

# Optimal Estimation of the Risk Premium for the Long Run and Asset Allocation: A Case of Compounded Estimation Risk

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## ABSTRACT:

It is well known that an unbiased forecast of the final value of a portfolio requires compounding at the arithmetic mean return over the investment horizon. However, the maximum likelihood practice, common with academics, of compounding at the *estimated* mean return, produces upward biased and highly inefficient estimates of long-term expected returns. We derive analytically two estimators of long-term expected returns for given sample sizes and horizons, one unbiased and one efficient in small samples. Both entail penalties that reduce the annual compounding rate as the investment horizon increases. The unbiased estimator, which is far lower than the compounded arithmetic average, is still very inefficient, often more so than a simple geometric estimator known to practitioners. Our small-sample efficient estimator is even lower. These results compound the sobering evidence in recent work that the equity risk premium is lower than suggested by post-1926 data. Our methodology and results are robust to extensions such as predictable returns. We also confirm analytically that parameter uncertainty, properly incorporated, produces optimal asset allocations in stark contrast to conventional wisdom. Longer investment horizons require *lower*, not higher, allocations to risky assets.

## KEYWORDS:

Risk premium, asset allocation, long-term returns, arithmetic mean, geometric mean, maximum likelihood, mean squared error, estimation risk, small sample.