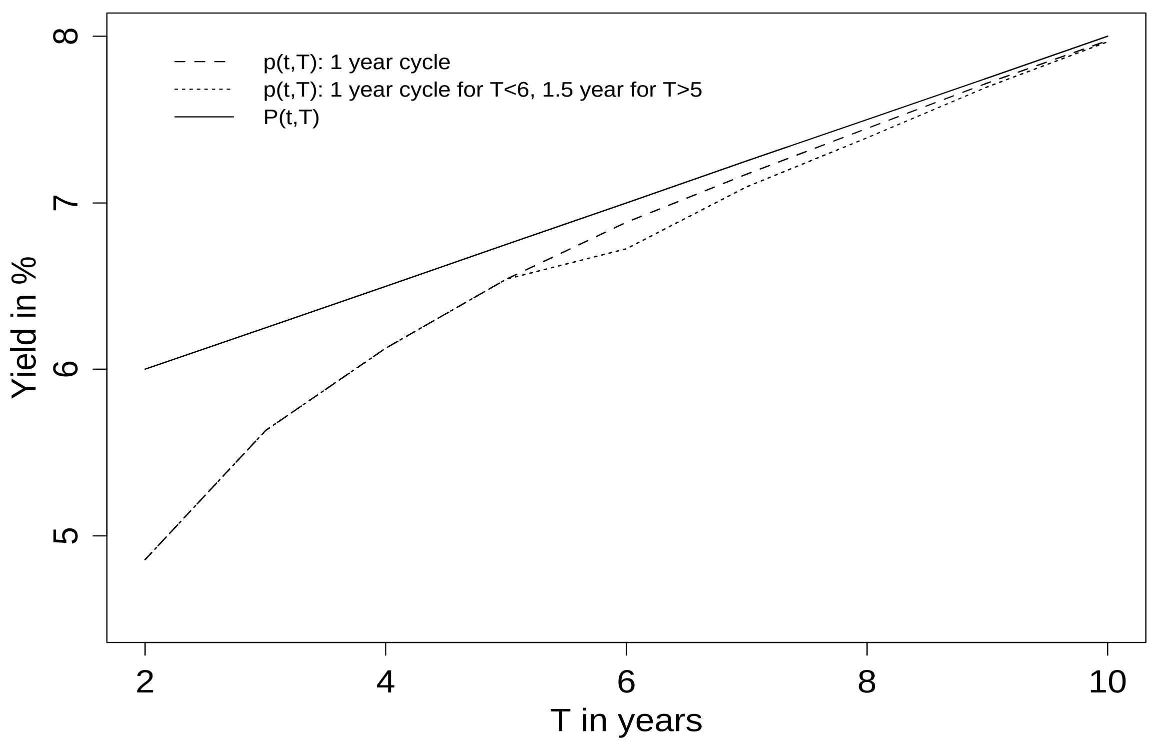

Figure 3: Model yield spreads between synthetic $\hat{P}(t,T)$ and on-the-run $p(t,T)$