Geometric or Arithmetic Mean: A New Take on an Old Controversy

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Abstract

It is well known that an unbiased forecast of the terminal value of a portfolio requires the initial value to be compounded at the arithmetic mean rate of return for the length of the investment period. However, an upward bias in forecasted values results if one uses a sample average to estimate the mean return, and uses that average to compound forward. This bias does *not* necessarily disappear even if the sample average return is unbiased, computed from long data series, and returns are generated according to a stable distribution. On the other hand, forecasts obtained by compounding at the geometric average generally will be *downward* biased. For typical investment horizons, the proper compounding rate is in-between these two values. Specifically, unbiased estimates of future portfolio value require that the current value be compounded forward at a weighted average of the two rates. The proper weight on the geometric average equals the ratio of the investment horizon to the sample estimation period. We demonstrate that these biases are empirically significant. For investment horizons of 20 years, the difference in forecasts of cumulative performance easily can exceed a factor of two. The percentage differences in forecasts grows along with the investment horizon, as well as with the imprecision in the estimate of the mean return.