Asset Allocation Models and Market Volatility

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Abstract

While asset allocation and risk management models all assume at least short-term stability of the covariance structure of asset returns, actual covariance and correlation relationships vary wildly, even over short horizons. Moreover, correlations increase in volatile periods, reducing the power of diversification when it might most be desired. We attempt to explain these phenomena and to present a framework for predicting short-horizon changes in correlation structure. We model correlations across assets as due to the common dependence of returns on a market-wide, systematic factor. Through this link, an increase in factor volatility increases the relative importance of systematic risk compared to the unsystematic component of returns. This in turn results in an increase in asset correlations.

We find that a very large fraction of the time-variation in cross-industry correlation can be explained solely by variation in the volatility of the market index. Moreover, there is enough predictability in index volatility to allow us to construct useful forecasts of covariance matrices in coming periods. Cross-country correlations behave in a qualitatively similar manner, but country-specific residuals are more pronounced than industry residuals, which reduces the efficacy of our approach.

Asset allocation and efficient diversification are at the heart of portfolio theory, and asset correlation structure is at the heart of efficient diversification. When that structure may be treated as stable over an investment horizon, the portfolio construction problem is well understood. In recent years, moreover, it has become clear that portfolio theory is also crucial to risk management. The risk manager uses the covariance matrix to compute probabilities of extreme outcomes or to minimize the risk of a portfolio of positions.

However, there is ample evidence that correlation and covariance structures vary dramatically over time, so much so and with such high frequency, that one must wonder whether asset allocation and risk management models can be of any use whatsoever. To illustrate the variation in correlation across U.S. industry groups, we use daily returns to calculate the correlation matrix of 12 industries for each quarter between 1962:3 and 1997:4. (See Table 1 for a description of the 12 industries.) Figure 1, Panel A presents box plots of the $12 \times 11/2 = 66$ cross-industry correlations for each quarter. The dramatic and unpredictable shifts in correlation structure over time suggest the difficulties one might encounter when attempting to use mean-variance analysis assuming a stable correlation structure. Instability seems to characterize international correlation structure as well. Figure 1, Panel B, shows analogous box plots for correlations of equity index returns across ten countries.[1] The same instability is evident.

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[1] The 10 countries are the U.S., U.K., Japan, France, Germany, Hong Kong, Canada, Australia, Switzerland, and Belgium. The returns are dollar-denominated, thus measuring the performance of each index from the perspective of an unhedged U.S. investor.

Moreover, correlation structure varies over time in ways that to a portfolio or risk manager must seem perverse. For example, it is well-documented that cross-country correlations increase markedly during periods of high global market volatility [see for example, Chow et al. (1999), Longin and Solnik (1995) or Solnik, Foucrelle, and Le Fur (1996)]. This means that the power of diversification is weakest precisely when it would be most desired. This phenomenon is often called *correlation breakdown*. While not as widely discussed, this pattern is, if anything, even more characteristic of cross-industry correlations in a domestic context.

Figures 2a and 2b illustrate this phenomenon in our data. The variation in cross-industry correlation is highly associated with market volatility. In Figure 2, Panel A, we plot the median of the 66 cross-correlations between industry sectors for each quarter against the volatility of the S&P 500 index computed from the daily returns of the S&P 500 in the same quarter.[2] Panel B presents the same plot of median cross-country correlation as a function of the standard deviation of the MSCI-World equity index in that quarter. In both contexts, correlation clearly rises with volatility. Correlation breakdown is obvious and dramatic.

These empirical patterns raise two questions. First, what drives such variation in correlation structure, and second, can we devise simple models of correlation/covariance

[2] The actual relationship between correlation and market volatility is non-linear. This is not surprising for a plot of correlation coefficients (which are bounded between −1 and 1) against volatility, which must be positive. Therefore, in Figure 2, we plot the relationship using a common transformation of the data. Calling ρ the correlation coefficient, and σ_M the standard deviation of the index return, the vertical axis is $\ln(\frac{1+\rho}{1-\rho})$, and the horizontal axis is $\log(\sigma_M)$, both of which have a range of $(-\infty, +\infty)$. The transformed relationship is apparently linear, and obviously strong.

structure that account for *and predict* such variation to an extent that is useful to a risk manager.

Our basic insight in this paper derives from a simple observation: Correlations will be higher when systematic macroeconomic factors, which affect all assets in tandem, dominate sector-specific factors. If variations in asset returns are driven by both systematic factors and idiosyncratic (i.e., sector-specific) risks, then periods of high factor volatility will coincide with periods of high correlation: during these periods, the dominant source of variation will be due to the common factor. Unless idiosyncratic volatility is strongly correlated across sectors (which is not the case in our sample), periods of high cross-sector correlation will coincide with periods of high overall market volatility. Thus, correlation breakdown – the strong observed association between correlation and volatility – is not simply bad luck. It is to be expected. Moreover, it is not evidence that the structure of security returns periodically changes perversely; "breakdown" can be part and parcel of a *stable* factor model of returns.

Moreover, this argument suggests that risk managers can use simple factor models of portfolio returns to better understand and predict time-variation in correlation.[3] In all

[3] Not surprisingly, factor models have been used and studied extensively. The most relevant test of such models for our purposes is Chan, Karceski, and Lakonishok (1999), who find that correlations derived from a three-factor model (with factors equal to the return on a value-weighted market index, and returns on size and book-to-market portfolios) are as effective as larger factor models in predicting covariance or correlation and managing portfolio risk, and in fact that none of the multi-factor models offer dramatic improvement in risk control compared to the one-factor market model. Our focus differs from theirs in several ways, however. Whereas CKL estimate models that predict unconditional covariances, we are more concerned with the *conditional* relationship between correlation and overall market volatility. As

of these models, correlation across sectors is due to their common dependence on shocks to systematic factors, and changes in correlation are driven by changes in the volatility of those shocks. During periods of large macroeconomic disturbances, the common factor dominates the volatility of individual sector returns, and leads to higher correlations. In quieter periods, sector-specific risks might dominate, with the results that correlations are lower and diversification eliminates a greater fraction of total volatility.

We find that a simple *univariate* factor model with only one systematic factor can explain a surprisingly large fraction of the short-horizon time variation in correlation structure. This suggests that univariate models of time variation in volatility, such as the ARCH model and its variants, which are already widely and successfully applied, can be integrated with the factor model to make useful short-horizon forecasts of cross-sector correlations. We find that short-term variation across time in the volatility of the "macro factor" can be used to forecast most of the time variation in correlation, and thus guide managers in dynamically updating portfolio positions. The results are qualitatively the same in the international and domestic settings. However, there is considerably more country-specific volatility than industry-specific volatility, implying that while the proposed methodology can be quite effective in the domestic setting, it will be less useful in the international setting.

The paper is organized as follows. In the next section, we briefly review the index model that serves to organize our approach. In Section 2, we examine the relation between correlation structure and market volatility for domestic portfolios, and examine

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a result, while monthly data is appropriate for the tests in CKL, we use higher frequency daily data that allows us to focus on shorter-term variation in market and asset volatility.

the contribution of variation in factor volatility to the time variation in correlation structure. We find that factor volatility almost solely determines the variation in correlation structure. We use this result to propose simple estimators of covariance matrices even for a large number of asset classes, and document the out-of-sample performance of this model. We find that models using only variation in volatility to predict changes in correlation perform extremely well. In Section 3, we present a parallel analysis for international asset classes. These results are qualitatively the same as in the domestic setting, but the higher level of country-specific risk makes the approach less effective. For international equity portfolios, factor volatility is important, but is not nearly as single-handedly determinative. The last section concludes.

1. THE INDEX MODEL

Sharpe (1963) was the first to show that an index model could potentially simplify the portfolio construction problem. His approach has become the textbook model for portfolio construction (literally – see for example, Bodie, Kane, Marcus (1998) or Reilly (1999), not to mention Sharpe's own text). Suppose we model returns for a particular sector i, as a function of returns on a market index, r_M, plus a sector-specific residual, ε_i

$$r_i = \alpha_i + \beta_i\, r_M + \varepsilon_i \tag{1}$$

If sector-specific risk is independent of macro risk, [i.e., Cov(r_M, ε_i) = 0] the total volatility of the sector return is

$$\sigma_i^2 = \beta_i^2\, \sigma_M^2 + \sigma_{\varepsilon_i}^2 \tag{2}$$

The first term on the right-hand side is systematic risk, the second is sector-specific volatility. Cross-sector covariances are

$$\sigma_{ij} = \beta_i \beta_j \sigma_M^2 + \mathrm{Cov}(\varepsilon_i, \varepsilon_j) \tag{3}$$

Notice that (3) implies that all else equal, covariances increase as σ_M^2 increases – this reflects the common dependence on the macro factor. Equations (2) and (3) imply that cross-sector correlations are

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j} = \frac{\beta_i \beta_j \sigma_M^2 + \mathrm{Cov}(\varepsilon_i, \varepsilon_j)}{\sqrt{[\beta_i^2 \sigma_M^2 + \sigma_{\varepsilon_i}^2] \times [\beta_j^2 \sigma_M^2 + \sigma_{\varepsilon_j}^2]}} \tag{4}$$

and that the squared correlation between the return of the sector and that of the index is

$$\rho_i^2 = R^2 = \frac{\beta_i^2 \sigma_M^2}{\beta_i^2 \sigma_M^2 + \sigma_{\varepsilon_i}^2} \tag{5}$$

which also increases monotonically with σ_M^2.

In Sharpe's exposition, the parameters of the model, β_i, σ_M^2, and $\sigma_{\varepsilon_i}^2$, are all taken as given parameters. However, in reality, all may vary and contribute to instability in the correlation structure. To simplify the portfolio problem, we will investigate how much of the variation in correlation can be explained *solely* by variation in the volatility of the macro-economic factor. In other words, does the time variation in σ_M^2 dominate the time variation in the other parameters? If β_i and $\sigma_{\varepsilon_i}^2$ are relatively stable compared to σ_M^2, then factor volatility will tend to drive correlations, and we can reduce the $N(N-1)/2$ entries in an N-dimensional correlation matrix to a function of a small number of variables

We examine the variation in correlation structure both internationally and domestically. In the international context, we will treat sectors as countries, and the common factor as the return on the MSCI World index; in the domestic U.S. context, we will interpret sectors as industries and the common factor as the return on the S&P 500.

2. DOMESTIC CORRELATIONS

2.1 In-Sample Relationships

In order to focus on methodology, we will focus initially on domestic correlation structure, and then present a parallel analysis for international correlations.

Figure 2, panel A, demonstrated that the typical cross-industry correlation is very closely related to the volatility of the S&P 500 in that quarter. Fitting a regression line through the scatter plot in Panel A, we find that the slope is 1.49, with a standard error of .067 (t-statistic = 22.1), and R-square = .78. Of course, the particular industry pair with the median correlation in any quarter will generally change from one quarter to another; therefore, by using the median correlation, we may obscure variation over time in the correlation of each industry pair and overstate the strength of the relationship with market volatility.

However, the strong relationship between correlation and volatility characterizes more than just the median cross correlation; it is true for *each* correlation individually. For each of the 66 pairs of industries, we regressed the time series of correlation in each quarter against volatility in that quarter.[4] In Table 2, we rank order the regressions by slope coefficient and report regressions for several percentiles. Consistent with the result

[4] As in Figure 2, we actually regress $\ln(\frac{1+\rho}{1-\rho})$ against $\log(\sigma_M)$, so as to fit a linear relationship.

for the median ρ_{ij}, these results also suggest that most of the variation in correlation structure can be attributed to variation in the volatility of the market factor. The R-square associated with the "median regression" is .66, meaning that for half the industry pairs in the sample, more than two thirds of the variance in cross-industry correlation was explained by market volatility. The *lowest* R-square of the 66 industry pairs was a still-respectable 0.43. These results thus imply that the index model with constant parameters may well capture most of the time variation in correlation structure. The dimensionality of the estimation problem may be greatly reduced without sacrificing much accuracy.

Rather than attempting to characterize all 66 correlation coefficients, a more economical way to describe the relationship between correlation and volatility is by focusing on the correlation of each industry group with the market index. Consider estimating the index model regression in equation (1) for each industry. The R-square of that regression, given in equation (5), equals the squared correlation of each industry with the market index. Figure 3, Panel A, plots the median of the 12 industry R-squares in each quarter as a function of the log of market variance in that quarter. The tightness of the fitted relationship again confirms the fact that market volatility overwhelmingly drives correlation. Panel B plots the R-squares for the individual industries against market volatility. There is obviously a wider scatter for the individual industries, but the positive association between correlation and volatility is clear. Panel A possibly overstates the fit by cross-sectionally averaging. Panel B possibly *understates* the fit by plotting on one graph 12 data sets for which the underlying relationships do not necessarily have equal coefficients.

To measure the relationship between index model R-square and market index volatility for each industry, take the log of both sides of equation (5), and rearrange to obtain:

$$\log\left(\frac{R_t^2}{1 - R_t^2}\right) = \log(\sigma_{Mt}^2) + \log(\beta_{it}^2/\sigma_{\varepsilon_{it}}^2) \tag{6}$$

If the "constrained" index model (where we use the term *constrained* to mean that we impose constant values for β_i and $\sigma_{\varepsilon_i}^2$) were exactly correct, a time series regression of $\log\left(\frac{R_t^2}{1 - R_t^2}\right)$ on $\log(\sigma_{Mt}^2)$ would have a slope of 1.0 and an intercept of $\log(\beta_{it}^2/\sigma_{\varepsilon_{it}}^2)$. The first three columns of Table 3 show the results of the estimation of equation (6). The average slope coefficient is .79, and with one exception, is above .80 for every industry. The average R-square of the regression plot is .58. This suggests that the majority of the variation in correlation structure can be explained solely by time variation in σ_{Mt}^2.

Equations (4) or (6) show that in principle, correlation structures may also be affected by variations in beta or residual variance. However, this seems not to be the case. The time series variation in either beta or residual variance is too small to affect correlation in an economically meaningful manner. For example, equations (4) and (5) show that the time variation of the product $\beta_{it}\sigma_{Mt}$ is a crucial factor for the time variation of correlations. We argue that $\beta_{it}\sigma_{Mt}$ can be modeled effectively by concentrating on the time variation of σ_M rather than of β. A similar case can be made for $\beta_i\beta_j\sigma_M^2$. Consider $\mathrm{var}(\beta_{it}\sigma_{Mt})$. If β is constant, this becomes $\beta^2\,\mathrm{var}(\sigma_t)$. Alternatively, if σ is constant, it becomes $\sigma_M^2 \mathrm{var}(\beta_t)$. From the quarterly time series of β_{it}'s and σ_{Mt} we can compute these three quantities for each of the 12 industries. The median value of $E(\beta)^2\mathrm{var}(\sigma_t)/\mathrm{var}(\beta_{it}\sigma_{Mt})$ over the 12 industries is 0.92. The median value of $E(\sigma_M)^2\mathrm{var}(\beta_t)/\mathrm{var}(\beta_{it}\sigma_{Mt})$ over the 12 industries is 0.08. This shows that allowing β to vary with time is a far less effective way to capture the time variation of $\beta_i\sigma_M$ than allowing σ_M to vary with time.

Of course, the covariation between β and σ may also affect the time variation of the product $\beta_i\sigma_M$. We now show that this covariation is negligible. We calculate the beta for each industry for each quarter, and estimate the (time series) regression between industry beta and market volatility. Table 3, Column 4, confirms that for each industry, there is effectively no relationship between beta and market volatility, with typical R-square of about 0.03. Similarly, in Table 3, Column 5, we disaggregate, and estimate the (time series) regression between residual variance for each industry, $\sigma_{\varepsilon_i}^2$, and market volatility. The typical R-squares of about 0.05 confirm that there is effectively no relationship between the two series.

Given the close fit between correlation and market volatility, and the fact that variation in beta or residual variance seems uncorrelated with market volatility, it is natural to ask how much of cross-industry correlation can be explained by changes in market volatility alone. In terms of equations (2) – (5), while, β_i and $\sigma_{\varepsilon_i}^2$ both may vary over time and contribute to instability in the correlation structure, it is possible that for practical purposes, treating them as constants and allowing only σ_M^2 to vary may provide nearly as accurate a forecast of correlation.

To test this notion, suppose that when we estimate correlation with the market index, we use equation (5), but allow *only* σ_M^2 to vary over time. That is, we recompute σ_M^2 each quarter, but we force both beta and residual standard deviation to be constant, and set them equal to their full-sample values.[5] Thus, this "constrained correlation" from equation (5) varies over time only because of quarter-by-quarter variation in σ_M^2. Figure 4 plots constrained squared correlation [transformed to $\log\{R^2/(1-R^2)\}$] for each industry in each quarter against its *realized* counterpart, i.e., against the *actual* squared correlation of industry returns with the S&P 500 in that quarter (also transformed). The tightness of the relationship is striking: the correlation between constrained and actual values is an impressive 0.82.

2.2 Forecasts of Domestic Correlations

The results presented in Section 2.1 use realized values of market volatility. They confirm that if we know just market volatility in a particular quarter, we can with surprising precision predict cross-industry correlation. In practice of course, one must forecast market volatility as well; to the extent that such forecasts are imperfect, this will degrade forecasts of cross-industry correlation. Therefore, we next examine how well one may predict cross-industry correlations and covariance using *predictions* of the volatility of the market index.

Following standard practice (e.g., Taylor, 1986), we use a simple autoregressive AR1 process to describe the evolution of the log of the variance of the S&P 500 index.

[5] Specifically, we estimate the index model regression [equation (1)] over the full sample period and use the

Volatility is computed each quarter, and an AR1 is fitted to those 142 quarterly observations.[6] We can then use the estimated AR1 process to predict market volatility in the next quarter based on the value we observe for it in the current quarter.

Figure 5 replicates Figure 2 except that it plots median cross-industry correlation as a function of *predicted* rather than *actual* market variance. While the relationship between (transformed) correlation and predicted market volatility is not as tight as it is for actual volatility, it is still highly significant. Fitting a regression line through the scatter diagram in Figure 6, we find the following relationship (with standard errors in parentheses):

$$\text{Median correlation} = \underset{(.1445)}{1.4264} + \underset{(.0585)}{.2596}\ \text{Predicted market variance}$$

R-square = .1242

Not surprisingly, there is considerably greater scatter than in Figure 2 (which had an R-square of .78). This is because market volatility itself exhibits considerable unpredictability as it evolves over time. All attempts to forecast correlation or covariance structure will run up against this problem. Nevertheless, we can capture a meaningful proportion of the variation in correlation using even a simple univariate model of the evolution of market volatility, and this may still be useful compared to other forecasts such as historical correlation.

Having established that predictions of market volatility are useful in predicting correlation structure, the next obvious question is the extent to which this methodology can be used in risk management applications. Can predictions of market volatility, in conjunction with the index model be used to efficiently diversify portfolio risk? We

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estimated values for the industry beta and residual variance.

begin to answer this question by comparing the predictive accuracy of several forecasts of covariance. In each quarter, with 12 industries, there are 66 industry pairs. Across the 142 quarters, therefore, there are 9,372 covariances to forecast. We consider several estimators:

1. *Current quarter covariance.* The actual covariance between two industries during the current quarter (estimated from daily returns) is used as the forecast of covariance in the following quarter.
2. *Full-sample "constant" covariance.* The full-sample covariance is the covariance estimate obtained by pooling all daily returns between 1962 and 1997, and calculating the single full-sample-period covariance matrix. This forecast obviously is not feasible for actual investors since it requires knowledge of returns over the full sample period. We include it primarily as an interesting benchmark, since it is the best *unconditional* covariance estimator.
3. *Index model, using <u>current</u> market variance.* The covariance implied from equation (3), using the full-sample estimates of betas and covariance structure of residual returns and the *current*-quarter market variance as the forecast of next-period market variance.
4. *Index model, using AR1 <u>forecast</u> of market variance.* The covariance implied from equation (3), using the full-sample estimates of betas and covariance structure of residual returns, and using the AR1 relationship to forecast next-quarter market variance from current quarter market variance.

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[6] We find that the first order autocorrelation of $\log(\sigma_M)$ is 0.41.

5. *Index model, using next period's market variance*. The covariance implied from equation (3), using the full-sample estimates of betas and covariance of residual returns, and using the *realized* value of next-period variance. This estimate obviously uses information not available at the current time, but it serves as an interesting upper bound on potential forecasting performance using the constrained index model.

Table 4 compares some properties of the predictions derived from each model. The grand mean and grand standard deviations of each model are obtained by pooling all 9,372 (i.e., 66×142) covariance estimates. Note that the full-sample covariance and the Index Model (AR1) estimates display far less variability than their competitors. This is because predictions from these models are based either on full-sample covariances or quarterly covariances attenuated by the AR1 relationship. In contrast, predictions from the competitor models are more volatile because they are based on actual quarterly covariances, which contain considerable noise.

Table 4 also reports on the forecasting accuracy of each model. We pool forecasts across industry and time. The grand mean errors, which are all essentially zero, document that none of the forecasting models shows any meaningful statistical bias. Not surprisingly, knowing next quarter's market volatility would be *very* valuable for forecasting covariances. The forecast errors, as measured either by root mean square error or mean absolute error are far lower for the constrained index model that uses *realized* volatility than for any other model. However, among *feasible* estimators, the constrained index model using the AR1 forecast of market volatility is the most accurate.

The full-sample covariance estimator performs about equally well, but as noted, this is not a feasible real-time forecasting method. In fact, it is interesting to note that the AR1 model using only information available to date is as accurate in forecasting covariance as the constant covariance estimator using the full sample of returns! Either method has a root mean square error that is only three-fourths as large as that of using the current quarter covariance to predict next quarter's covariance.[7]

3. International Correlations

We now repeat the analysis of the previous section. We start by noting that Figure 2 documents that the while the median cross-country correlation varies with index (i.e., MSCI-World) volatility, there is a wider scatter around the regression line than in the domestic context. Country residuals are larger and contribute more to portfolio volatility than do industry residuals in a domestic setting. The R-square of the relationship plotted in Panel B is only .06, compared to an R-square of .78 for the domestic relationship depicted in Panel A. Still, the slope coefficient is .180 with a standard error of .068, which is both economically and statistically significant.

Table 5 presents results of regressions of the quarterly time series of $\ln(\frac{1+\rho}{1-\rho})$ against $\log(\sigma_M)$. As in Table 2, we rank order the regressions by slope coefficient and report regressions for several percentiles. The slopes are consistently positive and significant, but again, the R-squares are far lower than in the domestic application. The

[7] For more details on the statistical methodology and forecast properties of these estimators, see: Erick Jacquier and Alan J. Marcus, "Market Volatility and Asset Correlation Structure," Boston College working paper, January 2000.

R-square associated with the “median regression” is only .12, indicating that only a small fraction of the variance in cross-country correlation was explained by index volatility.

Figure 6 demonstrates that the median R-square of the index model in the international setting varies positively with the volatility of the MSCI World index, although the fit between median R-square and index volatility is far looser than it was in the domestic setting (compare to Figure 3). Nevertheless, Figure 2 (Panel B) and Figure 6 together make the same qualitative case internationally as Figure 2 (Panel A) and Figure 3 do domestically, specifically, that part of the time-variation in international correlation is due to changes in factor volatility.

Figure 7 however, shows one interesting contrast between the international and domestic data. Whereas residual variance was not associated with market volatility in the domestic context (see Table 3, last column), it clearly rises with market volatility in the international setting. This suggests a missing factor governing returns.

Table 6 confirms these results on a country-by-country basis. The R-squares of the regressions of beta against market volatility are very small, but the R-squares of the regressions of residual risk against market volatility average .15 even if we exclude the Crash of 1987:4. This is actually double the average R-square of the relationship between (transformed) country R-squares and market volatility.

The ability of the index model to capture international correlation patterns is not as strong as in the domestic setting. As in Figure 4, Figure 8 plots the constrained R-square in each quarter for each country against the realized R-square. The positive association is obvious, but the fit is not as tight as in Figure 4.

Figure 9 shows that even with the greater variability in country returns, there may be some value to using the index model to forecast cross-country correlations. The median correlation does rise noticeably with the *forecast* of index variability derived from an AR1 process.

Table 7 documents that the index model provides predictions that are only marginally better than its competitors. We see in column 2 (Grand standard deviation) that the variability of the covariance prediction from the index model is slightly less than that of the current covariance estimator or the estimators based on actual market volatility. The values for root mean square error or mean absolute error show again that knowing market volatility would help in forecasting covariance structure. Here, however, the advantage it would convey is not nearly as large. The forecast errors for the predictions based on perfect foresight of market variance were only about one-fourth those of the feasible forecasts in the domestic setting. Here, they are more like three-fourths as large. The forecasts based on the index model are slightly better than the alternative feasible models, but not to an extent that would make much difference in practice.

4. Summary

This paper uses an index model to shed light on the variation through time of asset correlation structure. Our model captures the empirical tendency for cross-sector correlations tend to increase with the volatility of the market-wide, systematic factor.

We find that a large fraction of the time-variation in cross-industry correlation can be attributed to variation in the volatility of the market index. Moreover, there is enough predictability in index volatility to allow us to construct useful forecasts of covariance matrices in coming periods. Minimum-variance portfolios constructed from covariance matrices based on an index model and predicted market volatilities will perform substantially better than ones that do not account for the impact of time-varying volatility on correlation and covariance structure.

Our results for cross-country correlation structure are qualitatively the same as for the domestic setting. However, there is considerably more country-specific risk than industry-specific risk, implying that the proposed methodology to reduce the dimensionality of the forecasting problem will be less successful in the international setting.

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Table 1: US Industry Portfolio Groups

Portfolio	2-Digit SIC Code	Name
1	13,29	Petroleum
2	60-69	Finance/Real Estate
3	25, 30, 36, 37, 50, 55, 57	Consumer Durables
4	10,12,14,24,26,28,33	Basic Industries
5	1, 20, 21, 54	Food/Tobacco
6	15-17, 32, 52	Construction
7	34,35,38	Capital Goods
8	40-42,44,45,47	Transportation
9	46,48,49	Utilities
10	22,23,31,51,53,56,59	Textiles/Trade
11	72,73,75,80,82,89	Services
12	27,58,70,78-79	Leisure

Table 2: Regressions of (transformed) ρ_{ij} against $\log(\sigma_M)$

For each of the 66 pairs of industries, we used daily data within the quarter to calculate the correlation coefficient ρ_{ij}. We then regressed the time series of $\ln(\frac{1+\rho_{ij}}{1-\rho_{ij}})$ against $\log(\sigma_M)$. The regressions are rank-ordered by slope coefficient. The slope, t-statistic for that slope, and R-square for various percentile industry-pairs are reported in the table. (The first column reports the regression results for the median ρ_{ij} of each quarter. This median generally corresponds to different industry pairs each quarter, so the results for median ρ_{ij} differ from the fiftieth percentile regression, which is for a specific industry pair.

	Median ρ_{ij}	Min	10%	25%	50%	75%	90%	Max
Slope	1.49	1.08	1.22	1.41	1.51	1.56	1.62	1.78
t-statistic	22.1	10.4	11.6	12.8	16.5	17.7	18.6	20.0
R-square	0.78	0.43	0.49	0.54	0.66	0.69	0.71	0.74

Table 3: Time Series Regressions on log(market variance)

Dependent variable:	$\log\left(\frac{R_t^2}{1-R_t^2}\right)$			β_{it}	$\sigma^2_{\varepsilon_{it}}$
Estimate:	Slope	Std. Error	R-square	R-square	R-square
Median[a]:	0.82	NA	0.75	0.02	0.08
Industries:					
1	0.87	(0.10)	0.35	0.03	0.12
2	0.81	(0.07)	0.48	0.05	0.03
3	0.85	(0.04)	0.73	0.00	0.08
4	0.81	(0.06)	0.61	0.04	0.06
5	0.90	(0.05)	0.65	0.07	0.00
6	0.93	(0.06)	0.65	0.01	0.00
7	0.86	(0.05)	0.69	0.01	0.04
8	0.87	(0.06)	0.62	0.03	0.02
9	0.56	(0.07)	0.32	0.05	0.16
10	0.81	(0.05)	0.67	0.01	0.04
11	0.83	(0.08)	0.46	0.01	0.04
12	0.92	(0.05)	0.68	0.00	0.02
Average:	0.79		0.58	0.03	0.05

Notes:

a. This is the regression of the median value for $\log[R^2/(1-R^2)]$ in each quarter against market volatility. The identity of the median industry pair will differ across quarters.

Table 4: Properties of Covariance Estimators

Estimator	Grand Mean	Grand Standard Deviation	Grand Mean Error	Root Mean Square Error	Mean Absolute Error
Current covariance	0.0107	0.0169	-0.0001	0.0223	0.0086
Full-sample covariance	0.0113	0.0025	0.0007	0.0167	0.0080
Index model(t-1)	0.0105	0.0161	-0.0001	0.0219	0.0087
Index model(AR1)	0.0087	0.0035	-0.0200	0.0168	0.0066
Index model(t)	0.0106	0.0161	0.0000	0.0040	0.0019

Notes:

1. 141 quarters, 66 covariances each quarter

2. Current covariance: for each industry pair, the forecast of next period's covariance is this period's covariance.

   Full sample covariance is the estimate obtained by pooling all daily returns into one period and calculating the single full-sample-period covariance matrix.

   Index model estimate is the prediction from the constrained index model where only market variance is allowed to vary over time. Index model(*t*) denotes use of the next-period market variance to calculate next-period covariance, Index model(*t*-1) denotes the use of current market variance, and Index model(AR1) denotes the use of the prediction of the future market variance from the AR1 model.

Table 5

Regressions of (transformed) ρ_{ij} against $\log(\sigma_M)$: International data

For each of the 45 pairs of countries, we used daily data within the quarter to calculate the correlation coefficient ρ_{ij}. We then regressed the time series of $\ln(\frac{1 + \rho_{ij}}{1 - \rho_{ij}})$ against $\log(\sigma_M)$. The regressions are rank-ordered by slope coefficient. The slope, t-statistic for that slope, and R-square for various percentile country-pairs are reported in the table. (The first column reports the regression results for the median ρ_{ij} of each quarter. This median generally corresponds to different country pairs each quarter, so the results for median ρ_{ij} differ from the fiftieth percentile regression, which is for a specific country pair.

	Median ρ_{ij}	Min	10%	25%	50%	75%	90%	Max
Slope	0.44	0.17	0.30	0.38	0.43	0.53	0.70	0.89
t-stat	5.55	1.64	2.50	3.18	3.84	4.53	4.88	5.53
Rsquare	0.23	0.03	0.06	0.09	0.12	0.16	0.19	0.23

Table 6

Time Series Regressions on log(market variance): International data

Dependent variable:	$\log\left(\frac{R_t^2}{1-R_t^2}\right)$			β_{it}	$\sigma_{\varepsilon_{it}}^2$	$\sigma_{\varepsilon_{it}}^2$ w/o Crash
Estimate:	Slope	Std. Error	R-square	R-square	R-square	R-square
Median	0.62	(0.15)	0.14	0.06	0.46	0.38
Countries						
UK	0.66	(0.25)	0.06	0.00	0.07	0.05
Germany	0.99	(0.29)	0.10	0.05	0.30	0.20
France	1.10	(0.30)	0.11	0.03	0.16	0.05
Japan	0.56	(0.22)	0.06	0.02	0.40	0.32
Hong Kong	1.21	(0.37)	0.09	0.03	0.21	0.10
Canada	0.39	(0.16)	0.05	0.00	0.24	0.16
Australia	0.87	(0.36)	0.05	0.03	0.26	0.14
Belgium	0.70	(0.29)	0.05	0.04	0.33	0.22
Italy	1.13	(0.32)	0.11	0.06	0.02	0.02
US	0.10	(0.20)	0.00	0.02	0.25	0.21
Average	0.77		0.07	0.03	0.22	0.15

Notes

1. The second column for $\sigma_{\varepsilon_{it}}^2$ is the regression without 1987:4.

Table 7

Properties of Covariance Estimators: International Data

Estimator	Grand Mean	Grand Standard Deviation	Grand Mean Error	Root mean square error	Mean absolute error
Current covariance	0.0063	0.0159	-0.0001	0.0203	0.0078
Full-sample covariance	0.0065	0.0025	0.0002	0.0157	0.007
Index model(t-1)	0.0065	0.0058	0.0001	0.0158	0.0068
Index model(AR1)	0.006	0.0031	-0.0003	0.0155	0.0065
Index model(t)	0.0065	0.0058	0.0001	0.0122	0.0057

Notes:

1. 105 quarters, 45 covariances each quarter

2. 2. Current covariance: for each country pair, the forecast of next period's covariance is this period's covariance.

   Full sample covariance is the estimate obtained by pooling all daily returns into one period and calculating the single full-sample-period covariance matrix.

   Index model estimate is the prediction from the constrained index model where only market variance is allowed to vary over time. Index model(*t*) denotes use of the next-period market variance to calculate next-period covariance, Index model(*t*-1) denotes the use of current market variance, and Index model(AR1) denotes the use of the prediction of the future market variance from the AR1 model.